The fuzzy stability model: an interactive framework for measuring robustness and resiliency under uncertainty

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Abstract: The increasing complexity in workflow management systems (WMSs) has led to greater vulnerability due to system failure. Although system vulnerabilities cannot be completely eliminated, the accidental or anticipated failures have to be thoroughly understood and guarded. Traditionally, the failure in military command and control (C2) systems has been studied with robustness, the concept of self-protecting systems and resiliency, the concept of self-healing systems. Robustness and resiliency in C2 systems are generally measured with precise repair-recovery costs and repair-recovery times. However, the repair-recovery costs and repair-recovery times in real-world problems are often imprecise or uncertain. Fuzzy logic and fuzzy sets can represent imprecise or uncertain information formalising inaccuracy in human decision-making. We develop a stability model for simultaneous consideration of robustness and resiliency in fuzzy C2 systems. We measure robustness and resiliency with fuzzy repair-recovery times and fuzzy repair-recovery costs. The interactive method plots the fuzzy robustness and fuzzy resiliency measures in a Cartesian coordinate system and derives an overall fuzzy stability index for various processes in the C2 system based on the theory of displaced ideals.

Keywords: command and control system; workflow management system; WSM; resiliency; robustness; fuzzy sets theory.

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1 Introduction

The necessity for readiness and the ability to cope with the possibility and reality of failure in complex systems is an important area of research in workflow management systems (WMSs). Tavana et al. (2012) proposed the stability model for simultaneous consideration of robustness and resiliency in a military command and control (C2) WMS. Robustness and resiliency were measured with precise repair-recovery costs and repair-recovery times. However, the observed repair-recovery cost and repair-recovery time values in real-world WMSs are often imprecise or uncertain. In this study, we extend the stability model proposed by Tavana et al. (2012) into a fuzzy model which is capable of capturing the decision makers' (DMs) subjective judgments by representing imprecise or uncertain repair-recovery cost and repair-recovery time values with fuzzy numbers.

These fuzzy numbers, represented by fuzzy triangular numbers (FTNs) or fuzzy trapezoidal numbers (FTrNs), account for the variability of data while still enabling the calculation of robustness and resiliency. An ideal process and a nadir process are formed. The proximity to each of these performance poles is measured with the fuzzy Euclidean distance. The process should be as close to the fuzzy ideal process as possible and as far from the fuzzy nadir process as possible. The *fuzzy stability index*, a measure of fuzzy distance from the ideal and nadir states, is used in this study as a measure of stability of the C2 processes in a fuzzy environment. We study the four different states of *possession*, *preservation*, *restoration*, and *devastation* in military C2 systems in a fuzzy environment.

The remainder of this paper is organised as follows. We review the literature on fuzzy sets and WMSs in Section 2. In Section 3 we present a detailed description of the stability models with triangular and trapezoidal fuzzy numbers proposed in this study. In Section 4 we demonstrate the applicability of the proposed stability models with two numerical examples with triangular and trapezoidal fuzzy numbers. Finally, In Section 5 we present our conclusions and future research directions.

2 Literature review

A WMS is a set of activities involving the coordinated implementation of various tasks performed by different processing entities (Casati et al., 1995; Van der Aalst and Van Hee, 2002). Different techniques may be used for workflow modelling depending on the goals and objectives. While WMSs are popular, with wide-spread applications, they still suffer from lack of standards and an agreed-upon modelling method (Salimifard and Wright, 2001). These systems are complex artefacts; they are difficult and expensive to build and validate, especially when the components of the system exhibit complicated properties such as sequential synchronisation, merging, or prioritisation (Balduzzi et al., 2000; Mehrez et al., 1995).

Several theories have been developed to deal with imperfect data. Imperfect data can be characterised as being uncertain, imprecise, or both. Other imperfect data such as vague or incomplete data can be described as a special form of imprecision and (or) uncertainty (Smets, 1997). Bayesian theory deals with both uncertainty and imprecision (Fienberg, 2006; Howson and Urbach, 1993; Jaynes, 2003). Rough sets theory is used to handle imprecision when uncertainty is involved but not quantified (Pawlak, 1991). The theory of evidence deals with data that contains both imprecision and uncertainty at the same time (Shafer, 1976; Dempster, 1967). The theory of possibility handles incomplete data, which is a combination of imprecise and uncertain data (Zadeh, 1978). Although these theories are used to handle only one type of imperfection, random sets and the conditional event algebra are proposed to cope with all types of imperfection (Goodman et al., 1997). These fuzzy values will account for the variability of data while still enabling the calculation of robustness and resiliency. Fuzzy logic and approximate reasoning enable computation in the face of uncertainty, generating approximate results (Nedjah and Mourelle, 2005).

Uncertainty represents the state of knowledge about a piece of data, while imprecision is the characteristic of a piece of data that cannot be expressed with a single value. The theory of fuzzy sets deals with vague data which is a particular case of both imprecise and uncertain data (Zadeh, 1965). Fuzzy sets have been used to account for the vague data in WMSs (Lin et al., 2007; Tsai and Wang, 2008). The membership function of a fuzzy set defines the mapping of inputs to the degree or strength of membership, ranging from 0 to 1. The shape of this membership function can vary, as any function whose image is between 0 and 1 is a possible membership function. The simplest of these functions are those represented by straight lines, such as triangular and trapezoidal member functions. In this study, we use both FTNs and FTrNs to represent and quantify the vagueness associated with the decision variables and the input and output data.

3 The stability model

3.1 The stability model with triangular fuzzy numbers

Among the various types of fuzzy numbers, triangular and trapezoidal fuzzy numbers are the most widely used. Assuming that there are n processes ($i = 1,\ldots, n$), we measure two criteria (*time* and *cost*) for each process with respect to both *repair* and *recovery*:

Let:

$x_{i,rpt}$ the repair time for process i

$x_{i,rct}$ the recovery time for process i

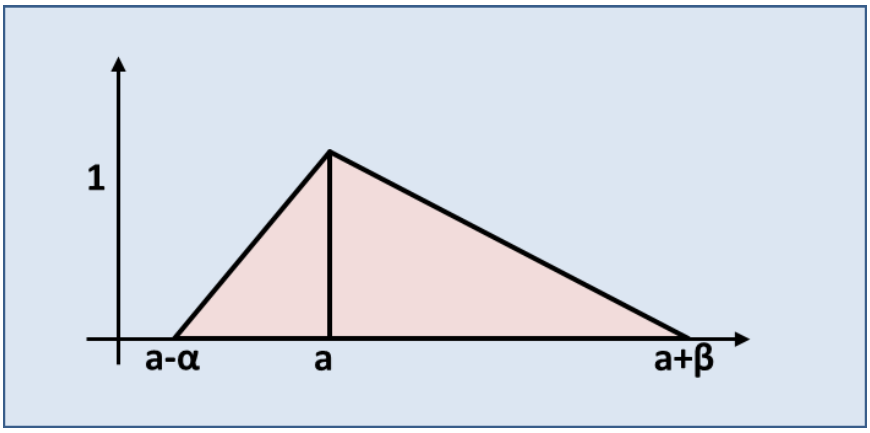
$x_{i,rpc}$ the repair cost for process i

$x_{i,rcc}$ the recovery cost for process i.

