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A game theoretic approach to modeling undesirable outputs and efficiency decomposition in data envelopment analysis

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A R T I C L E I N F O

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A B S T R A C T

The changing economic conditions have challenged many organizations to search for more effective performance measurement methods. Data envelopment analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogeneous decision making units (DMUs) by evaluating their relative efficiency. Performance measurement in the conventional DEA is based on the assumptions that inputs should be minimized and outputs should be maximized. However, there are circumstances in real-world problems where some output variables should be minimized. We consider the concepts of *technical efficiency* (the ratio of the desirable outputs to inputs) and *ecological efficiency* (the ratio of the desirable outputs to undesirable outputs) in DEA. We then introduce a new measure called *process environmental quality efficiency* (the ratio of the inputs to the undesirable outputs) and use game theory to integrate these three different efficiency scores into one overall efficiency score. The cooperative and non-cooperative game theory concepts are used to integrate different efficiency ratios into a linear model. We also present a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed models.

1. Introduction

Data envelopment analysis (DEA) was developed by Charnes et al. [3] to calculate the efficiency score of homogeneous decision-making units (DMUs). The original DEA models measure the efficiency score of the DMUs in terms of their used inputs and desirable (good) outputs. For example, CCR [3], BCC [1], the slack-based measure (SBM) [22] approach, the additive ([4]) and the range adjusted measure (RAM) [9] method are among the mostly used models of this type. The efficiency score measured by these models is known as the technical or operational efficiency index and is an indicator of how

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efficiently DMUs use their resources to produce desirable outputs and products. Technical efficiency (TE) is the ratio of the desirable outputs to inputs.

However, there might also be some undesirable by-products like CO_2 that are outputs in the production process. Chemical oxygen demand, sulphur dioxide emission, soot, dust and solid waste are some other examples of undesirable outputs. Some authors have studied the modeling of these factors in DEA. Färe et al. [11], Korhonen and Luptacik [14], Hailu and Veeman [12], Seiford and Zhu [19], Färe and Grosskopf [10], Yang and Pollitt [24] and Sueyoshi and Goto [21,20] have addressed different ways to model undesirable outputs. The models discussed by these authors measure overall efficiency (OE) of the DMUs by simultaneous consideration of the inputs, desirable and undesirable outputs. Overall efficiency can be seen as a combination of the technical and ecological efficiency (ECE) scores. The ecological efficiency can be measured as the ratio of the desirable outputs to undesirable outputs. The ecological efficiency, in short, is a measure that is maximized by creating more value with less environmental damage. Among others, Korhonen and Luptacik [14] addressed different ways to measure all three measures represented by the technical, ecological and overall efficiency scores.

The above mentioned researchers have made great strides in modeling undesirable outputs by considering the trade-off between the inputs and the desirable outputs (technical efficiency measurement) and also the trade-off between the desirable outputs and the undesirable outputs (ecological efficiency measurement). These advances have led to the overall efficiency measurement of DMUs which produce undesirable outputs in their production process. The technical efficiency is measured as the ratio of the weighted sum of the desirable outputs divided by the weighted sum of the inputs, while the ecological efficiency is calculated as the weighted sum of the desirable outputs divided by the weighted sum of the undesirable outputs.

However, our paper introduces a new measure of efficiency called "the process environmental quality efficiency (PEQE)" which is considered to be an overall efficiency measurement of the DMUs. The process environmental quality efficiency m is calculated as the ratio of the inputs to undesirable outputs. The process environmental quality efficiency is used to show how a DMU is efficiently using its resources to produce less undesirable outputs. This approach allows the practicing managers and decision makers to discover improvements in the system and decrease the amount of unwanted and undesirable outputs produced by the DMUs.

As an example, consider a performance evaluation problem with two DMUs. DMU 1 with 2 units of input produces 50 units of undesirable output while DMU 2 with 50 units of input produces 2 units of undesirable output. Both DMUs equally produce 1 unit of desirable output. This information suggests that there could be a problem with the quality of the fuel, technology or the process used by DMU 1 which leads to a relatively high emission of the pollution. DMU 1 can be seen as inefficient, since there is another DMU (DMU 2) which is able to produce less undesirable output with more inputs and the same amount of desirable output. This information can be useful to the management of the DMU. The process environmental quality efficiency uses this information to measure the inefficiency of DMU 1. The inefficiency can be due to the low quality of the inputs or the process which has created a high production of pollution.

The goal of our paper is to develop a model to measure the process environmental quality efficiency of the DMUs and also to integrate the technical, ecological and process environmental quality efficiency scores of the DMUs into a single model. For integration, cooperative and non-cooperative game theory concepts are used.

The main contributions of this paper to the performance measurement problems with undesirable outputs are threefold. For the first time: (1) the new concept called the process environmental quality efficiency is introduced and a new model is developed to measure it; (2) the efficiency score for DEA problems with undesirable outputs is decomposed into three different scores: technical, ecological and process environmental quality efficiency scores; and (3) some approaches are proposed to integrate these three efficiency scores into a single model.

The remainder of this paper is organized as follows. In Section 2, we first introduce a new model to measure the quality of the production process from an environmental perspective (PEQE). We then propose some linear and non-linear models to integrate the three different measures of efficiency into a single model. Finally, we show how to implement the proposed models under a common set of weights (CSW). In Section 3, we present a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed methods in a real-world case. In Section 4, we present our conclusions and future research directions.

2. Proposed model

2.1. Measuring process environmental quality efficiency

Assume we have n DMUs each consuming m inputs to produce s desirable and p undesirable outputs. The outputs corresponding to indices $1, 2, \ldots, s$ are desirable and the outputs corresponding to indices $1, 2, \ldots, p$ are undesirable. Let $X \in \boldsymbol{R}_+^{m\times n}$, $\boldsymbol{Y}^g \in \boldsymbol{R}_+^{s\times n}$ and $\boldsymbol{Y}^b \in \boldsymbol{R}_+^{p\times n}$ be the matrices, containing the observed input, desirable and undesirable output measures, respectively. x_{ij}, y_{rj}^g, and y_{kj}^b are the ith input, rth desirable and kth undesirable output of DMU_j, respectively. v_i, u_r, and μ_k are the weights of inputs, desirable and undesirable outputs, respectively. Subscript o indicates the DMU under observation. Model (1), known as the CCR model, developed by Charnes et al. [3] is used to measure the technical efficiency of such a DMU. Model (2) is used to measure the ecological efficiency of the DMU under observation. In this model, Korhonen and Luptacik [14] used the ratio of the weighted sum of the desirable outputs divided by the weighted sum of the undesirable outputs as a measure of ecological efficiency.

