# A multiple criteria decision support system for autonomous underwater vehicle mission planning and control

Madjid Tavana*

Management Information Systems,
Lindback Distinguished Chair of Information Systems,
La Salle University,
Philadelphia, PA 19141, USA
E-mail: tavana@lasalle.edu
Website: http://lasalle.edu/~tavana
*Corresponding author

Brian S. Bourgeois

Naval Research Laboratory,
Marine Geosciences Division,
Stennis Space Center, MS 39529, USA
E-mail: bsb2@nrlssc.navy.mil

**Abstract:** The growing use of autonomous air, surface, ground and underwater systems is continually demonstrating new military and commercial possibilities and applications. The state of the art in the planning and control of autonomous underwater vehicles (AUVs) is largely precarious because AUVs provide infrequent feedback, operate autonomously for long periods of time and yet have little knowledge of their dynamic environment. Consequently, mission planning and control is typically conducted based on human expert knowledge of vehicle capabilities, some level of observed environmental conditions and *ad hoc* optimisation with little assistance from computers. While the human expert offers a significant ability to mentally process data, the result typically lacks numeric and quantitative analysis of alternatives. Navigation of AUVs in the complex ocean environment involves time dependent dynamics, resulting in a problem that is computationally prohibitive for the use of brute force optimisation techniques. Although some research has been conducted for specific types of missions, and 'greedy' global-optimisation approaches have been investigated, no systematic and coherent approach to the requirement exists. We propose a dynamic multiple criteria decision support system that considers dynamic and episodic ocean phenomenon to provide reasonable and in-context recommendations with respect to the stated objective and subjective mission goals. Multi-criteria decision analysis, analytic network process and fuzzy sets are used in the model to reduce the vehicle routing solution space and maximise time-on station in adverse environments. The proposed system can be an added hierarchical layer on the top of a mission planning system currently under development by the United States Navy.

**Keywords:** ANP; analytic network process; AUV; autonomous underwater vehicles; DSS; decision support systems; fuzzy sets; geographical information system; mission planning and control; MCDA; multi-criteria decision analysis; vehicle routing.

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**Biographical notes:** Madjid Tavana is a Professor of Management Information Systems and the Lindback Distinguished Chair of Information Systems at the La Salle University, where he served as the Chairman of the Management Department and Director of the Center for Technology and Management. He has been a Distinguished Faculty Fellow at the NASA's Kennedy Space Center, NASA's Johnson Space Center, Naval Research Laboratory – Stennis Space Center and Air Force Research Laboratory. In 2005, he was awarded the prestigious Space Act Award by NASA. He holds an MBA, a PMIS and a PhD in Management Information Systems. He received his PostDoctoral Diploma in Strategic Information Systems from the Wharton School of the University of Pennsylvania. He has published in journals such as *Decision Sciences*, *Interfaces*, *Information Systems*, *Information and Management*, *Computers and Operations Research*, *Journal of the Operational Research Society* and *Advances in Engineering Software*, among others.

Brian Bourgeois is a Section Head in the Geosciences Division of the Naval Research Laboratory (NRL) at the Stennis Space Center. He holds a PhD in Electrical Engineering from Tulane University. His areas of research have spanned bathymetry sonar systems, unmanned underwater vehicles, underwater navigation systems, automated vehicle navigation and most recently GIS-based mission planning systems. He has received three NRL technology transition awards and the Berman publication award, has five patents and has published over 50 journal and conference papers. He is a senior member of IEEE and is on the editorial board of the *Int. J. Strategic Decision Sciences* and *Int. J. Applied Decision Sciences*.

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## 1 Introduction

We know more about the moon than we do about our own planet and the environment that covers over two-thirds of its surface. Observing the ocean is a challenging endeavour, made difficult by a wide range of time and environmental conditions and constraints. In recent years, autonomous underwater vehicles (AUVs) are being used to help conquer that challenge (Roemmich et al., 2004; Rudnick et al., 2004).

The growing use of autonomous air, surface, ground and underwater systems is continually demonstrating new military and commercial possibilities and applications. Underwater vehicles are used primarily by scientists as oceanographic tools to navigate autonomously and map features of the ocean. Government forces use AUVs for global war on terrorism. The United States Navy uses underwater vehicles to safely gain access to denied areas with revolutionary sensors and weapons for mine warfare; intelligence, surveillance, and reconnaissance; undersea environmental sensing and mapping.

The oil and gas industry uses AUVs to make detailed maps of the seafloor before building subsea infrastructure. These detailed maps allow cost effective installation of

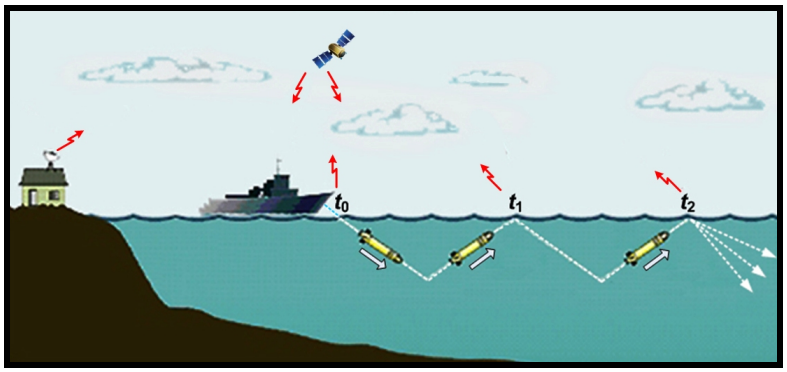
pipelines and communication cables with minimum disruption to the environment. Mining companies have long known the world's oceans and seas cover vast troves of commodities currently in high demand. However, unlike oil and gas companies, which have operated offshore for decades, miners lacked the technology to search the ocean floor and haul their bounty to the surface. Now, the global boom in commodity prices has encouraged mining companies to utilise AUVs for undersea mining. Other applications of underwater vehicles include: search for fish, surveying underwater pipes and cables, search and rescue operations, coastal drug control, tracking of pollutant sources and ocean floor studies prior to sewage outlet construction.

Most underwater vehicles in use today have limited energy powered by rechargeable batteries. Some vehicles use primary batteries which provide twice the endurance at a substantial extra cost per mission. Underwater gliders are a very low-power class of AUVs capable of operating unattended for weeks or months in littoral and open ocean areas in conjunction with surface vessels for navigational purposes. For gliders, energy use is highly dependent upon mission goals and environmental conditions. The depth, data sampling requirements and dive cycle period predominately determine the energy budget for a dive cycle (Eriksen et al., 2001). Gliders profile vertically by controlling buoyancy and move horizontally using wings. They employ energy-efficient buoyancy engines that alter the vehicles' displacement and thus its buoyancy relative to the water. Navigation control is achieved through the use of a rudder or the shifting of internal weights to alter pitch and roll. Each glider is equipped with global positioning system (GPS) and a phone/modem. With these devices, the glider knows its position from GPS when floating at the surface and also it can communicate to a computer through a satellite connection. This two-way communication allows the AUV operator to download measurement data from the glider and upload new instructions. With all these technologies, it is still not possible to escape the need for a launch and recovery (L&R) vehicle in the area of interest due to their very slow horizontal speeds (on the order of 0.25 m $sec^{-1}$).

Underwater vehicles operating autonomously relay data by satellite. However, GPS signals do not travel through water and gliders are required to periodically surface as programmed or needed for recalibration and obtaining precise position fixes before diving back down to continue their missions. These recalibration operations result in a reduction in mission efficiency and efficacy (DeAngelis and Whitney, 2000). The need for real-time navigation and guidance data requires that the glider accurately maintains its absolute and relative positional information. Absolute position refers to a globally unique location (the vehicle's latitude, longitude and depth) and the relative position refers to the location of the vehicle in the dive cycle with respect to its surroundings.

The glider begins a typical dive cycle by becoming slightly negatively buoyant and pitching down to leave the sea surface. As the antenna mast sinks below the surface, the glider attains its most extreme downward pitch. When the glider detects a depth greater than the target depth, it becomes slightly positively buoyant and pitches up to rise. Close to the sea surface, the glider crosses another user-specified depth threshold before pitching down again. This completes a dive cycle (see Figure 1). During each cycle, gliders have assigned tasks that may involve travelling a certain path and collecting information about the areas travelled.