A fuzzy set A is called a triangular fuzzy number where a is the peak or centre, α is the left width, and β is the right width if its membership function presented in Figure 1 is shown with the notation $A = (a, \alpha, \beta)$ and has the following form:

$$\begin{cases} 1-\dfrac{(a-t)}{\alpha} & if \quad a-\alpha \le t \le a \\ 1-\dfrac{(t-a)}{\beta} & if \quad a \le t \le a+\beta \\ 0 & otherwise \end{cases}$$

Figure 1 Triangular fuzzy number (see online version for colours)



Let:

$\tilde{x}_{i,rpt}$ the fuzzy set estimate for $x_{i,rpt}$

$\tilde{x}_{i,rct}$ the fuzzy set estimate for $x_{i,rct}$

$\tilde{x}_{i,rpc}$ the fuzzy set estimate for $x_{i,rpc}$

$\tilde{x}_{i,rcc}$ the fuzzy set estimate for $x_{i,rcc}$.

where

$$\tilde{x}_{i,rpt} = \left(a_{i,rpt}, \alpha_{i,rpt}, \beta_{i,rpt}\right),$$

$$\tilde{x}_{i,rct} = \left(a_{i,rct}, \alpha_{i,rct}, \beta_{i,rct}\right),$$

$$\tilde{x}_{i,rpc} = \left(a_{i,rpc}, \alpha_{i,rpc}, \beta_{i,rpc}\right),$$

and

$$\tilde{x}_{i,rcc} = \left(a_{i,rcc}, \alpha_{i,rcc}, \beta_{i,rcc}\right).$$

We normalise these fuzzy sets as follows. Considering $\tilde{x}_{i,rpt} = (a_{i,rpt}, \alpha_{i,rpt}, \beta_{i,rpt})$ and letting $a^-_{rpt} = \min\{a_{i,rpt}\}$ and $a^+_{rpt} = \max\{a_{i,rpt}\}$, we define the normalised peak value $a^{\bar{n}}_{i,rpt}$ as $a^{\bar{n}}_{i,rpt} = \dfrac{a_{i,rpt} - a^-_{rpt}}{a^+_{rpt} - a^-_{rpt}}$.

By defining $\alpha^{\bar{n}}_{i,rpt}$ such that $a^{\bar{n}}_{i,rpt} - \alpha^{\bar{n}}_{i,rpt} = \dfrac{(a_{i,rpt} - \alpha_{i,rpt}) - a^-_{rpt}}{a^+_{rpt} - a^-_{rpt}}$, we see that:

$$\alpha^{\bar{n}}_{i,rpt} = a^{\bar{n}}_{i,rpt} - \left[\frac{\left(a_{i,rpt} - \alpha_{i,rpt}\right) - a^-_{i,rpt}}{a^+_{i,rpt} - a^-_{i,rpt}}\right] = \frac{\alpha_{i,rpt}}{a^+_{i,rpt} - a^-_{i,rpt}} \tag{1}$$

where $\alpha^{\bar{n}}_{i,rpt}$ is the normalised left width. Similarly, $\beta^{\bar{n}}_{i,rpt}$ is the normalised right width:

$$\beta^{\bar{n}}_{i,rpt} = \left[\frac{\left(a_{i,rpt} - \beta_{i,rpt}\right) - a^-_{i,rpt}}{a^+_{i,rpt} - a^-_{i,rpt}} - a^{\bar{n}}_{i,rpt}\right] = \frac{\beta_{i,rpt}}{a^+_{i,rpt} - a^-_{i,rpt}} \tag{2}$$

These calculations are based on the fuzzy arithmetic defined by Dubois and Prade (1980). The normalised fuzzy sets are denoted as follows:

$$\tilde{x}^{\bar{n}}_{i,rpt} = \left(a^{\bar{n}}_{i,rpt}, \alpha^{\bar{n}}_{i,rpt}, \beta^{\bar{n}}_{i,rpt}\right)$$

$$\tilde{x}^{\bar{n}}_{i,rct} = \left(a^{\bar{n}}_{i,rct}, \alpha^{\bar{n}}_{i,rct}, \beta^{\bar{n}}_{i,rct}\right)$$

$$\tilde{x}^{\bar{n}}_{i,rpc} = \left(a^{\bar{n}}_{i,rpc}, \alpha^{\bar{n}}_{i,rpc}, \beta^{\bar{n}}_{i,rpc}\right)$$

$$\tilde{x}^{\bar{n}}_{i,rcc} = \left(a^{\bar{n}}_{i,rcc}, \alpha^{\bar{n}}_{i,rcc}, \beta^{\bar{n}}_{i,rcc}\right)$$

Now assume that c_1 and c_2 are the importance weights for the criteria time and cost, respectively; $\tilde{x}_{i,rob}$ is a measure of robustness for process i, and $\tilde{x}_{i,res}$ is a measure of resiliency for process i. The robustness for each process is assumed to be a weighted sum of the repair time and the repair cost, whereas the resiliency for each process is assumed to be a weighted sum of the recovery time and the recovery cost. $\tilde{x}_{i,rob}$ and $\tilde{x}_{i,res}$ are defined as follows:

$$\tilde{x}_{i,rob} = c_1\tilde{x}^{\overline{n}}_{i,rpt} + c_2\tilde{x}^{\overline{n}}_{i,rpc} \tag{3}$$

$$\tilde{x}_{i,res} = c_1\tilde{x}^{\overline{n}}_{i,rct} + c_2\tilde{x}^{\overline{n}}_{i,rcc} \tag{4}$$

Using the fuzzy arithmetic of Dubois and Prade (1980), we obtain the following for the robustness score:

$$\tilde{x}_{i,rob} = \left(a^{\overline{n}}_{i,rob}, \alpha^{\overline{n}}_{i,rob}, \beta^{\overline{n}}_{i,rob}\right)$$

where

$$a^{\overline{n}}_{i,rob} = c_1 a^{\overline{n}}_{i,rpt} + c_2 a^{\overline{n}}_{i,rpc},$$

$$\alpha^{\overline{n}}_{i,rob} = c_1\alpha^{\overline{n}}_{i,rpt} + c_2\alpha^{\overline{n}}_{i,rpc},$$

and

$$\beta^{\overline{n}}_{i,rob} = c_1\beta^{\overline{n}}_{i,rpt} + c_2\beta^{\overline{n}}_{i,rpc}.$$

Similarly, we obtain the following for the resiliency score:

$$\tilde{x}_{i,res} = \left(a^{\overline{n}}_{i,res}, \alpha^{\overline{n}}_{i,res}, \beta^{\overline{n}}_{i,res}\right)$$

where

$$a^{\overline{n}}_{i,res} = c_1 a^{\overline{n}}_{i,rct} + c_2 a^{\overline{n}}_{i,rcc},$$

$$\alpha^{\overline{n}}_{i,res} = c_1\alpha^{\overline{n}}_{i,rct} + c_2\alpha^{\overline{n}}_{i,rcc},$$

and

$$\beta^{\overline{n}}_{i,res} = c_1\beta^{\overline{n}}_{i,rct} + c_2\beta^{\overline{n}}_{i,rcc}.$$

The centre of gravity for the triangular fuzzy set $A = (a, \alpha, \beta)$ is $cg(A) = a + \frac{(\beta - \alpha)}{3}$ (aee the derivation in Appendix). Therefore, the centre of gravity for the fuzzy sets $\tilde{x}_{i,rob}$ and $\tilde{x}_{i,res}$ are as follows:

$$cg\left(\tilde{x}_{i,rob}\right) = a^{\overline{n}}_{i,rob} + \frac{\left(\beta^{\overline{n}}_{i,rob} - \alpha^{\overline{n}}_{i,rob}\right)}{3} \tag{5}$$

