

# An optimal sequential information acquisition model subject to a heuristic assimilation constraint

Sequential  
information  
acquisition  
model

937

Debora Di Caprio

*Department of Mathematics and Statistics, York University, Toronto, Canada*

Francisco J. Santos-Arteaga

*Departamento de Economía Aplicada II, Universidad Complutense de Madrid, Madrid, Spain, and*

Madjid Tavana

*Business Systems and Analytics Department, La Salle University, Philadelphia, Pennsylvania, USA*

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## Abstract

**Purpose** – The purpose of this paper is to study the optimal sequential information acquisition process of a rational decision maker (DM) when allowed to acquire  $n$  pieces of information from a set of bi-dimensional products whose characteristics vary in a continuum set.

**Design/methodology/approach** – The authors incorporate a heuristic mechanism that makes the  $n$ -observation scenario faced by a DM tractable. This heuristic allows the DM to assimilate substantial amounts of information and define an acquisition strategy within a coherent analytical framework. Numerical simulations are introduced to illustrate the main results obtained.

**Findings** – The information acquisition behavior modeled in this paper corresponds to that of a perfectly rational DM, i.e. endowed with complete and transitive preferences, whose objective is to choose optimally among the products available subject to a heuristic assimilation constraint. The current paper opens the way for additional research on heuristic information acquisition and choice processes when considered from a satisficing perspective that accounts for cognitive limits in the information processing capacities of DMs.

**Originality/value** – The proposed information acquisition algorithm does not allow for the use of standard dynamic programming techniques. That is, after each observation is gathered, a rational DM must modify his information acquisition strategy and recalculate his or her expected payoffs in terms of the observations already acquired and the information still to be gathered.

**Keywords** Competitive strategy, Decision support systems, Rationality, Sequential information acquisition, Utility theory

**Paper type** Research paper

## 1. Introduction

The current paper defines the optimal sequential information acquisition process of a rational utility maximizing decision maker (DM) when allowed to acquire  $n$  pieces of information from a set of bi-dimensional products whose characteristics have a continuum of variants.

The optimal allocation of the information available to the DM is based on two well-defined real-valued expected utility functions. One of the functions defines the

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expected utility obtained from continuing gathering information on one of the previously partially observed products. The other function describes the expected utility that follows from starting observing the characteristics of a new product. The crossing points, if any, between the graphs of both functions correspond to optimal thresholds for the information gathering process that define the dynamic behavior of the corresponding algorithmic search structure.

Moreover, we incorporate a heuristic mechanism to the information acquisition process of the DM. When deciding whether to start gathering information on a new product or continuing with any of the products whose characteristics have been observed, the DM will use as a reference point to define his optimal behavior the threshold value arising from the two observations setting. This value determines the final forward looking decision of the DM when calculating how to use his last observation and is, at the same time, carried backwards to determine his current information acquisition strategy. That is, it affects his current choice behavior.

The reason for imposing this mechanism is twofold. First, it allows us to represent the optimal behavior of the DM within an easily understandable setting that can be simulated numerically. Second, it accounts for the information capacity constraints with which we endow the DM. That is, a DM can only engage in pairwise comparisons when choosing between his two main options of continuing acquiring information on an observed product or starting acquiring information on a new one.

Note, however, that the optimal behavior exhibited by a DM should be determined by all previously observed characteristics as well as the number of observations remaining to be acquired. If this were the case and the heuristic mechanism was not imposed, then, even in the current bi-dimensional product environment, the DM would have to consider all possible combinations of information acquisition results simultaneously and include them within each subsequent continuation or starting function. This task is not particularly complex but the DM may only perform it when the sets describing the variants of the different characteristics are discrete. When the sets describing the variants of the different characteristics are defined on a continuum, as is the case in the setting of the current paper, then this task cannot be performed by the DM, who should calculate the resulting expected payoff for each and every variant within the continuum. We will elaborate further on this topic through the paper and provide additional intuition.

The paper proceeds as follows. The next section presents a review of the heterogeneous literature on sequential information acquisition. Section 3 deals with the standard notation and basic assumptions needed to develop the model. Section 4 defines the expected search utility functions and considers the two-observation case. Section 5 studies the behavior and properties that follow from the three-observation setting while Section 6 takes on the  $n$ -observation problem. Section 7 describes the main limitations and potential extensions of the current model and Section 8 summarizes the main findings and highlights their managerial significance. The Appendix analyses the three-observation setting when the heuristic mechanism is not implemented and proposes several modifications to the information acquisition process that widen the scope of the current paper.

## 2. Literature review

The sequential information acquisition process of DMs has been analyzed from very different perspectives by quite a heterogeneous group of literatures, including, but not limited to, economics, operations research, and information sciences.

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Consider first the approach of economists to the acquisition of information by rational DMs. The process of sequential information acquisition followed by rational DMs is generally analyzed when dealing with search environments defined within labor and money markets. In this case, the corresponding process is incorporated into a matching function determining the equilibrium outcome derived from the search. However, DMs are either assumed to acquire the information perfectly, see, for example, Guerrieri (2008), or required to observe the unique characteristic defining their corresponding search and matching processes, see McCall and McCall (2008), Rogerson *et al.* (2005) and Yashiv (2007) for several reviews of the literature. As a result, the information acquisition problem of DMs, either firms or workers, reduces to defining an optimal stopping rule while being constrained by a given search cost (see Bikhchandani and Sharma, 1996). The intuition behind these formal structures follows from the seminal papers of Moore and Whinston (1986, 1987), whose sequential information acquisition models were built to optimize expected costs or payoffs using standard dynamic programming techniques. However, these models do not consider relatively low dimensional environments and concentrate mainly on the existing trade-offs between the costs of acquiring information and its value. This is also the case in the operational research and information science literatures (see Mookerjee and Mannino, 1997; Mussi, 2002).

This type of optimal stopping problems defined within sequential search environments constitute also a common object of analysis in the management and operational research literatures leading to the design of expected value maximizing algorithms (see MacQueen, 1964; MacQueen and Miller, 1960). Both these literatures expanded the unknown characteristic environment analyzed by the economic literature within its search branch to multidimensional vectors of attributes (see Lim *et al.*, 2006). These research branches concentrate mainly on the optimal stopping properties of the decision algorithms designed to determine the introduction or dismissal of a new technology, see, among many others (Cho and McCardle, 2009; Ulu and Smith, 2009).

Similarly, the acquisition decision of new technologies by firms has been studied through Bayesian multidimensional search models that were initially introduced by Lippman and McCardle (1991) and McCardle (1985). In this case, return functions were required to be both convex increasing and continuous in order for the authors to prove the existence of optimal decision threshold values through dynamic programming techniques. Di Caprio and Santos Arteaga (2009) relaxed both these assumptions allowing for a generic information acquisition algorithm that accounts for any type of return function. This search algorithm is used as the base on which the current model is built.

While dealing with highly important technical aspects of the information acquisition process, the decision support and expert systems literatures on sequential search tend to relegate to a second plane the highly limited capacity of the DM to process and assimilate information and take subsequent decisions, a point, on the other hand, strongly emphasized by Simon (1997). His satisficing boundedly rational DMs were presented as an alternative to purely optimizers, who need impressive computational capacities in order to maximize their payoffs when facing sequential decision problems (see Bearden and Connolly, 2007; Bearden and Connolly, 2008). The motivation for the current paper arises from considering this bounded rationality perspective and applying a satisficing approach when defining the search strategy of DMs.

The common feature to all the models developed by the heterogeneous branches of the literature just described is the use of dynamic programming techniques to analyze

the sequential information acquisition behavior of DMs. As we will illustrate through the paper, the current information acquisition algorithm does not allow for the simplifying requirements of dynamic programming techniques. That is, the algorithm described through the paper should be redefined after each observation is gathered by the DM and recalculated in terms of all previously observed variables, their sets of possible combinations and corresponding expected payoffs.

We will bias our notation toward the economics branch of the literature and express the information acquisition algorithm in utility terms since we will be considering the point of view of an expected utility maximizing DM. This is the case despite the fact that there is not a value function being optimized but the expected outcome from a discrete sequential search process performed on a finite set of bi-dimensional products whose characteristics have a continuum of variants.

### 3. Basic notations and main assumptions

The notations and initial assumptions we refer to when constructing our model are those of Di Caprio and Santos Arteaga (2009). However, for the sake of completeness, this section partially reproduces some formal definitions and related comments already described in Section 2 (Preliminaries and basic notations) and Section 3 (Main assumptions) of Di Caprio and Santos Arteaga (2009).

Let  $X$  be a non-empty set and  $\succsim$  a preference relation defined on  $X$ . A utility function representing a preference relation  $\succsim$  on  $X$  (in short:  $u$  is a utility function on  $X$ ) is an order-preserving function  $u$  from  $X$  to  $\mathfrak{R}$ , that is:

$$\forall x, y \in X, x \succsim y \Leftrightarrow u(x) \geq u(y). \tag{1}$$

Let  $G$  denote the set of all products. For every  $i \leq 2$ , let  $X_i$  represent the set of all possible variants for the  $i$ th characteristic of any product in  $G$  and  $X$  stand for the Cartesian product  $X_1 \times X_2$ . Thus, every product in  $G$  is described by a pair  $\langle x_1, x_2 \rangle$  in  $X$ .  $X_i$  is called the  $i$ th characteristic factor space, while  $X$  stands for the characteristic space. It may initially seem that allowing DMs to acquire only two characteristics per product constitutes a substantial constraint on the set of information available. However, observations do not necessarily account for a unique property of the product, but a series of them whose combination may define a characteristic element endowed with a given subjective probability function. For example, when considering the purchase of a laptop computer, the first characteristic could be defined as portability and consist of the average of weight, size, and battery life, while the second characteristic set may include screen resolution together with processing and memory capacities, and be defined as manageability.

As Di Caprio and Santos Arteaga (2009), we follow the classical approach to information demand by economic agents proposed by Wilde (1980) and work under the assumption that each  $X_i$  is a compact and connected non-degenerate real subinterval of  $[0, +\infty)$ . Consequently, the topology and the preference relation on each  $X_i$  will be assumed to be those induced by the standard Euclidean topology and the standard linear order  $>$ , respectively.

The following assumptions are taken from Di Caprio and Santos Arteaga (2009):

*Assumption 1.* For every  $i \leq 2$ , there exist  $x_i^m, x_i^M > 0$ , with  $x_i^m \neq x_i^M$ , such that  $X_i = [x_i^m, x_i^M]$ , where  $x_i^m$  and  $x_i^M$  are the minimum and maximum of  $X_i$ .