**Figure 1** Dive cycle (see online version for colours)

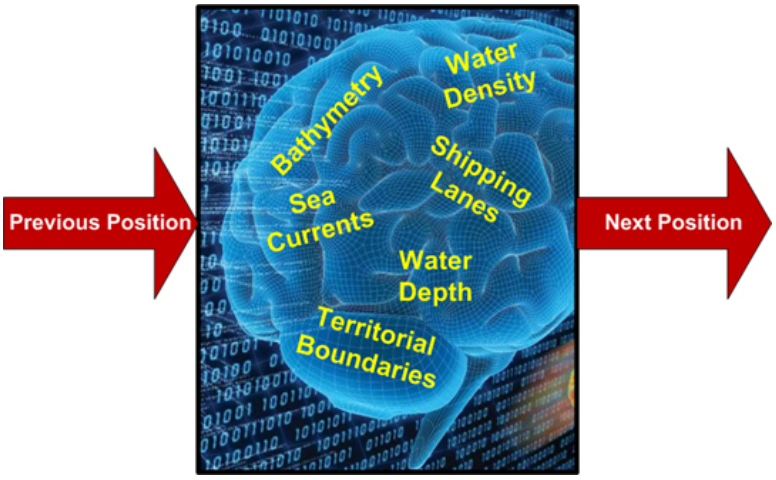



Mission planning for underwater vehicles requires the consideration of a multitude of static environmental factors such as bathymetry and shipping lanes and dynamic environmental factors such as weather, water density and sea currents. Depth considerations include having sufficient clearance to swim safely below ships, the minimum depth required for safe operation, the maximum pressure design of a vehicle, the ability to use acoustic velocity logs for navigation and bottom obstacles such as ship wrecks, reefs and shoals. Currents impact the time and power required for a vehicle to make a transit or stay-on station. Currents that are greater than the vehicles' maximum speed can deter mission feasibility. Water density can impact the maximum depth a vehicle can achieve and its ability to re-surface.

The mission planners must also consider factors ahead of time that would normally be handled by a human pilot with a paper or electronic chart during a mission. Most of these factors are boundaries or navigation hazards such as territorial and economic boundaries, shipping lanes, anchorages, buoys, oil rigs, naval exclusion zones, etc. Weather (winds, visibility), sun and moon rise/set and sea state must also be considered as they impact vehicle L&R and can potentially affect a vehicle's ability to surface and communicate with its host ship or command centre.

The current static and data-driven 'human-centric' mission planning and control system, as depicted in Figure 2, is conducted at a tactical level and lacks agility. This planning and control relies heavily on the execution of pre-planned actions limiting their effectiveness for measuring dynamic and episodic ocean phenomenon. Throughout the mission, gliders receive new directions from the human operator on the surface or ashore that change the original mission. The plethora of data that must be considered, much of which varies with time as well as position, puts a tremendous amount of cognitive burden on operators. While on the surface, there is little or no time for real-time data processing by a human operator. Consequently, the revised plan is usually based on the data transmitted by the vehicle during the previous re-surfing. The situation considered in Figure 1 shows a vehicle moving through time and space (ocean), where periodically, at fixed time intervals, a decision is made as to how to next control the vehicle. As shown in this figure, the next decision occurs at time $t_2$, as the decision-making process is conducted in the time interval $[t_1; t_2]$ considering data for the time interval $[t_0; t_1]$ transmitted by the vehicle during the resurfacing at $t_1$. There is a one period lag time in the current mission planning system.

**Figure 2** Current static 'human-centric' mission planning (see online version for colours)

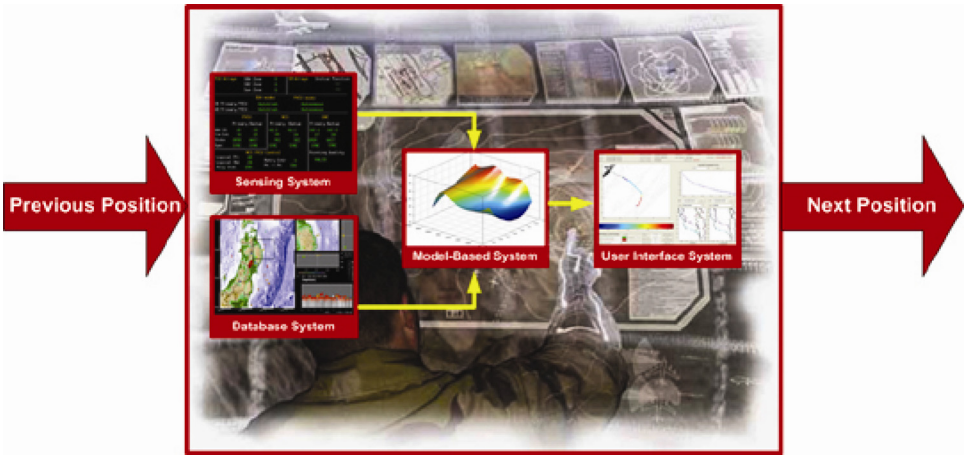



The computational tools currently used by human operators are all too often a calculator and a note pad. Computer displays data while the human operator is interpreting data, analysing alternative courses of action and developing plans. The common practice lacks adequate geospatial software to assist with mission planning. Creation, optimisation and execution of plans are entirely dependent on the mind of experienced operators. While the human expert offers a significant ability to mentally process data, the result typically lacks numeric and quantitative analysis of alternatives.

We propose a dynamic decision support system (DSS) for mission planning and control in realistic environments with real-world operational constraints and vehicle characteristics. Within constrained or simplified environment assumptions, various formal techniques have been developed, but they typically fall short of satisfying necessary operational capabilities. The proposed DSS, depicted in Figure 3, is comprised of four interrelated systems organising data into information and synthesising information into knowledge. The model-based component is based on a weighted-sum multi-criteria decision analysis (MCDA) model using the analytic network process (ANP) and fuzzy sets. The database component provides static factors such as bathymetry, territorial boundaries and shipping lanes and the sensing component provides the dynamic factors such as water depth and density and sea currents. The user interface system provides an interactive, transparent and user friendly manipulation of the DSS. This automated system does not replace the human operators, but reduces the cognitive burden on them since missions will now be specified as high-level goals and constraints rather than detailed task and behaviour sequences.

The remainder of the article is organised as follows. Section 2 reviews the most common AUV planning and control methods followed by a detailed description of our MCDA model in Section 3. In Section 4, we illustrate an AUV mission planning problem and in Section 5 we present our conclusions and future research directions.

**Figure 3** Mission planning decision support system (see online version for colours)




## 2 AUV planning and control methods

Navigation of autonomous vehicles in dynamic environments requires implementation of efficient real-time planning and control systems that imitate the way humans are operating manned or similar vehicles. Considering the underwater environmental uncertainties that are difficult to model, fuzzy logic, genetic algorithms and MCDA models have been used to plan and control AUV missions. The applicability of fuzzy logic in autonomous navigation is mainly required because a mathematical model of the dynamics of the vehicle is not needed (Driankov and Saffiotti, 2001; Kanakakis, Valavanis and Tsourveloudis, 2004; Loebis et al., 2004; Valavanis, 2006). Another reason that explains the popularity of fuzzy logic in autonomous navigation is the low computation time requirements.

Genetic algorithm is another approach commonly used for AUV navigation in large but static environments (Yanfei, 2000). Genetic algorithm searches the solution from a population of points, but requires effective memory management since the solution space is large. These algorithms have been shown in practice to be effective at efficiently searching large (multi-modal, discontinuous, etc.) spaces to find nearly global optima for AUV navigation in static environments (Zhang, 2006).

MCDA is the third approach commonly used for modelling and simulation of autonomous vehicles. Jian, Zheping and Xinqian (2007) developed a model for AUV decision-making with AHP where distance, safety, energy, time, uncertainty and proportion of finished mission were used to build a hierarchical weighted-sum model. However, their model assumed operations in a static environment with no dependency among the consideration factors. Tavana, Bailey and Busch (2008) proposed a dynamic MCDA model that considered four competing objectives (effort, effectiveness, efficiency and connectivity) to assess vehicle-target allocation and navigation of autonomous vehicles with entropy and AHP. While their model was dynamic, they also assumed no dependency among the four competing factors. The model proposed in this study is a dynamic MCDA model that allows dependencies and interdependencies among a comprehensive set of AUV mission planning and control factors.

