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ORIGINAL PAPER

Loyal customer bases as innovation disincentives for duopolistic firms using strategic signaling and Bayesian analysis

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Abstract In this paper we model the strategic behavior of firms competing in duopolistic environments with a loyal customer base and formalize their decision to delay the introduction of the most technologically developed product available. The proposed model extends and complements the partial approaches studied in the economic, management and operations research literatures. The former emphasizes the role of the strategic knowledge spillovers that may take place among competing firms because of their incentives to introduce technologically superior products while assuming the acceptance of such products by customers as given. The second defines its technology acceptance model based on the demand side of the economic system without considering the resulting strategic interactions that arise among the firms. The latter addresses the effect that signals about a new technology have on the information acquisition behavior of decision makers (DMs) but does not consider the capacity of DMs to account for several product characteristics and their interaction when acquiring

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information. Using a duopolistic innovation game model we illustrate how the existence of loyal customer bases allows for higher expected payoffs when generating monopolized markets but decreases the incentives of firms to introduce the most technologically developed product available. The signaling equilibria of the game are determined by demand-based factors and the incentives of customers to acquire information on the existing products in the market. Among the main implications of our model is also the fact that the availability of decision support systems that can be used by DMs through their information acquisition processes would improve the quality of the technology being introduced in the market and increase the firms' probability of success.

Keywords Multi-attribute sequential search · Customer loyalty · Strategic signaling · Technological evolution · Product introduction

1 Introduction

1.1 Literature review

Economists from different schools of thought have studied the incentives and disincentives of firms to introduce technologically advanced products in the market (Baumol 2010; Nelson and Winter 1985). The incentives to introduce a technologically superior product do not rely on the acceptance of willing customers but on the technological constraints and strategic knowledge spillovers that may take place among competing firms (Hanusch and Pyka 2007). Formal strategic models also concentrate on firms' costs and expected profits to justify the strategic delay in the introduction of technologically superior products within duopolistic scenarios (Li and Jin 2009; Su and Rao 2011). When consumers are introduced, they are either perfectly informed (Laksana and Hartman 2010) or consider a unique characteristic of the product to determine their purchasing decisions (Zhou et al. 2015). At the same time, when considering production processes from a managerial perspective, the heterogeneous quality of consumers is explicitly recognized (Herbon 2014) together with the capacity of firms to generate a loyal customer base (Abdolvand et al. 2015). In particular, postponement strategies on product development and production are suggested as instruments that can be used by the firms to manage the effects derived from the varying levels of uncertainty that they may be subject to (Yang et al. 2004).

On the other hand, management scholars have focused on the demand side of the economic system when defining their technology acceptance models, without considering the resulting strategic interactions that may arise among firms (Bagozzi 2007; Davis 1989; King and He 2006; Lee et al. 2003; Premkumar and Bhattacherjee 2008). The operations and product management literatures have recently focused on the strategic waiting behavior of loyal customers and its impact on the firm's pricing and inventory decisions (Zhang and Cooper 2008; Netessine and Tang 2009). Both streams of literature assume that customers know their exact valuations before purchasing the product and as a result, their information acquisition processes are not studied.

The operations research stream of the literature has addressed the introduction of technological improvements and the effect that signals about the new technology have on the information acquisition behavior of DMs (Smith and Ulu 2012). For example, Bohlmann et al. (2002) use game theory to model the advantages and disadvantages faced by early innovators while considering the customer valuation of product attributes. Smith and Ulu (2012) model the consumer decisions to adopt, reject or wait and the impact of those decisions on the firms' quality improvement and innovation strategies. However, these models do not consider the capacity of DMs to account for several product characteristics together with their interaction when acquiring information before making a choice.

The operations research literature is starting to account explicitly for the multiple characteristics defining the products, while emphasizing the substantial failure rates of the new products being introduced in the market (Yenipazarli 2015). Moreover, the shortening in the life-cycle of technological products due to rapid innovation and market competition has led to a reconsideration of the importance that demand has for the evolution of markets (Aytac and Wu 2013). Thus, a more sophisticated approach to the demand side of the market is required, together with a recognition of the capacity of DMs to assimilate (at least to a certain extent) the substantial amount of information available. In this regard, significant advances have been made in the design of decision support systems (DSS) that help DMs to assimilate information and consider several product characteristics before making a decision (Roberts 2008).

Marketing scholars have emphasized the relationship existing between the information acquisition process of customers, who consider products composed by several characteristics, and their satisfaction with the purchase made and posterior loyalty towards a brand. Consider, for example, the different stages of the consumer buying process defined in a standard marketing textbook (Ferrell and Hartline 2012, p. 118): need recognition—information search—evaluation of alternatives—purchase decision—postpurchase evaluation. Therefore, formal models analyzing consumer behavior should account for the search and evaluation process taking place before a purchase decision is made.

We formalize the strategic decisions of firms such as Apple, competing in duopolistic type environments and endowed with a loyal customer base. Within this type of setting, such firms may introduce small sequential improvements that differ from the most technologically developed product available, leading to the disappointment of their loyal customer base (Arruda-Filho et al. 2010, 2011). In order to formalize this product introduction strategy, we must explicitly incorporate the information acquisition process of consumers regarding multiattribute products into a technological duopolistic strategic scenario where different improvement levels can be introduced by the competing firms.

For this purpose, we define a duopolistic model of technological improvement where the characteristics of the products introduced by firms must be sequentially observed by the customers or decision makers (DMs) before purchasing. Each firm must decide the extent to which it improves its products given the extent to which its competitor is expected to improve its own products. The incentives of a firm to introduce different intensities of technological improvement in its products will therefore be based on the information acquisition incentives of the DMs as well as its expectations regarding the behavior of its competitor.

1.2 Contribution

Given our focus on the information acquisition process of the DMs, we will interpret customer loyalty as the willingness to obtain additional information about the products offered by a given firm whenever the initial observation is not considered to be satisfactory. As a result, loyal customers will exhibit a higher probability of purchasing a product from the firm to which they are loyal instead of switching to another (Hartung and Fisher 1965; Qiasi et al. 2012). This definition accounts also for those customers who are willing to learn more about the new products introduced by a given firm, endowed with an information acquisition advantage relative to its duopolistic rival (Arruda-Filho et al. 2010). As many economists have emphasized, this type of customers can provide a fertile environment for the introduction of product innovations (Malerba et al. 2007).

It should be noted that we will not consider search costs, since we want the information acquisition incentives of the DMs to be determined by their capacity to account for the interactions between product characteristics. Thus, loyalty effects will not be based on the existence of large search or switching costs (Jones et al. 2000; Feick et al. 2001; Burnham and Mahajan 2003).

Moreover, when considering customer satisfaction, the marketing literature highlights the fact that dissatisfied customers are more probable to consider and search information on the products of competitors than satisfied ones (Anderson and Srinivasan 2003; Santouridis and Trivellas 2010). At the same time, if dissatisfied with the initial information retrieved, loyal customers will take more time to evaluate the information available before considering the products offered by a competitor (Mittal and Lee 1989; Von Riesen and Herndon 2011).

Therefore, customer satisfaction can generally be considered as a reliable predictor of repurchase intentions (Wang et al. 2001). However, given the disappointment expressed by loyal customers regarding the upgrades introduced in several versions of the iPhone (Arruda-Filho and Lennon 2011), it could also be concluded that there are customers who are loyal to a company as long as it offers them some superior value (Salem Khalifa 2004). That is, even when dissatisfied, loyal customers are able to justify their purchase of a product while ignoring any problems (Belk et al. 2003).

The results derived from our model are based on the capacity of DMs to consider the potential interactions between the main characteristics of a product when acquiring information before making a choice.

- If customers do not consider the potential interactions between the product characteristics and use the corresponding certainty equivalents as their reference values when acquiring information, firms will be constrained in their incentives to introduce the most technologically developed product available.
- If customers consider these interactions, firms will have an incentive to introduce the most technologically developed product available. However, the emergence of a loyal customer base within this latter setting would decrease the incentives of firms to introduce the most technologically developed product. This result would explain the disappointment expressed by loyal customers regarding the upgrades introduced in several versions of the iPhone (Arruda-Filho and Lennon 2011).
- Moreover, if DMs do not consider the interactions and guide themselves by the certainty equivalent values, the rejection probability of the new products will be higher than the one obtained when DMs consider the interactions between the product characteristics.

Our model states that, when endowed with a loyal customer base, companies such as Apple operating in a very fast moving industry have incentives to introduce small sequential improvements instead of a large unique one. However, for this to be the case, customers must consider the interactions between the different characteristics of the products when acquiring information. If they do not, then the incentives of firms to introduce the most technologically developed product decrease further while the failure probability from the introduction of new technologically developed products increases. This latter setting is actually the one considered by the standard economics and operations research literature, where customers do not account for the interactions between the different characteristics of the products when acquiring information.

A direct consequence following from the model implies that in order for technology to be improved to the highest level available, we require the market to be composed by DMs who consider the potential interactions between the main characteristics of products. Therefore, the availability of DSS that can be used by DMs through the information acquisition process would actually improve the quality of the technology being introduced in the market while increasing the success probability of firms when doing so.

The paper proceeds as follows. Sections 2 and 3 present the basic assumptions and define the expected search utilities that determine the information acquisition behavior of the DMs. Sections 4 and 5 describe the effect that signals issued by the firms have on the behavior of the DMs both formally and numerically. The expected payoffs received by the firms based on their signaling strategies are described in Sect. 6, while Sect. 7 illustrates numerically the signaling incentives and equilibrium strategies of the firms. Section 8 concludes.

2 Assumptions

We will make the following assumptions throughout the paper:

(a) Products are represented by two characteristics, X_1 and X_2. The DMs have a well-defined preference order both *within* and *between* the characteristics. "Within characteristics" means that the DMs define two preference orders, one for each set of values that the characteristics X_1 and X_2 may take. "Between characteristic" means that the DMs define a preference order on the set formed by the two characteristics, i.e. $\{X_1,\ X_2\}$. In particular, the first characteristic is assumed to be more important and, consequently, provides a higher expected utility to the DMs than the second one.

(b) These characteristics can be interpreted as categories defined by several attributes of the product taxonomy. Each characteristic is evaluated numerically by the DMs after acquiring information on the corresponding attributes of the product.

(c) The distributions of the product characteristics are unknown to the DMs. However, the DMs know the best and worst potential realizations of each characteristic category, x_k^M and x_k^m, respectively, with $k = 1,\ 2$ and $x_k^m, x_k^M > 0$. To simplify the presentation, we assume X_k $(k = 1,\ 2)$ to be the compact real interval $[x_k^m,\ x_k^M]$.

