



## An optimal information acquisition model for competitive advantage in complex multiperspective environments



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### ARTICLE INFO

#### Keywords:

Multicriteria decision making  
Information acquisition  
Decision reversals  
Risk  
Linguistic reports  
Triangular fuzzy numbers

### ABSTRACT

The optimal information acquisition process is a major strategic task for sustaining a firm's competitive advantage. We define the optimal sequential information acquisition behavior of a rational decision maker (DM) when allowed to acquire two pieces of information from and observe positive credible signals on a set of multidimensional products. We illustrate how firm reputation affects the continuity of the expected utilities derived from a given search and may generate reversals in the information acquisition incentives of DMs when deciding whether or not to shift their search processes between different signal-induced markets. This study makes a number of important contributions to our understanding of a firm's information acquisition. First, it provides a formal analysis of the information acquisition process when the characteristics defining a product have a continuous set of variants. Second, it allows for the study of risk-averse DMs, while most of the literature concentrates on risk-neutral DMs. Third, it opens the way for strategic scenarios to be considered when analyzing the information acquisition processes of firms and creates a direct link to the game theoretical literature on strategic reporting. Fourth, it can be easily implemented within multicriteria decision making methods such as the analytic hierarchy process (AHP) to study the information acquisition behavior of DMs when the characteristics of the products are unknown.

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### 1. Introduction

"The fox knows many things, but the hedgehog knows one big thing," the Greek poet Archilochus once said. Perhaps Archilochus simply meant that the hedgehog's single defense defeats the fox's many tricks. Yet, the hedgehog and the fox were turned into metaphors by many thinkers and writers. We argue that, despite the increasing amount of information available nowadays, decision makers (DMs) within a firm should behave more like the hedgehog – that is, base their acquisition of

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information and subsequent decisions on a small but highly relevant amount of information. Formal models generally rely on *ad hoc* heuristic rules to simplify the information acquisition process of DMs within increasingly complex multidimensional environments, but do not consider the *seemingly* straightforward setting where little relevant information determines the behavior of DMs. In other words, foxes constitute the main subject of analysis nowadays while the more focused, though deemed simpler, hedgehog remains outside the scope of the literature.

We propose an information acquisition algorithm that intersects three different research lines linked by their respective analyses of the optimal information acquisition processes of rational DMs. In particular, it relates to the consumer choice literature, the economic one dealing with fads and herding phenomena, and the decision theoretical branch of operations research dedicated to study the optimal choice of technology by firm managers.

### 1.1. Literature review

Consider the problem faced by a rational DM regarding what information to gather given a limited capacity to do so. The consumer choice literature studies this problem mainly from a psychological perspective. In particular, this research line focuses on how the information given to DMs can be strategically designed in a way that some predetermined options appear more attractive than others (see e.g. [1,20,49]). These strategic considerations, together with the limited cognitive ability of DMs to assimilate information, allow for choice modifications to be induced through their information acquisition processes.

The previous research line provides the empirical counterpart to the search theoretical economic models that analyze fads and herds as rational phenomena, following the seminal works of Banerjee [4] and Bikhchandani et al. [7]. These models deal with the influence that signals and the observable choices made by other DMs have on the optimal (sequential) behavior of the remaining DMs. However, this stream of research does not study the influence that information transmission processes, and signals in particular, have on the optimal information *acquisition* behavior and subsequent choice structures of DMs, see [10] for a comprehensive review of the literature. It should be noted that the subject of information sharing and herding, and how it relates to decision making, has recently received ample attention in the game theoretical literature. Previous relevant work analyzing the impact of information sharing within a game theoretical environment is provided by Szolnoki and Perc [58]. In terms of collective phenomena, Chen et al. [14,15] have shown how decision making is affected by the fact that one is never alone in a crowd. Similarly, herding and the relevance of the wisdom of crowds have been studied in [30,57,59].

The design and analysis of algorithmic information acquisition processes remains outside the scope of the previous lines of research but within that of the operations research literature, which, at the same time, tends to overlook the strategic implications that different signaling and preference manipulation strategies have for the information acquisition and choice behavior of DMs.

Indeed, the management/operations research literature has been considering the optimal information acquisition and choice problems of firm managers for quite some time, in particular when analyzing the acquisition of a new technology. In this regard, the seminal models of McCardle [47] and Lippman and McCardle [40] limited their scope to return functions that were both convex increasing and continuous, a constraint removed by the most recent research models within this area, such as [63]. However, and despite the inclusion of Bayesian learning mechanisms into their algorithms, even the most recent models omit the strategic effects inherent to the information transmission process. This research line remains focused on the importance that search costs have on limiting the information processing capacity of generally risk-neutral DMs when determining the introduction or dismissal of a new technology (see e.g. [28,53]).

More precisely, the operational research/management theoretical literature on the demand for new technology can be classified as game theoretic, a research stream started by Reinganum [51], or decision theoretic, following Jensen [28]. Both approaches were initially developed within the economic literature but taken over by the operational research one, leading to two separate and clearly differentiated streams of research. The former approach concentrates on the strategic incentives implicit behind the adoption and *diffusion* of technology but does not deal with the information acquisition processes affecting technology adoption decisions, see [6]. At the same time, the decision theoretical branch of operations research has recently extended its scope to allow for comparisons between different technologies. In particular, Paulson Gjerde et al. [50], as well as Cho and McCardle [16], emphasize the cumulative and interdependent character of technological evolution and its effect on the corresponding adoption (or rejection) of new technologies. However, in both cases, the entire set of technological features is observable and its stochastic evolution defined by a known probability function. Therefore, the strategic effects resulting from signals received regarding the value of unknown technological characteristics as well as those derived from influencing the preferences of DMs remain unstudied.

### 1.2. Main results

The current paper formalizes and studies the optimal information acquisition behavior of a rational DM when acquiring information on and choosing among multidimensional products defined by vectors of characteristics. We analyze in detail the case where the decision process is based on the possibility of collecting two pieces of information before making a choice. This limit is imposed to account for existing information processing costs, either pecuniary or cognitive, and to allow for a simple numerical analysis illustrating the theoretical results obtained. Besides, the literature usually concentrates on a small

number of attributes when describing the products available to DMs, i.e. quality and preference in the consumer choice environment of [39], performance and cheapness in the economic setting of [44,45], and variety and quality in the operational research one of [8]. Bearden and Connolly [5] survey the literature on consumer choice, while Gaines [22] provides a review at the organizational level. In the current setting, the evolution of the information acquisition process will depend directly on the values of *all the characteristics observed previously*, which prevents the use of standard dynamic programming techniques in the design of the algorithm. We will show that the decision of how to allocate the second piece of information available depends on two well-defined real-valued functions. One of them describes the utility that the DM expects to derive from continuing acquiring information on the first product observed, while the other defines the expected utility obtained from starting checking the characteristics of a second product.

The intuition behind the basic postulates of the model may be easily interpreted from an economic viewpoint. In this sense, Malerba [43] highlights two key aspects of demand that are relevant for innovation in industries, namely, consumer behavior (including imperfect information with respect to new products and technologies, as well as inertia and habits concerning existing products and technologies) and consumer capabilities (absorptive capabilities and their distribution among consumers and users). Both these aspects will be considered when designing the sequential information acquisition process that defines the optimal behavior of (rational) DMs when comparing and choosing between competing technologies. In this regard, the utility-based decision theoretical perspective used to model the information acquisition and subsequent choice behavior of DMs should help complementing the agent-based evolutionary approach on which the current computational economic literature is based, see [13]. Moreover, the generality of the decision theoretical model presented allows for DMs to be interpreted as either firm managers or consumers, depending on the degree of sophistication that one wants to impose on them.

In addition, the model will be extended to account for the existence of publicly observable signals giving place to a standard Bayesian learning setting. Signals will be defined on the second characteristic space in order to separate the role played by observations from that played by expectations. The introduction of signals within the current multi-dimensional information acquisition framework allows us to account explicitly for the effects that different risk attitudes and types of signals have on the optimal information acquisition behavior of DMs.

We will illustrate how the willingness to search of DMs depends on their (highly malleable) degree of risk aversion and how it is influenced by the reception of signals defined on the distribution of unobserved characteristics. In doing so, we reach the same type of conclusion as Jensen [29], Cho and McCordle [16], and Ulu and Smith [63] regarding the effect that first order stochastic dominance expected utility improvements have on the incentives of DMs to adopt a new technology. That is, signaling the development of a more advanced technology does not necessarily lead to faster adoption. In other words, issuing a positive *credible* signal on the development of a technology leads to stricter continuation criteria in terms of the characteristic threshold value required to be observed in order to accept the more advanced technology.

Our basic initial approach to the information acquisition process of DMs imposes habits and consumption inertia implicitly, leading them to expect a minimum quality level or *certainty equivalent* product to be guaranteed from a random purchase. However, a more complex decision theoretical structure results if habits and consumption inertia are excluded from the information acquisition process of DMs and it is assumed that DMs do not purchase a product unless it's observed characteristics deliver a higher expected utility than those defining the reference certainty equivalent product. The intuition justifying this approach follows from the work of Christensen and Rosenbloom [18] on nested hierarchies and value networks. These authors consider products as systems comprised of components which relate to each other in a designed architecture. Specifically, Christensen and Rosenbloom [18] state that associated with each network is a unique rank-ordering of the importance of various performance attributes, i.e. two or three characteristics per product, that differs from those orderings employed in other value networks. Therefore, it could be assumed that the ability of an information sender [or a firm] to credibly guarantee DMs a given (quality level) certainty equivalent product depends on its position within a given network architecture or hierachic system, i.e. its reputation. In this sense, [reputation-based] quality expectations regarding newly introduced products and technologies will determine the incentives of DMs to shift their information acquisition (and choice) processes between different product markets.

We will indeed illustrate how being unable to guarantee a minimum certainty equivalent product, that is, lacking any reputation, decreases significantly the ability of firms (information senders) to introduce their products in a given market. This will be the case even if they signal the existence of technologically superior products and despite the credibility of the signal issued. Besides, the absence of a guaranteed certainty equivalent product may give place to information acquisition (and choice) reversals if DMs are allowed to reverse their information acquisition processes and compare the observations retrieved within different markets. This result follows from merging the comparability between competing technologies as well as the basic learning and memory capacities of DMs with the strategic elements derived from the economic and consumer choice literatures. The resulting demand based analysis can help complementing the supply side one usually undertaken in the operations research literature, where it is assumed that the stochastic product improvements that may result from a firm's (optimal) research and development decisions are immediately absorbed by the demand side of the market (see e.g. [21,36]). Moreover, by introducing basic information acquisition and decision structures within a risky environment when the characteristics defining a product have a continuous set of variants, the current analysis provides a complementary alternative to the models commonly used in the (utility-based) multi-attribute decision-making literature (see e.g. [33]).

Finally, it should be noted that the information acquisition and choice structure introduced in this paper can be elicited through linguistic evaluations of DMs using fuzzy numbers. This is worth mentioning since, as a result, a substantial amount of heterogeneous scenarios arises when considering the introduction of superior products by firms, which, as stated above, are supposed to be immediately assimilated by DMs according to the standard literature. Moreover, this elicitation property makes the resulting strategic environment compatible with multicriteria decision making methods such as the analytic hierarchy process (AHP). In this case, DMs must decide on which characteristic they acquire information, given the subjective weights assigned to each one of them. A similar remark applies to the selections of projects by managers when a limited amount of information is available. This is particularly the case during the initial stages of project development, where sophisticated evaluation models are generally unavailable to deal with a substantial degree of uncertainty (see [25,64]).

The remainder of the paper is organized in two differentiated parts as follows. The first part, composed by Sections 2–4, provides a formal analysis and numerical simulations describing the main characteristics of our information acquisition structure. In particular, Section 2 and its subsections define the basic information acquisition algorithm of DMs. Section 3 introduces signals and learning and analyzes formally their effect and that of reversibility on the optimal information acquisition behavior of DMs. Section 4 presents several numerical illustrations of the main formal results. The second part, composed by Section 5, describes how the information acquisition process can be implemented by managers using linguistic evaluations elicited from DMs. Section 6 concludes and suggests possible extensions.

## 2. The information acquisition algorithm'

### 2.1. Basic notations and main assumptions

The information acquisition algorithm described through the paper must be redefined after each observation is gathered by the DM and recalculated in terms of *all previously observed variables, their sets of possible combinations and corresponding expected payoffs*, which prevents the use of standard dynamic programming techniques, generally applied by the operations research literature within this type of decision theoretical environment, see [63]. This requisite is justified by the low dimensionality of the model and the memory capacity with which DMs are endowed when comparing products in basic information evaluation scenarios. While such an assumption may become prohibitive in larger dimensional settings, it is imposed here to account for basic *satisficing* capacity constraints, as those defined by Simon [54] within a fully rational environment. In this regard, a review of the literature describing how the complexity of decision support systems based on operational research models gives place to effort versus accuracy trade-offs among the DMs who utilize them is presented in [55].