$$\begin{aligned}
Max \quad & \mathrm{TE}^* = \sum_{r=1}^{s} u_r y_{ro}^g, \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, \ldots, n, \\
& v_i \geq 0, \qquad i = 1, 2, \ldots, m, \\
& u_r \geq 0, \qquad r = 1, 2, \ldots, s,
\end{aligned} \tag{1}$$

$$\begin{aligned}
Max \quad & \mathrm{ECE}^* = \sum_{r=1}^{s} u_r y_{ro}^g, \\
& \sum_{k=1}^{p} \mu_k y_{ko}^b = 1, \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, \quad j = 1, \ldots, n, \\
& v_i \geqslant 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geqslant 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{2}$$

We propose Model (3) to consider the trade-off between the inputs and the undesirable outputs and to measure the quality of the production process from an environmental perspective (process environmental quality efficiency).

$$\begin{aligned}
Max \quad & \mathrm{PEQE}^* = \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y_{ko}^b} \quad , \\
s.t. \quad & \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{k=1}^{p} \mu_k y_{kj}^b} \leqslant 1, \qquad j = 1, \ldots, n, \\
& v_i \geqslant 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geqslant 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{3}$$

Model (3) is intended to find the best possible weights for the inputs and the undesirable outputs of the DMU_o that maximize the ratio of the process environmental quality efficiency. This optimization process is performed under some constraints: (i) the weights v_i and μ_k should be equal or greater than zero, and (ii) with the optimal weights v_i^* and μ_k^*, the ratio of the process environmental quality efficiency should be less than or equal to one for DMU_j. Model (3) is written in its linear form as follows.

$$\begin{aligned}
Max \quad & \mathrm{PEQE}^* = \sum_{i=1}^{m} v_i x_{io}, \\
s.t. \quad & \sum_{k=1}^{p} \mu_k y_{ko}^b = 1, \qquad , \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, \quad j = 1, \ldots n, \\
& v_i \geq 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{4}$$

The objective function of Model (4) is between 0 and 1. DMUs with efficiency scores of 1 are recognized as efficient and those with efficiency scores of less than one are considered inefficient. This model can provide useful information on potential improvements in the quality of the process, fuel and technology to produce less undesirable outputs.

2.2. Integrating TE, PEQE and ECE into a non-linear model

At this stage, an effort is made to unify technical, ecological and process environmental quality efficiency scores and measure the overall efficiency score of the DMUs. The unification is done by two non-linear and linear procedures borrowed from [8,16], respectively. The non-linear unification leads to a one-step evaluation of technical, ecological and process environmental quality efficiency scores, while the linear formulation uses a two-stage procedure to solve the problem. First, the non-linear unification (Model 5) is developed as follows:

$$
\begin{aligned}
Max \quad & \mathrm{OE}^* = \frac{\sum_{r=1}^{s} u_r y_{ro}^g}{\sum_{i=1}^{m} v_i x_{io}} + \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y_{ko}^b}, \\
s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^g}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, && j = 1, \ldots, n, \\
& \frac{\sum_{i=1}^{m} v_i x_{ij}}{\sum_{k=1}^{p} \mu_k y_{kj}^b} \leqslant 1, && j = 1, \ldots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj}^g}{\sum_{k=1}^{p} \mu_k y_{kj}^b} \leqslant 1, && j = 1, \ldots, n, \\
& v_i \geq 0, && r = 1, 2, \ldots, s, \\
& u_r \geq 0, && i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, && k = 1, 2, \ldots, p.
\end{aligned} \tag{5}
$$

Model (5) can also be shown as Model (6) and is run n times to measure the overall efficiency of n DMUs.

$$
\begin{aligned}
Max \quad & \mathrm{OE}^* = \left(\sum_{r=1}^{s} u_r y_{ro}^g\right)\left(\sum_{k=1}^{p} \mu_k y_{ko}^b\right) + \left(\sum_{i=1}^{m} v_i x_{io}\right)\left(\sum_{i=1}^{m} v_i x_{io}\right), \\
s.t. \quad & \left(\sum_{i=1}^{m} v_i x_{io}\right)\left(\sum_{k=1}^{p} \mu_k y_{ko}^b\right) = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, && j = 1, \ldots, n, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, && j = 1, \ldots, n, \\
& \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, && j = 1, \ldots, n, \\
& v_i \geq 0, && r = 1, 2, \ldots, s, \\
& u_r \geq 0, && i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, && k = 1, 2, \ldots, p.
\end{aligned} \tag{6}
$$

The objective function of Model (6), which is an integrated measure of the technical and process environmental quality efficiencies, is between 0 and 2. The optimal weights of u_r^*, v_i^*, and μ_k^* are used in (7)–(10) to measure the technical, process environmental quality, ecological and overall efficiencies of DMU_o separately as follows:

$$\text{Technical efficiency} = \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{i=1}^{m} v_i^* x_{io}}, \tag{7}$$

$$\text{Process environmental quality efficiency} = \frac{\sum_{i=1}^{m} v_i^* x_{io}}{\sum_{k=1}^{p} \mu_k^* y_{ko}^b}, \tag{8}$$

$$\text{Ecological efficiency} = \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{k=1}^{p} \mu_k^* y_{ko}^b}, \tag{9}$$

$$\text{Overall efficiency} = 1/3\left(\frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{i=1}^{m} v_i^* x_{io}} + \frac{\sum_{i=1}^{m} v_i^* x_{io}}{\sum_{k=1}^{p} \mu_k^* y_{ko}^b} + \frac{\sum_{r=1}^{s} u_r^* y_{ro}^g}{\sum_{k=1}^{p} \mu_k^* y_{ko}^b}\right). \tag{10}$$

Note that constraints $\sum_{r=1}^{s} u_r y_{rj}^g - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0$, $\sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0$ and $\sum_{r=1}^{s} u_r y_{rj}^g - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0$ in Model (6) guarantee each single ratio of the technical, process environmental quality and ecological efficiencies to be less than or equal to one for each DMU.