*Assumption 2.* The characteristic space  $X$  is endowed with the product topology  $\tau_p$  and a strict preference relation  $\succ$ .

*Assumption 3.* There exist a continuous additive utility function  $u$  representing  $\succ$  on  $X$  such that each one of its components  $u_i: X_i \rightarrow \mathfrak{R}$ , where  $i \leq 2$ , is a continuous utility function on  $X_i$ .

A utility function  $u: X_1 \times X_2 \rightarrow \mathfrak{R}$  representing  $\succeq$  on  $X_1 \times X_2$  is called additive (Wakker, 1989) if there exist  $u_i: X_i \rightarrow \mathfrak{R}$ , where  $i \leq 2$ , such that  $\forall \langle x_1, x_2 \rangle \in X_1 \times X_2, u(\langle x_1, x_2 \rangle) = u_1(x_1) + u_2(x_2)$ :

*Assumption 4.* For every  $i \leq 2, \mu_i: X_i \rightarrow [0, 1]$  is a continuous probability density on  $X_i$ , whose support, the set  $\{x_i \in X_i; \mu_i(x_i) \neq 0\}$ , will be denoted by  $Supp(\mu_i)$ .

The probability densities  $\mu_1$  and  $\mu_2$  represent the subjective “beliefs” of the DM. For  $i \leq 2, \mu_i(Y_i)$  provides the subjective probability that the DM assigns to the value of the  $i$ th characteristic observed from a random product in  $G$  being an element  $x_i$  that belongs to the subinterval  $Y_i \subseteq X_i$ . In the current setting, the probability densities  $\mu_1$  and  $\mu_2$  will be assumed to be independent. However, the model allows for subjective correlations to be defined among different characteristic within a given product.

Clearly, the characteristics and probabilities defining the corresponding expected search utilities depend on the type of product under consideration. While the experimental economic literature tends to concentrate on discrete characteristics-based lotteries, many market products have characteristics defined in continuous scales. An immediate example could relate to the purchase of a house, whose evaluation depends on features such as size, distance to amenities, and public transportation, and different ratings of the neighborhood. These variables are usually represented in continuous scales. Another example would be given by a board of directors that must choose a next project to fund and has a fixed amount of time to devote to researching potential projects defined in terms of risk levels and expected returns. In this case, the board must choose between investigating a known option in detail and exploring further options. Similarly, when selecting personnel, corporations can apply different tests for qualifications. In this case, the problem would be whether to send all candidates through one test (which only tests one type of qualification), or test one candidate (or only a few candidates) thoroughly for all qualifications. This setting would also reinforce the assumption that the characteristics of all products follow from the same distribution[1]. Additional applications of sequential search or screening processes, particularly in online environments, whose alternatives have characteristics defined on continuous scales or a combination of both discrete and continuous ones, i.e. selection of a hotel, can always be found. The additive separability of the current model would allow us to incorporate this latter type of environments (see Di Caprio and Santos Arteaga, 2009) for an analysis and several numerical examples within the two observations setting. We will keep on referring to the objects of choice as products, while keeping in mind the wide array of possible interpretations available.

As in Di Caprio and Santos Arteaga (2009) and Mas-Colell *et al.* (1995), following the standard economic theory of choice under uncertainty, we assume that the  $i$ th certainty equivalent value induced by the probability density  $\mu_i$  and the utility function  $u_i$  (see definition below) works as the reference point against which the DM will compare the value  $x_i$  observed (or to be observed) for the  $i$ th characteristic of a certain product. The use of the certainty equivalent as a reference value is common when providing different types of relative valuations in the economics and operational research literatures,

see, for example, Cerdá Tena and Quiroga Gómez (2011). Indeed, generalized approximations to this concept may be generated even when core expected utility theory assumptions such as the continuity of the utility functions and the connectedness of their domains are relaxed (see Di Caprio and Santos-Arteaga, 2011).

Given  $i \leq 2$ , the certainty equivalent of  $\mu_i$  and  $u_i$ , denoted by  $ce_i$ , is a characteristic in  $X_i$  that the DM is indifferent to accept in place of the expected one to be obtained through  $\mu_i$  and  $u_i$ . That is, for every  $i \leq 2$ ,  $ce_i = u_i^{-1}(E_i)$ , where  $E_i$  denotes the expected value of  $u_i$ .

Since  $u_i$  is continuous,  $u_i^{-1}(E_i) \neq \emptyset$ . Also, the fact that  $u_i$  is strictly increasing implies that  $u_i^{-1}(E_i)$  consists of a single element. Thus, for every  $i \leq 2$ ,  $ce_i$  exists and it is unique.

#### 4. The two-observation setting: expected search utilities

The set of all products,  $G$ , is identified with a compact and convex sub set of the two-dimensional real space  $\mathfrak{R}^2$ . In the simplest non-trivial scenario,  $G$  consists of at least two products and the DM is allowed to collect two pieces of information, not necessarily from the same product. The DM must use the first observation to check the first characteristic of any of the products in  $G$ . After checking the value of the first characteristic from an initial product, the DM must decide whether to keep acquiring information on the same product and, hence, check the value of its second characteristic, or start collecting information on a new product.

Di Caprio and Santos Arteaga (2009) show that the choice between continuing with the initial product and starting with a new one relies on the comparison between the values taken at the observed point  $x_1$  by two real-valued functions defined on  $X_1$ . These functions, denoted by  $F$  and  $H$ , are defined by considering the expected utility values that the DM computes as the potential payoffs derived from his information acquisition process. The reference value for the DM's calculations is the sum  $E_1 + E_2$ , that is, the sum of the expected utility values defined by the pairs  $\langle u_1, \mu_1 \rangle$  and  $\langle u_2, \mu_2 \rangle$ . The function  $F$  provides the DM's expected utility value associated with the option "continuing with the initial product". The function  $H$  allows the DM to evaluate the expected utility value associated with the option "starting with a new product". For the sake of completeness, we reproduce the definition of  $F$  and  $H$ .

The function  $F: X_1 \rightarrow \mathfrak{R}$  is defined by:

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \quad (2)$$

The variable  $x_1$  is given by the value of the first characteristic from the initial product already observed. The integration sets:

$$P^+(x_1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \quad (3)$$

and:

$$P^-(x_1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}. \quad (4)$$

contain all the values  $x_2$  that the second characteristic of the initial product should have in order to deliver either a higher utility than a random product from  $G$ , see Equation (3), or a lower-equal utility than a random product from  $G$ , see Equation (4).

Suppose that, after the value  $x_1$  has been observed as the first characteristic for the initial product, the DM chooses the option "continuing with the initial product".

Then, a random product from  $G$  would give him an expected utility value of  $E_1 + E_2$ , while  $u_1(x_1) + u_2(x_2)$  would be the utility derived from the initial product provided that its second characteristic has value  $x_2$ . Thus, if  $u_1(x_1) + u_2(x_2) \leq E_1 + E_2$ , choosing a product from  $G$  randomly delivers the DM a higher expected utility than choosing the initially observed product.

The function  $H: X_1 \rightarrow \mathfrak{R}$  is defined as follows:

$$H(x_1) \stackrel{def}{=} \int_{Q^+(x_1)} \mu_1(x_1^n) (u_1(x_1^n) + E_2) dx_1^n + \int_{Q^-(x_1)} \mu_1(x_1^n) (\max\{u_1(x_1), E_1\} + E_2) dx_1^n. \tag{5}$$

The variable  $x_1$  is still given by the value of the first characteristic from the initial product already observed. The integration sets:

$$Q^+(x_1) = \{x_1^n \in X_1 \cap Supp(\mu_1) : u_1(x_1^n) > \max\{u_1(x_1), E_1\}\} \tag{6}$$

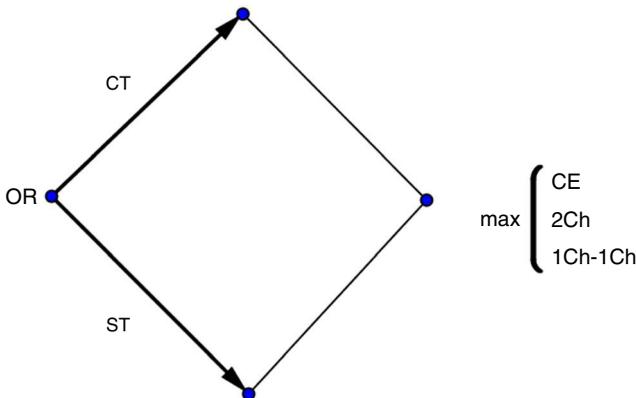
and:

$$Q^-(x_1) = \{x_1^n \in X_1 \cap Supp(\mu_1) : u_1(x_1^n) \leq \max\{u_1(x_1), E_1\}\}. \tag{7}$$

contain all the values  $x_1^n$  that the first characteristic of a new product should have in order to deliver either a higher utility than that of both a random product from  $G$  and the initial (partially observed) product, see Equation (6), or a lower-equal utility than that of either a random product from  $G$  or the initial (partially observed) product, see Equation (7).

Suppose that, after the value  $x_1$  has been observed as the first characteristic for the initial product, the DM chooses the option “starting with a new product”. Then, a random product from  $G$  would give him an expected utility value of  $E_1$ . At the same time, the utility of the new product would be  $u_1(x_1^n)$ , with  $x_1^n$  describing the value of the first characteristic observed from the new product. If, for instance,  $u_1(x_1^n) \leq \max\{u_1(x_1), E_1\}$ , then the DM would be better off choosing between the initial (partially observed) product and a randomly chosen one.

Figures 1 and 2 illustrate, respectively the sequential information acquisition process of DMs that follows from an initial observation point, OR, and the resulting expected continuation (CT) and starting (ST) payoffs. Figure 2, in particular, has been



**Figure 1.** Sequential information acquisition within a two-observation setting

$$\mu(x_1 \geq x_1^*) F(x_1)$$

$$\mu(x_1 < x_1^*) H(x_1)$$

**Figure 2.**  
Expected payoffs  
derived from a two-  
observation setting

introduced to highlight the relative simplicity of the two-observations environment when compared to the substantial complexities that follow from adding just one or two observations, as Figures 5 and 8, respectively illustrate. The final choice made by DMs is the maximum [in expected utility terms] between the certainty equivalent product, (CE), and either the initial fully observed product, (2 Ch), or the initial and the new ones, both being only partially observed (1 Ch – 1 Ch). This notation is maintained through the remaining figures representing the sequential information acquisition process of DMs. To simplify notations, in the figures representing the expected payoffs obtained by DMs, the probability subscripts have been eliminated from the functions  $\mu$ , as it is clear on which characteristics the probability functions are being defined.