Let  $X$  be a nonempty set and  $\succsim$  a *preference relation* defined on  $X$ , that is, a binary relation on  $X$  satisfying reflexivity ( $\forall x \in X, x \succsim x$ ), completeness ( $\forall x, y \in X$ , either  $x \succsim y$  or  $y \succsim x$ ) and transitivity ( $\forall x, y, z \in X, x \succsim y$  and  $y \succsim z$  imply  $x \succsim z$ ). A *utility function representing a preference relation*  $\succsim$  on  $X$  is a function  $u: X \rightarrow \mathbb{R}$  such that:

$$\forall x, y \in X, \quad x \succsim y \iff u(x) \geq u(y). \quad (1)$$

The symbol  $\geq$  denotes the standard partial order on the reals. When  $X \subseteq \mathbb{R}$  and  $\succsim$  coincides with  $\geq$ , we say that  $u$  is a *utility function* on  $X$ .

Let  $G$  denote the set of all products and fix a natural number  $n$ . For every  $i \leq n$ , with  $i$  natural number, let  $X_i$  represent the set of all possible variants for the  $i$ th characteristic of any product in  $G$  and  $X$  stand for the Cartesian product  $\prod_{i \leq n} X_i$ . Thus, every product in  $G$  is described by an  $n$ -tuple  $\langle x_1, \dots, x_n \rangle$  in  $X$ .  $X_i$  is called the  *$i$ th characteristic factor space*, while  $X$  stands for the *characteristic space*.

Henceforth, the parameter  $i$  will denote a natural number varying between 0 and  $n$ .

Following the classical approach to information demand by economic agents, see [67], we restrict our attention to the case where each  $X_i$  is identified with a compact and connected non-degenerate real subinterval of  $[0, +\infty)$ . The topology and the preference relation on each  $X_i$  are those induced by the standard Euclidean topology and the standard linear order  $>$ , respectively. Within this classical approach, we work under the following assumptions:

**Assumption 1.** For every  $i \leq n$ , there exist  $x_i^m, x_i^M > 0$ , with  $x_i^m \neq x_i^M$ , such that  $X_i = [x_i^m, x_i^M]$ , where  $x_i^m$  and  $x_i^M$  are the minimum and maximum of  $X_i$ .

**Assumption 2.** The characteristic space  $X$  is endowed with the product topology  $\tau_p$  and a strict preference relation  $\succ$ .

**Assumption 3.** There exist a continuous additive utility function  $u$  representing  $\succ$  on  $X$  such that each one of its components  $u_i: X_i \rightarrow \mathbb{R}$ , where  $i \leq n$ , is a continuous utility function on  $X_i$ . Let  $\succsim$  be a preference relation on  $\prod_{i \leq n} X_i$ . A utility function  $u: \prod_{i \leq n} X_i \rightarrow \mathbb{R}$  representing  $\succsim$  on  $\prod_{i \leq n} X_i$  is called *additive* [66] if there exist  $u_i: X_i \rightarrow \mathbb{R}$ , where  $i \leq n$ , such that  $\forall \langle x_1, \dots, x_n \rangle \in \prod_{i \leq n} X_i, u(\langle x_1, \dots, x_n \rangle) = u_1(x_1) + \dots + u_n(x_n)$ .

It should be noted that the information acquisition structure described through the paper can be easily incorporated into well-established multi criteria decision making methods such as the analytic hierarchy process (AHP), Saaty [52], and the technique for order preference by similarity to ideal solution (TOPSIS), Hwang and Yoon [27], to analyze situations where

the values of the product characteristics are unknown and the DM must select what information to acquire. In other words, the relative importance of the characteristics under consideration, i.e. the weights of the criteria, can be derived using AHP or TOPSIS and explicitly incorporated in the current information acquisition process, which will indicate on which product must the DM continue acquiring information. The additivity assumption imposed on the utilities allows for this combined information acquisition and decision structure to be directly applicable. Indeed, additivity is implicitly assumed within AHP and TOPSIS, since the characteristics of the products are defined separately and added after being assigned their relative weights. Moreover, the current model will allow us to define utility functions representing any degree of risk inherent in the preferences of DMs while accounting for the weights that indicate the relative importance of each attribute.

**Assumption 4.** For every  $i \leq n$ ,  $\mu_i: X_i \rightarrow [0, 1]$  is a continuous probability density on  $X_i$ , whose support, the set  $\{x_i \in X_i: \mu_i(x_i) \neq 0\}$ , will be denoted by  $Supp(\mu_i)$ .

The results introduced through the paper are derived for continuous  $\mu_1$  and  $\mu_2$  probability densities. The remaining cases, formally similar to the continuous one but providing less intuitive graphical presentations, are left to the reader. Refer to [19] for a formal and numerical analysis of the main properties defining both the continuous and discrete probability settings.

Moreover, we will consider, as the basic reference case and without loss of generality, the optimal information acquisition behavior of the DM when uniform probabilities are assumed on both  $X_1$  and  $X_2$ . Assuming a uniform probability, we maximize the information entropy (see [60]) implying that DMs face the highest possible level of uncertainty.

The probability densities  $\mu_1, \dots, \mu_n$  must be interpreted as the subjective “beliefs” of the DM. For  $i \leq n$ ,  $\mu_i(Y_i)$  is the subjective probability that a randomly observed product from  $G$  displays an element  $x_i \in Y_i \subseteq X_i$  as its  $i$ th characteristic. Kahneman and Tversky [32] provide an excellent introduction to the study of subjective expected utility and many of its variations developed by economists and psychologists. The probability densities  $\mu_1, \dots, \mu_n$  are assumed to be independent. However, the algorithm allows for subjective correlations to be defined among different characteristic within a given product.

Following the standard economic theory of choice under uncertainty, we assume that the DM elicits the  $i$ th certainty equivalent value induced by the subjective probability density  $\mu_i$  and the utility function  $u_i$  as the reference point against which to compare the information collected on the  $i$ th characteristic of a certain product.

Given  $i \leq n$ , the *certainty equivalent* of  $\mu_i$  and  $u_i$ , denoted by  $ce_i$ , is a characteristic in  $X_i$  that the DM is indifferent to accept in place of the expected one to be obtained through  $\mu_i$  and  $u_i$ . That is, for every  $i \leq n$ ,  $ce_i = u_i^{-1}(E_i)$ , where  $E_i$  denotes the expected value of  $u_i$ . The existence and uniqueness of the  $i$ th certainty equivalent value  $ce_i$  are guaranteed by the continuity and strict increasingness of  $u_i$ , respectively.

## 2.2. Expected search utilities

The set of all products,  $G$ , is identified with a compact and convex subset of the  $n$ -dimensional real space  $R^n$ . In the simplest non-trivial scenario,  $G$  consists of at least two products and the DM is allowed to collect two pieces of information, not necessarily from the same product. That is, once the value of the first characteristic from one of the products becomes known to the DM, she has to decide whether to check the second characteristic from the same product, or to check the first characteristic from a different product. Henceforth, we denote by  $J$  and  $K$  the two products that can be randomly checked by the DM.

We show below that the decision of how to allocate the second available piece of information depends on two real-valued functions defined on  $X_1$ . The DM considers the sum  $E_1 + E_2$ , corresponding to the expected utility values of the pairs  $\langle u_1, \mu_1 \rangle$  and  $\langle u_2, \mu_2 \rangle$ , as the main reference value when calculating both these functions.

Assume that the DM has already checked the first characteristic from product  $J$  and that she uses her remaining piece of information to observe the second characteristic from  $J$ . In this case, the expected utility gain over  $E_1 + E_2$  varies with the value  $x_1$  observed for the first characteristic. For every  $x_1 \in X_1$ , let

$$P^+(x_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \quad (2)$$

and

$$P^-(x_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}. \quad (3)$$

$P^+(x_1)$  and  $P^-(x_1)$  define the set of values for the second  $x_2$  characteristic from product  $J$  such that their combination with the observed first  $x_1$  characteristic delivers a respectively higher or lower-equal utility than a randomly chosen product from  $G$ .

Let  $F: X_1 \rightarrow R$  be defined by:

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2. \quad (4)$$

$F(x_1)$  describes the DM's expected utility derived from checking the second characteristic  $x_2$  of product  $J$  after observing that the value of the first characteristic is given by  $x_1$ .

Note that, if  $u_1(x_1) + u_2(x_2) \leq E_1 + E_2$ , then choosing a product from  $G$  randomly delivers an expected utility of  $E_1 + E_2$  to the DM, which is higher than the expected utility obtained from choosing product  $J$ , that is,  $u_1(x_1) + u_2(x_2)$ .

This basic initial approach to the information acquisition process of DMs assumes them to expect a minimum quality level or *certainty equivalent* product to be guaranteed from a random purchase within the set of products offered by a firm. The assumption relates directly to the reputation of the firm under consideration, i.e. the habits and consumption inertia defining its client base. Consequently, a null reputation level would lead DMs to expect a value of zero from a random purchase made from the set of products offered by an unknown firm, i.e. a newcomer. Note, however, that despite the formal postulates of expected utility theory, rational DMs are not obliged to choose randomly from a set of available products. That is, despite their expectations, DMs are aware of the fact that the certainty equivalent product is not necessarily obtained as the result of a random choice, thought it constitutes a useful reference point against which to compare firms. Therefore, it seems plausible to assume that if the information acquisition (search) process does not provide DMs with a product whose expected utility is higher than  $E_1 + E_2$ , then they refrain from making a random purchase from the corresponding set of available products. That is, quality verifiability is a precondition for purchase, despite the fact that the client base of a given firm may believe the signals it issues. This type of behavior is described by Christensen [17] when studying the demand-based evolution of the disk drive industry. We will return to these assumptions and their consequences when analyzing numerically the corresponding expected search utilities.

In order to simplify the presentation we will refer to the setting described above as the Guaranteed Certainty Equivalent Scenario (GCES).

**Definition.** The Guaranteed Certainty Equivalent Scenario (GCES) denotes any situation where DMs expect a minimum quality level or certainty equivalent product to be guaranteed from a random purchase within the set of products offered by a firm.

On the other hand, changes in the reputation level of firms would lead to different reference certainty equivalent products (to follow from a random purchase) when defining the corresponding expected search utilities of DMs. The lowest reference product expected gives place to the Refused Certainty Equivalent Scenario (RCES).

**Definition.** The Refused Certainty Equivalent Scenario (RCES) denotes any situation where DMs expect a utility of zero to follow from a random purchase within the set of products offered by a firm.

The  $F$ -function defined by DMs within the RCES simplifies to

$$F(x_1|rf) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2, \quad (5)$$

where the  $(P^+)$ -set is identical to the one defined within the GCES. Indeed,  $F(x_1|rf)$  is simply  $F(x_1)$  without the second right hand side expression.

Consider now the expected utility that the DM could gain over  $E_1 + E_2$  if the second available piece of information is employed to observe the first characteristic from product  $K$ . For every  $x_1 \in X_1$ , let

$$Q^+(x_1) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \quad (6)$$

and

$$Q^-(x_1) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}. \quad (7)$$

$Q^+(x_1)$  and  $Q^-(x_1)$  define the set of values for the first characteristic  $y_1$  from product  $K$  such that they deliver a respectively higher or lower-equal utility than the maximum between the observed first characteristic  $x_1$  from product  $J$  and a randomly chosen product from  $G$ .

Define  $H: X_1 \rightarrow \mathbb{R}$  as follows:

$$H(x_1) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)dy_1. \quad (8)$$

$H(x_1)$  describes the expected utility obtained from checking the first characteristic  $y_1$  of product  $K$  after having already observed the value of the first characteristic  $x_1$  from product  $J$ . If  $u_1(y_1) \leq \max\{u_1(x_1), E_1\}$ , then the DM must choose between  $J$  and a randomly chosen product from  $G$ , delivering an expected utility of  $E_1$ .

The corresponding  $H$ -function defined by DMs within the RCES is given by

$$H(x_1|rf) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)dy_1 + \int_{Q^-(x_1|rf)} \mu_1(y_1)(u_1(x_1) + E_2)dy_1 \quad (9)$$

with

$$Q^+(x_1) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \quad (10)$$

and

$$Q^-(x_1|rf) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\} \wedge x_1 \geq ce_1\}. \quad (11)$$

The second right hand side term of  $H(x_1|rf)$  together with  $Q^-(x_1|rf)$  illustrate the main differences with respect to the GCES. DMs will only account for the corresponding expected utility if  $x_1 \geq ce_1$ . That is, the expected utility derived from  $X_1$  realizations located below the certainty equivalent value equals zero.