Chiou et al. [8] provided a discussion regarding the global optimal solution (as opposed to a local optimum) of their developed non-linear model. The same discussion is valid for Models (5) and (6). Chiou et al. [8] have showed that their non-linear model provides a global optimal solution for the efficiency and effectiveness scores obtained by their model. They inspected two conditions of *concavity* (or *convexity*) with regards to the objective function and the feasible region and concluded that the necessary requirements for a global maximum are met in their non-linear model.

However, Lim and Zhu [18] showed that the conclusion made by Chiou et al. [8] is a false statement and Chiou et al.'s [8] DEA model is actually a non-convex optimization problem due to the misuse of the Hessian matrix in examining the concavity of the objective function. Lim and Zhu [18] concluded that Chiou et al.'s [8] DEA model is unusable in practice due to a lack of an efficient algorithm for this particular non-convex DEA model. Therefore, the same discussion can also be made for our proposed non-linear model.

Lim and Zhu [18] further showed some alternative linear formulations to solve Chiou et al.'s [8] model. They suggested linear formulations addressed in the two-stage DEA literature [17,13,16] to solve Chiou et al.'s [8] model, since it has a similar two-stage structure. We will adopt the approach developed by Liang et al. [16] to model the linear unification of the technical, ecological and process environmental quality efficiency scores.[1] This unification procedure is chosen for two reasons.

First, unlike separate modeling of the two efficiency ratios, it will reflect the potential conflict between technical and process environmental quality efficiency scores (this conflict exists since the weighted-sum of the inputs appears in the denominator of the technical efficiency ratio and in the numerator of the process environmental quality efficiency ratio and the larger values of the inputs shows a weak performance in the technical and a good performance in the process environmental quality efficiency ratio). Without this reflection of the conflict in two ratios when they are modeled separately, one criticism can arise; suppose the DMU under evaluation is a technically inefficient DMU but its process environmental quality efficiency is equal to 1. When the improvement target defined by the technical efficiency model suggests a decrement in input values of the technically inefficient DMU, this action may worsen process environmental quality efficiency of the DMU.

Second, by adopting the concepts of cooperative and non-cooperative games proposed by Liang et al. [16], the analyst can decide on the priority of the improvements on technical or process environmental quality efficiency scores.[2] If the focus is on improving technical efficiency, technical efficiency is considered as the leader and process environmental quality efficiency as the follower; otherwise, technical efficiency will be modeled as the follower. If both ratios of the technical and process environmental quality efficiencies have the same priorities, both ratios will be modeled in a cooperative manner.

2.3. Integrating TE, PEQE and ECE into a linear model

We model undesirable outputs in DEA similar to a two-stage DEA, where the operation in DMU is completed in two stages and the outputs of the first stage is used as the inputs of the second stage. These intermediate measures create the conflict between the two stages. We propose to model process environmental quality efficiency similar to the first stage of a two stage DEA and technical efficiency similar to the second stage of a two-stage DEA. Inputs are the intermediate measures of the developed models. Fig. 1 shows the conceptual model of the two-stage structured modeling of undesirable outputs in DEA.

Considering that the first stage (process environmental quality efficiency) is the leader and the second stage (technical efficiency) is the follower, process environmental quality efficiency is assumed more important in a non-cooperative game and the efficiency of the technical efficiency is computed, subject to the condition that the efficiency score of the first stage remains fixed. The process environmental quality efficiency score of the DMU under observation is calculated by Model (4) and process environmental quality efficiency (PEQE^*) is identified. As soon as the process environmental quality efficiency score of the DMU under evaluation is found, the technical efficiency of the DMU is measured subject to keeping to the process environmental quality efficiency score of the DMU equal to the PEQE^* obtained by Model (4). The model for measuring technical efficiency, the follower (second stage), is written as follows:

$$
\begin{aligned}
\mathit{Max}\quad & \text{TE}^* = \frac{\sum_{r=1}^{s} u_r y_{ro}^g}{\sum_{i=1}^{m} v_i x_{io}}, \\
s.t.\quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^g}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, \quad j = 1,\ldots,n, \\
& \sum_{k=1}^{p} \mu_k y_{ko}^b = 1, \\
& \sum_{i=1}^{m} v_i x_{io} = \text{PEQE}^*, \qquad j = 1,\ldots,n, \\
& v_i \geq 0, \qquad r = 1,2,\ldots,s, \\
& u_r \geq 0, \qquad i = 1,2,\ldots,m, \\
& \mu_k \geq 0, \qquad k = 1,2,\ldots,p.
\end{aligned}
\tag{11}
$$

Constraints $\sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0$, $\sum_{k=1}^{p} \mu_k y_{ko}^b = 1$ and $\sum_{i=1}^{m} v_i x_{io} = \text{PEQE}^*$ are intended to maintain the process environmental quality efficiency of the DMU under evaluation equal to PEQE^* in Model (4). Model (11) can equivalently be written in the following linear from:

[1] Lim and Zhu [18] showed how to adopt [17] model to solve [8] two-stage structured model.

[2] Consider for example a non-cooperative game between a supplier and manufacturer where the manufacturer plays the main role in defining price and order quantity to reach its maximum profit. In modeling this non-cooperative supply chain, the manufacturer is treated as the leader and the supplier as a follower.

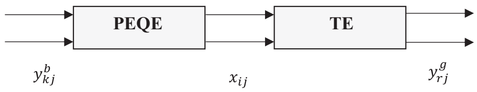

Fig. 1. Two-stage modeling of undesirable outputs.

$$
\begin{aligned}
Max \quad & \mathrm{TE}^* = \frac{\left(\sum_{r=1}^{s} u_r y^g_{ro}\right)}{\mathrm{PEQE}^*}, \\
s.t. \quad & \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y^b_{kj} \leqslant 0, \quad j = 1, \ldots n, \\
& \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{k=1}^{p} \mu_k y^b_{kj} \leqslant 0, \\
& \sum_{k=1}^{p} \mu_k y^b_{ko} = 1, \\
& \sum_{i=1}^{m} v_i x_{io} = \mathrm{PEQE}^*, \\
& v_i \geq 0, \qquad r = 1, 2, \ldots, s, \\
& u_r \geq 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned}
\tag{12}
$$

The outcome of Model (12) is the technical efficiency score of the DMU under evaluation. Ecological efficiency of the DMU can be calculated in either of the ways $\mathrm{ECE}^* = \mathrm{TE}^* \times \mathrm{PEQE}^*$ or $\mathrm{ECE}^* = \frac{\sum_{r=1}^{s} u^*_r y^g_{ro}}{\sum_{k=1}^{p} \mu^*_k y^b_{ko}}$ since: $\mathrm{ECE}^* = \frac{\sum_{r=1}^{s} u^*_r y^g_{ro}}{\sum_{i=1}^{m} v^*_i x_{io}} \times \frac{\sum_{i=1}^{m} v^*_i x_{io}}{\sum_{k=1}^{p} \mu^*_k y^b_{ko}} = \frac{\sum_{r=1}^{s} u^*_r y^g_{ro}}{\sum_{k=1}^{p} \mu^*_k y^b_{ko}}$. In addition, similar to the non-linear model, $\mathrm{OE}^* = 1/3(\mathrm{TE}^* + \mathrm{PEQE}^* + \mathrm{ECE}^*)$.

