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ORIGINAL RESEARCH

Analytical hierarchy process: revolution and evolution

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Abstract

The Analytical Hierarchy Process (AHP) is a reliable, rigorous, and robust method for eliciting and quantifying subjective judgments in multi-criteria decision-making (MCDM). Despite the many benefits, the complications of the pairwise comparison process and the limitations of consistency in AHP are challenges that have been the subject of extensive research. AHP revolutionized how we resolve complex decision problems and has evolved substantially over three decades. We recap this evolution by introducing five new hybrid methods that combine AHP with popular weighting methods in MCDM. The proposed methods are described and evaluated systematically by implementing a widely used example in the AHP literature. We show that (i) the hybrid methods proposed in this study require fewer expert judgments than AHP but deliver the same ranking, (ii) a higher degree of involvement in the hybrid voting AHP methods leads to higher acceptability of the results when experts are also the decision-makers, and (iii) experts are more motivated and attentive in methods requiring fewer pairwise comparisons and less interaction, resulting in a more efficient process and higher acceptability.

Keywords Decision support systems · Analytical hierarchy process multi-criteria decision-making · Weighting methods · Pairwise comparison · Consistency

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1 Introduction

Multi-Criteria Decision-Making (MCDM) is a general framework for solving complex decision-making problems with multiple conflicting criteria and decision-makers with different preferences. MCDM methods involve several evaluation criteria instead of a single measure of optimality. MCDM models are divided into Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM). The main difference between MODM and MADM is that the former is defined in a continuous decision-making space while the latter operates in a discrete decision-making space. The MADM methods include compensatory and non-compensatory approaches. In compensatory techniques, the exchange between criteria is allowed, and the strengths of one criterion may offset the weaknesses of another criterion. In non-compensatory approaches, the exchange between criteria is not allowed. Methods such as the Analytical Hierarchy Process (AHP), the LINear programming technique for Multidimensional Analysis of Preference (LINMAP), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and the ELimination Et Choice Translating REality (ELECTRE) are the most common compensatory methods. The interval, fuzzy, and probabilistic versions of these methods have been proposed to solve MCDM problems under uncertainty (Gál et al., 1999; Hwang & Yoon, 1981; Köksalan & Zionts, 2012; Tzeng & Huang, 2011).

The AHP is one of the most popular MCDM methods proposed by Saaty (1980). AHP is a structured technique for organizing and solving complex decision-making problems based on mathematics and psychology. AHP provides a comprehensive and logical framework to quantify each structural decision-making element within a hierarchical structure. The AHP begins with choosing the decision criteria. The alternatives are then evaluated based on the selected criteria. Figure 1 presents an example of a hierarchical structure in AHP.

AHP uses pairwise comparisons among the decision criteria and follows the principles of reciprocal condition, homogeneity, dependency, and expectations to prioritize each criterion.

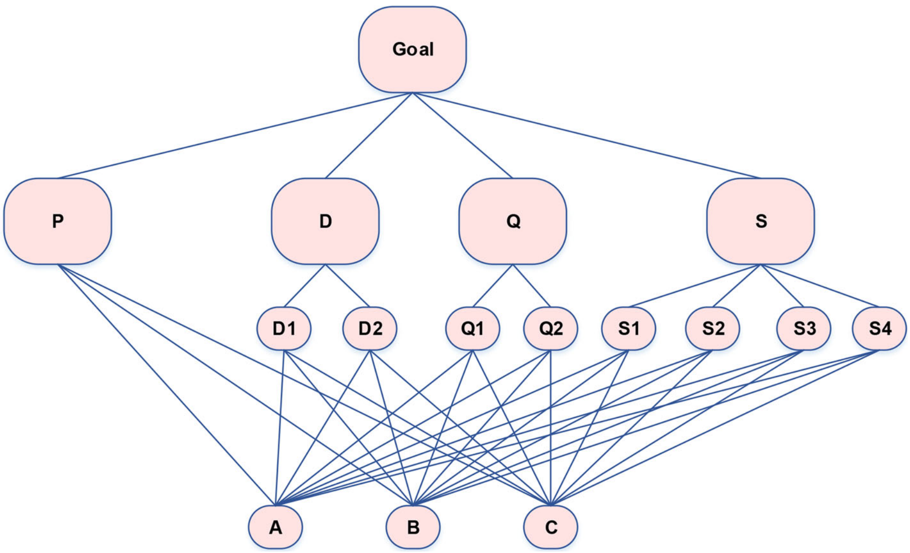


Fig. 1 An Example of Hierarchical Tree

Saaty (1980) highlights the following as the main advantages of AHP: unity in providing a model for problem-solving, analytical and systematic approach to solving complex problems, problem-solving power dealing with the interdependency of criteria, observance of hierarchical structures in decision making, measurement of intangible and qualitative cases, examination of consistency in priorities, synthesis desirability for alternatives, the trade-off in preferences, judgment, and consensus, and the possibility of improvement through repetition.

These advantages have led to the widespread use of AHP in business, science, healthcare, and education for applications from human exploration of Mars (Tavana, 2004, 2006) to selection of new production facilities (Ishizaka & Labib, 2011b), organizational evaluation (Ghamari et al., 2017), forest management (Darvishi et al., 2020; Zandebasiri & Pourhashemi, 2016), software selection (Mahmudova & Jabrailova, 2020), economic assessment (Wang & Deng, 2020), inventory management (Pérez Vergara et al., 2020; Sales et al., 2020), food safety assessment (Chaiyaphan & Ransikarbum, 2020), road selection (Han et al., 2020), sustainability assessment (Zand et al., 2020), tourism management(Çavmak & Çavmak, 2020), and environmental mapping (de Jesus França et al., 2020) among others.

AHP has also been applied as a weighting method for improving the performance of other decision-making approaches. In this regard, AHP has been combined with the Complex Proportional ASsessmen (COPRAS) and the Additive Ratio ASsessment (ARAS) (Goswami & Mitra, 2020), Dempster-Shafer theory (Wei & Liu, 2008), Combinatorial Mathematics-Based Approach (CMBA), ELECTRE (Jain & Ajmera, 2019), Data Envelopment Analysis (DEA) (Sinuany-Stern et al., 2000), Vlse Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (Güler et al., 2019), TOPSIS(Ban et al., 2020), Geographic Information System (GIS) (Bouroumine et al., 2020; Das et al., 2020), and genetic algorithms (İnce et al., 2020). The fuzzy versions of AHP have been developed to deal with the uncertainties inherent in many real-world problems (Calabrese et al., 2019; Khan et al., 2019; Ogundoyin & Kamil, 2020; Singh et al., 2020; Zhu et al., 2020). Additional applications of AHP can be found in the literature review presented by Ishizaka and Labib (2009, 2011a) and Vaidya and Kumar (2006).

One of the most critical features of AHP is the need for accuracy in forming the pairwise comparison matrices defined by the experts. Saaty (1980) described the consistency ratio criterion as an upper limit for each matrix and the hierarchical analytical process to test accuracy. The pairwise comparison matrices must be revised if they display a consistency value higher than a predetermined level. The complexity of the subsequent evaluation process may decrease participants' motivation and even reduce the accuracy of the results obtained.

Therefore, despite its many advantages, AHP users may encounter significant difficulties when performing extensive pairwise comparisons within a hierarchical structure. We investigate the opportunities to improve the pairwise comparison process in AHP by reviewing the preferential voting, AHP, and Voting AHP (VAHP) methods. We then describe the weighting approaches in the Best–Worst Method (BWM) and Stepwise Weight Assessment Ratio Analysis (SWARA) to introduce five new hybrid methods, including BWM-AHP, BWM-VAHP, SWARA-AHP, SWARA-VAHP, and Best Method (BM) AHP or BM-AHP (also called AHP-EXPRESS). These methods require judgments such as pairwise comparison and prioritization. We compare the methods according to the number of pairwise comparisons and the prioritization needed in the solution process. Figure 2 graphically depicts the new hybrid methods proposed in this study as the intersection of the existing weighting methods of preferential voting, AHP, BWM, and SWARA.

The remainder of the paper is organized as follows. Section 2 presents the weighting methods designed to replace the pairwise comparison step in AHP. Section 3 introduces five new hybrid models incorporating these methods within an AHP framework. Section 4

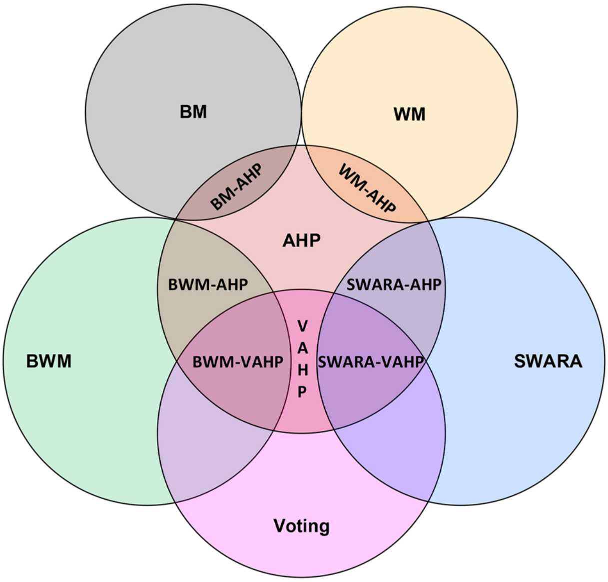


Fig. 2 Proposed methods

compares the new hybrid methods according to the amount of information retrieved from the expert(s). Section 5 compares the performance of the new methods with a widely used example of AHP in the literature. Finally, Sect. 6 concludes with our conclusion.

2 Research background

2.1 The AHP method

AHP helps decision-makers set priorities based on their goals, knowledge, and experience while considering their subjective feelings and judgments. The AHP algorithm is defined as follows:

Step 1. Create a hierarchical structure. Initially, the main criteria and alternatives defining the decision problem are determined, and then the problem is divided into target levels, criteria, sub-criteria, and alternatives. Each element of this hierarchy depends on its higher-level element, and this dependence continues linearly to the highest level. Furthermore, the evaluation process must be repeated whenever there is a change in the hierarchical structure.
Step 2. Form a pairwise comparison matrix. The elements of each level are compared in pairs, leading to the formation of paired comparison matrices. A 9-point scale is used to determine the importance and preference in pairwise comparisons. Preferences at this step must satisfy the reciprocal and homogeneity conditions. Readers should refer to Amenta et al. (2020b) for aggregation of individual judgments in group AHP and the formal method to aggregate judgments in a common matrix.
Step 3. Calculate the inconsistency rate. Given that experts' judgment may lead to the formation of inconsistent pairwise comparison matrices, an experimental rate has been proposed to evaluate their consistency and that of the hierarchical structure. The results will be returned to

Table 1 Inconsistency Index of Random Matrix (I.I.R.)

