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A soft multi-criteria decision analysis model with application to the European Union enlargement

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Abstract This paper proposes a new multi-criteria decision analysis (MCDA) model that uses a series of existing intuitive and analytical methods to systematically capture both objective and subjective beliefs and preferences from a group of decision makers (DMs). A defuzzification method that combines entropy and the theory of displaced ideal synthesizes crisp values from the DMs' subjective judgments. This approach assists the DMs in their selection process by plotting alternatives in a four quadrant graph and considering their Euclidean distance from the "ideal" choice. A pilot study illustrates the details of the proposed method. The DMs were a group of graduate students from the University of Paderborn in Germany. The pilot study concerned the addition of new members into the European Union (EU), a decision that has profound economic and political effects on both the entering and existing members of the Union. The DMs were required to consider a large number of internal strengths and weaknesses and external opportunities and threats in assessing the decision to enlarge the EU. Although the pilot study was not performed by actual DMs from the EU, it was an excellent platform for testing the proposed model.

Keywords Multi-criteria decision analysis · Soft computing · Fuzzy systems · SWOT · Analytic hierarchy process · European Union enlargement · Defuzzification · Entropy and theory of displaced ideal

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1 Introduction

A large body of intuitive and analytical models has evolved over the last several decades to assist decision makers (DMs) in multi-criteria decision analysis (MCDA). While these models have made great strides in multi-criteria decision-making, the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences. The literature and research show that the following difficulties may be encountered in MCDA:

(a) A decision may not be properly made without fully taking into consideration all subjective and objective criteria (Belton and Stewart 2002; Yang and Xu 2002).
(b) DMs often use verbal expressions and linguistic variables for subjective judgments which lead to ambiguity in human decision-making (Poyhonen et al. 1997).
(c) DMs often provide imprecise or vague information due to lack of expertise, unavailability of data, or time constraint (Kim and Ahn 1999).
(d) Meaningful and robust aggregation of subjective and objective judgments causes problems during the evaluation process (Valls and Torra 2000).

We propose a new MCDA model that uses a series of existing intuitive and analytical methods to systematically capture both objective and subjective beliefs and preferences from a group of DMs. A defuzzification method that combines entropy and the theory of displaced ideal is used to synthesize crisp values from the subjective judgments. This approach assists the DMs in their selection process by plotting alternatives in a four quadrant graph and considering their Euclidean distance from the "ideal" choice. We present the results of a pilot study to elucidate the details of the proposed method. The pilot study concerned the addition of new members into the European Union (EU), a decision that has profound economic and political effects on both the entering and existing members of the Union. In Sect. 2 we review the relevant literature and in Sect. 3 we present a detailed explanation of the mathematical model. In Sect. 4 we discuss the results of the pilot study and in Sect. 5 we present our concluding remarks and future research directions.

2 Literature review

The literature on MCDA contains hundreds of methods, including scoring methods, economic methods, portfolio methods, and decision analysis methods. Scoring methods use algebraic formulas to produce an overall score for each alternative (Osawa and Murakami 2002). Economic methods use financial models to calculate the monetary payoff of each alternative (Graves and Ringuest 1991). Portfolio methods evaluate the entire set of alternatives to identify the most attractive subset (Girotra et al. 2007; Lootsma et al. 1990; Mojsilovi et al. 2007; Wang and Hwang 2007). Cluster analysis, a more specific portfolio method, groups alternatives according to their support of the strategic positioning of the firm (Mathieu and Gibson 1993). Finally, decision analysis and simulation methods use random numbers and simulation to generate a large number of problems and pick the best outcome (Abacoumkin and Ballis 2004; Paisittanand and Olson 2006). Recently, strategic management researchers have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Bailey et al. 2003; Costa et al. 2003; Hsieh et al. 2004; Liesiö et al. 2007; Tavana 2006).

MCDA techniques require the determination of weights that reflect the relative importance of various competing criteria. Several approaches such as point allocation, paired comparisons, trade-off analysis, and regression estimates could be used to specify these weights (Kleindorfer et al. 1993). *Soft SWOT* utilizes the analytic hierarchy process (AHP) developed by Saaty (1977) to estimate the importance weight of the criteria. The process is simplified by confining the estimates to a series of pairwise comparisons. The measure of inconsistency provided by the AHP allows for the examination of inconsistent priorities. One of the advantages of the AHP is that it encourages DMs to be consistent in their pairwise comparisons. Saaty (1990a) suggests a measure of consistency for the pairwise comparisons. When the consistency ratio is unacceptable, the DMs are made aware that their pairwise comparisons are logically inconsistent, and they are encouraged to revise them. The AHP has been a very popular technique for determining weights in MCDA problems (Ho 2008; Vaidya and Kumar 2006; Saaty and Sodenkamp 2008). The advantage of the AHP is its capability to elicit judgments and scale them uniquely using a procedure that measures the consistency of these scale values (Saaty 1989).

There has been some criticism of the AHP in the operations research literature. Harker and Vargas (1987) show that the AHP does have an axiomatic foundation, the cardinal measurement of preferences is fully represented by the eigenvector method, and the principles of hierarchical decomposition and rank reversal are valid. On the other hand, Dyer (1990a) has questioned the theoretical basis underlying the AHP and argues that it can lead to preference reversals based on the alternative set being analyzed. In response, Saaty (1990b) explains how rank reversal is a positive feature when new reference points are introduced. In *Soft SWOT*, the geometric aggregation rule is used to avoid the controversies associated with rank reversal (Dyer 1990a, 1990b; Harker and Vargas 1990; Saaty 1990b).

Decision making in complex MCDA problems requires judgments on a large number of uncertain criteria. However, the estimation of uncertain criteria in MCDA is often very challenging. To address this issue, some researchers use fuzzy AHP to determine the weighting of subjective judgments for each criterion and to derive fuzzy synthetic utility values of alternatives. In *Soft SWOT* we do not use fuzzy AHP for the following reason. Fuzzy set theory has been introduced into AHP mainly to deal with uncertainty associated with pairwise comparison judgments. However, as Saaty and Tran (2007, p. 968) point out, "fuzzy AHP guarantees nothing and can foul up the outcome of a decision." The point is to use fuzzy numbers as a suitable tool to represent uncertain judgments. Judgments in the AHP are fuzzy themselves and additional fuzzifying of the numerical judgments will result in further fuzzification of the outcome. "Making good judgments gives good (valid) answers with the AHP and fuzzifying these judgments is simply a perturbation that leaves the results where they are without producing uniformly better outcomes" (Saaty and Tran 2007, p. 973).

Among the many tools and techniques in the strategic management literature, the strengths, weaknesses, opportunities, and threats or SWOT analysis has been widely used by both researchers and practitioners during the last several decades. SWOT is used to segregate the environmental factors and forces into internal strengths and weaknesses and external opportunities and threats (Valentin 2001; Duarte et al. 2006). Since its inception in the 1950s, SWOT has gained increasing success as a strategic management tool (Panagiotou 2003). SWOT is still alive and well as the popular framework for classifying environmental factors (Hitt et al. 2000; Anderson and Vince 2002). Despite its popularity, SWOT has remained a conceptual framework with limited prescriptive power for practice and minor significance for research (Novicevic et al. 2004).

Over the past few years, there has been an increasing application of the integrated SWOT with AHP (Ho 2008). Kurttila et al. (2000) proposed the combined SWOT-AHP approach to aid the decision-making in a Finnish forestry. Kajanus et al. (2004) proposed the combined approach to investigate the role of culture in rural tourism. Their approach was similar to the framework presented previously in Kurttila et al. (2000) where the AHP was used to measure the relative importance weightings of the individual SWOT factors. Shrestha et al. (2004) used the integrated framework to analyze the possibilities for silvopasture adoption in south-central Florida. Their approach was similar to those adopted by (Kurttila et al. 2000) and (Kajanus et al. 2004). Shrestha et al. (2004) applied the integrated framework to agricultural planning and Masozera et al. (2006) adopted the same approach to assess the suitability of community-based management method to the Nyungwe forest reserve in Rwanda. Shinno et al. (2006) presented the combined AHP–SWOT approach to analyze the global competitiveness of Japanese manufacturers of machine tools.

Recently, fuzzy concepts and theories have been applied to strategic management and SWOT analysis because of their complex and fuzzy nature (Buyukozkan and Feyzioglu 2002). Lin and Hsieh (2003) used fuzzy weighted average to defuzzify the industry attractiveness-business strength matrix. Pap et al. (2000) employed a fuzzy rule-base to handle the growth-share matrix. Ghazinoory et al. (2007) used the fuzzy approach to evaluate quantitative and qualitative factors and strategies in a SWOT matrix. Sodenkamp (2005) applied fuzzy sets to the network structured SWOT factors to measure their relative importance weights. More recently, the MCDA research community has extended their interest in fuzzy set theory into soft computing (Zadeh 1998). The integration of MCDA with soft computing for handling uncertainty is of major interest, both from a research and practical perspective (Kaliszewski 2006; Zopounidis and Doumpos 2001). Soft computing systematically applies the approximate "soft" treatments to MCDA methodologies to reduce the problems' computational complexity. Soft computing, unlike conventional (hard) computing, achieves tractability, robustness, low solution cost, and close resemblance with human like decision-making by exploiting the tolerance for imprecision, uncertainty, approximate reasoning and partial truth.

