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An ideal-seeking fuzzy data envelopment analysis framework

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ABSTRACT

Data envelopment analysis (DEA) is a widely used mathematical programming approach for evaluating the relative efficiency of decision making units (DMUs) in organizations. Crisp input and output data are fundamentally indispensable in traditional DEA evaluation process. However, the input and output data in real-world problems are often imprecise or ambiguous. In this study, we present a four-phase fuzzy DEA framework based on the theory of displaced ideal. Two hypothetical DMUs called the *ideal* and *nadir* DMUs are constructed and used as reference points to evaluate a set of information technology (IT) investment strategies based on their Euclidean distance from these reference points. The best relative efficiency of the fuzzy ideal DMU and the worst relative efficiency of the fuzzy nadir DMU are determined and combined to rank the DMUs. A numerical example is presented to demonstrate the applicability of the proposed framework and exhibit the efficacy of the procedures and algorithms.

1. Introduction

The intensity of global competition and ever-increasing economic uncertainties has led organizations to search for more efficient and effective ways to manage their business operations. Information technology (IT) investment has profound impact on different stages of business operations. Many organizations have turned to IT to better manage their business and cope with uncertainties in the business environment. While organizations continue to invest heavily in IT, there is much debate about the need for measuring the productivity impacts of IT investment. Many factors affect organizational productivity and it is difficult to establish causality between IT investments and organizational-level output performance [1].

In recent years, data envelopment analysis (DEA) has been widely used to evaluate the impact of IT investment on organizational productivity and performance. This is partly due to the fact that DEA does not need an a priori assumption on the functional form characterizing the relationships between IT investment and organizational performance measures [2]. DEA is a mathematical programming approach that was originally developed by Charnes et al. [3] and was extended by Banker et al. [4] to include variable returns to scale. The basic DEA models are known as Charnes, Cooper and Rhodes (CCR) and Banker, Charnes, and Cooper (BCC). DEA generalizes the Farrell [5] single-input/single-output technical efficiency measure to the multiple-input/multiple-output case to evaluate the relative efficiency of peer units with respect to multiple performance measures [3,6,7]. The units under evaluation in DEA are called decision making units (DMUs) and their performance measures are grouped into inputs and outputs. Through the optimization for each DMU, DEA yields an efficient frontier or tradeoff curve that represents the relations among the multiple performance measures. Unlike parametric methods which require detailed knowledge of the process, DEA is non-parametric and does not require an explicit functional form relating inputs and outputs.

We use DEA as the fundamental tool in our study for the following reasons. First, in performance evaluation, the use of single measures ignores any interactions among various firm performance measures. DEA has been proven effective in performance evaluation when multiple performance measures are present [2]. Second, DEA does not require a priori information about the relationship among multiple performance measures. Third, a number of studies about the IT impact on firm performance have successfully used DEA [8–19].

When evaluating an investment using DEA, it is required to estimate several parameters without any variations [6,7]. However, the estimation of uncertain parameters in this evaluation process is often very challenging. Some researchers have proposed various methods for dealing with imprecise and ambiguous data in DEA. Cooper et al. [20] has studied this problem in the context

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of interval data. However, many real-life problems use linguistic data such as good, fair or poor and cannot be used as interval data. Fuzzy logic and fuzzy sets can represent ambiguous, uncertain or imprecise information in DEA by formalizing inaccuracy in decision making [21]. Fuzzy set algebra developed by Zadeh [22] is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments.

We present a four-phase fuzzy DEA framework based on the theory of displaced ideal. Two hypothetical DMUs called the *ideal* and *nadir* DMUs are constructed and used as reference points to evaluate a set of information technology (IT) investment strategies based on their Euclidean distance from these reference points. The best relative efficiency of the fuzzy ideal DMU and the worst relative efficiency of the fuzzy nadir DMU are determined and combined to rank the DMUs. The remainder of this paper is organized as follows. The next section presents the preliminaries and the methodology used in this study. In Section 3, we illustrate the details of the proposed framework followed by a numerical example presented in Section 4 to demonstrate the applicability of the proposed framework and exhibit the efficacy of the procedures and algorithms. In Section 5, we conclude with our conclusions and future research directions.

2. Preliminaries and methodology

Consider a set of n DMUs, with each DMUj ($j = 1, \ldots, n$) using m inputs x_{ij} ($i = 1, \ldots, m$) and securing s outputs y_{rj} ($r = 1, \ldots, s$). Specifically, if DMUp is under consideration, the original CCR model for measuring the relative efficiency score is equivalent to the following fractional model:

$$\begin{aligned} \theta_p^* &= \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \\ \text{s.t.} & \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \forall j, \\ & u_r, v_i \ge \varepsilon, \quad \forall r, i. \end{aligned} \tag{1}$$

where u_r ($r = 1, \ldots, s$) and v_i ($i = 1, \ldots, m$) are the weights of the rth output and the ith input, respectively, and ε is a small non-Archimedean value [23,24,25]. Applying the Charnes and Cooper's [26] theory of fractional programming, model (1) can be replaced by the following linear program (LP):

$$\begin{aligned} \theta_p^* &= \max \sum_{r=1}^{s} u_r y_{rp} \\ \text{s.t.} & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad \forall j, \\ & u_r, v_i \ge \varepsilon, \quad \forall r, i. \end{aligned} \tag{2}$$

Note that DMU$_p$ is efficient if $\theta_p^* = 1$, where θ_p^* is the optimal value of (2). Next, we introduce the concept of the *ideal* and *nadir* DMUs, IDMU and NDMU, respectively, followed by reviewing some models with precise data proposed by Wu [27] and Wang and Luo [28].