Finally, note that the domain of all the  $F$  and  $H$  functions is the support of  $\mu_1$ .

Pointwise comparisons between the functions cannot be undertaken analytically due to the variety of possible domains and functional forms that may define their behavior. However, for a given set of probability densities and utilities, changes in the curvature of the  $F$ -function and the  $H$ -function (degree of risk aversion) and their effect on the optimal information acquisition behavior of DMs can be analyzed numerically, see [19]. In the current paper, we will also illustrate numerically how changes in the degree of risk aversion of DMs affect their optimal information acquisition behavior within a RCES. It should be already emphasized that the resulting functions  $F(x_1|rf)$  and  $H(x_1|rf)$  describe a more complex choice environment than  $F(x_1)$  and  $H(x_1)$ , due mainly to the discontinuities generated by  $H(x_1|rf)$ .

### 2.3. Existence of optimal thresholds

Clearly, the expected utility functions  $F$  and  $H$  guide the DM's optimal information acquisition process. Assume that the information search on product  $J$  has produced  $x_1$  as first result. Then, the DM will choose whether to continue checking product  $J$  or switching to product  $K$  according to which function, either  $F$  or  $H$ , takes the highest value at  $x_1$ . It may also happen that she is indifferent between continuing with  $J$  and switching to  $K$ . It is reasonable to think of these indifference values as optimal information acquisition thresholds. It should be noted that our definition of information acquisition threshold values does not include the domain limit points,  $x_1^m$  and  $x_1^M$ , since no switch in the information acquisition behavior of decision makers can be guaranteed at these points. Thus,  $X_1$  turns out to be partitioned in subintervals whose values induce the DM either to continue checking the initial product  $J$  or to switch and start checking  $K$ . In this regard, introducing fuzzy preference relations and probability functions may allow us to establish a direct link between the current setting and the three-way decision-theoretical rough set environments studied by Yao [69].

Di Caprio and Santos Arteaga [19] illustrate how the existence of optimal threshold values, or reversing points, in the DM's information acquisition process can be guaranteed under common non-pathological assumptions within the GCES. For example, it can be easily shown that  $H(x_1^M) \leq F(x_1^M)$ , with  $H(x_1^M) = F(x_1^M)$  if and only if  $u_1(x_1^M) + u_2(x_2^m) \geq E_1 + E_2$ . Therefore,  $u_1(x_1^M) + u_2(x_2^m) < E_1 + E_2$  suffices to guarantee the existence of at least one threshold value whenever  $P^+(x_1^m) = \emptyset$ .

On the other hand, the condition  $u_1(x_1^M) + u_2(x_2^m) < E_1 + E_2$  does not suffice to guarantee the existence of an optimal threshold value within the RCES. If this condition is imposed, we would obtain  $F(x_1^M|rf) < H(x_1^M|rf)$ . It is easy to show that  $F(x_1^m|rf) < H(x_1^m|rf)$  whenever  $P^+(x_1^m) = \emptyset$ . However,  $H(x_1^M|rf) = F(x_1^M|rf)$  if and only if  $u_1(x_1^M) + u_2(x_2^m) \geq E_1 + E_2$ . Thus, we have that  $F(x_1^M|rf) \leq H(x_1^M|rf)$ . As a result, the existence of a threshold value within the RCES can only be guaranteed if we assume  $u_1(x_1^M) + u_2(x_2^m) < E_1 + E_2$  and impose  $F(x_1^m|rf) > H(x_1^m|rf)$ .

The numerical simulations will shed some light to the main differences between the information acquisition thresholds generated by both scenarios.

## 3. Signals and learning

We proceed now to analyze the effect that positive signals regarding the distribution of characteristics on  $X_2$  and the resulting learning process have on the optimal information acquisition behavior of rational DMs. Signals are introduced on the second characteristic space in order to intuitively separate the role played by actual observations from that played by expectations. We will denote by  $\theta = k$  the observation of a total of  $k$  signals by DMs, with  $k = 0, \dots, n$ , for any  $n$  positive and finite. As stated in Section 2.1, we can restrict our attention to the case when uniform probabilities are assumed on both  $X_1$  and  $X_2$ . That is, even though we will only analyze the effect that the first-order stochastic dominance resulting from the signal has for the uniform density case defined in the paper, the analysis could be generalized to any other density function, see Chapter 6 in [46].

We will assume that receiving a credible positive signal  $\theta$  regarding the distribution of characteristics on  $X_2$  implies that the probability mass accumulated on the lower half of the distribution halves. At the same time, the probability mass eliminated from the lower half of the distribution is shifted to the upper one. In other words, the introduction of a superior product in the market involves the announcement of an improvement in a subset of its characteristics. Such an improvement should bring the corresponding characteristics closer to an ideal maximum value, in this case  $x_2^M$ . The degree to which this objective is expected to be achieved by the firm is represented by the initial beliefs of DMs and the credibility assigned to the signal issued by the firm. We assume in this section that DMs believe the information contained in the signal to be a truthful description of the distribution of characteristics on  $X_2$ . However, we will modify this assumption in Section 5, allowing DMs to confer different subjective degrees of credibility to the signal. Thus, given the distribution of  $X_2$  characteristics defined by  $\mu_2(x_2) = \frac{1}{x_2^M - x_2^m}$  for  $x_2 \in X_2 = [x_2^m, x_2^M]$ , the corresponding conditional density function is given by

$$\pi(\theta, \delta|x_2) = \begin{cases} \frac{1}{(x_2^M - x_2^m)} + \delta \frac{1}{(x_2^M - x_2^m)} & \text{if } x_2 \in \left[ \frac{x_2^m + x_2^M}{2}, x_2^M \right], \\ \frac{1}{(x_2^M - x_2^m)} - \delta \frac{1}{(x_2^M - x_2^m)} & \text{if } x_2 \in \left[ x_2^m, \frac{x_2^m + x_2^M}{2} \right], \end{cases} \quad (12)$$

where  $\delta \in [0, 1]$ . As described above, we will work with  $\delta = \frac{1}{2}$  throughout the paper.

After receiving a positive signal, rational DMs update their initial beliefs, given by  $\mu_2(x_2)$ , following Bayes' rule. Therefore, if one signal is received, i.e.  $\theta = 1$ , the updated beliefs of DMs will be given by

$$\mu_2(x_2|\theta = 1) = \frac{\pi(\theta, \delta|x_2)\mu_2(x_2)}{\int_{X_2} \pi(\theta, \delta|x_2)\mu_2(x_2)dx_2}. \quad (13)$$

This process can be assumed to continue as rational DMs keep on updating their beliefs using Bayes' rule after receiving further signals. For example, a second signal, providing DMs with the same qualitative information, i.e.  $\theta = 2$ , would lead to a second Bayesian updating process and the following distribution of beliefs on  $X_2$

$$\mu_2(x_2|\theta = 2) = \frac{\pi(\theta, \delta|x_2)\mu_2(x_2|\theta = 1)}{\int_{X_2} \pi(\theta, \delta|x_2)\mu_2(x_2|\theta = 1)dx_2}. \quad (14)$$

The corresponding (Bayesian) updated  $F$ -function and  $H$ -function defined by DMs after receiving one credible signal within the GCES are given by

$$F(x_1|\theta = 1) \stackrel{\text{def}}{=} \int_{P^+(x_1|\theta=1)} \mu_2(x_2|\theta = 1)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1|\theta=1)} \mu_2(x_2|\theta = 1)(E_1 + E_{2|\theta=1})dx_2 \quad (15)$$

and

$$H(x_1|\theta = 1) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1})dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_{2|\theta=1})dy_1 \quad (16)$$

with

$$P^+(x_1|\theta = 1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) > E_1 + E_{2|\theta=1} - u_1(x_1)\}, \quad (17)$$

$$P^-(x_1|\theta = 1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) \leq E_1 + E_{2|\theta=1} - u_1(x_1)\} \quad (18)$$

and the same ( $Q^+$ )-set and ( $Q^-$ )-set as those defined in the unsignaled case, since  $X_1$  and  $\mu_1(X_1)$  remain unaffected by the signal received.

**Remark.** It should be noted that the analysis performed through the rest of this section is independent of the probability mass shifted between intervals, with larger shifts, i.e. a higher number of signals, simply strengthening the initial effects derived from observing a signal.

With this remark in mind, we show that

**Proposition 1.** Given the GCES, if  $\mu_2(x_2|\theta = 1)$  first-order stochastically dominates  $\mu_2(x_2)$ , then,  $F(x_1|\theta = 1) \geq F(x_1)$  and  $H(x_1|\theta = 1) \geq H(x_1)$ , for every  $x_1 \in X_1$ .

**Proof.** If  $\mu_2(x_2|\theta = 1)$  first-order stochastically dominates  $\mu_2(x_2)$ , then, by definition,  $E_{2|\theta=1} \geq E_2$ . This is the effect that the updated  $\mu_2(x_2|\theta = 1)$  density generated by the signal has on the  $F(x_1)$  and  $H(x_1)$  functions through the new induced value  $E_{2|\theta=1}$ . It trivially follows that

$$\frac{dH(x_1)}{dE_2} = 1 > 0. \quad (19)$$

Therefore, the direct dependence of  $H(x_1)$  on  $E_2$  leads to an upward shift of the function if the value of  $E_2$  increases after the signal is received.

Regarding the  $F(x_1)$  case, the increase in  $E_{2|\theta=1}$  results in the set  $P^+(x_1|\theta = 1)$  shrinking with respect to  $P^+(x_1)$ , while  $\mu_2(x_2|\theta = 1) \geq \mu_2(x_2)$  over the newly defined interval  $P^+(x_1|\theta = 1)$ . Applying Leibnitz's rule to  $\frac{dF(x_1)}{dE_2}$ , while keeping  $\mu_2(x_2)$  fixed, allows to isolate the effect that changes in the  $E_2$  value have on the function  $F(x_1)$ . It follows that

$$\left. \frac{dF(x_1)}{dE_2} \right|_{\mu_2(x_2)} = \int_{x_2^M}^{u_2^{-1}(E_1 + E_2 - u_1(x_1))} \mu_2(x_2)dx_2 > 0, \quad \forall x_1 \in X_1, \quad \text{iff} \quad P^-(x_1^M) \neq \emptyset. \quad (20)$$

Thus, coupling a signal-based increment in  $E_2$  with a first-order stochastic dominance spread on  $\mu_2(x_2)$  leads to an increase of the function  $F(x_1) \forall x_1 \in X_1$ .

Note that, if either  $E_{2|\theta=1} \geq E_2$  or  $P^-(x_1^M) = \emptyset$ , or both, the first order stochastic dominance on  $\mu_2(x_2)$  guarantees that  $F(x_1|\theta = 1) \geq F(x_1)$  for all  $x_1$  values in  $X_1$ .  $\square$

**Proposition 2.** Given the RCES, if  $\mu_2(x_2|\theta = 1)$  first-order stochastically dominates  $\mu_2(x_2)$ , then  $H(x_1|\theta=1|rf) \geq H(x_1|rf)$  but  $F(x_1|\theta=1|rf) \geq F(x_1|rf)$  does not necessarily hold.

**Proof.** The (Bayesian) updated  $F(x_{1|\theta=1}|rf)$  and  $H(x_{1|\theta=1}|rf)$  functions defined by DMs after receiving a credible signal within the RCES follow directly from  $F(x_1|\theta=1)$  and  $H(x_1|\theta=1)$  in the exact same way  $F(x_1|rf)$  and  $H(x_1|rf)$  followed from  $F(x_1)$  and  $H(x_1)$ . However, the effect that the updated  $\mu_2(x_2|\theta=1)$  density has on the  $F(x_1|rf)$  and  $H(x_1|rf)$  functions differs significantly from that of the GCES. For example, it is trivial to show that

$$\frac{dH(x_1|rf)}{dE_2} = \int_{Q^+(x_1)} \mu_1(y_1) dy_1 + \int_{Q^-(x_1|rf)} \mu_1(y_1) dy_1 \in (0, 1). \quad (21)$$

However, the effect that the signal has on  $F(x_1|rf)$  is slightly more cumbersome. Once again, applying Leibnitz's rule to the definition of  $F(x_1|rf)$  while keeping  $\mu_2(x_2)$  fixed allows us to isolate the effect that changes in the  $E_2$  value have on the  $F(x_1|rf)$  function

$$\left. \frac{dF(x_1|rf)}{dE_2} \right|_{\mu_2(x_2)} = -[\mu_2(u_2^{-1}(E_1 + E_2 - u_1(x_1)))(E_1 + E_2)] \frac{d}{dE_2}(u_2^{-1}(E_1 + E_2 - u_1(x_1))) < 0. \quad \square \quad (22)$$

Note that the initial shock induced by the signal on  $F(x_1|rf)$  is negative. That is, an increase in the value of  $E_2$  has a negative effect on the incentives of DMs to gather the next piece of information from the product whose first characteristic has been observed. The intuition for this result is straightforward. Requiring relatively higher  $X_2$  realizations to compensate for the higher value of  $E_{2|\theta=1}$  constitutes a serious drawback when the certainty equivalent product is not guaranteed. This is the case despite the first order stochastic dominance exhibited by the  $\mu_2(x_2|\theta=1)$  probability function. This negative effect will become evident in the corresponding numerical simulations, where further intuition will be provided.