(d) Firms may introduce and signal improvements of different magnitudes, depending on the technological development level of the new products. The first characteristics are harder to improve, since its attributes have experienced a higher degree of competition. Thus, the domain defining X_1 will be smaller than the one defining X_2.

(e) The DMs must acquire information on the characteristics X_k $(k = 1,\ 2)$ following a specific heuristic rule. The implementation of a heuristic mechanism takes into account the limited capacity of the DMs to acquire and process information (Simon 1997). The heuristic rule that the DMs use in the current paper is determined by two expected search utility functions.

(f) The DMs are willing to acquire at least two pieces of information from the set of products offered by a firm. Three types of DMs will be considered depending on their incentives to acquire information from a given firm when it introduces a technologically improved product in a monopolized market. Unless highly loyal to the firm, the DMs will observe each product completely, i.e. verify both characteristics, before purchasing it (Christensen 1997).

We will also make the following more technical assumption (see Tavana et al. 2014).

(g) The DMs define a continuous additive utility function on $X = X_1 \times X_2$, that is, $u : X_1 \times X_2 \to \Re$, such that $\forall < x_1, x_2 >\in X_1 \times X_2, u(< x_1, x_2 >) = u_1(x_1) + u_2(x_2)$. The pair $< x_1, x_2 >$ in X is used to describe a product.

(h) For every $k = 1, 2$, the DMs define a continuous probability density on X_k, $\mu_k : X_k \to [0, 1]$, whose support is denoted by $Supp(\mu_k)$. That is, for $k = 1, 2$, $\mu_k(x_k)$ is the subjective probability that a randomly observed product has $x_k \in X_k$ as its k-th characteristic. The probability densities μ_1 and μ_2 are assumed to be independent, i.e. a high realization of the first characteristic does not necessarily guarantee a better realization of the second. Without loss of generality, we will assume that $Supp(\mu_k) = X_k = [x_k^m, x_k^M]$.

(i) The DMs face complete initial uncertainty, i.e. maximum information entropy (Tavana 2004), regarding the distribution of characteristics on X_k (with $k = 1, 2$). As a result, the DMs will be assumed to assign uniform probabilities to the distribution of characteristics on X_1 and X_2. That is, for every $x_k \in X_k = [x_k^m, x_k^M]$, $\mu_k(x_k) = \frac{1}{x_k^M - x_k^m}$.

(j) The DMs elicit the k-th certainty equivalent (ce_k) value induced by μ_k and u_k as the reference point against which to compare the information collected on the k-th characteristic of a given product. That is, for each $k = 1, 2$, $ce_k = u_k^{-1}(E_k)$, where $E_k \overset{def}{=} \int_{X_k} \mu_k(x_k)\, u_k(x_k)\, dx_k$ is the expected value of u_k.

3 Information acquisition process: expected search utilities

At each point within the search process, after observing the value of the first characteristic from a product, the DMs have to decide whether to check the second characteristic from the same product or to start checking the first characteristic from a new product. The decision of how to allocate the next piece of information depends on two real-valued functions defined on X_1, which will be referred to as *expected search utilities*. The DMs will use the sum $E_1 + E_2$, corresponding to the expected utility values of the pairs $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$, as the main reference value when defining these functions.

Assume that the DM has already checked the first characteristic from an initial product, x_1, and that he decides to observe the second characteristic from this product, x_2. The expected utility gain over $E_1 + E_2$ varies with the observed value x_1. Thus, for every $x_1 \in X_1$, let:

$$P^+(x_1) = \{x_2 \in X_2 : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \tag{1}$$

and

$$P^-(x_1) = \{x_2 \in X_2 : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}. \tag{2}$$

$P^+(x_1)$ and $P^-(x_1)$ define the set of values x_2 from the initial product such that their combination with x_1 delivers a respectively higher or lower-equal utility than a randomly chosen product.

Let $F : X_1 \to \Re$ be defined by:

$$F(x_1) \overset{def}{=} \int_{P^+(x_1)} \mu_2(x_2)\,(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)\,(E_1 + E_2)\,dx_2. \tag{3}$$

$F(x_1)$ describes the DM's expected utility derived from checking the second characteristic of the initial product after observing x_1 as the value of its first characteristic. Note that, if $u_1(x_1) + u_2(x_2) \leq E_1 + E_2$, then choosing a product randomly delivers an expected utility of $E_1 + E_2$ to the DM, which is higher than the expected utility obtained from choosing the product initially observed, that is, $u_1(x_1) + u_2(x_2)$.

Consider now the expected utility that the DM could gain over $E_1 + E_2$ if the second piece of information is employed to observe the first characteristic from a different new product, $y_1 \in X_1$. For every $x_1 \in X_1$, let:

$$Q^+(x_1) = \{y_1 \in X_1 \ : \ u_1(y_1) > \max\{u_1(x_1), E_1\}\} \tag{4}$$

and

$$Q^-(x_1) = \{y_1 \in X_1 \ : \ u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}. \tag{5}$$

$Q^+(x_1)$ and $Q^-(x_1)$ define the set of values y_1 from the new product such that they deliver a respectively higher or lower-equal utility than the maximum between the observed value x_1 from the initial product and a randomly chosen product.

Define $H : X_1 \to \Re$ as follows:

$$H(x_1) \overset{def}{=} \int\limits_{Q^+(x_1)} \mu_1(y_1)\,(u_1(y_1)+E_2)\,dy_1 + \int\limits_{Q^-(x_1)} \mu_1(y_1)\,(\max\{u_1(x_1), E_1\} + E_2)\,dy_1. \tag{6}$$

$H(x_1)$ represents the expected utility obtained from checking the first characteristic of a new product after having already observed the value of the first characteristic from an initial product. If $u_1(y_1) \leq \max\{u_1(x_1), E_1\}$, then the DM must choose the highest between the initial (partially observed) product and a randomly chosen one when computing the expected utility derived from using this information acquisition strategy.

Note that, after observing $x_1 \in X_1$, the DM will either continue acquiring information on the product that has been partially observed or start acquiring information on a different product. His decision will depend on which function, either $F(x_1)$ or $H(x_1)$, delivers the higher value at x_1. It therefore follows that whenever $F(x_1^*) = H(x_1^*)$ at a given $x_1^* \in X_1$, the DM would be indifferent between continuing with the partially observed product and starting with a different one. These x_1^* values behave as information acquisition thresholds that partition X_1 in subintervals inducing the DM to either continue acquiring information on the partially observed product or to switch and start observing a different one.

We will assume throughout the paper that the DMs apply this information acquisition mechanism when gathering any number of observations from the firms. This heuristic mechanism is based on the satisficing level of X_1 computed by the DMs, which determines their information acquisition behavior given their expectations on the set of potential realizations of both characteristics.

4 Signaling the introduction of technological improvements

Firms can signal the technological improvements introduced in a given characteristic. These signals modify the distribution of the product characteristics considered by the DMs.

Henceforth, θ will denote a number of signals being issued, that is, $\theta = ns$ means 0 signals issued, $\theta = 1s$ (or simply $\theta = 1$) means 1 signal issued, $\theta = 2s$ (or simply $\theta = 2$) means 2 signals issued, and so on. To indicate that a generic number of signals is issued we will write $\theta = s$. We will assume that receiving a *credible* positive signal shifts a fraction $\gamma \in [0, 1]$ of the probability mass accumulated on the lower half of the distribution to the upper half. Thus, since by Assumption i, we have:

$$\mu_k(x_k) = \frac{1}{x_k^M - x_k^m} \text{ for every } x_k \in X_k = [x_k^m, x_k^M] \text{ and } k = 1,\ 2 \tag{7}$$

the corresponding conditional density function (conditional on the value of x_k) is given by:

$$\pi_1(\theta, \gamma | x_k) = \frac{1}{x_k^M - x_k^m} + \gamma \frac{1}{x_k^M - x_k^m}, \text{ for every } \gamma \in [0,\ 1], \tag{8}$$

when $x_k \in \left(\frac{x_k^m + x_k^M}{2},\ x_k^M \right]$, and by:

$$\pi_2(\theta, \gamma | x_k) = \frac{1}{x_k^M - x_k^m} - \gamma \frac{1}{x_k^M - x_k^m}, \text{ for every } \gamma \in [0,\ 1] \tag{9}$$

when $x_k \in \left[x_k^m,\ \frac{x_k^m + x_k^M}{2} \right]$.

We will assume that $\gamma = 1/2$ throughout the paper and allow for subsequent signals to continue shifting half of the remaining mass accumulated on the lower half of the distribution to the upper one. In this respect, we can interpret the variable γ as a proxy for the quality of the innovation introduced by the firm, with improvements in quality being reinforced as additional signals are issued. It should be emphasized that our main results do not depend on the value of γ chosen. That is, as will be illustrated in the numerical simulations, the strategic patterns arising from the resulting game theoretical environment wouldnot depend upon the initial values of γ, although the expected payoffs obtained by the firms would.

After receiving a positive signal, the DMs use Bayes' rule to update their initial beliefs, given by $\mu_k(x_k)$. Therefore, if a signal is received, i.e. $\theta = 1$, on the distribution of X_k (with $k = 1,\ 2$) the updated beliefs of DMs will be given by:

$$\mu_k(x_k|\theta = 1) = \begin{cases} \frac{\pi_1(\theta,\gamma|x_k)\mu_k(x_k)}{\int_{X_k} \pi_1(\theta,\gamma|x_k)\mu_k(x_k)dx_k} & \text{if } x_k \in \left(\frac{x_k^m + x_k^M}{2},\ x_k^M \right] \\ \frac{\pi_2(\theta,\gamma|x_k)\mu_k(x_k)}{\int_{X_k} \pi_2(\theta,\gamma|x_k)\mu_k(x_k)dx_k} & \text{if } x_k \in \left[x_k^m,\ \frac{x_k^m + x_k^M}{2} \right] \end{cases} \tag{10}$$

This updating process can be assumed to continue as the DMs update their beliefs using Bayes' rule after receiving further signals. For instance, after receiving a second signal on the same characteristic on which the first signal was issued, the DM updates the beliefs $\mu_k(x_k|\theta = 1)$ induced by the first signal as follows:

$$\mu_k(x_k|\theta = 2) = \begin{cases} \frac{\pi_1(\theta,\gamma|x_k)\mu_k(x_k|\theta=1)}{\int_{X_k} \pi_1(\theta,\gamma|x_k)\mu_k(x_k|\theta=1)dx_k} & \text{if } x_k \in \left(\frac{x_k^m + x_k^M}{2},\ x_k^M \right] \\ \frac{\pi_2(\theta,\gamma|x_k)\mu_k(x_k|\theta=1)}{\int_{X_k} \pi_2(\theta,\gamma|x_k)\mu_k(x_k|\theta=1)dx_k} & \text{if } x_k \in \left[x_k^m,\ \frac{x_k^m + x_k^M}{2} \right] \end{cases} \tag{11}$$

We will concentrate our analysis on the number of potential signals that can be issued on the most important characteristic for the DMs, that is, X_1. This is the characteristic on which the firms compete with more intensity and is also the first one to be observed by the DMs when acquiring information on a given product. Thus, the most important characteristic for the DMs will be prioritized by the firms when introducing technological improvements.