If the technical efficiency is assumed to be more important and the leader, is first calculated by Model (1) and the process environmental quality efficiency is then calculated by the following model:

$$
\begin{aligned}
Max \quad & \mathrm{PEQE}^* = \frac{1}{\sum_{k=1}^{p} \mu_k y^b_{ko}}, \\
s.t. \quad & \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y^b_{kj} \leqslant 0, \quad j = 1, \ldots, n, \\
& \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{k=1}^{p} \mu_k y^b_{kj} \leqslant 0, \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y^g_{ro} = \mathrm{TE}^*, \\
& v_i \geq 0, \qquad r = 1, 2, \ldots, s, \\
& u_r \geq 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned}
\tag{13}
$$

The ecological and overall efficiency scores are also calculated in a similar manner when the process environmental quality efficiency is the leader.

An alternative approach to the non-cooperative game approach of modeling undesirable outputs is measuring technical, process environmental quality and ecological efficiencies from a cooperative game perspective or centralized perspective, as [16] called it. In the cooperative game approach, optimal weights of the inputs (as intermediate factors) are determined in a way to maximize the aggregate efficiency score and the efficiency of both stages (technical and process environmental quality efficiencies) are evaluated simultaneously. In the cooperative game approach, the objective function is similar to the non-linear model, but instead of maximizing the arithmetic mean of the two objectives the geometric mean of them is maximized. Since $\frac{\sum_{r=1}^{s} u_r y^g_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \times \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y^b_{ko}} = \frac{\sum_{r=1}^{s} u_r y^g_{ro}}{\sum_{k=1}^{p} \mu_k y^b_{ko}}$, the cooperative game approach will maximize $\frac{\sum_{r=1}^{s} u_r y^g_{ro}}{\sum_{k=1}^{p} \mu_k y^b_{ko}}$ instead of maximizing

$\frac{\sum_{r=1}^{s} u_r y_{ro}^g}{\sum_{i=1}^{m} v_i x_{io}} + \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{k=1}^{p} \mu_k y_{ko}^b}$. In other words, the cooperative model will measure ecological efficiency of the DMU under evaluation first and then the optimal weights of the inputs and outputs will be used to calculate technical and process environmental quality efficiencies. Model (14) is the cooperative model.

$$\begin{aligned}
&Max \quad \text{Cooperative efficiency} = \text{EE}^* = \frac{\sum_{r=1}^{s} u_r y_{ro}^g}{\sum_{k=1}^{p} \mu_k y_{ko}^b}, \\
&s.t. \quad \text{TE} \leqslant 1, \\
&\qquad \text{PEQE} \leqslant 1, \\
&\qquad \text{EE} \leqslant 1, \\
&\qquad v_i \geq 0, \qquad r = 1, 2, \ldots, s, \\
&\qquad u_r \geq 0, \qquad i = 1, 2, \ldots, m, \\
&\qquad \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{14}$$

Model (14) in its linear formulation can be written as follows:

$$\begin{aligned}
&Max \quad \text{Cooperative efficiency} = \text{ECE}^* = \sum_{r=1}^{s} u_r y_{ro}^g, \\
&s.t. \quad \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \qquad j = 1, \ldots, n, \\
&\qquad \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, \qquad j = 1, \ldots, n, \\
&\qquad \sum_{r=1}^{s} u_r y_{rj}^g - \sum_{k=1}^{p} \mu_k y_{kj}^b \leqslant 0, \qquad j = 1, \ldots, n, \\
&\qquad \sum_{k=1}^{p} \mu_k y_{ko}^b = 1, \\
&\qquad v_i \geq 0, \qquad r = 1, 2, \ldots, s, \\
&\qquad u_r \geq 0, \qquad i = 1, 2, \ldots, m, \\
&\qquad \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{15}$$

2.4. A CSWs approach for the developed cooperative and non-cooperative model

The models developed in the previous section are run n times to find the most optimal weights that maximize the technical, process environmental quality, ecological and overall efficiency scores of each DMU. The separate runs of the models lets each DMU to freely choose its own optimal weights which sometimes may lead to unrealistically high efficiency scores for some DMUs. The problem is more severe when the freedom of this weighting system causes a lack of discrimination among efficient DMUs. Moreover, sometimes it is not acceptable for a factor to assume different weights among different DMUs. In order to find the CSWs for all DMUs, we suggest considering all the DMUs as a subset of a main entity with a goal of maximizing its efficiency score. Therefore, there will be a main entity with n DMUs. The main entity is a virtual DMU which is made by aggregating n DMUs. To find three sets of technical, process environmental quality and ecological efficiencies, i.e. (i) when process environmental quality efficiency is the leader and the technical efficiency the follower, (ii) when technical efficiency is the leader and the process environmental quality efficiency the follower and (iii) when the technical, process environmental quality and ecological efficiencies are determined by a cooperative game approach) all the models introduced in previous section are modified to measure the efficiency scores of the aggregated virtual DMU. The optimal weights of the inputs and outputs of the aggregated virtual DMU are then used as CSW to measure efficiency scores of the n DMUs. By using these common set of weights, efficiency scores of n DMUs are found in one step, discrimination between DMUs will increase and all the DMUs will be evaluated on a common base (weights).

A similar approach is proposed by Chen [5], where the CCR model is modified to consider the aggregated virtual DMU and to find CSW. We list all the modified models below to measure the efficiency scores of the aggregated virtual DMU and to find a set of CSW for n DMUs.