An IDMU is a hypothetical DMU consuming the least inputs to secure the most outputs and a NDMU is a hypothetical DMU which consumes the most inputs but produces the least outputs. The IDMU and the NDMU are defined as:

$$\begin{aligned} \text{IDMU} &= (X^{\min}, Y^{\max}) \\ \text{NDMU} &= (X^{\max}, Y^{\min}) \end{aligned} \tag{3}$$

where

$$x_i^{\min} = \min_j \{x_{ij}\}, \quad y_r^{\max} = \max_j \{y_{rj}\} \qquad \forall i, r,$$

$$x_i^{\max} = \max_j \{x_{ij}\}, \quad y_r^{\min} = \min_j \{y_{rj}\} \qquad \forall i, r.$$

The IDMU and the NDMU have the best and the worst performance, respectively, among all the DMUs. Recently, several researchers have proposed combining TOPSIS (technique for order preference by similarity to the ideal solution) for measuring qualitative performance with DEA for measuring quantitative performance [29,30]. According to TOPSIS, the favorite DMU should have the shortest distance from the ideal DMU and the farthest distance from the nadir DMU. Given n DMUs under consideration, the TOPSIS procedure can be summarized as follows:

(1) Collect the preliminary data for making the decision matrix.
(2) Determine the ideal DMU (θ_I^*) and the nadir DMU (φ_N^*).
(3) Find the distance of each DMU from the ideal DMU (θ_j^*) and the nadir DMU (φ_j^*) where $j = 1, \ldots, n$.
(4) Determine the relative closeness of each DMU to the ideal DMU. Specifically, if DMUj is under evaluation, the relative closeness is defined as:

$$RC_j = \frac{\varphi_j^*}{\varphi_j^* + \theta_j^*} \quad (j = 1, \ldots, n)$$

(5) Rank the preference order of all DMUs under consideration.

In the first phase, we determine the best relative efficiency of the ideal DMU (θ_I^*) and the worst relative efficiency of the nadir DMU (φ_N^*). Suppose θ_I^* is the best relative efficiency of the ideal DMU. We use the following fractional programming model suggested by Wang and Luo [28] to find θ_I^*:

$$\begin{aligned} \theta_I^* &= \max \frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}} \\ \text{s.t.} & \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad \forall j, \\ & u_r, v_i \ge \varepsilon, \quad \forall r, i. \end{aligned} \tag{4}$$

which can be transformed into the following LP model using the Charnes-Cooper's method:

$$\begin{aligned} \theta_I^* &= \max \sum_{r=1}^{s} u_r y_r^{\max} \\ \text{s.t.} & \sum_{i=1}^{m} v_i x_i^{\min} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad \forall j, \\ & u_r, v_i \ge \varepsilon, \quad \forall r, i. \end{aligned} \tag{5}$$

It should be noted that the efficiency of none of the DMUs can be larger than θ_I^*.

The worst efficiency of the nadir DMU denoted by φ_N^* can be determined by solving the following model as suggested by Wu

[27]:

$$\begin{aligned}
\varphi_N^* &= \min \frac{\sum_{r=1}^{s} u_r y_r^{\min}}{\sum_{i=1}^{s} v_i x_i^{\max}}\\
\text{s.t.}\ &\frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}} \geq \theta_I^*,\\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j,\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{6}$$

Note that φ_N^* is less than the efficiencies of all DMUs. Similarly, the above model can be solved through the following LP model:

$$\begin{aligned}
\varphi_N^* &= \min \sum_{r=1}^{s} u_r y_r^{\min}\\
\text{s.t.}\ &\sum_{i=1}^{m} v_i x_i^{\max} = 1,\\
&\sum_{r=1}^{s} u_r y_r^{\max} - \sum_{i=1}^{m} v_i x_i^{\min} \theta_I^* \geq 0,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad \forall j,\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{7}$$

In the second phase, the best relative efficiency of DMU$_p$ (θ_p^*) is identified using the efficiency of θ_I^*, and the worst relative efficiency of DMU$_p$ (φ_p^*) is identified using the efficiency of φ_N^*. Next, we use the following model as suggested by Wang and Luo [28] to determine the best relative efficiency of each DMU using the optimum efficiency of the IDMU,

$$\begin{aligned}
\theta_p^* &= \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}\\
\text{s.t.}\ &\frac{\sum_{r=1}^{s} u_r y_r^{\max}}{\sum_{i=1}^{m} v_i x_i^{\min}} = \theta_I^*\\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{8}$$

The purpose of model (8) is to maximize the efficiency of DMU$_p$ ($p = 1, 2, \ldots, n$) while the optimal efficiency of the IDMU is fixed by the first constraint in model (8). Next, we transform model (8) into the following LP model:

$$\begin{aligned}
\theta_p^* &= \max \sum_{r=1}^{s} u_r y_{rp}\\
\text{s.t.}\ &\sum_{i=1}^{m} v_i x_{ip} = 1,\\
&\sum_{r=1}^{s} u_r y_r^{\max} - \sum_{i=1}^{m} v_i \theta_I^* x_i^{\min} = 0,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{9}$$

Similarly, the following model suggested by Wu [27] is used to obtain the worst relative efficiency of DMU$_p$ ($p = 1, 2, \ldots, n$):

$$\begin{aligned}
\varphi_p^* &= \min \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}\\
\text{s.t.}\ &\frac{\sum_{r=1}^{s} u_r y_r^{\min}}{\sum_{i=1}^{m} v_i x_i^{\max}} = \varphi_N^*,\\
&\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \quad \forall j\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{10}$$

The purpose of model (10) is to minimize the efficiency of DMU$_p$ ($p = 1, 2, \ldots, n$) while the optimal efficiency of the NDMU is unchanged by the first constraint in model (10). Model (10) can be solved using the following linear model:

$$\begin{aligned}
\varphi_p^* &= \min \sum_{r=1}^{s} u_r y_{rp}\\
\text{s.t.}\ &\sum_{i=1}^{m} v_i x_{ip} = 1,\\
&\sum_{r=1}^{s} u_r y_r^{\min} - \sum_{i=1}^{m} v_i \varphi_N^* x_i^{\max} = 0,\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j,\\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned} \tag{11}$$

Therefore, each DMU is evaluated by two different efficiencies: the best and the worst possible relative efficiencies.