#### 4.1 Existence of optimal thresholds

Clearly, the expected utility functions  $F$  and  $H$  determine the optimal information acquisition process followed by the DM. Assume that the DM observes  $x_1$  after acquiring information on the initial product. Then, the DM will either continue gathering information on the initial product or switch to a new one depending on whether it is  $F$  or  $H$  the one achieving the highest value at  $x_1$ . If both functions have the same value at  $x_1$ , then the DM would be indifferent between continuing acquiring information on the initial product and switching to a new one. These indifference threshold values partition  $X_1$  in subintervals that determine whether the DM continues observing the initial product or switches and starts acquiring information on a new one.

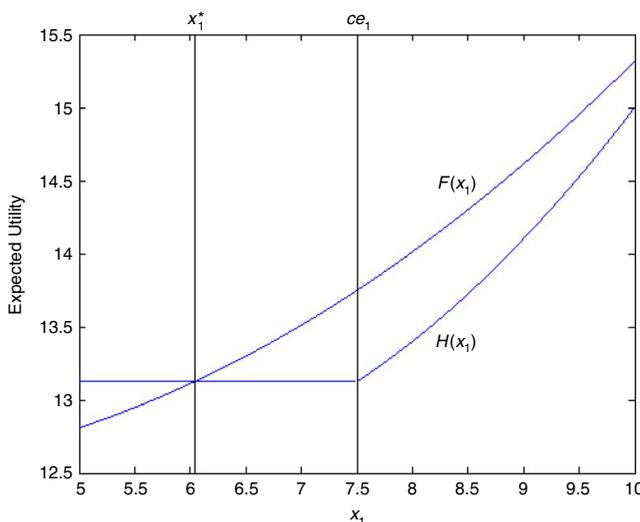
We will denote by  $x_1^*$  the value of the first characteristic determining the optimal threshold within the two observations setting. For expositional simplicity, and following the results of the numerical simulation, we will consider a unique threshold value,  $x_1^* < ce_1$ , through the numerical analysis. However, it should be noted that the analysis can be easily extended to account for a larger number of threshold values. In order to prevent any loss of generality, we will allow the threshold value to be located within any of the intervals defined by  $ce_1$  when analyzing formally the different information acquisition settings.

4.2 Numerical simulations

Decision theoretical models, mostly in their economic and operational research variants, tend to assume risk neutral DMs. Thus, even though the current model allows for any level of risk aversion to be imposed on the DM's utility function, we will maintain the risk neutrality assumption. The effects of an increase in the (relative) risk aversion coefficient of DMs within the current two observations setting are presented in Di Caprio and Santos Arteaga (2009). In addition, DMs will be assumed to be endowed with a well-defined preference order within and among characteristics. In other words, the first characteristic defined by the DM will be assumed to be more important and, consequently, provide the DM with a higher expected utility than the second one.

Consider, as the basic reference case, the optimal information acquisition behavior that follows from a standard risk neutral utility function, i.e.  $u_i(x_i) = x_i$ , with  $i = 1, 2, \dots$  when uniform probabilities are assumed on both  $X_1$  and  $X_2$ , i.e.  $\forall x_1 \in X_1 = [5, 10]$ ,  $\mu_1(x_1) = 1/5$ , and  $\forall x_2 \in X_2 = [0, 10]$ ,  $\mu_2(x_2) = 1/10$ . This case is represented in Figure 3, where the main features of the functions  $F$  and  $H$  obtained through the above theoretical analysis can be easily verified. In all the numerical figures, the horizontal axis represents the set of possible  $x_1$  realizations that may be observed by the DM, with the corresponding subjective expected utility values defined on the vertical axis and the certainty equivalent and threshold values explicitly identified through a vertical line.

We have assumed uniform densities on the distribution of characteristics through the paper since they allow for the highest information entropy (lowest information content) on the side of DMs (see Tavana, 2004). In other words, DMs are unaware of the differences existing between the characteristics in terms of distributional properties and must therefore focus on their subjective weights. However, any other density can be incorporated when defining the expected search utilities and the mass of the probability function can also be shifted in response to exogenous signals or changes in the subjective degree of optimism or pessimism exhibited by DMs. Note that DMs tend to differ in their subjective evaluations of probability functions (see Kahneman and



**Figure 3.**  
Continuation and  
starting regions  
within a two-  
observation setting

Tversky, 2000). In this case, DMs may be asked how optimistic or pessimistic they are about the potential realizations of the characteristics from different products and these distinctions incorporated when defining their respective information acquisition processes.

The current model may also account explicitly for concepts such as loss and ambiguity aversion (Kahneman and Tversky, 2000), when allowing for different degrees of risk aversion and introducing signals and learning processes among DMs. We have considered both these extensions in a related environment with two observations, where different types of risk attitudes as well as signals and learning were introduced to identify a formal phenomenon that was defined as search aversion (see Di Caprio and Santos Arteaga, 2009, 2014).

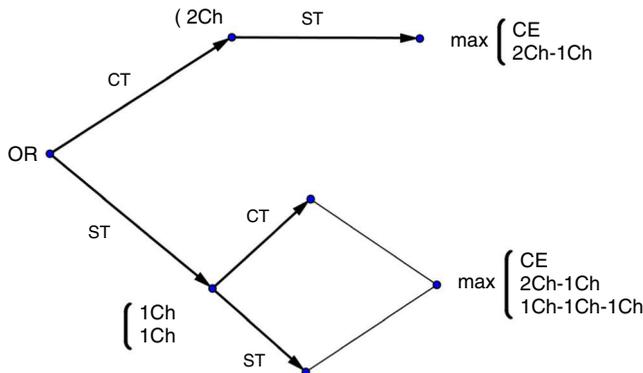
**5. Further observations**

*5.1 The three-observation setting: expected search utilities*

The sequential information acquisition process defined within a three observations setting is represented in Figure 4. The intuition is quite similar to the two observations case. After acquiring the first (initial) observation, the DM must choose between continuing gathering information on the product observed or starting gathering information on a second product:

- (1) (CT): the former option will lead the DM to fully observe the first product and partially a second one. His final choice will be made between both these products and, if these products do not deliver a sufficiently high expected utility, the CE one.
- (2) (ST): the latter option leads the DM to partially observe two products. He will face an additional decision regarding whether to observe any of these products fully or start gathering information on a third one. His final choice set will comprise either one fully and one partially observed product or three partially observed products. If all fail to deliver a sufficiently high expected utility, the CE product will be chosen.

In this section, we define the expected utility values derived from the Continuation and Starting information acquisition options before the first observation is actually gathered. This is done to provide the reader with a more intuitive presentation of the theoretical structure based on the corresponding decision trees. However,



**Figure 4.**  
Sequential  
information  
acquisition within a  
three-observation  
setting

when illustrating the results numerically, the first observation on a given product will be assumed to have already been acquired, which conditions the posterior behavior exhibited by the DM.

Consider first the Continuation expected value. Denote by  $x_1^n$  the first characteristic observed from a new product different from the initial one. In this case, it refers to the third observation gathered by the DM, after two characteristics have been observed from a first [initial] product. Divide the domain on which  $x_1^n$  is defined in two  $ce_1$ -based sub sets, i.e.  $\mu_1(x_1^n \geq ce_1)$  and  $\mu_1(x_1^n < ce_1)$ . The expected value from continuing acquiring information on the first product observed is given by:

$$\int_{x_1^m}^{x_1^M} \mu_1(x_1) \left[ \int_{ce_1}^{x_1^{nM}} \mu_1(x_1^n) \left[ \int_{P^+(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \right. \\ \left. \left. + \int_{P^-(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1^n) + E_2) dx_2 \right] dx_1^n + \int_{x_1^m}^{ce_1} \mu_1(x_1^n) F(x_1) dx_1^n \right] dx_1 \quad (8)$$

where  $P^+(x_1|x_1^n) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > u_1(x_1^n) + E_2 - u_1(x_1)\}$  and similarly for  $P^-(x_1|x_1^n)$ . Note that  $x_1^{nM}$  and  $x_1^{nm}$  are, respectively, the maximum and minimum realizations of  $x_1^n$ , which, in the current setting, coincide with  $x_1^M$  and  $x_1^m$ .

The continuation expected payoff builds on the fact that if the DM continues gathering information on the product whose first characteristic he has observed then he will end up observing both characteristics from this product plus one from a new product. As a result, the ability of the DM to acquire this final piece of information must be taken into account when defining his final choice. That is, the product whose both characteristics have been observed,  $(x_1, x_2)$ , will only be chosen if it improves upon the partially observed second product,  $(x_1^n, ce_2)$ , and the CE product. In this respect, we have defined  $F(x_1|x_1^n)$  as a variation of  $F(x_1)$  with such a modification incorporated:

$$F(x_1|x_1^n) = \int_{P^+(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + \int_{P^-(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1^n) + E_2) dx_2. \quad (9)$$

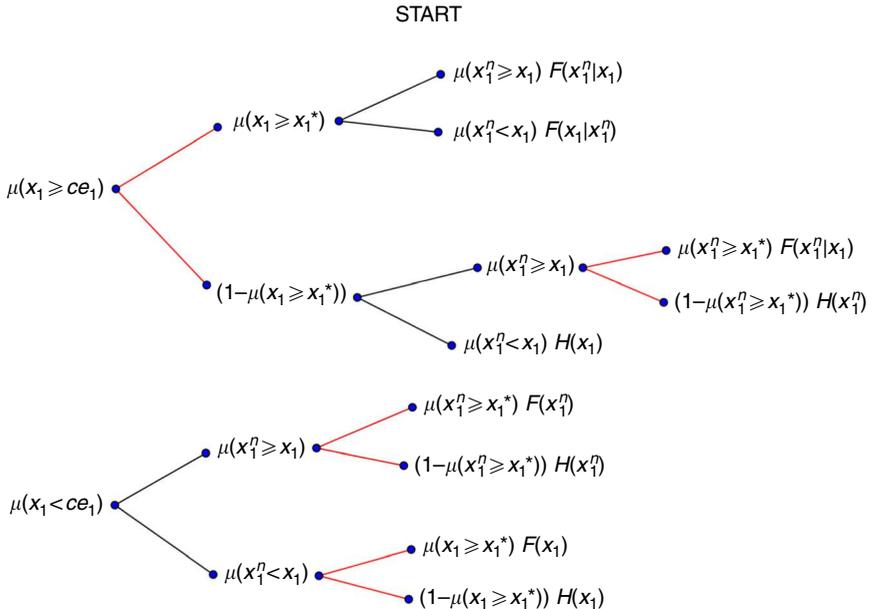
Clearly, the second product becomes a relevant option only when  $x_1^n > ce_1$ , which is accounted for by the  $\int_{ce_1}^{x_1^{nM}} \mu_1(x_1^n) dx_1^n$  term within the Continuation expression. Otherwise, the CE product would be used as the reference one, leading to the  $F(x_1)$ -based term.