Models (16) and (17) are developed to measure the process environmental quality and technical efficiencies of the aggregated virtual DMU when the process environmental quality efficiency is the leader. Subscript av represents the aggregated virtual DMU. The inputs, desirable and undesirable outputs of the aggregated virtual DMU are made by the summation of the inputs, desirable and undesirable outputs of all DMUs.

$$
\begin{aligned}
Max \quad & \mathrm{PEQE}^{*}_{av} = \sum_{i=1}^{m} v_i x_{iav}, \\
s.t. \quad & \sum_{k=1}^{p} \mu_k y^{b}_{kav} = 1, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y^{b}_{kj} \leqslant 0, \qquad j = 1, \ldots, n, \\
& \sum_{i=1}^{m} v_i x_{iav} - \sum_{k=1}^{p} \mu_k y^{b}_{kav} \leqslant 0, \\
& v_i \geq 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, \qquad k = 1, 2, \ldots, p,
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
Max \quad & \mathrm{TE}^{*}_{av} = \frac{\left(\sum_{r=1}^{s} u_r y^{g}_{rav}\right)}{\mathrm{PEQE}^{*}_{av}}, \\
s.t. \quad & \sum_{r=1}^{s} u_r y^{g}_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \\
& \sum_{r=1}^{s} u_r y^{g}_{rav} - \sum_{i=1}^{m} v_i x_{iav} \leqslant 0, \\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y^{b}_{kj} \leqslant 0, \\
& \sum_{i=1}^{m} v_i x_{iav} - \sum_{k=1}^{p} \mu_k y^{b}_{kav} \leqslant 0, \\
& \sum_{r=1}^{s} u_r y^{g}_{rj} - \sum_{k=1}^{p} \mu_k y^{b}_{kj} \leqslant 0, \\
& \sum_{r=1}^{s} u_r y^{g}_{rav} - \sum_{k=1}^{p} \mu_k y^{b}_{kav} \leqslant 0, \\
& \sum_{k=1}^{p} \mu_k y^{b}_{kav} = 1, \\
& \sum_{i=1}^{m} v_i x_{iav} = \mathrm{PEQE}^{*}_{av}, \\
& v_i \geq 0, \\
& u_r \geq 0, \qquad i = 1, 2, \ldots, m, \\
& \mu_k \geq 0, \qquad k = 1, 2, \ldots, p.
\end{aligned} \tag{17}
$$

Models (18) and (19) measure the technical and process environmental quality efficiencies of the aggregated virtual DMU when the technical efficiency is the leader.

$$
\begin{aligned}
Max \quad & \mathrm{TE}^{*}_{av} = \sum_{r=1}^{s} u_r y^{g}_{rav}, \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{iav} = 1, \\
& \sum_{r=1}^{s} u_r y^{g}_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \\
& \sum_{r=1}^{s} u_r y^{g}_{rav} - \sum_{i=1}^{m} v_i x_{iav} \leqslant 0, \\
& v_i \geq 0, \qquad i = 1, 2, \ldots, m, \\
& u_r \geq 0, \qquad r = 1, 2, \ldots, s,
\end{aligned} \tag{18}
$$

Table 1
Data set for 30 provinces.

Area	Provinces	x_1 (100 Million m^3)	x_2 (100 Million Yuan)	x_3 (10,000 Units)	x_4 (10,000 Tons of SCE*)	y_1^g (100 Million Yuan)	y_1^b (10,000 Tons)	y_2^b (10,000 Tons)	y_3^b (10,000 Tons)	y_4^b (10,000 Tons)	y_5^b (10,000 Tons)
North	Beijing	23.08	5402.95	38.45	6954.05	14113.58	9.20	5.68	2.13	1.66	1269
	Tianjin	9.20	6278.09	16.85	6818.08	9224.46	13.20	21.76	5.38	0.80	1862
	Hebei	138.92	15083.35	34.58	27531.11	20394.26	54.61	99.42	32.26	32.09	31688
	Shanxi	91.55	6063.17	20.52	16808.03	9200.86	33.31	114.71	43.23	36.51	18270
	Inner Mongolia	388.54	8926.46	13.85	16820.30	11672.00	27.51	119.30	47.59	16.04	16996
Northeast	Liaoning	606.67	16043.03	36.90	20946.52	18457.27	54.16	85.94	39.79	16.73	17273
	Jilin	686.68	7870.38	11.80	8297.31	8667.58	35.22	30.06	20.96	5.30	4642
	Heilongjiang	853.48	6812.56	17.58	11233.51	10368.60	44.44	41.71	29.68	5.71	5405
East	Shanghai	36.81	5108.90	39.95	11201.13	17165.98	21.98	22.15	4.18	0.97	2448
	Jiangsu	383.53	23184.28	86.22	25773.71	41425.48	78.80	100.24	29.91	15.11	9064
	Zhejiang	1398.55	12376.04	68.55	16865.29	27722.31	48.68	65.39	16.54	13.92	4268
	Anhui	922.82	11542.94	24.26	9706.60	12359.33	41.11	48.39	20.74	26.37	9158
	Fujian	1652.71	8199.12	30.78	9808.52	14737.12	37.26	39.12	10.00	14.01	7487
	Jiangxi	2275.49	8772.27	18.46	6354.88	9451.26	43.11	47.10	13.90	22.36	9407
	Shandong	309.12	23280.52	77.36	34807.77	39169.92	62.05	138.29	29.12	18.94	16038
South	Henan	534.89	16585.86	40.07	21437.76	23092.36	61.97	116.29	47.37	22.70	10714
	Hubei	1268.72	10262.70	34.58	15137.59	15967.61	57.23	51.60	14.51	14.65	6813
	Hunan	1906.61	9663.58	29.72	14880.06	16037.96	79.81	62.74	23.50	39.44	5773
	Guangdong	1998.79	15623.70	80.28	26908.02	46013.06	85.84	98.91	25.33	10.43	5456
	Guangxi	1823.57	7057.56	20.50	7918.97	9569.85	93.69	84.80	25.01	31.77	6232
	Hainan	479.82	1317.04	3.76	1358.51	2064.50	9.23	2.82	0.65	0.66	212
Southwest	Chongqing	464.30	6688.91	17.79	7855.52	7925.58	23.45	57.27	10.21	8.36	2837
	Sichuan	2575.29	13116.72	35.30	17891.83	17185.48	74.08	93.76	25.97	14.14	11239
	Guizhou	956.54	3104.92	11.03	8175.43	4602.16	20.79	63.78	11.32	8.64	8188
	Yunnan	1941.45	5528.71	17.09	8674.17	7224.18	26.83	43.96	8.92	9.18	9392
Northwest	Shaanxi	507.50	7963.67	20.67	8882.11	10123.48	30.77	70.70	11.50	18.55	6892
	Gansu	215.25	3158.34	10.84	5923.13	4120.75	16.76	45.25	9.81	9.26	3745
	Qinghai	741.11	1016.87	2.83	2568.26	1350.43	8.31	13.31	5.19	9.75	1783
	Ningxia	9.32	1444.16	3.60	3681.10	1689.65	12.17	28.04	13.62	6.09	2465
	Xinjiang	1113.14	3423.24	9.67	8290.20	5437.47	29.60	51.84	24.84	18.46	3914

* Standard coal equivalent.

