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An integrated multi-objective framework for solving multi-period project selection problems

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ABSTRACT

Investment managers are multi-objective decision-makers (DMs) who make portfolio decisions by maximizing profits and minimizing risks over a multi-period planning horizon. Portfolio decisions are complex multi-objective problems which include both tangible and intangible factors. We propose an integrated multi-objective framework for project portfolio selection with respect to both the profits and risks objectives. The proposed method is based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and an efficient version of the epsilon-constraint method. TOPSIS is used to reduce the Multi-Objective Decision Making (MODM) problem into a bi-objective problem. The efficient epsilon-constraint method is used to generate non-dominated solutions with a pre-defined and arbitrary resolution on the Pareto front of the aforementioned bi-objective problem. The results from the integrated framework proposed in this study are compared with the results from the conventional epsilon-constraint method based on a series of simulated benchmark cases. A sensitivity analysis is performed to study the sensitivity of the relative importance weights of the objective functions in re-generating the Pareto front. The practical application of the proposed framework illustrates the efficacy of the procedures and algorithms.

1. Introduction

The selection among various capital investment projects is a laborious task involving a simultaneous optimization of multiple conflicting or competing objectives. The complexities inherent in capital investment projects in particular make Multi-objective Decision Making (MODM) a valuable tool in the decision-making process. MODM problems often defy traditional methods of problem solving because of the numerous causal and interwoven objectives. The key to solving these difficult problems lies in the Decision Makers' (DMs') ability to formulate them with precision.

Formally, a MODM model considers a vector of decision variables, objective functions and constraints. DMs are expected to choose a solution from a set of efficient solutions because MODM problems rarely have a unique solution. Generally, a MODM problem with maximum objective functions can be formulated as follows:

$$(MODM)\begin{cases} \max & f(x) \\ s.t. & x \in S = \{x \in R^n | g(x) \leqslant b, x \geqslant 0\} \end{cases} \tag{1}$$

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where, $f(x)$ represents k conflicting objective functions, $g(x) \leqslant b$ represents m constraints, S is the feasible solution space and x is the n-vector of the decision variables, $x \in R^n$ [1].

For special kinds of MODM problems (mostly linear problems), there are several methods that produce the entire efficient set [2]. These methods can provide a representative subset of the Pareto set which in most cases is adequate. The ε-constraint Method proposed by Chankong and Haimes [3] is a one of those techniques. In this method, the DM chooses one objective out of n to be optimized while the remaining objectives are constrained to be less than or equal to given target values. One advantage of the ε-constraint method is its ability to achieve efficient points in a non-convex Pareto curve. Therefore, as proposed by Steuer [4], the DM can vary the upper bounds ε_i to obtain weak Pareto optima. This method also has some drawbacks in choosing the appropriate upper bounds for the ε_i values and the efficiency of calculations as the number of objective functions increases. Mavrotas [5] has proposed a novel version of the ε-constraint method (i.e. augmented ε-constraint method – *AUGMECON*) that avoids the production of weakly Pareto optimal solutions and accelerates the entire process by avoiding redundant iterations.

In this paper, we propose a MODM framework to generate a set of non-dominated solutions with a pre-defined resolution. The proposed framework has two main phases. The first phase reduces the MODM problem into a bi-objective problem based on the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). In the second phase, an efficient ε-constraint procedure is utilized to solve the resultant bi-objective model. The trimming of the MODM problem to a bi-objective problem significantly reduces the computational efforts of the ε-constraint procedure.

A new method called Multi-objective Project Selection Programming with Multi-Period Planning Horizon (*MOPSP-MPPH*) is proposed to select an optimum portfolio of projects. Benchmark cases are used in a simulation process to survey the performance of the proposed framework. A sensitivity analysis is performed to study the sensitivity of the relative importance weights of the objective functions in re-generating the Pareto front. A case study is presented to show the applicability of the proposed framework and illustrate the efficacy of the procedures and algorithms.

The remainder of the paper is organized as follows. We present a brief review of the capital investment project selection literature in Section 2. The proposed integrated framework is developed in Section 3. In Section 4, we present a new mathematical programming for the *MOPSP-MPPH*. The simulation experiment and the sensitivity analysis results along with a case study are presented in Section 5. We complete our paper with conclusions and future research directions in Section 6.

2. Brief review of the capital investment project selection literature

Khalili-Damghani et al. [6] developed a modular decision support system to select an optimum portfolio of investment projects in the presence of uncertainty. Their proposed system has two main modules. The first module included a fuzzy binary programming model of the capital budgeting problem. It involved finding the optimum combination of the investment projects with a multi-objective measurement function and subject to several set of constraints. The outputs of first module plus a managerial confidence level value were used as the input of the fuzzy rule-based system. The procedure was used to determine the risks associated with capital investments.

Shakhsi-Niaei et al. [7] proposed a comprehensive framework for project selection problem under uncertainty and subject to a set of real-world constraints. Their proposed framework consisted of two main phases. In the first phase, a Monte Carlo simulation linked to a multi-criteria approach was used to rank the candidate projects. In the second phase, the overall complete preorder of the projects was first determined and then used in another Monte Carlo simulation linked to an integer programming module to effectively drive the final portfolio selection. Farzipoor Saen [8] proposed a data envelopment analysis approach for technology investment project selection. Liu and Gao [9] used the mean-absolute deviation for the portfolio optimization problem in a frictional market with additional constraints. An algorithm was proposed to solve the optimization problem based on linear programming.

Wei and Ye [10] considered a multi-period mean–variance portfolio selection model imposed by a bankruptcy constraint in a stochastic market. The random returns of risky assets were modeled using a Markov chain and dynamic programming was used to derive an optimal portfolio policy. Bilbao-Terol et al. [11] proposed a new fuzzy compromise programming approach based on the minimum fuzzy distance to the fuzzy ideal solution of the portfolio selection problem.

Golmakani and Fazel [12] presented a novel heuristic method for solving an extended Markowitz mean–variance portfolio selection problem based on particle swarm optimization. The extended model included four sets of constraints: bounds on holdings, cardinality, minimum transaction lots and sector (or market/class) capitalization constraints. They compared particle swarm optimization with a genetic algorithm on different sets of cases. The proposed particle swarm optimization outperformed genetic algorithm.

Zhu et al. [13] used a meta-heuristic approach based on particle swarm optimization technique to solve a non-linear constrained portfolio optimization problem with multi-objective functions. Their approach outperformed genetic algorithm on different sets of cases.

Chan et al. [14] developed a goal seeking model to address different variations of the capital budgeting problem. They proposed a multi-criteria optimization model which considered a diverse set of functions in one organization. Coldrick et al. [15] developed a project selection and evaluation tool that could be used for solving a wide range of research, technology and investment decisions. Their proposed evaluation tool considered important research and development factors such as uncertainty, interrelationships between projects, changes over time and success factors that were difficult to measure with the optimization project selection models in practice. Steuer and Paul [16] surveyed the applications of methodologies such as goal programming, multiple objective programming, and the analytic hierarchy process among others in capital budgeting, working capital management and portfolio analysis. Antonio et al. [17] considered optimal capital allocation and managerial compensation mechanisms in decentralized firms when division managers had an incentive to misrepresent project quality and to minimize costly but value-enhancing efforts. Badri Masood et al. [18] developed a 0–1 goal programming model for information system project selection. They considered several factors that impacted the decision to select an information system project. These factors were DMs' preferences and priorities, profits, risks, costs, project durations and the availability of other scarce resources. Padberg and Wilczak [19] used mathematical programming to obtain an optimal decision rule for project selection in capital budgeting in a non-perfect capital market. Timothy and Kalu [20] utilized goal programming for capital budgeting under uncertainty. They proposed the necessary and sufficient conditions for the acceptance of a set of investment projects by a business enterprise.

3. Proposed integrated MODM framework

The TOPSIS method, introduced by Hwang and Yoon [21], ranks the alternative choices in multi-attribute decision making problem according to an algorithmic procedure. The alternatives are sorted in decreasing order of their Closeness Coefficients (CCs) which is calculated with respect to the distance of a given alternative from both positive and negative ideal solutions concurrently. The application of TOPSIS for MODM problems was first introduced by Lai et al. [22]. They used the compromise property of TOPSIS to generate solutions. They reduced a k-dimensional objective space to a two-dimensional objective space by a first-order compromise procedure.