### 3.1. Shifting between markets

DMs shift from unsignaled to signaled markets in order to try to improve upon an observed product through their information acquisition processes. As will be emphasized later on, when modeling the transition between markets we will be implicitly assuming that DMs gather their first observation from the unsignaled one due to habits and inertia in their information acquisition processes. Thus, given the existing set of products composing the unsignaled market, the incentives to shift the information acquisition process to the signaled market depend on whether or not the improvements introduced allow the DM to actually improve upon the existing product characteristics. As a result, two different types of decision processes will be analyzed when defining the transition between markets. Each process leads to its own  $H: X_1 \rightarrow R$  functional form based on the improvements upon the observed characteristics that may be guaranteed when DMs shift between markets.

#### 3.1.1. Decision irreversibility

Consider first the GCES. In this case, if the DM shifts her information acquisition process to the signaled market, she will have to forego any information obtained in the unsignaled one. Hence, a shift to the signaled market constitutes an irreversible decision, and her final choice, if any, must be made within the set of products available in the signaled market. The corresponding  $H$ -function is given by the expected value derived from observing one characteristic in the signaled market

$$H(x_1|nr) \stackrel{\text{def}}{=} \int_{ce_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1}) dy_1 + \int_{x_1^m}^{ce_1} \mu_1(y_1)(E_1 + E_{2|\theta=1}) dy_1. \quad (23)$$

Note that the integration intervals differ with respect to the  $H(x_1)$  case defined previously for the unsignaled market. That is, the DM aims at observing a characteristic above  $E_1$  in the signaled market and is unable to guarantee a given  $x_1$  value due to the irreversibility assumption and the unique observation she has left. If  $E_{2|\theta=1} > E_2$ , then  $\frac{\partial H(x_1)}{\partial E_2} > 0$ , implying that  $H(x_1|nr) > H(x_1), \forall x_1 \leq ce_1$ . However, as the numerical simulations will show, this effect does not suffice to generate a shift to the signaled market for all  $x_1$  values in  $X_1$ . The utility loss caused by the irreversibility effect dominates the transition incentives triggered by a higher  $E_{2|\theta=1}$  value for high  $X_1$  realizations.

Trivially, the irreversible version of  $H(x_1)$  defined within the RCES is given by

$$H_{rf}(x_1|nr) \stackrel{\text{def}}{=} \int_{ce_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1}) dy_1. \quad (24)$$

#### 3.1.2. Decision reversibility

The second GCES assumes that the transition of the information acquisition process between markets is reversible. That is, after gathering an observation from the unsignaled market the DM may shift her information acquisition process to the signaled one and, if the observation attained in this market is not sufficiently good, return to the unsignaled market, where the observed  $x_1$  is guaranteed (though coupled with a lower  $E_2$  value). This assumption leads to the following  $H$ -function

$$H(x_1|r) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1}) dy_1 + \int_{Q^-(x_1)} \mu_1(y_1) \max\{(\max\{u_1(x_{1|\theta=1}), E_1\} + E_{2|\theta=1}), (\max\{u_1(x_1), E_1\} + E_2)\} dy_1. \quad (25)$$

Note that the integration intervals are identical to those of  $H(x_1)$  in the unsignaled case, since the point of reference defining the  $(Q^+)$ -set and the  $(Q^-)$ -set remains the observed  $x_1$  value in the unsignaled market.

The first expression on the right hand side of  $H(x_1|r)$  resembles the one defined for  $H(x_1)$  in the unsignaled market. In this case, if the DM observes a first characteristic in the signaled market higher than the  $x_1$  from the unsignaled one, then, since  $E_{2|\theta=1} > E_2$ , she will shift her information acquisition process to the product observed in the signaled market (and purchase it).

The second right hand side of Eq. (25) states that the DM prefers (to gather information on) the product whose first characteristic has been observed in the signaled market as long as its expected utility value remains above that of the product whose  $x_1$  has been observed in the unsignaled one. If its expected utility falls below, the DM will shift her information acquisition process back to the unsignaled market. This expression requires some additional explanations.

Consider the case where the first characteristic observed in the unsignaled market is below  $ce_1$ . If this were the case, then the product that the DM expects to observe in the unsignaled market (if she uses her second piece of information to gather an observation from this market) provides her with an expected utility given by

$$H(x_1|nru) \stackrel{\text{def}}{=} \int_{ce_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_2) dy_1 + \int_{x_1^m}^{ce_1} \mu_1(y_1)(E_1 + E_2) dy_1, \quad (26)$$

which is lower than  $H(x_1|nr)$ . Note that  $H(x_1|nr)$  defines the expected utility derived from the product that the DM expects to observe in the signaled market (if she uses her second piece of information to gather an observation from this market). It therefore follows that

**Proposition 3.** *Given the GCES, if the  $x_1$  observed in the unsignaled market is below  $ce_1$ , the DM has an incentive to shift her information acquisition process to the signaled one.*

**Proof.** If  $x_1 < ce_1$  within the GCES, we have that  $H(x_1|r) = H(x_1|nr) > H(x_1|nru)$ .  $\square$

Consider now the case where the first characteristic observed in the unsignaled market is above  $ce_1$ . If this were the case, gathering a first observation from the signaled market, denoted by  $x_{1|\theta=1}$ , such that

$$u_1(x_{1|\theta=1}^*) = u_1(x_1) + E_2 - E_{2|\theta=1} \quad (27)$$

would leave the DM indifferent between acquiring an additional piece of information from either the signaled or the unsignaled market. Observing a  $x_{1|\theta=1}$  value (below) above  $x_{1|\theta=1}^*$  would shift her acquisition process towards the (un)signaled market. Clearly, the higher expected utility derived from the second signaled characteristic generates an interval defined by the difference  $E_{2|\theta=1} - E_2$  such that any distance  $u_1(x_1) - u_1(x_{1|\theta=1}^*)$  smaller than this difference would not suffice to compensate for the signal effect and would lead the DM to bias her information acquisition process towards the signaled market.

The reversible version of  $H(x_1)$  defined within the RCES is given by

$$H_{rf}(x_1|r) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1}) dy_1 + \int_{Q^-(x_{1|\theta=1}|rf)} \mu_1(y_1) \max\{(u_1(x_{1|\theta=1}) + E_{2|\theta=1}), (u_1(x_1) + E_2)\} dy_1 \quad (28)$$

with

$$Q^-(x_{1|\theta=1}|rf) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) \leqslant \max\{u_1(x_1), E_1\} \wedge x_{1|\theta=1} \geqslant ce_1\}. \quad (29)$$

Clearly, given the definition of  $Q^-(x_{1|\theta=1}|rf)$ , at least one of the observations gathered must be above  $ce_1$  in order for the DM to account for its expected utility. The sets  $Q^-(x_1)$  and  $Q^-(x_{1|\theta=1}|rf)$  have the same interpretation as those defined in the unsignaled case, since the main reference point guiding the transition between markets remains the  $x_1$  observed in the unsignaled one. It should however be highlighted that the second right hand side term of  $H_{rf}(x_1|r)$  states that for DMs to choose a product from the unsignaled market it must provide them with an expected utility higher than  $E_1 + E_{2|\theta=1}$ . The explanation for this constraint is simple. When searching within a given market, we have assumed that DMs follow as reference points the certainty equivalent values determined within that market and do not consider those values defined within any of the other markets. That is, DMs consider  $E_1 + E_2$  when searching within the unsignaled market, independently of the value taken by  $E_{2|\theta=1}$ . However, when dealing with two markets simultaneously, DMs consider the best available option as long as it provides an expected utility higher than that provided by the best reference [certainty equivalent] product, in this case  $E_1 + E_{2|\theta=1}$ . This assumption could be modified and different reference points assumed. However, changing the certainty equivalent reference values would modify the corresponding  $H$  functions as well as the optimal behavior of DMs and is therefore left as a possible extension of the current paper.

#### 4. Numerical simulations and analysis

This section presents several numerical simulations that illustrate the behavior of the optimal threshold values and information acquisition intervals within both certainty equivalent scenarios and for both types of decision processes as the number of signals received by DMs indicating the existence of an improved or technologically superior set of products increases. DMs will be assumed to have a well-defined preference order both *within* and *among* characteristics. That is, the first characteristic will be assumed to be more important for DMs and, therefore, lead to a higher expected utility than the second one. In other words, rational DMs subject to information acquisition constraints of any type will be assumed to base their information acquisition process on the subjective importance of the characteristics that can be observed. The intuition justifying this assumption follows from the consumer choice literature dealing with multi-attribute sequential search processes, see [5]. Besides, in order to facilitate comparisons among the threshold values and information acquisition continuation areas generated by different numbers of signals within all possible settings, the support of all the probability functions will be kept unchanged through the simulations. In all figures the horizontal axis will represent the set of  $X_1$  realizations that may be observed by the DM, with the corresponding subjective expected utility values defined on the vertical axis and the certainty equivalents explicitly identified through a vertical line whenever intuitively necessary.

When referring to the risk-neutral scenario, the following parameter values have been assumed:

- Characteristic spaces:  $X_1 = [5, 10]; X_2 = [0, 10]$ ;
- Utility functions:  $u_1(x_1) = x_1; u_2(x_2) = x_2$ ;
- Continuous and uniform probability densities:  $\forall x_1 \in X_1, \mu_1(x_1) = \frac{1}{5}; \forall x_2 \in X_2, \mu_2(x_2) = \frac{1}{10}$ .

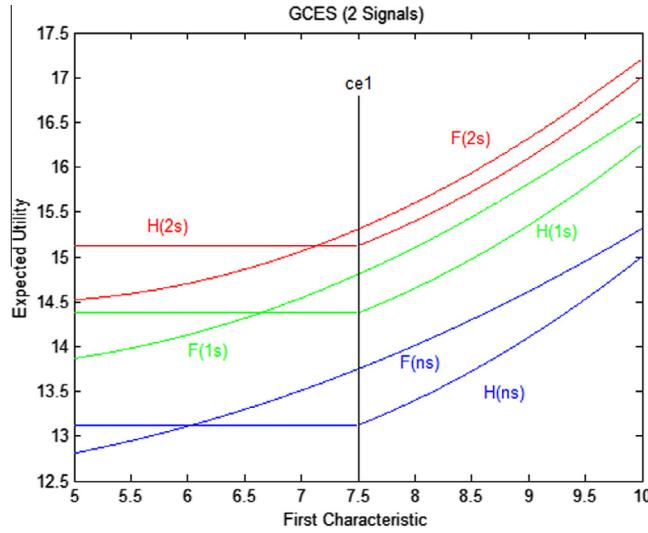
The risk-averse setting shares the characteristic spaces and probability densities but endows the DM with the concave utility functions  $u_1(x_1) = \sqrt{x_1}$  and  $u_2(x_2) = \sqrt{x_2}$ .

There exist multiple ways to select the intervals on which the characteristic spaces are defined as well as the ranking of preferences among characteristics. We may assume characteristics distributed uniformly on identical interval domains whose ranking is determined through the weights calculated using AHP or TOPSIS. As previously stated, this uniform approach assumes maximum information entropy (see [60]) and would lead to a setting where DMs are unaware of the differences existing between the characteristics in terms of distributional properties and must therefore focus on their subjective weights. Managers may use multicriteria decision tools such as AHP to determine the weights of the characteristics while maintaining an identical uniform distribution over the same domains for both variables.

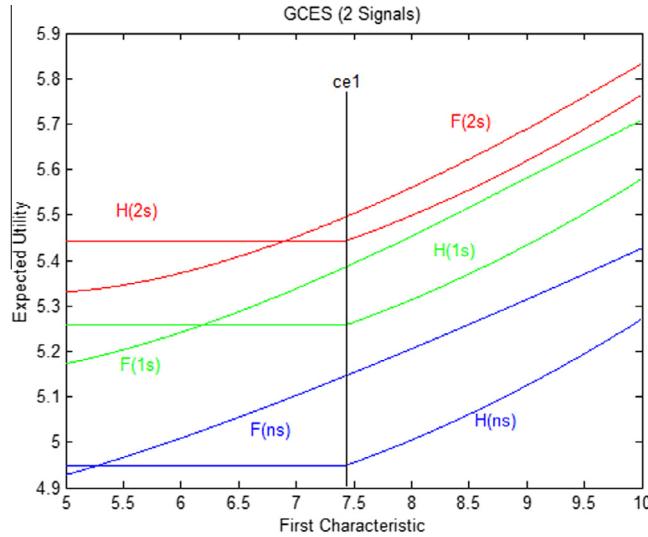
On the other hand, we have the current situation, where dominance relations among characteristics are established through changes in their probability densities. That is, DMs may have a highly limited amount of information about the characteristics of the products in terms of probabilities and their domains but expect one of them to perform better than the other. In this regard, we may assume that two characteristics start with the same probability function defined on identical domains and that performance improvements take place in terms of domain restrictions. These improvements can be calculated using linguistic evaluations of DMs by slightly modifying the setting described in Section 5.3, with the likelihood variable defining restrictions in the domain of the first characteristic relative to the second one.