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I.I.R	0	0	0.58	0.90	1.12	1.24	1,32	1.41	1.45	1.45	1.51	1.52	1.56	1.57	1.59

the experts for reconsideration if the judgments are inconsistent. The algorithm for calculating the inconsistency rate of a pairwise comparison matrix (D) is defined as follows:

a. *Calculate the weighted sum vector*. Calculate the overall priority vector by multiplying the pairwise comparison matrix (D) by the local priority vector.
b. *Calculate the consistency vector*. Calculate the consistency vector by dividing the elements of the overall priority vector coordinate-wise by those of the local priority vector. That is, each element of the consistency vector is obtained by dividing the corresponding element of the weighted sum vector by that of the local priority vector. The components of the consistency vector are actually $\lambda_{\max}$ estimates.
c. *Calculate the largest Eigenvalue of the pairwise comparison matrix* ($\lambda_{\max}$). The average of the elements of the consistency vector is equal to $\lambda_{\max}$.
d. *Calculate the inconsistency index (II)*. Assuming that the pairwise comparison matrix (D) is an $m \times m$ matrix, the inconsistency index equals $\frac{\lambda_{\max}-m}{m-1}$.
e. *Define the inconsistency ratio (IR)*. The IR is given by $\frac{II}{IRI}$, where IRI is the inconsistency random index, whose value is extracted from Table 1. The values in this table are determined via simulation.

Saaty (1980) suggested that if the inconsistency ratio is less than or equal to 0.1, the results of the pairwise comparisons are acceptable. Otherwise, they are returned to the expert(s) for review and reconsideration. In this regard, it should be highlighted that transitivity thresholds, proposed by Amenta et al. (2020a), constitute a useful technique since they provide meaningful information about the degree of misclassification and the reliability of preferences while avoiding the need to revise the judgments. Alternatively, Aguarón et al. (2020) have proposed a procedure to reduce the inconsistency measured by the geometric consistency index.

Note that the acceptability of the results derived from the paired comparison matrices alone is insufficient, and a hierarchical inconsistency ratio should be calculated by considering the inconsistency ratios of the matrices, the hierarchical structure of the model, and Saaty's experimental formula (Saaty, 1980).

Step 4. Calculate local priorities. The local priorities of the criteria and the alternatives relative to each criterion are obtained using different weighting methods. The most common weighting methods include the sum of rows, columns, arithmetic mean, geometric mean, eigenvector, ordinary least squares, and logarithmic least squares.

Step 5. Calculate the overall priority of the alternatives. The overall priority of each alternative is equal to the sum of the product of the local priority of the alternative relative to each weighted criterion.

Step 6. Rank the alternatives**.** The alternatives are ranked based on their overall priorities. The higher the overall priority of an alternative, the better its ranking position.

The implementation of AHP can sometimes become very time-consuming, particularly with increasing the number of elements in each level. In such situations, decision criteria are usually subdivided into sub-criteria, though in many cases, this does not solve the problem. Furthermore, the formation of consistent pairwise comparison matrices may be a task beyond the actual expertise of experts. As a result, the structure of AHP should be revised in terms of interacting with the decision-maker and determining the local priorities of the criteria and the alternatives. We summarize below several methods for weighing the selection criteria that can be incorporated into the AHP structure to define enhanced hybrid ranking techniques.

2.2 Weighting methods

2.2.1 Preferential voting

The problem of "selection" using the aggregation of votes is one of the most common group decision-making problems for which several solution models have been proposed. For example, consider a group of decision-makers who need to solve a problem. Given that people have different preferences and opinions, how can they agree on a solution considering different views? Voting is a way of aggregating individual preferences to reach a group decision.

The ballot structure is generally divided into two categories. In the first category, voters vote for one candidate, while voters vote for more than one candidate in the second category. At the same time, the second category is divided into two sub-categories. In the first one, only the names of a few candidates are written on the ballots, while, in the second, in addition to selecting several candidates, the voters also express their preferences among them. In the voting mechanism without voters' preferences, r $(r < n)$ individuals are chosen from n candidates. Thus, each voter maximally votes for the r candidate, and in the end, the candidate with the most votes wins. One of the disadvantages of this mechanism is that voters cannot hand over their preferences to the community. That is, in classical voting models, the aggregation of votes is performed regardless of the voting position. Therefore, the result does not reflect the collective will of the voters.

In preferential voting, more information is used from the voters' opinions than in other electoral systems. In this type of voting, voters are asked to choose their preferred candidate and nominate a second and third candidate if their first and second choices are not selected. Therefore, in the preferential voting system, each voter chooses a subset of candidates and arranges them according to his/her priorities. In classical voting models, votes are aggregated regardless of the voting position, while in preferential voting, subjective priorities play a crucial role in the final result.

Different methods have been developed to aggregate votes in a preferential scenario. Although no single method is the best, some methods are superior to others. A popular vote aggregation technique is the weighting of priorities based on Borda's count (Borda, 1781), in which fixed weights are assigned to different priorities. Suppose v_{rj} represents the number of priority votes r, $(r = 1, 2, ..., k)$, received by the candidate j, $(j = 1, 2, ..., n)$. The overall utility index for each candidate is defined by Eq. (1).

$$\begin{cases} Z_j = \sum_{r=1}^{k} w_r v_{rj} \\ w_1 > w_2 > \ldots > w_k. \end{cases} \tag{1}$$

Each priority's relative weight or importance must be greater than that of the next one. Cook and Kress (1990) applied an optimistic policy in selecting weights and determined the multiples of w_r, $(r - 1, 2, \ldots, k)$, in such a way that the overall utility index was maximized for each candidate. They transformed the weight limitations into linear constraints, adapting the idea of Thompson et al. (1986) and Thompson et al. (1989) to define the assurance region described in Eq. (2)

$$
\begin{aligned}
&w_r - w_{r+1} \geq d(r, \varepsilon), \ \ r = 1, 2, ..., k-1 \\
&w_k \geq d(k, \varepsilon)
\end{aligned} \tag{2}
$$

where $d(0, \varepsilon) : \mathrm{N} \times R^{\geq 0} \rightarrow R^{\geq 0}$ is a non-decreasing and non-negative discrimination intensity function, ε is called the discrimination factor, and $d(r, \varepsilon)$ is the minimum distance between the priority weights of r and $r+1$. Model (3) was developed by Cook and Kress (1990) to aggregate the votes across voters based on an optimistic policy.

$$
\begin{aligned}
&Z_p = \max \sum_{r=1}^{k} w_r v_{rp} \\
&s.t. \\
&\sum_{r=1}^{k} w_r v_{rj} \leq 1, \qquad j = 1, 2, ..., n. \\
&w_r - w_{r+1} \geq d(r, \varepsilon), \quad r = 1, 2, ..., k-1 \\
&w_k \geq d(k, \varepsilon)
\end{aligned} \tag{3}
$$

This DEA model is a particular case of the Charnes, Cooper, and Rhodes (CCR) model (Charnes et al., 1978). Each candidate plays the role of a homogeneous decision-making unit, and the aggregated votes of each candidate represent the corresponding output. Furthermore, all decision-making units have a single input equal to “1”. The weight control constraints, imposed based on an appropriate discrimination intensity function, are introduced to account for the differences between voting positions. In Model (3), each candidate chooses the best weight vector for his voting position, such that any candidate whose value function equals one is considered efficient. First, since a separate model is solved for each candidate and Model (3) selects the best weight vector, it is sometimes impossible to rank the candidates. As is also the case in DEA, several candidates could display an optimal value of “1”, creating a tie in the ranking. Second, modifying the weight difference between voting priorities could change the winning candidate.

Cook and Kress (1990) showed that the rankings obtained were sensitive to discrimination intensity function and ε. Moreover, the assumption of an optimistic policy, where each candidate chooses the best weight vector for himself, was also questioned. Applying the Sexton cross-efficiency method (Sexton et al., 1986), Green et al. (1996) ranked the candidates using optimistic and pessimistic views of the multiple-choice set. They also suggested a weak weight order based on accumulation. Hashimoto (1997) built on the super efficiency method of Andersen and Petersen (1993) to propose the elimination of the candidate under evaluation and introduced a constraint category in which the difference between two consecutive weights should be greater than or equal to that of the subsequent two successive weights. Noguchi et al. (2002) defined weight constraints using a strong order and provided a way to rank candidates when considering multiple criteria.

Obata and Ishii (2003) proposed a model to discriminate reasonably between efficient candidates considering the same size weights. Foroughi and Tamiz (2005) extended this latter model to efficient and inefficient candidates and reduced its computational complexity through an algorithmic expression. Finally, Llamazares and Pena (2009) showed that the top candidate obtained by Obata and Ishii (2003) could be modified through different soft selections and incorporated the weight constraint of Green et al. (1996) to the resulting decision environment.

Contreras (2011) defined a two-step method for deciding the group ranking of candidates. In the first step, a weight vector model determines which candidate under evaluation has the best rank. A compromise answer is obtained in the second based on the best candidate rank derived from the first step. Lotfi et al. (2013) built on the Contreras (2011) model to define the ranking of candidates in the worst possible case, namely, the anti-ideal rank, and considered it as the upper limit of each candidate's group ranking.

Preferential voting has been widely used in recent studies. For instance, Liu and Hai (2005) used preferential voting to select suppliers within an analytical hierarchy structure. Thus, the suppliers played the role of candidates and managers that of voters. An advantage of this method is that it does not need to form matrices of pairwise comparisons or check their consistency but requires the opinion of the experts. Preferential voting has also been used to rank decision-making units in DEA (Soltanifar et al., 2010), measure cross-efficiency (Soltanifar & Shahghobadi, 2013), classify inputs and outputs (Soltanifar & Shahghobadi, 2014), rank companies (Sharafi et al., 2019), and select suppliers (Soltanifar, 2020).

2.2.2 SWARA method

The SWARA method is an index weighting method developed by Keršuliene et al. (2010). Applications of this method include the selection of suppliers (Alimardani et al., 2013; Jamali et al., 2017; Narayanan & Jinesh, 2018), packaging designers(Stanujkic et al., 2015), research and development projects (Zolfani et al., 2015), cold storages (Katranci & Kundakci, 2020), and internal safety auditors (Prasad, 2019) among others. SWARA has been used for environmental assessment and mapping (Juodagalvienė et al., 2017; Panahi et al., 2017), planning high-tech industry priorities (Ghorshi Nezhad et al., 2015), ranking solutions to reduce the risks of sustainable supply chain production (Ansari et al., 2020), and evaluating the performance of environmentally friendly thermal power plants (Rani & Mishra, 2020) or the sustainability of bioenergy production processes (Mishra et al., 2020).