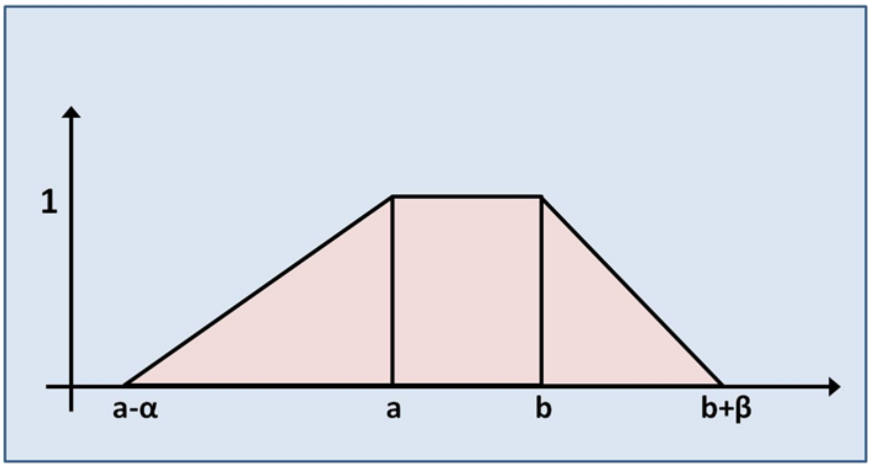
$$cg\left(\tilde{x}_{i,res}\right) = a^{\overline{n}}_{i,res} + \frac{\left(\beta^{\overline{n}}_{i,res} - \alpha^{\overline{n}}_{i,res}\right)}{3} \tag{6}$$

3.2 The stability model with trapezoidal fuzzy numbers

Trapezoidal fuzzy numbers are used most often for characterising imprecise, vague and ambiguous information in practical applications (Klir and Yuan, 1995; Yeh and Deng, 2004). The common use of trapezoidal fuzzy numbers is mainly attributed to their simplicity in both concept and computation. A fuzzy set A is called a trapezoidal fuzzy number with tolerance interval $[a, b]$, left width α and right width β if its membership function presented in Figure 2 is shown with the notation $A = (a, b, \alpha, \beta)$ and has the following form:

$$\begin{cases} 1 - \dfrac{a-t}{\alpha} & \text{if } a - \alpha \le t \le a \\ 1 & \text{if } a \le t \le b \\ 1 - \dfrac{t-b}{\beta} & \text{if } b \le t \le b + \beta \\ 0 & \text{otherwise} \end{cases}$$

Figure 2 Trapezoidal fuzzy number (see online version for colours)



Let

$$\tilde{x}_{i,rpt} = \left(a_{i,rpt}, b_{i,rpt}, \alpha_{i,rpt}, \beta_{i,rpt}\right),$$

$$\tilde{x}_{i,rct} = \left(a_{i,rct}, b_{i,rct}, \alpha_{i,rct}, \beta_{i,rct}\right),$$

$$\tilde{x}_{i,rpc} = \left(a_{i,rpc}, b_{i,rpc}, \alpha_{i,rpc}, \beta_{i,rpc}\right),$$

and

$$\tilde{x}_{i,rcc} = \left(a_{i,rcc}, b_{i,rcc}, \alpha_{i,rcc}, \beta_{i,rcc}\right).$$

We normalise these fuzzy sets as follows. Considering $\tilde{x}_{i,rpt} = \left(a_{i,rpt}, b_{i,rpt}, \alpha_{i,rpt}, \beta_{i,rpt}\right)$ and letting $c_{i,rpt} = \frac{a_{i,rpt} + b_{i,rpt}}{2}, c_{rpt}^{-} = \min\left\{c_{i,rpt}\right\}$ and $c_{rpt}^{+} = \max\left\{c_{i,rpt}\right\}$, the normalised peak values *a* and *b* are defined as

$$a_{i,rpt}^{\overline{n}} = \frac{a_{i,rpt} - c_{rpt}^{-}}{c_{rpt}^{+} - c_{rpt}^{-}}$$

and

$$b_{i,rpt}^{\overline{n}} = \frac{b_{i,rpt} - c_{rpt}^{-}}{c_{rpt}^{+} - c_{rpt}^{-}}.$$

By defining $\alpha_{i,rpt}^{\overline{n}}$ such that

$$a_{i,rpt}^{\overline{n}} - \alpha_{i,rpt}^{\overline{n}} = \frac{\left(a_{i,rpt} - \alpha_{i,rpt}\right) - c_{rpt}^{-}}{c_{rpt}^{+} - c_{rpt}^{-}},$$

we see that:

$$\alpha_{i,rpt}^{\overline{n}} = a_{i,rpt}^{\overline{n}} - \left[\frac{\left(a_{i,rpt} - \alpha_{i,rpt}\right) - c_{rpt}^{-}}{c_{rpt}^{+} - c_{rpt}^{-}}\right] = \frac{\alpha_{i,rpt}}{c_{rpt}^{+} - c_{rpt}^{-}} \tag{7}$$

where $\alpha_{i,rpt}^{\overline{n}}$ is the normalised left-width.

Similarly:

$$\beta_{i,rpt}^{\overline{n}} = \frac{\beta_{i,rpt}}{c_{rpt}^{+} - c_{rpt}^{-}} \tag{8}$$

where $\beta_{i,rpt}^{\overline{n}}$ is the normalised right-width.

These calculations are based on the fuzzy arithmetic defined by Dubois and Prade (1980). The normalised fuzzy sets are defined as follows:

$$\tilde{x}_{i,rpt}^{\overline{n}} = \left(a_{i,rpt}^{\overline{n}}, b_{i,rpt}^{\overline{n}}, \alpha_{i,rpt}^{\overline{n}}, \beta_{i,rpt}^{\overline{n}}\right)$$

$$\tilde{x}_{i,rct}^{\overline{n}} = \left(a_{i,rct}^{\overline{n}}, b_{i,rct}^{\overline{n}}, \alpha_{i,rct}^{\overline{n}}, \beta_{i,rct}^{\overline{n}}\right)$$

$$\tilde{x}_{i,rpc}^{\overline{n}} = \left(a_{i,rpc}^{\overline{n}}, b_{i,rpc}^{\overline{n}}, \alpha_{i,rpc}^{\overline{n}}, \beta_{i,rpc}^{\overline{n}}\right)$$

$$\tilde{x}_{i,rcc}^{\overline{n}} = \left(a_{i,rcc}^{\overline{n}}, b_{i,rcc}^{\overline{n}}, \alpha_{i,rcc}^{\overline{n}}, \beta_{i,rcc}^{\overline{n}}\right)$$

Using the fuzzy arithmetic of Dubois and Prade (1980), we obtain the following for the robustness score:

$$\tilde{x}_{i,rob}^{\overline{n}} = \left(a_{i,rob}^{\overline{n}}, b_{i,rob}^{\overline{n}}, \alpha_{i,rob}^{\overline{n}}, \beta_{i,rob}^{\overline{n}}\right)$$

where

$$a_{i,rob}^{\bar{n}} = c_1 a_{i,rpt}^{\bar{n}} + c_2 a_{i,rpc}^{\bar{n}},$$

$$b_{i,rob}^{\bar{n}} = c_1 b_{i,rpt}^{\bar{n}} + c_2 b_{i,rpc}^{\bar{n}},$$

$$\alpha_{i,rob}^{\bar{n}} = c_1 \alpha_{i,rpt}^{\bar{n}} + c_2 \alpha_{i,rpc}^{\bar{n}},$$

and

$$\beta_{i,rob}^{\bar{n}} = c_1 \beta_{i,rpt}^{\bar{n}} + c_2 \beta_{i,rpc}^{\bar{n}}.$$