4.1 The effect of positive X_1 signals on the expected search utilities

We analyze here how observing a positive signal on the first characteristic affects the expected search utilities of the DMs. For what concerns the function F, the analysis is similar to the one presented in Di Caprio et al. (2014), who studied the effects of signals on the second characteristic. However, in the case of function H, the description differs significantly from the one described in their paper. The main definitions required to perform the analysis are restated in order to keep the current paper self-contained.

The definition of the updated functions F and H after one signal is issued are:

$$F(x_1|\theta=1) \overset{def}{=} \int\limits_{P^+(x_1|\theta=1)} \mu_2(x_2)\,(u_1(x_1)+u_2(x_2))\,dx_2 + \int\limits_{P^-(x_1|\theta=1)} \mu_2(x_2)\left(E_{(1|\theta=1)}+E_2\right)dx_2 \tag{12}$$

$$H(x_1|\theta=1) \overset{def}{=} \int\limits_{Q^+(x_1|\theta=1)} \mu_1(y_1|\theta=1)\,(u_1(y_1)+E_2)\,dy_1 + \int\limits_{Q^-(x_1|\theta=1)} \mu_1(y_1|\theta=1)\left(\max\left\{u_1(x_1), E_{(1|\theta=1)}\right\}+E_2\right)dy_1 \tag{13}$$

where

$$E_{(1|\theta=1)} = \int\limits_{X_1} \mu_1(x_1|\theta=1)u_1(x_1)dx_1 \tag{14}$$

$$P^+(x_1|\theta=1) = \left\{x_2 \in X_2 \;:\; u_2(x_2) > E_{(1|\theta=1)} + E_2 - u_1(x_1)\right\} \tag{15}$$

$$P^-(x_1|\theta=1) = \left\{x_2 \in X_2 \;:\; u_2(x_2) \le E_{(1|\theta=1)} + E_2 - u_1(x_1)\right\} \tag{16}$$

$$Q^+(x_1|\theta=1) = \left\{y_1 \in X_1 \;:\; u_1(y_1) > \max\left\{u_1(x_1), E_{(1|\theta=1)}\right\}\right\} \tag{17}$$

$$Q^-(x_1|\theta=1) = \left\{y_1 \in X_1 \;:\; u_1(y_1) \le \max\left\{u_1(x_1), E_{(1|\theta=1)}\right\}\right\} \tag{18}$$

The updated definition of the functions F and H after more signal are issued is similar.

Consider first the function $F(x_1)$. Whenever a signal is observed, the updated Bayesian density $\mu_1(x_1|\theta=1)$ should lead to $E_{(1|\theta=1)} \ge E_1$ through the interval X_1. This increment in E_1 should, at the same time, result in the set $P^+(x_1|\theta=1)$ shrinking relative to $P^+(x_1)$. Thus, receiving a positive signal leads to a more restricted interval $P^+(x_1)$ containing a lower probability mass than its presignal counterpart but based on a higher value for E_1. The intuitive effect of the signal on the function $F(x_1)$ remains ambiguous.

Regarding the function $H(x_1)$, the type of positive signal received implies that $\mu_1(x_1|\theta=1) \ge \mu_1(x_1)$ over the newly defined interval $Q^+(x_1|\theta=1)$. As in the $P^+(x_1|\theta=1)$ case, $Q^+(x_1|\theta=1)$ shrinks relative to its presignal counterpart due to $E_{(1|\theta=1)} \ge E_1$. Therefore, the intuitive effect of the signal on the function $H(x_1)$ remains also ambiguous.

In order to shed some light on the previous intuitive description, we provide a formal evaluation of the effects that signals have on both functions $F(x_1)$ and $H(x_1)$. We will make use of the following definition and proposition throughout the subsequent analysis.

Definition 4.1 (*Definition 6.D.1 in Mas-Colell et al.* 1995) The distribution $f(\cdot)$ first-order stochastically dominates $g(\cdot)$ if, for every non-decreasing function $u{:}\Re \to \Re$, we have

$$\int u(x)df(x) \ge \int u(x)dg(x). \tag{19}$$

Proposition 4.2 (*Proposition 6.D.1 in Mas-Colell et al.*1995) *The distribution of monetary payoffs $f(\cdot)$ first-order stochastically dominates the distribution $g(\cdot)$ if and only if $f(x) \le g(x)$ for every x.*

We are able to show now the following two results.

Proposition 4.3 *For every* $x_1 \in X_1$, $F(x_1|\theta = 1) \geq F(x_1)$.

Proposition 4.4 *For every* $x_1 \in X_1$, $H(x_1|\theta = 1) \geq H(x_1)$.

To show Propositions 4.3 and 4.4, we will use the following Lemmas. The proof of Lemma 4.5 is relatively straightforward and has therefore been omitted.

Lemma 4.5 $\mu_1(x_1|\theta = 1)$ *first-order stochastically dominates* $\mu_1(x_1)$.

Lemma 4.5 together with Definition 4.1 imply that

Corollary 4.6 $E_{(1|\theta=1)} \geq E_1$.

The signal received affects the function $F(x_1)$ both through the new induced value of E_1 and its effect on the sets $P^+(\cdot)$ and $P^-(\cdot)$ defining the integral limits. It follows that

Lemma 4.7 (a) $\frac{dF(x_1)}{dE_1} > 0, \forall x_1 \in X_1$, *if and only if* $P^-(x_1^M) \neq \varnothing$.
(b) $\frac{dF(x_1)}{dE_1} = 0, \forall x_1 \in X_1$, *if and only if* $P^-(x_1^M) = \varnothing$.

Proof We start by expressing $F(x_1)$ as a function of E_1.

$$F(x_1) \stackrel{def}{=} \int_{u_2^{-1}(E_1+E_2-u_1(x_1))}^{x_2^M} \mu_2(x_2)\,(u_1(x_1)+u_2(x_2))\,dx_2 + \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)(E_1+E_2)dx_2 \tag{20}$$

Applying Leibnitz's rule to the above definition we obtain

$$\begin{aligned}\frac{dF(x_1)}{dE_1} &= \int_{u_2^{-1}(E_1+E_2-u_1(x_1))}^{x_2^M} \frac{\partial}{\partial E_1}\left[\mu_2(x_2)\,(u_1(x_1)+u_2(x_2))\right]dx_2 \\ &+ \left[\mu_2\left(x_2^M\right)\left(u_1(x_1)+u_2\left(x_2^M\right)\right)\right]\frac{dx_2^M}{dE_1} \\ &- \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(u_1(x_1)+u_2(u_2^{-1}(E_1+E_2-u_1(x_1))))\right] \\ &\quad \frac{d}{dE_1}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] \\ &+ \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)dx_2 + \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(E_1+E_2)\right] \\ &\quad \frac{d}{dE_1}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] \\ &- \left[\mu_2(x_2^m)(E_1+E_2)\right]\frac{dx_2^m}{dE_1} = \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)dx_2.\end{aligned} \tag{21}$$

Therefore, increments in E_1 have a strictly positive effect on $F(x_1)$, $\forall x_1 \in X_1$, if and only if $P^-(x_1^M) \neq \varnothing$, and no effect on $F(x_1)$, $\forall x_1 \in X_1$, if and only if $P^-(x_1^M) = \varnothing$. □

Proof of Proposition 4.3 First note that, by Corollary 4.6, if $E_{(1|\theta=1)} \ngtr E_1$, then $E_{(1|\theta=1)} = E_1$. Hence, consider the following two complementary cases:

(i) $P^-(x_1^M) \neq \varnothing$ and $E_{(1|\theta=1)} > E_1$;
(ii) $P^-(x_1^M) = \varnothing$ or $E_{(1|\theta=1)} = E_1$.

Suppose that (i) holds. By Lemma 4.7, we have $\frac{dF(x_1)}{dE_1} > 0, \forall x_1 \in X_1$, which, together with $E_{(1|\theta=1)} > E_1$, implies $F(x_1|\theta = 1) > F(x_1)$.

Suppose now that (ii) holds. We have two subcases.

(ii.a) If $P^-(x_1^M) = \varnothing$, then, again by Lemma 4.7, we have $\frac{dF(x_1)}{dE_1} = 0, \forall x_1 \in X_1$, which implies $F(x_1|\theta = 1) = F(x_1)$.
(ii.b) If $E_{(1|\theta=1)} = E_1$, then $P^+(x_1|\theta = 1) = P^+(x_1)$ and $P^-(x_1|\theta = 1) = P^-(x_1)$. Therefore:

$$\begin{aligned} F(x_1|\theta = 1) &\overset{def}{=} \int\limits_{P^+(x_1|\theta=1)} \mu_2(x_2)\,(u_1(x_1) + u_2(x_2))\,dx_2 \\ &\quad + \int\limits_{P^-(x_1|\theta=1)} \mu_2(x_2)\,\big(E_{(1|\theta=1)} + E_2\big)\,dx_2 \\ &= F(x_1) \overset{def}{=} \int\limits_{P^+(x_1)} \mu_2(x_2)\,(u_1(x_1) + u_2(x_2))\,dx_2 \\ &\quad + \int\limits_{P^-(x_1)} \mu_2(x_2)\,(E_1 + E_2)\,dx_2. \end{aligned} \tag{22}$$

□

Proof of Proposition 4.4 Note that the signal received affects the function $H(x_1)$ both through the new induced value of E_1 and the updated density $\mu_1(x_1|\theta = 1)$. Thus, in order to prove this proposition we have to split the function $H(x_1|\theta = 1)$ over two separate intervals:

(i) If $x_1 \in \left[x_1^m,\ ce_{(1|\theta=1)}\right]$

$$\begin{aligned} H(x_1|\theta = 1) &\overset{def}{=} \int\limits_{ce_{(1|\theta=1)}}^{x_1^M} \mu_1(y_1|\theta = 1)\,(u_1(y_1) + E_2)\,dy_1 \\ &\quad + \int\limits_{x_1^m}^{ce_{(1|\theta=1)}} \mu_1(y_1|\theta = 1)(E_{(1|\theta=1)} + E_2)dy_1 \end{aligned} \tag{23}$$

(ii) If $x_1 \in \left[ce_{(1|\theta=1)},\ x_1^M\right]$

$$H(x_1|\theta = 1) \overset{def}{=} \int\limits_{x_1}^{x_1^M} \mu_1(y_1|\theta = 1)\,(u_1(y_1) + E_2)\,dy_1$$

$$+\int_{x_1^m}^{x_1} \mu_1(y_1|\theta=1)(u_1(x_1)+E_2)dy_1. \tag{24}$$

Case (i). Let $x_1 \in [x_1^m,\ ce_{(1|\theta=1)}]$. Since $E_{(1|\theta=1)} \geq E_1$ implies $ce_{(1|\theta=1)} \geq ce_1$, the interval $[x_1^m, ce_{(1|\theta=1)}]$ splits in two subintervals $[x_1^m, ce_1]$ and $[ce_1, ce_{(1|\theta=1)}]$, corresponding to the following two subcases.