Suppose a decision-making problem consists of m criteria $C_1, C_2, ..., C_m$. The SWARA algorithm is defined as follows:

Step 1. The criteria are prioritized according to their importance, which is based on the opinion of the decision-maker(s). Suppose that $\overline{C}_1, \overline{C}_2, ..., \overline{C}_m$ are redesigned criteria so that $\overline{C}_1 \succ \overline{C}_2 \succ ... \succ \overline{C}_m$.
Step 2. Starting from the second criterion and interacting with the decision-maker(s), the relative difference between criterion j and $(j-1)$ is determined and assigned the value s_j, $(j = 2, ..., m)$. s_j is known as the comparative importance of the average value.
Step 3. Calculate the coefficient k_j, $(j = 1, 2, ..., m)$ through Eq. (4).

$$k_j = \begin{cases} 1 & j = 1 \\ s_j + 1 & j > 1 \end{cases} \tag{4}$$

Step 4. Calculate the weight q_j, $(j = 1, 2, ..., m)$ through Eq. (5).

$$q_j = \begin{cases} 1 & j = 1 \\ \dfrac{k_{j-1}}{k_j} & j > 1 \end{cases} \tag{5}$$

If we assume $s_1 = 0$ in the second step, we will have $q_j = \frac{1}{\prod_{k=1}^{j}(1+s_k)}$, $(j = 1, 2, ...m)$.
Step 5. Calculate the local priority of criterion w_j, $(j = 1, 2, ..., m)$ using Eq. (6).

$$w_j = \frac{q_j}{\sum_{k=1}^{m} q_k} \tag{6}$$

Note that the local priority assigned to the criteria is determined through interactions with the decision-maker(s). At the same time, the final relative weight is computed via the geometric mean of the decision makers' weights.

2.2.3 BWM

The BWM, presented by Rezaei (2015), constitutes another criteria weighting technique that has become quite popular in the literature (Ahmadi et al., 2017; Delice & Can, 2020; Liang et al., 2020; Rezaei, 2016; Rezaei et al., 2015, 2016). After determining the best and worst criteria, a pairwise comparison between the other criteria and the reference ones becomes the basis for defining a mathematical programming model. The weights assigned to the criteria correspond to the solutions of this mathematical programming model. Furthermore, a formula for calculating the inconsistency ratio is provided to verify the validity of the comparisons. The BWM algorithm is defined as follows:

Step 1. Identify the influential criteria for the purpose of the problem by interacting with the decision-maker(s), $C_1, C_2, ..., C_m$.
Step 2. Determine the best (C_B) and the worst (C_W) criteria and sub-criteria among those selected based on the opinion of the decision-maker.
Step 3. After interacting with the decision-maker, determine the preferences of the best criterion over the other criteria based on a 9-point scale, $\left(a_{Bj},\ \ j = 1, 2, ..., m\right)$.
Step 4. After interacting with the decision-maker, determine the preferences of the other criteria over the worst criterion using a 9-point scale, $\left(a_{jW},\ \ j = 1, 2, ..., m\right)$.
Step 5. Obtain the weights of the criteria by solving the mathematical programming Model (7).

$$\begin{aligned}
&\min\ \xi \\
&s.t. \\
&\left|w_B - a_{Bj} w_j\right| \le \xi w_j,\quad j = 1, 2, ..., m \\
&\left|w_j - a_{jw} w_w\right| \le \xi w_w,\quad j = 1, 2, ..., m \\
&\sum_{j=1}^{m} w_j = 1 \\
&w_j \ge 0,\quad j = 1, 2, ..., m
\end{aligned} \tag{7}$$

Denote by $\left(w_1^*, w_2^*, \ldots, w_m^*\right)$ the optimal weights of the criteria obtained when solving Model (7). The results obtained are validated through the inconsistency ratio of the system $C.R. = \frac{\xi^*}{C.I.}$, where ξ^* is the optimal value of the objective and $C.I.$ is extracted from Table 2. If the inconsistency ratio is close to zero, then it is plausible to rely on the judgments of the experts.

Table 2 Consistency Index (C.I.)

m	1	2	3	4	5	6	7	8	9
(C.I.)	0	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

3 Improved versions of AHP

3.1 VAHP method

One of the shortcomings inherent in the AHP is that it may fail to form paired comparison matrices as the number of elements in each level increases. Moreover, obtaining a pairwise comparison matrix or a hierarchical structure with acceptable inconsistency ratios may turn into a complex task beyond the expertise of experts. Liu and Hai (2005) calculated the priority of the criteria and alternatives by substituting Model (3) in place of the pairwise comparison matrices and presented the VAHP method. Among its applications, we highlight the selection of suppliers (Hadi-Vencheh & Niazi-Motlagh, 2011) and ranking efficient decision-making units in DEA (Soltanifar & Lotfi, 2011). Furthermore, VAHP retrieves the priority of criteria and the alternatives per criterion from the experts and the discrimination intensity functions to determine the distance between priorities.

To properly determine the discrimination intensity functions while balancing the information required from the experts, Green et al. (1996) considered the number of votes for each candidate as cumulative standings. They defined $V_{rq} = \sum_{k=1}^{r} v_{kq}$, with v_{kq} representing the number of votes of candidate q for the k priority. The use of accumulation in the number of votes weakly regulates the weights. Therefore, there is no need to write the constraints of the assurance region, and Model (3) becomes Model (8), that is, assuming we want to rank m candidates

$$
\begin{aligned}
& Z_p = \max \sum_{r=1}^{m} W_r V_{rq} \\
& s.t. \\
& \sum_{r=1}^{m} W_r V_{rj} \leq 1, \quad j = 1, 2, ..., m \\
& W_r \geq 0, \qquad r = 1, 2, ..., m
\end{aligned}
\tag{8}
$$

where $W_r = \sum_{k=r}^{m} W_k$ represents the weight of the voting position r. It can now be observed how the model determines the overall priority of the alternatives by considering the priority of the criteria and that of the alternatives per criterion. Pishchulov et al. (2019) introduced a process to solve the same type of problem.

Even though AHP has been extended into a group decision-making framework in the form of the Group AHP method, the lack of consideration for the characteristics of decision-makers remains one of its main shortcomings. On the other hand, VAHP has sufficient flexibility to formalize decision-making processes in groups with unequal power levels among their members (Soltanifar, 2017). The algorithm for implementing VAHP is defined as follows.

Step 1. Create a hierarchical tree.
Step 2 Determine the priority of criteria and that of the alternatives per criterion. The elements composing each level are compared to other related elements located at a higher level, and their priorities are determined.
Step 3 Calculate the local priorities: After solving Model (8) and obtaining the optimal $Z_j^*, \ j = 1, 2, ..., m$, values, the relative local priority weights of the criteria and alternatives are computed using Eq. (9).

$$
w_j = \frac{Z_j^*}{\sum_{k=1}^{m} Z_k^*}, \quad j = 1, 2, ..., m \tag{9}
$$

Step 4. Calculate the overall priority of the alternatives.
Step 5. Rank the alternatives.
One of the main advantages of this method is that due to the minimal information retrieved from the experts, there is no need to check the consistency of the information obtained.

3.2 SWARA-AHP method

The VAHP replaces the process of forming pairwise comparison matrices at each AHP level with a preferential voting model designed to calculate the local priority of the criteria and the alternatives. This process retrieves minimum information from the experts. Suppose that the SWARA method is applied to calculate the local priorities in place of the preferential voting model. In addition to the priorities, we show that the experts must provide the comparative importance between criteria and alternatives. The corresponding SWARA-AHP algorithm is defined as follows.

Step 1. Create a hierarchical tree.
Step 2. Prioritize criteria and alternatives over each criterion. The elements composing each level are compared to other related elements located at a higher level, and their priorities are determined. Criteria and alternatives are reevaluated after this step so that the criteria or alternatives with the smaller index values are given a higher priority.
Step 3. Determine the comparative importance of the average value. At each level, starting from the first element of the redesigned criteria or alternatives and after interacting with the decision-maker(s), the relative difference between each criterion or alternative and the previous one must be determined. The resulting variables are denoted s_j, $(j = 1, 2, ..., m)$, and known as the comparative importance of the average value. Note that $s_1 = 0$ for each level.
Step 4. Calculate the Local Priority: Eq. (6) is applied to compute the local priorities of the criteria and alternatives for each criterion while noting that $q_j = \frac{1}{\prod_{k=1}^{j}(1+s_k)}$, $(j = 1, 2, ..., m)$.
Step 5. Calculate the overall priority of the alternatives.
Step 6. Rank the alternatives.
As was the case with VAHP, the minimum amount of information retrieved from the experts implies that there is no need to check the consistency of the information obtained.

3.3 SWARA-VAHP Method

The VAHP retrieves minimum information from the experts to compute the local priorities of the criteria and alternatives. Therefore, VAHP requires experts to prioritize the criteria and alternatives. This basic requirement motivates experts to participate and provide information while decreasing their confidence in the ranking results' accuracy.

SWARA-AHP requires experts also to define the relative difference between priorities. In this regard, the information obtained from the experts in SWARA-AHP can be combined with the flexibility of VAHP to determine the local priorities of the criteria and alternatives for each criterion. Consequently, the weights will be defined within a flexible structure, and experts' confidence in the results obtained should increase. However, to achieve this goal, experts must provide the minimum relative difference between priorities.

Consider the comparative importance of the average value of priority j to priority $(j-1)$, s_j, $(j = 2, ..., m)$. According to Sect. (3.2), we must have $\frac{q_j}{q_{j+1}} \geq 1 + s_{j+1}$, $(j = 1, 2, ..., m-1)$. Thus, Model (10) can be defined to calculate the local priorities

$$
\begin{aligned}
& Z_p = \max \sum_{r=1}^{m} q_r v_{rp} \\
& s.t. \\
& \sum_{r=1}^{k} q_r v_{rj} \leq 1, \qquad\qquad j = 1, 2, ..., m \\
& q_r \geq q_{r+1}(1 + s_{r+1}), \qquad r = 1, 2, ..., m-1 \\
& q_m \geq \varepsilon_{\max}
\end{aligned}
\tag{10}
$$

where v_{rj} denotes the number of votes obtained by criterion (alternative) j in voting position r and $\varepsilon_{\max}$ defines the maximum positive value for which Model (10) is feasible. The SWARA-VAHP algorithm is defined as follows.

Step 1. Create a hierarchical tree.
Step 2 Determine the priority of criteria and alternatives per criterion. The elements composing each level are compared to other related elements located at a higher level, and their priorities are determined. The criteria and alternatives are re-indexed after this step so that the criterion or alternative with the smaller index is assigned a better priority.
Step 3. Determine the minimum comparative importance of the average value. At each level, starting from the first element of the redesigned criteria or alternatives and after interacting with the decision-maker(s), the relative difference between each criterion or alternative and the previous one must be determined.
Step 4. Calculate the local priorities. Equation (9) is applied to obtain the local priority of the criteria and alternatives per criterion, where Z_j^*, $(j = 1, 2, ..., m)$ the optimal values are obtained from Model (10).
Step 5. Calculate the overall priority of the alternatives.
Step 6. Rank the alternatives.
Similar to the previous algorithms, the minimum information retrieved from the experts implies that there is no need to check the consistency of the information obtained.

3.4 AHP-express method

The AHP-express method was presented by Leal (2020) and can be summarized as follows:

Step 1. Create a hierarchical tree.
Step 2. Determine the best elements of each level. The elements of each level are compared to other related elements located at a higher level, and the best ones are identified.
Step 3. Determine the preferences of the best element at each level: after interacting with the decision-maker, determine the preferences of the best criterion (best alternative per criterion) relative to the other criteria (alternatives at the level selected) based on a 9-point scale, $(a_{Bj},\ j = 1, 2, ..., m)$.