##### 4.1. Signaling induced herds and managerial insights

Consider, as the basic reference case, the optimal information acquisition behavior that follows from the standard risk-neutral utility function represented in Fig. 1. This figure illustrates the unsignaled, one and two signals cases, denoted by  $ns$ ,  $1s$  and  $2s$ , respectively, and the evolution of the corresponding threshold values within a risk-neutral GCES. Clearly, positive signals generating first order stochastic dominant beliefs lead to higher expected utility levels for all possible  $X_1$  realizations while shifting the respective optimal thresholds towards higher  $x_1$  values. The intuition arising from these results states that positive signals should generate immediate herds of DMs towards the subset of products on which they are defined. However, at the same time, DMs, who expect a higher  $E_2$  value to be guaranteed from their information acquisition process, become less search-averse within the corresponding subset of signaled products, i.e. the area where the  $H$  function remains above the  $F$  one increases in the number of signals. As a result, DMs would require relatively higher realizations from the first characteristic space in order to continue acquiring information on the observed product. Such an effect can also be observed in the risk-averse setting illustrated in Fig. 2. This figure describes the same environment as Fig. 1 but within a risk-averse setting, where the utilities with which DMs are endowed have been shifted from basic linear functions to square roots. Clearly, as the degree of risk aversion increases, the support area on which the function  $H(x_1)$  remains above the function  $F(x_1)$  vanishes. Thus, the (subjectively defined) degree of risk aversion does not only affect the calculation of the corresponding certainty equivalent values but also the willingness to search of rational DMs. That is, as the coefficient of relative risk aversion increases, DMs become more reluctant to start a new search for a product better than the one whose first characteristic has already been observed.



**Fig. 1.** GCES and two signals: threshold values with risk-neutral utilities. Continuation and starting intervals for the unsignaled, one and two signals cases, denoted by ns, 1s and 2s, respectively, and the increasing evolution of the corresponding threshold values within a risk-neutral GCES. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = x_1$ ,  $u_2(x_2) = x_2$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .

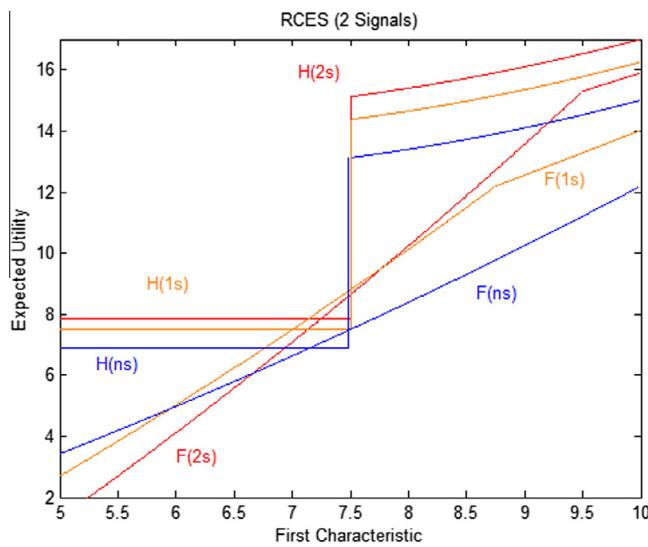


**Fig. 2.** GCES and two signals: threshold values with risk-averse utilities. Continuation and starting intervals for the unsignaled, one and two signals cases, denoted by ns, 1s and 2s, respectively, and the increasing evolution of the corresponding threshold values within a risk-averse GCES. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = \sqrt{x_1}$ ,  $u_2(x_2) = \sqrt{x_2}$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .

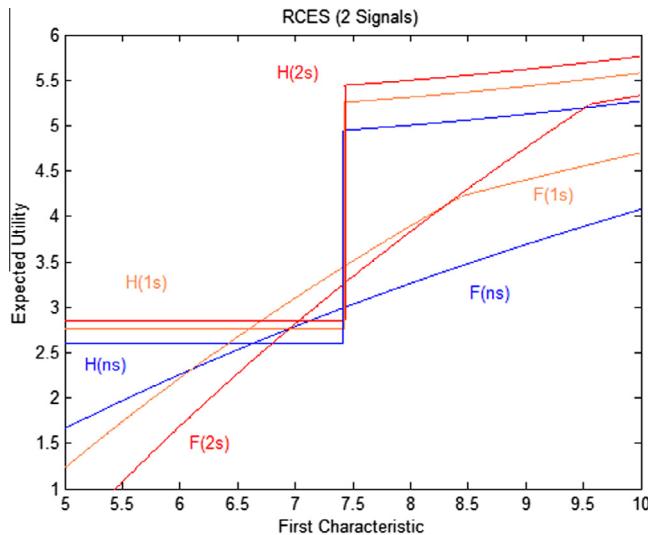
**Managerial insight 1.** In markets where DMs trust firms to deliver a quality product when choosing randomly, that is, without or with very little information regarding the main characteristics of the product, increments in the degree of risk aversion of DMs will bias their information acquisition process towards the first product observed.

**Managerial insight 2.** Consider a market where DMs trust firms to deliver a quality product when choosing randomly. Announcing *credibly* the introduction of an improved set of products implies that DMs must be shown better realizations of their most preferred characteristic in order to continue acquiring information on the product observed.

The optimal behavior described in Figs. 1 and 2 corresponds to the GCES. Figs. 3 and 4 illustrate how, when considering the RCES, the set of  $X_1$  realizations such that  $F(x_1) > H(x_1)$  within a given market, i.e. such that DMs prefer to remain acquiring information on the product whose first characteristic has been observed instead of starting to acquire information on a new product, becomes a proper subset of the one defined within the GCES. Besides, refusing to make a random purchase from the



**Fig. 3.** RCES and two signals: threshold values and information acquisition intervals with risk-neutral utilities. Continuation and starting intervals for the unsignaled, one and two signals cases, denoted by ns, 1s and 2s, respectively, and the increasing evolution of the corresponding threshold values within a risk-neutral RCES. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = x_1$ ,  $u_2(x_2) = x_2$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .



**Fig. 4.** RCES and two signals: threshold values and information acquisition intervals with risk-averse utilities. Continuation and starting intervals for the unsignaled, one and two signals cases, denoted by ns, 1s and 2s, respectively, and the increasing evolution of the corresponding threshold values within a risk-averse RCES. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = \sqrt{x_1}$ ,  $u_2(x_2) = \sqrt{x_2}$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .

set of products available within a given market generates three different information acquisition subintervals and a discontinuity in the  $H$ -function, both in the risk-neutral and risk-averse environments. It should be noted that the vertical lines joining the discontinuous pieces of the corresponding  $H$ -functions have been added to allow for a more intuitive graphical presentation and to simplify comparisons among markets. The main conclusion that may be initially derived from this basic environment is that any (endogenous or exogenous) constraint imposed on the reputation level of a firm generates a much stricter set of continuation criteria within its corresponding set of products. Indeed, the negative effect of  $E_2$  on  $F(x_1|rf)$ , derived analytically in the previous section, can also be observed in these figures. Note how as the observed realizations of  $X_1$  improve they manage to compensate for the initial negative shock on  $F(x_1|rf)$ , i.e.  $F(1s)$  and  $F(2s)$  in Figs. 3 and 4.

**Managerial insight 3.** If DMs lose their confidence and do not trust firms to deliver a quality product when choosing randomly, they will almost completely ignore the first product observed through their information acquisition process. This will be the case even if firms announce credibly the introduction of an improved set of products.

#### 4.2. Reversing decisions and managerial insights

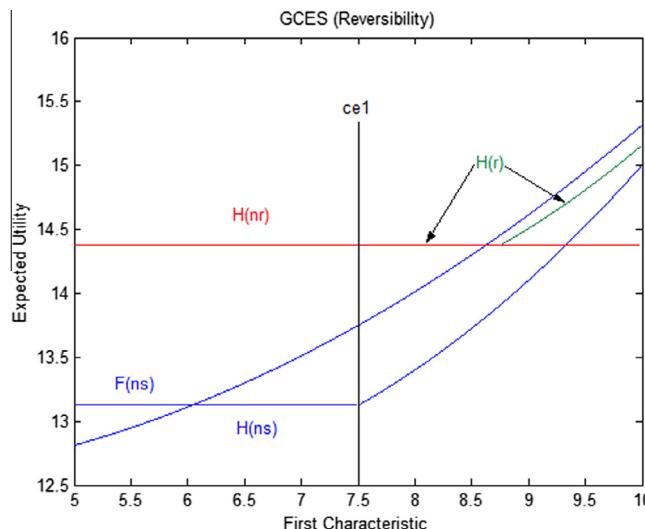
In this subsection, DMs will be allowed to compare the products observed when acquiring information from two different markets. Therefore, they must consider the effects derived from this reversibility property when deciding whether or not to shift their information acquisition processes between markets *after they have gathered an observation from the unsignaled one*. This last constraint is imposed to account for inertia and habits in the information acquisition processes of DMs, see Geroski [23] for a review of the literature on these subjects. Such a ‘basic memory capacity’ effect remains unstudied in the recent operations research literature when dealing with the (memoryless) adoption of new technology, even though, as we shall see, it generates substantial differences in the (optimal) behavior of DMs. We will compare this reversible setting with the standard irreversible one, where DMs must forego the observation gathered in the unsignaled market when shifting to the signaled one.

Consider the risk-neutral GCES presented in Fig. 5, where the transition between the unsignaled and the one-signal markets may be undertaken through irreversible,  $H(nr)$ , or reversible,  $H(r)$ , decision processes. The threshold point defined by both markets results from comparing the expected utility derived from checking the second characteristic of the product observed in the unsignaled market with that obtained from checking the first characteristic of a product in the signaled one. Note that the realizations of  $X_1$  for which the reversible and irreversible  $H$ -functions overlap are those where the value of the  $x_1$  being observed in the unsignaled market does not suffice to compensate for the higher  $E_{2|0=1}$  of the signaled one. The reversible  $H$ -function becomes strictly increasing when the realizations of  $X_1$  observed in the unsignaled market start compensating for the  $E_{2|0=1} - E_2$  differential created by the signal.

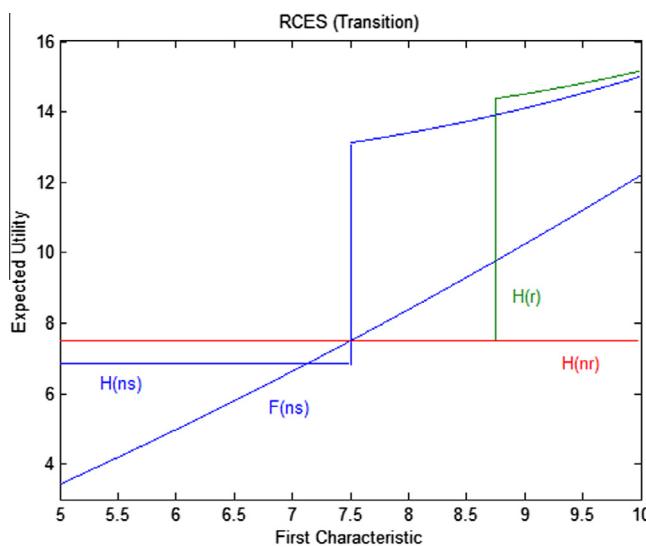
**Managerial insight 4.** Consider a market where DMs trust firms to deliver a quality product when choosing randomly and where DMs are allowed to shift their information acquisition process between markets costlessly. Announcing credibly the introduction of an improved set of products does not guarantee a shift of the DMs to the improved market. DMs may remain faithful to the original, less developed, product if a sufficiently high realization of their most preferred characteristic is observed.