(i.a) If $x_1 \in [ce_1,\ ce_{(1|\theta=1)}]$, then applying Lemma 4.5 and using the definition of first order stochastic dominance (Definition 4.1) and the fact $E_{(1|\theta=1)} \geq u_1(x_1)$ we obtain:

$$\begin{aligned} H(x_1|\theta=1) &\overset{def}{=} \int_{ce_{(1|\theta=1)}}^{x_1^M} \mu_1(y_1|\theta=1)\,(u_1(y_1)+E_2)\,dy_1 \\ &+ \int_{x_1^m}^{ce_{(1|\theta=1)}} \mu_1(y_1|\theta=1)(E_{(1|\theta=1)}+E_2)dy_1 \\ &\geq H(x_1) \overset{def}{=} \int_{x_1}^{x_1^M} \mu_1(y_1)\,(u_1(y_1)+E_2)\,dy_1 + \int_{x_1^m}^{x_1} \mu_1(y_1)(u_1(x_1)+E_2)dy_1. \end{aligned} \tag{25}$$

(i.b) If $x_1 \in [x_1^m, c_1]$, then we can concentrate here on the effect that changes in the value of E_1 has on $H(x_1)$ for a given constant $\mu_1(x_1)$. If this effect is positive, then coupling a signal-based increment in E_1 with a first-order stochastic dominant spread on $\mu_1(x_1)$ would lead to an increment of the function $H(x_1)$.

Claim $\left.\frac{dH(x_1)}{dE_1}\right|_{\mu_1(x_1)} > 0, \forall x_1 \in [x_1^m, c_1]$.

Proof of Claim We start by expressing $H(x_1)$ as a function of ce_1 and noting that ce_1 and E_1 are positively related within the current environment

$$H(x_1) \overset{def}{=} \int_{ce_1}^{x_1^M} \mu_1(y_1)\,(u_1(y_1)+E_2)\,dy_1 + \int_{x_1^m}^{ce_1} \mu_1(y_1)(E_1+E_2)dy_1. \tag{26}$$

Applying Leibnitz's rule to the definition above while keeping $\mu_1(x_1)$ fixed allows us to isolate the effect that shifting the value of E_1 has on the function $H(x_1)$.

$$\begin{aligned} \left.\frac{dH(x_1)}{dE_1}\right|_{\mu_1(x_1)} &= \int_{ce_1}^{x_1^M} \frac{\partial}{\partial E_1}\left[\mu_1(y_1)\,(u_1(y_1)+E_2)\right]dy_1 \\ &+ \left[\mu_1\left(x_1^M\right)\left(u_1\left(x_1^M\right)+E_2\right)\right]\frac{dx_1^M}{dE_1} - \left[\mu_1(ce_1)(u_1(ce_1)+E_2)\right]\frac{d}{dE_1}[ce_1] \\ &+ \int_{x_1^m}^{ce_1} \mu_1(y_1)dy_1 + \left[\mu_1(ce_1)(E_1+E_2)\right]\frac{d}{dE_1}[ce_1] \\ &- \left[\mu_1(x_1^m)(E_1+E_2)\right]\frac{dx_1^m}{dE_1} \end{aligned}$$

$$= 0 + 0 - [\mu_1(ce_1)(E_1 + E_2)] \frac{d}{dE_1}[ce_1] + \int_{x_1^m}^{ce_1} \mu_1(y_1)dy_1$$
$$+ [\mu_1(ce_1)(E_1 + E_2)] \frac{d}{dE_1}[ce_1] - 0$$
$$= \int_{x_1^m}^{ce_1} \mu_1(y_1)dy_1 > 0. \tag{27}$$

Therefore, increments in E_1 have a strictly positive effect on $H(x_1)$, $\forall x_1 \in [x_1^m, c_1]$ and the claim is proved.

Corollary 4.6 together with Lemmas 4.5 and the claim provide the required conclusion, i.e.,$H(x_1|\theta = 1) \geq H(x_1)$, $\forall x_1 \in [x_1^m, c_1]$.

Case (ii). Let $x_1 \in [ce_{(1|\theta=1)}, \ x_1^M]$. By Lemma 4.5 and the definition of first order stochastic dominance (Definition 4.1) we immediately conclude that $H(x_1|\theta = 1) \geq H(x_1)$. □

Reasoning as in the proof of Propositions 4.3 and 4.4 we can show a more general result for any number $\theta = s$ of issued signals.

Proposition 4.8 *For every $x_1 \in X_1$ and every number $\theta = s$ of signals on X_1, we have $F(x_1|\theta = s) \geq F(x_1)$ and $H(x_1|\theta = s) \geq H(x_1)$.*

It should be noted that, the current analysis is not only valid for the signal-induced first-order stochastic dominant uniform density case described in this paper but also for any density function whose probability mass is redistributed to generate higher expected utilities (refer to Chapter 6 in Mas-Colell et al. (1995)).

5 Numerical simulations: information acquisition thresholds

We provide several numerical simulations illustrating the behavior of the threshold values defined by the functions $F(\cdot)$ and $H(\cdot)$ as the DMs receive an increasing number of signals regarding the distribution of X_1. The following parameter values have been assumed throughout the numerical simulations in order to derive the information acquisition incentives of the DMs and the resulting signaling strategies of the firms.

(i) Characteristic spaces: $X_1 = [5, 10]$, $X_2 = [0, 10]$.
(ii) Utility functions, risk neutral case: $u_1(x_1) = x_1$, $u_2(x_2) = x_2$.
(iii) Utility functions, risk averse case: $u_1(x_1) = \sqrt{x_1}$, $u_2(x_2) = \sqrt{x_2}$.
(iv) Probability densities:$\forall x_1 \in X_1$, $\mu_1(x_1) = 1/5$; $\forall x_2 \in X_2$, $\mu_2(x_2) = 1/10$.

The expected search utilities for the risk neutral and risk averse cases are represented in Figs. 1 and 2.

In both figures, the horizontal axis represents the set of realizations of $x_1 \in X_1$ that may be observed by the DMs, while the vertical axis accounts for their subjective expected utilities derived from the information acquisition process. We have illustrated the zero, one and two signals cases, denoted by ns, $1s$ and $2s$, respectively, and the evolution of the corresponding threshold values generated by both functions.

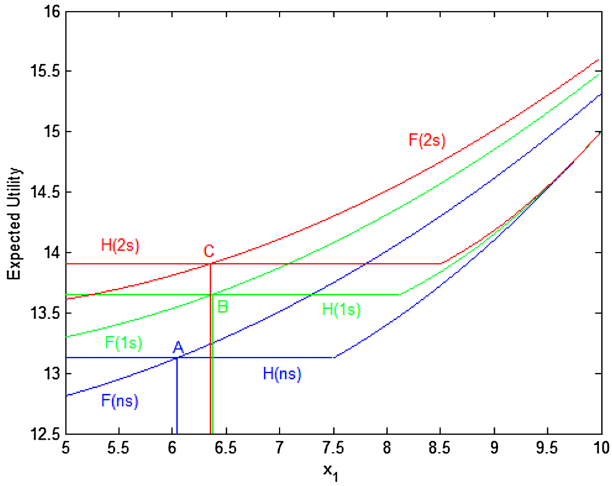

Fig. 1 Signals on X_1 with risk-neutral DMs: expected search utilities and threshold values. (Color figure online)

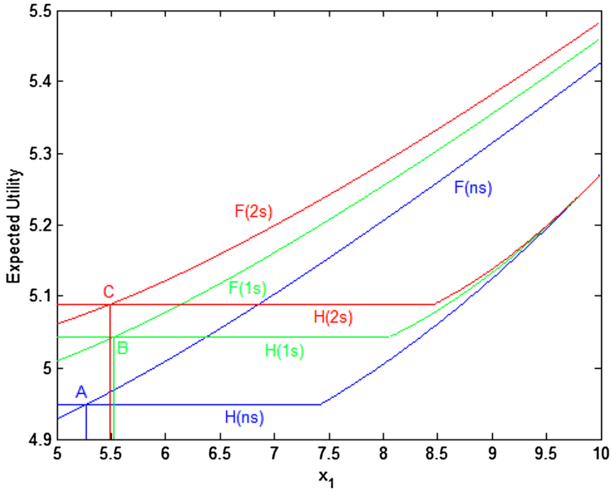

Fig. 2 Signals on X_1 with risk-averse DMs: expected search utilities and threshold values. (Color figure online)

6 Heuristic search process

Given the information acquisition thresholds obtained in Figs. 1 and 2 for the DMs, firms must account for the resulting rejection probabilities when deciding how many signals to issue. In this section, we will compute the changes in the rejection probabilities that result from

(a) The different signaling strategies of firms, which are defined in terms of the number of signals issued on the X_1 characteristic, i.e. the intensity of the improvement, and
(b) The loyalty of the DMs composing the market, i.e. the incentives of the DMs to acquire information from a given firm when it introduces a technologically superior product.

6.1 Information acquisition from a monopoly

Consider first the information acquisition/choice structure of a DM with no loyalty ties to the firm. We will assume that the DM observes a maximum of two characteristics from the set of available products, i.e. the DM is willing to fully observe a product, and denote by x_1^* the threshold value determined by the corresponding expected search utilities for a given number $\theta = s$ of signals issued. Moreover, if he is not initially satisfied (that is, if $x_1 < x_1^*$), he will not consider purchasing any product from the monopolized market. In the current heuristic setting, we have the following expression for the rejection probability faced by the unique firm monopolizing a newly created technological market

$$\mu_1(x_1 < x_1^* | \theta = s) + \mu_1(x_1 > x_1^* | \theta = s)\mu_2(x_2 < x_2^* | x_1) \tag{28}$$

where x_2^* is such that

$$u_1(x_1) + u_2(x_2^*) = E_{(1|\theta=s)} + E_2, \text{ for } s = 1, \ldots, 3. \tag{29}$$

This last equation leads to

$$\mu_1(x_1 > x_1^* | \theta = s)\mu_2(x_2 < x_2^* | x_1) = \int_{x_1^*}^{x_1^M} \mu_1(x_1 | \theta = s) \left[\int_0^{ce_{(1|\theta=s)} + ce_2 - x_1} \mu_2(x_2) dx_2 \right] dx_1, \tag{30}$$

That is, rejection takes place when the initial characteristic observed is located below x_1^* or whenever the first characteristic is located above x_1^* but its combination with the second one does not provide an acceptable utility level, i.e., $\mu_1(x_1 > x_1^* | \theta = s)\mu_2(x_2 < x_2^* | x_1)$. Otherwise, the DM will purchase a product from the monopolistic firm.