Consider now the Starting expected utility value. Note that, in the Starting setting,  $x_1^n$  is the second observation gathered, while it was the third one in the Continuation case. Divide the domain of  $x_1$  in two different sub sets relative to the value of  $ce_1$ . Figure 5 illustrates the division of the expected utility derived from starting acquiring information on a second product in two  $ce_1$ -based intervals. The first interval describes the case where the first observation from the first product,  $x_1$ , corresponds to a value above the certainty equivalent, i.e.  $\mu(x_1 \geq ce_1)$ . In this case, two possible subcases must be considered:

- (1)  $(x_1 \geq x_1^*)$ : which would lead the DM to favor the continuation option on the first product observed; and
- (2)  $(x_1 < x_1^*)$ : which would lead the DM to favor either the continuation option on the second product observed or the starting option on a new (third) product.

$$\mu(x_1^n \geq ce_1) F(x_1 | x_1^n)$$

$$\mu(x_1^n < ce_1) F(x_1)$$



**Figure 5.**  
Expected payoffs  
derived from a three-  
observation setting

In order to describe both these subcases we have normalized the probability [mass] contained within  $\mu(x_1 \geq ce_1)$  in two sub sets defined relative to  $x_1^*$ ,  $\mu(x_1 \geq x_1^*)$  and  $1 - \mu(x_1 \geq x_1^*)$ . The former accounts for the probability that  $x_1$  falls within the  $[x_1^*, x_1^M]$  interval while  $1 - \mu(x_1 \geq x_1^*)$  denotes the probability that  $x_1$  falls within  $[ce_1, x_1^*]$ . The same type of reasoning applies to the normalization of the probability contained within  $\mu(x_1 < ce_1)$ . When normalizing the probability contained within a given interval, we will make use of the previous notation and indicate it through a red linking line in the corresponding figure. If the probability is not normalized, then the notation will be given by  $\mu(x_1 \geq x_1^*)$  and  $\mu(x_1 < x_1^*)$ , with a black line joining the respective options within the figure. Note that the starting option has already been chosen, so  $x_1$  provides the DM with a remanent observation to which he may return if the additional observations do not deliver a product whose expected utility is at least as high as the CE one.

Clearly, the information acquisition behavior of the DM will be conditioned by the value of  $x_1^n$  observed. In this regard, we should note that a heuristic mechanism has been applied to the behavioral process illustrated in Figure 5 (and through the rest of

the paper). That is, we are considering the threshold value obtained in the two observations case,  $x_1^*$ , when determining the information acquisition behavior of DMs. However, when deciding the use given to his final observation available, i.e. the third one, the DM should consider both the values of  $x_1$  and  $x_1^n$  when defining the corresponding optimal threshold value. In this case, the threshold “value” should be given by a plane within a three-dimensional space. This type of reasoning implies that an additional dimension must be considered each time the number of observations available increases, which constitutes a serious representability problem when analyzing any environment with four or more observations. We return to this point in Section 7.

Consider the two possible subcases arising within the current setting in more detail:

- (1) If  $(x_1 \geq x_1^*)$ : then the first characteristic observed from a second product,  $x_1^n$ , may be either above the observed  $x_1$  or below. If it is above,  $(x_1^n \geq x_1)$ , then the DM will gather a second piece of information from the new product,  $x_2^n$ . If the newly observed product  $(x_1^n, x_2^n)$  does not provide a utility higher than  $(x_1, ce_2)$ , which constitutes the reference product to beat, then this latter product will be chosen by the DM over the CE one (since  $x_1 \geq ce_1$ ).
- (2) If  $(x_1 < x_1^*)$ : then the first characteristic observed from a second product,  $x_1^n$ , may be either above the observed  $x_1$  or below. If it is above,  $(x_1^n \geq x_1)$ , then two new subcases arise depending on the normalized probability defined within  $[x_1, x_1^M]$ , namely,  $x_1^n$  may be above or below  $x_1^*$ :
  - If  $(x_1^n \geq x_1^*)$ : then the DM will acquire a second piece of information from the new product,  $x_2^n$ . If the newly observed product  $(x_1^n, x_2^n)$  does not provide a utility higher than  $(x_1, ce_2)$ , which constitutes the reference product to beat, then this latter product will be chosen by the DM over the CE one (since  $x_1 \geq ce_1$ ).
  - If  $(x_1^n < x_1^*)$ : then the DM will start acquiring information on a new (third) product,  $x_1^{n+1}$ . The new observation on the third product must improve upon  $x_1^n$ , since  $x_1^n \geq x_1 \geq ce_1$ . If it does not, then  $(x_1^n, ce_2)$  will be chosen.
  - If it is below,  $(x_1^n < x_1)$ , then the DM will start gathering information on a new (third) product,  $x_1^{n+1}$ . The new observation on the third product must improve upon  $x_1$ , since  $x_1 \geq ce_1$ . If it does not, then  $(x_1, ce_2)$  will be chosen.

The resulting expected utility is therefore given by:

$$\begin{aligned} &\mu(x_1 \geq ce_1) \{ \mu(x_1 \geq x_1^*) [ \mu(x_1^n \geq x_1) F(x_1^n | x_1) + \mu(x_1^n < x_1) F(x_1 | x_1^n) ] \\ &+ (1 - \mu(x_1 \geq x_1^*)) [ \mu(x_1^n \geq x_1) [ \mu(x_1^n \geq x_1^*) F(x_1^n | x_1) + (1 - \mu(x_1^n \geq x_1^*)) H(x_1^n) ] \\ &+ \mu(x_1^n < x_1) H(x_1) ] \} \end{aligned} \tag{10}$$

where:

$$F(x_1^n | x_1) = \int_{P^+(x_1^n | x_1)} \mu_2(x_2^n) (u_1(x_1^n) + u_2(x_2^n)) dx_2^n + \int_{P^-(x_1^n | x_1)} \mu_2(x_2^n) (u_1(x_1) + E_2) dx_2^n \tag{11}$$

with  $P^+(x_1^n | x_1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2^n) > u_1(x_1) + E_2 - u_1(x_1^n)\}$  and similarly for  $P^-(x_1^n | x_1)$ , and:

$$H(x_1^n) = \int_{Q^+(x_1^n)} \mu_1(x_1^{n+1}) (u_1(x_1^{n+1}) + E_2) dx_1^{n+1} + \int_{Q^-(x_1^n)} \mu_1(x_1^{n+1}) (\max\{u_1(x_1^n), E_1\} + E_2) dx_1^{n+1} \quad (12)$$

with  $Q^+(x_1^n) = \{x_1^{n+1} \in X_1 \cap \text{Supp}(\mu_1) : u_1(x_1^{n+1}) > \max\{u_1(x_1^n), E_1\}\}$  and similarly for  $Q^-(x_1^n)$ , where  $x_1^{n+1}$  denotes the value of the third observation gathered, which, in this case, corresponds to the first characteristic from a new third product.

We proceed now with the second interval, which describes the case where the first observation from the first product,  $x_1$ , corresponds to a value below the certainty equivalent. The *ex ante* probability of this event is  $\mu(x_1 < ce_1)$ . Given such an event, the observation gathered from a new [second] product,  $x_1^n$ , could be either higher than the first observation from the initial product,  $\mu(x_1^n \geq x_1)$ , or lower,  $\mu(x_1^n < x_1)$ :

- (1) If  $(x_1^n \geq x_1)$ : then we must consider the threshold value  $x_1^*$  as the one determining the optimal behavior of the DM. The next step requires the normalization of the probability mass contained within the interval  $[x_1, x_1^*]$ . This gives place to two probability densities being defined within this interval,  $\mu(x_1^n \geq x_1^*)$ , which defines the probability of the new observation being above the threshold value, and  $(1 - \mu(x_1^n \geq x_1^*))$ , which accounts for the  $x_1^n \in [x_1, x_1^*]$  domain:
  - If  $(x_1^n \geq x_1^*)$ : then the DM will gather a second observation from the new (second) product observed, leading to an expected utility value given by  $F(x_1^n)$ , as in the two observations case but with respect to  $x_1^n$ .
  - If  $(x_1^n < x_1^*)$ : then the DM will start acquiring the first observation from a new (third) product,  $x_1^{n+1}$ , leading to an expected utility value given by  $H(x_1^n)$ , as in the two observations case but relative to  $x_1^n$ . That is, when defining  $H(x_1^n)$ , if  $x_1^{n+1}$  is below the  $ce_1$ , then the DM considers the maximum between the  $ce_1$  value and the first characteristic of the second product observed,  $x_1^n$ , which is higher than  $x_1$ .
- (2) If  $(x_1^n < x_1)$ : then the first observation prevails as the highest one, and the resulting expected utility depends on whether this observation is above or below the threshold value. The normalization of the probability mass contained within the interval  $[x_1^n, x_1]$  will be required, with the resulting densities being defined relative to  $x_1^*$ :
  - If  $(x_1 \geq x_1^*)$ : then the DM will use his last observation to continue gathering information on the first product observed, leading to an expected utility of  $F(x_1)$ , as defined in the two observations case.
  - If  $(x_1 < x_1^*)$ : then the DM will start acquiring a new observation from a third product,  $x_1^{n+1}$ , leading to an expected utility defined by  $H(x_1)$ , as in the two observations case, but with  $x_1^{n+1}$  instead of  $x_1^n$  and given  $x_1^n < x_1 < ce_1$ .

The resulting expected utility is therefore given by:

$$\begin{aligned} \mu(x_1 < ce_1) & \left[ \mu(x_1^n \geq x_1) [\mu(x_1^n \geq x_1^*)F(x_1^n) + (1 - \mu(x_1^n \geq x_1^*))H(x_1^n)] \right. \\ & \left. + \mu(x_1^n < x_1) \begin{cases} F(x_1) & \text{if } x_1 \geq x_1^* \\ H(x_1) & \text{if } x_1 < x_1^* \end{cases} \right] \end{aligned} \quad (13)$$

Finally, note that the expected utility derived from Starting gathering information on a new product is defined by the sum of the expected utilities derived from both the upper and lower  $ce_1$ -based intervals.