## 3 Multi-criteria decision analysis model and procedure

The mission planning DSS developed in this study is a combined hierarchical/network model with multiple hierarchies and interdependency loops. As depicted in Figure 4, the DSS provides mission planners with the navigation points and depths; given the present vehicle state and sampling objectives and based on a series of objective and subjective goal factors (i.e. water density, sea current, safety, priority, etc.). The navigation points are represented by a vector of velocity $v_t = [x_t\ y_t\ z_t]$ and time ($t$, measured in minutes since start of mission). Three buoys: $x$, $y$ and $z$ (latitude, longitude and depth) represent the horizontal and vertical positions of the vehicle. Latitude and longitude are measured in degrees and depth is measured in meters (zero at sea level).

We consider a comprehensive set of mission planning and control factors and classify them into environmental and non-environmental factors. Environmental factors include static and dynamic factors. As shown in Figure 5, static factors are divided into man-made boundaries and natural boundaries. Dynamic factors are grouped into weather, water density, moving boundaries and sea current. Non-environmental factors are divided into safety, priority and cost. Cost factors include L&R and time-off station. The solid lines in Figure 5 represent the hierarchical dependencies and the dotted lines represent the network dependencies among the factors considered in this study. All interdependencies in this figure are one way except for the dependency between priority and L&R cost shown by a double-headed arrow.

Man-made maritime boundaries include such things as the 12 mile territorial limit, the 200 mile economic exclusion zone, shipping lanes, anchorages, etc. These boundaries impact where a vehicle is allowed to legally and safely operate. For example, we would want to avoid working in shipping lanes and anchorages, and we would only operate within another country's territorial waters with explicit permission.

Natural boundaries in the ocean are primarily bathymetry (shape of the seafloor) including obstacles such as reefs, wrecks, etc. Near shore, this would also include the tide since tides can significantly influence actual water depth at a specific location. This factor impacts where an underwater vehicle is capable of operating safely. For a glider, sufficient depth of water is required to conduct its up/down transits, and a depth buffer below shipping is also desired.

**Figure 4** Mission planning system (see online version for colours)

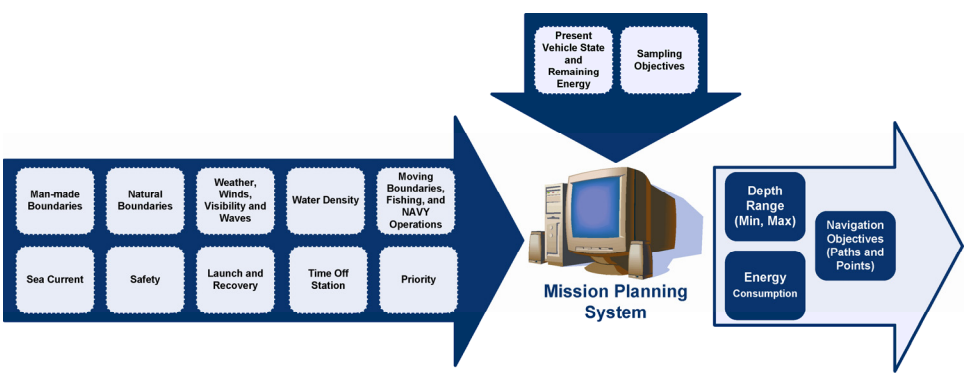
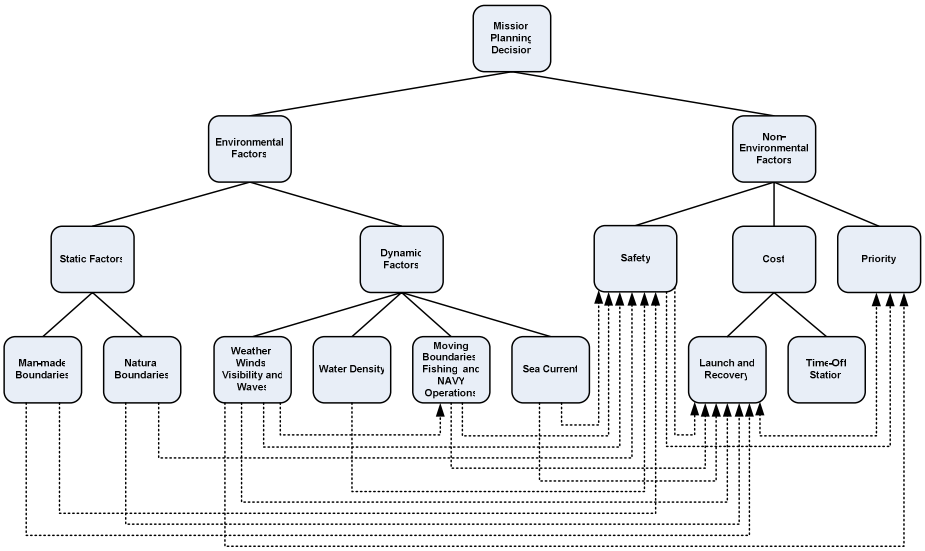


**Figure 5** Mission planning network (see online version for colours)




Weather includes conditions such as winds, wave heights, humidity, rain, fog, sunset/rise and moonset/rise. These factors affect visibility which is a primary concern for L&R, as is wind and waves which impact safety during L&R. Thunderstorms and heavy seas may also impact the ability of an AUV to communicate or obtain GPS fixes with radio while on the surface.

Water density impacts the ability of AUVs to descend and ascend. A vehicle can go no deeper than its maximum density, or no shallower than its minimum density. Underwater gliders have active buoyancy control, but have limited buoyancy range. A common density hazard for AUVs is river fresh water outflows or large amounts of rainfall that lower the surface density of the water to the extent that the vehicle cannot surface to communicate and obtain a GPS fix. Likewise, if the water mass is denser than expected, this could prohibit the AUV from descending to the desired mission depth.

Moving boundaries are those primarily associated with man-made activities, such as naval operation areas or fishing activity. These boundaries have a direct impact on the safety of AUVs and are avoided if known. Moving boundaries could also include the edges of a spill or the extent of an algae plume, in which case the mission objective may be to find and track the boundary.

Ocean current is a dominant factor in the underwater navigation of a vehicle, dramatically affecting the vehicle's knowledge of its own position, since most AUVs operate at speeds of 3 m $sec^{-1}$ or slower and surface currents may approach 2 m $sec^{-1}$ in some areas. When operating in the open ocean, an AUV can only determine its position when on the surface, and must dead reckon using heading and speed through the water while submerged.

The characterisation of mission safety is used on a scaled weighting of its dependent factors including man-made boundaries, natural boundaries, weather, moving boundaries and currents. The weighting is largely subjective, but it is clear that perceived safety is diminished as multiple factors migrate from favourable to adverse.

The two dominant cost factors considered in our model are L&R and time-off station. L&R requires the use of an open ocean vessel which incurs significant expense, so the glider operations should be conducted in a way that minimises the frequency of L&R of all deployed AUVs as well as the transit distance of the L&R ship. In addition, L&R is impacted by all factors that affect the ability of a ship to navigate and conduct over-the-side operations including man-made boundaries, natural boundaries, weather, moving boundaries, current and safety.

Time-off station refers to the time spent by the AUV not collecting data along the desired path. Mission objectives are typically specified as collecting data along a path, with the primitive building block consisting of a straight transect. Considering a simple mission that involves moving back and forth between two points, the ocean currents may be such that an AUV is unable to go along this path in one direction or the other. In this case, the vehicle may be able to return to the other end by going deeper or by moving horizontally to get out of the current. In both cases, the vehicle is spending time 'off-station', not collecting data in the desired area.

### *3.1 The weighting system*

Initially, we define the importance weight of the mission planning decision factors using the ANP. The ANP is a more general form of the analytic hierarchy process (AHP) used in MCDA. Saaty (1980) developed the AHP to capture the intuitive judgments in multi-criteria decision problems. AHP assumes unidirectional hierarchical relationships among the decision elements in a problem. However, in many real-life problems, there are interdependencies among the elements in a hierarchy. ANP does not require independence and allows for decision elements to 'influence' or 'be influenced' by other elements in the model. Both processes have been widely used on a practical level and numerous applications have been published in the literature (Saaty, 1996, 2001, 2005).