Similarly, we obtain the following for the resiliency score:

$$\tilde{x}_{i,res} = \left(a_{i,res}^{\bar{n}}, b_{i,res}^{\bar{n}}, \alpha_{i,res}^{\bar{n}}, \beta_{i,res}^{\bar{n}}\right)$$

where

$$a_{i,res}^{\bar{n}} = c_1 a_{i,rct}^{\bar{n}} + c_2 a_{i,rcc}^{\bar{n}},$$

$$b_{i,res}^{\bar{n}} = c_1 b_{i,rct}^{\bar{n}} + c_2 b_{i,rcc}^{\bar{n}},$$

$$\alpha_{i,res}^{\bar{n}} = c_1 \alpha_{i,rct}^{\bar{n}} + c_2 \alpha_{i,rcc}^{\bar{n}},$$

and

$$\beta_{i,res}^{\bar{n}} = c_1 \beta_{i,rct}^{\bar{n}} + c_2 \beta_{i,rcc}^{\bar{n}}.$$

The centre of gravity for the trapezoidal fuzzy set $A = (a, b, \alpha, \beta)$ is $cg(A) = \dfrac{\alpha(3a-\alpha)+\beta(\beta+3b)+3(b^2-a^2)}{3\left[\alpha+\beta+2(b-a)\right]}$ (see the derivation in Appendix). Therefore, the centre of gravity for $\tilde{x}_{i,rob}$ and $\tilde{x}_{i,res}$ are as follows:

$$cg\left(\tilde{x}_{i,rob}\right) = \frac{\alpha_{i,rob}^{\bar{n}}\left(3a_{i,rob}^{\bar{n}} - \alpha_{i,rob}^{\bar{n}}\right) + \beta_{i,rob}^{\bar{n}}\left(3b_{i,rob}^{\bar{n}} + \beta_{i,rob}^{\bar{n}}\right) + 3\left({b_{i,rob}^{\bar{n}}}^2 - {a_{i,rob}^{\bar{n}}}^2\right)}{3\left[\alpha_{i,rob}^{\bar{n}} + \beta_{i,rob}^{\bar{n}} + 2\left(b_{i,rob}^{\bar{n}} - a_{i,rob}^{\bar{n}}\right)\right]}$$

$$cg\left(\tilde{x}_{i,res}\right) = \frac{\alpha_{i,res}^{\bar{n}}\left(3a_{i,res}^{\bar{n}} - \alpha_{i,res}^{\bar{n}}\right) + \beta_{i,res}^{\bar{n}}\left(3b_{i,res}^{\bar{n}} + \beta_{i,res}^{\bar{n}}\right) + 3\left({b_{i,res}^{\bar{n}}}^2 - {a_{i,res}^{\bar{n}}}^2\right)}{3\left[\alpha_{i,res}^{\bar{n}} + \beta_{i,res}^{\bar{n}} + 2\left(b_{i,res}^{\bar{n}} - a_{i,res}^{\bar{n}}\right)\right]}$$

3.3 A graphical perspective

After both the robustness scores $cg(\tilde{x}_{i,rob})$ and resiliency scores $cg(\tilde{x}_{i,res})$ are calculated for all the processes, we can plot them on a plane. The mean robustness score and the mean resiliency score of the *i* processes divides the plane into four quadrants, identified as the possession quadrant, the preservation quadrant, the restoration quadrant, and the devastation quadrant:

- *Possession state:* processes in this state are both robust and resilient. They are unlikely to fail. However, if they do fail, the system can recover without significant difficulties.
- *Preservation state:* processes in this state are robust but not resilient. They are unlikely to fail. However, if they do fail, the road to recovery will be long and hard.
- *Restoration state:* processes in this state are resilient but not robust. They are likely to fail. However, when they fail, they can recover without significant difficulties.
- *Devastation state:* processes in this state are neither robust nor resilient. They are very likely to fail, and once they fail, recovery is difficult.

3.4 Euclidean distance

Let

$$cg_{rob}^{+} = \text{Min } cg\left(\tilde{x}_{i,rob}\right),$$

$$cg_{rob}^{-} = \text{Max } cg\left(\tilde{x}_{i,rob}\right),$$

$$cg_{res}^{+} = \text{Min } cg\left(\tilde{x}_{i,res}\right),$$

and

$$cg_{res}^{-} = \text{Max } cg\left(\tilde{x}_{i,res}\right).$$

$(cg_{rob}^{+}, cg_{res}^{+})$ represents the *ideal point* on the *robustness* and *resiliency* graph and $(cg_{rob}^{-}, cg_{res}^{-})$ represents the nadir point on the *robustness* and *resiliency* graph. The distance of process *i* from the ideal point is:

$$D_i^{+} = \sqrt{\left[cg\left(\tilde{x}_{i,rob}\right) - cg_{rob}^{+}\right]^2 + \left[cg\left(\tilde{x}_{i,res}\right) - cg_{res}^{+}\right]^2} \tag{9}$$

And the distance of process *i* from the nadir point is:

$$D_i^{-} = \sqrt{\left[cg\left(\tilde{x}_{i,rob}\right) - cg_{rob}^{-}\right]^2 + \left[cg\left(\tilde{x}_{i,res}\right) - cg_{res}^{-}\right]^2} \tag{10}$$

The process should be as close to the fuzzy ideal process as possible and as far from the fuzzy nadir process as possible. The *fuzzy stability index*, S_i, a measure of fuzzy distance from the ideal and nadir states, is used in this study as a measure of stability of the C2 processes in a fuzzy environment as follows:

$$S_i = \frac{D_i^{-}}{D_i^{-} + D_i^{+}} \tag{11}$$

The higher the stability index, the closer the process resiliency and robustness are to the ideal.

4 The stability model example

4.1 The stability model example with triangular fuzzy numbers

In this model, each fuzzy set for the repair-recovery costs and repair-recovery times are assumed to be triangular. The following table represents the triangular fuzzy sets for the repair time, recovery time, repair cost and recovery cost for each process. Each fuzzy set is characterised by the peak value, the left width and the right width. For example, the fuzzy set corresponding to the repair time for process 1 is (20, 5, 5)hours.

Table 1 Triangular fuzzy sets for repair and recovery time and cost

Process (i)	*Repair and recovery time and cost*			
	$\tilde{x}_{i,rpt}$	$\tilde{x}_{i,rct}$	$\tilde{x}_{i,rpc}$	$\tilde{x}_{i,rcc}$
1	(20, 5, 5)	(50, 10, 10)	(1,000, 100, 200)	(3,000, 200, 400)
2	(15, 5, 3)	(30, 5, 7)	(500, 100, 200)	(2,000, 100, 200)
3	(30, 10, 10)	(40, 10, 20)	(600, 50, 100)	(1,000, 200, 300)
4	(40, 5, 7)	(30, 5, 7)	(400, 100, 150)	(800, 100, 200)
5	(18, 2, 3)	(20, 2, 5)	(1,200, 100, 200)	(2,000, 200, 400)

The peak values of the triangular fuzzy sets for the repair-recovery costs and repair-recovery times provided in Table 1 are normalised in Table 2 by using the maximum and minimum peak values.