### 5.2 Numerical simulations

This section illustrates numerically the optimal information acquisition behavior of a DM as the number of observations increases from two to three. For consistency and comparability purposes, the numerical values assumed remain as in the two observations risk neutral case.

The Continuation function simulated differs slightly from the expected Continuation payoff defined in the previous subsection. This is due to the fact that the DM observes the first characteristic from a first (initial) product,  $x_1$ , before deciding whether to continue with the same or start over with a new product. That is, the value of  $x_1$  determines his subsequent information gathering behavior. Thus, the continuation value obtained after having observed the first characteristic from an initial product reads as follows:

$$\begin{aligned} & \int_{ce_1}^{x_1^{nM}} \mu_1(x_1^n) \left[ \int_{P^+(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1^n) + E_2)dx_2 \right] dx_1^n \\ & \quad + \int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2 \right] dx_1^n. \end{aligned} \quad (14)$$

Note that this expression corresponds to the expected Continuation payoff when the uncertainty regarding  $x_1$  has been resolved.

Consider now the two subcases composing the Starting option. First, we describe the Starting above  $ce_1$  scenario. In this case, if we were not aware of the optimal threshold value being defined below  $ce_1$  when two characteristics are observable, we should have considered the following function, which allows for thresholds to be defined above  $ce_1$ .

If  $(x_1 \geq x_1^*)$ :

$$\left[ \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n)F(x_1^n|x_1) dx_1^n + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n)F(x_1|x_1^n) dx_1^n \right] \quad (15)$$

If  $(x_1 < x_1^*)$ :

$$\left[ \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left\{ \left( \frac{x_1^{nM} - x_1^*}{x_1^{nM} - x_1} \right) F(x_1^n | x_1) + \left( \frac{x_1^* - x_1}{x_1^{nM} - x_1} \right) H(x_1^n) \right\} dx_1^n + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) H(x_1) dx_1^n \right] \quad (16)$$

Clearly, when normalizing the probability mass within the corresponding interval, a uniform (density) function has been assumed, as in the numerical case. This simplifies the presentation and relates the theoretical setting to the numerical simulation without triggering any generality loss. Moreover, we know from the two-observation case that  $x_1^* < ce_1$ , implying that the second part of the above expression happens with zero probability, which allows for a simplified simulation expression given by:

$$\left[ \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) F(x_1^n | x_1) dx_1^n + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) [F(x_1 | x_1^n) \vee F(x_1)] dx_1^n \right] \quad (17)$$

where  $[F(x_1 | x_1^n) \vee F(x_1)]$  implies that, since  $x_1^n$  may be either above or below  $ce_1$  when  $x_1^n < x_1$ , the improvement on the first product observed through  $x_2$  may be relative to either  $x_1^n$  [if  $x_1^n \geq ce_1$ ] or  $ce_1$  (if  $x_1^n < ce_1$ ). Note that this expression is explicitly developed within the Continuation expected payoff of the previous subsection.

Finally, consider the Starting below  $ce_1$  scenario. Theoretically,  $x_1^*$  may be located either above or below  $x_1$  within the current scenario. Thus, both situations are initially allowed for in the simulation equation, whose intuition follows directly from the description presented in the previous subsection[2]:

$$\left[ \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left( \frac{x_1^{nM} - x_1^*}{x_1^{nM} - x_1} \right) F(x_1^n) dx_1^n + \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left( \frac{x_1^* - x_1}{x_1^{nM} - x_1} \right) H(x_1^n) dx_1^n \right] + \left[ \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left( \frac{x_1 - x_1^*}{x_1 - x_1^{nm}} \right) F(x_1) dx_1^n + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left( \frac{x_1^* - x_1^{nm}}{x_1 - x_1^{nm}} \right) H(x_1) dx_1^n \right] \quad (18)$$

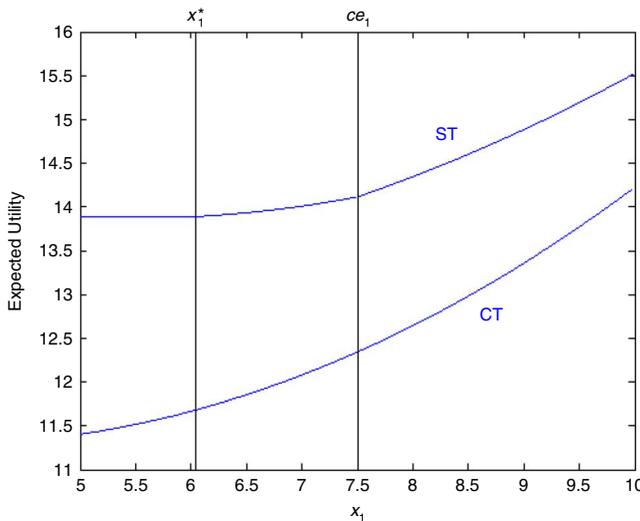
where:

$$F(x_1^n) \stackrel{def}{=} \int_{P^+(x_1^n)} \mu_2(x_2^n) (u_1(x_1^n) + u_2(x_2^n)) dx_2^n + \int_{P^-(x_1^n)} \mu_2(x_2^n) (E_1 + E_2) dx_2^n \quad (19)$$

with  $P^+(x_1^n) = \{x_2^n \in X_2 \cap Supp(\mu_2) : u_2(x_2^n) > E_1 + E_2 - u_1(x_1^n)\}$  and similarly for  $P^-(x_1^n)$ . Clearly, the above expression simplifies to:

$$\int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left\{ \left( \frac{x_1^{nM} - x_1^*}{x_1^{nM} - x_1} \right) F(x_1^n) + \left( \frac{x_1^* - x_1}{x_1^{nM} - x_1} \right) H(x_1^n) \right\} dx_1^n + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left\{ \left( \frac{x_1 - x_1^*}{x_1 - x_1^{nm}} \right) F(x_1) + \left( \frac{x_1^* - x_1^{nm}}{x_1 - x_1^{nm}} \right) H(x_1) \right\} dx_1^n \quad (20)$$

Figure 6 illustrates the optimal information acquisition behavior that follows from a standard risk neutral utility function when uniform probabilities are assumed on both  $X_1$  and  $X_2$  and the DM is allowed to gather three observations. In the figure, the



**Figure 6.**  
Continuation and  
starting regions  
within a three-  
observation setting

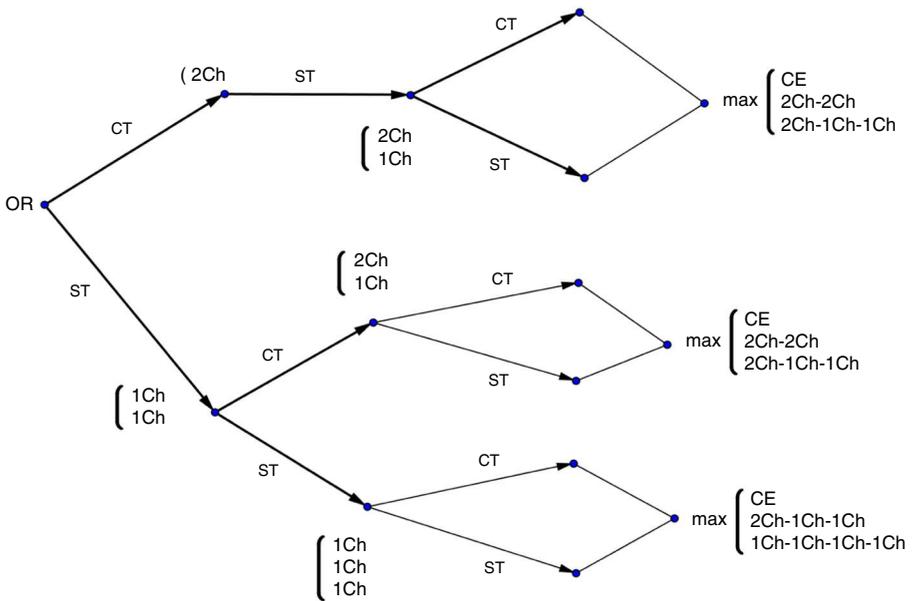
horizontal axis represents the set of possible  $x_1$  realizations that may be observed by the DM, with the corresponding expected utilities defined on the vertical axis and the  $ce_1$  and  $x_1^*$  values explicitly identified through vertical lines. Clearly, the Starting expected payoff dominates the Continuation option through the entire domain on which  $X_1$  is defined. This result differs substantially from the one obtained in the two observations case. In the current setting, the ability of DMs to acquire an additional piece of information translates into an increase in their preference for variety. This preference for variety contrasts sharply with the much stricter continuation criteria exhibited by DMs when only two observations are left to gather.

## 6. Toward the $n$ observations case

### 6.1 Four observations

The decision tree representing the four observations case is illustrated in Figure 7. The basic intuition determining the behavior of the DM reads as follows. After observing the first characteristic from a first product,  $x_1$ , the DM must decide whether to continue observing the second characteristic from the first product,  $x_2$ , or start gathering information on a new product,  $x_1^n$ :

- (1) (CT): If he continues gathering information on the first product, he will observe both characteristics from the first product,  $(x_1, x_2)$ , and must therefore start with the first characteristic from a new (second) product,  $x_1^n$ . Depending on whether this new observation is above or below  $x_1^*$ , the DM will either continue acquiring information on the new observed product,  $x_2^n$ , or start with a third one,  $x_1^{n+1}$ . At the end, he will choose the highest one among the alternatives composing the final choice set, which will consist of either  $\{(x_1, x_2), (x_1^n, x_2^n)\}$  or  $\{(x_1, x_2), (x_1^n, ce_2), (x_1^{n+1}, ce_2)\}$ . If none of these products delivers a sufficiently high utility, then the CE product will be chosen.
- (2) (ST): If he starts gathering information on a new (second) product, then he will observe the first characteristic from two products,  $x_1$  and  $x_1^n$ , and must therefore



**Figure 7.**  
Sequential  
information  
acquisition within a  
four-observation  
setting

decide whether to continue acquiring information about either one of these products or start gathering information on a new (third) product,  $x_1^{n+1}$ .