Soft SWOT is a new MCDA model that captures the DMs' beliefs through a series of intuitive and analytical methods such as the analytic hierarchy process (AHP) and SWOT analysis. A defuzzification method is used to obtain crisp values from the subjective judgments provided by multiple decision groups (DGs). These crisp values are synthesized with entropy and the theory of displaced ideal to assist the DMs in their selection process. Two aggregated opportunity/threat and strength/weakness indexes are used to plot the candidate states in a four quadrant graph based on their Euclidean distance from the ideal state. We do not introduce additional fuzziness to the problem. On the contrary, we use fuzzy sets as a tool for aggregating and analyzing uncertain group judgments. Representation of group estimates by means of fuzzy sets is not new and it was used by Tavana and Sodenkamp (2009) to facilitate advanced technology assessment by multiple DGs at the Kennedy space center.

Soft SWOT is a normative MCDA model with multiple criteria representing different dimensions from which the candidate states are viewed. When the number of criteria in a MCDA problem is large, they may be arranged hierarchically (Saaty 2003; Triantaphyllou 2000). Six groups of DMs, all from the University of Paderborn in Germany, were selected to participate in the EU expansion pilot study. We held several brainstorming sessions within and between groups to classify the 169 Copenhagen criteria into internal and external categories. Internal criteria reflect domestic affairs of a candidate state and external criteria are the environmental factors that influence the entire EU membership. Internal criteria are essentially controllable and external criteria are uncontrollable. The internal and external criteria include political, economic, and community standards. Each standard was divided into several criteria and each criterion was further divided into multiple sub-criteria.

Next, we used a scoring system to capture the intensity of each criterion and further classify the internal criteria into strengths and weaknesses and the external criteria into opportunities and threats. According to this scoring system, the DMs assigned a score from -10 to $+10$ to each internal and external criterion. A positive score to an internal criterion indicated strength and a negative score indicated weakness. In addition, a positive score to an external criterion indicated opportunity and a negative score indicated threat. Higher scores were preferred to lower scores for both internal and external criteria. In practice, two aggregation techniques are used to compute two aggregated indexes and evaluate the alternatives when criteria are divided into positive and negative forces. The first approach is the positive to negative ratio approach (Tavana and Banerjee 1995) and the second is the positive minus negative approach (Tavana 2004). The former approach is a ratio scale and the latter approach is an interval scale.

Soft SWOT is a weighted-sum MCDA model with strengths, weaknesses, opportunities and threats as conflicting criteria. Triantaphyllou (2000) has discussed the mathematical properties of weighted-sum MCDA models. Many weighted-sum models have been developed to help DMs deal with the strategy evaluation process (Gouveia et al. 2008; Leyva-Lopez and Fernandez-Gonzalez 2003). Triantaphyllou and Baig (2005) have examined the use of four key weighted-sum MCDA methods when benefits and costs (opportunities and threats) are used as conflicting criteria. They compared the simple weighted-sum model, the weighted-product model, and the analytic hierarchy process (AHP) along with some of its variants, including the multiplicative AHP. Their extensive empirical analysis revealed some ranking inconsistencies among the four methods, especially, when the number of alternatives was high. Although they were not able to show which method results in the "correct" ranking, they did prove multiplicative AHP is immune to ranking inconsistencies.

The weighted-sum scores in *Soft SWOT* are used to compare potential candidate states among themselves and with the *ideal state*. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance.

3 Mathematical model and procedure

The evaluation process begins with a preliminary review of M candidate states. The DMs from the K DGs consider H set of standards for the screening and evaluation of the candidate states. A series of weights and scores are used in *Soft SWOT* to estimate the importance weight of the selection criteria and their performance degree for each alternative. Initially, DMs use the AHP independently to weight their importance of the standards (q_h^k). We do not describe the technical details of the AHP because the procedure is well-documented in numerous research papers and literature sources (e.g., see Saaty 2006 or Saaty and Sodenkamp 2008 for a detailed discussion of pairwise comparisons and priority derivations in the AHP).

Next, each DG independently decides what internal and external criteria should be considered in the evaluation process. Both internal and external criteria have hierarchical structures with L levels. Once the DGs agree on a hierarchical structure, they define their importance of the internal and external criteria and sub-criteria ($w_{u_{hi}}^k$, $w_{v_{hi}}^k$, u_{hij}^k and v_{hij}^k).

Miller (1956) has shown that an individual can simultaneously compare not more than seven criteria (±2). In agreement with this proposition, we use the AHP in *Soft SWOT* for clusters with nine or less criteria. It is a common practice to divide clusters with more than nine criteria into smaller ones. When it is not justified to change the criteria structure, the DMs can assign direct priorities ($w'^k_{u_{hi}}$, $w'^k_{v_{hi}}$, u'^k_{hij} and v'^k_{hij}) to the criteria on a 1 to 10 scale and normalize those using (1) and (2) to unify their dimensions with the priority dimensions derived through the AHP.

$$\begin{aligned} w^k_{u_{hi}} &= \frac{w'^k_{u_{hi}}}{\sum_{i=1}^{Ih} w'^k_{u_{hi}}} \\ w^k_{v_{hi}} &= \frac{w'^k_{v_{hi}}}{\sum_{i=1}^{Ih} w'^k_{v_{hi}}} \end{aligned} \tag{1}$$

$$\begin{aligned} u^k_{hij} &= \frac{u'^k_{hij}}{\sum_{j=1}^{Jhi} u'^k_{hij}} \\ v^k_{hij} &= \frac{v'^k_{hij}}{\sum_{j=1}^{Jhi} v'^k_{hij}} \end{aligned} \tag{2}$$

where:

q^k_h = the hth standard importance weight for the kth DG; ($h = 1, 2, \ldots, H$; $k = 1, 2, \ldots, K$),

$w^k_{u_{hi}}(w^k_{v_{hi}})$ = the importance weight of the ith group of internal (external) criteria within the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $k = 1, 2, \ldots, K$),

$u^k_{hij}(v^k_{hij})$ = the weight of the jth internal (external) criterion in the ith group of the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$),

$w'^k_{u_{hi}}(w'^k_{v_{hi}})$ = the importance weight of the ith group of internal (external) criteria within the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $k = 1, 2, \ldots, K$; $I_h > 9$),

$u'^k_{hij}(v'^k_{hij})$ = the weight of the jth internal (external) criterion in the ith group of the standard h for the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $J_{hi} > 9$).

Next, the DMs provide their judgment on the intensity of the internal and external factors for each candidate state using a 1 to 10 scale. Positive scores represent strengths for the internal criteria and opportunities for the external criteria. Negative scores represent weaknesses for the internal criteria and threats for the external criteria. In those situations where the DMs are not sure about their estimates or cannot render judgments with full confidence, triangular or trapezoidal fuzzy numbers could be used in lieu of crisp scores. For example, Tavana et al. (2009) used triangular fuzzy numbers instead of exact judgments when evaluating alternative military bases in a base realignment and closure problem at the Department of Defense. However, using triangular or trapezoidal fuzzy numbers requires additional information about the dispersion of the scores (spreads of fuzzy numbers) and ultimately increasing the total number of judgments required from the DMs. The real-world decision problems can include hundreds of criteria. In order to use triangular fuzzy numbers, each DG has to make two additional judgments for each candidate on each criterion signifying the left and right spreads. In other words, three sets of judgments are required in comparison with one set of judgments required by *Soft SWOT*. As we shall see in the pilot study presented in Sect. 4, each DG has to judge 8 candidates on 169 criteria which in our case generates 1352 estimates. Whereas, for the scores expressed with triangular fuzzy numbers, a total of 2704 additional judgments are required from each DG. Furthermore, in order to use trapezoidal fuzzy numbers, each DG has to assign their scores as intervals, along with uncertainty spreads. The trapezoidal representation demands 5408 judgments by each DG in our pilot study. Therefore, in large problems with many uncertainties, it is not justified to introduce additional fuzziness and increase the number of DMs' judgments. Alternatively, a verbal judgment scale might be used to rate the alternatives where each verbal term is expressed with a fuzzy number. In this case, the DMs have to use predefined linguistic terms and cannot assign their own confidence bounds (left and right spreads of the fuzzy numbers or intervals). Such an approach supposes an equal uncertainty level for each DG when evaluating an alternative on a criterion, which sufficiently restricts the DMs' rights to decide on their own.

We then use a defuzzification method to obtain crisp values from the subjective judgments and estimates provided for the H standards and the M candidate states. These crisp values are then synthesized in a MCDA model to produce an overall performance score for each of the M countries under consideration. Table 1 presents the mathematical notations of a defuzzified decision matrix for a general problem.

Decision theory generally deals with three types of uncertainty: stochastic uncertainty, subjective uncertainty and informational uncertainty. Stochastic uncertainty is treated by probability theory and subjective and informational uncertainties are treated by fuzzy logic theory. Although fuzzy logic and probability theory are similar, they are not identical. Probability refers to the likelihood that something is true while fuzzy logic establishes the degree to which something is true. Probability is not a special case of fuzziness, but leads us to consider the probability of fuzzy events. Dubois and Prade (1993) provide an analysis of correlation between fuzzy sets and probability theory. They argue that the existence of mathematical objects in probability theory does not suggest that fuzziness is reducible to randomness and it is possible to approach fuzzy sets and possibility theory without any probability considerations. Their study emphasizes the interpretation multiplicity of probability and fuzzy set theories and shows that fuzzy set-theoretic operations can be categorized according to their membership in the upper probability, the one-point coverage of a random set, or a likelihood function.