Fig. 6 introduces the corresponding risk-neutral RCES. Besides illustrating the discontinuous behavior distinguishing the GCESs and RCESs scenarios, the numerical simulations presented in Figs. 5 and 6 describe a fundamental difference between both settings induced on the optimal information acquisition behavior of DMs. That is, in the reversible RCES, DMs have an incentive to reverse their search between markets and shift their information acquisition processes to the signaled one for relatively good realizations of  $X_1$ , a behavioral option that remains absent in the reversible GCES. The intuition justifying this result is straightforward. Note that, in the RCES, relatively low  $X_1$  realizations lead to an expected search utility value of zero, while in the GCES DMs are able to compensate for low realizations through the (expected) certainty equivalent product. In this case, the higher value of  $E_{2|0=1}$  relative to  $E_2$  reinforces the certainty equivalent reputation effect significantly within the GCES, while much less so in the RCES.

Similarly, a relatively high  $x_1$  observation from the unsignaled market within the RCES guarantees the DM a product with an expected utility at least as high as  $E_1 + E_{2|0=1}$ . If the DM shifts the information acquisition process to the signaled market, this reversal possibility must be added to whatever observation may be obtained together with its corresponding higher



**Fig. 5.** GCES and one signal: market transition thresholds with risk-neutral utilities. Continuation and starting intervals for the unsignaled and the one signal cases within a risk-neutral GCES. The transition between the unsignaled and the one-signal markets may be undertaken through irreversible,  $H(nr)$ , or reversible,  $H(r)$ , decision processes. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = x_1$ ,  $u_2(x_2) = x_2$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .



**Fig. 6.** RCES and decision reversibility: threshold values and information acquisition intervals with risk-neutral utilities. Continuation and starting intervals for the unsignaled and the one signal cases within a risk-neutral RCES. The transition between the unsignaled and the one-signal markets may be undertaken through irreversible,  $H(nr)$ , or reversible,  $H(r)$ , decision processes. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = x_1$ ,  $u_2(x_2) = x_2$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .

$E_{2|0=1}$  value. This reversal effect provides the DM with a higher expected utility from shifting than continuing to acquire information in the unsignaled market and observing any possible  $X_2$  realization, some of them leading to an expected search utility of zero. Thus, given identical distributions of characteristics, signals and  $X_1$  realizations, DMs may not reverse their information acquisition processes within the reversible case in the GCES, though they could do so in the RCES.

Note that,  $E_{2|0=1} > E_2$  implies  $H(r) > H(ns)$  for sufficiently high  $X_1$  realizations within the RCES. However, if the  $X_1$  realizations located above  $ce_1$  are not sufficiently high so as to guarantee a product with an expected utility at least as high as  $E_1 + E_{2|0=1}$ , then the DM will remain acquiring information within the unsignaled market, as can be observed in Fig. 6. That is, as stated at the end of the previous section, DMs consider different  $E_2$  values when searching within different markets. As a result, all the  $x_1 > ce_1$  observations retrieved from the unsignaled market that do not manage to compensate for the  $E_{2|0=1} - E_2$  differential lead to a payoff of zero if the information acquisition process is shifted to the signaled market. This is due to the fact that, in this case,  $E_1 + E_{2|0=1}$  would provide the decision maker with a higher expected search utility. However, the same observations deliver a product within the unsignaled market whose expected utility is higher than  $E_1 + E_2$ . This ‘reference certainty equivalent value’ effect is the one responsible for the differences observed between the  $H(r)$  and  $H(ns)$  functions within this subset of  $x_1$  values.

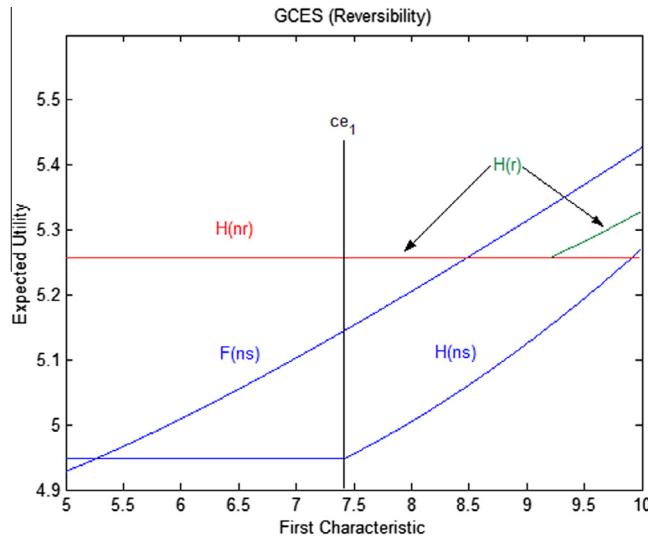
**Managerial insight 5.** Consider a market where DMs *do not* trust firms to deliver a quality product when choosing randomly and where DMs are allowed to shift their information acquisition process between markets costlessly. Announcing credibly the introduction of an improved set of products does not guarantee a shift of the DMs to the improved market. However, DMs may shift to the improved market if:

- they observe a relatively low realization of their most preferred characteristic in the original, less developed, market;
- they are able to secure a sufficiently high realization of their most preferred characteristic in the original, less developed, market.

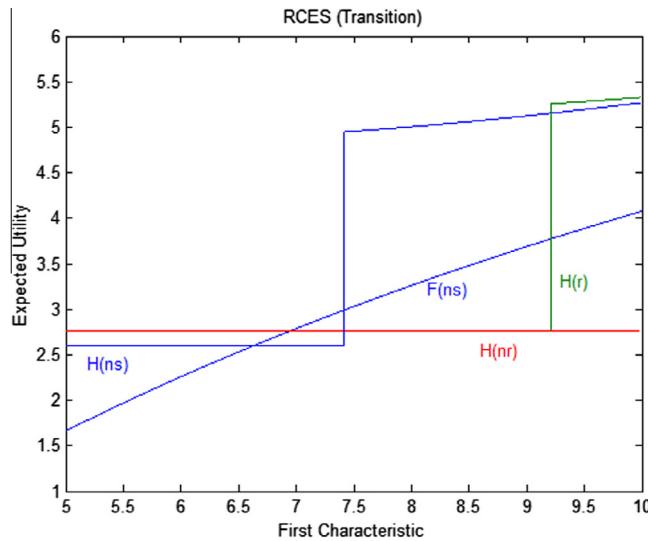
Finally, Figs. 7 and 8 represent the market transition GCESs and RCESs, respectively, within a risk-averse setting. An almost identical analysis to the one developed for the risk-neutral case could be performed here. The main characteristic to highlight from the risk-averse environment is the generation of four different information acquisition intervals within the RCES. Other than that, the intuition justifying the observed results is identical to the one used in the risk-neutral case described above.

## 5. Operative applicability of the model

This section illustrates how to implement the model described through the paper using linguistic evaluation variables gathered from DMs. The main scenario where the current model may become operative and highly relevant is that where



**Fig. 7.** GCES and one signal: market transition thresholds with risk-averse utilities. Continuation and starting intervals for the unsignaled and the one signal cases within a risk-averse GCES. The transition between the unsignaled and the one-signal markets may be undertaken through irreversible,  $H(nr)$ , or reversible,  $H(r)$ , decision processes. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = \sqrt{x_1}$ ,  $u_2(x_2) = \sqrt{x_2}$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .



**Fig. 8.** RCES and decision reversibility: threshold values and information acquisition intervals with risk-averse utilities. Continuation and starting intervals for the unsignaled and the one signal cases within a risk-averse RCES. The transition between the unsignaled and the one-signal markets may be undertaken through irreversible,  $H(nr)$ , or reversible,  $H(r)$ , decision processes. The parameters used are:  $X_1 = [5, 10]$ ,  $X_2 = [0, 10]$ ,  $u_1(x_1) = \sqrt{x_1}$ ,  $u_2(x_2) = \sqrt{x_2}$ ,  $\mu_1(x_1) = 1/5$ ,  $\mu_2(x_2) = 1/10$ .

a limited amount of information is available among DMs when deciding what product to choose. In particular, as stated in the introduction, the main reason why a demand based environment has been chosen relates directly to the secondary role played by it in the operational research and management literatures. This is not only the case when dealing with consumers but also when managers consider implementing new projects with a very limited amount of information (see [68]). In this regard, the current model applies to settings where a manager must choose between projects being developed while being subject to a severe information constraint but having a subjective expectation on the potential set of outcomes. This rational modelization hides a powerful strategic component, that is, the capacity to forecast the behavior of managers or consumers based on their degree of risk aversion, their degree of optimism or pessimism regarding the product or outcome being delivered and the degree of confidence that the DM has on the information sender. This latter one is particularly relevant in online search and purchase environments, see [38].

In order to illustrate the potential operative applicability of the paper we implement a fuzzy-based elicitation procedure that will be used to determine the parametric and functional values of the expected search utilities. We remain confined to a

fuzzy environment due to the limited information available to DMs and their corresponding imprecise linguistic statements when relating to novel technology acceptance; for example, the data retrieved by Arruda-Filho et al. [2] and Arruda-Filho and Lennon [3] on iPhone usage and acceptance relies on evidence derived from netnographic linguistic variables. Similarly, vague evaluations are received by DMs, even at the managerial level, when having to take a decision at the initial stages of a given project (see [68,70]). Moreover, the subjective evaluations regarding risky decisions considered by DMs and their degree of trust on a firm are generally expressed through linguistic imprecise variables, see [35,12], respectively.

Finally, a fuzzy approach is the most natural way to proceed when dealing with decision making under risk since, when eliciting the preferences of DMs, the economic literature has encountered a substantial amount of errors and imprecision due to violations of its standard basic axioms, see [56]. The economic theories of choice under risk developed to explain violations of these axioms rely on random errors made by DMs during their choice process, which may appear when selecting preferences [41], making calculations [26], or taking actions [24]. In particular, and in direct relation to our paper, MacCrimmon and Smith [42] showed how DMs have difficulties providing a single precise certainty equivalent and must approach their valuations through equivalence intervals, a tendency also observed in psychology by Krahn et al. [37]. The inclusion of fuzzy variables determining the parameter values of the expected search utilities allows us to accommodate these empirical facts described by economists within our theoretical environment.

### 5.1. Implementing a fuzzy based decision environment

Fuzzy methods are commonly used to evaluate the assessments made by DMs when describing linguistically the behavior or value of variables relating to the risk inherent in the decisions they must make, see [11,70]. The analysis presented through the current paper allows for any degree of risk being assumed on DMs and reflected in their utility functions. We have already described the problems faced by economists when trying to elicit the utility functions of DMs. However, DMs may be asked to describe linguistically the subjective degree of risk that they are willing to face inherent in a given decision. As a result, utility functions based on the degrees of risk aversion implied by the reports of DMs can be immediately obtained. This approximation can also be applied to the likelihood of the improvements reported by the firm and the degree of confidence that DMs have on the firm issuing the signals. DMs may provide linguistic judgments about all these variables that will allow us to define entirely any of the functions  $F$  and  $H$  introduced through the paper.

We consider seven linguistic values, each associated to a corresponding triangular fuzzy number (TFN). The criteria ratings for risk, likelihood and confidence are linguistic variables with linguistic values  $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ . The first two columns of Tables 1–3 describe these linguistic values and the linguistic variables associated with each one of them for risk, likelihood and confidence, respectively. The linguistic variables are then identified with TFNs with membership functions defined as follows:

$$V_1 = \left(0, 0, \frac{1}{6}\right) \quad \xi_1(x) = \begin{cases} 1 - 6x, & 0 \leq x \leq \frac{1}{6}, \\ 0, & \frac{1}{6} \leq x \leq 1, \end{cases}$$

$$V_n = \left(\frac{n-2}{6}, \frac{n-1}{6}, \frac{n}{6}\right) \quad \xi_n(x) = \begin{cases} 0, & 0 \leq x < \frac{n-2}{6}, \\ 6x - (n-2), & \frac{n-2}{6} \leq x < \frac{n-1}{6}, \\ n - 6x, & \frac{n-1}{6} \leq x \leq \frac{n}{6}, \\ 0, & \frac{n}{6} \leq x \leq 1, \end{cases} \quad (30)$$

$n = 2, 3, 4, 5, 6$  and

$$V_7 = \left(\frac{5}{6}, 1, 1\right) \quad \xi_7(x) = \begin{cases} 0, & 0 \leq x < \frac{5}{6}, \\ 6x - 5, & \frac{5}{6} \leq x \leq 1, \end{cases}$$

Using the centroid method we observe that the seven qualitative scales  $V_1, V_2, V_3, V_4, V_5, V_6, V_7$  have centroids  $VG(1) = 0.0556$ ,  $VG(2) = 0.1667$ ,  $VG(3) = 0.3333$ ,  $VG(4) = 0.5000$ ,  $VG(5) = 0.6667$ ,  $VG(6) = 0.8334$ , and  $VG(7) = 0.9444$ , respectively. The corresponding set of fuzzy numbers is represented in Fig. 9. These fuzzy numbers, each one of them associated

**Table 1**

Linguistic values, linguistic variables and corresponding TFNs: subjective degree of risk aversion.

| Ranking level | Degree of risk | Triangular fuzzy number |
|---------------|----------------|-------------------------|
| 1             | Extra high     | (0, 0, 0.167)           |
| 2             | Very high      | (0, 0.167, 0.333)       |
| 3             | High           | (0.167, 0.333, 0.5)     |
| 4             | Middle         | (0.333, 0.5, 0.667)     |
| 5             | Low            | (0.5, 0.667, 0.833)     |
| 6             | Very low       | (0.667, 0.833, 1)       |
| 7             | Extra low      | (0.833, 1, 1)           |

**Table 2**

Linguistic values, linguistic variables and corresponding TFNs: signal credibility.

| Ranking level | Degree of likelihood | Triangular fuzzy number |
|---------------|----------------------|-------------------------|
| 1             | Extremely low        | (0, 0, 0.167)           |
| 2             | Very low             | (0, 0.167, 0.333)       |
| 3             | Low                  | (0.167, 0.333, 0.5)     |
| 4             | Moderate             | (0.333, 0.5, 0.667)     |
| 5             | High                 | (0.5, 0.667, 0.833)     |
| 6             | Very high            | (0.667, 0.833, 1)       |
| 7             | Extremely high       | (0.833, 1, 1)           |

**Table 3**

Linguistic values, linguistic variables and corresponding TFNs: degree of confidence in the firm.