- (CT): If he continues, he will acquire information on the highest of both products observed, either  $x_1$  or  $x_1^n$ , and afterwards he must decide whether to continue with the remaining product whose first characteristic has been observed or start with a new (third) product,  $x_1^{n+1}$ . This final decision is identical to the one faced in the above continuation case. Indeed, the final choice set comprises the same type of elements in both cases, though not necessarily the same products. Clearly, the decision process leading to the final choice set is different. In the above continuation case, the DM always proceeds with the first product observed,  $x_1$ , while in the current setting, he chooses between  $x_1$  and  $x_1^n$  and may actually discard continuing gathering information on  $x_1$ .
- (ST): If he starts, he will observe three first characteristics from three different products,  $x_1$ ,  $x_1^n$  and  $x_1^{n+1}$ . Thus, he must consider the maximum of these three and decide whether to continue with the product with the highest first characteristic or start acquiring information on a new (fourth) product,  $x_1^{n+2}$ . The final choice set is based on both these possibilities, where, besides the CE option, the DM may face either three products, one of them with both characteristics observed, or four different products with their first characteristics observed.

*6.2 The four-observation simulation process explained*

This subsection describes the equations required to illustrate numerically the behavior of the optimal threshold values as the number of observations acquired by the DM on a given set of products increases to four. In order to simplify the presentation, all

previous assumptions on the uniformity of the distribution of characteristics remain valid here. Moreover, the analytical description of any additional notation representing equations that follow an identical logic to the definitions presented in the previous sections will be omitted.

After observing the first characteristic from a first (initial) product,  $x_1$ , the DM must decide whether to continue gathering information on the first product,  $x_2$ , or start with a new [second] product,  $x_1^n$ . We analyze first the Continuation expected payoff. When continuing, the DM will be observing the first product entirely,  $(x_1, x_2)$ , and the first characteristic from a second product,  $x_1^n$ . This new characteristic determines the improvements upon the  $F(x_1)$  function defined on the first product observed and the resulting optimal information acquisition path. Thus, the continuation value after having observed the first characteristic from an initial product reads as follows.

If  $(x_1^n \geq ce_1)$ :

$$\begin{aligned} & \int_{P^+}(x_1^n) \mu_2(x_2^n) \int_{ce_1}^{x_1^{nm}} \mu_1(x_1^n) \left[ \int_{P^+(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1^n) + u_2(x_2^n)) dx_2 \right] dx_1^n dx_2^n \\ & \quad + \int_{P^-(x_1^n)} \mu_2(x_2^n) \int_{ce_1}^{x_1^{nm}} \mu_1(x_1^n) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \right] dx_1^n dx_2^n. \end{aligned} \tag{21}$$

The first term represents the combinations of  $(x_1, x_2)$  and  $(x_1^n, x_2^n)$  improving upon the CE product, while the second term considers those of  $(x_1, x_2)$  improving upon the CE product with  $(x_1^n, x_2^n) < (ce_1, ce_2)$ . Note that the above expression assumes that  $x_1^* < ce_1$ , as it is known from the two observations heuristic being applied. The corresponding information acquisition process described in Figure 8(a) allows for  $x_1^*$  to be contained within  $[ce_1, x_1^{nm}]$ .

If  $(x_1^n < ce_1)$  and  $x_1^n \in [x_1^*, ce_1]$ :

$$\begin{aligned} & \int_{P^+}(x_1^n) \mu_2(x_2^n) \int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) \left( \frac{ce_1 - x_1^*}{ce_1 - x_1^{nm}} \right) \left[ \int_{P^+(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1^n) + u_2(x_2^n)) dx_2 \right] dx_1^n dx_2^n \\ & \quad + \int_{P^-(x_1^n)} \mu_2(x_2^n) \int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) \left( \frac{ce_1 - x_1^*}{ce_1 - x_1^{nm}} \right) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \quad \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \right] dx_1^n dx_2^n. \end{aligned} \tag{22}$$

The normalized probability terms introduced in this and the following expression correspond to realizations of  $x_1^n$  located both above and below the  $x_1^*$  value, respectively.

The current setting represents the former set of possibilities. This expression describes the same type of scenario as the  $(x_1^n \geq ce_1)$  case, with potential improvements of  $(x_1, x_2)$  over  $(x_1^n, x_2^n)$  and the CE product, respectively. The final continuation term presented below describes the remaining set of possible realizations for the new observed product, that is, those located within the interval  $[x_1^{nm}, ce_1]$  but below  $x_1^*$ .

If  $(x_1^n < ce_1)$  and  $x_1^n \in [x_1^{nm}, x_1^*]$ :

$$\int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) \left( \frac{x_1^* - x_1^{nm}}{ce_1 - x_1^{nm}} \right) \left[ \int_{ce_1}^{x_1^{(n+1)M}} \mu_1(x_1^{n+1}) \left[ \int_{P^+(x_1|x_1^{n+1})} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \right. \\ \left. \left. + \int_{P^-(x_1|x_1^{n+1})} \mu_2(x_2)(u_1(x_1^{n+1}) + E_2) dx_2 \right] dx_1^{n+1} + \mu_1(x_1^{n+1} < ce_1) F(x_1) \right] dx_1^n. \quad (23)$$

This final term represents the choice made by the DM between the initial product observed and a second newly observed one, from which only its first characteristic,  $x_1^{n+1}$ , is known. The intuition is identical to that defining the continuation area within the three observations case, where choices were made between the initial totally observed product and the third partially observed one, as long as they delivered an expected utility higher than the CE product. Figure 8(a) illustrates the set of possible expected payoffs derived from the continuation option.

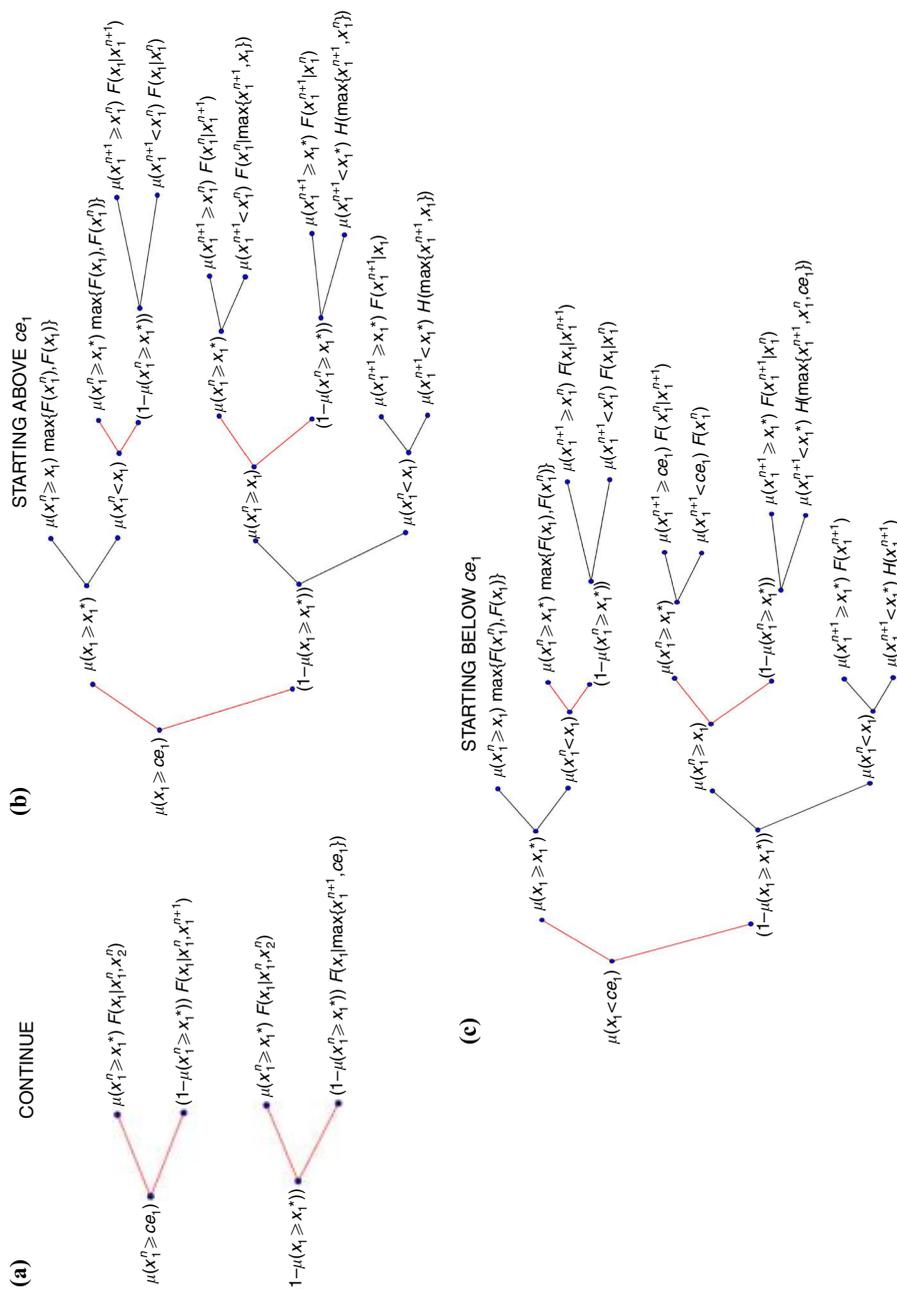
Consider now the two possible subcases defining the Starting option. First, we describe the Starting above  $ce_1$  scenario. As in the three observations setting, if we were not aware of the optimal threshold value being located below  $ce_1$  when two characteristics are observable, we should allow for threshold values to be defined both above and below  $ce_1$ , a general scenario that is indeed described in Figure 8(b).

The intuition defining the following expressions is almost identical to that of the continuation case just analyzed, with the main difference between both scenarios being the reference value delimiting the respective subcases. That is, the reference value determining the expected payoffs derived from the newly observed first characteristic,  $x_1^n$ , was given by  $ce_1$  in the Continuation setting, while here the reference value has been shifted to  $x_1$ , which is higher than both  $ce_1$  and  $x_1^*$  within the current scenario.

If  $(x_1^n \geq x_1)$ :

$$\int_{P^+(x_1^n)} \mu_2(x_2^n) \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left[ \int_{P^+(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ \left. + \int_{P^-(x_1|x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1^n) + u_2(x_2^n)) dx_2 \right] dx_1^n dx_2^n \\ + \int_{P^-(x_1^n)} \mu_2(x_2^n) \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \right] dx_1^n dx_2^n. \quad (24)$$

As in the Continuation setting, the first term represents the combinations of  $(x_1, x_2)$  over  $(x_1^n, x_2^n)$  improving upon the CE product, while the second term considers those of  $(x_1, x_2)$



**Figure 8.** Expected payoffs derived from a four-observation setting

improving upon the CE product with  $(x_1^n, x_2^n) < (ce_1, ce_2)$ . Clearly, the above expression is based on  $x_1^n \geq x_1 \geq ce_1 \geq x_1^*$ . On the other hand, we have the following possibilities arising from the  $x_1^n < x_1$  scenario described below.