Table 2
Efficiency scores obtained from non-linear Model (6).

Provinces	TE	PEQE	EE	OE
Beijing	1.000	1.000	1.000	1.000
Tianjin	1.000	0.916	0.916	0.944
Hebei	0.984	0.209	0.206	0.467
Shanxi	0.842	0.181	0.152	0.392
Inner Mongolia	0.933	0.260	0.242	0.478
Liaoning	0.748	0.260	0.195	0.401
Jilin	0.937	0.166	0.155	0.419
Heilongjiang	0.856	0.161	0.137	0.385
Shanghai	1.000	1.000	1.000	1.000
Jiangsu	1.000	0.404	0.404	0.603
Zhejiang	0.891	0.656	0.584	0.710
Anhui	0.835	0.178	0.148	0.387
Fujian	0.869	0.297	0.258	0.474
Jiangxi	0.875	0.163	0.143	0.394
Shandong	0.873	0.471	0.411	0.585
Henan	0.899	0.237	0.213	0.449
Hubei	0.662	0.314	0.208	0.395
Hunan	0.808	0.234	0.189	0.410
Guangdong	1.000	0.758	0.758	0.839
Guangxi	0.775	0.135	0.105	0.338
Hainan	0.876	1.000	0.876	0.917
Chongqing	0.666	0.372	0.248	0.429
Sichuan	0.725	0.158	0.114	0.332
Guizhou	0.667	0.147	0.098	0.304
Yunnan	0.629	0.211	0.133	0.324
Shaanxi	0.749	0.286	0.214	0.417
Gansu	0.627	0.215	0.135	0.326
Qinghai	0.710	0.101	0.072	0.294
Ningxia	0.818	0.094	0.077	0.329
Xinjiang	0.841	0.100	0.084	0.342
Mean efficiency	0.837	0.356	0.316	0.503

$$\begin{aligned}
Max\quad & \mathrm{PEQE}^*_{av} = \frac{1}{\sum_{k=1}^{p}\mu_k y^b_{kav}},\\
& \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p}\mu_k y^b_{kj} \leqslant 0,\\
& \sum_{i=1}^{m} v_i x_{iav} - \sum_{k=1}^{p}\mu_k y^b_{kav} \leqslant 0,\\
& \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0,\\
& \sum_{r=1}^{s} u_r y^g_{rav} - \sum_{i=1}^{m} v_i x_{iav} \leqslant 0,\\
& \sum_{r=1}^{s} u_r y^g_{rj} - \sum_{k=1}^{p}\mu_k y^b_{kj} \leqslant 0,\\
& \sum_{r=1}^{s} u_r y^g_{rav} - \sum_{k=1}^{p}\mu_k y^b_{kav} \leqslant 0,\\
& sum^m_{i=1} v_i x_{iav} = 1,\\
& \sum_{r=1}^{s} u_r y^g_{rav} = \mathrm{TE}^*_{av},\\
& v_i \geq 0, \qquad r = 1,2,\ldots,s,\\
& u_r \geq 0, \qquad i = 1,2,\ldots,m,\\
& \mu_k \geq 0, \qquad k = 1,2,\ldots,p.
\end{aligned} \tag{19}$$

Model (20) measures technical, process environmental quality and ecological efficiencies of the aggregated virtual DMU with a cooperative game approach.

$$
\begin{aligned}
&\textit{Max} \quad \text{Cooperative efficiency} = \mathrm{ECE}^{*}_{av} = \sum_{r=1}^{s} u_r y^{g}_{rav}, \\
&\text{s.t.} \quad \sum_{r=1}^{s} u_r y^{g}_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, && j = 1,\ldots,n, \\
&\qquad \sum_{r=1}^{s} u_r y^{g}_{rav} - \sum_{i=1}^{m} v_i x_{iav} \leqslant 0, \\
&\qquad \sum_{i=1}^{m} v_i x_{ij} - \sum_{k=1}^{p} \mu_k y^{b}_{kj} \leqslant 0, && j = 1,\ldots,n, \\
&\qquad \sum_{i=1}^{m} v_i x_{iav} - \sum_{k=1}^{p} \mu_k y^{b}_{kav} \leqslant 0, \\
&\qquad \sum_{r=1}^{s} u_r y^{g}_{rj} - \sum_{k=1}^{p} \mu_k y^{b}_{kj} \leqslant 0, \\
&\qquad \sum_{r=1}^{s} u_r y^{g}_{rav} - \sum_{k=1}^{p} \mu_k y^{b}_{kav} \leqslant 0, \\
&\qquad \sum_{k=1}^{p} \mu_k y^{b}_{kav} = 1, \\
&\qquad v_i \geqslant 0, && r = 1,2,\ldots,s, \\
&\qquad u_r \geqslant 0, && i = 1,2,\ldots,m, \\
&\qquad \mu_k \geqslant 0, && k = 1,2,\ldots,p.
\end{aligned}
\tag{20}
$$

In the next section, the applicability of the developed models is tested by a case study.

Table 3
Efficiency scores obtained from the cooperative and non-cooperative game approaches.