Table 2 Normalised peak values for triangular example

Process (i)	*Normalised peak values*			
	$a_{rpt}^{+}=40$ $a_{rpt}^{-}=15$	$a_{rct}^{+}=50$ $a_{rct}^{-}=20$	$a_{rpc}^{+}=1{,}200$ $a_{rpc}^{-}=400$	$a_{rcc}^{+}=3{,}000$ $a_{rcc}^{-}=800$
1	0.200	1	0.750	1
2	0	0.330	0.125	0.550
3	0.600	0.670	0.250	0.090
4	1	0.330	0	0
5	0.120	0	1	0.550

The left width values of the triangular fuzzy sets for the repair-recovery costs and repair-recovery times provided in Table 1 are normalised in Table 3 according to equation (1).

Table 3 Normalised left width values for triangular fuzzy example

Process (i)	*Normalised left width values*			
	$\alpha_{i,rpt}^{\bar{n}}$	$\alpha_{i,rct}^{\bar{n}}$	$\alpha_{i,rpc}^{\bar{n}}$	$\alpha_{i,rcc}^{\bar{n}}$
1	0.200	0.330	0.125	0.090
2	0.200	0.167	0.125	0.041
3	0.400	0.330	0.063	0.090
4	0.200	0.167	0.125	0.045
5	0.080	0.067	0.125	0.090

The right width values of the triangular fuzzy sets for the repair-recovery costs and repair-recovery times provided in Table 1 are normalised in Table 4 according to equation (2).

Table 4 Normalised right width values for triangular fuzzy example

Process (i)	*Normalised right width values*			
	$\beta^{\bar{n}}_{i,rpt}$	$\beta^{\bar{n}}_{i,rct}$	$\beta^{\bar{n}}_{i,rpc}$	$\beta^{\bar{n}}_{i,rcc}$
1	0.200	0.330	0.250	0.180
2	0.120	0.233	0.250	0.090
3	0.400	0.670	0.125	0.136
4	0.280	0.233	0.188	0.090
5	0.120	0.167	0.250	0.180

The fuzzy sets for the repair-recovery costs and repair-recovery times are normalised in Table 5 by using the normalised values for the peak and the left and right width as shown in Table 2, Table 3 and Table 4.

Table 5 Normalised triangular fuzzy sets

Process (i)	*Normalised repair and recovery time and cost*			
	$\tilde{x}^{\bar{n}}_{i,rpt}$	$\tilde{x}^{\bar{n}}_{i,rct}$	$\tilde{x}^{\bar{n}}_{i,rpc}$	$\tilde{x}^{\bar{n}}_{i,rcc}$
1	(0.2, 0.2, 0.2)	(1, 0.33, 0.33)	(0.75, 0.125, 0.25)	(1, 0.09, 0.18)
2	(0, 0.2, 0.12)	(0.33, 0.167, 0.233)	(0.125, 0.125, 0.25)	(0.55, 0.045, 0.09)
3	(0.6, 0.4, 0.4)	(0.67, 0.33, 0.67)	(0.25, 0.063, 0.125)	(0.09, 0.09, 0.136)
4	(1, 0.2, 0.28)	(0.33, 0.167, 0.233)	(0, 0.125, 0.188)	(0, 0.045, 0.09)
5	(0.12, 0.08, 0.12)	(0, 0.067, 0.167)	(1, 0.125, 0.25)	(0.55, 0.09, 0.18)

Table 6 presents the fuzzy sets for robustness and resiliency which are computed as a linear combination of the repair-recovery costs and repair-recovery times as in equation (3) and equation (4). The weights for time and cost are given as $c_1 = 0.40$ and $c_2 = 0.60$.

Table 6 Triangular fuzzy sets for robustness and resiliency

Process (i)	*Robustness and resiliency*	
	$\tilde{x}_{i,rob}$	$\tilde{x}_{i,res}$
1	(0.53, 0.155, 0.23)	(1, 0.186, 0.24)
2	(0.075, 0.155, 0.198)	(0.462, 0.094, 0.147)
3	(0.39, 0.198, 0.235)	(0.322, 0.186, 0.35)
4	(0.4, 0.155, 0.225)	(0.132, 0.0938, 0.147)
5	(0.648, 0.107, 0.198)	(0.33, 0.081, 0.175)

In Table 7, the centre of gravity for the robustness and resiliency for each process is calculated using equations (5) and (6) (or equation A.3). The ideal point is assumed to be the minimum centre of gravity for both robustness and resiliency, and the nadir point is assumed to be the maximum centre of gravity for both robustness and resiliency.

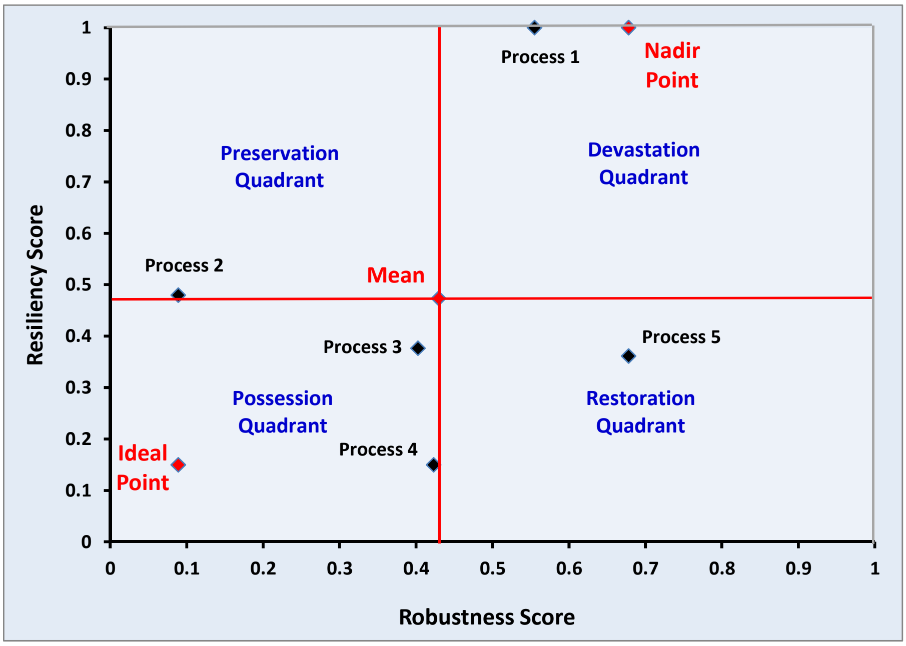
Table 7 Centre of gravity, ideal point and nadir point for triangular fuzzy example

Process (i)	*Centre of gravity for robustness and resiliency*	
	$cg(\tilde{x}_{i,rob})$	$cg(\tilde{x}_{i,res})$
1	0.555	1
2	0.089	0.480
3	0.403	0.377
4	0.423	0.150
5	0.678	0.361
Mean	*0.430*	*0.473*
(cg^+_{rob}, cg^+_{res})	0.089	0.150
(cg^-_{rob}, cg^-_{res})	0.678	1

Table 8 Euclidean distance and stability index for triangular fuzzy example

Process (i)	*Euclidean distance and stability index*		
	D_i^+	D_i^-	S_i
1	0.970	0.123	0.113
2	0.330	0.786	0.704
3	0.387	0.682	0.638
4	0.334	0.888	0.727
5	0.626	0.639	0.505

Figure 3 Robustness-resiliency graph for triangular fuzzy example (see online version for colours)



In Table 8, the Euclidean distance of each process from the ideal point and from the nadir point is calculated using equation (7) and equation (8). The stability index for each process is calculated using equation (9).