The compromise property of TOPSIS helps in generating desired solutions which are far from the Negative Ideal Solution (NIS) and near the Positive Ideal Solution (PIS), simultaneously. This property is a clear advantage in real MODM problems where the DM is interested in low-risk and high-return solutions, simultaneously. All objective functions directly affect the generation of the resulting bi-objective problem. In other words, no objective is completely omitted from consideration. The relative importance of the objectives in the original MODM problem can be easily controlled through the weights and the order of compromise which are determined by the DM.

3.1. TOPSIS method for MODM problems

In this section we extend the concept of TOPSIS for MODM problems to obtain a compromise (non-dominated) solution. Table 1 presents the notations used in the proposed algorithm.

Step 1. Solve the single objective optimization problems using the same constraints of the original MODM problem (1) as (2):

$$Max\, f_i(X), \quad i = 1,2,\ldots,k; \quad g_j(X) \leqslant B_j, \quad j = 1,2,\ldots,m \tag{2}$$

Step 2. Consider the original MODM problem (1) and calculate Z^+ and Z^- vectors as (3) and (4):

$$Z^- = (z_1^-, z_2^-, \ldots, z_i^-, \ldots, z_{k-1}^-, z_k^-) \tag{3}$$

Table 1
The notations used in the TOPSIS for MODM.

Indices	
i	Index of the objective functions
j	Index of the constraints
k	Number of the objective functions
m	Number of the constraints
Parameters	
$f_i(X)$	The ith objective function of the MODM problem
$g_j(X)$	The jth constraint of the MODM problem
B_j	The right-hand-side value of jth constraint of the MODM problem
Z^-	Nadir vector of objective functions of the MODM problem
z_i^-	Nadir value of ith objective function of the MODM problem
z_i^+	Ideal value of ith objective function of the MODM problem
W_i	The relative importance of ith objective
P	The compromising order of the algorithm
d_p^{PIS}	Distance of pth compromise degree from Positive Ideal Solution (PIS)
d_p^{NIS}	Distance of pth compromise degree from Negative Ideal Solution (NIS)
S	Feasible Space of MODM problem
Decision variables	
X	Vector of Positive decision variables
x_n	The nth positive decision variable

$$Z^{+} = (z_1^{+}, z_2^{+}, \ldots, z_i^{+}, \ldots, z_{k-1}^{+}, z_k^{+}) \tag{4}$$

where Z^{+} is the ideal vector in the original MODM problem (1) (i.e. PIS) and Z^{-} is the nadir vector in the original MODM problem (1) (i.e. NIS).

Step 3. Use the NIS, the PIS and the DM's opinion about the relative importance of the objective functions, calculate the distance from the NIS and the distance from the PIS as (5) and (6):

$$d_p^{PIS} = \left[\sum_{\text{for all min obj.}} \left[W_i \times \frac{(f_i(X) - z_k^{+})}{z_k^{-} - z_k^{+}} \right]^p + \sum_{\text{for all max obj.}} \left[W_i \times \frac{(z_k^{+} - f_i(X))}{z_k^{+} - z_k^{-}} \right]^p \right]^{\frac{1}{p}} \tag{5}$$

$$d_p^{NIS} = \left[\sum_{\text{for all min obj.}} \left[W_i \times \frac{(z_k^{-} - f_i(X))}{z_k^{-} - z_k^{+}} \right]^p + \sum_{\text{for all max obj.}} \left[W_i \times \frac{(f_i(X) - z_k^{-})}{z_k^{+} - z_k^{-}} \right]^p \right]^{\frac{1}{p}} \tag{6}$$

where $\sum_{i=1}^{k} W_i = 1$, and $p = 1, 2, \ldots, \infty$. we should note that d_p^{PIS} and d_p^{NIS} are scale independent measures.

Step 4. Solve the following resultant bi-objective problem:

$$\begin{aligned} &Min \quad d_p^{PIS} \\ &Max \quad d_p^{NIS} \\ &s.t. \quad\; X \in S \end{aligned} \tag{7}$$

where S means the feasible space of the original MODM problem (1).

In Model (7), we intend to re-generate a special part of the Pareto front which has desirable properties for the project selection problem. In practice, non-dominated solutions, which are concurrently near the PIS and far from the NIS, are potentially useful for multi-objective project selection problems. The nadir point in project selection problems can be interpreted as the point with high risks and low profits whereas the ideal point can be interpreted as the point with low-risks and high-profits. Investment managers prefer low-risk high-profit portfolios. Our proposed framework allows investment managers to attain both objectives concurrently in the MODM problem.

Fig. 1 presents the symbolic effects of applying TOPSIS to a bi-objective minimization MODM problem. The TOPSIS for MODM focuses on the Pareto front and ignores the non-dominated solutions which do not consider the risks and profits objectives, simultaneously. Moreover, the DMs in real-life MODM problems might be interested in guiding the solutions to a specific area of the feasible region in which non-dominated solutions on the Pareto front have low-risks and high-profits, simultaneously.

The proposed method results in a restricted Pareto front. While generating non-dominated solutions with an unrestricted or wide range of solutions on the Pareto front of a MODM problem is preferred in some cases, a restricted or narrow range of solutions on the Pareto front is preferable in real-life capital investment projects. It is much easier for a DM to choose among a limited number of solutions versus a large number of solutions.

3.2. Efficient epsilon-constraint method

Let us reconsider the MODM problem (1). In the ε-constraint method, one of the objective functions (e. g.$f_j(x)$) is optimized while the other objective functions are formulated as constraints in the model. Following are the necessary steps involved in solving a MODM problem with the efficient epsilon-constraint method.

Step1. Calculate the payoff table by using lexicographic optimization of the objective functions.

Step2. Divide the ranges of the objective functions into T equal intervals and use the $T + 1$ grid points as the values of the RHS of the constrained objective functions.

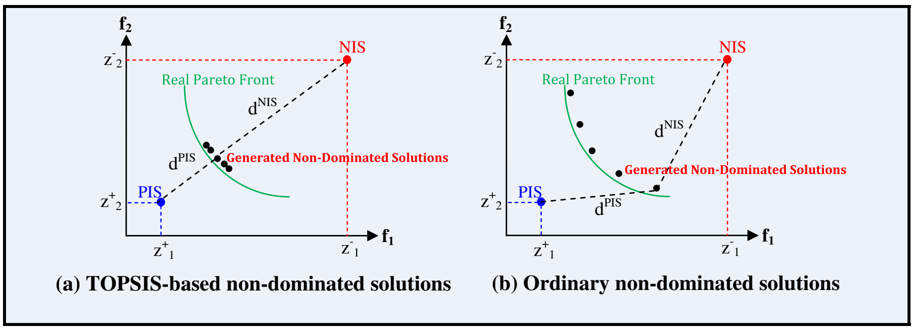

Fig. 1. A pictorial comparison between the TOPSIS-based and ordinary solutions.

Step3. Solve the set of resultant single objective problems.

The number of required single optimization problems for a full analysis in conventional ε-constraint method is equal to $k \times (T+1)^{k+1}$, where k is the number of objective functions and T is the interval numbers. This number is effectively reduced in the *AUGMECON* method.

It is obvious that the optimal solution of the resultant single objective problem is guaranteed to be efficient if and only if the value of the slack or the surplus variables of the entire associated (k-1) constrained objective functions are equal to zero. In order to overcome this problem the following slack-based models are solved [5]:

$$\begin{aligned}
&\max f_j(x) - \beta \times (s_1 + \ldots + s_{j-1} + s_{j+1} + \ldots + s_k) \\
&s.t. \\
&f_i(x) - s_i = \varepsilon_i, \quad \forall i \in \{1,\ldots,k\}, \quad i \neq j \\
&X \in S \\
&s_i \in \Re^+ \quad \forall i \in \{1,\ldots,k\}, \quad i \neq j
\end{aligned} \tag{8}$$

where β is a small number usually between 0.001 and 0.000001.