In phase 3, the efficiencies obtained from models (9) and (11) are aggregated into a combined index called the relative closeness (RC) index. The RC index is calculated for each DMU using the following equation:

$$RC_j = \frac{\varphi_j^* - \varphi_N^*}{(\varphi_j^* - \varphi_N^*) + (\theta_I^* - \theta_j^*)}, \quad \forall j \tag{12}$$

Distinctly, DMUj is closer to θ_I^* and farther from φ_N^* as RC_j approaches 1. In phase 4, we determine the ranking order of all DMUs according to their RC index.

3. The proposed method

There have been some attempts in the literature to use DEA for understanding the impacts of IT investments on performance and productivity. Banker et al. [9] used basic DEA models with statistical tests to compare the performance of Hardee's restaurants which had invested heavily in IT in those restaurants with no special investment in IT. Their DEA model showed that IT helped to reduce input material costs at Hardee's restaurants. Kauffman and Weill [12] showed how IT, primarily used for reservations in the airline industry, directly impacts market share, which in turn, together with other factors impacts profitability. The three inputs related to investments in their DEA model included: IT budget as a percentage of sales, an organization's total processor value as a percentage of sales, and the percentage of the IT budget allocated to training.

Researchers studying the indirect impact of IT on productivity, argue that investments in IT should provide information that can be used to increase revenues and/or permit the organizations to improve their operating efficiency leading to higher profitability and performance measures. After determining efficient and

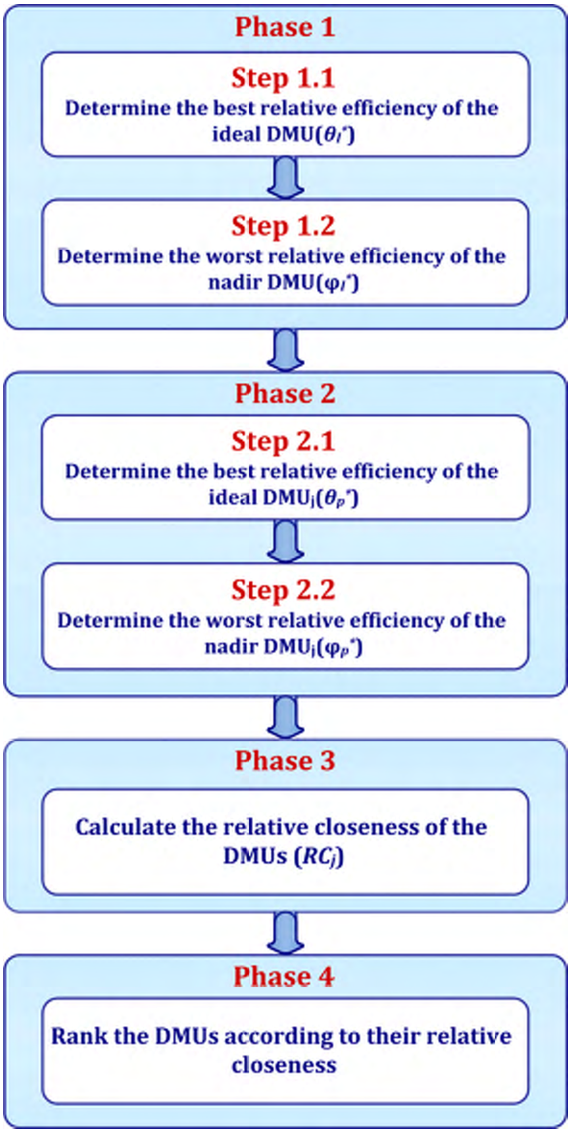

Fig. 1. The proposed framework.

inefficient organizations, these organizations can then be studied in-depth based on the input and output criteria. Such research studies should help researchers work toward the development of a comprehensive theory of IT investment and its relationship to other organizational variables. Wang et al. [17] utilized DEA to study the marginal benefits of IT with respect to a two-stage process. In their approach, they viewed firm-level outputs in the context of a series of value added activities. They identified IT effects on intermediate output variables and then related the effects of IT-produced intermediate output variables to firm performance. In an attempt to improve the former method, Chen and Zhu [10] presented a single DEA model that identified the efficient frontier of a two-stage process linked by intermediate measures.

Dasgupta et al. [11] showed some limitations of the parametric techniques in investigating the impact of IT investment on firm productivity. Shafer and Byrd [16] developed a framework for evaluating the efficiency of IT investment based on a DEA model. Their proposed methodology considered the issue of time lags between investments in IT and the accrual of benefits associated with these investments. Seiford and Zhu [14] used a two-stage DEA method and divided a commercial bank's production process into marketability and profitability stages. The marketability stage focused on three inputs (employees, assets and shareholders' equity) and two outputs (revenues and profits). The profitability stage used the outputs of the first stage (revenues and profits) as its inputs, and used market value, total investment returns and earnings per share as the outputs to measure the profitability. Zhu [19] followed up the two-stage DEA method introduced by Seiford and Zhu [14] and used it to analyze the financial efficiency of the best 500 companies ranked by Fortune magazine. Sexton and Lewis [15] used a two-stage DEA model for major league baseball and demonstrated its advantages over the standard DEA models. Their model detected inefficiencies that standard DEA models missed. Abad et al. [8] developed a two-stage multi-criteria procedure based on DEA to investigate the relationship between the financial data of stock to firm value in two consecutive steps: a predictive information step tying current financial data to future earnings, and a valuation step, tying future earnings to firm value.