The robustness-resiliency graph presented in Figure 3 illustrates the relative distance of the various processes from the ideal and nadir point.

4.2 The stability model example with trapezoidal fuzzy numbers

In this model, each fuzzy set for the repair-recovery costs and repair-recovery times are assumed to be trapezoidal. The trapezoidal sets presented in Table 9 consists of a tolerance interval and a left width and right width. For example, the repair time for process 1 is (15, 25, 5, 5) hours.

Table 9 Trapezoidal fuzzy sets for repair and recovery time and cost

Process (i)	*Repair and recovery time and cost*			
	$\tilde{x}_{i,rpt}$	$\tilde{x}_{i,rct}$	$\tilde{x}_{i,rpc}$	$\tilde{x}_{i,rcc}$
1	(15, 25, 5, 5)	(45, 55, 10, 10)	(900, 1,100, 100, 200)	(2,800, 3,200, 200, 400)
2	(10, 20, 5, 3)	(25, 35, 5, 7)	(400, 600, 100, 200)	(1,800, 2,200, 100, 200)
3	(27, 33, 10, 10)	(38, 42, 10, 20)	(550, 650, 50, 100)	(800, 1,200, 200, 300)
4	(30, 50, 5, 7)	(28, 32, 5, 7)	(300, 500, 100, 150)	(700, 900, 100, 200)
5	(16, 20, 2, 3)	(17, 23, 2, 5)	(1,000, 1,400, 100, 200)	(1,700, 2,300, 200, 400)

The mean of the tolerance interval presented in Table 10 is computed as the average of the lower and upper level of the tolerance interval (i.e., the average of *a* and *b*).

Table 10 Mean of tolerance interval for trapezoidal fuzzy example

Process (i)	*Mean of tolerance interval*			
	$c_{i,rpt}$	$c_{i,rct}$	$c_{i,rpc}$	$c_{i,rcc}$
1	20	50	1,000	3,000
2	15	30	500	2,000
3	30	40	600	1,000
4	40	30	400	800
5	18	20	1,200	2,000

Table 11 Normalised lower tolerance levels for trapezoidal fuzzy example

Process (i)	$c^+_{rpt} = 40$ $c^-_{rpt} = 15$	$c^+_{rct} = 50$ $c^-_{rct} = 20$	$c^+_{rpc} = 1{,}200$ $c^-_{rpc} = 400$	$c^+_{rcc} = 3{,}000$ $c^-_{rcc} = 800$
	$a^{\bar{n}}_{i,rpt}$	$a^{\bar{n}}_{i,rct}$	$a^{\bar{n}}_{i,rpc}$	$a^{\bar{n}}_{i,rcc}$
1	0	0.830	0.625	0.909
2	–0.200	0.167	0	0.455
3	0.480	0.600	0.188	0
4	0.600	0.267	–0.125	–0.045
5	0.040	–0.100	0.750	0.409

The lower tolerance levels (a) are normalised in Table 11 using the maximum and minimum values of the mean tolerance levels.

The upper tolerance levels (b) are normalised in Table 12 using the maximum and minimum values of the mean tolerance levels.

Table 12 Normalised upper tolerance levels for trapezoidal fuzzy example

Process (i)	$c_{rpt}^{+} = 40$ $c_{rpt}^{-} = 15$	$c_{rct}^{+} = 50$ $c_{rct}^{-} = 20$	$c_{rpc}^{+} = 1,200$ $c_{rpc}^{-} = 400$	$c_{rcc}^{+} = 3,000$ $c_{rcc}^{-} = 800$
	$b_{i,rpt}^{\overline{n}}$	$b_{i,rct}^{\overline{n}}$	$b_{i,rpc}^{\overline{n}}$	$b_{i,rcc}^{\overline{n}}$
1	0.400	1.670	0.875	1.090
2	0.200	0.500	0.250	0.636
3	0.720	0.733	0.313	0.180
4	1.400	0.400	0.125	0.045
5	0.200	0.100	1.250	0.680

In Table 13, the left width values are normalised using equation (10).

Table 13 Normalised left width values for trapezoidal fuzzy example

Process (i)	*Normalised left width values*			
	$\alpha_{i,rpt}^{\overline{n}}$	$\alpha_{i,rct}^{\overline{n}}$	$\alpha_{i,rpc}^{\overline{n}}$	$\alpha_{i,rcc}^{\overline{n}}$
1	0.200	0.330	0.125	0.090
2	0.200	0.167	0.125	0.045
3	0.400	0.330	0.063	0.090
4	0.200	0.667	0.125	0.045
5	0.080	0.067	0.125	0.090

In Table 14, the right width values are normalised using equation (11).

Table 14 Normalised right width values for trapezoidal fuzzy example

Process (i)	*Normalised right width values*			
	$\beta_{i,rpt}^{\overline{n}}$	$\beta_{i,rct}^{\overline{n}}$	$\beta_{i,rpc}^{\overline{n}}$	$\beta_{i,rcc}^{\overline{n}}$
1	0.200	0.330	0.250	0.180
2	0.120	0.233	0.250	0.090
3	0.400	0.670	0.125	0.136
4	0.280	0.233	0.188	0.090
5	0.120	0.167	0.250	0.180

In Table 15, the fuzzy sets for the repair-recovery costs and repair-recovery times are normalised using the normalised values given in Table 11, Table 12, Table 13 and Table 14.

Table 15 Normalised trapezoidal fuzzy sets for repair and recovery time and cost

Process (i)	*Repair and recovery time and cost*			
	$\tilde{x}^{\bar{n}}_{i,rpt}$	$\tilde{x}^{\bar{n}}_{i,rct}$	$\tilde{x}^{\bar{n}}_{i,rpc}$	$\tilde{x}^{\bar{n}}_{i,rcc}$
1	(0.0.4, 0.2, 0.2)	(0.83, 1.67, 0.33, 0.33)	(0.625, 0.875, 0.125, 0.25)	(0.909, 1.09, 0.09, 0.18)
2	(–0.2, 0.2, 0.2, 0.12)	(0.167, 0.5, 0.167, 0.233)	(0, 0.25, 0.125, 0.25)	(0.455, 0.636, 0.045, 0.09)
3	(0.48, 0.72, 0.4, 0.4)	(0.6, 0.733, 0.33, 0.67)	(0.188, 0.313, 0.063, 0.125)	(0, 0.18, 0.09, 0.136)
4	(0.6, 1.4, 0.2, 0.28)	(0.267, 0.4, 0.167, 0.233)	(–0.125, 0.125, 0.125, 0.188)	(–0.045, 0.045, 0.045, 0.09)
5	(0.04, 0.2, 0.08, 0.12)	(–0.1, 0.1, 0.067, 0.167)	(.75, 1.25, 0.125, 0.25)	(0.409, 0.68, 0.09, 0.18)

The fuzzy sets for robustness and resiliency presented in Table 16 are computed as a linear combination of the normalised fuzzy sets for the repair-recovery costs and repair-recovery times. Note that $c_1 = .4$ and $c_2 = .6$.