There are two different kinds of dependencies in a hierarchy, within level or between levels dependencies. The directions of the arrows (or arcs) signify dependence (or influence). An example of a between level dependency (or outer dependency) in Figure 5 is the dependency between sea current and safety and an example of a within level dependency (or inner dependency) is the interdependency between weather and moving boundaries. With such interactions, the hierarchical structure becomes a network and a matrix manipulation approach developed by Saaty and Takizawa (1986) is used to measure the relative importance or strength of the impacts on a given element in the network using a ratio scale similar to AHP.

According to Saaty (1996), the ANP comprises problem structuring, pairwise comparisons, super-matrix formation and weight determination. In ANP, similar to AHP, decision-makers are asked to provide a series of pairwise comparisons of the elements at each level of the hierarchy with respect to a control element. The control element can be an element at the upper or lower levels of the hierarchy. This is the fundamental requirement for developing the super-matrix in the ANP. The pairwise comparison for the elements at one level with respect to the control element at another level is expressed in a matrix form (A) with Saaty's 1–9 scale shown in Table 1.

**Table 1** The fundamental scale used in analytic hierarchy process and analytic network process

| *Intensity of importance* | *Definition* | *Explanation* |
|---|---|---|
| 1 | Equal importance | Two activities contribute equally to the objective |
| 2 | Weak or slight | |
| 3 | Moderate importance | Experience and judgment slightly favour one activity over another |
| 4 | Moderate plus | |
| 5 | Strong importance | Experience and judgment strongly favour one activity over another |
| 6 | Strong plus | |
| 7 | Very strong or demonstrated importance | An activity is favoured very strongly over another; its dominance demonstrated in practice |
| 8 | Very, very strong | |
| 9 | Extreme importance | The evidence favouring one activity over another is of the highest possible order of affirmation |

A reciprocal value is assigned to the inverse comparison; that is, $a_{ij} = 1/a_{ji}$, where $a_{ij}$ ($a_{ji}$) represents the importance weight of the $i$th ($j$th) element. Once the pairwise comparisons are completed, the local priority vector $w$ is computed as the unique solution to $A \times w = \lambda_{\max} w$ where $A$ is the matrix of pairwise comparison, $w$ is the eigenvector and $\lambda_{\max}$ is the largest eigenvalue of $A$. There are several algorithms available for approximating the vector $w$ (Saaty and Takizawz, 1986; Saaty, 1988). We use a two-stage algorithm proposed by Meade and Sarkis (1998) for averaging normalised columns and approximating the vector $w$.

$$w_i = \left( \sum_{j=1}^{n} \left( A_{ij} \Big/ \sum_{i=1}^{n} A_{ij} \right) \right) \Big/ n \quad \text{for } i = 1, \ldots, n. \tag{1}$$

The deviation from consistency of the pairwise comparisons must be addressed in the assessment process. Saaty (1980) provides a consistency index (CI) defined as $\mathrm{CI} = (\lambda_{\max} - n)/(n-1)$ for this test in which $\lambda_{\max}$ is approximated by $\sum_{i=1}^{n} [(Aw)_i / w_i] / n$. The acceptable consistency index is $\mathrm{CI} \le 0.10$.

Next, the super-matrix is formed. The super-matrix concept is similar to a Markov chain process (Saaty, 1996). The local priority vectors developed earlier are entered in the appropriate columns of a matrix to obtain global priorities in a problem with interdependencies. As a result, a partitioned matrix called a super-matrix is created, where each matrix segment represents a relationship between two elements in the model. When there is an interrelationship between the elements of a component or two components, zeros can be replaced by a matrix in the super-matrix. Let the components of a decision system be $C_k$, $k = 1,2, \ldots, n$; each component $k$ is assumed to have $m_k$ elements, denoted by $e_{k1}$, $e_{k2}$, …, $e_{kmk}$. The standard form of a super-matrix is shown in Figure 6 (Saaty, 1996).

**Figure 6** The standard form of a super-matrix

$$
W = \begin{array}{cc}
 & \begin{array}{ccccc} C_1 & \cdots\cdots & C_k & \cdots\cdots & C_n \\ e_{11}e_{12}\ldots e_{1m1} & \cdots\cdots & e_{k1}e_{k2}\ldots e_{kmk} & \cdots\cdots & e_{n1}e_{n2}\ldots e_{nmn} \end{array} \\
\begin{array}{cc} & e_{11} \\ C_i & e_{12} \\ & \vdots \\ & e_{1m1} \\ \cdots\cdots & \cdots\cdots \\ & e_{k1} \\ C_k & e_{k2} \\ & \vdots \\ & e_{kmk} \\ \cdots\cdots & \cdots\cdots \\ & e_{n1} \\ C_n & e_{n2} \\ & \vdots \\ & e_{nmn} \end{array} &
\begin{bmatrix} W_{ii} & \cdots\cdots & W_{ik} & \cdots\cdots & W_{in} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ W_{k1} & \cdots\cdots & W_{kk} & \cdots\cdots & W_{kN} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ W_{n1} & \cdots\cdots & W_{nk} & \cdots\cdots & W_{nn} \end{bmatrix}
\end{array}
$$

The super-matrix is then converged to obtain a long-term stable set of weights. For convergence to occur, the super-matrix needs to be column stochastic. In other words, the sum of each column of the super-matrix needs to be one. Saaty (1996) suggests raising the weighted super-matrix to the power of $2k + 1$, where $k$ is an arbitrarily large number, to achieve a convergence on the importance weights. This new matrix is called the limit super-matrix. The limit super-matrix has the same form as the weighted super-matrix, but all the columns of the limit super-matrix are the same. By normalising each block of the limit super-matrix, the final importance weights of all the elements in the matrix can be obtained. The limit may not converge unless the matrix is column stochastic (i.e. each of its columns sums to one). Note that $\lambda_{\max}(T) = 1$ for the super-matrix. Since $\text{Max}\sum_{j=1}^{n} a_{ij} \geq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i}\lambda_{\max}$ for max $w_i$ and $\text{Min}\sum_{j=1}^{n} a_{ij} \leq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i}\lambda_{\max}$ for min $w_i$, the eigenvalue of the matrix ($\lambda_{\max}$), lies between its largest and smallest column sums $\left(1 = \text{Min}\sum_{j=1}^{n} a_{ij} \leq \lambda_{\max} \leq \text{Max}\sum_{j=1}^{n} a_{ij} = 1\right)$. When the eigenvalues of the matrix $W$ are distinct, then the power series expansion of $f(x)$ converges for all finite values of $x$ with $x$ replaced by $W$:

$$
f(W) = \sum_{i=1}^{n} f(\lambda_i)Z(\lambda_i), Z(\lambda_i) = \frac{\prod_{j\neq i}(\lambda_j I - A)}{\prod_{j\neq i}(\lambda_j - \lambda_i)}, \sum_{i=1}^{n} Z(\lambda_i) = I, Z(\lambda_i)Z(\lambda_j) = 0, Z^2(\lambda_i) = Z(\lambda_i), \quad (2)
$$

where $I$ and 0 are the identity and the null matrices, respectively.

A similar expression is also available when some or all of the eigenvalues have multiplicities. When $f(W) = W^k$, then $f(\lambda_i) = \lambda_i^k$ and as $k \to \infty$, the only terms that give a finite non-zero value are those for which the modulus of $\lambda_i$ is equal to one. The priorities of the clusters (or any set of elements in a cluster) are obtained by normalising the corresponding values in the appropriate columns of the limit matrix. For complete treatment, see Saaty and Ozdemir (2005) and Saaty (2005).