Provinces	PEQE as the leader				TE as the leader				Cooperative			
	TE	PEQE	ECE	OE	TE	PEQE	ECE	OE	TE	PEQE	ECE	OE
Beijing	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Tianjin	0.916	1.000	0.916	0.944	1.000	0.916	0.916	0.944	0.916	1.000	0.916	0.944
Hebei	0.362	0.673*	0.243	0.426	1.000	0.176	0.176	0.451	0.437	0.557	0.243	0.413
Shanxi	0.267	0.675	0.180	0.374	0.842	0.181	0.152	0.392	0.534	0.337	0.180	0.350
Inner Mongolia	0.328	0.666	0.218	0.404	1.000	0.242	0.242	0.495	0.535	0.517	0.277	0.443
Liaoning	0.295	0.594	0.175	0.355	0.748	0.260	0.195	0.401	0.378	0.587	0.222	0.396
Jilin	0.211	0.627	0.132	0.324	0.937	0.149	0.140	0.409	0.592	0.311	0.184	0.362
Heilongjiang	0.274	0.609	0.167	0.350	0.856	0.161	0.137	0.385	0.549	0.361	0.198	0.369
Shanghai	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Jiangsu	0.532	0.590	0.314	0.479	1.000	0.404	0.404	0.603	0.921	0.446	0.411	0.593
Zhejiang	0.521	0.833	0.434	0.596	0.891	0.656	0.584	0.710	0.887	0.658	0.584	0.710
Anhui	0.232	0.668	0.155	0.351	0.835	0.178	0.148	0.387	0.468	0.419	0.196	0.361
Fujian	0.232	0.834	0.193	0.419	0.869	0.297	0.258	0.474	0.399	0.646	0.258	0.434
Jiangxi	0.114	0.885	0.101	0.367	0.885	0.122	0.108	0.372	0.262	0.546	0.143	0.317
Shandong	0.539	0.763	0.411	0.571	0.951	0.366	0.348	0.555	0.831	0.495	0.411	0.579
Henan	0.367	0.523	0.192	0.360	0.899	0.237	0.213	0.449	0.830	0.293	0.243	0.455
Hubei	0.297	0.605	0.180	0.361	0.731	0.218	0.160	0.370	0.651	0.324	0.211	0.395
Hunan	0.284	0.568	0.162	0.338	0.828	0.205	0.170	0.401	0.677	0.369	0.250	0.432
Guangdong	0.843	0.866	0.729	0.813	1.000	0.758	0.758	0.839	1.000	0.758	0.758	0.839
Guangxi	0.160	0.381	0.061	0.201	0.775	0.135	0.105	0.338	0.716	0.193	0.138	0.349
Hainan	0.876	1.000	0.876	0.917	0.934	0.710	0.663	0.769	0.876	1.000	0.876	0.917
Chongqing	0.256	0.704	0.180	0.380	0.703	0.271	0.190	0.388	0.625	0.402	0.251	0.426
Sichuan	0.205	0.722	0.148	0.359	0.725	0.158	0.114	0.332	0.300	0.504	0.151	0.318
Guizhou	0.161	0.864	0.139	0.388	0.667	0.147	0.098	0.304	0.210	0.688	0.144	0.347
Yunnan	0.174	1.000	0.174	0.449	0.650	0.183	0.119	0.317	0.194	0.906	0.176	0.425
Shaanxi	0.293	0.578	0.169	0.347	0.784	0.237	0.186	0.402	0.544	0.394	0.214	0.384
Gansu	0.294	0.549	0.161	0.335	0.627	0.217	0.136	0.327	0.411	0.390	0.161	0.321
Qinghai	0.106	1.000	0.106	0.404	0.710	0.101	0.072	0.294	0.106	1.000	0.106	0.404
Ningxia	0.226	0.400	0.091	0.239	0.822	0.080	0.065	0.322	0.490	0.185	0.091	0.255
Xinjiang	0.169	0.720	0.122	0.337	0.841	0.101	0.085	0.342	0.442	0.283	0.125	0.283
Mean efficiency	0.384	0.730	0.304	0.473	0.850	0.329	0.298	0.492	0.593	0.552	0.337	0.494

* Please note that, when the PEQE* score of DMU#3 (0.673) from the first stage is found and used with 6 decimal places in the second stage, the second stage TE model faces an infeasible solution. To avoid this problem, we used optimal efficiency scores found from the first stage with 7 decimal places in the second stage models.

3. Case study

In this section, we demonstrate the applicability of all the linear and non-linear models through a real-life case study of overall efficiency measurement for 30 provinces in China similar to studies by Zhang et al. [25], Bian and Yang [2] and Wang et al. [23]. Zhang et al. [25] divided China's provinces and municipalities into six areas (i.e. north, northeast, east, south, southwest, and northwest) as presented in Table 1.

Zhang et al. [25] used a three-step process to find an overall efficiency for each of the 30 provinces in these six regions. They first used the CCR model with inputs and desirable outputs and measured the technical efficiency or as they referred to, the resource efficiency. They then ignored the inputs and solved the model once again by considering the undesirable outputs as inputs. They referred to the resulting efficiencies as environmental efficiencies. In the last step of the process, they considered the inputs, undesirable outputs, and desirable outputs to measure an eco-efficiency for each city. The input and output factors used in our study are similar to the input and output factors used by Zhang et al. [25].

We used the following four input factors in our study: water resource (x_1), fixed assets (x_2), the number of entities (x_3), and energy (x_4). The gross regional product (y_1^g) is the only desirable output while chemical oxygen demand (COD) (y_1^b), sulphur dioxide emission (SO_2) (y_2^b), soot (y_3^b), dust (y_4^b), and solid waste (y_5^b) are considered as undesirable outputs. Table 1 shows the data set used in our case study which are all collected from [7,6]. The technical, process environmental quality, ecological and overall efficiency scores of the provinces presented in Table 1 were first measured by using the proposed non-linear Model (6). The model is run separately for each DMU and the purpose of each separate run is to find the most optimal weights that maximize different efficiency scores for each particular province. The results are shown in Table 2. Beijing and Shanghai with an overall efficiency score of 1 were identified as the best DMUs.

Model (6) allowed us to study the efficiency of the provinces from different perspectives. At this stage, the developed cooperative and non-cooperative models are tested on the same data set and the results are shown in Table 3. In all the three scenarios reported in Table 3 (process environmental quality efficiency as the leader, technical efficiency as the leader and cooperative model) the same provinces, i.e. Beijing and Shanghai got the full overall efficiency score of 1, therefore, recognizing them as efficient provinces can be validated.

Based on the results reported in Tables 2 and 3, two important features of the non-linear model can be deduced; (i) the non-linear model was able to correctly construct the overall efficiency frontier, since it recognized the same efficient provinces as the linear models; and (ii) the ranking results of the provinces by the non-linear model is significantly correlated by the non-cooperative model when the second stage (technical efficiency) is considered as the leader (the correlation

Table 4
Efficiency scores obtained from the cooperative and non-cooperative game approaches with CSW.