Table 16 Trapezoidal fuzzy sets for robustness and resiliency

Process (i)	Robustness and resiliency	
	$\tilde{x}_{i,rob}$	$\tilde{x}_{i,res}$
1	(0.375, 0.685, 0.155, 0.23)	(0.877, 1.322, 0.186, 0.24)
2	(–0.08, 0.23, 0.155, 0.198)	(0.34, 0.5816, 0.0938, 0.1472)
3	(0.305, 0.476, 0.198, 0.235)	(0.24, 0.4012, 0.186, 0.35)
4	(0.165, 0.635, 0.155, 0.225)	(0.08, 0.187, 0.094, 0.147)
5	(0.466, 0.83, 0.107, 0.198)	(0.205, 0.448, 0.081, 0.175)

The centre of gravity for the robustness and resiliency of each process given in Table 17 is computed using equation (A.5). The ideal point is assumed to be the minimum centre of gravity for both robustness and resiliency. The nadir point is assumed to be the maximum centre of gravity for both robustness and resiliency.

Table 17 Centre of gravity, ideal point and nadir point for trapezoidal fuzzy example

Process (i)	Centre of gravity for robustness and resiliency	
	$cg(\tilde{x}_{i,rob})$	$cg(\tilde{x}_{i,res})$
1	0.551	1
2	0.087	0.475
3	0.401	0.37
4	0.419	0.149
5	0.673	0.353
Mean	*0.426*	*0.469*
(cg^+_{rob}, cg^+_{res})	0.087	0.149
(cg^-_{rob}, cg^-_{res})	0.673	1

The Euclidean distance of each point from the ideal point and the nadir point given in Table 18 is computed using equation (9) and equation (10). The stability index is computed using equation (11).

Table 18 Euclidean distance and stability index for trapezoidal fuzzy example

Process (i)	Euclidean distance and stability index		
	D_i^+	D_i^-	S_i
1	0.969	0.122	0.112
2	0.326	0.787	0.707
3	0.384	0.686	0.641
4	0.332	0.888	0.728
5	0.620	0.647	0.511

Table 19 Stability index of days 1 through 20 for triangular fuzzy example

Process	Day																			
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
1	0.113	0.117	0.117	0.118	0.119	0.122	0.124	0.124	0.127	0.130	0.131	0.133	0.136	0.138	0.142	0.144	0.147	0.148	0.149	0.150
2	0.704	0.726	0.726	0.738	0.739	0.762	0.770	0.771	0.791	0.813	0.817	0.829	0.849	0.859	0.886	0.895	0.915	0.921	0.927	0.931
3	0.638	0.658	0.658	0.669	0.670	0.691	0.698	0.699	0.717	0.737	0.741	0.752	0.769	0.778	0.803	0.811	0.829	0.834	0.840	0.844
4	0.727	0.750	0.750	0.762	0.764	0.787	0.796	0.796	0.817	0.839	0.844	0.856	0.877	0.887	0.915	0.924	0.944	0.951	0.957	0.962
5	0.505	0.521	0.521	0.529	0.530	0.547	0.553	0.553	0.567	0.583	0.586	0.595	0.609	0.616	0.636	0.642	0.656	0.661	0.665	0.668

The robustness-resiliency graph presented in Figure 4 illustrates the relative distance of the various process from the ideal and nadir points.

Figure 4 The robustness-resiliency graph for trapezoidal fuzzy example (see online version for colours)

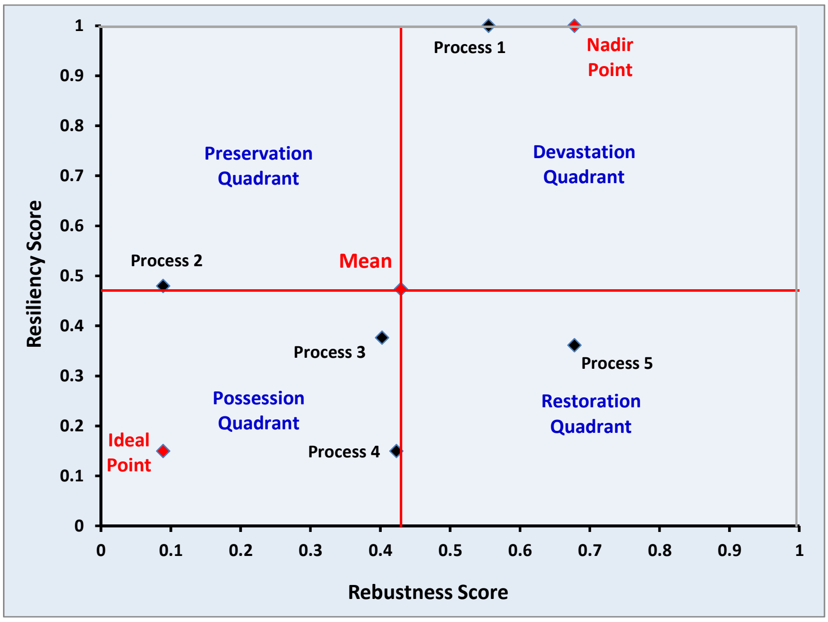


Figure 5 Stability index of days 1 through 20 for triangular fuzzy example (see online version for colours)

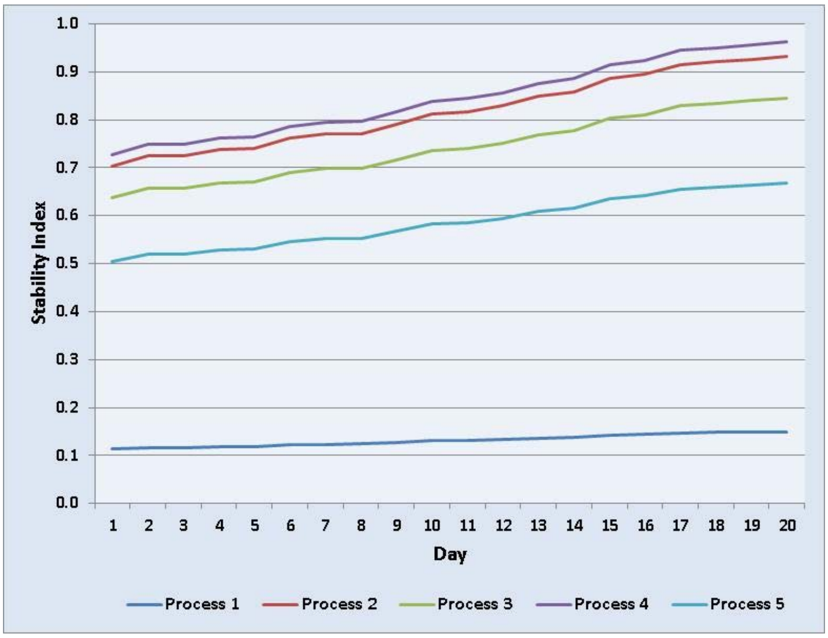


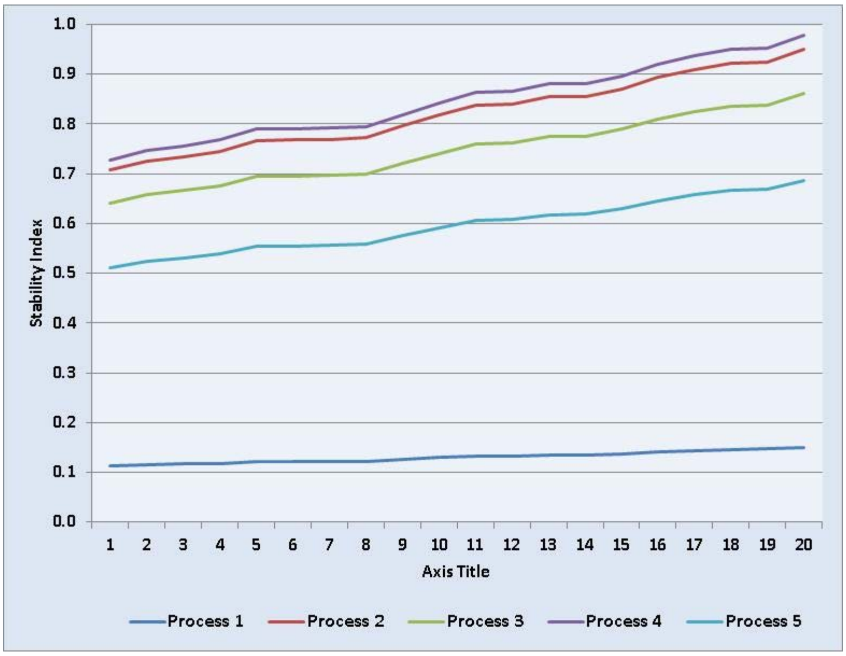
Table 20 Stability index of days 1 through 20 for trapezoidal fuzzy example