### *3.2 The scoring system*

In order to navigate through the environment, we use a grid-based approach for map representation and modelling the environment and partition the geographical map of the area into uniform grid squares referred to as 'cells'. This mapping approach is easy to build and facilitates the computation of the shortest path with the Voronoi graph (Aurenhammer and Klein, 2000; Bailey, Tavana and Busch, 2006). The grid-based approach is the most commonly used map representation for AUV navigation (Dissanayake et al., 2001). The size of the cells is dependent on the mission objectives and requirements. For example, in the problem provided in Section 4, a 16.5 × 18.5 km decision space is divided into 1,221 square cells, each measuring 500 × 500 m. The cells within a specific mission operations region are scored according to the environmental and non-environmental factors identified earlier. Different factors require different scoring schemes. The static features and piloting of vehicles are represented by binary representation regarding transiting areas with man-made maritime boundaries. A '1' indicates 'presence' and a '0' indicates 'absence' of man-made boundaries in a cell. Similarly, a binary representation is used to indicate presence or absence of natural boundaries in a cell. A 0–10 scaled grading system (0 = 'no weather conditions' to 10 = 'extreme weather conditions') is used to characterise the influences of weather, winds, visibility and waves in each cell. We use a binary score based on the minimum/maximum density capability of the vehicle and the forecasted ocean density to capture the density hazards in a cell. A '1' indicates 'hazardous conditions' and a '0' indicates 'non-hazardous conditions' in a cell.

Binary representation is also used to capture the effects of moving boundaries in the system. Similar to man-made boundaries, a '1' indicates 'presence' and a '0' indicates 'absence' of moving boundaries in a cell. The impact of ocean current in a particular cell is measured with respect to the stated mission goals, i.e. whether the current assists or conflicts with the desired direction of motion. We use a three-state representation for ocean currents, but a scaled approach based on current speed and direction could also be used to capture the effect of sea currents in a cell. In our model, a '–1' indicates 'assisting currents', a '0' indicates 'no currents' and a '+1' indicates 'conflicting currents'.

A 0–10 scaled grading scheme is used for safety (0 = 'safe conditions' to 10 = 'extremely unsafe conditions'). This scoring is largely subjective, but it is clear that perceived safety is diminished as multiple factors migrate from favourable to adverse. A 0–10 metric scale (0 = 'none' to 10 = 'high costs') is also used to account for the transit distance and cost of the L&R ship to selected locations. To capture the time-off station score of each cell, we use a 0–10 scale by assigning higher scores (undesirable) to cells further away from the planned transect and lower scores (desirable) to the cells closer to the planned transect. The scoring for the time-off station is done without considering specific proposed paths. Finally, a 0–10 scaled scoring system (0 = 'low priority' to

10 = 'high priority') is used to capture the priority score of each cell. The mission goals and objectives directly influence the priority score of a cell.

Next, all scores are transformed into a 0–10 scale for meaningful comparisons. Lower scores are more desirable than higher scores in the transformed scale. Scale transformation adjusts the range of data, rather than the centre of the distribution. This eliminates differences in the range of scores and makes data more comparable across arrays. The following equation is used to perform a linear transformation on the original data values ($O_{\text{Value}}$) into new data values ($N_{\text{Value}}$) within a specified new data interval [New minimum ($N_{\text{Min}}$), New maximum ($N_{\text{Max}}$)]:

$$N_{\text{Value}} = N_{\text{Max}} - O_{\text{Max}}\left(\frac{N_{\text{Max}} - N_{\text{Min}}}{O_{\text{Max}} - O_{\text{Min}}}\right) + O_{\text{Value}}\left(\frac{N_{\text{Max}} - N_{\text{Min}}}{O_{\text{Max}} - O_{\text{Min}}}\right) \tag{3}$$

Table 2 presents a summary of the scoring system used in our model.

### 3.3 The weighted-sum model

Defuzzification is the process of producing a quantifiable result in fuzzy logic. A series of importance weights and scores are used in our model which may require defuzzification. Some examples include multiple importance weights or scores provided by multiple decision-makers or multiple weights or scores provided by a single decision-maker who is unsure about his or her judgment. Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar and crisp representations. The process of condensing the information captured by fuzzy sets into numerical values is similar to that of the transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process here is similar to an averaging procedure. Special defuzzification methods can be used to increase the numerical efficiency and transparency of the computations.

**Table 2** The scoring system

| *Factor* | *Initial scores* | *Transformed scores* |
|---|---|---|
| Man-made boundaries | 0,1 | 0,10 |
| Natural boundaries | 0,1 | 0,10 |
| Weather, winds, visibility and waves | 0–10 | 0–10 |
| Water density | 0,1 | 0,10 |
| Moving boundaries, fishing and navy operations | 0,1 | 0,10 |
| Sea current | –1, 0 and +1 | 0, 5 and 10 |
| Safety | 0–10 | 0–10 |
| Launch and recovery | 0–10 | 0–10 |
| Time-off station | 0–10 | 0–10 |
| Priority | 0–10 | 0–10 |

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo, Wall and Wang, 2008), financial modelling (Muzzioli and Reynaerts, 2007), information systems (Rolly Intan and Mukaidono, 2004), auditing (Friedlob and Schleifer, 1999), manufacturing cost estimation (Jahan-Shahi, Shayan and Masood, 1999) and water quality management (Benoit, 1994). Many defuzzification techniques have been proposed in the literature. The most commonly used method is the centre of gravity (COG). Other methods include: random choice of maximum, first of maximum, last of maximum, middle of maximum, mean of maxima, basic defuzzification distributions, generalised level set defuzzification, indexed COG, semi-linear defuzzification, fuzzy mean, weighted fuzzy mean, quality method, extended quality method, centre of area, extended centre of area, constraint decision defuzzification and fuzzy clustering defuzzification. Dubois and Prade (2000) and Roychowdhury and Pedrycz (2001) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions in MCDA (Runkler, 1997; Van Leekwijk and Kerre, 1999; Ali and Zhang, 2001; Roychowdhury and Pedrycz, 2001). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. While the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir und Yuan (1995). We use COG in our model which is highly popular and is often used as a standard defuzzification method. COG calculates the centroid of a distribution function. Defining $W_{ij}$ as the weight of factor $j$ defined by decision-makers $I$ ($i = 1,2, \ldots, I$; $j = 1,2, \ldots, J$) and $S_{ij}^{m}$ as the score of $j$th factor for cell $m$ provided by the $i$th decision-maker ($i = 1,2, \ldots, I$; $j = 1,2, \ldots, J$; $m = 1,2, \ldots, M$), we find $R^m$, the 'navigation index' of the $m$th cell. With factor weights as membership grades, $\mu_{ij}(S_{ij}^{m}) = W_{ij}$, Equations (4) and (5) are used to aggregate the $j$th factor weights and scores given by $I$ decision-makers for cell $m$:

$$\mathrm{COG}_j^m = \frac{\sum_{i=1}^{I} S_{ij}^m \mu\left(S_{ij}^m\right)}{\sum_{i=1}^{I} \mu\left(S_{ij}^m\right)} \tag{4}$$

$$R^m = \sum_{j=1}^{J} \mathrm{COG}_j^m \quad \text{where } 0 \le R^m \le 100 \tag{5}$$

The navigation indices are next translated into a colour-coded system reflecting the overall risk level in a cell. The system is comprised of green, yellow, orange and red categories. Green is the lowest risk level representing cells with $0 \le R^m \le 25$. Yellow indicates a moderate risk level and represents cells with $25 < R^m \le 50$. Orange signifies high risk and represents cells with $50 < R^m \le 75$ and red symbolises severe risk and represents cells with $75 < R^m \le 100$.

## 4 A mission planning problem

In this section, we illustrate our model through a mission planning example where a glider is deployed to travel along a path defined by the mission objectives. Initially, mission planners identified the relative priorities of the mission factors in a series of pairwise comparison matrices described in Section 3. The ANP was used to develop the importance weights of environmental and non-environmental factors described earlier. Table 3 presents the importance weights for the end nodes in the hierarchy.

In this example, the map of a 300 $km^2$ mission operations area is divided into 1,221 squares each measuring $500 \times 500$ m. Figure 7 displays the reference position at deployment ($t_0$). Our objective is to travel along the $t_0$–$t_1$–$t_2$–$t_3$ path shown by a solid line on top of the green/yellow region. This requirement is captured in the mission planning allowing deviation from the path, but using a lower scoring for points further away from the desired path. Besides this horizontal path, an additional specification for an underwater glider trajectory will be its desired maximum and minimum depth. The red in the figure along the coastline corresponds to our make-believe country's territorial seas, in which we are prohibited to operate without permission. This particular constraint is time independent and thus this geographic area will always be red. The L&R vessel is launching the underwater vehicle at point $t_0$. The green and yellow regions at time 0 identify the cumulative conditions defined by the MCDA model, indicating that for the moment we have a green zone encompassing our desired path. The navigation indices ($R^m$) for $t_0$ in Table 4, ranging from 4.26 to 48.32, are used to define the green and yellow spaces in Figure 7. Labels $t_1$, $t_2$ and $t_3$ indicates the anticipated times the vessel will arrive at different locations.