Model (8) produces only efficient solutions. Some considerations of the commensurability in objective functions may be desirable, so that the objective function of Model (8) will be $f_j(x) - \beta \times (s_1/r_1 + \ldots + s_{j-1}/r_{j-1} + s_{j+1}/r_{j+1} + \ldots + s_k/r_k)$, where $r_i, i = 1,\ldots,k$ represents the range of the objective i which is calculated based on the lexicographic payoff table.

3.3. Proposed framework

The following model is proposed by setting $(-d_p^{PIS}) = d_p^1$, $d_p^{NIS} = d_p^2$, and by considering Model (8) and the efficient ε-constraint method:

$$\begin{aligned}
&\max \quad d_p^1 + \beta \times \tfrac{s_2}{r_2} \\
&s.t. \\
&\qquad d_p^2 - s_2 = \varepsilon_2 \\
&\qquad X \in S \\
&\qquad s_2 \in \Re^+
\end{aligned} \tag{9}$$

where r_2 represents the ranges of the second objective which is calculated using the payoff table. This helps the commensurability of Model (10). The parametric nature of the proposed model can help DMs to generate several non-dominated solutions with desirable properties.

4. Multi-objective project selection problem formulation

In this section, a new multi-objective mathematical programming is proposed to select a portfolio of independent investments in the form of projects in a multi-planning period. The model is considered in an environment where all the parameters of the projects are assumed to differ during the planning horizon. Suppose that an organization is facing several investment projects. The notations used in the proposed multi-objective mathematical programming are presented in Table 2.

4.1. Objective functions

The objective function (10) is used to maximize the net profit of the selected projects:

$$\text{Max } Z_1 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \times p_{jt} \tag{10}$$

The objective function (11) is intended to minimize the total cost of the selected projects:

$$\text{Min } Z_2 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{k=1}^{s} m_{kj}.C_{kt} + \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt}\sum_{o=1}^{z} r_{oj}\cdot C_{ot} \tag{11}$$

The objective function (12) is proposed to maximize the total internal rate of return of the selected projects:

$$\text{Max } Z_3 = \sum_{t=1}^{T}\sum_{j=1}^{n} x_{jt} \times I_{jt} \tag{12}$$

Finally, the objective function (13) is designed to minimize the total unused resources of the optimum portfolio:

$$\text{Min } Z_4 = \sum_{t=1}^{T}\left[\sum_{i=1}^{m}(H_{it} - \sum_{j=1}^{n} h_{ij}.x_{jt}) + \sum_{k=1}^{s}(M_{kt} - \sum_{j=1}^{n} m_{kj}.x_{jt}) + \sum_{o=1}^{z}(R_{ot} - \sum_{j=1}^{n} r_{oj}.x_{jt})\right] \tag{13}$$

Table 2
The notations used in the proposed multi-objective mathematical programming.

Indices		
j	Number of projects	$j = 1, 2, \ldots, n.$
i	Type of human resources	$i = 1, 2, \ldots, m.$
k	Kind of machines	$k = 1, 2, \ldots, s.$
o	Type of raw material	$o = 1, 2, \ldots, z.$
t	The planning horizon	$t = 1, 2, \ldots, T.$
Parameters		
H_{it}	Maximum available human resource of type i in period t (person-hour)	
h_{ij}	Requirement of human resource i in project j (person-hour)	
M_{kt}	Maximum available machine-hour of type k in period t	
m_{ij}	Requirement of machine-hour of type k in project j	
R_{ot}	Maximum available raw material of type o in period t	
r_{oj}	Requirement of raw material o in project j	
B_{jt}	Maximum available budget for project j in period t	
C_{it}	Per hour cost of human resource i in period t	
C_{kt}	Per hour cost of machine type k in period t	
C_{ot}	Unit cost material o in period t	
p_{jt}	Total net profit of project j in period t	
I_{jt}	Rate of return of project j in period t	
MARR_t	Minimum attractive rate of return in period t	
d_{jt}	Duration of project j in period t	
Decision variables		
x_{jt}	$\begin{cases} 1 & \text{if project } j \text{ is selected for investement in period } t \\ 0 & \text{otherwise} \end{cases}$	

Table 3
The test problems.

Case	Project	Period	H_{it}, M_{kt}, R_{ot}	h_{ij}, m_{ij}, r_{oj}	$\mathrm{P_{jt}}$	$\mathrm{B_{jt}}$	$\mathrm{C_{it}, C_{kt}, C_{ot}}$	I_{jt}	$\mathrm{MARR_t}$	d_{jt}
I	15	2	U[1,1000]	U[0,20]	U[1,10000]	U[1,100000]	U[0,5]	U[1,10]	U[1,5]	U[0,1]∗
II	10	3	U[1,1000]	U[0,20]	U[1,10000]	U[1,100000]	U[0,5]	U[1,10]	U[1,5]	U[0,2]∗
III	7	4	U[1,1000]	U[0,20]	U[1,10000]	U[1,100000]	U[0,5]	U[1,10]	U[1,5]	U[0,3]∗
IV	10	2	U[1,1000]	U[0,20]	U[1,10000]	U[1,100000]	U[0,5]	U[1,10]	U[1,5]	U[0,1]∗

The parameters are generated using a discrete uniform destiny function except for the cases marked with ∗ sign which are continuous.

Table 4
The payoff matrix of the original and bi-objective TOPSIS-based problems.

Case	Upper bound of ideal objective functions						Lower bound of objective functions					
	Z_1	Z_2	Z_3	Z_4	d_{PIS}	d_{NIS}	Z_1	Z_2	Z_3	Z_4	d_{PIS}	d_{NIS}
I	49517	1828	30	14636	3	2.96	0	0	0	13926	1.038	1
II	23420	845	29	30291	3	2.98	0	0	0	29815	1.046	1
III	26797	978	23	31642	3	2.96746	0	0	0	31298	1.03254	1
IV	19493	1236	24	14636	3	2.923	0	0	0	14129	1.077	1

4.2. Constraints

Constraints (14) are defined for all the projects and are used to ensure that the chosen project is selected only one time throughout the planning horizon:

$$\sum_{t=1}^{T} x_{jt} \leqslant 1, \quad j = 1, 2, \ldots, n \tag{14}$$

Constraints (15) are also defined for all the projects and are proposed to ensure that each selected project is completed during the planning horizon:

$$\sum_{t=1}^{T} (t + d_{jt}).x_{jt} \leqslant T + 1, \quad j = 1, 2, \ldots, n \tag{15}$$

Table 5
The objective values of the solutions generated by the proposed framework.

Case		E_2 values										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I	d_{PIS}	1.038	1.038	1.038	1.038	1.038	1.038	1.038	1.038	1.038	1.038	1.038
	d_{NIS}	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96	2.96
	Z_1	47683	47683	47683	47683	47683	47683	47683	47683	47683	47683	47683
	Z_2	1828	1828	1828	1828	1828	1828	1828	1828	1828	1828	1828
	Z_3	30	30	30	30	30	30	30	30	30	30	30
	Z_4	13926	13926	13926	13926	13926	13926	13926	13926	13926	13926	13926
	O.F.V.	1.0391	1.0391	1.0389	1.0388	1.0387	1.0386	1.0385	1.0384	1.0383	1.0382	1.0381
II	d_{PIS}	1.046126	1.046126	1.046126	1.046126	1.046126	1.046126	1.046126	1.046126	1.046126	1.046126	Infeasible
	d_{NIS}	2.976195	2.976195	2.976195	2.976195	2.976195	2.976195	2.976195	2.976195	2.976195	2.976195	Infeasible
	Z_1	23420	23420	23420	23420	23420	23420	23420	23420	23420	23420	Infeasible
	Z_2	820	820	820	820	820	820	820	820	820	820	Infeasible
	Z_3	28	28	28	28	28	28	28	28	28	28	Infeasible
	Z_4	29824	29824	29824	29824	29824	29824	29824	29824	29824	29824	Infeasible
	O.F.V.	1.047138	1.047036	1.046935	1.046834	1.046732	1.046631	1.046530	1.046428	1.046327	1.046226	Infeasible
III	d_{PIS}	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541	1.032541
	d_{NIS}	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459	2.967459
	Z_1	25925	25925	25925	25925	25925	25925	25925	25925	25925	25925	25925
	Z_2	978	978	978	978	978	978	978	978	978	978	978
	Z_3	23	23	23	23	23	23	23	23	23	23	23
	Z_4	31298	31298	31298	31298	31298	31298	31298	31298	31298	31298	31298
	O.F.V.	1.033541	1.033441	1.033341	1.033241	1.033141	1.033041	1.032941	1.0328	1.0327	1.0326	1.0325
IV	d_{PIS}	1.077146	1.077146	1.077146	1.077146	1.077146	1.077146	1.077146	1.077146	1.077146	1.077146	Infeasible
	d_{NIS}	2.922854	2.922854	2.922854	2.922854	2.922854	2.922854	2.922854	2.922854	2.922854	2.922854	Infeasible
	Z_1	17658	17658	17658	17658	17658	17658	17658	17658	17658	17658	Infeasible
	Z_2	1215	1215	1215	1215	1215	1215	1215	1215	1215	1215	Infeasible
	Z_3	24	24	24	24	24	24	24	24	24	24	Infeasible
	Z_4	14129	14129	14129	14129	14129	14129	14129	14129	14129	14129	Infeasible
	O.F.V.	1.078146	1.078046	1.0779	1.0778	1.0777	1.0776	1.0775	1.077446	1.077346	1.077246	Infeasible

Table 6
Objective values of the solutions generated by the *AUGMECON* method.