Consider now an extended search framework where a monopoly enjoys an additional information acquisition advantage, with the DMs acquiring (at least) two observations from its set of products even if the first observation is located below x_1^*.That is, consider those DMs who, due to exogenous effects such as brand loyalty or devotion, acquire at least two pieces of information from the set of products offered by a monopolist. That is, even if the initial product observed has a realization of x_1 located below x_1^*, these DMs will acquire information on a new product from the set offered by the technological monopoly. We will define two different potential information acquisition scenarios depending on the degree of loyalty exhibited by the DMs.

(i) *Complete verification setting* In this case, the DM exhibits a certain degree of preference for the products of the firm, but requires the verification of both characteristics before purchasing a product. That is, the DM requires observing a product completely and verifying its suitability before purchasing it. Otherwise, he stops acquiring information after partially observing two products. This behavior leads to the following version of Eq. (28):

$$\begin{aligned}&[\mu_1(x_1 < x_1^* | \theta = s)]^2 + [\mu_1(x_1 < x_1^* | \theta = s)]\mu_1(x_1 > x_1^* | \theta = s)\mu_2(x_2 < x_2^* | x_1)\\ &\quad + \mu_1(x_1 > x_1^* | \theta = s)\mu_2(x_2 < x_2^* | x_1)\end{aligned} \tag{31}$$

(ii) *Partial verification setting* In this case, we consider the behavior of a DM with a high degree of preference for the productsof the firm. Thus, given an initial observation located below x_1^*, if the first characteristic from the second product is higher than the threshold value, then the DM acquires the product without observing its second characteristic. That is, the DM observes a total of two characteristics and requires only partial verifiability if a second product is observed. The corresponding version of Eq. (28) is given by

$$[\mu_1(x_1 < x_1^*|\theta = s)]^2 + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) \tag{32}$$

In all cases, if the utility of a completely observed product is not sufficiently high, the DM will stop searching and will not purchase any product. Loyalty does not necessarily lead to suboptimal purchases, it simply gives the firm a second chance if the initial observation is not sufficiently good. Note, however, that we have also considered DMs whose loyalty leads them to purchase a product after only partially observing it, which may result in a suboptimal choice.

6.2 Information acquisition from a duopoly: symmetric scenario

Consider now the scenario where two *identical* firms compete within a given market as a duopoly. The information acquisition process of DMs is assumed to proceed as follows:

(a) The DMs can observe (if required) as many as four characteristics from both firms in order to find a satisficing product.
(b) The DMs stop acquiring information if a completely observed product provides a higher utility than the certainty equivalent one.
(c) The DMs will not purchase any product if after observing a product from each firm, either partially or fully, their characteristics do not deliver a satisfying level of utility.

Given this setting, the rejection probability faced by a duopolist when all customers are identical and equally distributed between both firms is given by

$$\begin{aligned}
&\frac{1}{2}\mu_1(x_1 < x_1^*|\theta = s) + \frac{1}{2}\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)\\
&\quad+\frac{1}{2}\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' > x_2^*|x_1')\\
&\quad+\frac{1}{2}\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' < x_2^*|x_1')\\
&\quad[\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]\\
&\quad+\frac{1}{2}\mu_1'(x_1' < x_1^*|\theta = s)\\
&\quad[\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]
\end{aligned} \tag{33}$$

where the prime superindex refers to the rival firm. Assuming that both firms are identical when competing as duopolists implies assuming that the improvements introduced on the products are also identical (*but not the other way around*). This leads the DMs to define the same distribution on the characteristics of the products of both firms after receiving each signal, i.e. $\mu_1'(x_1' < x_1^*|\theta = s) = \mu_1(x_1 < x_1^*|\theta = s)$ and $\mu_2'(x_2' < x_2^*|x_1') = \mu_2(x_2 < x_2^*|x_1)$ for all $x_1 \in X_1$. Note that the model allows us to relax this assumption and study the behavior of other potential types of DMs, such as those behaving optimistically or pessimistically while basing the search on their subjectively perceived brand reputation (Lee et al. 2009; Liu et al. 2015).

The following expression can be easily derived from the previous equation:

$$\frac{1}{2} + \frac{1}{2}[\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) + \mu_1(x_1 < x_1^*|\theta = s)]$$

$$[\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)] \tag{34}$$

We have assumed that the DMs divide themselves equally between firms if both firms compete within the same market. However, we could also assume that each firm is endowed with a (known or potential) customer base that modifies its duopolistic payoffs. As a result, the expected payoffs in all signaling duopolistic scenarios should account for the percentage of loyal customers composing the respective bases and their incentives to acquire more than one observation from the corresponding firm. In other words, the resulting duopolistic game becomes asymmetric.

6.3 Information acquisition from a duopoly: asymmetric scenario

We differentiate now between the two main types of DMs who can compose the customer base of a firm within the market, namely, regular, i.e. non-loyal, and loyal. Each firm will be endowed with a different share of customers and with different proportions of loyal customers within its corresponding share. Both types of customers start acquiring information on the products offered by the firm to whose customer base they belong. However, regular customers are not willing to acquire a second observation from the set of products offered by a firm if the first observation is lower than x_1^*. Thus, the only advantage provided by the regular customers composing the customer base is their preference for a given firm in terms of the initial observation acquired. We will denote by α the percentage of regular customers and by ϕ the percentage of loyal ones composing the customer base of one of the firms. As in the previous section, the prime superindex will be used to refer to the rival firm, and the standard requirement $\alpha + \phi + \alpha' + \phi' = 1$ will be imposed throughout the rest of the paper.

6.3.1 Complete verification setting

Consider a market composed by both regular and loyal customers within a complete verification setting. We must modify Eq. (33), representing the payoff received by each one of the firms when competing as a duopoly, so as to allow for both types of customers and the corresponding customer bases assigned to each firm. The resulting equation consists of three main parts determined by the type of customers being considered.

We must first account for the payoff derived from the different percentages of regular customers within the respective customer bases, α and α', which leads to

$$\begin{aligned}
&\alpha\mu_1(x_1 < x_1^*|\theta = s) + \alpha\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)\\
&\quad + \alpha'\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' > x_2^*|x_1')\\
&\quad + \alpha'\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' < x_2^*|x_1')\\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]\\
&\quad + \alpha'\mu_1'(x_1' < x_1^*|\theta = s)\\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]
\end{aligned} \tag{35}$$

We have to add to the previous expression the payoff determined by the percentage ϕ of customers who are loyal to the firm, which is based on the monopolistic payoffs described in Eq. (31).

$$\begin{aligned}
&\phi[\mu_1(x_1 < x_1^*|\theta = s)]^2 + \phi[\mu_1(x_1 < x_1^*|\theta = s)]\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)\\
&\quad + \phi\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)
\end{aligned} \tag{36}$$

and the payoff derived from the rejection of the rival's product, which is based on the percentage ϕ' of customers loyal to the rival firm

$$\begin{aligned}
&\phi'\mu'_1(x'_1 > x^*_1|\theta = s)\mu'_2(x'_2 > x^*_2|x'_1) + \phi'\mu'_1(x'_1 > x^*_1|\theta = s)\mu'_2(x'_2 < x^*_2|x'_1)\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)]\\
&\quad +\phi'\mu'_1(x'_1 < x^*_1|\theta = s)\mu'_1(x'_1 > x^*_1|\theta = s)\\
&\quad [\mu'_2(x'_2 > x^*_2|x'_1) + \mu'_2(x'_2 < x^*_2|x'_1)\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)]]\\
&\quad +\phi'\mu'_1(x'_1 < x^*_1|\theta = s)\mu'_1(x'_1 < x^*_1|\theta = s)\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)] \qquad (37)
\end{aligned}$$

Assuming that both firms provide identical improvements when competing as duopolists, the above equations can be integrated and simplified as follows

$$\begin{aligned}
&\alpha\mu_1(x_1 < x^*_1|\theta = s) + \alpha\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)\\
&\quad +\alpha'\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 > x^*_2|x_1)\\
&\quad +\alpha'[\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1) + \mu_1(x_1 < x^*_1|\theta = s)]\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)]\\
&\quad +\phi[\mu_1(x_1 < x^*_1|\theta = s)]^2 + \phi[\mu_1(x_1 < x^*_1|\theta = s)]\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)\\
&\quad +\phi\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)\\
&\quad +\phi'\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 > x^*_2|x_1)[1 + \mu_1(x_1 < x^*_1|\theta = s)]\\
&\quad +\phi'[\mu_1(x_1 > x^*_1|\theta{=}s)\mu_2(x_2 < x^*_2|x_1)(1{+}\mu_1(x_1 < x^*_1|\theta{=}s)) + \mu_1(x_1 < x^*_1|\theta = s)^2]\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)] \qquad (38)
\end{aligned}$$

6.3.2 *Partial verification setting*

Consider now a market composed by both regular and loyal customers within a partial verification setting. The version of Eq. (33) representing the payoff received by each one of the firms when competing as duopolists is similar to the one used to describe the complete verification setting above. In particular, the payoff derived from the percentages of regular customers composing the respective customer bases of the firms is given by

$$\begin{aligned}
&\alpha\mu_1(x_1 < x^*_1|\theta = s) + \alpha\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)\\
&\quad +\alpha'\mu'_1(x'_1 > x^*_1|\theta = s)\mu'_2(x'_2 > x^*_2|x'_1)\\
&\quad +\alpha'\mu'_1(x'_1 > x^*_1|\theta = s)\mu'_2(x'_2 < x^*_2|x'_1)\\
&\quad [\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)]\\
&\quad +\alpha'\mu'_1(x'_1 < x^*_1|\theta = s)[\mu_1(x_1 < x^*_1|\theta = s) + \mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1)] \qquad (39)
\end{aligned}$$