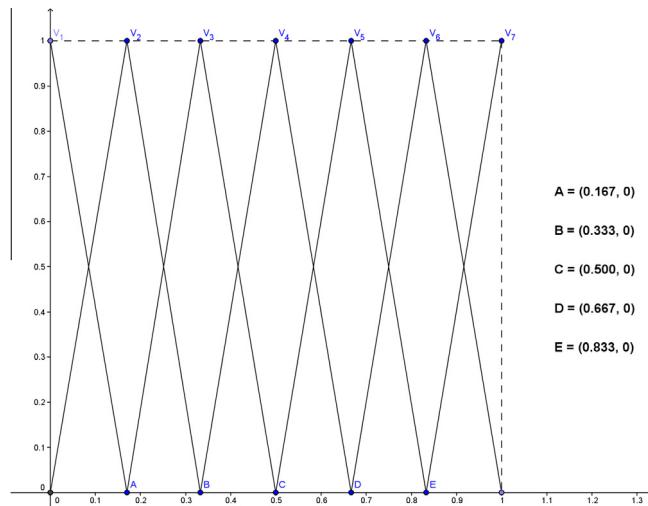
| Ranking level | Degree of confidence | Triangular fuzzy number |
|---------------|----------------------|-------------------------|
| 1             | Extremely low        | (0, 0, 0.167)           |
| 2             | Very low             | (0, 0.167, 0.333)       |
| 3             | Low                  | (0.167, 0.333, 0.5)     |
| 4             | Moderate             | (0.333, 0.5, 0.667)     |
| 5             | High                 | (0.5, 0.667, 0.833)     |
| 6             | Very high            | (0.667, 0.833, 1)       |
| 7             | Extremely high       | (0.833, 1, 1)           |

to its respective linguistic variable, are introduced in the third column of Tables 1–3, where the three main variables determining the behavior of DMs in the current setting, i.e. the degree of risk aversion, the credibility of the signal and the confidence in the firm delivering a quality product, are defined in terms of linguistic criteria.

Tables 1–3 have been generated following standard textbook guidelines (see [35]). Clearly, we could have followed different linguistic value classifications or base the approach on trapezoidal fuzzy numbers, such as [70], while implementing variations of the impact, detection and valuation scales suggested by Carbone and Tippett [9]. In other words, the current setting constitutes an example of how to implement our model using a fuzzy hierarchical scale, with several different alternatives existing in the literature (see [11]).

We will use the standard center of area method to defuzzify the triangular fuzzy numbers representing the subjective valuations of DMs. That is, the defuzzified value of the triangular fuzzy grading  $g_i = (Lg_i, Mg_i, Ug_i)$ , with  $Lg_i, Mg_i, Ug_i$  representing respectively the lower, middle and upper values of the fuzzy grade provided by the  $i$ th decision maker, is given by

$$g_i = \frac{[(Ug_i - Lg_i) + (Mg_i - Lg_i)]}{3} + Lg_i. \quad (31)$$



**Fig. 9.** Set of TFNs associated with the linguistic values. The figure represents the membership functions  $\xi_1, \dots, \xi_7$  associated with the fuzzy numbers  $V_1, \dots, V_7$ , defined in Eq. (30).

**Remark.** The numerical examples described through this section will be based on the valuations of a unique DM. However, group decisions could be incorporated into the analysis using the fuzzy geometric mean,  $\tilde{g} = (g_1 \otimes g_2 \otimes \cdots \otimes g_n)^{1/n}$ , when considering the valuations n DMs. The symbol  $\otimes$  denotes the standard product between fuzzy numbers. Refer to [61] for an alternative approach regarding the aggregation of individual judgments into a group one.

### 5.2. Accounting for the degree of risk aversion

We start the implementation analysis of our model considering the utility functions with which DMs are endowed. Our theoretical setting allows these utility functions to be defined in terms of the degree of risk aversion of DMs. This constitutes a substantial advantage since it allows us to consider different scenarios based on the potential degree of risk aversion exhibited by DMs. If the model could only accommodate risk neutral DMs, we will be constrained to operate under such an (unrealistic) assumption. DMs are not required to describe their utility functions but to state linguistically the subjective degree of risk they are willing to face when making a purchase or developing a project. These reports relate directly to the triangular fuzzy numbers assigned to account for the subjective valuations of DMs in Table 1. Note that the higher the degree of risk a DM is willing to face the lower her degree of risk aversion.

Therefore, we can always provide a set of utility functions based on the risk aversion reports of DMs. That is, once the triangular fuzzy numbers are derived from the reports and the resulting valuation is defuzzified, we can obtain the corresponding utility functions using basic microeconomic theory. For example, the utility function

$$u(x) = 1 - e^{-\varphi x} \quad (32)$$

exhibits a constant absolute degree of risk aversion equal to  $\varphi$  (the calculations and basic intuition relating to the corresponding differential equation can be found in standard microeconomic textbooks such as [65]). In other words, following Eq. (32), we can assign a utility function to a DM given the coefficient of risk aversion that follows directly from the linguistic variables reported. It should be noted that we have chosen the family of functions described in Eq. (32) to provide some contrast with respect to the decreasing absolute risk aversion utilities, i.e. the square root functions, illustrated in the previous numerical simulations. These latter ones are considered to be a more realistic assumption (see [32]), while economists tend to prefer the operational simplicity of the former type. In this regard, note that a hierarchical fuzzy scale can be easily defined, from lower to higher reported risk aversion levels, on the  $1/n$  power to which  $x_i$ ,  $i = 1, 2$ , is raised. Furthermore, it is quite simple to infer whether DMs exhibit constant or decreasing absolute risk aversion, since the latter one is decreasing in  $x_i$ ,  $i = 1, 2$ .

A similar approach is followed to account for the credibility of the signal issued and the degree of confidence in the firm. The subjective reports will range from zero to full signal credibility and confidence in the firm, respectively, leading to different valuation-based scenarios. The former set of reports will determine the shift in the signal-induced density and the latter one the expected value of the product obtained from a random purchase. The required data can be easily elicited from linguistic questionnaires and translated into crisp values leading to precise expected search utility functions.

### 5.3. Accounting for signal credibility

Consider now the likelihood of the improvements reported by the firm on the unobserved second characteristic, i.e. the credibility of the shift in the probability density. We may start from a uniform probability function defined on the set of possible realizations considered by the DM. As already stated, the choice of a uniform density is not random, but reflects the fact that it has maximum (information) entropy and therefore provides the least possible information content to the DM. The mass of the probability function can be shifted from the lower to the upper end of the domain based both on the credibility of the signal and the degree of optimism or pessimism exhibited by the DM. Note that DMs tend to differ in their subjective evaluations of probability functions and signals, see [32]. In this case, the DM may be asked how optimistic or pessimistic is about the potential realizations of the characteristics of the product constituting a substantial improvement over the current ones. As a result, based on the credibility of the signal or the degree of optimism expressed by the DM, an equivalent weighted probability mass  $\frac{\gamma}{x_2^M - x_2^m}$ , with  $\gamma \in [0, 1]$ , would be shifted from the lower to the upper part of the function. This shift leads to an updated density function reflecting the capacity of the signal to modify the beliefs of DMs.

Therefore, considering the framework described when defining Eq. (12), we will assume that DMs believe the information contained in a signal to be a truthful description of the distribution of characteristics on  $X_2$  with a degree of likelihood  $\gamma$ , where  $\gamma \in [0, 1]$ . Consequently, given the distribution of  $X_2$  characteristics defined by  $\mu_2(x_2) = \frac{1}{x_2^M - x_2^m}$  for  $x_2 \in X_2$ , the corresponding conditional density function reads as follows:

$$\pi(\theta, \delta, \gamma | x_2) = \begin{cases} \frac{1}{(x_2^M - x_2^m)} + \gamma \delta \frac{1}{(x_2^M - x_2^m)} = \frac{1+\gamma\delta}{(x_2^M - x_2^m)} & \text{if } x_2 \in \left(\frac{x_2^m+x_2^M}{2}, x_2^M\right], \\ \frac{1}{(x_2^M - x_2^m)} - \gamma \delta \frac{1}{(x_2^M - x_2^m)} = \frac{1-\gamma\delta}{(x_2^M - x_2^m)} & \text{if } x_2 \in \left[x_2^m, \frac{x_2^m+x_2^M}{2}\right]. \end{cases} \quad (33)$$

The degree of trust regarding the credibility of the signal is given by  $\gamma$ , with the DM deciding what proportion of probability mass should be shifted from the lower to the upper side of the distribution relative to the announcement made by the firm.

Thus, the fuzzy scale associated to the linguistic variable together with an equivalent shift in the probability mass allows us to provide an estimate of the expectations of the DM.

#### 5.4. Accounting for the degree of confidence and summarizing

Finally, and similarly to the risk and credibility cases, the DM may provide a linguistic evaluation of the degree of confidence that she has on the firm, ranging from a complete lack to total confidence. The level of confidence in the firm will have a numerical value ranging from zero to one and will be denoted by  $\sigma \in [0, 1]$ . In terms of the expected payoff received from a random purchase, this scale ranges from zero to the certainty equivalent product.

The fuzzy valuations of the reports provided by DMs and their posterior defuzzification allow us to use or model to evaluate the expected behavior of DMs in terms of the resulting threshold values. Therefore, maintaining the notational approach introduced through the paper, the expected search utility functions to be considered given the linguistic reports of DMs are described below.

$$F(x_1, \varphi, \sigma | \theta = 1, \gamma) \stackrel{\text{def}}{=} \int_{P^+(x_1 | \theta=1, \gamma)} \mu_2(x_2 | \theta = 1)(u_1(x_1) + u_2(x_2)) dx_2 + \int_{P^-(x_1 | \theta=1, \gamma)} \mu_2(x_2 | \theta = 1)\sigma(E_1 + E_{2|\theta=1,\gamma}) dx_2 \quad (34)$$

with

$$P^+(x_1 | \theta = 1, \gamma) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) > E_1 + E_{2|\theta=1,\gamma} - u_1(x_1)\}, \quad (35)$$

$$P^-(x_1 | \theta = 1, \gamma) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) \leq E_1 + E_{2|\theta=1,\gamma} - u_1(x_1)\}, \quad (36)$$

where  $\varphi$  is the degree of absolute risk aversion and  $\sigma$  the level of confidence in the firm. Similarly,

$$\begin{aligned} H(x_1, \varphi, \sigma | \theta = 1, \gamma) \stackrel{\text{def}}{=} & \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_{2|\theta=1,\gamma}) dy_1 + \int_{Q^-(x_1 | rf)} \mu_1(y_1)(u_1(x_1) + E_{2|\theta=1,\gamma}) dy_1 \\ & + \int_{Q^-(\sigma E_1 | rf)} \mu_1(y_1)\sigma(E_1 + E_{2|\theta=1,\gamma}) dy_1 \end{aligned} \quad (37)$$

with

$$Q^-(x_1 | rf) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\} \wedge x_1 \geq ce_1\} \quad (38)$$

and

$$Q^-(\sigma E_1 | rf) = \{y_1 \in X_1 \cap \text{Supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\} \wedge x_1 < ce_1\}. \quad (39)$$

Clearly, as the set  $Q^-(\sigma E_1 | rf)$  indicates, DMs may consider initial realizations located below  $ce_1$  in the current setting since the product obtained from a random purchase does not necessarily deliver an expected value of zero.