Case		Epsilon values*										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I	d_{PIS}	1.061210	1.061210	1.218997	1.455217	1.718686	1.892864	1.996588	2.170765	2.600392	2.774570	3
	d_{NIS}	2.938790	2.938790	2.781003	2.544783	2.281314	2.107136	2.003412	1.829235	1.399608	1.225430	1
	Z_1	49517	49517	47164.00	43155	38835	33578	26823	21566	13300	8043	0
	Z_2	1582	1582	1377	1201	1041	866	706	531	318	143	0
	Z_3	28	28	25	21	16	14	14	12	4	2	0
	Z_4	14017	14017	14104	14188	14257	14326	14365	14434	14514	14583	14636
II	d_{PIS}	1.046126	1.303204	1.686018	1.579500	1.911399	2.294214	2.187696	2.570510	2.878143	3	3
	d_{NIS}	2.976195	2.716260	2.330587	2.434818	2.101150	1.715477	1.819708	1.434036	1.125940	1	1
	Z_1	23420	22593	18940	17979	15845	12192	11231	7578	1408	0.0001	0
	Z_2	820	715	610	526	461	356	272	167	150	0	0
	Z_3	28	23	17	19	14	8	10	4	2	0	0
	Z_4	29824	29908	29978	29989	30059	30129	30140	30210	30208	30291	30291
III	d_{PIS}	1.069915	1.069915	1.503729	1.503729	1.503729	1.893988	1.893988	2.535322	2.535322	3	3
	d_{NIS}	2.930085	2.930085	2.496271	2.496271	2.496271	2.106012	2.106012	1.464678	1.464678	1	1
	Z_1	26797	26797	22607	22607	22607	15900	15900	6811	6811	0	0
	Z_2	802	802	571	571	571	387	387	183	183	0	0
	Z_3	22	22	15	15	15	12	12	5	5	0	0
	Z_4	31369	31369	31441	31441	31441	31509	31509	31580	31580	31642	31642
IV	d_{PIS}	1.083333	1.264638	1.577484	1.577484	1.805887	1.805887	2.265531	2.424068	2.613059	3	3
	d_{NIS}	2.916667	2.735362	2.422516	2.422516	2.194113	2.194113	1.734469	1.575932	1.386941	1	1
	Z_1	19493	19221	18929	18929	16278	16278	12256	7908	6999	0	0
	Z_2	1236	1015	774	774	617	617	409	332	234	0	0
	Z_3	22	18	11	11	9	9	3	4	1	0	0
	Z_4	14129	14220	14322	14322	14391	14391	14478	14498	14547	14636	14636

* The value of E_3 & E_4 are assumed to be equal to zero while the E_2 is increased through a step-size equal to 0.1.

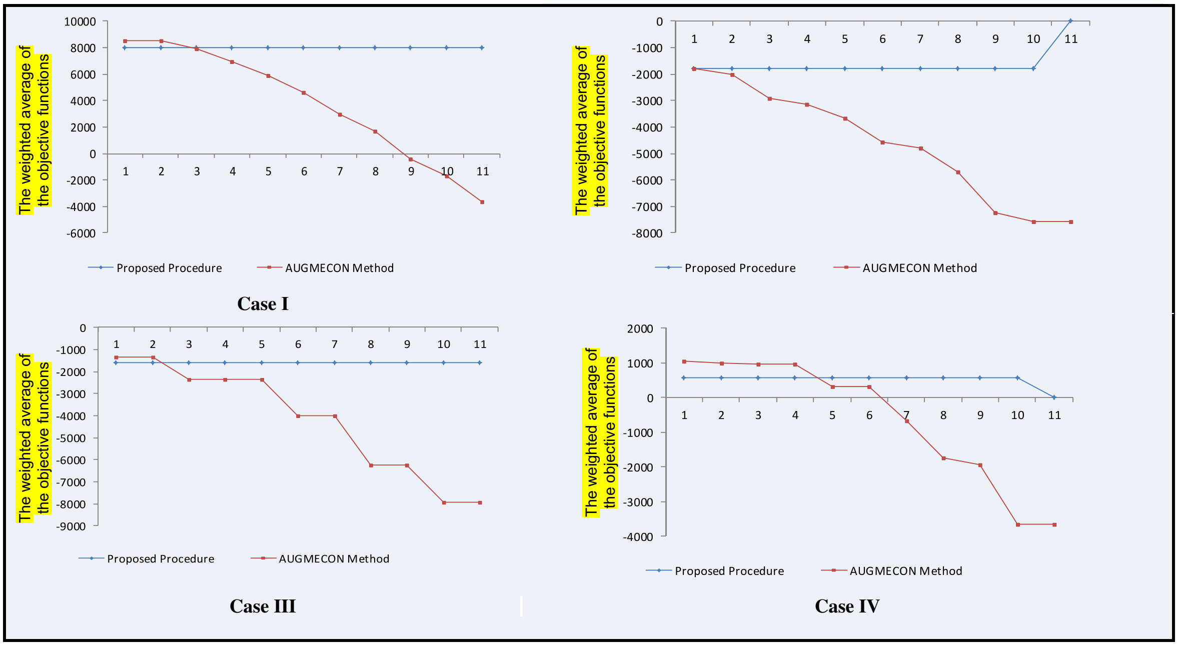

Fig. 2. A graphical comparison of the objective function values in the proposed framework and the *AUGMECON* method.

Constraints (16) are defined for all the human resources of the projects in all the planning horizons. These constraints ensure that human resources availability is met during the project selection process:

$$\sum_{j=1}^{n} h_{ij} x_{jt} \leqslant H_{it}, \quad i = 1, 2, \ldots, m, \quad t = 1, 2, \ldots, T \tag{16}$$

Constraints (17) and (18) have the same description as constraints (16) but they represent machine-hours and raw materials, respectively:

$$\sum_{j=1}^{n} m_{kj} x_{jt} \leqslant M_{kt}, \quad k = 1, 2, \ldots, s, \quad t = 1, 2, \ldots, T \tag{17}$$

$$\sum_{j=1}^{n} r_{oj} x_{jt} \leqslant R_{ot}, \quad o = 1, 2, \ldots, z, \quad t = 1, 2, \ldots, T \tag{18}$$

Constraints (19) are defined for all the projects in all the planning horizons. These constraints check the budget availability:

$$\left(\sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{k=1}^{s} m_{kj}.C_{kt} + \sum_{o=1}^{z} r_{oj}.C_{ot} \right) \times x_{jt} \leqslant B_{jt}, \quad j = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, T \tag{19}$$

Constraints (20) are also held for all projects in all planning horizons. They ensure that the total cost of a selected project is less than its profit:

$$\left(\sum_{i=1}^{m} h_{ij}.C_{it} + \sum_{k=1}^{s} m_{kj}.C_{kt} + \sum_{o=1}^{z} r_{oj}.C_{ot} \right) \times x_{jt} < P_{jt}, \quad j = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, T \tag{20}$$

Constraints (21) ensure that the selected projects have a rate of return greater than or equal to the Minimum Attractive Rate of Return (MARR):

$$\sum_{j=1}^{n} (x_{jt}.(MARR_t - I_{jt})) \leqslant 0, \quad t = 1, 2, \ldots, T \tag{21}$$

Constraints (22) refer to the projects that might be selected in each planning horizon:

Table 7
The structure of the solution vectors in the proposed framework and the AUGMECON method.