Provinces	PEQE as the leader				TE as the leader				Centralized			
	TE	PEQE	ECE	OE	TE	PEQE	ECE	OE	PEQE	TE	ECE	OE
Beijing	1.000	1.000	1.000	1.000	0.757	1.000	0.757	0.838	1.000	1.000	1.000	1.000
Tianjin	0.679	0.703	0.477	0.620	0.893	0.386	0.345	0.541	0.750	0.607	0.456	0.604
Hebei	0.359	0.408	0.147	0.305	0.705	0.262	0.184	0.383	0.557	0.437	0.243	0.413
Shanxi	0.264	0.371	0.098	0.245	0.527	0.259	0.136	0.307	0.464	0.388	0.180	0.344
Inner Mongolia	0.289	0.563	0.163	0.338	0.783	0.268	0.209	0.420	0.754	0.367	0.277	0.466
Liaoning	0.351	0.505	0.177	0.344	0.708	0.237	0.168	0.371	0.587	0.378	0.222	0.396
Jilin	0.296	0.552	0.164	0.337	0.937	0.130	0.121	0.396	0.503	0.319	0.160	0.327
Heilongjiang	0.272	0.583	0.158	0.338	0.789	0.146	0.115	0.350	0.439	0.347	0.152	0.312
Shanghai	0.755	0.737	0.557	0.683	0.793	0.486	0.385	0.555	0.509	1.000	0.509	0.673
Jiangsu	0.715	0.489	0.350	0.518	0.870	0.298	0.259	0.476	0.504	0.680	0.343	0.509
Zhejiang	0.466	0.808	0.376	0.550	0.775	0.363	0.281	0.473	0.665	0.558	0.371	0.532
Anhui	0.339	0.451	0.153	0.314	0.835	0.178	0.148	0.387	0.585	0.335	0.196	0.372
Fujian	0.292	0.767	0.224	0.428	0.849	0.230	0.195	0.425	0.714	0.361	0.258	0.444
Jiangxi	0.171	0.682	0.116	0.323	0.885	0.122	0.108	0.372	0.706	0.202	0.143	0.350
Shandong	0.527	0.658	0.347	0.511	0.793	0.393	0.312	0.499	0.706	0.583	0.411	0.567
Henan	0.442	0.493	0.218	0.384	0.838	0.219	0.184	0.414	0.517	0.470	0.243	0.410
Hubei	0.297	0.605	0.180	0.361	0.731	0.188	0.138	0.352	0.495	0.367	0.182	0.348
Hunan	0.246	0.505	0.124	0.292	0.808	0.123	0.099	0.343	0.410	0.319	0.131	0.287
Guangdong	0.508	0.770	0.391	0.557	1.000	0.265	0.265	0.510	0.532	0.657	0.349	0.513
Guangxi	0.192	0.353	0.068	0.204	0.775	0.065	0.050	0.297	0.276	0.241	0.067	0.195
Hainan	0.176	1.000	0.176	0.451	0.934	0.118	0.110	0.387	0.638	0.229	0.146	0.337
Chongqing	0.328	0.639	0.209	0.392	0.703	0.237	0.167	0.369	0.619	0.356	0.220	0.398
Sichuan	0.205	0.722	0.148	0.359	0.725	0.158	0.114	0.332	0.582	0.260	0.151	0.331
Guizhou	0.135	0.774	0.104	0.338	0.518	0.211	0.109	0.279	0.723	0.200	0.144	0.356
Yunnan	0.135	1.000	0.135	0.423	0.629	0.211	0.133	0.324	0.956	0.184	0.176	0.438
Shaanxi	0.375	0.451	0.169	0.332	0.781	0.208	0.162	0.384	0.542	0.396	0.214	0.384
Gansu	0.262	0.490	0.128	0.293	0.548	0.222	0.121	0.297	0.475	0.338	0.160	0.324
Qinghai	0.071	1.000	0.071	0.381	0.530	0.151	0.080	0.254	1.000	0.106	0.106	0.404
Ningxia	0.228	0.330	0.075	0.211	0.486	0.141	0.069	0.232	0.281	0.322	0.091	0.231
Xinjiang	0.146	0.699	0.102	0.316	0.645	0.141	0.091	0.292	0.563	0.213	0.120	0.298

coefficient between its technical, process environmental quality, ecological and overall efficiency ranking results and those of the linear model when technical efficiency is the leader, at significant level of 0.01, is 0.98).

As the last analysis, the developed cooperative and non-cooperative models with CSW are run and the results are shown in Table 4. These models are developed and applied to avoid unrealistically high efficiency scores, increase the discrimination power among the provinces and to carry out the efficiency analysis of the provinces based on a common base. Unlike the non-linear model and the cooperative and non-cooperative models, the weights determined for the input and output factors are not the best possible weights for each particular province (DMU_o) but for all the provinces (DMU_j). The developed cooperative and non-cooperative models with CSW provide a complete ranking among all provinces and based on all the scenarios, Beijing is recognized as the most efficient province with the highest overall efficiency score.

4. Conclusions and future research directions

In DEA, there are a number of producers known as DMUs. The production process for each DMU involves using a set of inputs to produce a set of outputs. Each producer has varying levels of inputs and generates varying levels of outputs. DEA assumes that either making more output with the same input or making the same output with less input is a criterion of efficiency. In the presence of undesirable outputs, DMUs with better (more desirable) outputs and worse (less undesirable) outputs (relative to less input resources) should be recognized as efficient. For example, if there are inefficiencies in the production process, the outputs of wastes and pollutants (which are undesirable) should be reduced to improve the performance [19,15].

We considered the concepts of technical efficiency and ecological efficiency in performance measurement problems. We also introduced the concept of process environmental quality efficiency and proposed several new linear and non-linear models to integrate these three different efficiency scores into an overall efficiency score. We presented a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed models.

The model proposed in this study is intended to find the best possible input and output weights for maximizing the efficiency of the entire performance evaluation system. It is often important to seek experts' opinions on the importance of the factors and incorporate these opinions as weight restrictions in the model. These restrictions will limit the feasible region of the weights and lead to an integration of the objective and subjective weighting systems. Also, in this paper, we studied undesirable outputs in the performance evaluation systems. We are currently studying the effects of desirable inputs in the model proposed in this study. We also plan to study the overall simultaneous effects of both undesirable outputs and desirable inputs in our model.

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