**Table 3** The analytic network process results

| *Factor* | *Importance weight* |
|---|---|
| Man-made boundaries | 0.028 |
| Natural boundaries | 0.077 |
| Weather, winds, visibility and waves | 0.081 |
| Water density | 0.051 |
| Moving boundaries, fishing and navy operations | 0.112 |
| Sea current | 0.201 |
| Safety | 0.173 |
| Launch and recovery | 0.132 |
| Time-off station | 0.048 |
| Priority | 0.097 |

**Figure 7** Reference position at $t_0$ or launch point (distance = 0, depth = 0) (see online version for colours)

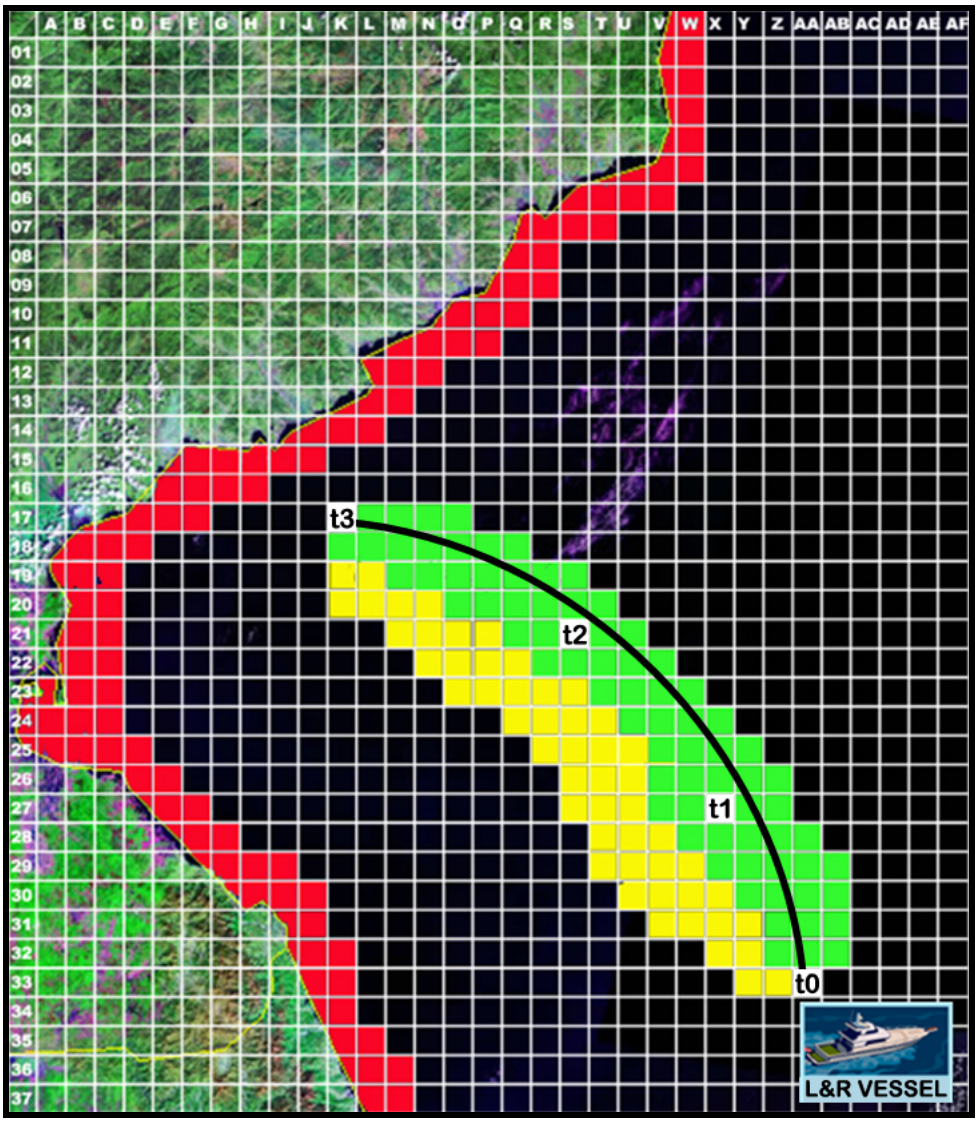


**Table 4** Navigation indices ($R^m$) for $t_0$, $t_1$ and $t_2$ (see online version for colours)

| $t_0$ | | | | $t_1$ | | | | $t_2$ | |
|---|---|---|---|---|---|---|---|---|---|
| K18 | 4.26 | U22 | 12.43 | I16 | 82.45 | Q23 | 39.24 | J16 | 53.29 |
| K19 | 36.35 | U23 | 12.55 | J16 | 2.35 | Q24 | 40.56 | J17 | 4.87 |
| K20 | 32.68 | U24 | 12.98 | J17 | 2.43 | Q25 | 8.24 | J18 | 5.21 |
| L17 | 6.34 | U25 | 47.02 | K15 | 32.11 | Q26 | 8.41 | J19 | 5.34 |
| L18 | 6.65 | U26 | 47.11 | K16 | 3.05 | Q27 | 8.89 | J20 | 5.88 |
| L19 | 36.65 | U27 | 47.23 | K17 | 3.87 | R17 | 86.01 | J21 | 6.03 |
| L20 | 36.74 | U28 | 47.34 | K18 | 4.01 | R18 | 86.34 | K15 | 54.11 |
| M17 | 6.87 | U29 | 47.36 | L15 | 33.21 | R19 | 86.74 | K16 | 54.98 |
| M18 | 7.93 | U30 | 47.44 | L16 | 33.25 | R20 | 86.88 | K17 | 6.23 |
| M19 | 7.95 | V22 | 13.03 | L17 | 4.05 | R21 | 63.14 | K18 | 6.87 |
| M20 | 37.18 | V23 | 13.46 | L18 | 4.23 | R22 | 63.29 | K19 | 7.04 |
| M21 | 37.87 | V24 | 14.05 | L19 | 4.44 | R23 | 63.87 | K20 | 7.12 |
| N17 | 8.03 | V25 | 14.07 | M14 | 82.54 | R24 | 40.74 | K21 | 7.22 |
| N18 | 8.09 | V26 | 14.67 | M15 | 57.12 | R25 | 41.12 | K22 | 7.64 |
| N19 | 8.12 | V27 | 14.68 | M16 | 34.21 | R26 | 9.11 | L15 | 55.23 |
| N20 | 37.92 | V28 | 47.56 | M17 | 35.02 | R27 | 9.25 | L16 | 55.36 |
| N21 | 38.65 | V29 | 47.62 | M18 | 4.67 | S18 | 87.88 | L17 | 55.97 |
| N22 | 38.79 | V30 | 47.69 | M19 | 4.82 | S19 | 87.94 | L18 | 56.65 |
| O17 | 8.23 | V31 | 47.72 | M20 | 4.95 | S20 | 88.21 | L19 | 27.14 |
| O18 | 8.34 | W23 | 14.89 | N13 | 83.98 | S21 | 88.45 | L20 | 28.56 |
| O19 | 8.45 | W24 | 14.96 | N14 | 57.34 | S22 | 88.76 | L21 | 7.89 |
| O20 | 8.56 | W25 | 15.11 | N15 | 57.48 | S23 | 64.11 | L22 | 8.03 |
| O21 | 39.22 | W26 | 15.23 | N16 | 58.23 | S24 | 64.34 | M14 | |
| O22 | 39.85 | W27 | 15.43 | N17 | 35.23 | S25 | 41.29 | M15 | |
| O23 | 40.03 | W28 | 15.76 | N18 | 35.29 | S26 | 10.23 | M16 | 56.78 |
| P18 | 8.75 | W29 | 47.78 | N19 | 5.11 | S27 | 10.44 | M17 | 56.89 |
| P19 | 8.84 | W30 | 47.83 | N20 | 5.63 | S28 | 10.89 | M18 | 57.73 |
| P20 | 8.94 | W31 | 47.84 | N21 | 5.68 | T19 | 89.23 | M19 | 57.93 |
| P21 | 40.78 | X24 | 15.87 | O12 | 84.65 | T20 | 89.37 | M20 | 32.13 |
| P22 | 40.82 | X25 | 15.92 | O13 | 84.78 | T21 | 89.56 | M21 | 33.71 |
| P23 | 41.65 | X26 | 16.02 | O14 | 84.83 | T22 | 89.68 | M22 | 9.12 |
| Q18 | 9.04 | X27 | 16.77 | O15 | 84.92 | T23 | 89.76 | N13 | 86.12 |
| Q19 | 9.07 | X28 | 16.85 | O16 | 58.35 | T24 | 65.88 | N14 | 86.34 |
| Q20 | 9.34 | X29 | 16.92 | O17 | 58.64 | T25 | 41.66 | N15 | 86.41 |
| Q21 | 9.89 | X30 | 47.89 | O18 | 37.09 | T26 | 41.87 | N16 | 87.04 |
| Q22 | 41.78 | X31 | 47.92 | O19 | 37.23 | T27 | 11.03 | N17 | 87.11 |
| Q23 | 41.79 | X32 | 47.97 | O20 | 37.29 | T28 | 11.34 | N18 | 58.14 |
| Q24 | 42.03 | Y25 | 17.23 | O21 | 5.95 | U21 | 90.03 | N19 | 58.23 |