Case	E_2	Proposed Framework	*AUGMECON* Method
I (j = 1,2,...,15, & t = 1,2)	0	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{21}, x_{32}, x_{61}, x_{11,1}, x_{12,1}, x_{14,2}, x_{15,1}$
	0.1	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{21}, x_{32}, x_{61}, x_{11,1}, x_{12,1}, x_{14,2}, x_{15,1}$
	0.2	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{21}, x_{32}, x_{11,1}, x_{12,1}, x_{14,1}, x_{15,1}$
	0.3	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{21}, x_{32}, x_{12,1}, x_{14,1}, x_{15,1}$
	0.4	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{32}, x_{12,1}, x_{14,2}, x_{15,1}$
	0.5	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{11}, x_{12,1}, x_{14,2}, x_{15,1}$
	0.6	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{21}, x_{32}, x_{12,1}, x_{14,1}, x_{15,1}$
	0.7	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{21}, x_{14,1}, x_{15,1}$
	0.8	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{32}, x_{14,2}$
	0.9	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	$x_{14,2}$
	1	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{11,2}, x_{12,1}, x_{14,2}, x_{15,1}$	–
II (j = 1,2,...,10, & t = 1,2,3)	0	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$
	0.1	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	$x_{12}, x_{22}, x_{43}, x_{52}, x_{10,2}$
	0.2	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	$x_{12}, x_{43}, x_{52}, x_{10,2}$
	0.3	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	$x_{12}, x_{22}, x_{52}, x_{10,2}$
	0.4	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	x_{22}, x_{43}, x_{52}
	0.5	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	x_{43}, x_{52}
	0.6	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	x_{22}, x_{52}
	0.7	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	x_{52}
	0.8	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	x_{12}
	0.9	$x_{12}, x_{22}, x_{43}, x_{52}, x_{92}, x_{10,2}$	–
	1	Infeasible	–
III (j = 1,2,...,7, & t = 1,2,3,4)	0	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	$x_{21}, x_{31}, x_{51}, x_{61}$
	0.1	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	$x_{21}, x_{31}, x_{51}, x_{61}$
	0.2	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	$x_{21}, x_{31}, x_{51}, x_{61}$
	0.3	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{31}, x_{61}
	0.4	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}, x_{31}, x_{61}
	0.5	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}, x_{31}, x_{61}
	0.6	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}, x_{31}
	0.7	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}, x_{31}
	0.8	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}
	0.9	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	x_{11}
	1	$x_{21}, x_{32}, x_{41}, x_{51}, x_{61}$	–
IV (j = 1,2,...,10, & t = 1,2)	0	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	$x_{11}, x_{21}, x_{32}, x_{61}, x_{91}, x_{10,1}$
	0.1	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	$x_{11}, x_{21}, x_{32}, x_{61}, x_{10,1}$
	0.2	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	$x_{11}, x_{21}, x_{32}, x_{61}$
	0.3	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	$x_{11}, x_{21}, x_{32}, x_{61}$
	0.4	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	x_{11}, x_{32}, x_{61}
	0.5	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	x_{11}, x_{32}, x_{61}
	0.6	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	x_{11}, x_{32}
	0.7	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	x_{21}, x_{32}
	0.8	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	x_{11}
	0.9	$x_{11}, x_{21}, x_{31}, x_{61}, x_{91}, x_{10,1}$	–
	1	Infeasible	–

$$\sum_{j=1}^{n} x_{jt} \geqslant 0, \quad t = 1, 2, \ldots, T \tag{22}$$

Finally, constraints (23) represent the decision variables in the model:

$$x_{jt} \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \quad t = 1, 2, \ldots, T \tag{23}$$

4.3. Application of the proposed method on MOPSP-MPPH

The application of the *AUGMECON* method [5] on the *MOPSP-MPPH* with profit as the main objective function will result in models (24)–(28).

$$\text{Max}\, Z_1 + \beta \times (S_2/r_2 + S_3/r_3/ + S_4/r_4) \tag{24}$$

s.t.

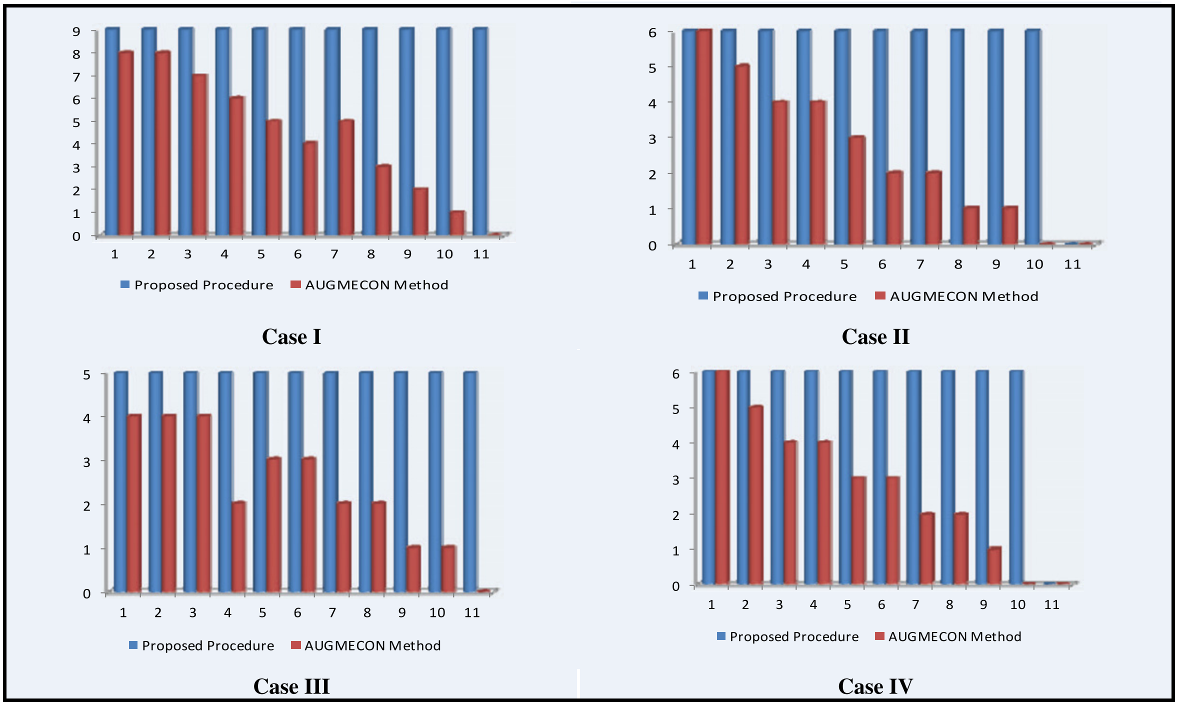

Fig. 3. The number of selected projects in the final solution for the proposed framework and the *AUGMECON* method.