As we did in the previous setting, we must add to the previous expression the payoff determined by the percentage ϕ of customers loyal to the firm

$$\phi[\mu_1(x_1 < x^*_1|\theta = s)]^2 + \phi\mu_1(x_1 > x^*_1|\theta = s)\mu_2(x_2 < x^*_2|x_1) \qquad (40)$$

and the payoff derived from the rejection of the rival's product, which is based on the percentage ϕ' of customers loyal to the rival firm

$$
\begin{aligned}
&\phi'\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' > x_2^*|x_1') \\
&\quad + \phi'\mu_1'(x_1' > x_1^*|\theta = s)\mu_2'(x_2' < x_2^*|x_1') \\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)] \\
&\quad + \phi'\mu_1'(x_1' < x_1^*|\theta = s)\mu_1'(x_1' > x_1^*|\theta = s) \\
&\quad + \phi'\mu_1'(x_1' < x_1^*|\theta = s)\mu_1'(x_1' < x_1^*|\theta = s) \\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]
\end{aligned} \tag{41}
$$

Assuming that both firms provide identical improvements when competing as duopolists, we obtain the following simplified expression

$$
\begin{aligned}
&\alpha\mu_1(x_1 < x_1^*|\theta = s) + \alpha\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) \\
&\quad + \alpha'\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 > x_2^*|x_1) \\
&\quad + \alpha'[\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) + \mu_1(x_1 < x_1^*|\theta = s)] \\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)] \\
&\quad + \phi[\mu_1(x_1 < x_1^*|\theta = s)]^2 + \phi\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) \\
&\quad + \phi'\mu_1(x_1 > x_1^*|\theta = s)[\mu_2(x_2 > x_2^*|x_1) + \mu_1(x_1 < x_1^*|\theta = s)] \\
&\quad + \phi'[\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) + \mu_1(x_1 < x_1^*|\theta = s)^2] \\
&\quad [\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)]
\end{aligned} \tag{42}
$$

7 Numerical simulations: strategic signaling

The monopolistic, $[r(\theta)]_j$, and duopolistic, $(> [r(\theta)]_j)$, rejection probabilities derived from the threshold values computed by the DMs, $j = A, B, C, D$, define the technological transition matrix represented in Table 1. Note that D refers to the threshold value resulting from a third signal, which, to simplify the presentation, has not been illustrated in Figs. 1 or 2. This matrix will be used to determine the strategic signaling behavior of the firms.

We have considered risk-neutral and risk-averse DMs when computing the corresponding entries of the technological transition matrices presented in Tables 2, 3 and 4. The technological transition incentives describing the scenario without a loyal customer base follow from Table 2.

Similarly, Tables 3 and 4 illustrate the enhanced monopolistic environment, where a loyal customer base is introduced. In particular, Table 3 describes the strategic environment that results from the stricter Eq. (31) determining the probability of purchase in monopoly, while Table 4 describes the partial verification setting.

Table 1 Technological transition game based on the number of signals issued by the firms

	$3s$	$2s$	$1s$	ns
$3s$	$(>[r(\theta)]_D)$	$[r(\theta)]_D, 0$	–	–
$2s$	$0, [r(\theta)]_D$	$(>[r(\theta)]_C)$	$[r(\theta)]_C, 0$	–
$1s$	–	$0, [r(\theta)]_C$	$(>[r(\theta)]_B)$	$[r(\theta)]_B, 0$
ns	–	–	$0, [r(\theta)]_B$	$[r(\theta)]_A$

Table 2 Market transition incentives without a loyal customer Base

Rival	Firm			
	Signal	No signal		
Risk neutrality				
Signal	(0.6268, 0.6268)	(0.5036, 1)	–	–
No Signal	(1, 0.5036)	(0.6309, 0.6309)	(0.5117, 1)	–
	–	(1, 0.5117)	(0.6432, 0.6432)	(0.5352, 1)
	–	–	(1, 0.5352)	(0.6582, 0.6582)
Risk aversion				
Signal	(0.6004, 0.6004)	(0.4481, 1)	–	–
No Signal	(1, 0.4481)	(0.6025, 0.6025)	(0.4528, 1)	–
	–	(1, 0.4528)	(0.6084, 0.6084)	(0.4656, 1)
	–	–	(1, 0.4656)	(0.6104, 0.6104)

Table 3 Market transition incentives in the complete verification setting

Rival	Firm			
	Signal	No signal		
Risk neutrality				
Signal	(0.6268, 0.6268)	(0.4945, 1)	–	–
No Signal	(1, 0.4945)	(0.6309, 0.6309)	(0.4854, 1)	–
	–	(1, 0.4854)	(0.6432, 0.6432)	(0.4714, 1)
	–	–	(1, 0.4714)	(0.6582, 0.6582)
Risk aversion				
Signal	(0.6004, 0.6004)	(0.4448, 1)	–	–
No Signal	(1, 0.4448)	(0.6025, 0.6025)	(0.4423, 1)	–
	–	(1, 0.4423)	(0.6084, 0.6084)	(0.4374, 1)
	–	–	(1, 0.4374)	(0.6104, 0.6104)

These tables illustrate how the presence of a loyal customer base does not necessarily increase the incentives of firms to improve upon the characteristics of a product to their highest levels available.

As expected, the rejection probabilities faced by a monopolist are all considerably reduced when endowed with a loyal customer base, though this difference vanishes as the number of signals issued increases. However, the resulting strategic environment does not necessarily provide incentives for the firms to issue the highest possible number of signals. That is, given a set of potential signaling strategies that may be followed by the rival, a firm may not have sufficient incentives to issue the highest possible number of signals.

Proposition 7.1 *The dominant strategy for both firms in the signaling game without loyal customer bases consists of issuing the maximum number of signals, i.e. introducing the most technologically developed product.*

Proof Consider the expected payoffs received from not issuing any signals and issuing one, two and three signals, respectively

Number of signals	Expected payoffs
ns	$P_i(X_1\|0s)[r(\theta)]_A + [P_i(X_1\|1s) + P_i(X_1\|2s) + P_i(X_1\|3s)] \times 0$
1s	$P_i(X_1\|0s)[r(\theta)]_B + P_i(X_1\|1s)(> [r(\theta)]_B) + [P_i(X_1\|2s) + P_i(X_1\|3s)] \times 0$
2s	$[P_i(X_1\|0s) + P_i(X_1\|1s)][r(\theta)]_C + P_i(X_1\|2s)(> [r(\theta)]_C) + P_i(X_1\|3s) \times 0$
3s	$[P_i(X_1\|0s) + P_i(X_1\|1s) + P_i(X_1\|2s)][r(\theta)]_D + P_i(X_1\|3s)(> [r(\theta)]_D)$

with the following condition holding

$$1 - [P_i(X_1|0s) + P_i(X_1|1s) + P_i(X_1|2s)] = P_i(X_1|3s) \tag{43}$$

where $P_i(X_1|ls)$ is the probability assigned by firm i to the rival issuing $l = 0, 1, 2, 3$ signals. We have implicitly assumed that the payoff obtained by a firm from selling a product equals one.

Given the entrances of the matrices presented in Table 2, it follows that

$[r(\theta)]_B > [r(\theta)]_A \rightarrow$ payoff (ns) < payoff (1s), $\forall P_i(X_1|\cdot)$.
$[r(\theta)]_C > [r(\theta)]_B > (> [r(\theta)]_B) \rightarrow$ payoff (1s) < payoff (2s), $\forall P_i(X_1|\cdot)$.
$[r(\theta)]_D > [r(\theta)]_C > (> [r(\theta)]_C) \rightarrow$ payoff (2s) < payoff (3s), $\forall P_i(X_1|\cdot)$.

Thus, issuing three signals constitutes a dominant strategy for both firms in the game without a loyal customer base, both in the risk neutral and risk averse cases. □

Note that the dominance of the above signaling strategy does not hold when a loyal customer base is introduced in the market, either within a complete or a partial verification setting. The following corollary illustrates this claim.

Corollary 7.2 *Issuing the maximum number of signals, i.e. introducing the most technologically developed product, is not a dominant strategy for the firms in the signaling game with loyal customer bases.*

Proof Tables 3 and 4 respectively illustrate how either within the complete or the partial verification setting $[r(\theta)]_B$ provides the highest payoff to a firm. Therefore, there exists a value of $P_i(X_1|0s)$, denoted by $\bar{P}_i$, such that for any $P_i(X_1|0s) > \bar{P}$ issuing one signal constitutes a dominant strategy for the firm. □

Table 4 Market transition incentives in the partial verification setting

Rival	Firm			
	Signal	No signal		
Risk neutrality				
Signal	(0.6268, 0.6268)	(0.4855, 1)	–	–
No Signal	(1, 0.4855)	(0.6309, 0.6309)	(0.4608, 1)	-
	–	(1, 0.4608)	(0.6432, 0.6432)	(0.4168, 1)
	–	–	(1, 0.4168)	(0.6582, 0.6582)
Risk aversion				
Signal	(0.6004, 0.6004)	(0.4421, 1)	–	–
No Signal	(1, 0.4421)	(0.6025, 0.6025)	(0.4340, 1)	–
	–	(1, 0.4340)	(0.6084, 0.6084)	(0.4156, 1)
	–	–	(1, 0.4156)	(0.6104, 0.6104)

Note that it is the substantial decrease in $\mu_1(x_1 < x_1^*|\theta = s)$ per signal issued what generates the incentives to issue the highest number of signals when introducing a technologically superior product. This decrease in rejection probability compensates for and modifies the trend exhibited by the increasing $\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)$. As a result, acquiring a second observation from X_1 whenever $x_1 < x_1^*$ mitigates this compensation effect, which allows the increasing rejection probability trend defined by $\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1)$ to dominate the signaling incentives of firms. Hence, loyal customers willing to acquire additional information decrease the rejection probability of the firm when issuing a given number of signals but modify its signaling incentives since the resulting probabilities increase in the number of signals issued.

Thus, the introduction of loyal customer bases within the initial duopolistic setting allows for the existence of a set of beliefs on the side of firms such that issuing the maximum number of signals stops being a dominant strategy. We will illustrate numerically the existence of this set as well as its limit values for different beliefs of the firm when solving numerically the Bayesian Nash equilibria of the corresponding games. The following proposition constitutes one of the main results of the paper.