Yang [18] used a two-stage DEA model to provide valuable managerial insights when assessing the impacts of operating and business strategies on the Canadian life and health insurance industry. Yang's [18] model allowed integration of the production performance and investment performance for the insurance companies and provided management with overall performance evaluation needed to achieve efficiency. Shafer and Byrd [16] proposed a DEA framework for measuring the efficiency of IT investments. Chen and Zhu [10] developed a DEA model that identified the efficient firms in a two-stage production process and measured the marginal benefits of IT on productivity. Färe and Grosskopf [31] applied a network DEA model in a similar two-stage process setting. Chen et al. [32] developed a DEA model to evaluate the impact of IT investment on multiple stages of the business process along with the information on how to distribute the IT-related resources to maximize efficiency.

In real-world situations, the representation and manipulation of inexact, incomplete, vague, ambiguous or imprecise information is a major concern. Prior to Zedeh's pioneering work in fuzzy sets, probability theory based on Boolean logic was used to deal with uncertainties (or randomness) of real events and activities. Fuzzy set theory developed by Zadeh [22] provided a valuable conceptual tool for dealing with imprecise or vague information. We propose a fuzzy DEA framework which incorporates the concept of TOPSIS to measure the efficiencies of the DMUs. The strengths of our framework are based on the following: (1) we consider uncertainty in our model, (2) we analyze the sensitivity of the DMUs in our model, and (3) we integrate the concepts of fuzzy IDMU and fuzzy NDMU in our model.

Consider n DMUs, each of which consumes m different fuzzy inputs to secure s different fuzzy outputs. Among the several types of fuzzy numbers, triangular fuzzy numbers are commonly used in managerial decision making because of their conceptual and computational simplicities. Hence, the fuzzy data are assumed to be triangular fuzzy numbers. $\tilde{x}_{ij} = (x_{ij}^M, x_{ij}^L, x_{ij}^R)$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $\tilde{y}_{rj} = (y_{rj}^M, y_{rj}^L, y_{rj}^R)$, $r = 1, 2, \ldots, s$, $j = 1, 2, \ldots, n$, denote, respectively, the fuzzy input and fuzzy output values of the jth DMU. Note that the first, second and third components represent the center, left and right point of the related triangular fuzzy number, respectively [33]. We use an input-oriented CCR model to evaluate the relative efficiency of the DMUs. The following four-phase procedure depicted in Fig. 1 is used to extend the DEA model using fuzzy inputs and outputs based on the TOPSIS method.

In the first phase, we identify the best relative efficiency of the fuzzy ideal DMU (θ_I^*) and the worst relative efficiency of the fuzzy nadir DMU (φ_N^*) using models (15) and (16) (with fuzzy input-output data). Therefore, the best relative efficiency of the fuzzy ideal DMU can be obtained by the following fuzzy LP problem:

$$
\begin{aligned}
\theta_I^* &= \max \sum_{r=1}^{s} u_r \tilde{y}_r^{\max} \\
\text{s.t.} &\sum_{i=1}^{m} v_i \tilde{x}_i^{\min} = \tilde{1}, \\
&\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le \tilde{0}, \quad \forall j \\
&u_r, v_i \ge \varepsilon, \quad \forall r, i.
\end{aligned} \tag{13}
$$

where "~" indicates fuzziness. $\tilde{x}_i^{\min}$ and $\tilde{y}_r^{\max}$ of the fuzzy IDMU in model (13) are obtained as follows:

$$
\tilde{x}_i^{\min} = \left(x_i^{M\min}, x_i^{L\min}, x_i^{R\min}\right) = \left(\min_j\{x_{ij}^M\}, \min_j\{x_{ij}^L\}, \min_j\{x_{ij}^R\}\right) \quad \forall i,
$$

$$
\begin{aligned}
\tilde{y}_r^{\max} &= \left(y_r^{M\max}, y_r^{L\max}, y_r^{R\max}\right) \\
&= \left(\max_j\{y_{rj}^M\}, \max_j\{y_{rj}^L\}, \max_j\{y_{rj}^R\}\right) \quad \forall r.
\end{aligned}
$$

The DEA literature reports on several methods for dealing with fuzzy data (see, e.g.; [34–42]). These methods can be classified into four groups: (1) the fuzzy ranking methods, (2) the defuzzification methods, (3) the tolerance methods, and (4) the α-*cut* based methods. We use the α-*cut* based approach suggested by Saati et al. [43] to define the fuzzy CCR model as a possibilistic programming problem and transform it into an interval programming problem. The resulting model can be solved as a crisp LP model with some variable substitutions to produce a crisp efficiency score for each DMU and for a given α. Moreover, this model can be solved for various values of α to monitor how the efficiency scores of the DMUs change when the possibility level α varies. It means we could show the decision maker a table with the efficiency scores of different α values for each DMU. One of the tangible advantages of such a table is providing the decision maker with the necessary information to consider the sensitivity of the results to small variations in various linguistic data. Using the approach suggested by Saati et al. [43], model (13) can be written as follows:

$$
\begin{aligned}
&\theta_{I(\alpha)}^* = \max \sum_{r=1}^{s} u_r \hat{y}_r^{\max} \\
&\text{s.t.} \sum_{i=1}^{m} v_i \hat{x}_i^{\min} = \hat{1}, \\
&\sum_{r=1}^{s} u_r \hat{y}_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \le \hat{0}, \quad \forall j, \\
&\alpha x_{ij}^M + (1-\alpha) x_{ij}^L \le \hat{x}_{ij} \le \alpha x_{ij}^M + (1-\alpha) x_{ij}^R, \quad \forall i, j, \\
&\alpha y_{rj}^M + (1-\alpha) y_{rj}^L \le \hat{y}_{rj} \le \alpha y_{rj}^M + (1-\alpha) y_{rj}^R, \quad \forall r, j, \\
&\alpha x_i^{M\min} + (1-\alpha) x_i^{L\min} \le \hat{x}_i^{\min} \le \alpha x_i^{M\min} + (1-\alpha) x_i^{R\min}, \quad \forall i, \\
&\alpha y_r^{M\max} + (1-\alpha) y_r^{L\max} \le \hat{y}_r^{\max} \le \alpha y_r^{M\max} + (1-\alpha) y_r^{R\max}, \quad \forall r, \\
&\alpha 1^M + (1-\alpha) 1^L \le \hat{1} \le \alpha 1^M + (1-\alpha) 1^R, \\
&\alpha 0^M + (1-\alpha) 0^L \le \hat{0} \le \alpha 0^M + (1-\alpha) 0^R, \\
&\hat{y}_r^{\max}, \hat{x}_i^{\min}, \hat{y}_{rj}, \hat{x}_{ij}, \hat{1}, \hat{0} \ge 0, \quad \forall r, i, j, \\
&u_r, v_i \ge \varepsilon, \quad \forall r, i.
\end{aligned} \tag{14}
$$

The right-hand side of the first constraint in model (14) must be equal to 1 because of the normalization of the efficiency scores of the DMUs. Note that model (14) is a non-linear programming model because of the first two constraints. Next, we transform the non-linear model to a linear programming model through the following substitutions:

$$
\begin{aligned}
&\bar{x}_{ij} = v_i \hat{x}_{ij} \quad, \bar{y}_{rj} = u_r \hat{y}_{rj}, \quad \forall i, j, r, \\
&\bar{x}_i^{\min} = v_i \hat{x}_i^{\min} \quad, \bar{y}_r^{\max} = u_r \hat{y}_r^{\max}, \quad \forall i, j, r.
\end{aligned}
$$

With the above transformations, model (14) is finally transformed into the following LP model:

$$
\begin{aligned}
&\theta_{I(\alpha)}^* = \max \sum_{r=1}^{s} \bar{y}_r^{\max} \\
&\text{s.t.} \sum_{i=1}^{m} \bar{x}_i^{\min} = 1, \\
&\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} \le 0, \quad \forall j, \\
&\xi_x^L \le \bar{x}_{ij} \le \xi_x^U, \quad \forall i, j, \\
&\xi_y^L \le \bar{y}_{rj} \le \xi_y^U, \quad \forall r, j, \\
&\psi_x^L \le \bar{x}_i^{\min} \le \psi_y^L, \quad \forall i, \\
&\psi_y^L \le \bar{y}_r^{\max} \le \psi_y^L, \quad \forall r, \\
&\bar{y}_r^{\max}, \bar{x}_i^{\min}, \bar{y}_{rj}, \bar{x}_{ij} \ge 0, \quad \forall r, i, j, \\
&u_r, v_i \ge \varepsilon, \quad \forall r, i.
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
&\xi_x^L = v_i \left(\alpha x_{ij}^M + (1-\alpha) x_{ij}^L\right), \quad \xi_x^U = v_i \left(\alpha x_{ij}^M + (1-\alpha) x_{ij}^R\right) \\
&\xi_y^L = u_r \left(\alpha y_{rj}^M + (1-\alpha) y_{rj}^L\right), \quad \xi_y^U = u_r \left(\alpha y_{rj}^M + (1-\alpha) y_{rj}^R\right) \\
&\psi_x^L = v_i \left(\alpha x_i^{M\min} + (1-\alpha) x_i^{L\min}\right) \\
&\psi_x^L = v_i \left(\alpha x_i^{M\min} + (1-\alpha) x_i^{R\min}\right) \\
&\psi_y^L = u_r \left(\alpha y_r^{M\max} + (1-\alpha) y_r^{L\max}\right) \\
&\psi_y^L = u_r \left(\alpha y_r^{M\max} + (1-\alpha) y_r^{R\max}\right)
\end{aligned}
$$

Model (15) is a parametric linear programming problem, with $\alpha \in [0,1]$ being a parameter. Model (15) provides an evaluation of the efficiency of the fuzzy IDMU with different α in [0,1], which we denote by $\theta_{I(\alpha)}^*$. A similar minimization model may be solved to determine the worst relative efficiency of the fuzzy nadir DMU as follows:

$$
\begin{aligned}
&\varphi^*_{N(\alpha)} = \min \sum_{r=1}^{s} \bar{y}_r^{\min} \\
&\quad \text{s.t.} \sum_{i=1}^{m} \bar{x}_i^{\max} = 1, \\
&\quad \sum_{r=1}^{s} \bar{y}_r^{\max} - \sum_{i=1}^{m} \theta^*_{I(\alpha)} \bar{x}_i^{\min} \geq 0, \\
&\quad \sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \quad \forall j, \\
&\quad \xi_x^L \leq \bar{x}_{ij} \leq \xi_x^U, \quad \forall i, j, \\
&\quad \xi_y^L \leq \bar{y}_{rj} \leq \xi_y^U, \quad \forall r, j, \\
&\quad \psi_x^L \leq \bar{x}_i^{\min} \leq \psi_y^L, \quad \forall i, \\
&\quad \psi_y^L \leq \bar{y}_r^{\max} \leq \psi_y^L u_r, \quad \forall r, \\
&\quad \vartheta_x^L \leq \bar{x}_i^{\max} \leq \vartheta_x^U, \quad \forall i, \\
&\quad \vartheta_y^L \leq \bar{y}_r^{\min} \leq \vartheta_y^U, \quad \forall r, \\
&\bar{y}_r^{\min}, \bar{x}_i^{\max}, \bar{y}_r^{\max}, \bar{x}_i^{\min}, \bar{y}_{rj}, \bar{x}_{ij} \geq 0, \quad \forall r, i, j, \\
&u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned}
\tag{16}
$$

In model (16), ϑ_x^L, ϑ_x^U, ϑ_y^L and ϑ_y^U are defined as:

$$
\begin{aligned}
\vartheta_x^L &= v_i \left(\alpha x_i^{M\max} + (1-\alpha) x_i^{L\max} \right) \\
\vartheta_x^U &= v_i \left(\alpha x_i^{M\max} + (1-\alpha) x_i^{R\max} \right) \\
\vartheta_y^L &= u_r \left(\alpha y_r^{M\min} + (1-\alpha) y_r^{L\min} \right) \\
\vartheta_y^U &= u_r \left(\alpha y_r^{M\min} + (1-\alpha) y_r^{R\min} \right)
\end{aligned}
$$

Moreover, $\tilde{x}_i^{\max}$ and $\tilde{y}_r^{\min}$ of the fuzzy NDMU are obtained as follows:

$$
\begin{aligned}
\tilde{x}_i^{\max} &= \left(x_i^{M\max}, x_i^{L\max}, x_i^{R\max} \right) \\
&= \left(\max_j \{x_{ij}^M\}, \max_j \{x_{ij}^L\}, \max_j \{x_{ij}^R\} \right), \quad \forall i, \\
\tilde{y}_r^{\min} &= \left(y_r^{M\min}, y_r^{L\min}, y_r^{R\min} \right) \\
&= \left(\min_j \{y_{rj}^M\}, \min_j \{y_{rj}^L\}, \min_j \{y_{rj}^R\} \right), \quad \forall r.
\end{aligned}
$$

Note that ξ_x^L, ξ_x^U, ξ_y^L, ξ_y^U, ψ_x^L, ψ_x^U, ψ_y^L and ψ_y^U in the above model are defined in model (15). Therefore, the optimal value of model (16) determines the worst relative efficiency for a given $\alpha \in [0,1]$, which we denote by $\varphi^*_{N(\alpha)}$.

In the second phase, the best efficiency score of DMU$_p$ is evaluated while the best relative efficiency is kept fixed for a given α. Furthermore, the worst efficiency score of DMU$_p$ is assessed while the worst relative efficiency remains unchanged for a given α.

When input-output data are triangular fuzzy numbers, the best relative efficiency of DMU$_p$ ($p = 1, 2, \ldots, n$) may be determined by:

$$
\begin{aligned}
&\theta^*_{p(\alpha)} = \max \sum_{r=1}^{s} \bar{y}_{rp} \\
&\quad \text{s.t.} \sum_{i=1}^{m} \bar{x}_{ip} = 1, \\
&\quad \sum_{r=1}^{s} \bar{y}_r^{\max} - \sum_{i=1}^{m} \theta^*_{I(\alpha)} \bar{x}_i^{\min} = 0, \\
&\quad \sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \quad \forall j, \\
&\quad \xi_x^L \leq \bar{x}_{ij} \leq \xi_x^U, \quad \forall i, j, \\
&\quad \xi_y^L \leq \bar{y}_{rj} \leq \xi_y^U, \quad \forall r, j, \\
&\quad \psi_x^L \leq \bar{x}_i^{\min} \leq \psi_y^L, \quad \forall i, \\
&\quad \psi_y^L \leq \bar{y}_r^{\max} \leq \psi_y^L, \quad \forall r, \\
&\quad \bar{y}_r^{\max}, \bar{x}_i^{\min}, \bar{y}_{rj}, \bar{x}_{ij} \geq 0, \quad \forall r, i, j, \\
&\quad u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned}
\tag{17}
$$

where $\theta^*_{I(\alpha)}$ is obtained by model (15) for a given α.

The worst relative efficiency of each DMU can be determined through the following model:

$$
\begin{aligned}
&\varphi^*_{p(\alpha)} = \min \sum_{r=1}^{s} \bar{y}_{rp} \\
&\quad \text{s.t.} \sum_{i=1}^{m} \bar{x}_{ip} = 1, \\
&\quad \sum_{r=1}^{s} \bar{y}_r^{\min} - \sum_{i=1}^{m} \varphi^*_{N(\alpha)} \bar{x}_i^{\max} = 0, \\
&\quad \sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \quad \forall j, \\
&\quad \xi_x^L \leq \bar{x}_{ij} \leq \xi_x^U, \quad \forall i, j, \\
&\quad \xi_y^L \leq \bar{y}_{rj} \leq \xi_y^U, \quad \forall r, j, \\
&\quad \vartheta_x^L \leq \bar{x}_i^{\max} \leq \vartheta_x^U, \quad \forall i, \\
&\quad \vartheta_y^L \leq \bar{y}_r^{\min} \leq \vartheta_y^U, \quad \forall r, \\
&\quad \bar{y}_{rj}, \bar{x}_{ij}, \bar{x}_i^{\max}, \bar{y}_r^{\min} \geq 0, \quad \forall r, i, j, \\
&\quad u_r, v_i \geq \varepsilon, \quad \forall r, i.
\end{aligned}
\tag{18}
$$

where $\varphi^*_{N(\alpha)}$ is obtained by model (16) for a given α.