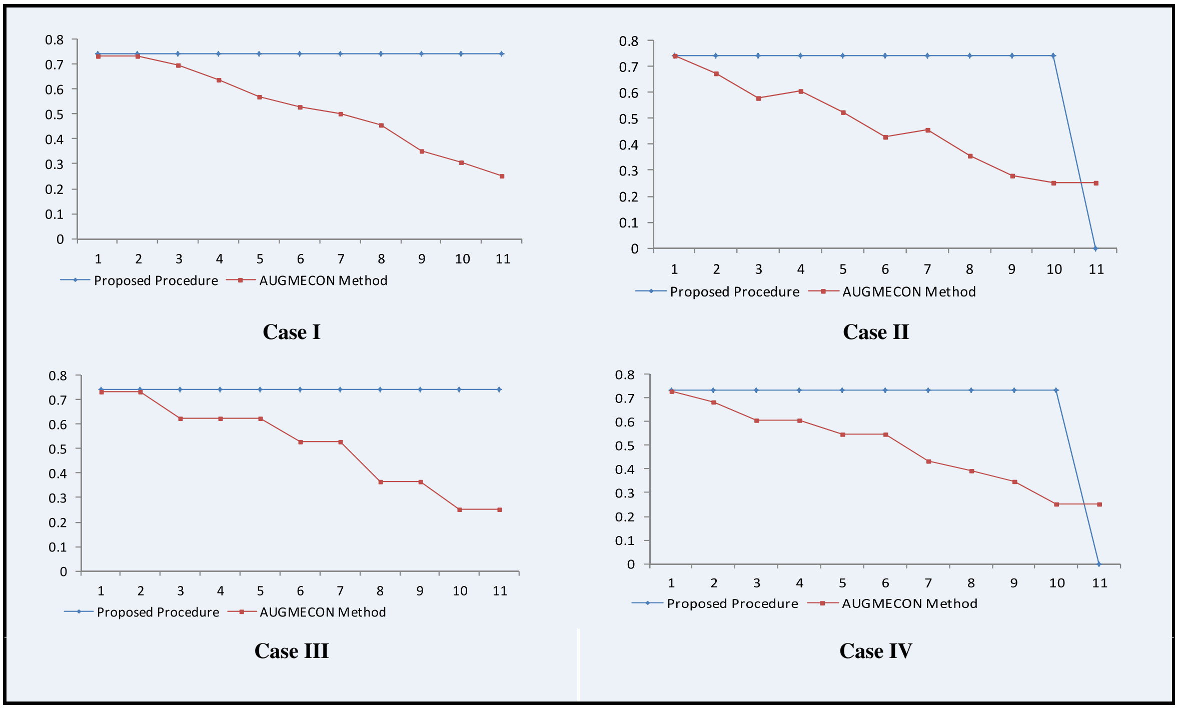

Fig. 4. A graphical comparison of the comparison index (i.e., CC) in the proposed framework and the *AUGMECON* method.

$$Z_2 - S_2 = \varepsilon_2, \quad \varepsilon_2 \in [Z_3^+, Z_3^-] \tag{25}$$

$$Z_3 + S_3 = \varepsilon_3, \quad \varepsilon_3 \in [Z_3^-, Z_3^+] \tag{26}$$

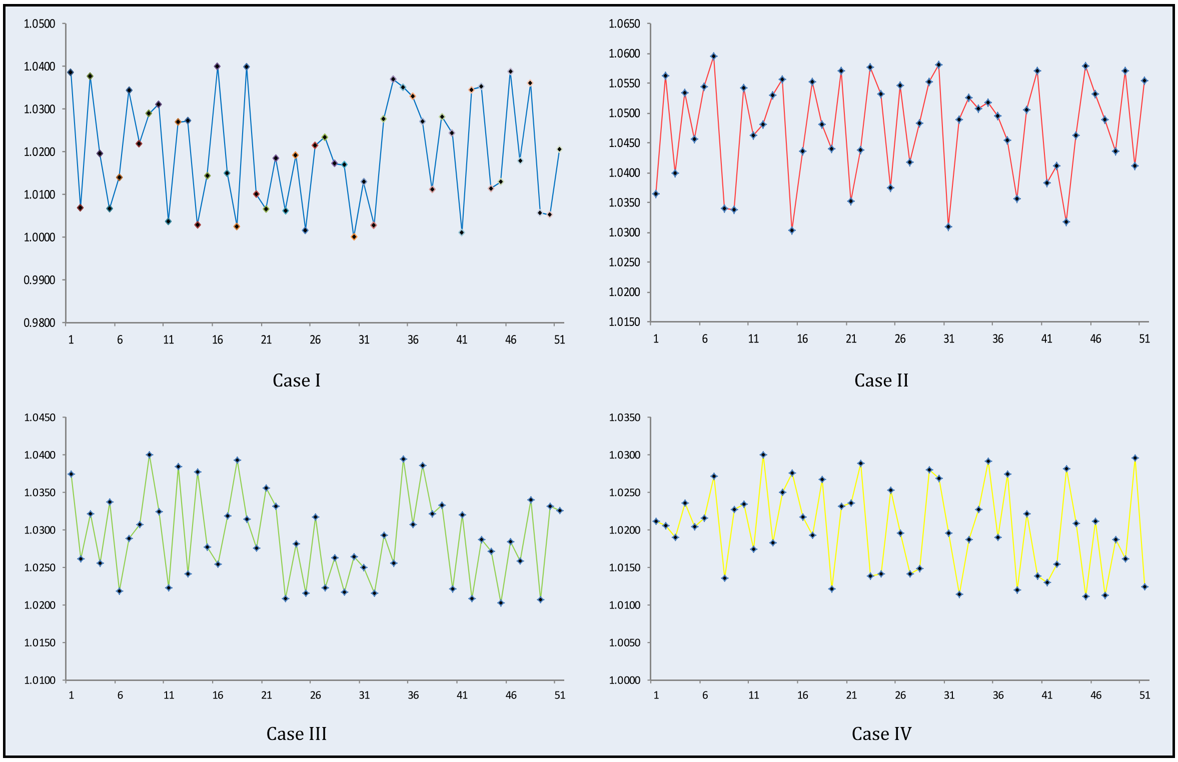

Fig. 5. The sensitivity analysis of the weights of the objectives for the proposed framework.

$$Z_4 - S_4 = \varepsilon_4, \quad \varepsilon_4 \in [Z_4^+, Z_4^-] \tag{27}$$

$$X \in S \tag{28}$$

where $r_i, \; i = 1, 2, 3$ represent the range of the ith objective function which is calculated using the lexicographic payoff table of *MOPSP-MPPH*. The relation (28) represents the feasible region of the *MOPSP-MPPH* (i.e., constraints (14)–(23)).

The application of the proposed framework on the *MOPSP-MPPH* preferring minimization of d_{PIS} as the main objective function with $p = 1$ and equal relative importance for the objectives, results in Eqs. (29)–(31):

$$Min\ d_{PIS} + \beta \times \left(\frac{S_2}{r_{d_{NIS}}}\right) \tag{29}$$

$$s.t.$$

$$d_{NIS} + S_2 = \varepsilon_2, \quad \varepsilon_2 \in [d_{NIS}^-, d_{NIS}^+] \tag{30}$$

$$X \in S \tag{31}$$

where, d_{NIS}^- and d_{NIS}^+ are calculated using the lexicographic payoff table. $r_{d_{NIS}}$ represents the range of the objective d_{NIS} . The relation (31) represents the feasible region for the *MOPSP-MPPH*.

5. Experimental results

We used simulation to evaluate the performance of the proposed framework. All data presented in this section are generated by the simulation model. The proposed framework and the *AUGMECON* method were experimented on four test cases of the *MOPSP-MPPH*. Different categories of the test problem are provided in this section. A uniform probability distribution was used for the simulation parameters. Table 3 presents the parameters of the simulated test problems. We should note that the types of human resources, machines and materials which are associated with the i, k, and o indices are assumed to be equal to four for all cases. The step-size of all ε-constraint parameters were set equal to 0.25 for both algorithms.

We developed a series of generic codes in LINGO 11.0 and linked them to MS-Excel 12.0 to analyze the simulated cases.

Table 8
The budgets, profits, and project durations of the investment projects.