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo et al. 2008), financial modeling (Muzzioli and Reynaerts 2007), information systems (Rolly Intan and Mukaidono 2004), auditing (Friedlob and Schleifer 1999), manufacturing cost estimation (Jahan-Shahi et al. 1999), and water quality management (Benoit 1994).

We use fuzzy logic in a defuzzification process to collapse H sets of standard weights, criteria weights and subjective scores into one set of crisp values for K groups of DMs. Consider discrete fuzzy sets A^m_{hij} represented by the pairs:

$$A^m_{hij} = \{(p^{km}_{hij}, \mu_{A^m_{hij}}(p^{km}_{hij}))\}, \quad \forall p^{km}_{hij} \in P^m_{hij} \tag{3}$$

where:

P^m_{hij} = the set of judgments of the DGs on criterion j in the ith group of standard h given the choice of the mth candidate; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $m = 1, 2, \ldots, M$),

Table 1 The defuzzified decision matrix

h	q_h^k	i_h	w_{hi}^k	j_{hi}	$u_{hij}^k(v_{hij}^k)$	μ_{hij}	EU membership candidate, m						
							1		2		…	M	
1	$\{q_1^k\}$	1	$\{w_{11}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	…	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{111}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	…	$\{p_{112}^{kM}\}$	r_{112}^M
				…	…	…	…	…	…	…	…	…	…
				J_{11}	$\{u_{11J_{11}}^k\}(\{v_{111}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	…	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{11}^k\}$	1	$\{u_{121}^k\}(\{v_{121}^k\})$	μ_{121}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	…	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{122}^k\}(\{v_{122}^k\})$	μ_{122}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	…	$\{p_{122}^{kM}\}$	r_{122}^M
				…	…	…	…	…	…	…	…	…	…
				J_{12}	$\{u_{12J_{12}}^k\}(\{v_{111}^k\})$	$\mu_{12J_{12}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	…	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		…	…	…	…	…	…	…	…	…	…	…	…
		I_1	$\{w_{1I_1}^k\}$	1	$\{u_{1I_11}^k\}(\{v_{1I_11}^k\})$	μ_{1I_11}	$\{p_{1I_11}^{k1}\}$	$r_{1I_11}^1$	$\{p_{1I_11}^{k2}\}$	$r_{1I_11}^2$	…	$\{p_{121}^{kM}\}$	$r_{1I_11}^M$
				2	$\{u_{1I_12}^k\}(\{v_{1I_12}^k\})$	μ_{1I_12}	$\{p_{1I_12}^{k1}\}$	$r_{1I_12}^1$	$\{p_{1I_12}^{k2}\}$	$r_{1I_12}^2$	…	$\{p_{1I_12}^{kM}\}$	$r_{1I_12}^M$
				…	…	…	…	…	…	…	…	…	…
				J_{1I1}	$\{u_{1I_1J_{12}}^k\}(\{v_{1I_1J_{12}}^k\})$	$\mu_{1I_1J_{12}}$	$\{p_{12I_1J_{12}}^{k1}\}$	$r_{1I_1J_{12}}^1$	$\{p_{1I_1J_{12}}^{k2}\}$	$r_{1I_1J_{12}}^2$	…	$\{p_{1I_1J_{12}}^{kM}\}$	$r_{1I_1J_{12}}^M$
2	$\{q_2^k\}$	1	$\{w_{21}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	…	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{112}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	…	$\{p_{112}^{kM}\}$	r_{112}^M
				…	…	…	…	…	…	…	…	…	…
				J_{11}	$\{u_{11J_{11}}^k\}(\{v_{111}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	…	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{22}^k\}$	1	$\{u_{221}^k\}(\{v_{221}^k\})$	μ_{121}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	…	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{222}^k\}(\{v_{222}^k\})$	μ_{122}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	…	$\{p_{122}^{kM}\}$	r_{122}^M
				…	…	…	…	…	…	…	…	…	…
				J_{22}	$\{u_{12J_{22}}^k\}(\{v_{11I_{22}}^k\})$	$\mu_{12J_{12}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	…	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		…	…	…	…	…	…	…	…	…	…	…	…

Table 1 (*Continued*)

h	q_h^k	i_h	w_{hi}^k	j_{hi}	$u_{hij}^k(v_{hij}^k)$	μ_{hij}	EU membership candidate, m						
							1		2		...	M	
		I_2	$\{w_{2I_2}^k\}$	1	$\{u_{2I_21}^k\}(\{v_{2I_21}^k\})$	μ_{2I_21}	$\{p_{2I_21}^{k1}\}$	$r_{2I_21}^1$	$\{p_{2I_21}^{k2}\}$	$r_{2I_21}^2$	...	$\{p_{2I_21}^{kM}\}$	$r_{2I_21}^M$
				2	$\{u_{2I_22}^k\}(\{v_{2I_22}^k\})$	μ_{2I_22}	$\{p_{2I_22}^{k1}\}$	$r_{2I_22}^1$	$\{p_{2I_22}^{k2}\}$	$r_{2I_22}^2$	...	$\{p_{2I_22}^{kM}\}$	$r_{2I_22}^M$
				...	...	...	...	...	...	...	...	...	...
				J_{2I_2}	$\{u_{2I_2J_{12}}^k\}(\{v_{2I_2J_{12}}^k\})$	$\mu_{2I_2J_{I22}}$	$\{p_{2I_2J_{22}}^{k1}\}$	$r_{2I_2J_{22}}^1$	$\{p_{2I_2J_{22}}^{k2}\}$	$r_{2I_2J_{22}}^2$	...	$\{p_{2I_2J_{22}}^{kM}\}$	$r_{2I_2J_{22}}^M$
...	...	...	...	...	...	...	...	...	...	...	...	...	...
H	$\{q_H^k\}$	1	$\{w_{H1}^k\}$	1	$\{u_{111}^k\}(\{v_{111}^k\})$	μ_{111}	$\{p_{111}^{k1}\}$	r_{111}^1	$\{p_{111}^{k2}\}$	r_{111}^2	...	$\{p_{111}^{kM}\}$	r_{111}^M
				2	$\{u_{112}^k\}(\{v_{112}^k\})$	μ_{112}	$\{p_{112}^{k1}\}$	r_{112}^1	$\{p_{112}^{k2}\}$	r_{112}^2	...	$\{p_{112}^{kM}\}$	r_{112}^M
				...	...	...	...	...	...	...	...	...	...
				J_{H1}	$\{u_{11J_{11}}^k\}(\{v_{1J_{11}}^k\})$	$\mu_{11J_{11}}$	$\{p_{11J_{11}}^{k1}\}$	$r_{11J_{11}}^1$	$\{p_{11J_{11}}^{k2}\}$	$r_{11J_{11}}^2$	...	$\{p_{11J_{11}}^{kM}\}$	$r_{11J_{11}}^M$
		2	$\{w_{H2}^k\}$	1	$\{u_{H21}^k\}(\{v_{H21}^k\})$	μ_{H21}	$\{p_{121}^{k1}\}$	r_{121}^1	$\{p_{121}^{k2}\}$	r_{121}^2	...	$\{p_{121}^{kM}\}$	r_{121}^M
				2	$\{u_{H22}^k\}(\{v_{H22}^k\})$	μ_{H22}	$\{p_{122}^{k1}\}$	r_{122}^1	$\{p_{122}^{k2}\}$	r_{122}^2	...	$\{p_{122}^{kM}\}$	r_{122}^M
				...	...	...	...	...	...	...	...	...	...
				J_{H2}	$\{u_{H2J_{22}}^k\}(\{v_{H1I_{H2}}^k\})$	$\mu_{H2J_{H2}}$	$\{p_{12J_{12}}^{k1}\}$	$r_{12J_{12}}^1$	$\{p_{12J_{12}}^{k2}\}$	$r_{12J_{12}}^2$	...	$\{p_{12J_{12}}^{kM}\}$	$r_{12J_{12}}^M$
		...	...	...	...	...	...	...	...	...	...	...	...
		I_H	$\{w_{HI_H}^k\}$	1	$\{u_{HI_{H1}1}^k\}(\{v_{HI_H1}^k\})$	μ_{HI_H1}	$\{p_{HI_H1}^{k1}\}$	$r_{HI_H1}^1$	$\{p_{HI_H1}^{k2}\}$	$r_{HI_H1}^2$	...	$\{p_{HI_H1}^{kM}\}$	$r_{HI_H1}^M$
				2	$\{u_{HI_H2}^k\}(\{v_{HI_H2}^k\})$	μ_{HI_H2}	$\{p_{HI_H2}^{k1}\}$	$r_{HI_H2}^1$	$\{p_{HI_H2}^{k2}\}$	$r_{HI_H2}^2$	...	$\{p_{HI_H2}^{kM}\}$	$r_{HI_H2}^M$
				...	...	...	...	...	...	...	...	...	...
				J_{HI1}	$\{u_{HI_HJ_{12}}^k\}(\{v_{HI_HJ_{12}}^k\})$	$\mu_{HI_HJ_{I22}}$	$\{p_{H2I_HJ_{22}}^{k1}\}$	$r_{HI_HJ_{22}}^1$	$\{p_{HI_HJ_{22}}^{k2}\}$	$r_{HI_HJ_{22}}^2$	...	$\{p_{HI_HJ_{22}}^{kM}\}$	$r_{HI_HJ_{22}}^M$
							$U^1(V^1)$		$U^2(V^2)$		...	$U^M(V^M)$	

p_{hij}^{km} = the judgment given by the kth DG on criterion j in the ith group of standard h given the choice of the mth candidate; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$),

$\mu_{A_{hij}^m}(p_{hij}^{km})$ = the membership grade of judgment of the kth DG; ($h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, I_h$; $j = 1, 2, \ldots, J_{hi}$; $k = 1, 2, \ldots, K$; $m = 1, 2, \ldots, M$).

Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar, and crisp representations. The process of condensing the information captured by fuzzy sets into numerical values is similar to that of transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process in *Soft SWOT* is similar to an averaging procedure. Special defuzzification methods can be used to increase the numerical efficiency and transparency of the computations. Many defuzzification techniques have been proposed in the literature. The most commonly used method is the center of gravity (COG). Other methods include: random choice of maximum, first of maximum, last of maximum, middle of maximum, mean of maxima, basic defuzzification distributions, generalized level set defuzzification, indexed center of gravity, semi-linear defuzzification, fuzzy mean, weighted fuzzy mean, quality method, extended quality method, center of area, extended center of area, constraint decision defuzzification, and fuzzy clustering defuzzification. Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions (Ali and Zhang 2001; Roychowdhury and Pedrycz 2001; Runkler 1996, Van Leekwijk and Kerre 1999). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. While the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir and Yuan (1995). We use COG, also referred to as the center of area method, in *Soft SWOT*. This method is highly popular and is often used as a standard defuzzification method. COG calculates the centroid of a possibility distribution function using (4) for continuous cases and (5) for discontinuous cases:

$$\mathrm{COG}(N) = \frac{\int_{-\infty}^{\infty} x\mu N(x)dx}{\int_{-\infty}^{\infty} \mu N(x)dx} \tag{4}$$

$$\mathrm{COG}(N) = \frac{\sum_{k=1}^{K} x_k \mu(x_k)}{\sum_{k=1}^{K} \mu(x_k)} \tag{5}$$

The procedure for converting the fuzzy numbers into a set of crisp values in *Soft SWOT* can be divided into the following three steps:

Step 1 Evaluation of the membership functions related to the subjective scores for the internal ($\mu_{u_{hij}}^k$) and external criteria ($\mu_{v_{hij}}^k$):

$$\begin{aligned} \mu_{u_{hij}}^k &= q_h^k \cdot w_{u_{hij}}^k \cdot u_{hij}^k \\ \mu_{v_{hij}}^k &= q_h^k \cdot w_{v_{hij}}^k \cdot v_{hij}^k \end{aligned} \tag{6}$$

We should note that, even though we assume an equal voting power for all the DGs, alternatively, different voting weights could be assigned to different DGs in the model.

Step 2 Calculation of the overall weighted scores of the strengths ($r_{hij}^m(S)$), weaknesses ($r_{hij}^m(W)$), opportunities ($r_{hij}^m(O)$) and threats ($r_{hij}^m(T)$) for the M candidate countries as the summed product of the scores on their grades of membership:

$$
\begin{aligned}
r_{hij}^{m}(S) &= \sum_{k=1}^{K} \mu_{U_{hij}}^{k} p_{hij}^{km}(S) \\
r_{hij}^{m}(W) &= -\sum_{k=1}^{K} \mu_{U_{hij}}^{k} p_{hij}^{km}(W) \\
r_{hij}^{m}(O) &= \sum_{k=1}^{K} \mu_{V_{hij}}^{k} p_{hij}^{km}(O) \\
r_{hij}^{m}(T) &= -\sum_{k=1}^{K} \mu_{V_{hij}}^{k} p_{hij}^{km}(T)
\end{aligned} \tag{7}
$$

where:

$p_{hij}^{km}(S)$ = the intensity of the jth strength for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^S; i = 1, 2, \dots, I_h^S; j = 1, 2, \dots, J_{hi}^S; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p_{hij}^{km}(W)$ = the intensity of the jth weakness for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^W; i = 1, 2, \dots, I_h^W; j = 1, 2, \dots, J_{hi}^W; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p_{hij}^{km}(O)$ = the intensity of the jth opportunity for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^O; i = 1, 2, \dots, I_h^O; j = 1, 2, \dots, J_{hi}^O; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$,

$p_{hij}^{km}(T)$ = the intensity of the jth threat for the ith group of the hth standard given the choice of the mth candidate state by DG k; $(h = 1, 2, \dots, H^T; i = 1, 2, \dots, I_h^T; j = 1, 2, \dots, J_{hi}^T; k = 1, 2, \dots, K; m = 1, 2, \dots, M)$.

and:

$H^S(H^W, H^O, H^T)$ = the number of standards in the cluster of strengths (weaknesses, opportunities, threats),

$I_h^S(I_h^W, I_h^O, I_h^T)$ = the number of criteria groups in the hth standard of strengths (weaknesses, opportunities, threats),

$J_{hi}^S(J_{hi}^W, J_{hi}^O, J_{hi}^T)$ = the number of criteria in the ith group of the hth standard of strengths (weaknesses, opportunities, threats).

Step 3 On the final step of the defuzzification process, we divide the overall weighted scores of the internal and external factors by their summed membership grades. These calculations result in M vectors of non-fuzzy values characterizing strengths, weaknesses, opportunities and threats for M countries:

$$
\begin{aligned}
\mathrm{COG}(S_{hij}^{m}) &= \frac{r_{hij}^{m}(S)}{\mu_{u_{hij}}} \\
\mathrm{COG}(W_{hij}^{m}) &= \frac{r_{hij}^{m}(W)}{\mu_{u_{hij}}} \\
\mathrm{COG}(O_{hij}^{m}) &= \frac{r_{hij}^{m}(O)}{\mu_{v_{hij}}} \\
\mathrm{COG}(T_{hij}^{m}) &= \frac{r_{hij}^{m}(T)}{\mu_{v_{hij}}}
\end{aligned} \tag{8}
$$

where $\mu_{u_{hij}}$ and $\mu_{v_{hij}}$ define the membership functions for the internal and external criteria with aggregated results for all DGs:

$$\mu_{u_{hij}} = \sum_{k=1}^{K} \mu^k_{u_{hij}}$$
$$\mu_{v_{hij}} = \sum_{k=1}^{K} \mu^k_{v_{hij}} \tag{9}$$

Next, we find the defuzzified value of the importance weights of the strengths, weaknesses, opportunities and threats, as well as the total defuzzified values of the internal (U^m) and external (V^m) criteria for all countries under consideration:

$$S^m = \sum_{h=1}^{H^S} \sum_{i=1}^{I_h^S} \sum_{j=1}^{J_{hi}^S} \mathrm{COG}(S^m_{hij})$$
$$W^m = \sum_{h=1}^{H^W} \sum_{i=1}^{I_h^W} \sum_{j=1}^{J_{hi}^W} \mathrm{COG}(W^m_{hij})$$
$$O^m = \sum_{h=1}^{H^O} \sum_{i=1}^{I_h^O} \sum_{j=1}^{J_{hi}^O} \mathrm{COG}(O^m_{hij})$$
$$T^m = \sum_{h=1}^{H^T} \sum_{i=1}^{I_h^T} \sum_{j=1}^{J_{hi}^T} \mathrm{COG}(T^m_{hij}) \tag{10}$$

$$U^m = S^m - W^m$$
$$V^m = O^m - T^m \tag{11}$$

Finally, we revise the importance weight of the strengths, weaknesses, opportunities and threats determined through the defuzzification process with the entropy concept. Each internal or external criterion is an information source; therefore, the more information each criterion reveals, the more relevant it is to the decision analysis. The level of entropy $e(P)$ as a measure of fuzziness, indicates the variance of the assigned preference relation. The concept of entropy originated in physics and statistics and has become increasingly popular in computer science and information theory. Shannon (1948) has defined the entropy of a probability distribution where the total probability for all elements must add up to 1. However, De Luca and Termini (1972) show that this restriction is unnecessary. They define a fuzzy entropy formula on a finite universal set $X = \{x_1, \ldots, x_n\}$ as:

$$e_{LT}(A) = -\beta \sum_{i=1}^{n} [\mu_A(x_i) \ln \mu_A(x_i) + (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))] \tag{12}$$

where $\beta > 0$ is a normalization constant, ln is the natural logarithm and $\mu_A(x_i)$ is the membership function for each preference intensity.