**Table 4** Navigation indices ($R^m$) for $t_0$, $t_1$ and $t_2$ (see online version for colours) (continued)

| $t_0$ | | | | | $t_1$ | | | | $t_2$ |
|---|---|---|---|---|---|---|---|---|---|
| R19 | 10.12 | Y26 | 17.25 | O22 | 5.98 | U22 | 90.16 | N20 | 59.01 |
| R20 | 10.45 | Y27 | 17.37 | O23 | 6.03 | U23 | 90.34 | N21 | 36.12 |
| R21 | 10.66 | Y28 | 17.45 | O24 | 6.11 | U24 | 66.48 | N22 | 10.23 |
| R22 | 10.69 | Y29 | 17.66 | O25 | 6.24 | U25 | 66.89 | O12 | 88.13 |
| R23 | 43.11 | Y30 | 17.74 | P12 | 85.32 | U26 | 42.08 | O13 | 88.23 |
| R24 | 43.87 | Y31 | 48.03 | P13 | 85.34 | U27 | 11.87 | O14 | 88.54 |
| R25 | 44.21 | Y32 | 48.09 | P14 | 85.45 | U28 | 12.87 | O15 | 89.02 |
| S19 | 11.02 | Y33 | 48.21 | P15 | 85.56 | V22 | 90.87 | O16 | 90.34 |
| S20 | 11.11 | Z26 | 18.11 | P16 | 85.74 | V23 | 91.03 | O17 | 89.55 |
| S21 | 11.23 | Z27 | 18.45 | P17 | 59.23 | V24 | 91.09 | O18 | 88.74 |
| S22 | 11.34 | Z28 | 18.72 | P18 | 60.04 | V25 | 67.24 | O19 | 90.12 |
| S23 | 44.55 | Z29 | 18.79 | P19 | 60.24 | V26 | 43.12 | O20 | 59.08 |
| S24 | 44.68 | Z30 | 18.81 | P20 | 37.33 | V27 | 13.98 | O21 | 59.23 |
| S25 | 44.96 | Z31 | 18.88 | P21 | 37.48 | V28 | 14.09 | O22 | 10.54 |
| S26 | 45.03 | Z32 | 18.93 | P22 | 37.61 | W23 | 91.76 | P12 | 92.23 |
| S27 | 45.07 | Z33 | 48.32 | P23 | 6.76 | W24 | 92.05 | P13 | 92.76 |
| T20 | 11.69 | AA28 | 19.03 | P24 | 6.81 | W25 | 92.14 | P14 | 92.77 |
| T21 | 11.72 | AA29 | 20.07 | P25 | 6.92 | W26 | 45.23 | P15 | 93.02 |
| T22 | 11.88 | AA30 | 20.34 | P26 | 7.23 | W27 | 45.69 | P16 | 93.19 |
| T23 | 11.94 | AA31 | 20.45 | Q15 | 85.78 | W28 | 15.93 | P17 | 93.26 |
| T24 | 45.22 | AA32 | 20.56 | Q16 | 85.94 | X24 | 93.87 | P18 | 93.78 |
| T25 | 45.54 | AA33 | 20.68 | Q17 | 85.98 | X25 | 94.23 | P19 | 94.12 |
| T26 | 45.88 | AB29 | 20.73 | Q18 | 61.29 | X26 | 95.06 | P20 | 94.39 |
| T27 | 46.91 | AB30 | 20.77 | Q19 | 61.34 | X27 | 16.04 | P21 | 95.27 |
| T28 | 46.98 | AB31 | 20.85 | Q20 | 61.78 | | | | |
| T29 | 46.99 | AB32 | 20.91 | Q21 | 62.12 | | | | |
| U21 | 12.02 | | | Q22 | 38.88 | | | | |

The coloured areas in Figure 8 show the output of the MCDA model at $t_1$. Again, these colours are known from the navigation indices presented in Table 4 for $t_1$, ranging from 2.35 to 95.06. By this time, the L&R vessel has moved and is engaged in other work. The dotted line shows the original trajectory. We see that the vessel has arrived at the desired point $t_1$, but conditions have changed such that we can no longer reach point $t_2$. Consequently, we modify our mission to stay within the green zone and plan on arriving at point $t_2'$. The new navigation path ($t_1$–$t_2'$–$t_3$) at time 1 is presented with a solid line.