Process	Day																			
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
1	0.112	0.115	0.116	0.118	0.121	0.122	0.122	0.122	0.126	0.130	0.133	0.133	0.135	0.136	0.138	0.142	0.144	0.146	0.147	0.150
2	0.707	0.725	0.735	0.746	0.767	0.767	0.769	0.772	0.796	0.818	0.838	0.840	0.855	0.856	0.870	0.894	0.910	0.922	0.925	0.949
3	0.641	0.657	0.666	0.676	0.695	0.696	0.698	0.700	0.721	0.741	0.760	0.762	0.775	0.776	0.789	0.810	0.825	0.836	0.839	0.861
4	0.728	0.746	0.756	0.768	0.789	0.790	0.792	0.795	0.819	0.842	0.863	0.865	0.880	0.881	0.896	0.920	0.937	0.949	0.953	0.978
5	0.511	0.524	0.531	0.539	0.554	0.555	0.556	0.558	0.575	0.591	0.606	0.607	0.618	0.619	0.629	0.646	0.657	0.666	0.669	0.686

Next, we conducted a simulation study of the triangular model for 20 days. The stability indices of the five processes for days 1 through 20 are presented in Table 19 and depicted in Figure 5.

We also conducted a simulation study of the trapezoidal model for 20 days. The stability indices of the five processes for days 1 through 20 are presented in Table 20 and depicted in Figure 6.

Figure 6 Stability index of days 1 through 20 for trapezoidal fuzzy example (see online version for colours)



The simulation results show how the stability indices could be tracked for several processes in the C2 system over a period of time.

5 Conclusions and future research directions

The ability of a system to avoid failure (robustness) and the ability to recover from failure once it occurs (resiliency) are essential elements of good WFMs. Nevertheless, there has been very little discussion of these properties in the workflow management literature. The necessity for readiness and the ability to cope with the possibility of failure in complex systems are important areas for future workflow management studies, especially in highly critical areas such as military C2 operations. The WFMs in conventional military C2 systems generally measure robustness and resiliency with precise repair-recovery costs and repair-recovery times. However, the repair-recovery costs and repair-recovery times in real-world problems are often imprecise or uncertain. We used fuzzy sets to represent imprecise or uncertain information in military C2 systems and developed an interactive stability model for simultaneous consideration of robustness and resiliency in fuzzy environments. The interactive model plotted the fuzzy robustness and fuzzy resiliency measures in a Cartesian coordinate system and derived an overall fuzzy stability index for various processes in the C2 system based on the theory of displaced ideals.

This study increases the practical application of the conventional WFMs by considering imprecise or uncertain data through the use of fuzzy numbers. The fuzzy stability model proposed in this study allows broader and more flexible inputs while still generating well-defined and useful results. The framework proposed in this paper could be expanded to:

- Include additional dimensions in the stability index to track changes to a system over time.
- Consider the interdependency of resiliency and robustness. These two features may not necessarily function independently, as changes to one may affect the other.
- Develop automated systems to provide the capability for continuous monitoring of the resiliency and robustness in large systems.

We also hope that the concept introduced here provides the groundwork for studying robustness and resiliency in complex WFMs.

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Appendix

Derivation of the centre of gravity for the triangular and trapezoidal fuzzy sets

Let A be a fuzzy set defined in $X = \{x_1,\ldots, x_n\}$ with membership function $\mu_A(x_i)$. Then, the centre of gravity of A is defined as follows:

$$cg(A) = \frac{\sum_{i=1}^{n} x_i \mu_A(x_i)}{\sum_{i=1}^{n} \mu_A(x_i)} \tag{A.1}$$

The triangular fuzzy set $A = (a, \alpha, \beta)$ has a continuous membership function $\mu_A(t_i)$ defined as follows:

$$\begin{cases} 1-\dfrac{(a-t)}{\alpha} & \text{if } \; a-\alpha \le t \le a \\ 1-\dfrac{(t-a)}{\beta} & \text{if } \; a \le t \le a+\beta \\ 0 & \text{otherwise} \end{cases}$$

Applying the centre of gravity Formula (1) to the continuous case, we obtain the following for the centre of gravity of $A = (a, \alpha, \beta)$:

$$cg(A) = \frac{\int_{a-\alpha}^{a} t\left(1-\frac{(a-t)}{\alpha}\right)dt + \int_{a}^{a+\beta} t\left(1-\frac{(t-a)}{\beta}\right)dt}{\int_{a-\alpha}^{a} \left(1-\frac{(a-t)}{\alpha}\right)dt + \int_{a}^{a+\beta} \left(1-\frac{(t-a)}{\beta}\right)dt} \tag{A.2}$$

Calculating these integral expressions and collecting terms will result in the following formula for $cg(A)$:

$$cg(A) = a + \frac{\beta-\alpha}{3} \tag{A.3}$$

The trapezoidal fuzzy set $A = (a, b, \alpha, \beta)$ has a continuous membership function $\mu_A(t)$ defined as follows:

$$\begin{cases} 1-\dfrac{a-t}{\alpha} & \text{if } a-\alpha \le t \le a \\ 1 & \text{if } a \le t \le b \\ 1-\dfrac{t-b}{\beta} & \text{if } b \le t \le b+\beta \\ 0 & \text{otherwise} \end{cases}$$

Applying the centre of gravity Formula (1) to the continuous case, we obtain the following expression for the centre of gravity of $A = (a, b, \alpha, \beta)$:

$$cg(A) = \frac{\int_{a-\alpha}^{a} t\left(1 - \frac{(a-t)}{\alpha}\right)dt + \int_{a}^{b} t\,dt + \int_{b}^{b+\beta} t\left(1 - \frac{(t-b)}{\beta}\right)dt}{\int_{a-\alpha}^{a} \left(1 - \frac{(a-t)}{\alpha}\right)dt + \int_{a}^{b} dt + \int_{b}^{b+\beta} \left(1 - \frac{(t-b)}{\beta}\right)dt} \tag{A.4}$$

Calculating these integral expressions and collecting terms, we find the following formula for $cg(A)$:

$$cg(A) = \frac{\alpha(3a-\alpha) + \beta(\beta+3b) + 3\left(b^2 - a^2\right)}{3\left[\alpha + \beta + 2(b-a)\right]} \tag{A.5}$$

Note that the triangular case corresponds to $a = b$, implying that $cg(A) = a + \frac{\beta - \alpha}{3}$ (which is what we obtained for the triangular case).