Table 1
The numerical example.

DMU	Inputs		Outputs	
	1	2	1	2
A	(4,3.5,4.5)	(2.1,1.9,2.3)	(2.6,2.4,2.8)	(4.1,3.8,4.4)
B	(2.9,2.9,2.9)	(1.5,1.4,1.6)	(2.2,2.2,2.2)	(3.5,3.3,3.7)
C	(4.9,4.4,5.4)	(2.6,2.2,3)	(3.2,2.7,3.7)	(5.1,4.3,5.9)
D	(4.1,3.4,4.8)	(2.3,2.2,2.4)	(2.9,2.5,2.3)	(5.7,5.5,5.9)
E	(6.5,5.9,7.1)	(4.1,3.6,4.6)	(5.1,4.4,5.8)	(7.4,6.5,8.3)
Fuzzy IDMU	(2.9,2.9,2.9)	(1.5,1.4,1.6)	(5.1,4.4,5.8)	(7.4,6.5,8.3)
Fuzzy NDMU	(6.5,5.9,7.1)	(4.1,3.6,4.6)	(2.2,2.2,2.2)	(3.5,3.3,3.7)

Table 2
The Guo and Tanaka [36] results.

α	A	B	C	D	E
0	(0.81,0.66,0.99)	(0.98,0.88,1.09)	(0.82,0.60,1.12)	(0.93,0.71,1.25)	(0.79,0.61,1.02)
0.5	(0.83,0.75,0.92)	(0.97,0.94,1.00)	(0.83,0.71,0.97)	(0.97,0.85,1.12)	(0.82,0.72,0.93)
0.75	(0.84,0.80,0.88)	(0.99,0.96,1.02)	(0.83,0.77,0.90)	(0.98,0.92,1.05)	(0.83,0.78,0.89)
1	0.85	1.00	0.86	1.00	1.00

Let $\theta^*_{p(\alpha)}$ and $\varphi^*_{p(\alpha)}$ be the best and the worst possible relative efficiencies of each DMU for a given α, respectively. These two distinctive efficiency assessments may lead to completely different results. Hence, it is necessary to consider them together to give an overall assessment of each DMU for a given α.

In the third phase, we calculate the relative closeness of each DMU as follows:

$$RC_{p(\alpha)} = \frac{\varphi^*_{p(\alpha)} - \varphi^*_{N(\alpha)}}{\left(\varphi^*_{p(\alpha)} - \varphi^*_{N(\alpha)}\right) + \left(\theta^*_{I(\alpha)} - \theta^*_{p(\alpha)}\right)}, \quad \forall j \tag{19}$$

A larger difference between $\varphi^*_{p(\alpha)}$ and $\varphi^*_{N(\alpha)}$ and a smaller difference between $\theta^*_{p(\alpha)}$ and $\theta^*_{I(\alpha)}$ mean a better performance for DMU$_p$ ($p = 1, 2, \ldots, n$). Finally, in the fourth phase, we determine the ranking order of the DMUs according to their $RC_{p(\alpha)}$ for each $\alpha \in [0,1]$.

4. Numerical example

In this section, we present a numerical example to demonstrate the applicability of the proposed framework and exhibit the efficacy of the procedures and algorithms. This example was originally introduced by Guo and Tanaka [36]. Lertworasirikul et al. [37] also used this example to illustrate their approach. Five DMUs are considered with two inputs and two outputs, where all the input and output data are symmetrical triangular fuzzy numbers as listed in Table 1.

Guo and Tanaka [36] provide the efficiency score of the DMUs for different α-cuts in Table 2. They show that $S_0 = \{B,C,D,E\}$, $S_{0.5} = \{B,D\}$, $S_{0.75} = \{B,D\}$ and $S_1 = \{B,D,E\}$ are the nondominated sets for different α values. Note that a DMU is said to be α-possibilistic efficient if the maximum value of the fuzzy efficiency at that α level is greater than or equal to 1. The set of all possibilistic efficient DMUs is called the α-possibilistic nondominated set, denoted by S_α.

For comparison, in Table 3 we present the results obtained by using the possibility approach developed by Lertworasirikul et al. [37]. As shown in Table 3, the possibilistic efficiency values of DMU C and DMU D are 0.861 and 1 at the possibility level 1, respectively. Moreover, DMUs B, D and E are possibilistically efficient at all possibility levels, whereas DMUs A and C are possibilistically efficient only at some possibility levels. In their paper, a DMU becomes α-possibilistic efficient if its objective value is greater than or equal to 1 at the specified α level.

We now examine this numerical example using the framework proposed in this study. The fuzzy IDMU and fuzzy NDMU are shown in the last two rows of Table 1. The best efficiency of the fuzzy IDMU ($\theta^*_{I(\alpha)}$) and the worst efficiency of the fuzzy NDMU ($\varphi^*_{N(\alpha)}$) are obtained by using models (15) and (16) for four different possibility levels (0, 0.5, 0.75, 1), which are shown in the last column of Tables 4 and 5. Then, models (17) and (18) are used to obtain the best efficiency ($\theta^*_{p(\alpha)}$) and the worst efficiency ($\varphi^*_{p(\alpha)}$) scores for each DMU, which are also given in Tables 4 and 5.