**Figure 8** Reference position at $t_1$ (distance = 3.12 km, depth = 0) (see online version for colours)

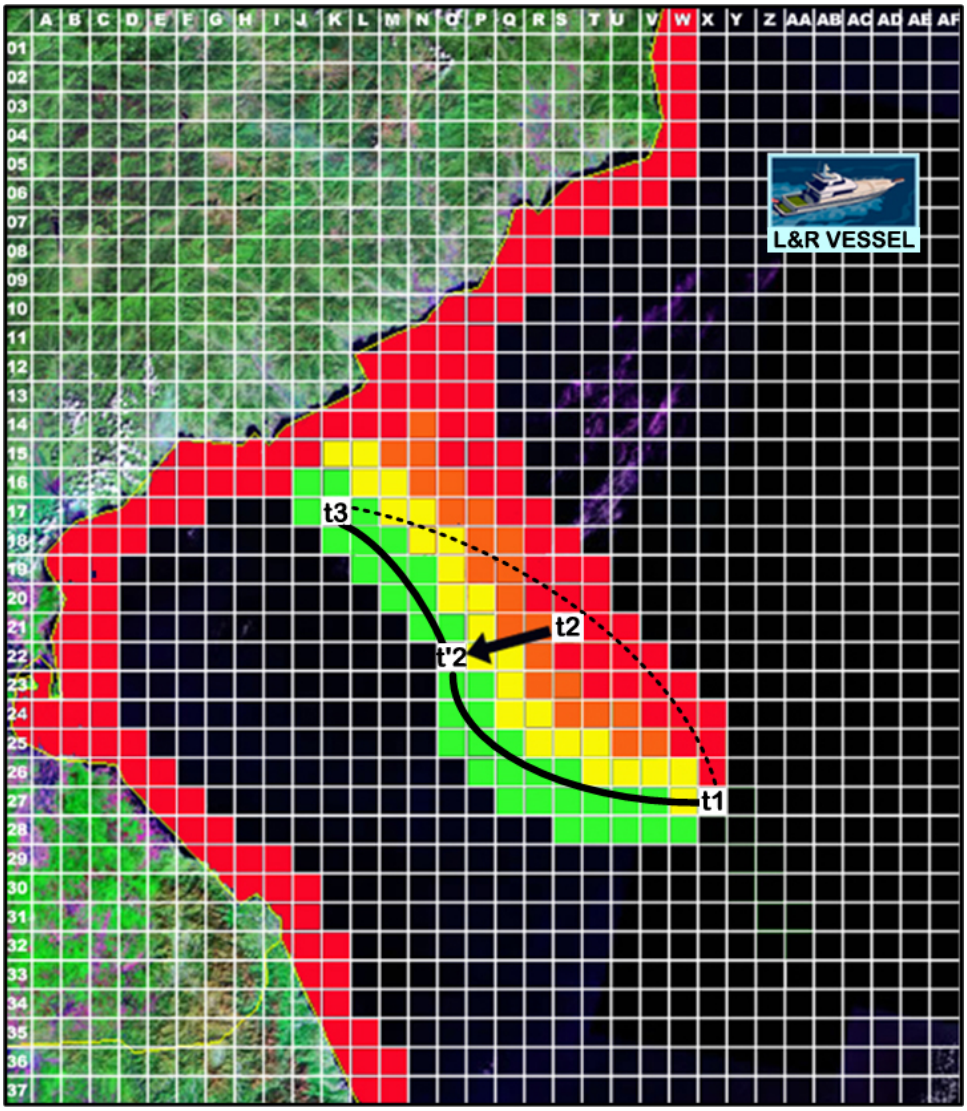
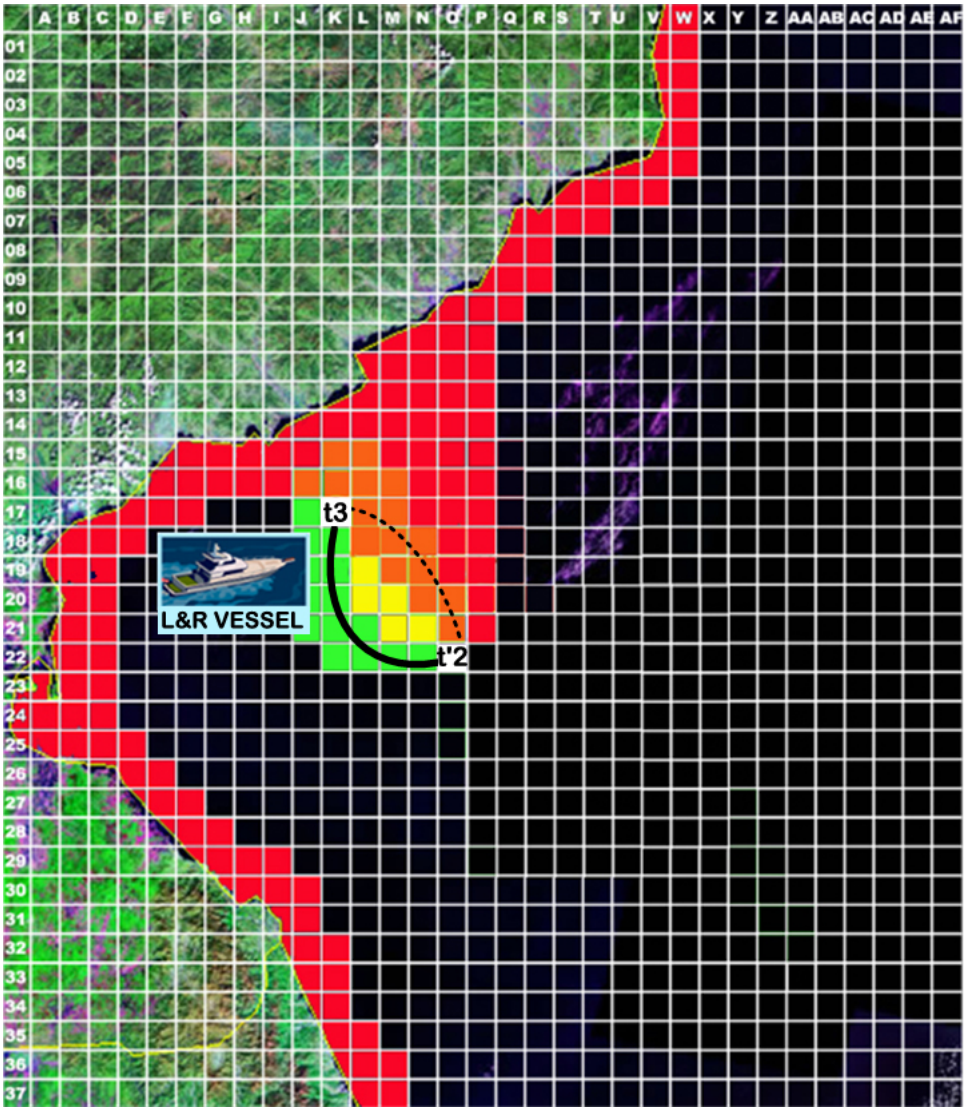



Figure 9 shows the conditions at $t_2$. Again, we are unable to traverse along our desired path shown with the dotted line due to a change in conditions since the start of the mission, but the result for the MCDA model shows that we are able to achieve our planned pick-up point $t_3$ following the new trajectory presented by a solid line. Similarly, these colours are known from the navigation indices presented in Table 4 for $t_2$, ranging from 4.87 to 95.27.

**Figure 9** Reference position at $t_2$ (distance = 6.48 km, depth = 0) (see online version for colours)




## 5 Conclusions and future research directions

The existing AUV mission planning and control models are either static, single factor or lack capabilities to consider the dependencies and interdependencies among the competing factors. In contrast, the model presented in this study is

1 dynamic, since the outcome of previous dive cycles are used in planning for future dive cycles

2 multi-factor, since it considers a series of competing objective and subjective factors

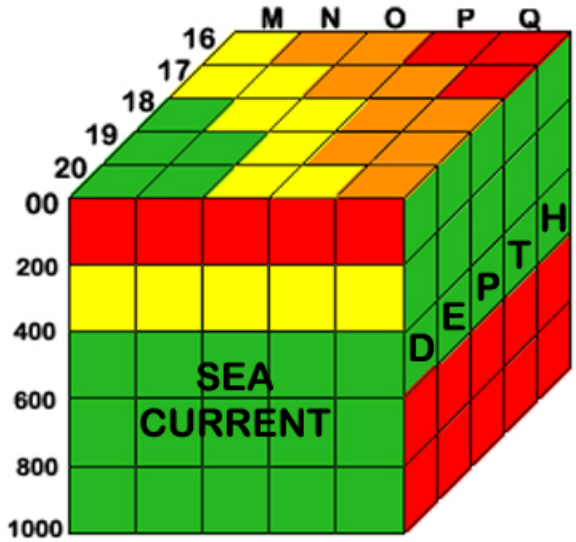
3 networked, since it takes into account the dependencies and interdependencies among the competing factors with the ANP.

The proposed model provides in-context recommendations with respect to a series of hierarchical and networked mission goals with subjective and objective measures. Although our approach is applied to planning and control of underwater gliders, it has much broader applications to unmanned vehicles in general. The model allows for human specification of high-level mission goals and constraints and provides numeric and quantitative analysis of alternatives. These capabilities represent a significant step forward compared to the existing manual-intensive approaches.

The example considers only the current at the surface of the ocean. As mentioned earlier, transect for an underwater vehicle includes not only the desired horizontal path, but also the depth range that the vehicle will operate in. Figure 10 depicts how the model could be extended in future research to include considerations that change vertically, i.e. with different ocean depths. This is done with a cube representing the conditions at a single time. In this figure, the top of the cube represents all those conditions affecting our mission with the exception of ocean currents below the surface. The front face of the cube depicts a vertical and horizontal weighting based on the current speed and/or direction. As seen in the figure for this period of time, the current is prohibitive for depth 0 to 200, marginal for 200 to 400 and satisfactory for depths 400–1,000. The right face of the cube defines our mission objective that is to operate between the surface and 600. Consequently, for this time period, we can only operate in the green zone between 400 and 600. These factors would be combined with those shown on the surface to generate a solution for each depth 'slice'. Thus, instead of a surface at each time, the result of the model is a volume defining regions of green, yellow and red.

The proposed DSS lends itself well to 'process improvement' – as experience is gained and codified from experts into deterministic approaches, the model can be easily updated to incorporate the new paradigm. It also supports expansion to cooperative teams of AUVs. Working together will allow underwater vehicles to complete tasks that could not be completed by a single vehicle. The DSS will suggest a group assignment that could be changed throughout the mission, as mission progress information is received. For instance, if the goals change during the mission, some vehicles may leave the original area to enter new territories while other vehicles may pick-up the remaining tasks left incomplete by departing vehicles. Cooperative AUVs would also allow a human operator to re-task the group as a whole, rather than each vehicle individually, making it easier to manage the vehicles.

**Figure 10** The 3D depth-determination model (see online version for colours)

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