If  $(x_1^n < x_1)$  and  $x_1^n \in [x_1^*, x_1]$ :

$$\begin{aligned} & \int_{P^+(x_1^n)} \mu_2(x_2^n) \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left( \frac{x_1 - x_1^*}{x_1 - x_1^{nm}} \right) \left[ \int_{P^+(x_1 | x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \left. + \int_{P^-(x_1 | x_1^n, x_2^n)} \mu_2(x_2)(u_1(x_1^n) + u_2(x_2^n)) dx_2 \right] dx_1^n dx_2^n \\ & + \int_{P^-(x_1^n)} \mu_2(x_2^n) \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left( \frac{x_1 - x_1^*}{x_1 - x_1^{nm}} \right) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \\ & \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \right] dx_1^n dx_2^n. \end{aligned} \tag{25}$$

As in the  $(x_1^n \geq x_1)$  Starting above  $ce_1$  setting, this expression accounts for the improvements of  $(x_1, x_2)$  over  $(x_1^n, x_2^n)$  and the CE product, respectively. The final Starting above  $ce_1$  term presented below describes the remaining set of possible realizations for the new observed product, that is, those located within the interval  $[x_1^{nm}, x_1]$  but below  $x_1^*$ .

If  $(x_1^n < x_1)$  and  $x_1^n \in [x_1^{nm}, x_1^*]$ :

$$\begin{aligned} & \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left( \frac{x_1^* - x_1^{nm}}{x_1 - x_1^{nm}} \right) \left[ \int_{ce_1}^{x_1^{(n+1)M}} \mu_1(x_1^{n+1}) \left[ \int_{P^+(x_1 | x_1^{n+1})} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 \right. \right. \\ & \left. \left. + \int_{P^-(x_1 | x_1^{n+1})} \mu_2(x_2)(u_1(x_1^{n+1}) + E_2) dx_2 \right] dx_1^{n+1} + \mu_1(x_1^{n+1} < ce_1) F(x_1) \right] dx_1^n. \end{aligned} \tag{26}$$

This final term represents the choice made by the DM between the initial product observed and a second newly observed one, from which only its first characteristic,  $x_1^{n+1}$ , is known. The intuition behind this expression is identical to that defining the continuation area within the three observations case and the final term in the current (four observations) Continuation setting, where choices are made between the initial totally observed product and the third partially observed one, as long as they deliver an expected utility higher than the CE product.

Finally, consider the Starting below  $ce_1$  scenario. The intuition defining the current setting follows directly from the description presented in the previous section when analyzing the corresponding scenario within the three observations case. The expected payoffs that result from the information acquisition process of DMs are presented in Figure 8(c).

Two possible subcases arise based on the location of the  $x_1^*$  value. First, if  $x_1^n \in [x_1^*, ce_1]$ , then the information acquisition behavior of the DM is identical to the

one just described for Starting above  $ce_1$  when  $x_1^n \geq x_1$ . Second, if  $x_1^n \in [x_1^{nm}, x_1^*]$ , then we have the following expected payoff:

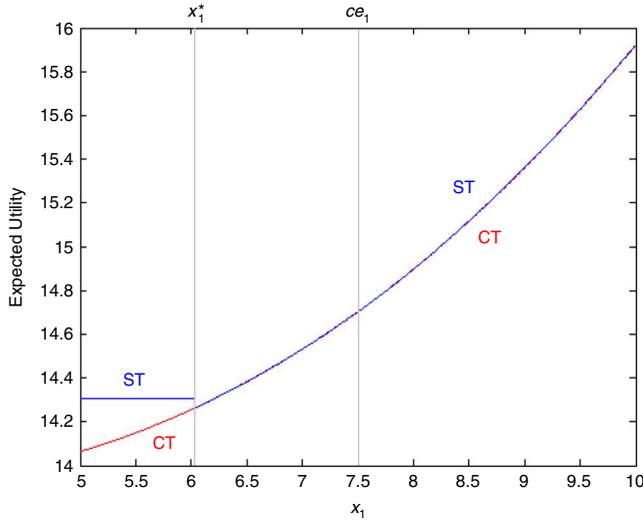
$$\begin{aligned} & \int_{x_1}^{x_1^{nM}} \mu_1(x_1^n) \left[ \left( \frac{x_1^{nM} - x_1^*}{x_1^{nM} - x_1} \right) [F(x_1^n | x_1^{n+1}) \vee F(x_1^n)] \right. \\ & \quad + \left( \frac{x_1^* - x_1}{x_1^{nM} - x_1} \right) \left\{ \int_{x_1^*}^{x_1^{(n+1)M}} \mu_1(x_1^{n+1}) F(x_1^{n+1}) dx_1^{n+1} \right. \\ & \quad \left. \left. + \int_{x_1^{(n+1)m}}^{x_1^*} \mu_1(x_1^{n+1}) H(x_1^{n+1}) dx_1^{n+1} \right\} dx_1^n \right. \\ & \quad + \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) \left\{ \int_{x_1^*}^{x_1^{(n+1)M}} \mu_1(x_1^{n+1}) F(x_1^{n+1}) dx_1^{n+1} \right. \\ & \quad \left. \left. + \int_{x_1^{(n+1)m}}^{x_1^*} \mu_1(x_1^{n+1}) H(x_1^{n+1}) dx_1^{n+1} \right\} dx_1^n. \end{aligned} \quad (27)$$

with:

$$\begin{aligned} F(x_1^n | x_1^{n+1}) \vee F(x_1^n) &= \int_{ce_1}^{x_1^{(n+1)M}} \mu_1(x_1^{n+1}) \left[ \int_{P^+(x_1^n | x_1^{n+1})} \mu_2(x_2^n) (u_1(x_1^n) + u_2(x_2^n)) dx_2^n \right. \\ & \quad \left. + \int_{P^-(x_1^n | x_1^{n+1})} \mu_2(x_2^n) (u_1(x_1^{n+1}) + E_2) dx_2^n \right] dx_1^{n+1} \\ & \quad + \mu_1(x_1^{n+1} < ce_1) F(x_1^n). \end{aligned} \quad (28)$$

The intuition defining the first term within the above expression is similar to the one used to describe the Starting above  $ce_1$  case when  $(x_1^n < x_1)$  and  $x_1^n \in [x_1^{nm}, x_1^*]$ , and has therefore been omitted. Moreover, note that the  $x_1^n$  observations located within  $[x_1^{nm}, x_1^*]$  imply that the DM starts acquiring information on a new third product,  $x_1^{n+1}$ . This new observation may be located above or below  $x_1^*$ , which conditions the final continuation decision of the DM. In this case, the definitions of  $F(x_1^{n+1})$  and  $H(x_1^{n+1})$  follow directly from those of  $F(x_1)$  and  $H(x_1)$  and have also been omitted. The resulting numerical simulation is presented in Figure 9, where it can be observed how the continuation and starting functions overlap for values of  $x_1 \geq x_1^*$ .

The jump in the Starting function at  $x_1 = x_1^* = 6.0355$  can be explained by the carrying over of the suboptimal product observations defined within  $(6.0355, 7.5)$ . In other words, this jump did not appear in the scenario with three observations because all the suboptimal product observations contained within  $(6.0355, 7.5)$  were eliminated. That is, if  $x_1 \in [6.0355, 7.5]$  and  $x_1^n < x_1$ , then the  $x_1^n$  product was not observed again, which allowed only for improvements on  $x_1$  when defining the corresponding Starting expected payoff. However, in the four observations setting, if  $x_1 \in [6.0355, 7.5]$  ( $x_1^n \in [6.0355, 7.5]$ ) and  $x_1^n \in [6.0355, x_1]$  ( $x_1 \in [6.0355, x_1^n]$ ), then  $x_1^n(x_1)$  determines the next product on which information is acquired after observing  $x_2(x_2^n)$ . These suboptimal products are also carried over at exactly  $x_1 = x_1^* = 6.0355$  for  $x_1 \in [6.0355, 7.5]$ , while they are trivially eliminated when  $x_1 \in [5, 6.0355]$ .



**Figure 9.**  
Continuation and starting regions within a four-observation setting

### 6.3 Five observations

We present here the decision tree defining the five observations case, describe some patterns that relate this case to the previous ones and provide some intuition regarding the  $n$ -observation case within the current [heuristic] environment. It should be already clear that a standard algorithmic structure cannot be defined, as the set of possible combinations increases in the number of observations. Thus, even though some basic patterns relating different information acquisition settings may be elicited, a general recursive algorithmic structure in the standard dynamic programming vein cannot be defined. We will return to this point in the following section.

The basic decision tree representing the five observations case is illustrated in Figure 10. The intuition determining the behavior of the DM reads as follows.

After observing the first characteristic from a product,  $x_1$ , the DM must decide whether to continue observing the second characteristic from the first product,  $x_2$ , or start gathering information on a new product,  $x_1^n$ :

- (1) (CT): If he continues, he will have observed two characteristics from the first product,  $(x_1, x_2)$ , and must therefore start with the first characteristic from a new (second) product,  $x_1^n$ . After acquiring this observation, he must decide whether to continue gathering information on the second product observed or start acquiring information on a new [third] product:
  - (CT)': If he continues, he will have observed two products completely,  $\{(x_1, x_2), (x_1^n, x_2^n)\}$ , and has to acquire the first characteristic from a new third product,  $x_1^{n+1}$ . Consequently, he will finally choose among the observed products and the CE one.
  - (ST)': If he starts, he will have observed completely the first product,  $(x_1, x_2)$ , and the first characteristic from another two,  $x_1^n$  and  $x_1^{n+1}$ . Therefore:
    - (CT)'': If he continues acquiring information, it will be on the maximum of the two partially observed products, either  $x_1^n$  or  $x_1^{n+1}$ . In this case, he will have observed completely two products and partially a third one.