Step 4. Calculate the local priorities: The local priorities of the criteria and alternatives per criterion are obtained by applying Eq. (11).

$$w_j = \frac{1/a_{Bj}}{\sum_{k=1}^{m} 1/a_{Bk}}, \quad j = 1, 2, ..., m \tag{11}$$

Step 5. Calculate the overall priority of the alternatives.
Step 6. Rank the alternatives.

Note that the information that AHP-express retrieves from the expert(s) is limited to determining the best element per level and performing pairwise comparisons between the best and other elements. In other words, the amount of information retrieved is substantially lower (slightly higher) than that required by AHP (VAHP). Given a total of m elements per level, AHP-express retrieves m-1 judgments, instead of the $\frac{(m^2-m)}{2}$ required by AHP.

3.5 BWM-AHP method

As the previous extensions, BWM-AHP computes the local priorities of the criteria and alternatives per criterion using a method different from AHP. In this case, the method applied is BWM. The information retrieved is neither as complex as that required by AHP nor as simple as the one gathered via SWARA or VAHP. The BWM-AHP algorithm is defined as follows:

Step 1. Create a hierarchical tree.
Step 2. Determine the worst and best elements per level. The elements composing each level are compared to other related elements located at a higher level, and the best and worst are identified.
Step 3. Determine the preferences of the best element per level. After interacting with the decision-maker, determine the preferences of the best criterion (best alternative per criterion) relative to the other criteria (alternatives at the level selected) based on a 9-point scale, $(a_{Bj},\ j = 1, 2, ..., m)$.
Step 4. Determine the preferences of the worst element per level. After interacting with the decision-maker, determine the preferences of the other criteria (alternatives at the level selected) relative to the worst criterion (worst alternative per criterion) based on a 9-point scale, $(a_{jW},\ j = 1, 2, ..., m)$.
Step 5. Calculate the local priorities. Derive the local priorities of the criteria and alternatives per criterion by solving Model (7).
Step 6. Calculate the inconsistency ratio of the judgments. A rate has been proposed to accept some degree of inconsistency in experts' judgments both per level and hierarchically.
As stated in Sect. 2.2.3, the inconsistency ratio of the judgments per hierarchical level is defined by Eq. (12)

$$C.R = \frac{\xi^*}{C.I} \tag{12}$$

where ξ^* denotes the optimal objective value derived from Model (7) and $C.I$ can be extracted from Table 2 (Rezaei, 2015). Clearly, $a_{BW} = a_{Bj} \times a_{jW},\ \ j = 1, 2, ..., m$, implies $\xi^* = 0$, with all the judgments becoming consistent. Indeed, the assurance of the final results increases as ξ^* approaches zero.

Note that BWM-AHP requires validating the inconsistency ratio of both each level and the entire hierarchical structure. If either one is not acceptable, the results will be returned to the experts for review.
Step 7 Calculate the overall priority of the alternatives.
Step 8 Rank the alternatives.

In this hybrid model, the implementation of the BWM bears some resemblance to the models developed within the branch of the literature focusing on incomplete AHP (Brunelli, 2018; Faramondi et al., 2020; Harker, 1987; Wedley, 1993). Incomplete AHP models were introduced to deal with situations where a subset of the pairwise comparisons could not be used in the analysis due to mistakes of the evaluators or any potential exogenous factor. The more straightforward evaluation setting imposed by the BWM, where the number of pairwise comparisons is purposely reduced, delivers a similar analysis framework. In this regard, the fact that the BWM deals with fewer pairwise comparisons than AHP implies that some information would be lost from the original set of pairwise comparisons. We elaborate on this feature of the model below.

The reduction in the number of comparisons implied by the BWM constitutes both one of the main advantages of the current hybrid method and, at the same time, one of its main disadvantages. By imposing the BWM on the pairwise comparison matrices, we will be losing information regarding the existing relationships among the elements composing each level. Indeed, we are converting an $m \times m$ matrix into a vector of dimension m. That is, information will be lost regarding the relations among alternatives.

Intuitively, this loss follows from the fact that we must select one of the m relative interdependencies among the elements as the main one on which to base the comparisons performed by the BWM. Clearly, this narrowing also constitutes its main advantage as the dimension of the comparison matrix increases, providing the main motivation for implementing the BWM over other MCDM techniques. For instance, ten alternatives requiring 45 explicit pairwise comparisons per criterion represent a substantially more complex scenario than a vector with ten entries describing the basic relationships between the alternatives or criteria analyzed.

For intuitive purposes, consider the numerical example analyzed in Sect. 5. The pairwise comparison matrix describing the relationships among criteria is presented in Table 4, while the priority vector illustrating the relative importance of each criterion is defined in the first row of Table 5. We know from the priority vector that the criteria should be ranked as follows based on their relative importance: Price, Comfort, Miles Per Gallon (MPG), and Prestige. This is also the ranking relation obtained by considering the row of pairwise comparisons defined by the Price criterion in Table 4. However, if we were to use the Prestige row as the reference one, the ranking would be different, with Price and Prestige remaining as the best and worst criterion, respectively, but MPG shifting to the second position in place of Comfort.

The numerical example is sufficiently simple to allow for the criteria within the BWM to be defined in terms of the priority ranking – and it's reversed when considering the worst criterion. However, differences can also arise whenever the alternatives are compared with respect to each criterion. A cumulative number of modifications across the different comparison matrices could lead to modifications in the final ranking. Such an evaluation problem, which can be contained in relatively simple scenarios, constitutes an advantage in complex settings with a large number of criteria or alternatives. This is particularly the case when entries are missing from the pairwise comparison matrices, or the evaluators face requirements that cannot be fulfilled. Moreover, the BWM should generally display a higher consistency ratio merely because of reducing the number of comparisons relative to AHP.

3.6 BWM-VAHP method

BWM-VAHP combines BWM and VAHP. In this method, the experts prioritize the criteria and compare the best and the worst ones with the other criteria based on a 9-point scale. Thus, the amount of information retrieved from the experts increases compared to BWM-AHP, as does the results' accuracy. The BWM-VAHP algorithm is defined as follows:

Step 1. Create a hierarchical tree.
Step 2. Determine the priority of each level. The elements of each level are compared to other related elements located at a higher level, and their priorities are defined from best to worst. The presentation can be simplified by assuming that the priority order matches the index of each element.
Step 3. Determine the preferences of the best element per level. After interacting with the decision-maker, determine the preferences of the best criterion (the best alternative per criterion) relative to the other criteria (alternatives at the level selected) based on a 9-point scale. Without loss of generality, assume that the best element is the first element of the level $(a_{j1},\ j = 1, 2, ..., m)$.
Step 4. Determine the preferences of the worst element per level. After interacting with the decision-maker, determine the preferences of the other criteria (alternatives at the level selected) to the worst criterion (worst alternative per criterion) based on a 9-point scale. Without loss of generality, assume that the worst element is the last element of the level $(a_{jm},\ j = 1, 2, ..., m)$.
Step 5. Calculate the local priorities. The criteria and alternatives per criterion are obtained by solving the Model (13). The discrimination intensity function $d(., \varepsilon)$, which determines the difference between the weight of each element and the immediately better one, can either be determined by an expert or following the steps provided in Sect. 2.2.1.

$$\begin{aligned}
&\min\ \xi \\
&s.t. \\
&\left|w_1 - a_{1j}w_j\right| \le \xi w_j, \quad j = 1, 2, ..., m \\
&\left|w_j - a_{jm}w_m\right| \le \xi w_m, \quad j = 1, 2, ..., m \\
&\sum\nolimits_{j=1}^{m} w_j = 1 \\
&w_j - w_{j+1} \ge d(j, \varepsilon), \quad j = 1, 2, ..., m-1 \\
&w_m \ge d(m, \varepsilon)
\end{aligned} \tag{13}$$

Step 6. Calculate the inconsistency ratio of the judgments per level and the hierarchical inconsistency ratio. Since the judgments provided by the experts may be inconsistent, the inconsistency ratio is calculated following the sixth step described within Sect. 3.5. If the inconsistency ratio derived from the comparisons per level or the hierarchical inconsistency ratio is unacceptable, the results are returned to the experts for review.
Step 7. Calculate the overall priority of the alternatives.
Step 8. Rank the alternatives.

4 A comparative analysis of the different methods

AHP is one of the most effective MCDM techniques, whose intuitive appeal has led to its widespread use among researchers. As a result, a prevalent debate has taken place regarding its strengths and weaknesses. Among its main strengths we must highlight the following ones:

a. AHP displays higher flexibility than other methods and a substantial capacity to identify inconsistencies (Ramanathan, 2001). Moreover, pairwise comparisons constitute a simple and convenient data inputting technique.
b. AHP restructures decision problems into hierarchical categories of criteria, highlighting the relative importance assigned to each element composing the problem (Macharis et al., 2004).
c. AHP incorporates objective and subjective evaluations into the analysis while validating their consistency, reducing potential biases in the decision-making process.
d. AHP can analyze group decision-making problems through the geometric mean of the individual pairwise comparisons (Zahir, 1999).
e. AHP can easily account for risky and uncertain situations due to its capacity to derive scales in settings where measures generally do not exist (Millet & Wedley, 2002).
f. Among its main weaknesses, the following ones have been emphasized by researchers:
g. Rank reversals may arise when a copy or a close copy of one of the alternatives being ranked is added to the set of potential choices. Multiplicative variants of AHP have been defined to prevent rank reversal (Triantaphyllou, 2001). The interpretation of the weights allocated to the criteria plays a prominent role in explaining the phenomenon (Belton & Stewart, 2002).
h. The complete additive aggregation defined by AHP allows for the compensation between good and bad scores across criteria while also foregoing specific information that may be important when ranking the alternatives.
i. AHP divides any decision into several subsystems requiring the performance of pairwise comparisons both within their elements and across systems. The resulting number of pairwise comparisons equals $\frac{m^2-m}{2}$, becoming an arduous task as m increases (Macharis et al., 2004).
j. The limitations imposed on decision-makers through the 9-point scale and the subsequent relative importance that must be defined constitute another drawback of the model. Hajkowicz et al. (2000) proposed the use of a binary scale as a potential solution, with decision-makers describing whether criteria were more, less, or equally important than others.

We have tackled some of the weaknesses displayed by AHP through the introduction of different weighting methods to calculate the local priorities. Some of these hybrid methods, such as VAHP and AHP-express, have already been introduced in the literature, while the others constitute novel approaches.

AHP requires $\frac{m^2-m}{2}$ pairwise comparisons per level with m elements, reducing the willingness of the expert(s) to provide information and, therefore, requiring the validation of the inconsistency ratio. BWM-AHP reduces this requirement to $(2m - 3)$ pairwise comparisons per level, while AHP-express requires only $m - 1$ comparisons to determine the best element per level and compare it with the other elements within the same level.

Note that if we remove the second constraint of Model (7) when computing BWM-AHP, the solutions obtained will be equal to the weights suggested by AHP-express, and the inconsistency ratio will be zero. Therefore, AHP-express is a special case of BWM-AHP that reduces the amount of information retrieved from experts. As a result, this method is also called BM-AHP (Best Method-AHP). A similar logic applies when, instead of the best, the worst element per level is selected within AHP-express and compared with the other elements composing the level, leading to WM-AHP (Worst Method-AHP).