B_{jt} ($10 Million)										
Project	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Machinery parts	6	8	8	9	6	8	6	9	6	7
Crude oil distillation	2	4	2	2	4	3	4	3	3	4
Telecommunication	2	2	1	2	2	1	2	1	1	1
New-energies	2	2	1	1	1	2	1	2	1	1
Water resource	2	1	2	1	1	1	1	2	1	1
Cement industries	1	1	1	1	2	2	1	2	2	2
Steel industries	4	4	4	3	3	4	4	2	2	3
Road construction	13	14	14	11	11	12	14	11	10	10
Rail transportation	2	1	3	2	1	1	1	3	2	2
Information technology	2	2	2	1	2	2	1	1	2	1
Agriculture	2	3	2	3	3	2	3	3	2	2
P_{jt} ($10 Million)										
Project	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Machinery parts	12	16	16	18	12	16	12	18	12	14
Crude oil distillation	6	12	6	6	12	9	12	9	9	12
Telecommunication	10	10	5	10	10	5	10	5	5	5
New-energies	4	4	2	2	2	4	2	4	2	2
Water resource	12	6	12	6	6	6	6	12	6	6
Cement industries	4	4	4	4	8	8	4	8	8	8
Steel industries	20	20	20	15	15	20	20	10	10	15
Road construction	130	140	140	110	110	120	140	110	100	100
Rail transportation	6	3	9	6	3	3	3	9	6	6
Information technology	30	30	30	15	30	30	15	15	30	15
Agriculture	10	15	10	15	15	10	15	15	10	10
d_{jt} (Years)										
Project	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Machinery parts	1	1	2	3	5	3	2	5	3	5
Crude oil distillation	1	1	1	5	5	2	3	4	3	3
Telecommunication	0.5	0.5	0.3	0.5	0.5	0.4	0.4	0.2	0.1	0.4
New-energies	0.8	0.4	0.6	0.3	0.5	0.8	0.7	0.4	0.2	0.7
Water resource	0.3	0.7	0.3	0.5	0.8	0.8	1	0.2	0.8	0.2
Cement industries	0.2	0.5	1	0.9	1	0.7	1	0.5	0.8	0.9
Steel industries	0.5	0.3	0.8	0.3	0.8	0.6	0.9	1	0.2	0.2
Road construction	1	3	1	1	3	1	3	1	2	1
Rail transportation	0.8	0.2	0.4	0.2	0.2	0.2	0.4	0.6	0.6	0.6
Information technology	2	1	2	3	1	2	3	2	2	2
Agriculture	1	2	3	1	2	3	3	3	1	3
I_{jt} (*Rate of Return*)										
Project	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Machinery parts	5	7	6	8	6	7	7	7	7	8
Crude oil distillation	9	10	12	12	3	9	11	6	3	9
Telecommunication	11	17	9	10	12	14	14	9	10	13
New-energies	15	5	17	8	19	2	13	19	18	9
Water resource	6	7	7	7	5	7	6	8	6	5
Cement industries	3	9	11	6	9	10	12	12	3	9
Steel industries	12	14	14	9	11	17	9	10	12	11
Road construction	19	2	13	19	15	5	17	8	19	15
Rail transportation	17	8	19	2	13	19	12	3	9	11
Information technology	7	7	5	7	6	8	10	12	14	14
Agriculture	11	6	9	10	12	12	8	19	2	13
$MARR_t$	4	5	4	4	5	6	6	5	6	7

5.1. Results

Table 4 presents the ideal and nadir point of four test cases, distinctively.

Tables 5 and 6 present the objective values of the generated solutions for the proposed framework and the *AUGMECON* method, respectively.

As shown in Table 5, the proposed method generates a special part of the Pareto front which is far from the NIS and near the PIS, simultaneously. We should note that the proposed framework was developed for this purpose.

Table 6 presents the range of the generated solutions on the Pareto front. Although this is desirable in most MODM problems, it is not appropriate in project selection problems where the goal is to generate non-dominated solutions with low-risks and high-profits, concurrently. In these problems, DMs are required to post-screen the generated non-dominated solutions by employing additional analysis and finding the solutions that satisfy the low-risk high-profit requirements.

Fig. 2 plots the weighted average of the objective functions achieved by both procedures for all benchmark cases.

The minimum objective functions (i.e. Z_2 and Z_4) were transformed into maximum functions through multiplying by (-1). It can be concluded from Fig. 2 that the weighted average of all objective functions in the proposed framework represents relative dominance in comparison with the *AUGMECON* method. Consequently, Table 7 represents the solutions for both procedures for all four cases.

It can be concluded from Table 7 that the proposed framework implies approximately the same combination of investments (i.e., selected projects) for different values of epsilons.

On the other hand, changing the right-hand-side of the constraints in the proposed mathematical model cannot undermine the procedure. As mentioned earlier, only a restricted part of the Pareto front concurrently has low-risks and high-profits. Our procedure can identify the aforementioned area of the Pareto front in different conditions. This is a useful property that also highlights the robustness of our approach. Fig. 3 presents the number of selected projects in the final portfolio of each method for all benchmark cases.

The following points are illustrated by Fig. 3 for all benchmark cases:

- Given that the decision maker is able to apply weights to the objective functions, a narrower Pareto front is re-generated by TOPSIS in comparison with the application of AUGMECON in the full criteria cone.
- In spite of narrowing the re-generated Pareto front by the proposed framework, the number of selected portfolios in the final solution is higher than the number of selected portfolios in the *AUGMECON* method. In other words, searching for feasible investment projects, which concurrently minimize risks and maximize profits, in the bi-objective space of the proposed framework is more successful than searching in the multi-objective space in the original MODM problem. Consequently, decision making in the restricted bi-objective space is easier (and more effective) than decision making in the wide multi-objective space.
- The combination of selected portfolios is approximately fixed in the proposed method while a considerable variation is illustrated in the *AUGMECON* method. This means better robustness of the proposed framework for different epsilon values (i.e., the right-hand-sides of the constraints).
- Both methods were not able to find any solutions for some epsilon values. But again the number of infeasible cases is smaller in the proposed framework in comparison with the *AUGMECON* method. Although the proposed framework seeks a restricted part of the Pareto front but it is relatively successful in finding the feasible solutions. This is a direct result of forming the bi-objective space using all objective functions in the original MODM problem and concentrating on a simpler bi-objective space.

5.2. Comparison index

We also used the CC index of the TOPSIS procedure to compare the performance of the procedures. This index has been calculated by Eq. (31) for all generated solutions:

$$CC_{ij} = \frac{d^{ij}_{NIS}}{d^{ij}_{PIS} + d^{ij}_{NIS}}, \quad i \in [0,1], \quad j \in \{1,2\} \tag{31}$$

where, CC_{ij} represents the CC of procedure j for parameter $E_2 = i$. It is obvious that the CC_{ij} value is between zero and one. The higher value of CC_{ij} represents a farther distance from the NIS and a closer distance to PIS, simultaneously. Fig. 4 presents the CC measurement for all cases in both procedures.

Table 9
The available resources for the investment projects.

Available resources	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Labor (Man/Period)										
H_{1t}	921	841	561	722	824	673	912	737	947	758
H_{2t}	589	888	977	858	726	705	695	597	641	679
H_{3t}	706	679	575	709	870	831	579	745	883	843
Machine (Quantity/Period)										
M_{1t}	120	124	75	164	188	142	88	199	198	97
M_{2t}	151	158	160	193	124	169	187	118	199	76
M_{3t}	82	132	182	149	67	65	198	173	123	109
Material (m^3/*Period*)										
R_{1t}	5441	5004	5058	5185	5487	5530	5272	5116	5747	5087
R_{2t}	5273	5853	5679	5214	5752	5593	5248	5334	5568	5833
R_{3t}	5588	5391	5790	5955	5140	5760	5627	5450	5496	5457

Table 10
The required resources for the investment projects.

Project	Labor			Machine			Material		
	H_{1j}	H_{2j}	H_{3j}	M_{1j}	M_{2j}	M_{3j}	R_{1j}	R_{2j}	R_{3j}
Machinery parts	67	67	97	9	6	5	588	587	584
Crude oil distillation	71	70	67	9	9	6	547	505	558
Telecommunication	88	74	66	6	8	8	588	510	549
New-energies	68	90	51	8	5	9	541	548	560
Water resource	52	59	54	8	5	9	523	582	517
Cement industries	74	73	63	5	9	5	593	508	594
Steel industries	99	91	74	7	6	6	545	505	538
Road construction	79	80	96	8	7	7	528	567	535
Rail transportation	77	75	62	7	9	9	552	531	580
Information technology	71	69	97	5	8	5	545	554	593
Agriculture	76	76	75	8	7	7	564	539	521

As is shown in Fig. 4, the CC values are better for the proposed framework in comparison with the *AUGMECON* method. We should note that the proposed framework was developed for this purpose.

5.3. Sensitivity analysis

We also performed a sensitivity analysis to study the effects of changing the weights of the objective functions in the proposed method. We simulated the weights of each objective function for the *MOPSP-MPPH* 51 times. We then implemented the proposed algorithm and plotted the objective function values. The results are presented in Fig. 5.