The larger the difference between the subjective weights and scores of criteria The more different the subjective weights and scores of criteria are, the larger is the contrast intensity of the strength, weakness, opportunity or threat, and the greater is the amount of information transmitted by that criteria. Assuming that the vector $P^m_{hji} = \{P^{km}_{hji}\}$ characterizes the set of weighted scores in terms of the jth factor for the ith group of standard h (hijth factor) given the choice of the mth candidate state, the entropy measure of the hijth factor is:

$$e(A^m_{hij}) = -\beta \sum_{k=1}^{K} [\mu(p^{km}_{hij}) \ln \mu(p^{km}_{hij}) + (1 - \mu(p^{km}_{hij})) \ln(1 - \mu(p^{km}_{hij}))] \tag{13}$$

where $0 \le \mu(p^{km}_{hij}) \le 1$ and $e(A^m_{hij}) \ge 0$. The smaller $e(A^m_{hij})$ is, the more information the hijth criterion transmits, and the larger $e(A^m_{hij})$ is, the less information it transmits. Taking $\beta = 0$, the entropy measure of the hijth strength, weakness, opportunity or threat is:

$$\begin{aligned}
e(A^m_{hij}(S)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(S) \ln \mu^k_{hij}(S) + (1 - \mu^k_{hij}(S)) \ln(1 - \mu^k_{hij}(S))] \\
e(A^m_{hij}(W)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(W) \ln \mu^k_{hij}(W) + (1 - \mu^k_{hij}(W)) \ln(1 - \mu^k_{hij}(W))] \\
e(A^m_{hij}(O)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(O) \ln \mu^k_{hij}(O) + (1 - \mu^k_{hij}(O)) \ln(1 - \mu^k_{hij}(O))] \\
e(A^m_{hij}(T)) &= -\sum_{k=1}^{K} [\mu^k_{hij}(T) \ln \mu^k_{hij}(T) + (1 - \mu^k_{hij}(T)) \ln(1 - \mu^k_{hij}(T))]
\end{aligned} \tag{14}$$

The total entropies of strengths (E^m_S), weaknesses (E^m_W), opportunities (E^m_O) and threats (E^m_T) for candidate m are defined as $E^m_S = \sum_{h=1}^{H^S} \sum_{i=1}^{I^S_h} \sum_{j=1}^{J^S_{hi}} e(A^m_{hij}(S))$, $E^m_W = \sum_{h=1}^{H^W} \sum_{i=1}^{I^W_h} \sum_{j=1}^{J^W_{hi}} e(A^m_{hij}(W))$, $E^m_O = \sum_{h=1}^{H^O} \sum_{i=1}^{I^O_h} \sum_{j=1}^{J^O_{hi}} e(A^m_{hij}(O))$ and $E^m_T = \sum_{h=1}^{H^T} \times \sum_{i=1}^{I^T_h} \sum_{j=1}^{J^T_{hi}} e(A^m_{hij}(T))$, respectively.

In order to calculate the overall entropy in the groups of internal and external criteria, we add up the entropies of strengths and weaknesses, and the entropies of opportunities and threats:

$$\begin{aligned}
E^m_U &= E^m_S + E^m_W \\
E^m_V &= E^m_O + E^m_T
\end{aligned} \tag{15}$$

Using the Euclidean measure suggested by Zeleny (1982), *Soft SWOT* synthesizes the results by determining the ideal internal and external criteria values. The ideal overbalance of strengths and weaknesses (U^*) is the highest defuzzified importance weight of the internal criteria among the set U^m and the ideal overbalance between opportunities and threats (V^*) is the lowest defuzzified importance weight of the external criteria among the set V^m. We then find the Euclidean distance of each candidate state from the ideal state. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the mth indices of the internal and external characteristics. To formulate *Soft SWOT* algebraically, let us assume:

D_U^m = total Euclidean distance from the ideal internal characteristic for the mth candidate state; $(m = 1, 2, \dots, M)$,
D_V^m = total Euclidean distance from the ideal external characteristic for the mth candidate state; $(m = 1, 2, \dots, M)$,
D^m = overall Euclidean distance of the mth candidate state; $(m = 1, 2, \dots, M)$,
$U^*(V^*)$ = the ideal defuzzified value of the internal (external) criteria,
$E_U^*(E_V^*)$ = the entropy of the ideal internal (external) characteristic,
$DE_U^m(DE_V^m)$ = the Euclidean distance from the entropy of the ideal internal (external) characteristic for the mth candidate state; $(m = 1, 2, \dots, M)$,
D^m = overall Euclidean distance of the entropy for the mth candidate state; $(m = 1, 2, \dots, M)$.

$$D^m = \sqrt{(D_U^m)^2 + (D_V^m)^2} \tag{16}$$

$$DE^m = \sqrt{(DE_U^m)^2 + (DE_V^m)^2} \tag{17}$$

$$\begin{aligned} U^* &= \text{Max}\{U^m\} \\ V^* &= \text{Max}\{V^m\} \end{aligned} \tag{18}$$

$$\begin{aligned} E_U^* &= \text{Min}\{E_U^m\} \\ E_V^* &= \text{Min}\{E_V^m\} \end{aligned} \tag{19}$$

where:

$$\begin{aligned} D_U^m &= U^* - U^m \\ D_V^m &= V^m - V^* \\ DE_U^m &= E_U^m - E_U^* \\ DE_V^m &= E_V^m - E_V^* \end{aligned}$$

Candidate states with smaller D^m are closer to the ideal state and are preferred to candidate states with larger D^m which are further away from the ideal state. Next, we plot the candidate state on a graph where the x-axis is represented by the overbalance between strengths and weaknesses (U) and the y-axis is represented by the overbalance between opportunities and threats (V). The position of the point corresponding to candidate state m has Cartesian coordinates (U^m, V^m) on the graph.

Soft SWOT also considers the overall Euclidean distance of the entropy for the mth candidate state (DE^m). States with smaller Euclidean distance of entropy (minimal measure of uncertainty) are preferred to states with larger Euclidean distance of entropy (higher measure of uncertainty). The resulting SWOT graph contains four quadrants: *exploitation*, *challenge*, *discretion*, and *desperation*:

- *Exploitation quadrant*: In this quadrant, the EU membership candidate state has a positive overbalance of strengths over weaknesses and opportunities over threats. This area represents the greatest possible advantage for the EU. States falling into this quadrant should be considered seriously.

- *Challenge quadrant*: In this quadrant, the state has prevalence of strengths over weaknesses within internal scope. However, the condition of its external issues is unsatisfactory for the EU acceptance since threats exceed opportunities. A state falling in this quadrant should utilize their strengths to reduce their vulnerability to external threats. This quadrant requires full use of the EU's abilities and resources.
- *Discretion quadrant*: In this quadrant, the state has a high level of the EU integration and cooperation which is reflected by a positive overbalance of opportunities over threats. Strong weaknesses in this quadrant point to a negative internal situation. An improvement plan must be developed for reducing weaknesses and increasing strengths where possible. This quadrant represents the area where the EU has freedom or power to act or judge on its own.
- *Desperation quadrant*: This is the most risky quadrant. States falling into this quadrant have negative results with respect to the internal and external evaluation criteria. These should be considered as a last resort since they are characterized by high weaknesses and threats.

Once the model is developed, sensitivity analyses can be performed to determine the impact on the ranking of projects for changes in various model assumptions. Some sensitivity analyses that are usually of interest are on the weights and scores. The weights representing the relative importance of the standard and the internal and external evaluation criteria are occasionally a point for discussion among the various DMs. In addition, scores which reflect the degree of performance of an uncertain criterion are sometimes a matter of contention.

4 The pilot study

The European Union (EU) is a geo-political and economic community covering a large portion of the European continent. It was founded upon numerous treaties and has undergone several expansions taking it from its six founder states to twenty-seven member states. To join the EU, a country must go through an extensive screening process and conform to a series of fairly demanding criteria established by the European Council in Copenhagen. The Copenhagen criteria require a stable democracy which respects human rights and the rule of law (political standard); a functioning market economy capable of competition within the EU (economic standard); and the acceptance of the obligations of membership, including EU law (community standard). Each standard has several criteria and each criterion is comprised of multiple sub-criteria. The screening process is intended to determine how well a candidate state is prepared to join the EU. The Commission issues a report to the Council on the screening of the political, economic and community standards. This leads to open negotiations or a requirement that the candidate state must first meet these benchmarks. It is the responsibility of the European Council to determine whether the candidate state has fulfilled the Copenhagen criteria. The current selection process is ambiguous and unstructured. One problem is that the accession criteria adopted at the Copenhagen summit of EU leaders in 1993 are not all quantitative and precise.

We illustrate the application of *Soft SWOT* in a pilot study conducted for the European Commission to screen candidates for membership in the EU. The pilot study involved 30 graduate students from the University of Paderborn in Germany. Six groups of five DMs with equal voting power were formed. Each group included at least one economist, one political scientist, one social scientist, and one business student. Decisive thresholds and benchmarks were used to assess the candidate states and their progress towards fulfilling the accession requirements. Currently, accession negotiations are underway with eight countries identified by the European Commission as either "candidate" or "potential candidate" states. Candidate states which include Croatia, Macedonia and Turkey have already applied for membership and are involved in current negotiations. Potential candidate states which include Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia are interested in membership and are promised a prospect of EU membership. To join the EU, each country must conform to a series of criteria established by the European Council in Copenhagen. These criteria are combined in three large groups ($L = 3$) of political, economic and community standards. In our study, the political, economic and community standards have been considered to be of equal importance. Hence, we could operate with the three respective hierarchies independently where the weight of the top-element is 1. Altogether, 38 criteria divided into 169 sub-criteria were considered in this study.