Table 3
The Lertworasirikul et al. [37] results.

α	A	B	C	D	E
0	1.107	1.238	1.276	1.520	1.296
0.5	0.963	1.112	1.035	1.258	1.159
0.75	0.904	1.055	0.932	1.131	1.095
1	0.855	1.000	0.861	1.000	1.000

Table 4
The best efficiency scores of the DMUs.

α	A	B	C	D	E	$\theta^*_{I(\alpha)}$
0	1	1	1	1	1	3.0130
0.5	0.9511	1	1	0.9707	0.9973	2.6481
0.75	0.899	1	0.9265	0.9168	0.9252	2.4895
1	0.8548	1	0.8575	0.9231	1	2.3182

Finally, the relative closeness values and the ranking of the DMUs for a given $\alpha \in [0,1]$ are presented in Tables 6 and 7.

As an example, let us consider units B and C in Table 4. These DMUs have an efficiency score of 1 according to the fuzzy IDMU for $\alpha = 0.5$. On the other hand, in Table 5, if we evaluate the performance of the units with respect to the fuzzy NDMU, we can see that only DMU B has the best performance among the DMUs for $\alpha = 0.5$. As shown in Table 7, when these results are merged with the relative closeness indices, a complete ranking for the DMUs is achieved.

In Table 7, DMU B is possibilistically efficient at all possibility levels and DMU E has the second ranking order at some α-cuts. When using Guo and Tanaka's approach, DMUs B and D fall in all α-possibilistic nondominated sets. Moreover, when using Lertworasirikul et al.'s approach, DMUs B, D and E are always efficient.

Tables 6 and 7 present a comprehensive evaluation of the five DMUs for different values of α, which are also portrayed in Fig. 2. Fig. 2 is useful for identifying sensitive DMUs in the problem. Table 6 shows that as the value of α increases, the RC index becomes larger. In fact, decreasing α results in more opportunities for some DMUs.

Table 5
The worst efficiency scores of the DMUs.

α	A	B	C	D	E	$\varphi^*_{N(\alpha)}$
0	0.4684	0.6311	0.4417	0.4858	0.4782	0.3478
0.5	0.5703	0.7195	0.5600	0.6093	0.6088	0.3563
0.75	0.6321	0.7706	0.6310	0.6838	0.6883	0.3621
1	0.6964	0.8200	0.7059	0.6883	0.4782	0.3659

Table 6
The relative closeness of the DMUs.

α	Relative closeness				
	A	B	C	D	E
0	0.0565	0.1234	0.0445	0.0642	0.0608
0.5	0.1120	0.1806	0.1100	0.1310	0.1327
0.75	0.1451	0.2152	0.1468	0.1698	0.1725
1	0.1843	0.2563	0.1888	0.2223	0.2362

Table 7
The final ranking order of the DMUs.

α	Ranking order				
	A	B	C	D	E
0	4	1	5	2	3
0.5	4	1	5	3	2
0.75	5	1	4	3	2
1	5	1	4	3	2

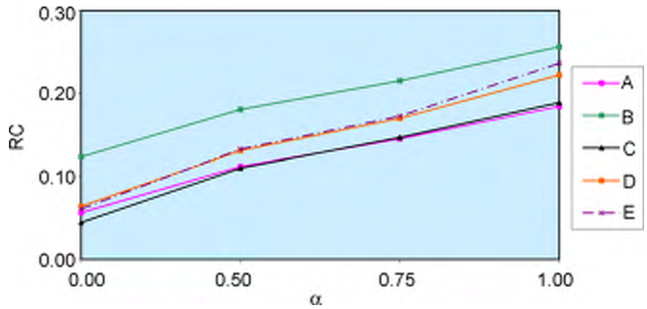

Fig. 2. A comprehensive analysis of the four DMUs.

Beside this feature of α, a decision maker shows his/her attitude on uncertainty.

5. Conclusions and future research directions

IT managers have recently come under a great deal of scrutiny to show the impact of IT investment on firm productivity and profitability. Therefore, it is incumbent upon them to effectively assess IT investment projects. DEA is a methodology for measuring the relative efficiencies of a set of DMUs that use multiple inputs to produce multiple outputs. Due to its solid underlying theoretical basis and wide application to real-world problems, DEA has been widely used to assess IT investment projects. While crisp input and output data are fundamentally indispensable in the traditional DEA evaluation process, input and output data in real-world problems are often imprecise or ambiguous. Fuzzy logic and fuzzy sets can represent ambiguous, uncertain or imprecise information in DEA by formalizing inaccuracy in human decision making.

In this paper, the concept of the TOPSIS is integrated into a four-phase fuzzy DEA framework to measure the efficiencies of a set of DMUs and rank them with fuzzy input–output levels. The basic idea is to introduce two hypothetical DMUs, the fuzzy ideal DMU and the fuzzy nadir DMU. In the first phase, the best relative efficiency of the fuzzy ideal DMU and the worst relative efficiency of the fuzzy nadir DMU are identified. These efficiencies are measured by using the optimal efficiency of the fuzzy ideal and nadir DMUs in the second phase. Subsequently, an integrated index called the RC index in the third phase is constructed and used to develop a ranking order of all DMUs in phase 4. The proposed framework is generic, structured and comprehensive and can be applied to analyze various DMU evaluation problems in fuzzy environments.

The application of the fuzzy DEA framework to hierarchical structures is an important area for future research. Many organizational problems tend to exhibit such a profile. The framework developed in this study can potentially lend itself to other areas of study, such as supply chain management. The principles are also somewhat related to the concepts and structures studied in the network DEA model of Färe and Grosskopf [31].

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