AHP-express is also quite similar to SWARA-AHP. If the comparisons between the best elements of each level and the other elements composing the level (a_{Bj}, $j = 1, 2, ..., m$) are equal to the inverse of the non-normalized weight within SWARA, i.e., ($\frac{1}{q_j}$; $j = 1, 2, ..., m$), the results of both methods coincide. In other words, if we assume that the elements of each row are arranged from best to worst and ($a_{Bj} = \prod_{k=1}^{j} (1 + s_k)$; $j = 1, 2, ..., m$), then the results of both methods will be the same. In this regard, the main difference between SWARA-AHP and SWARA-VAHP lies in the number of experts. SWARA-AHP ranks the alternatives using the judgment of one expert, while SWARA-VAHP requires several experts and the geometric mean of the judgments.

VAHP considers the priority assigned by the expert(s) to the elements composing each level and solves an optimistic model such as Model (8) to extract the local priority of each element, requiring the lowest amount of information from the expert(s) compared to the other methods. This quality does not imply that this method is better. By focusing less on expert(s) opinions, the latter may display an insufficient motivation to consider the results if acting as decision-maker(s), increasing uncertainty about the quality of the results obtained.

4.1 Comparing the judgments required across methods

Figure 3 provides a comparative summary of the different extensions of AHP discussed in the previous sections and ranks these methods based on the amount of information retrieved

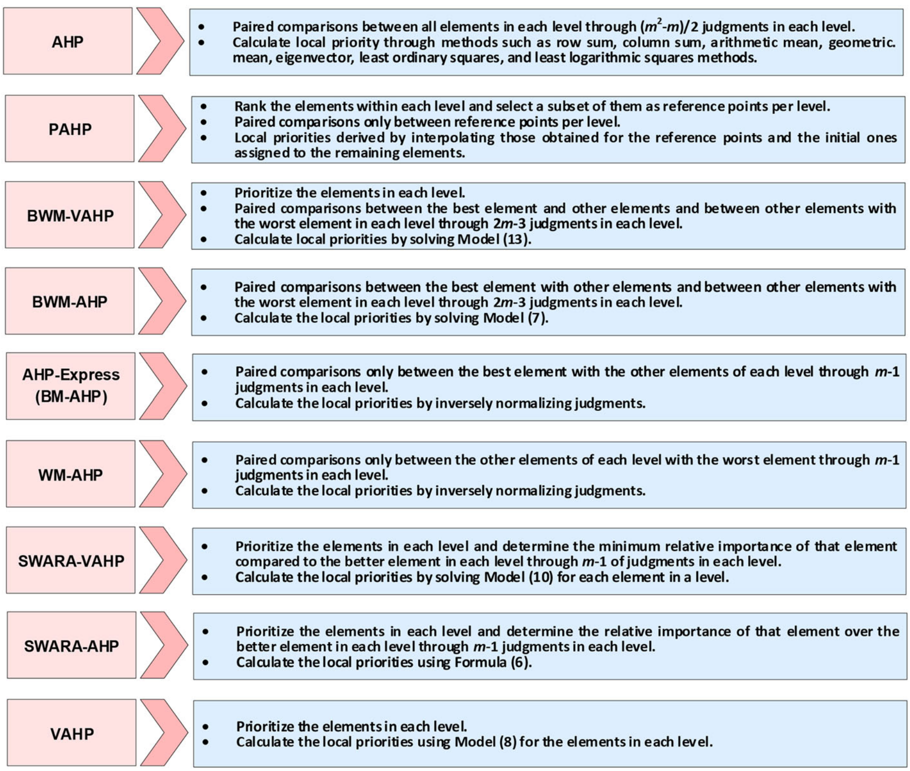


Fig. 3 Hybrid AHP methods according to the amount of information obtained from the experts

from the expert(s). The number of judgments required per level, both in terms of pairwise comparisons and prioritizations, by each of the enhanced methods analyzed is summarized in Table 3. The methods have been ordered according to the number of judgments required, with pairwise comparisons constituting a more demanding evaluation scenario than prioritizations Fig. 4.

Even though some methods require the same number of judgments, both the type of judgment and the number of calculations performed differ. For example, AHP-express is based on pairwise comparisons, while VAHP focuses on prioritizations. Moreover, SWARA-AHP requires determining both the priority of the elements and the relative distance between the different priority positions. More precisely,

a. AHP requires only pairwise comparison judgments but no prioritizations. Clearly, as the value of m increases, the number of judgments needed by AHP becomes substantially larger than those of the other methods.
b. The judgments in BWM-AHP are pairwise comparisons, with *m-1* judgments used to compare the best element with the other elements and *m-2* judgments required to compare the other elements with the worst element. The best and worst elements are paired in the initial set of *m-1* judgments and do not need to be compared again.
c. BWM-VAHP requires two types of judgments. The first one corresponds to the standard pairwise comparisons of the best element with the others and the other elements with the worst one, as also required by BWM-AHP. In contrast, the second is a prioritization judgment between elements, as in VAHP. Thus, the number of pairwise comparisons is the same as in BWM-AHP, while the number of prioritization judgments equals *m-3*, since the first and last priorities are determined in the initial comparisons. The remaining *m-2* elements require *m-3* judgments, one to determine the priority of the second element, one for the third element, until reaching the $(m\text{-}2)^{th}$ element. The remaining element has a priority of *m-1*.
d. AHP-express and WM-AHP require only *m-1* pairwise comparisons.
e. SWARA-VAHP, SWARA-AHP, and VAHP do not require pairwise comparisons but *m-1* prioritizations. This number of judgments suffices for VAHP to implement a linear programming model and determine the weights of the elements. However, SWARA-VAHP and SWARA-AHP also require defining the distance between priority positions in the form of s factor determination, which is not a judgment between elements and can be specified via a predetermined standard.

We have added to the analysis the Parsimonious AHP (PAHP) model, developed by Abastante et al. (2019), which constitutes a recent evaluation technique developed to reduce the number of pairwise comparisons. Their model is structured as follows. First, the authors ask the decision-makers (DMs) to rank each alternative against each criterion by assigning a weight defined within a common scale. Then, they ask the DMs to select several alternatives as reference points per criterion. We denote these reference points by r. Pairwise comparisons are made between reference points, allowing for applying AHP to the set of reference evaluations. The inconsistency ratio of the pairwise comparison matrices is then calculated. If the corresponding values cannot be accepted (according to Saaty's criterion), the matrices are returned to the DMs for revision. This process continues until an acceptable inconsistency ratio is reached for each level. Then, using the relative weights (local priorities) calculated for the reference points and the initial weights assigned to the remaining alternatives, an interpolation procedure is applied to derive the relative weights (local priorities) of the latter alternatives. Duleba (2020) extends the same intuition to a multi-level structure.

Table 3 Pairwise comparisons and prioritization judgments per method

Methods	Number of judgments per level			Total number of judgments in the example		
	Pairwise comparisons	Prioritizations	Weight allocation	Pairwise comparisons	Prioritizations	Weight allocation
AHP	$(m^2\text{-}m)/2$	0	0	$4[(3^2–3)/2] + (4^2–4)/2 = 18$	0	0
PAHP (Abastante et al., 2019)	$(r^2\text{-}r)/2^*$	0	m	$4[(2^2–2)/2] + (2^2–2)/2 = 5^{**}$	0	$4[3] + [4] = 16$
BWM-VAHP	2 m-3	m-3	0	4[2(3)-3] + [2(4)-3] = 17	4[3–3] + [4–3] = 1	0
BWM-AHP	2 m-3	0	0	4[2(3)-3] + [2(4)-3] = 17	0	0
AHP-Express (BM-AHP)	m-1	0	0	4[3–1] + [4–1] = 11	0	0
WM-AHP	m-1	0	0	4[3–1] + [4–1] = 11	0	0
SWARA-AHP	0	m-1	0	0	4[3–1] + [4–1] = 11	0
SWARA-VAHP	0	m-1	0	0	4[3–1] + [4–1] = 11	0
VAHP	0	m-1	0	0	4[3–1] + [4–1] = 11	0

*The term r refers to the number of reference points per level

**Assuming two reference points per level, i.e., the best and the worst one

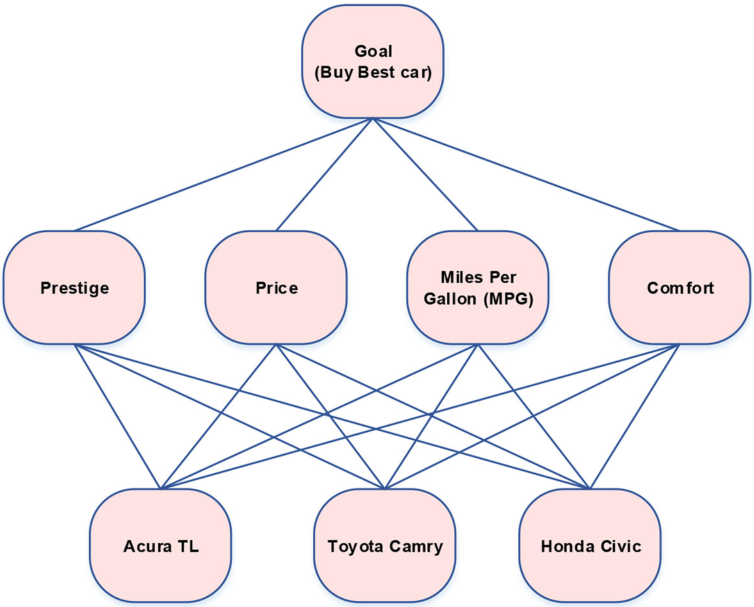


Fig. 4 Three-level hierarchy to choose the best car

To better compare the enhanced versions of AHP, we implement these methods using a numerical example analyzed many times in the AHP literature. We have applied the PAHP model to solve this numerical example assuming two reference points per level, namely, those determined by the best and the worst elements.

5 Illustrative example

This section analyzes and compares the results obtained from different AHP extensions by performing a familiar numerical example in the AHP literature (Bodin & Gass, 2004). Saaty (2013) introduced this numerical example by asking, "How do we choose the best car from among three alternatives by considering different importance priorities for the four criteria, some intangible and some tangible: prestige, price, MPG, and comfort?" The hierarchy in Fig. 3 is used to represent this decision. Note that, in this example, we have one level with $m = 4$ criteria elements and four levels, one per criterion, with $m = 3$ elements, namely, alternatives, within each one of them.

The corresponding pairwise comparison matrices are presented in Table 4. The local priorities of the criteria and alternatives and the overall priority of the alternatives are given in Table 5. The process of weighting, adding, and normalizing priorities to "1" is called the distributive mode of synthesis. If the synthesized values are divided by the largest priority, the synthesis result is called the ideal mode. For additional intuition regarding synthesis modes, see Saaty (2006, 2013). This example has also been considered in the ANP literature, and the same results have been obtained (Saaty, 2018).