It should be noted that in both the  $F(x_1, \varphi, \sigma | \theta = 1, \gamma)$  and  $H(x_1, \varphi, \sigma | \theta = 1, \gamma)$  expressions  $E_{2|\theta=1,\gamma}$  and the corresponding integral limits are functions of  $\gamma$ , since this variable determines the density function of the signal and therefore the resulting expected reference value.

#### 5.5. Numerical examples and further results

In this subsection we introduce a second set of numerical simulations to illustrate the behavior of the expected search utilities when DMs exhibit constant absolute risk aversion. We will concentrate the analysis on the behavior of the threshold values when the linguistic reports of DMs are modified. These simulations allow for constructive comparisons with the risk neutral and decreasing absolute risk-averse environments described in the previous section.

**Remark.** Note that the analysis introduced in the previous section regarding the reversibility of the information acquisition process of DMs could be also be performed here. However, this would extend the length of the paper considerably without providing any additional significant insight and is therefore left to the interested reader.

In the our setting, DMs report three different linguistic values ( $R, L, C$ ), which stand for their subjective degree of risk aversion, the credibility (likelihood) assigned to the improvements announced and the level of confidence in the firm. We present several numerical simulations that start from a reference ranking triple  $(4, 4, 4)$  and allow for variations in each one of the reported variables as follows:  $(6, 4, 4)$ ,  $(4, 6, 4)$  and  $(4, 4, 6)$ . The reference triple provided corresponds to the middle risk and moderate likelihood and confidence case. Clearly, different valuations and combinations may be considered. However, we are interested in isolating the individual effects derived from increments in the degree of risk aversion of DMs, the likelihood of product improvements and the confidence in the firm under consideration. The corresponding triangular fuzzy numbers are given by  $(0.5, 0.5, 0.5)$ , as the reference one, and  $(0.833, 0.5, 0.5)$ ,  $(0.5, 0.833, 0.5)$  and  $(0.5, 0.5, 0.833)$  for each respective variation.

The numerical simulations presented in Figures 10 to 13 illustrate the effects that changes in the subjective valuations of DMs have on the threshold values derived from the corresponding functions  $F(R, L, C)$  and  $H(R, L, C)$ . The threshold values

defined in the figures coincide with the certainty equivalent values of each corresponding scenario, given by 6.7220 and 7.0065 in Fig. 10 and by 7.0065 in Figs. 11 and 12. The main numerical-based results, which complement and widen the scope of those obtained in the previous section, are described through the following propositions and managerial insights.

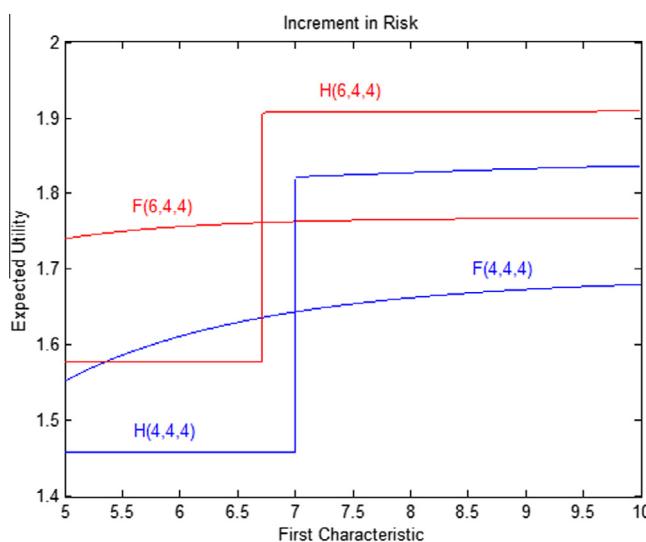
**Proposition 4.** *In markets composed by DMs with a constant degree of absolute risk aversion, increments in the degree of risk aversion of DMs lead to a decrease in the threshold value.*

**Managerial insight 6.** Consider a scenario where DMs maintain the same risk attitude independently of the payoff derived from the product or project. If DMs do not completely trust firms to deliver a quality product when choosing randomly and signals are not fully credible, increments in the degree of risk aversion of DMs will bias their information acquisition process against the first product observed.

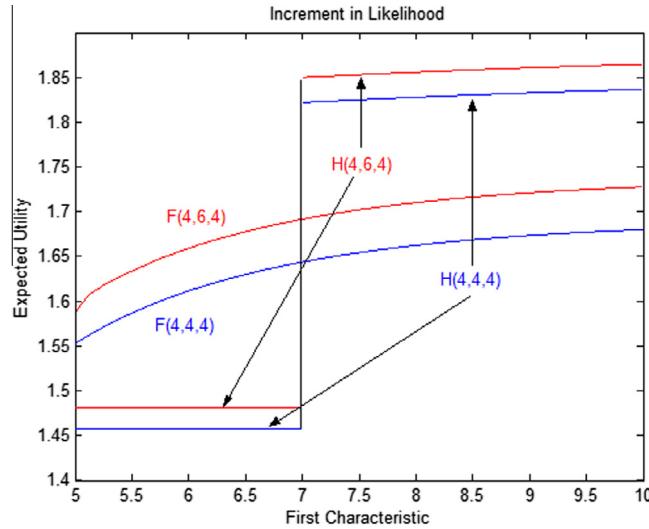
The same effect was illustrated numerically in the previous section, when DMs shifted from risk neutrality to (decreasing absolute) risk aversion. Thus, increments in the degree of risk aversion of DMs shift the corresponding threshold values leftwards. However, there is a fundamental difference between both scenarios. In the risk neutral and decreasing absolute risk aversion settings, the leftward shift of the threshold value implied an increment in the incentives of DMs to continue acquiring information on the initial good observed. However, under constant absolute risk aversion the leftward shift of the threshold value increments the incentives of DMs to acquire information from more than one firm. This result emphasizes the substantial degree of heterogeneity arising from the potential set of scenarios that may be considered by managers.

**Proposition 5.** *In markets composed by DMs with a constant degree of absolute risk aversion and absent full credibility and confidence, improvements in either the signal's likelihood or the degree of confidence in the firm may not suffice to modify the information acquisition incentives of DMs.*

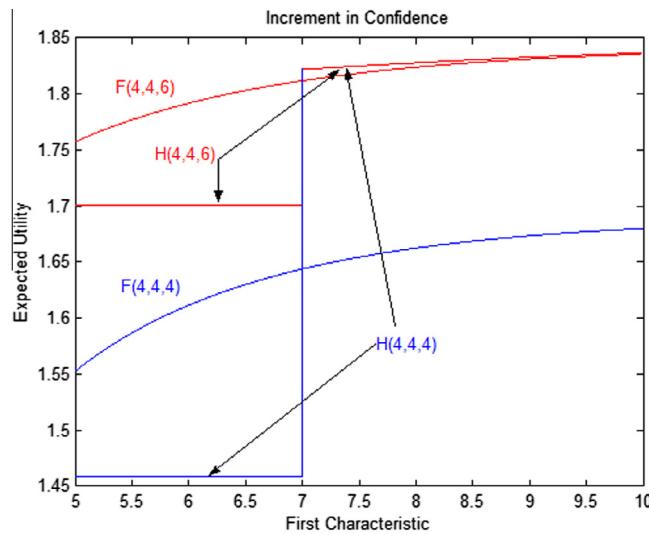
Once again, these results contrast with those obtained following signal-based improvements in the previous section. Clearly, the final effect derived from a signal depends on the  $(R,L,C)$  triple under consideration, which follows directly from the reports provided by DMs. Note, for example, that, in the current setting, a complete degree of confidence in the firm generates an immediate herd towards the initial product independently of the characteristic observed, a result absent in the previous section. Fig. 13 illustrates this point.



**Fig. 10.** Threshold values with constant risk-averse utilities: increments in risk aversion. Continuation and starting intervals for the unsignaled market when DMs increment their subjective degree of risk aversion. The functions reflect the reports of DMs regarding three different linguistic values  $(R,L,C)$ :  $R$  stands for the subjective degree of risk aversion;  $L$  is the credibility assigned to the improvements announced by the firm;  $C$  measures the level of confidence in the firm.

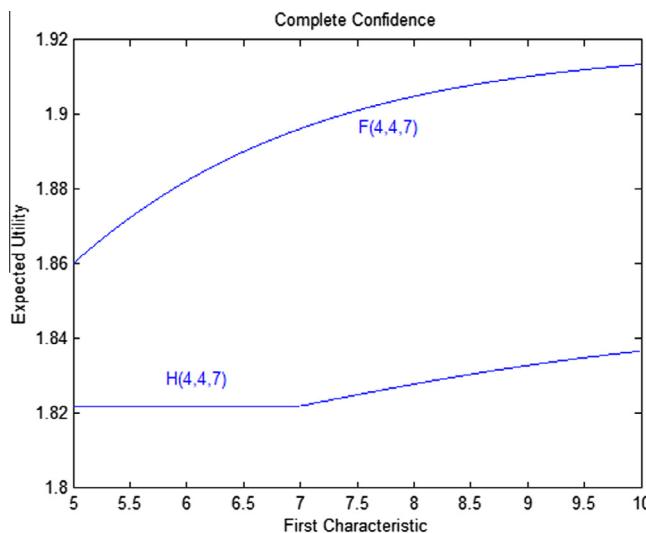


**Fig. 11.** Threshold values with constant risk-averse utilities: increments in the likelihood of product improvement. Continuation and starting intervals for the unsignalized market when DMs increment the credibility assigned to the improvements announced by the firm. The functions reflect the reports of DMs regarding three different linguistic values ( $R,L,C$ ):  $R$  stands for the subjective degree of risk aversion;  $L$  is the credibility assigned to the improvements announced by the firm;  $C$  measures the level of confidence in the firm.



**Fig. 12.** Threshold values with constant risk-averse utilities: increments in DM's confidence. Continuation and starting intervals for the unsignalized market when DMs increment their level of confidence in the firm. The functions reflect the reports of DMs regarding three different linguistic values ( $R,L,C$ ):  $R$  stands for the subjective degree of risk aversion;  $L$  is the credibility assigned to the improvements announced by the firm;  $C$  measures the level of confidence in the firm.

**Managerial insight 7.** Consider a scenario where DMs maintain the same risk attitude independently of the payoff derived from the product or project. If DMs do not completely trust firms to deliver a quality product when choosing randomly and signals are not fully credible, increments in either confidence or credibility may not modify the behavior of DMs. However, a sufficiently high degree of confidence may lead DMs to consider exclusively the initial product observed independently of the value of its first characteristic.



**Fig. 13.** Threshold values with constant risk-averse utilities: full DM's confidence in the firm. Continuation interval for the unsignaled market when DMs have full confidence in the firm. The functions reflect the reports of DMs regarding three different linguistic values ( $R, L, C$ ):  $R$  stands for the subjective degree of risk aversion;  $L$  is the credibility assigned to the improvements announced by the firm;  $C$  measures the level of confidence in the firm.

## 6. Conclusions and future research directions

DMs shift from unsignaled to signaled technological markets in order to improve upon the products that may be observed through their information acquisition process. As a result, if a firm (information sender) is unable to guarantee a minimum quality level (a given certainty equivalent product) because of its reputation, stricter acceptance criteria will be (optimally) imposed among DMs when considering the set of products offered by the firm. Thus, improving the (technological) characteristics of a product and credibly announcing its introduction in the market does not necessarily guarantee its immediate adoption, a phenomenon already identified by the [empirical] economic and business literatures, see Malerba et al. [44,45] and Christensen [17], respectively.

The current model allows us to calculate a considerable amount of alternative scenarios, assign a probability to each one of them, derive the resulting threshold values and generate intervals regarding the value of the characteristics expected to be observed by a DM in order to continue acquiring information on a given product or project. In this respect, managers may consider the reports provided by DMs and weight strategically the resulting interval values according to the importance assigned to the DM providing the reports. The potential direct implementation of the introduced information acquisition model within the AHP and TOPSIS methods allows for the inclusion of uncertainty and strategic reporting within these multicriteria decision tools.

In addition, information acquisition (and choice) reversals may arise if DMs are allowed to reverse their information acquisition processes and compare the observations retrieved within different markets. The strategic dimension arising from these results should be developed further and its consequences for the behavior of firms and managers when deciding whether or not to enter a given market or create a technological monopoly analyzed in future research. That is, the strategic dimension arising from either the subjective beliefs of DMs regarding the reputation of the information sender or their interactions with other DMs, endowed with their own subjective criteria, see [62,71], should constitute an exciting extension of the current research.

Finally, the existence of self-reinforcing mechanisms in the information acquisition processes of DMs, as illustrated empirically by Jones and Sugden [31], as well as the strategic dimension generated by the subjective choice of sources and channels of information acquisition, see [34,48], respectively, constitute potential extensions of the current model to scenarios that remain barely explored.

## Acknowledgments

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

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