The process began with an initial meeting of the six groups of DMs who used Expert Choice (Expert Choice 2006) to weight the importance of each standard. The DMs worked within and between groups to classify 169 Copenhagen sub-criteria into internal criteria which were believed somewhat controllable by the candidate states, and external criteria which were less controllable depending on the environmental conditions and forces. Internal and external criteria were further classified into favorable and unfavorable categories. Favorable internal criteria were characterized as "strengths" and favorable external criteria as "opportunities." Unfavorable internal criteria were characterized as "weaknesses" and unfavorable external criteria as "threats." Such classification allows for the simultaneous inclusion of favorable and unfavorable internal and external criteria in an integrated model. Tables 2 and 3 present selected Copenhagen criteria along with their associated classification. A complete listing of the criteria and their descriptions are provided in the official progress reports published by the European Commission and presented in Appendix. The criteria used in this study were identical to those listed in the progress reports.

Next, the DGs held separate meetings to weight 38 criteria and 169 sub-criteria. Theoretically, $N(N-1)/2$ pairwise comparisons are needed for each set of N criteria. However, the psychological experiments conducted by Miller (1956) have shown that an individual cannot simultaneously compare more than seven criteria (± 2) without being confused. In agreement with Miller (1956), we used the AHP and Expert Choice (Expert Choice 2006) for sets with nine or less criteria. The DMs recorded their consistency ratios and made sure it was below 0.10 as suggested by Saaty (1977, 1980). The Copenhagen criteria classification is strictly defined by the European Council. Therefore, we did not break down the large groups of standards to operationalize the AHP pairwise comparison process. Instead, we used the scoring and normalization process described in the previous section for clusters with more than nine criteria. Altogether, each decision-making group made 365 judgments. However, we were able to effectively manage the pairwise comparison process by dividing the judgments into manageable groups and assigning them to those with relevant background and expertise in economics, political science, social science, and business.

Next, the six DGs identified their performance score of each internal and external criterion for each of the eight candidate states using the data provided in the official progress report (see Appendix). They used a -10 to -1 scale to assign a score to those internal and external criteria which had a negative performance and used a $+1$ to $+10$ scale to assign a score to those internal and external criteria which had a positive performance. The scoring of the criteria resulted in the identification of the strengths, weaknesses, opportunities and threats. Higher scores were preferred to lower scores for both internal and external criteria.

The importance weight of the six standards along with the importance weight of the criteria and sub-criteria and the performance scores were all collapsed using discrete fuzzy

Table 2 The selected internal evaluation criteria

Standard	Criterion	Sub-criterion
Political	Guarantee of democracy and rule of law	Parliament Public administration Anti-corruption policy ...
	Guarantee of human right and respect for and protection of minorities	Civil and political rights Economic and social rights Minority rights, cultural rights and protection of minorities ...
	...	...
Economic	Functioning market economy	Legal system Financial sector development ...
	Ability to cope with the pressure of competition and the market forces at work inside the Union	Existence of a functioning market economy Human and physical capital Sector and enterprise structure State influence on competitiveness ...
	...	...
Acceptance of the Community *Acquis*	Free movement of goods	Administrative capacity Metrology ...
	Freedom of movement for workers	Access to the labour market Coordination of social security systems ...
	Company law	Company law Corporate accounting Auditing ...
	Competition policy	State aid/state aid enforcement Fiscal aid ...
	Financial services	Banks and financial conglomerates Financial market infrastructure ...
	Agriculture	Rural development Organic farming ...
	Food safety, veterinary and phytosanitary policy	Veterinary checks Phytosanitary issues Animal welfare ...
	...	...

Table 3 The selected external evaluation criteria

Standard	Criterion	Sub-criterion
Political	Guarantee of human right and respect for and protection of minorities	Observance of international human rights law
		…
	Regional issues and international obligations	Dayton/Paris and Erdut peace agreements
		Bilateral relations with other enlargement countries and neighboring member states
		…
	…	…
Economic	Functioning market economy	Market entry and exit
		…
	Ability to cope with the pressure of competition and the market forces at work inside the Union	Economic integration with the EU
		…
	…	…
Acceptance of the Community *Acquis*	Freedom of movement for workers	Participation in the EURES network
		European health insurance card
		…
	Free movement of capital	Capital movements
		AML directives/standards of the financial action task force
		…
	Agriculture	Integrated administration and control system (IACS)/Land parcel identification system (LPIS)
		Common market organizations
		…
	Energy	Gas market
		Energy community treaty
		Nuclear safety and radiation protection
		…
	Trans-European networks	Development of the trans-European networks
		Conformity with TEN guidelines
		…
	Justice, freedom and security	Schengen and external borders
		Police cooperation
		Organized crime/terrorism
		…
	Science and research	EC framework programmes
		Integration into the European research area
		…
	…	…

Table 4 A membership grade example

A. The criterion weights for *guarantee of democracy and rule of law*

Standard	Criterion	Criterion weights					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Political	Guarantee of democracy and rule of law	0.50	0.40	0.40	0.45	0.40	0.38

B. The sub-criteria weights for *guarantee of democracy and rule of law*

Criterion	Sub-criteria	Sub-criteria weights					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy	Parliament	0.30	0.20	0.15	0.20	0.15	0.23
and rule of law	Public administration	0.20	0.25	0.20	0.20	0.20	0.22
	Anti-corruption policy	0.10	0.10	0.10	0.10	0.10	0.13
	...	...	...	...	...	...	...

C. The sub-criteria scores for Albania

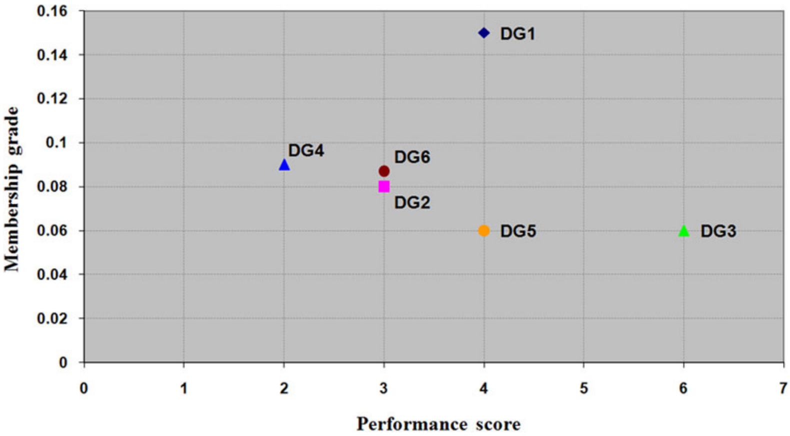
Criterion	Sub-criteria	Scores for Albania					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy	Parliament	4	3	6	2	4	3
and rule of law	Public administration	6	4	3	3	2	4
	Anti-corruption policy	4	4	2	2	3	4
	...	...	...	...	...	...	...

D. The sub-criteria weights for Albania

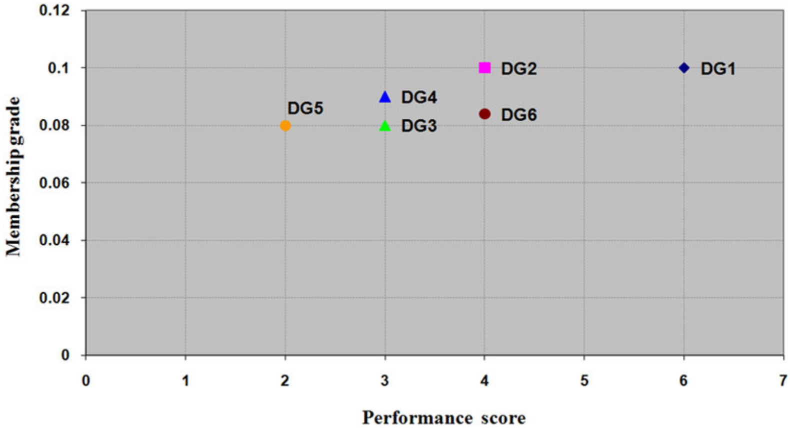
Criterion	Sub-criteria	Grades of membership					
		DG 1	DG 2	DG 3	DG 4	DG 5	DG 6
Guarantee of democracy	Parliament	0.15	0.08	0.06	0.09	0.06	0.087
and rule of law	Public administration	0.10	0.10	0.08	0.09	0.08	0.084
	Anti-corruption policy	0.05	0.04	0.04	0.045	0.04	0.05
	...	...	...	...	...	...	...

sets with a [0, 1] membership function range. The fuzzy sets are composed of criteria and sub-criteria where their intensities are assigned by the DMs and their grades of membership are obtained from (5) and (6). Let us exemplify this idea through a set of sample judgments provided by the DMs from the six DGs for the sub-criteria *Parliament*, *public administration* and *anti-corruption* for Albania. These sub-criteria belong to the *guarantee of democracy and rule of law* criterion and *political* standard. The results are presented in Table 4.