Proposition 7.3 *Consider a symmetric duopoly whose firms must determine the number of technological improvements to introduce given the probability of the rival introducing any number of potential improvements. The heuristic information acquisition framework defined in the paper implies that the existence of a loyal customer base will*

(i) *reduce the rejection probabilities faced by a technological monopolist;*
(ii) *decrease the incentives of firms to introduce the highest number of product improvements available and issue the corresponding number of signals.*

Proof Part (i) follows directly from Eqs. (28), (31) and (32). Part (ii) requires some additional analysis. Consider a standard signaling game, whose set of Bayesian Nash equilibria is determined by the subjective beliefs of a firm regarding the capacity of its rival to introduce a technological improvement of a given intensity in the market and issue the corresponding number of signals. The equilibrium payoff conditions that weaken the dominant strategy consisting of issuing the highest number of signals could be easily derived analytically. However, numerical analysis is the best way to gather insights regarding the signaling incentives of the firms when a loyal customer base is assumed.

Consider Figs. 3 and 4, which illustrate the combinations of equilibrium probabilities making the firm indifferent between issuing 1 or 2 signals (Fig. 3) and 1 or 3 signals (Fig. 4) when DMs are risk neutral, though the same results would follow from a risk averse environment. These figures describe numerically the incentives of the firm to issue more than one signal, given the subjective probability assigned by the firm to a rival issuing any potential number of signals.

That is, Fig. 3 represents the payoff equality condition

$$\begin{aligned} &P_i(X_1|0s) \cdot (1 - [r(\theta)]_B) + P_i(X_1|1s) \cdot (1 - (> [r(\theta)]_B)) \\ &\quad = P_i(X_1|0s) \cdot (1 - [r(\theta)]_C) + P_i(X_1|1s) \\ &\qquad \cdot (1 - [r(\theta)]_C) + P_i(X_1|2s) \cdot (1 - (> [r(\theta)]_C)) \end{aligned} \tag{44}$$

while the equation represented in Fig. 4 is given by

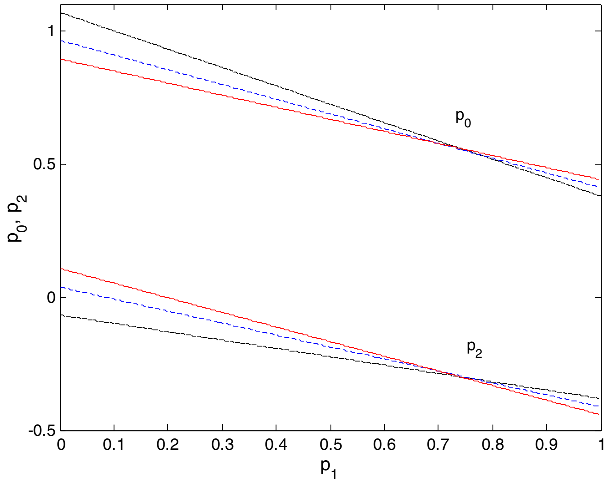

Fig. 3 Signaling incentives of firms and loyal customers: 1 versus 2 signals. (Color figure online)

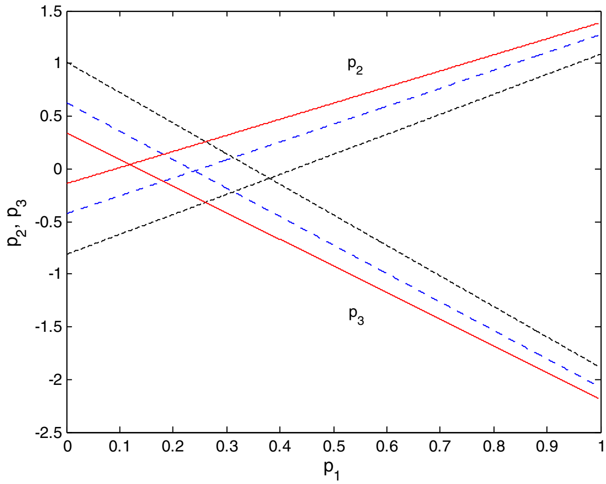

Fig. 4 Signaling incentives of firms and loyal customers: 1 versus 3 signals. (Color figure online)

$$\begin{aligned}
&P_i(X_1|0s) \cdot (1 - [r(\theta)]_B) + P_i(X_1|1s) \cdot (1 - (> [r(\theta)]_B)) \\
&\quad = P_i(X_1|0s) \cdot (1 - [r(\theta)]_D) + P_i(X_1|1s) \cdot (1 - [r(\theta)]_D) + P_i(X_1|2s) \cdot (1 - [r(\theta)]_D) \\
&\qquad + P_i(X_1|3s) \cdot (1 - (> [r(\theta)]_D))
\end{aligned} \tag{45}$$

The dotted black functions in both figures describe the environment without a loyal customer base, while the dashed blue functions represent the complete verification setting and the solid red ones illustrate the partial verification setting. The interpretation of these functions within the figures is given as follows.

The functions represented in Fig. 3 provide the (subjective) probability combinations such that firms are indifferent between issuing 1 and 2 signals. In order to simplify the presentation, we will denote these probabilities by p_j, with $j = 0, 1, 2, 3$, representing the *jth*–signal scenario. That is, given a probability of $p_1 = 0$, the firm requires a probability

Table 5 Market transition incentives in the complete verification setting with risk neutrality and different customer bases

Signals	Duopolistic payoffs
ns	$0.5625\alpha + 0.7539\alpha' + 0.4719\phi + 0.7935\phi'$
1s	$0.5352\alpha + 0.7512\alpha' + 0.4714\phi + 0.7809\phi'$
2s	$0.5117\alpha + 0.7501\alpha' + 0.4854\phi + 0.7630\phi'$
3s	$0.5036\alpha + 0.7499\alpha' + 0.4945\phi + 0.7544\phi'$
4s	$0.5012\alpha + 0.7500\alpha' + 0.4981\phi + 0.7515\phi'$
Signals	**Monopolistic payoffs**
1s	$0.5352\alpha(1+\xi) + 0.7512(\alpha' - \xi\alpha) + 0.4714\phi(1+\xi) + 0.7809(\phi' - \xi\phi)$
2s	$0.5117\alpha(1+\xi) + 0.7501(\alpha' - \xi\alpha) + 0.4854\phi(1+\xi) + 0.7630(\phi' - \xi\phi)$
3s	$0.5036\alpha(1+\xi) + 0.7499(\alpha' - \xi\alpha) + 0.4945\phi(1+\xi) + 0.7544(\phi' - \xi\phi)$

combination of $p_0 = 0.8935$ and $p_2 = 0.1065$ in order to be indifferent between issuing 1 and 2 signals within the partial verification setting. A value of $p_0 > 0.8935$ and, consequently, of $p_2 < 0.1065$ would lead firms to issue only one signal in equilibrium, while a value of $p_0 < 0.8935$ together with $p_2 > 0.1065$ would result in two signals being issued. The corresponding probability values for the complete verification setting whenever $p_1 = 0$ are given by $p_0 = 0.9635$ and $p_2 = 0.0365$.

The logic is identical when describing the incentives of firms to issue either 1 or 3 signals as presented in Fig. 4. We have assumed a value of $p_0 = 0.8$ in order to allow for the existence of an equilibrium where only one signal is issued. Note, for example, that within the partial verification setting described in Fig. 4 firms are indifferent between issuing 1 and 3 signals when $p_1 = 0.1$, $p_2 = 0.0132$ and $p_3 = 0.0868$.

Thus, given the basic requirement defined in Eq. (43), we can observe in both figures that the partial verification setting allows for a larger set of probabilities where issuing only one signal instead of two (Fig. 3) or three (Fig. 4) is an equilibrium of the signaling game. This concludes the proof. □

The same type of result follows from the asymmetric scenario where firms are endowed with different customer bases composed by regular and loyal DMs. In this case, the expected payoffs defined by the firms depend on the relative percentages of DMs composing their respective customer bases. Tables 5 and 6 describe the expected monopolistic and duopolistic payoffs received by a firm when customers are risk neutral in the complete and partial verification settings, respectively. It should be emphasized that the same results would be obtained with risk averse customers.

Note how, in this scenario, the firm monopolizing a given niche market does not gain control over the whole market but increments the size of its customer base, while the rival suffers an equivalent decrement in the size of its customer base. In this regard, the variable ξ represents the increment in the customer base of the firm introducing a technological improvement and monopolizing a given niche market. It could be claimed that this scenario is closer to a real-life one, since product improvements tend to increase the market quota of the signaling firm but do not grant control over the entire market. We will assume that $\xi = 0.10$ through the simulations, though the main results remain unchanged if we modify this percentage.

Figure 5a and b illustrate the combinations of equilibrium probabilities making the firm indifferent between issuing 1 or 2 signals when $\alpha' + \phi' = 0.5$ and $\alpha' + \phi' = 0.2$, respectively.

Table 6 Market transition incentives in the partial verification setting with risk neutrality and different customer bases

Signals	Duopolistic payoffs
ns	$0.5625\alpha + 0.7539\alpha' + 0.3983\phi + 0.8257\phi'$
1s	$0.5352\alpha + 0.7512\alpha' + 0.4168\phi + 0.8063\phi'$
2s	$0.5117\alpha + 0.7501\alpha' + 0.4608\phi + 0.7750\phi'$
3s	$0.5036\alpha + 0.7499\alpha' + 0.4855\phi + 0.7589\phi'$
4s	$0.5012\alpha + 0.7500\alpha' + 0.4950\phi + 0.7531\phi'$
Signals	Monopolistic payoffs
1s	$0.5352\alpha(1+\xi) + 0.7512(\alpha' - \xi\alpha) + 0.4168\phi(1+\xi) + 0.8063(\phi' - \xi\phi)$
2s	$0.5117\alpha(1+\xi) + 0.7501(\alpha' - \xi\alpha) + 0.4608\phi(1+\xi) + 0.7750(\phi' - \xi\phi)$
3s	$0.5036\alpha(1+\xi) + 0.7499(\alpha' - \xi\alpha) + 0.4855\phi(1+\xi) + 0.7589(\phi' - \xi\phi)$

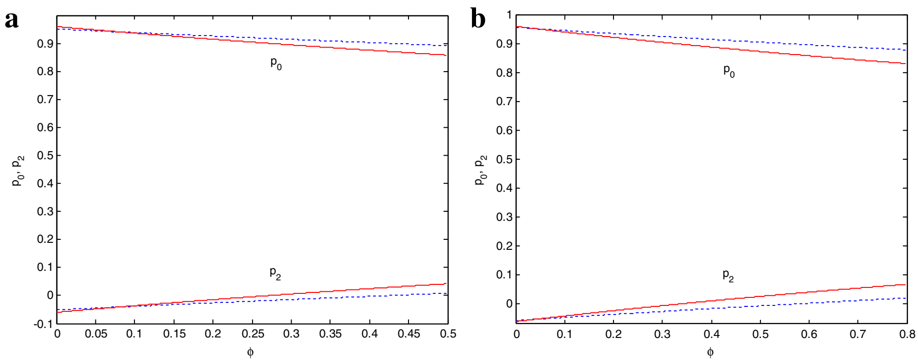

Fig. 5 Signaling incentives of firms with different customer bases: 1 versus 2 signals. **a** $\alpha' + \phi' = 0.5$; $\alpha' = 0.3$; $\phi' = 0.2$; $p_1 = 0.1$; $\alpha = 0.5 - \phi$, **b** $\alpha' + \phi' = 0.2$; $\alpha' = 0.1$; $\phi' = 0.1$; $p_1 = 0.1$; $\alpha = 0.8 - \phi$. (Color figure online)

The resulting intuition is identical to the one derived from the proof of Proposition 7.3. There is however an additional result obtained from these figures. Firms endowed with a larger customer base require a lower proportion of loyal customers relative to the size of their bases in order to be indifferent between issuing 1 and 2 signals. That is, in the partial verification setting defined within the $\alpha' + \phi' = 0.5$ case, $\phi = 0.2719$ out of $\alpha + \phi = 0.5$ in order for $p_2 = 0$. However, in the partial verification setting defined within the $\alpha' + \phi' = 0.2$ case, $\phi = 0.3384$ out of $\alpha + \phi = 0.8$ in order for $p_2 = 0$. The same result follows from the complete verification setting, with $\phi = 0.4253$ and 0.5767, respectively. Thus, firms endowed with larger customer bases will tend to require a lower relative proportion of loyal DMs in order to issue a lower number of signals. Similar results are obtained in the 1 versus 3 signals case, as Fig. 6a and b illustrate.