Table 4 Pairwise comparison matrices

Goal (Buy best car)	Prestige	Price	MPG	Comfort
Prestige	1	1/4	1/3	1/2
Price	4	1	3	3/2
MPG	3	1/3	1	1/3
Comfort	2	2/3	3	1

Prestige	Acura TL	Toyota camry	Honda civic	Price	Acura TL	Toyota camry	Honda civic
Acura TL	1	8	4	Acura TL	1	1/4	1/9
Toyota camry	1/8	1	1/4	Toyota Camry	4	1	1/5
Honda civic	1/4	4	1	Honda Civic	9	5	1

MPG	Acura TL	Toyota camry	Honda civic	Comfort	Acura TL	Toyota camry	Honda civic
Acura TL	1	2/3	1/3	Acura TL	1	4	7
Toyota camry	3/2	1	1/2	Toyota camry	1/4	1	3
Honda Civic	3	2	1	Honda civic	1/7	1/3	1

Table 5 Synthesis of the priorities of the alternatives

Goal (Buy best car)	Prestige	Price	MPG	Comfort	Synthesis of overall priorities
Priorities	**0.099**	**0.425**	**0.169**	**0.308**	
Acura TL	0.707	0.063	0.182	0.705	0.342
Toyota camry	0.070	0.194	0.273	0.211	0.204
Honda civic	0.223	0.743	0.545	0.084	0.454

AHP requires performing pairwise comparisons at each level. As emphasized within Sect. 3.5, BWM-AHP incorporates pairwise comparisons between the best and worst elements per level and the other elements within the same level. Thus, for instance, to calculate the local priorities of the criteria, “price” being the best criterion and “prestige” being the worst one, it suffices to obtain pairwise comparisons of the best criterion compared to the other criteria, and the other criteria compared to the worst one.

BWM-VAHP requires the same information as BWM-AHP and prioritizes the elements composing each level, as was the case with VAHP. In addition, BWM-VAHP is sensitive to the type of discrimination intensity function defined in Model (13), and the decision-maker must be careful in determining it. For instance, if all the discrimination intensity functions in Model (13) are equal to zero, the same results as those of BWM-AHP would be obtained.

AHP-express (BM-AHP) requires comparing the best elements in each level with the other elements composing the level. While comparing the other elements composing each level with the worst element of the level is required to apply WM-AHP. That is, both methods require the same amount of information.

AHP, BWM-AHP, AHP-express (BM-AHP), and WM-AHP retrieve information from the experts in the form of pairwise comparison matrices. However, in SWARA-AHP, the information received from the experts will be different, first prioritizing the elements of each level and then determining the relative difference between each element and the previous, i.e., better one. Nevertheless, the structure of the information received and analyzed by SWARA-VAHP is the same as SWARA-AHP.

AHP, BWM-AHP, AHP-express, WM-AHP, and SWARA-AHP support group decision-making. It suffices to obtain the required information from each expert and combine it through geometric means. However, SWARA-VAHP and VAHP require the cooperation of a team of experts that supplies the necessary information. To provide additional intuition, Tables 6 and 7 present a detailed implementation of SWARA-VAHP when ranking the set of alternatives based on the judgments of a team of experts.

In this example, the opinions of five experts regarding the prioritization of the elements composing each level and the difference between each priority relative to the previously better one per level have been retrieved. If a tie arises when solving Models (8) and (10), the cross-efficiency ranking method of Sexton et al. (1986) has been applied. All linear and nonlinear programming models have been solved using GAMS software. The output from the software together with the algorithm of each method, have then been implemented in Microsoft Excel software to obtain the final rankings.

The results derived from implementing the set of different ranking methods are presented in Fig. 5. The inconsistency ratios of the pairwise comparisons for AHP and BWM-AHP are acceptable, and the rate of hierarchical inconsistency equals 0.062 and 0.331, respectively. Figure 6 has been introduced to highlight two important features of these hybrid models. First, identical rankings may be obtained based on very different information requirements. For instance, note how AHP and VAHP deliver the same rankings, with VAHP requiring much less information from the experts than AHP. Note also how the ranking delivered by the PAHP model is consistent with those of the main hybrid techniques proposed in the current paper.

More interestingly, the two hybrid techniques are improved by VAHP. The BWM-VAHP and the SWARA-VAHP methods deliver rankings that differ from those of the other methods. In addition to the judgments provided, a higher, more detailed involvement of the experts in the decision-making process leads to different ranking results. When defining relative comparisons, as required by VAHP, the additional level of detail leads to potential ranking modifications that should be considered when determining the level of involvement required from the experts in the evaluation process.

6 Conclusion

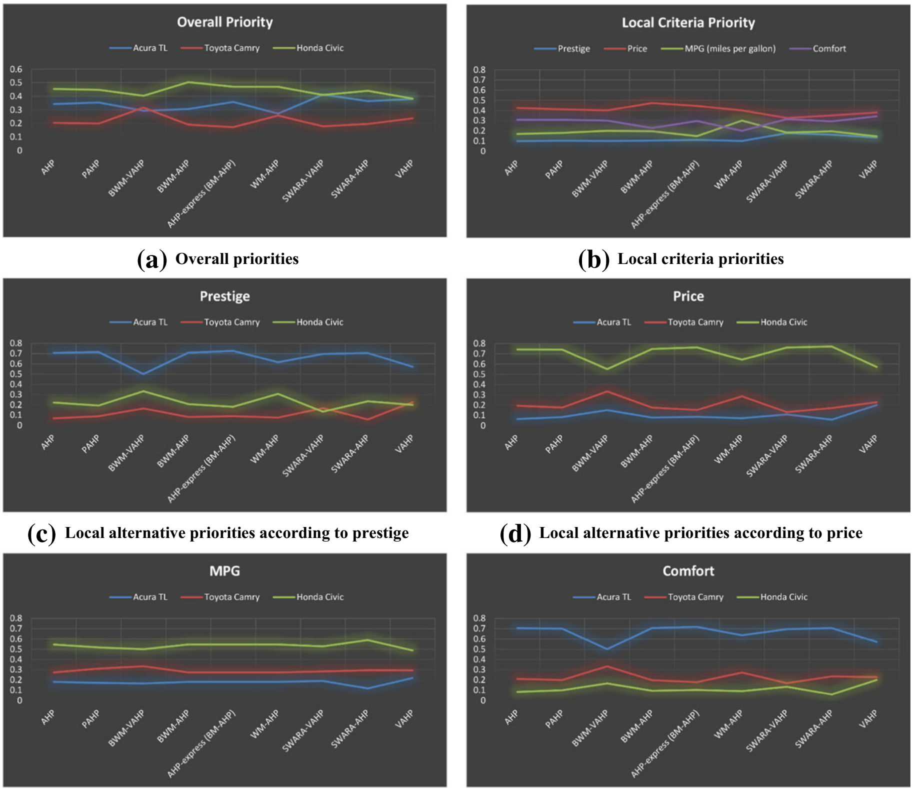
This paper examines the features, strengths, and weaknesses of the AHP method, which uses pairwise comparisons in determining relative weights. One of the weaknesses of this method is the inconsistency in judgments when working with experts. Interaction with DM is often prolonged, stressful, and tiring. To overcome this problem, we proposed five new methods of

Table 6 Detailed implementation of SWARA-VAHP

Level	Goal (Buy best car)	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	s1	s2	s3	s4	s5	Geometric mean	Local Priorities	Normalized
Goal	Prestige	4	4	4	3	3	0	0	0	0	0	0	0.5432	0.1769
	Price	1	2	1	1	2	0.2	0.3	0.2	0.1	0.2	0.1888	1	0.3256
	MPG	3	3	3	4	4	0.5	0.4	0.5	0.6	0.5	0.4959	0.5623	0.1831
	Comfort	2	1	2	2	1	0.2	0.2	0.3	0.1	0.2	0.1888	0.9661	0.3145
Prestige	Acura TL	1	1	1	1	1	0	0	0	0	0	0	1	0.6972
	Toyota Camry	3	3	2	3	2	2	2	1	3	2	1.8882	0.2430	0.1694
	Honda Civic	2	2	3	2	3	3	3	4	2	3	2.9302	0.1914	0.1334
Price	Acura TL	3	3	2	3	2	0	0	0	0	0	0	0.1413	0.1075
	Toyota Camry	2	2	3	2	3	3.5	3.5	3	4	2.5	3.2588	0.1725	0.1313
	Honda Civic	1	1	1	1	1	2	2	2.5	1.5	2	1.9744	1	0.7612
MPG	Acura TL	3	3	2	3	2	0	0	0	0	0	0	0.3618	0.1907
	Toyota Camry	2	1	3	2	3	1	1	1.5	2	0.5	1.0845	0.5358	0.2824
	Honda Civic	1	2	1	1	1	1.5	1.5	1	0.5	2	1.1761	1	0.5270
Comfort	Acura TL	1	1	1	1	1	0	0	0	0	0	0	1	0.6960
	Toyota Camry	2	3	2	3	2	2	2	2.5	1.5	1.5	1.8640	0.2446	0.1702
	Honda Civic	3	2	3	2	3	3	3	2.5	3	3.5	2.9831	0.1923	0.1338

Table 7 Ranking results obtained from SWARA-VAHP

Goal (Buy the best car)	Synthesis of overall priorities
Acura TL	0.4121
Toyota camry	0.1779
Honda civic	0.4100



(a) **Overall priorities**

(b) **Local criteria priorities**

(c) **Local alternative priorities according to prestige**

(d) **Local alternative priorities according to price**

(e) **Local alternative priorities according to MPG**

(f) **Local alternative priorities according to comfort**

Fig. 5 A comparison of the local and overall priorities obtained by different AHP versions

BWM-AHP, BWM-VAHP, WM-AHP, SWARA-AHP, and SWARA-VAHP; and systematically compared them with three existing methods of AHP, VAHP, and AHP-express according to their volume of pairwise comparisons and prioritization efforts. We then implemented the methods on a familiar example in the AHP literature. In this example, we obtained the same ranking of the AHP method with five methods of BWM-AHP, AHP-express, WM-AHP, SWARA-AHP, and VAHP, but requiring far fewer judgments and effort. Furthermore, we obtained different results with BWM-VAHP and SWARA-VAHP, as these methods required more interaction with the experts compared with the previous five methods. We conclude that methods requiring more interaction with experts are less efficient and produce less acceptable results. In contrast, experts are more motivated and attentive in methods requiring fewer

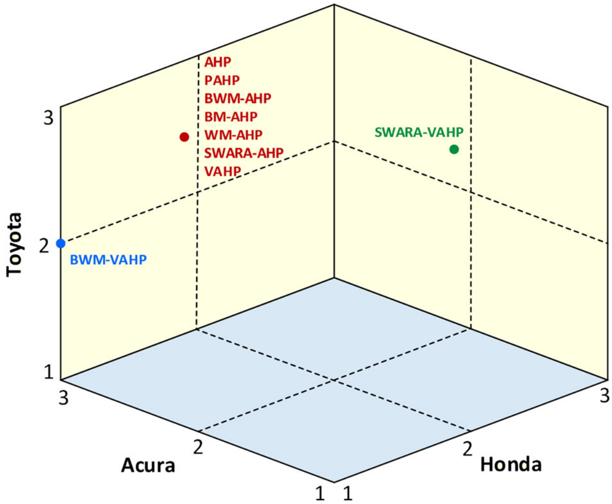


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Fig. 6 A comparison of the ranking results derived from the different versions of AHP

pairwise comparisons and less interaction, resulting in more efficient processes and more acceptable results.