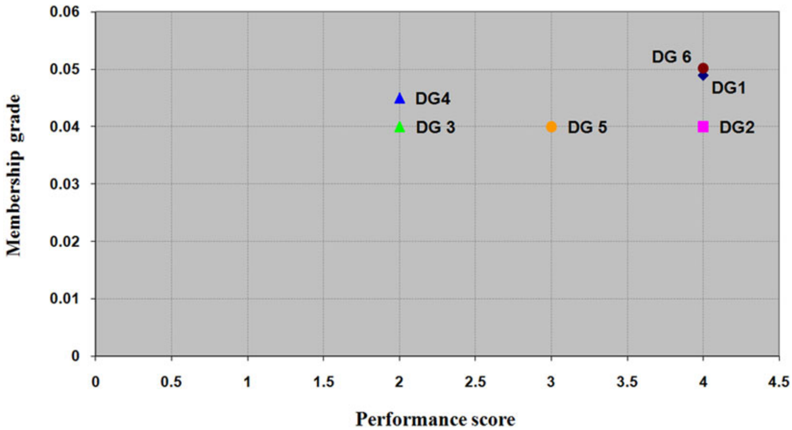
Table 4A shows the importance weights of the guarantee *of democracy and rule of law* criteria group provided by the DMs from the six DGs. Table 4B shows the sub-criteria weights for *Parliament*, *public administration* and *anti-corruption policy*. Table 4C shows the sub-criteria scores provided by the DMs from the six DGs for Albania. The mem-



(a)



(b)



(c)

Fig. 1 The fuzzy set results of judgments for Albania on criterion (**a**) Parliament, (**b**) public administration, (**c**) anti-corruption policy

Table 5 A tabular representation of the opportunities/threats and strengths/weaknesses

Country	Croatia	Turkey	Macedonia	Serbia	Montenegro	Albania	B&H	Kosovo
Strengths	252.54	206.43	156.9	122.55	112.73	116.43	78.18	124.81
Weaknesses	42.57	65.28	97.31	123.3	132.17	137.7	213.69	249.76
Opportunities	288.26	200.43	183.4	132.02	102.46	115.69	120.92	67.05
Threats	48.09	82.73	105.13	132.42	133.11	162.91	152.03	211.62

bership grades for the *Parliament*, *public administration* and *anti-corruption policy* sub-criteria, shown in Table 4D, were calculated using (6). A graphical representation of the results for this example is presented in Fig. 1. The fuzzy sets for the three sub-criteria are presented in Figs. 1A, 1B and 1C. The fuzzy judgments of the DM groups for Albania on the three sub-criteria are presented in Fig. 1A, 1B and 1C. For example, the fuzzy set of the DGs' judgments for Albania on the criterion *Parliament*, *Public Administration* and *Anti-corruption Policy* is depicted on Fig. 1A and can be formulated as $\tilde{A}_{Parlament}^{Albania} = \{2/0.09, 3/0.08, 3/0.087, 4/0.06, 4/0.15, 6/0.06\}$. The set is composed of six pairs that represent the scores for the DGs and their corresponding membership grades calculated using (6). Each pair corresponds to a point on the graph. Intuitively, such a set can be interpreted as the vague score of the country on a criterion treated using fuzzy sets. The aim is to integrate the discrete elements of the scores, defuzzify them and extract additional characteristics assisting in the decision making process (see Dubois and Prade 2000 for further notations on fuzzy sets).

After computations within each hierarchy, the outcomes were integrated into the SWOT groups for each candidate using (10) and (11) to obtain the total values, as well as the strengths/weaknesses and opportunities/threats overbalances. Table 5 presents the opportunities/threats and strengths/weaknesses of the eight candidate states. As it is shown in this table, Croatia has the highest opportunities and strengths and the lowest threats and weaknesses. Contrary to Croatia, Kosovo has low opportunities and strengths and the highest threats and weaknesses. This information is also depicted in the radar diagram presented in Fig. 2 where Croatia is shown in green and Kosovo in red. Similarly, other candidate states could be analyzed through this table and figure.

Table 6 presents the Euclidean distances (D^m) of the eight candidate states from the ideal state using (16). Croatia with a Euclidean distance of 0.00 was closest to the ideal state followed by Turkey and Macedonia. Kosovo was judged farthest away from the ideal state by our DMs.

The entropy was calculated next to evaluate the level of uncertainty in the DMs' estimations. $E_U^* = 10.494$ and $E_V^* = 8.092$ were the ideal entropies of the internal and external characteristics for the ideal state. We then used (17) to calculate the Euclidean distances of the entropies (DE^m) for the eight candidate states from the ideal state. The entropies of the strengths (E_S^m), weaknesses (E_W^m), opportunities (E_O^m) and threats (E_T^m), and the Euclidean distances of the entropies (DE^m) are shown in Table 7. Croatia, Turkey and Macedonia had the largest entropies indicating the disagreement among the DMs for these candidate states while Montenegro, Serbia and Kosovo had the smallest entropies indicating the agreement among the DMs.

Next, we present the final results in Fig. 3. Each bubble in this figure represents a candidate state and the size of the bubble is directly proportional to its entropy level. Croatia has the maximum overbalance of strengths and weaknesses, as well as, opportunities and

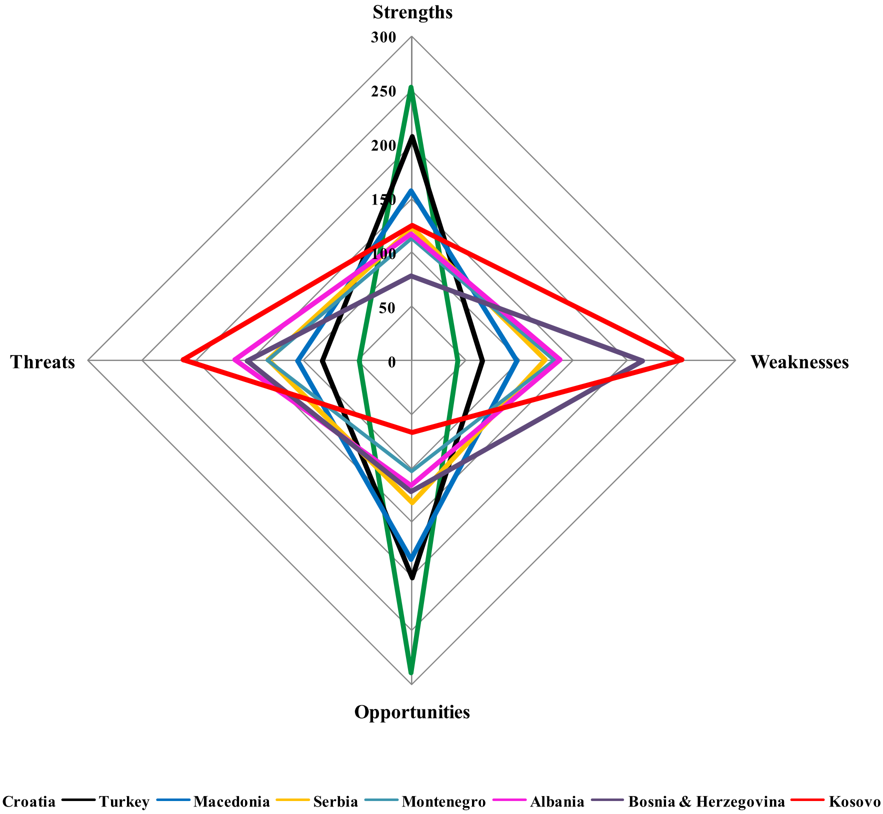


Fig. 2 A graphical representation of the opportunities/threats and strengths/weaknesses

Table 6 The internal and external characteristics and the Euclidean distances

	Albania	B&H	Croatia	Kosovo	Macedonia	Montenegro	Serbia	Turkey	Ideal	
U^m	−2.66	−16.94	26.25	−15.62	7.45	−2.43	−0.09	17.64	U^*	26.2
V^m	−5.90	−3.89	30.02	−18.07	9.78	−3.83	−0.05	14.71	V^*	30.0
D^m	5.76	6.86	0.00	7.97	3.45	5.55	5.00	2.20		

Table 7 The entropies of the candidate states

	Albania	B&H	Croatia	Kosovo	Macedonia	Montenegro	Serbia	Turkey
E_S^m	14.991	10.025	25.706	5.821	16.772	15.623	13.736	22.264
E_W^m	12.452	10.962	4.522	22.736	8.758	11.991	10.740	7.204
E_O^m	10.034	10.081	19.560	6.088	14.006	10.042	10.531	13.385
E_T^m	8.269	8.066	2.513	11.360	5.173	6.142	7.215	4.829
DE^m	13.002	12.911	19.721	12.888	14.931	11.772	12.766	14.230