As shown in Fig. 5, the sensitivity analysis reveals small variations in the final objective function of the proposed framework. This can be interpreted as the robustness of our proposed framework in finding the restricted Pareto front.

5.4. Case study

Investment bankers are fundamentally concerned with the profit, cost, rate of return, and the resource utilization of their invested portfolios. Investment is not a trivial job considering all of these highly conflicting objectives. The framework proposed in this study can help investment bankers select lucrative portfolios of projects in multi-period planning horizons.

The projects are investment opportunities with pre-determined profit, cost, rate of return, and resource requirements in a wide variety of services and production sectors such as agriculture, banking, petroleum, crude oil distillation, road construction, dam construction, automotive, machinery parts, telecommunication, information technology, steel industries, cement industries, new energies, and water resource among others.

The People's Bank is considering 11 projects presented in Table 8 during a ten-year planning period (P1, P2, ..., P10).

As is shown in Table 8, all properties of the investment projects, including available budgets, profits, project durations and the rates of return are assumed to be flexible during the planning horizon. This is the case in most real-life investment projects.

Tables 9 and 10 present the available and required resources for each project during the ten-year planning period.

Table 11 shows the unit cost of the resources for the investment projects for each project during the planning horizon.

Table 11
The unit costs of the resources for the investment projects.

Resources	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Labor[*]										
1	24	11	17	24	26	17	23	17	19	12
2	28	22	20	17	25	10	28	19	25	19
3	18	14	12	25	15	18	29	15	24	17
Machine[**]										
1	1130	1077	1093	1148	1182	1129	1080	1187	1083	1114
2	1002	1039	1020	1092	1124	1044	1016	1162	1113	1031
3	1046	1196	1142	1070	1011	1054	1045	1166	1077	1005
Material[***]										
1	7	8	7	9	8	9	7	6	5	9
2	7	5	6	9	5	7	7	8	5	5
3	5	8	9	8	7	5	9	5	7	6

[*] The unit cost of labor has been calculated per hour ($10/hr) – Approximately four clusters are considered for labor types.
[**] The unit cost of machines has been calculated per hour ($1000/hr) – Approximately four clusters are considered for machine types.
[***] The unit cost of materials has been calculated per m3 ($500/m3) – Approximately four clusters are considered for material types.

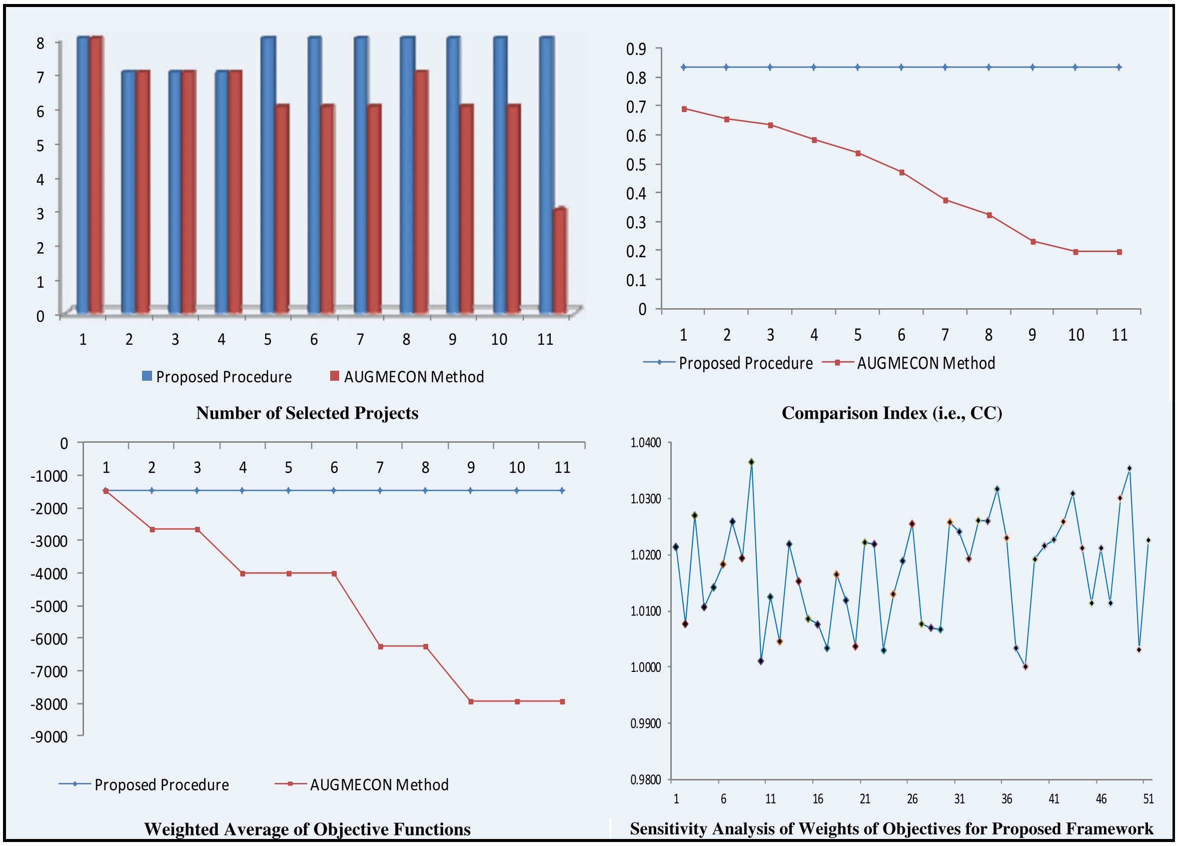

Fig. 6. The result of the case study for the proposed framework and the *AUGMECON* method.

Both methods were applied to the data presented in this case study. The results for the proposed framework and the *AUGMECON* method are presented in Fig. 6.

The following points result from Fig. 6:

- Although the proposed framework identifies a restricted part of the Pareto front, the number of selected projects in the final solution is higher than the proposed framework for all epsilon-levels.
- The higher comparison index (i.e., CC) in the proposed framework for all epsilon-levels shows the relative dominance of the proposed framework in finding the solutions which are near the PIS and far from the NIS, simultaneously.
- The weighted average of the maximization objective functions has relative dominance in the proposed framework in comparison with the *AUGMECON* method for all epsilon-levels.
- The sensitivity analysis of the objective values for the proposed framework for the 51 different weights of the objectives revealed that the results of the proposed method are relatively unperturbed.

Finally, the dominance of the proposed framework, which was demonstrated by the simulation experiments, was also confirmed in this case study.

6. Conclusions and future research directions

In this paper, an integrated framework was proposed to solve MODM problems efficiently by generating solutions on the Pareto front that have the minimum distance from the ideal solution and the maximum distance from the nadir solution, concurrently. The proposed framework is based on the TOPSIS method for MODM problems and an extended version of the efficient ε-constraint method.

The proposed framework reduced a MODM problem to a bi-objective problem using the TOPSIS concepts. This simplification results in several benefits. First, the objective function space is restricted. Therefore, the search procedure is more effective and the implementation time of the algorithm is more manageable. Second, all the objectives in the original MODM problem are utilized in the formation of the bi-objective model. Consequently, the DMs are satisfied with the high-quality solutions which are close to the ideal solution and far from the nadir solution, simultaneously. Third, we then used an extended version of the efficient ε-constraint method to generate non-dominated solutions with a pre-defined and arbitrary resolution on the Pareto front of the aforementioned bi-objective problem. The procedure generated non-dominated solutions on a restricted part of the Pareto front in which the minimum distance from the ideal solution and the maximum distance from the nadir solution was met. This property is an essential factor in real-life problems such as capital investment project selection.

A new mathematical model for solving the multi-objective project selection problem with multi-period planning horizon (*MOPSP-MPPH*) was also developed. The proposed framework and the conventional ε-constraint method were applied to different benchmark cases of the *MOPSP-MPPH* generated based on a simulation experiment. The proposed framework efficiently generated higher-quality solutions which were closer to the PIS and further from the NIS, simultaneously. Furthermore, a real case study in investment banking was considered. Both methods were applied to the data provided in the case study. The results also confirmed the relative dominance of the framework proposed in this study.

Future research will concentrate on the comparison of results obtained with those that might be obtained with other methods. We hope that the concepts introduced here will provide some motivation for future research.

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