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Declaration

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Abastante, F., Corrente, S., Greco, S., Ishizaka, A., & Lami, I. M. (2019). A new parsimonious AHP methodology: Assigning priorities to many objects by comparing pairwise few reference objects. *Expert Systems with Applications*. https://doi.org/10.1016/j.eswa.2019.02.036

Aguarón, J., Escobar, M. T., & Moreno-Jiménez, J. M. (2020). Reducing inconsistency measured by the geometric consistency index in the analytic hierarchy process. *European Journal of Operational Research, 288*(2), 576–583.

Ahmadi, H. B., Kusi-Sarpong, S., & Rezaei, J. (2017). Assessing the social sustainability of supply chains using Best Worst Method. *Resources, Conservation and Recycling, 126*, 99–106.

Alimardani, M., Zolfani, S. H., Aghdaie, M. H., & Jolanta, T. (2013). A novel hybrid SWARA and VIKOR methodology for supplier selection in an agile environment. *Technological and Economic Development of Economy, 19*(3), 533–548. https://doi.org/10.3846/20294913.2013.814606

Amenta, P., Lucadamo, A., & Marcarelli, G. (2020a). On the transitivity and consistency approximated thresholds of some consistency indices for pairwise comparison matrices. *Information Sciences, 507*, 274–287.

Amenta, P., Lucadamo, A., & Marcarelli, G. (2020b). On the choice of weights for aggregating judgments in non-negotiable AHP group decision making. *European Journal of Operational Research, 288*(1), 294–301.

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science, 39*(10), 1261–1264.

Ansari, Z. N., Kant, R., & Shankar, R. (2020). Evaluation and ranking of solutions to mitigate sustainable remanufacturing supply chain risks: A hybrid fuzzy SWARA-fuzzy COPRAS framework approach. *International Journal of Sustainable Engineering*. https://doi.org/10.1080/19397038.2020.1758973

Ban, A. I., Ban, O. I., Bogdan, V., Popa, D. C. S., & Tuse, D. (2020). Performance evaluation model of Romanian manufacturing listed companies by fuzzy AHP and TOPSIS. *Technological and Economic Development of Economy, 26*(4), 808–836.

Belton, V., & Stewart, T. (2002). *Multiple criteria decision analysis: an integrated approach*. Springer Science & Business Media.

Bodin, L., & Gass, S. I. (2004). Exercises for teaching the analytic hierarchy process. *Informstransactions on Education, 4*(2), 1–13.

Borda, J.-C. de. (1781). Mémoire sur les élections au scrutin: Histoire de l'Académie Royale des Sciences. *Paris, France*, *12*.

Bouroumine, Y., Bahi, L., Ouadif, L., Elhachmi, D., & Errouhi, A. A. (2020). Sitting MSW landfill combining GIS and analytic hierarchy process (AHP), case study: Ajdir, Morocco. *International Journal of Advanced Research in Engineering and Technology (IJARET)*, *11*(5).

Brunelli, M. (2018). A survey of inconsistency indices for pairwise comparisons. *International Journal of General Systems, 47*(8), 751–771.

Calabrese, A., Costa, R., Levialdi, N., & Menichini, T. (2019). Integrating sustainability into strategic decision-making: A fuzzy AHP method for the selection of relevant sustainability issues. *Technological Forecasting and Social Change, 139*, 155–168.

Çavmak, D., & Çavmak, Ş. (2020). Using AHP to prioritize barriers in developing medical tourism: Case of Turkey. *International Journal of Travel Medicine and Global Health, 8*(2), 73–79.

Chaiyaphan, C., & Ransikarbum, K. (2020). Criteria analysis of food safety using the analytic hierarchy process (AHP) - A case STUDY of Thailand's fresh markets. *E3S Web of Conferences, 141*, 02001.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444.

Contreras, I. (2011). A DEA-inspired procedure for the aggregation of preferences. *Expert Systems with Applications, 38*(1), 564–570.

Cook, W. D., & Kress, M. (1990). A data envelopment model for aggregating preference rankings. *Management Science, 36*(11), 1302–1310.

Darvishi, L., Daryaei, M. G., & Kouchi, A. H. S. (2020). Comparison of statistical modeling and AHP Methods in fire risk assessment in Oak forests of Iran. *Fores Res, 9*, 229.

Das, B., Bordoloi, R., Thungon, L. T., Paul, A., Pandey, P. K., Mishra, M., & Tripathi, O. P. (2020). An integrated approach of GIS, RUSLE and AHP to model soil erosion in West Kameng watershed, Arunachal Pradesh. *Journal of Earth System Science, 129*(1), 1–18.

de Jesus França, L. C., Mucida, D. P., Santana, R. C., de Morais, M. S., Gomide, L. R., & de Meneses Bateira, C. V. (2020). AHP Approach applied to multi-criteria decisions in environmental fragility mapping. *Floresta, 50*(3), 1623–1632.

Delice, E. K., & Can, G. F. (2020). A new approach for ergonomic risk assessment integrating KEMIRA, best–worst and MCDM methods. *Soft Computing, 24*(19), 15093–15110. https://doi.org/10.1007/s00500-020-05143-9

Duleba, S. (2020). Introduction and comparative analysis of the multi-level parsimonious AHP methodology in a public transport development decision problem. *Journal of the Operational Research Society*. https://doi.org/10.1080/01605682.2020.1824553

Faramondi, L., Oliva, G., & Bozóki, S. (2020). Incomplete analytic hierarchy process with minimum weighted ordinal violations. *International Journal of General Systems, 49*(6), 574–601.

Foroughi, A. A., & Tamiz, M. (2005). An effective total ranking model for a ranked voting system. *Omega, 33*(6), 491–496.

Gál, T., Stewart, T. J., & Hanne, T. (1999). *Multicriteria decision making: Advances in MCDM models, algorithms, theory, and applications, volume 21 of international series in operations research & management science*. Kluwer Academic Publishers, Boston, Dordrecht, London.

Ghamari, A., Abdollahi, B., Zeinabadi, H. R., & Tabeshfar, G. H. (2017). Assessment of organizational excellence based on analytical hierarchy process (AHP) emphasizing on the development of Bank Shahr economic capabilities. *Journal of Urban Economics and Management, 5*(19), 1–13.

Ghorshi Nezhad, M. R., Zolfani, S. H., Moztarzadeh, F., Zavadskas, E. K., & Bahrami, M. (2015). Planning the priority of high tech industries based on SWARA-WASPAS methodology: The case of the nanotechnology industry in Iran. *Economic Research-Ekonomska Istraživanja, 28*(1), 1111–1137. https://doi.org/10.1080/1331677X.2015.1102404

Goswami, S., & Mitra, S. (2020). Selecting the best mobile model by applying AHP-COPRAS and AHP-ARAS decision making methodology. *International Journal of Data and Network Science, 4*(1), 27–42.

Green, R. H., Doyle, J. R., & Cook, W. D. (1996). Preference voting and project ranking using DEA and cross-evaluation. *European Journal of Operational Research, 90*(3), 461–472.

Güler, M., Mukul, E., & Büyüközkan, G. (2019). Business intelligence system selection with AHP-VIKOR methodology. *6th International Conference on New Ideas in Management, Economics and Accounting, Paris, France*, 57–72.

Hadi-Vencheh, A., & Niazi-Motlagh, M. (2011). An improved voting analytic hierarchy process–data envelopment analysis methodology for suppliers selection. *International Journal of Computer Integrated Manufacturing, 24*(3), 189–197.

Hajkowicz, S. A., McDonald, G. T., & Smith, P. N. (2000). An evaluation of multiple objective decision support weighting techniques in natural resource management. *Journal of Environmental Planning and Management, 43*(4), 505–518.

Han, Y., Wang, Z., Lu, X., & Hu, B. (2020). Application of AHP to Road Selection. *ISPRS International Journal of Geo-Information, 9*(2), 86. https://doi.org/10.3390/ijgi9020086

Harker, P. T. (1987). Incomplete pairwise comparisons in the analytic hierarchy process. *Mathematical Modelling, 9*(11), 837–848.

Hashimoto, A. (1997). A ranked voting system using a DEA/AR exclusion model: A note. *European Journal of Operational Research, 97*(3), 600–604.

Hwang, C.-L., & Yoon, K. (1981). *Multiple Attribute Decision Making* (Vol. 186). Springer.

İnce, M., Yiğit, T., & Hakan Işik, A. (2020). A novel hybrid fuzzy AHP-GA method for test sheet question selection. *International Journal of Information Technology & Decision Making, 19*(02), 629–647. https://doi.org/10.1142/S0219622020500054

Ishizaka, A., & Labib, A. (2009). Analytic hierarchy process and expert choice: Benefits and limitations. *Or Insight, 22*(4), 201–220.

Ishizaka, A., & Labib, A. (2011a). Review of the main developments in the analytic hierarchy process. *Expert Systems with Applications, 38*(11), 14336–14345.

Ishizaka, A., & Labib, A. (2011b). Selection of new production facilities with the group analytic hierarchy process ordering method. *Expert Systems with Applications, 38*(6), 7317–7325.

Jain, V., & Ajmera, P. (2019). Evaluation of performance factors of FMS by combined decision making methods as AHP, CMBA and ELECTRE methodology. *Management Science Letters, 9*(4), 519–534.

Jamali, G., Asl, E. K., Zolfani, S. H., & Šaparauskas, J. (2017). *Analysing LARG supply chain management competitive strategies in Iranian cement industries [Analyzování konkurenčních strategií LARG řízení dodavatelského řetězce v Íránském cementárenském odvětví]*.

Juodagalvienė, B., Turskis, Z., Šaparauskas, J., & Endriukaitytė, A. (2017). Integrated multi-criteria evaluation of house's plan shape based on the edas and swara methods. *Engineering Structures and Technologies, 9*(3), 117–125. https://doi.org/10.3846/2029882X.2017.1347528

Katranci, A., & Kundakci, N. (2020). SWARA Temelli Bulanık COPRAS Yöntemi ile Soğuk Hava Deposu Seçimi. *Optimum: Journal of Economics & Management Sciences/Ekonomi ve Yönetim Bilimleri Dergisi*, *7*(1), 63–80

Keršuliene, V., Zavadskas, E. K., & Turskis, Z. (2010). Selection of rational dispute resolution method by applying new step-wise weight assessment ratio analysis (SWARA). *Journal of Business Economics and Management, 11*(2), 243–258.

Khan, A. A., Shameem, M., Kumar, R. R., Hussain, S., & Yan, X. (2019). Fuzzy AHP based prioritization and taxonomy of software process improvement success factors in global software development. *Applied Soft Computing, 83*, 105648.