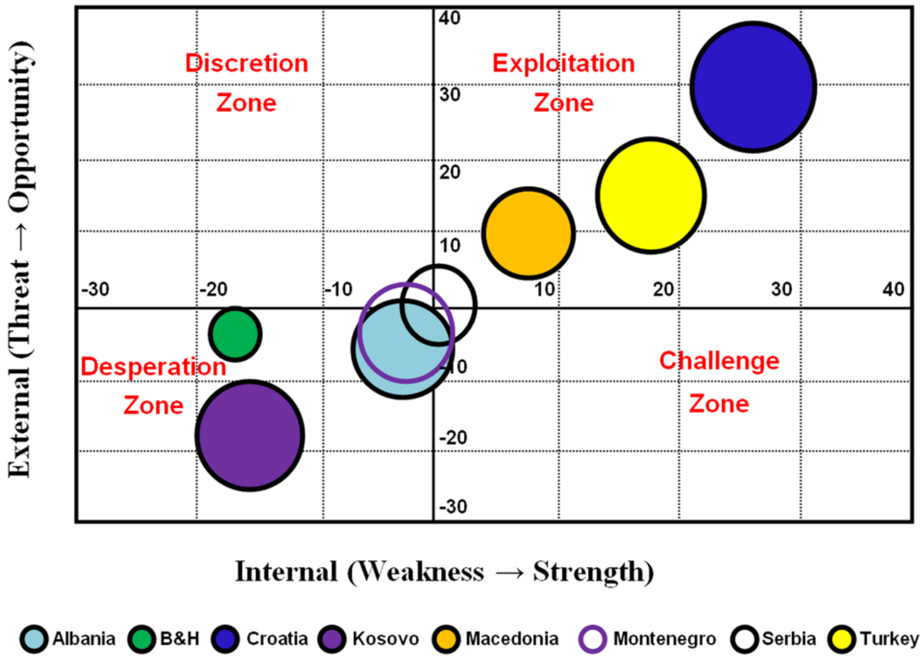


Fig. 3 A graphical representation of the candidate states and their scores

Table 8 The final ranking and classification of the candidate states

Candidate state	Priority	Quadrant			
		Exploitation	Discretion	Challenge	Desperation
Croatia	1	Completely			
Turkey	2	Completely			
Macedonia	3	Completely			
Serbia	4	Partially	Partially	Partially	Mainly
Montenegro	5		Partially	Partially	Mainly
Albania	6		Partially	Partially	Mainly
Bosnia & Herzegovina	7				Completely
Kosovo	8				Completely

threats. However, Croatia has large entropy indicating the DMs' disagreement for this candidate state. Turkey is also characterized by a positive outcome for the internal and external criteria. The overbalance of negative and positive internal and external factors is favourable for Macedonia. This overbalance is not as strong for Croatia and Macedonia.

Finally, we present the overall rankings of the candidate states in Table 8. The top-three countries of Croatia, Turkey and Macedonia all lie in the exploitation quadrant which means they should be considered seriously for the EU membership. It should be noted that these countries also had large entropies indicating the DMs' discords of their opinion. Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia all had prevalence of weaknesses over strengths and prevalence of threats over opportunities. These states lie in the desperation quadrant. Serbia had the best internal and external characteristics while Kosovo has the worst characteristics. These results were consistent with the EU's classification of "candidate" and "potential candidate" states described earlier.

5 Conclusions

The addition of new members into the EU is a strategic problem with profound economic and political effects on both the entering and existing members of the Union. The EU enlargement problem is a complex MCDA problem that embraces qualitative and quantitative internal strengths and weaknesses as well as external opportunities and threats. Candidates seeking membership in the EU must conform to a large number of quantitative and qualitative Copenhagen criteria established by the Copenhagen European Council. The current selection process is ambiguous and lacks structure. *Soft SWOT* w*as* developed in response to the need for a meaningful and robust aggregation of subjective and objective judgments concerning a large number of competing and conflicting criteria. *Soft SWOT* uses the AHP, subjective probabilities, defuzzification, entropy, and the theory of displaced ideal to reduce these complexities by decomposing the evaluation process into manageable steps. This decomposition is achieved without overly simplifying the process.

The results from this pilot study shows that Croatia, Turkey and Macedonia should be considered seriously for the EU membership. These countries had a positive overbalance of strengths over weaknesses and opportunities over threats. The results also reveals that Albania, Bosnia and Herzegovina, Kosovo, Montenegro and Serbia all had prevalence of weaknesses over strengths and prevalence of threats over opportunities. These countries are risky and should be avoided for the time being since they are characterized by high weaknesses and threats. The results of this study are consistent with the EU's current classification of "candidate" and "potential candidate" states.

Soft SWOT promotes consistent and systematic alternative evaluation and selection throughout the organization. Judgments captured as separate importance weights and performance scores are used uniformly across all alternatives in the evaluation process. In the absence of separate value judgments, it is difficult to apply a set of importance weights and performance scores consistently among the strengths, weaknesses, opportunities and threats when evaluating alternatives. *Soft SWOT* provides a consistent combination of all assessments among all the alternatives. Whether the assessments faithfully represent real-world circumstances depends on the competence and degree of effort the DMs exert in making the assessments.

Soft SWOT is also useful in examining how sensitive the overall Euclidean scores are to changes in the portfolio of selected alternatives. *Soft SWOT* also addresses questions about the sensitivity of the portfolio of selected alternatives to changes in the relative importance of the organizations, the relative importance of the strengths, weaknesses, opportunities and threats, and the performance scores.

Soft SWOT is not intended to replace human judgment in alternative evaluation and selection. In fact, human judgment is the core input in the process. *Soft SWOT* helps the DMs to think systematically about complex MCDA problems and improves the quality of the decisions. It is almost impossible to obtain objective data on the complex strategic problems because of inherent uncertainties. However, experienced DMs can often make fairly accurate estimates of values. *Soft SWOT* decomposes the alternative evaluation process into manageable steps and integrates the results to arrive at a solution consistent with managerial goals and objectives. This decomposition encourages DMs to carefully consider the elements of uncertainty.

Using a structured framework like *Soft SWOT* does not imply a deterministic approach in MCDA. While *Soft SWOT* enables DMs to crystallize their thoughts and organize their beliefs, it should be used very carefully. Managerial judgment is an integral component of *Soft SWOT*; therefore, the effectiveness of the model relies heavily on the DM's cognitive abilities to provide sound judgments. As with any decision analysis model, the researchers and practicing managers must be aware of the limitations of subjective estimates and use them carefully.

Finally, although the pilot study was not carried out with real DMs from the European Commission, it was an excellent platform for testing the practicality of the proposed framework with student subjects. The use of student subjects has been opposed by some researchers who argue that students may not be representative of real DMs. A central question regarding the legitimacy of using student subjects is whether the research findings obtained from student subjects can be generalized to actual organizational settings? In other words, do the findings obtained from the student subjects have external validity? Critics of student subject research argue that students are different from practicing managers and the dissimilarity between them precludes the validity of student subject research. This reasoning implies two broad assumptions. First, it assumes that external validity is the most important determinant of the value of the research project. Second, it assumes that practicing managers and students are different and that any differences between them will influence the results of the research. Below we scrutinize these two assumptions:

- The first assumption can be challenged by the fact this study is more concerned with the application process rather than the application outcome. The application process required the DMs to classify 169 Copenhagen sub-criteria into strengths, weaknesses, opportunities and threats, weigh these sub-criteria and determine their performance scores. This process was effectively simulated by a group of graduate students from the University of Paderborn in Germany with a good grasp of economics, political, social and business issues. Another aspect of the process required a series of numerical calculations involving a large number of equations aggregating a large number of weights with defuzzified subjective and objective sores. We argue this study is not concerned with the particular outcome of the case but rather with the process and its effective implementation with a group of mature and knowledgeable students who played the role of the DMs.
- The second assumption assumes there could be differences between the practitioners and the students and this difference could significantly affect the results of the research. We acknowledge the differences between the practitioners and the student subjects could change the particular judgments and potentially the final outcome. However, we argue these dissimilarities are not relevant in this study. These differences would most likely derive from the fact that practitioners may have more experience and better access to additional information. While these differences might change the particular results, they do not decrease the overall validity of the process.

We also argue there might be sufficient similarity between the students and the practicing managers which allows the students to be a representative sample from the intended population. In conclusion, we contend the use of students is a valid way to carry out a test of the model and is comparable with the role playing exercises that are generally carried out in graduate management classes to simulate decision making processes. These points are also clarified by Walters-York and Curatola (2000):

> Research relying on student subjects is likely no less valid than research relying on non-student subject groups; student samples provide no greater threat to external validity than typical real-world samples. The customary real-world sample can be placed

under the same scrutiny for lack of formal representativeness and atypicality as the customary student sample. Moreover, even when the sample is formally representative, real-world subjects in experimental settings are likely to be poor surrogates for real-world subjects in the real world due to a lack of experimental realism. As such, there is no universally valid basis for automatically privileging real-world samples and dismissing student samples as inherently inferior.”

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Appendix: The European Commission enlargement strategy and progress reports

- The main page:
  http://ec.europa.eu/enlargement/press_corner/key-documents/reports_nov_2008_en.htm

The links to the progress reports for the candidate states are available at:

- Croatia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/croatia_progress_report_en.pdf
- The former Yugoslav Republic of Macedonia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/the_former_yugoslav_republic_of_macedonia_progress_report_en.pdf
- Turkey:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/turkey_progress_report_en.pdf

The links to the progress reports of the potential candidate states are available at:

- Albania:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/albania_progress_report_en.pdf
- Bosnia and Herzegovina:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/bosnia_herzegovina_progress_report_en.pdf
- Montenegro:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/montenegro_progress_report_en.pdf
- Serbia:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/serbia_progress_report_en.pdf
- Kosovo under UNSCR 1244/99:
  http://ec.europa.eu/enlargement/pdf/press_corner/key-documents/reports_nov_2008/kosovo_progress_report_en.pdf

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