7.1 Using the certainty equivalents as reference values

We shift now the focus of the analysis towards the standard framework considered in the economics and operations research literatures. In order to do so, we must first introduce a fourth type of customer, who will base his purchase decision on the first characteristic of the

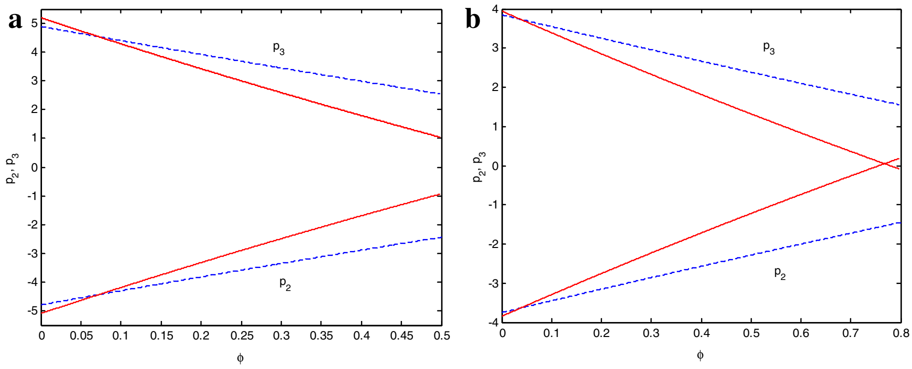

Fig. 6 Signaling incentives of firms with different customer bases: 1 versus 3 signals. **a** $\alpha' + \phi' = 0.5$; $\alpha' = 0.3$; $\phi' = 0.2$; $p_0 = 0.8$; $p_1 = 0.1$; $\alpha = 0.5 - \phi$, **b** $\alpha' + \phi' = 0.2$; $\alpha' = 0.1$; $\phi' = 0.1$; $p_0 = 0.8$; $p_1 = 0.1$; $\alpha = 0.8 - \phi$; $p_2 = 0$ for $\phi = 0.7574$ and $p_3 = 0$ for $\phi = 0.7793$. (Color figure online)

products while completely ignoring the second. Assume that this type of customer is willing to acquire a total of two observations from the set of products offered by the firm to whose customer base he belongs. In order to differentiate this case from the previous ones analyzed through the paper, we define this type of customer as a completely loyal one.

Consider now the differences in rejection probabilities arising between the settings where the completely loyal customers use either x_1^* or ce_1 as the reference value to determine their choices. This distinction leads to two different monopolistic verification settings, whose corresponding rejection probabilities within the risk neutral environment are defined by either $[\mu_1(x_1 < x_1^* | \theta = s)]^2 = 0.0189, 0.0029$ and 0.0003, or $[\mu_1(x_1 < ce_1 | \theta = s)]^2 = 0.1914, 0.2116$ and 0.2337, in the one, two and three signals cases, respectively. Note that the same type of patterns are obtained in the risk averse setting.

The intuition behind these results is based on the redistribution of probability implied by the signals together with its effect on x_1^* through the corresponding expected search utilities. Note that the probability mass accumulated below x_1^* keeps on decreasing, leading to the decreasing pattern obtained for $\mu_1(x_1 < x_1^* | \theta = s)$. However, the pattern differs when considering the certainty equivalent value, with the probability mass accumulated below ce_1 increasing, which leads to a different set of incentives for the DMs. Clearly, this difference in information acquisition incentives is due to the fact that $ce_1 > x_1^*$ together with the larger increase in the value of ce_1 relative to that of x_1^* resulting from each signal received on X_1. In particular, the probability mass accumulated below ce_1 is given by

$$\int_{x_1^m}^{\overbrace{u_1^{-1}(E_1)}^{\uparrow}} \underbrace{\mu_1(x_1)}_{\downarrow} \, dx_1 \tag{46}$$

Thus, if the decrease in the value of the density function does not suffice to compensate for the increase in the certainty equivalent value, then a positive signal issued on the first characteristic would lead to an increase in the probability of rejection. This is the case in the numerical examples described in Tables 7 and 8 for the risk neutral and risk averse cases, respectively. These tables present the rejection probabilities obtained in each one of the other monopolistic settings defined through the paper when the DMs do not consider the

Table 7 Monopolistic rejection probabilities with certainty equivalents as reference values: risk neutral case

Number of signals	$\mu_1(x_1 < ce_1 \mid \theta = s)$	$\overline{\mu_2}(x_2 < ce_2 \mid \theta = s)$	Absent loyal cus tomers	Complete verifica tion setting	Partial ver ification setting
1s	0.4375	0.5	0.7188	0.5957	0.4727
2s	0.46	0.5	0.7300	0.6058	0.4816
3s	0.4834	0.5	0.7417	0.6168	0.4920

Table 8 Monopolistic rejection probabilities with certainty equivalents as reference values: risk averse case

Number of sig- nals	$\mu_1(x_1 < ce_1 \mid \theta = s)$	$\overline{\mu_2}(x_2 < ce_2 \mid \theta = s)$	Absent loyal cus tomers	Complete verifica tion setting	Partial ver ification setting
1s	0.4207	0.4444	0.6781	0.5427	0.4344
2s	0.4475	0.4444	0.6930	0.5557	0.4458
3s	0.4748	0.4444	0.7082	0.5697	0.4588

interactions between the first and the second characteristic of the products when acquiring information.

Two main implications follow from the above numerical results. Consider the case where the DMs do not account for the interactions between the first and the second characteristic when acquiring information, as is generally assumed by the economics and operations research literature. That is, assume that the choice made by a DM is determined by $\mu_1(x_1 < ce_1 \mid \theta = s)$ and $\overline{\mu_2}(x_2 < ce_2 \mid \theta = s)$, $\forall s$, i.e. the rejection probability defined by the second characteristic, which, in this case, is independent from the realization of the first and remains constant for all signals received on the first characteristic. Thus, the threshold value employed by the DMs to determine their information acquisition incentives is given by the certainty equivalent value, ce_1, whose corresponding rejection probability increases in the number of signals. As a result, *ignoring the interactions between both characteristics restricts the incentives of the firms to introduce the most technologically developed product available*.

On the other hand, the decreasing rejection probability obtained as the number of signals issued increases illustrates how *those DMs who account for the potential interactions between both characteristics when defining* x_1^* *foster the introduction of the most technologically advanced product by the firms*. Therefore, the availability of DSS that can be used by DMs through the information acquisition process would improve the quality of the technology being introduced in the market while increasing the success probability of firms when doing so. This latter result prevails even when introducing loyal customer bases in the certainty equivalent setting, as Tables 2, 7 and 8 illustrate. In particular, note that the monopolistic rejection probabilities of the certainty equivalent setting with complete verification are higher than those of the potential interactions setting without a loyal customer base.

Finally, for the sake of completeness, we have computed the differences arising between the rejection probabilities defined by Eq. (32) in the partial verification setting and those obtained when the DMs use the certainty equivalent value as a reference when acquiring the second observation from the first characteristic

$$[\mu_1(x_1 < x_1^*|\theta = s)][\mu_1(x_1 < ce_1|\theta = s)] + \mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) \quad (47)$$

The rejection probability values derived from Eq. (47) within the risk neutral setting are equal to 0.4580, 0.4826 and 0.4941 in the one, two and three signals cases, respectively. The corresponding values obtained in the risk averse setting are equal to 0.4350, 0.4422 and 0.4449. Note that the resulting pattern of expected payoffs is not different from that of Eq. (32). Therefore, the signaling incentives of firms remain similar to those obtained in the case where the DMs consider only x_1^* as the reference value.

8 Conclusion

In this paper, we have illustrated how the existence of a loyal customer base does not necessarily provide the incentives required for firms to introduce the most technologically developed product available. Moreover, if the DMs do not consider the potential interactions between the product characteristics and use the certainty equivalents as their reference values when acquiring information, the rejection probability of the new products will be higher than the one obtained when DMs consider these interactions.

On the other hand, the existence of a loyal customer base within this latter setting decreases the incentives to introduce the most technologically developed product available while also decreasing the rejection probability faced by the firms when generating monopolized niche markets. It can be assumed that companies such as Apple consider the main consequences derived from the sequential improvement strategy that becomes available due to the existence of loyal customers. That is, partial improvements prevent subsequent delays in product introduction, since further improvements are already available within the resulting sequential structure. This is indeed quite significant, given the fact that these delays may lead to loss of market value (Hendricks and Singhal 2008).

It should be noted that we have concentrated on pure incentive mechanisms and considered only profits, leaving costs aside and thus ignoring the fact that developing more advanced technological products would imply a higher cost for the firms. Moreover, we have not analyzed the transformation process between different customer types, that is, the possibility of generating a loyal customer base as innovations are introduced in the market. In this regard, the model can be extended to define loyalty as a function of the quality of the product improvements introduced in the market (Tellis et al. 2009). Analyzing the equilibrium effects resulting from firms expecting a given proportion of customers to become loyal as innovations are introduced would constitute a natural extension of the current paper. Finally, the integration of our demand-based game theoretic structure with that defined by the different supplier development investment strategies of firms, see Bai and Sarkis (2014), could also be considered as a potential extension of the model presented in this paper.

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