Köksalan, M., & Zionts, S. (2012). *Multiple criteria decision making in the new millennium: Proceedings of the Fifteenth International Conference on Multiple Criteria Decision Making (MCDM) Ankara, Turkey, July 10–14, 2000* (Vol. 507). Springer Science & Business Media.

Leal, J. E. (2020). AHP-express: A simplified version of the analytical hierarchy process method. *MethodsX, 7*, 100748.

Liang, F., Brunelli, M., & Rezaei, J. (2020). Consistency issues in the best worst method: Measurements and thresholds. *Omega, 96*, 102175.

Liu, F.-H.F., & Hai, H. L. (2005). The voting analytic hierarchy process method for selecting supplier. *International Journal of Production Economics, 97*(3), 308–317.

Llamazares, B., & Pena, T. (2009). Preference aggregation and DEA: An analysis of the methods proposed to discriminate efficient candidates. *European Journal of Operational Research, 197*(2), 714–721.

Lotfi, F. H., Rostamy-Malkhalifeh, M., Aghayi, N., Beigi, Z. G., & Gholami, K. (2013). An improved method for ranking alternatives in multiple criteria decision analysis. *Applied Mathematical Modelling, 37*(1–2), 25–33.

Macharis, C., Springael, J., De Brucker, K., & Verbeke, A. (2004). PROMETHEE and AHP: The design of operational synergies in multi-criteria analysis: Strengthening PROMETHEE with ideas of AHP. *European Journal of Operational Research, 153*(2), 307–317.

Mahmudova, S., & Jabrailova, Z. (2020). Development of an algorithm using the AHP method for selecting software according to its functionality. *Soft Computing, 24*(11), 8495–8502. https://doi.org/10.1007/s00500-020-04902-y
Millet, I., & Wedley, W. C. (2002). Modelling risk and uncertainty with the analytic hierarchy process. *Journal of Multi-Criteria Decision Analysis, 11*(2), 97–107.
Mishra, A. R., Rani, P., Pandey, K., Mardani, A., Streimikis, J., Streimikiene, D., & Alrasheedi, M. (2020). Novel multi-criteria intuitionistic fuzzy SWARA–COPRAS approach for sustainability evaluation of the bioenergy production process. *Sustainability, 12*(10), 4155. https://doi.org/10.3390/su12104155
Narayanan, A. K., & Jinesh, N. (2018). Application of SWARA and TOPSIS methods for supplier selection in a casting unit. *International Journal of Engineering Research & Technology, 7*(5), 456–458.
Noguchi, H., Ogawa, M., & Ishii, H. (2002). The appropriate total ranking method using DEA for multiple categorized purposes. *Journal of Computational and Applied Mathematics, 146*(1), 155–166.
Obata, T., & Ishii, H. (2003). A method for discriminating efficient candidates with ranked voting data. *European Journal of Operational Research, 151*(1), 233–237.
Ogundoyin, S. O., & Kamil, I. A. (2020). A Fuzzy-AHP based prioritization of trust criteria in fog computing services. *Applied Soft Computing, 97*, 106789.
Panahi, S., Khakzad, A., & Afzal, P. (2017). Application of stepwise weight assessment ratio analysis (SWARA) for copper prospectivity mapping in the Anarak region, central Iran. *Arabian Journal of Geosciences, 10*(22), 484.
Pérez Vergara, I. G., Arias Sánchez, J. A., Poveda-Bautista, R., & Diego-Mas, J.-A. (2020). Improving distributed decision making in inventory management: A combined ABC-AHP approach supported by teamwork. *Complexity, 2020*, 1–13. https://doi.org/10.1155/2020/6758108
Pishchulov, G., Trautrims, A., Chesney, T., Gold, S., & Schwab, L. (2019). The voting analytic hierarchy process revisited: A revised method with application to sustainable supplier selection. *International Journal of Production Economics, 211*, 166–179. https://doi.org/10.1016/j.ijpe.2019.01.025
Prasad, R. (2019). Selection of internal safety auditors in an Indian construction organization based on the SWARA and ARAS methods. *Journal of Occupational Health and Epidemiology, 8*(3), 134–140.
Ramanathan, R. (2001). A note on the use of the analytic hierarchy process for environmental impact assessment. *Journal of Environmental Management, 63*(1), 27–35.
Rani, P., & Mishra, A. R. (2020). Single-valued neutrosophic SWARA-VIKOR framework for performance assessment of eco-industrial thermal power plants. *ICSES Trans. Neural Fuzzy Comput, 3*, 1–9.
Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega, 53*, 49–57.
Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega, 64*, 126–130. https://doi.org/10.1016/j.omega.2015.12.001
Rezaei, J., Nispeling, T., Sarkis, J., & Tavasszy, L. (2016). A supplier selection life cycle approach integrating traditional and environmental criteria using the best worst method. *Journal of Cleaner Production, 135*, 577–588.
Rezaei, J., Wang, J., & Tavasszy, L. (2015). Linking supplier development to supplier segmentation using Best Worst Method. *Expert Systems with Applications, 42*(23), 9152–9164. https://doi.org/10.1016/j.eswa.2015.07.073
Saaty, T. L. (1980). The analytic hierarchy process Mcgraw Hill, New York. *Agricultural Economics Review*, *70*.
Saaty, R. (2018). A validation of the effectiveness of inner dependence in an ANP model. *International Journal of the Analytic Hierarchy Process*. https://doi.org/10.13033/ijahp.v10i2.594
Saaty, T. L. (2006). *Fundamentals of decision making with the analytic hierarchy process*. RWS Publications.
Saaty, T. L. (2013). The modern science of multicriteria decision making and its practical applications: The AHP/ANP approach. *Operations Research, 61*(5), 1101–1118.
Sales, A. C. M., de Guimarães, L. G. A., Veiga Neto, A. R., El-Aouar, W. A., & Pereira, G. R. (2020). Risk assessment model in inventory management using the AHP method. *Gestão & Produção*. https://doi.org/10.1590/0104-530x4537-20
Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. *New Directions for Program Evaluation, 1986*(32), 73–105.
Sharafi, H., Lotfi, F. H., Jahanshahloo, G., Rostamy-malkhalifeh, M., Soltanifar, M., & Razipour-GhalehJough, S. (2019). Ranking of petrochemical companies using preferential voting at unequal levels of voting power through data envelopment analysis. *Mathematical Sciences, 13*(3), 287–297.
Singh, R., Avikal, S., Rashmi, R., & Ram, M. (2020). A Kano model, AHP and TOPSIS based approach for selecting the best mobile phone under a fuzzy environment. *International Journal of Quality & Reliability Management*, *ahead-of-p*(ahead-of-print). https://doi.org/10.1108/IJQRM-01-2020-0022
Sinuany-Stern, Z., Mehrez, A., & Hadad, Y. (2000). An AHP/DEA methodology for ranking decision making units. *International Transactions in Operational Research, 7*(2), 109–124.

Soltanifar, M. (2017). *A* new group voting analytical hierarchy process method using preferential voting. *Journal of Operational Research and Its Applications, 14*(3540016), 1–13.
Soltanifar, M. (2020). A new voting model for groups with members of unequal power and proficiency. *International Journal of Industrial Mathematics, 12*(2), 121–134.
Soltanifar, M., Ebrahimnejad, A., & Farrokhi, M. M. (2010). Ranking of different ranking models using a voting model and its application in determining efficient candidates. *International Journal of Society Systems Science, 2*(4), 375–389.
Soltanifar, M., & Lotfi, F. H. (2011). The voting analytic hierarchy process method for discriminating among efficient decision making units in data envelopment analysis. *Computers & Industrial Engineering, 60*(4), 585–592.
Soltanifar, M., & Shahghobadi, S. (2013). Selecting a benevolent secondary goal model in data envelopment analysis cross-efficiency evaluation by a voting model. *Socio-Economic Planning Sciences, 47*(1), 65–74.
Soltanifar, M., & Shahghobadi, S. (2014). Classifying inputs and outputs in data envelopment analysis based on TOPSIS method and a voting model. *International Journal of Business Analytics (IJBAN), 1*(2), 48–63.
Stanujkic, D., Karabasevic, D., & Zavadskas, E. K. (2015). A framework for the selection of a packaging design based on the SWARA method. *Engineering Economics, 26*(2), 181–187.
Tavana, M. (2004). A subjective assessment of alternative mission architectures for the human exploration of mars at NASA using multicriteria decision making. *Computers and Operations Research, 31*(7), 1147–1164.
Tavana, M. (2006). A priority assessment multi-criteria decision model for human spaceflight mission planning at NASA. *Journal of the Operational Research Society, 57*(10), 1197–1215.
Thompson, R G, Langemeiar, L. N., Lee, C. T., & Thrall, R. M. (1989). The measurement of productive efficiency with an application to Kansas Royland wheat farming. *Jesse H. Jones Graduate School of Administration Working Paper*, *65*.
Thompson, R. G., Singleton, F. D., Jr., Thrall, R. M., & Smith, B. A. (1986). Comparative site evaluations for locating a high-energy physics lab in Texas. *Interfaces, 16*(6), 35–49.
Triantaphyllou, E. (2001). Two new cases of rank reversals when the AHP and some of its additive variants are used that do not occur with the multiplicative AHP. *Journal of Multi-Criteria Decision Analysis, 10*(1), 11–25.
Tzeng, G.-H., & Huang, J.-J. (2011). *Multiple attribute decision making: Methods and applications*. CRC Press.
Vaidya, O. S., & Kumar, S. (2006). Analytic hierarchy process: An overview of applications. *European Journal of Operational Research, 169*(1), 1–29.
Wang, J., & Deng, X. (2020). Comprehensive economic benefit evaluation method of coastal enterprises based on AHP. *Journal of Coastal Research, 103*, 24–28.
Wedley, W. C. (1993). Consistency prediction for incomplete AHP matrices. *Mathematical and Computer Modelling, 17*(4–5), 151–161.
Wei, G., & Liu, J. (2008). A DS/AHP method for comprehensive decision-making in urban power system planning. *China International Conference on Electricity Distribution, 2008*, 1–5. https://doi.org/10.1109/CICED.2008.5211715
Zahir, S. (1999). Clusters in a group: Decision making in the vector space formulation of the analytic hierarchy process. *European Journal of Operational Research, 112*(3), 620–634.
Zand, A., Arfaee, M., & Eslami, K. (2020). Studying effects of consumed fertilizer on sustainable rural development by using AHP method. *Agricultural Marketing and Commercialization Journal, 4*(1), 1–12.
Zandebasiri, M., & Pourhashemi, M. (2016). The place of AHP method among multi criteria decision making methods in forest management. *International Journal of Applied Operational Research-an Open Access Journal, 6*(2), 75–89.
Zhu, G.-N., Hu, J., & Ren, H. (2020). A fuzzy rough number-based AHP-TOPSIS for design concept evaluation under uncertain environments. *Applied Soft Computing, 91*, 106228. https://doi.org/10.1016/j.asoc.2020.106228
Zolfani, S. H., Salimi, J., Maknoon, R., & Kildiene, S. (2015). Technology foresight about R&D projects selection; application of SWARA method at the policy making level. *Engineering Economics, 26*(5), 571–580.