- (ST)'': If he starts with another product, he will end up with a fully observed product,  $(x_1, x_2)$ , and three partially observed ones,  $x_1^n, x_1^{n+1}$  and  $x_1^{n+2}$ . At the end, he will choose the product providing him the highest expected utility among the final choice possibilities together with the CE product.
- (2) (ST): If he starts acquiring information on a new (second) product, then he will observe the first characteristic from two products,  $x_1$  and  $x_1^n$ , and must therefore decide whether to continue gathering information about either one of these products or start acquiring information on a new [third] product,  $x_1^{n+1}$ :
- (CT)': If he continues, he will gather information on the highest of the two products observed, either  $x_1$  or  $x_1^n$ , and afterwards he must decide whether to continue with the remaining product whose first characteristic has been observed or start with a new [third] product,  $x_1^{n+1}$ . This decision is identical to the one faced in the previous (initial) continuation case, and indeed the final set of choices comprises the same type of elements though not necessarily the same products. Note that, in fact, the decision process leading to the final set of choices is different. In the initial continuation case the DM always proceeds with the first product observed, while in the start case, he chooses between the first two products observed. The following possibilities remain:
    - (CT)'': If he continues, he will have observed two products completely,  $\{(x_1, x_2), (x_1^n, x_2^n)\}$ , and has to acquire the first characteristic from a new third product,  $x_1^{n+1}$ . Consequently, his final choice will be made among the observed products and the CE one.
    - (ST)'': If he starts with another product, he will end up with a fully observed product and two partially observed ones. Therefore, he must consider the maximum of the two partially observed ones and decide whether to continue with the product with the highest first characteristic or start gathering information on a new (fourth) product,  $x_1^{n+2}$ . The final choice set is based on both these possibilities, where, besides the CE option, the DM may face either four products, one of them with both characteristics observed, or three different products, two of which have been completely observed.
  - (ST)': If he starts, then he will have observed three first characteristics from three different products,  $x_1, x_1^n$  and  $x_1^{n+1}$ . Thus, he must consider the maximum of these three observations and decide whether to continue with the product with the highest first characteristic or start acquiring information on a new (fourth) product,  $x_1^{n+2}$ :
    - (CT)'': If he continues, then he will have observed one product completely and two partially. The maximum of the latter ones will therefore face the standard two observations decision scenario regarding whether to start with a new (fourth) product,  $x_1^{n+2}$ , or continue acquiring information on the maximum of the partially observed ones. The final choice set is based on both these possibilities, where, besides the CE option, the DM may face either three products, two of them with both characteristics observed, or four, with only one of them having both its characteristics observed.

- (ST)'': If he starts again, he will observe four first characteristics from four different products,  $x_1, x_1^n, x_1^{n+1}$  and  $x_1^{n+2}$ . Thus, once again, he must consider the maximum of these four and decide whether to continue with the product with the highest first characteristic or start acquiring information on a new (fifth) product,  $x_1^{n+3}$ . The final choice set is based on both these possibilities, where, besides the CE option, the DM may face either four products, one of them with both characteristics observed, or five different products with their first characteristics observed.

It should be emphasized that within the current heuristic setting, the decision regarding the optimal allocation of the observations depends on whether the maximum of the  $X_1$  realizations defining the set of partially observed products is above or below  $x_1^*$ , where  $x_1^*$  is the threshold value defined within the two observations case. As previously stated, an exact calculation of the corresponding threshold (hyperplane) requires an increasing number of characteristics (dimensions) being accounted for, which relate directly to the total number of observations that the DM may acquire. Finally, note the increment in the uncertainty regarding the products on which information has been acquired as the number observations gathered approaches five in the information acquisition process just described. This phenomenon constitutes a serious estimation problem when product suppliers try to elicit the expected information acquisition and choice behavior of DMs. We elaborate on this problem in the following section.

### 7. Evolution and limitations

The information acquisition process described through the paper is only partially complete. That is, when more than two observations are acquired, subsequent recalculations are required on the side of the DM. Consider, for example, the three observations setting. In this case, starting is the optimal behavior of the DM after observing  $x_1$ . Thus, after the DM observes  $x_1$  and  $x_1^n$ , he must decide how to allocate the third piece of information remaining. Following the approach described in the paper, he should first select the highest observation of the two acquired, which will be denoted by  $\bar{x}_1 = \max\{x_1, x_1^n\}$ . Then, he must calculate the respective  $F(\cdot)$  and  $H(\cdot)$  functions. These functions are determined by both  $\bar{x}_1$  and the other observation acquired, denoted  $\bar{x}_1$ . Moreover, the point of reference when calculating both functions should be  $ce_1$ . That is, the DM must consider whether the maximum of both these observations is above or below  $ce_1$  and then see where  $\bar{x}_1$  is located. Note that  $\bar{x}_1$  determines also the calculation of the  $F(\cdot)$  and  $H(\cdot)$  functions, particularly if it is above  $ce_1$ . The variable on the x-axis when simulating the information acquisition behavior of the DM should therefore be  $\bar{x}_1$ .

The following possibilities must be considered depending on the  $x_1$  and  $x_1^n$  characteristics observed:

- (1) If  $\bar{x}_1 < ce_1$ : then we have the standard  $F(x_1)$  and  $H(x_1)$  functions.
- (2) If  $\bar{x}_1 \geq ce_1$ : then the DM must check whether  $\bar{x}_1$  is above or below  $ce_1$ . This gives place to two possible situations:
  - If  $\bar{x}_1 < ce_1$ : then we have the standard  $F(\bar{x}_1)$  and  $H(\bar{x}_1)$  functions with the corresponding certainty equivalent values being used as reference points.
  - If  $\bar{x}_1 \geq ce_1$ : then we also have the standard  $F(\bar{x}_1)$  and  $H(\bar{x}_1)$  functions but, in this case, it is  $\bar{x}_1$  the variable acting as a reference point.

The DM should therefore be able to design an optimal information acquisition mechanism when searching among bi-dimensional products while endowed with a finite number of observations. However, it may already be intuitively clear that two considerable problems arise. The first one is obvious: as the dimension of the products (vectors) under consideration increases, so do the graphical and computational requirements imposed on the DM. In this case, an heuristic mechanism that decreases the dimensionality of the products should be designed. Moreover, a second equally complex problem arises from the supply side. Note, for example, that in order for a supplier to visualize the behavior of the DM when three observations can be acquired, a three-dimensional representation of the optimal behavior of the DM, which must be based on the possible values of  $x_1$  and  $x_1^n$  acquired, is required. Similarly, a four dimensional representation is required in the final stage of the four observations case. As a result, suppliers must decompose the behavior of DMs, based on the expected set of observations that they may acquire, and behave accordingly on a subjectively expected way. The first and second Appendix sections consider both these problems. In particular, the first section analyzes the three-observation setting when the heuristic mechanism is not imposed and proposes several modifications to the information acquisition process that widen the scope of the model introduced through the paper. The second section complements the current description of the four-observation setting while providing additional intuition.

Finally, acquisition patterns may be inferred when considering the  $n$ -observation setting relative to the  $(n-1)$ -observations one, but no recursivity can be defined between them. The easiest way to see this is in the three observations case when compared to that with two observations. Consider the three observations case. After observing two characteristics, the optimal information acquisition behavior of the DM is determined by both observations,  $x_1$  and  $x_1^n$ . Thus, when defining his optimal information acquisition behavior, the DM will face a similar scenario to the two observations case, but not identical. As stated above, it is only identical when both observations are below the  $ce_1$  value. If, on the other hand, they are both above, the resulting  $F(\cdot)$  and  $H(\cdot)$  functions differ from those calculated in the two observations case, where  $ce_1$  is always used as the reference value.

Similarly, if after acquiring the first observation in the four observations case, the DM decides to start acquiring information on a second product, then he will have acquired two observations and will be missing another two. Once again, this case is similar but not identical to the three observations case, where only one piece of information is assumed to have been acquired before analyzing the behavior of the DM. Clearly, if after acquiring the first observation in the four (three) observations case, the DM decides to continue acquiring information on the first product observed, then he will be adding a new observation to the analysis performed in the initial continuation setting of the three (two) observations case. Thus, the current setting does not allow for a recursive structure that simplifies the behavior of DMs. Though similar, each additional observation defines a totally new information acquisition setting.

Note that each new observation acquired adds two possible paths to each element composing the set of potential scenarios. This leads to additional potential scenarios based on the relative values of the new observation and those already acquired by the DM. Note also that we are unable to calculate the exact value of  $\bar{x}_1$ , since each additional observation may be higher than the  $x_1$  reference value whenever located above  $x_1^*$ . Thus, we are only able to calculate and operate with the corresponding expected utility values defined in terms relative to  $x_1^*$ , without knowing the exact threshold realization.

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Clearly, as the DM advances through the decision tree, he should be able to define a threshold value based on the uncertainty resolved through the previous observations. Even though this learning effect simplifies the information acquisition process, the definition of the  $n$ th-dimensional setting remains unattainable in the current theoretical environment. Refer to the third Appendix section for the description of an alternative theoretical framework that allows for the definition of an information acquisition structure accounting for an environment with  $n$  observations.

## 8. Conclusions and managerial significance

The information acquisition behavior modeled in this paper corresponds to that of a perfectly rational DM, i.e. endowed with complete and transitive preferences, whose objective is to choose optimally among the products available subject to a heuristic assimilation constraint. The current paper opens the way for additional research on heuristic information acquisition and choice processes when considered from a satisficing perspective that accounts for cognitive limits in the information processing capacities of DMs. In this regard, the reference threshold value defining the heuristic mechanism and the resulting information acquisition process of DMs may be interpreted as the level of satisficing required by a DM when designing his information acquisition strategy. This heuristic mechanism has been imposed to simplify a process that would otherwise require an increasingly larger calculation and information processing capacity on the side of the DM as the number of observations that may be acquired or the number of characteristics defining the products increases. Despite the heuristic mechanism imposed, a risk neutral DM has been shown to exhibit an uncertain and almost completely random information acquisition behavior when four observations are allowed to be gathered. This result illustrates the difficulties that arise when trying to forecast the behavior of rational DMs even in the current heuristic setting with a relatively small number of observations being acquired.

Decision making is a crucial component in many real life applications such as organization management, financial planning, products evaluation and recommendation. Thus, a considerable range of extensions from the current paper may be considered. For example, modifications in the preference formation process of DMs (see Chen *et al.*, 2013; Häubl and Murray, 2003), or their perception of the environment (see Johansson and Xiong, 2003; Kardes *et al.*, 2004), and their effect on their subsequent information acquisition and choice processes could be analyzed. Impatient DMs display this type of features when searching through an increasingly larger amount of information available on the world wide web (see Carr, 2011; Yao *et al.*, 2008).

Moreover, DMs tend to gather a relatively small amount of observations when subject to external time constraints and required to make immediate decisions within extremely different environments, see, for example, Kott *et al.* (2011) and Mulder and Voorbraak (2003). Therefore, it follows from the dynamic nature defining information acquisition that the design of information and expert systems should account for time pressure and its effects on the information gathering process of DMs. For example, when resources are limited, strategies requiring a high degree of analysis may be excluded from consideration, which may lead the DM to make suboptimal decisions (see Hwang, 1994) for a review of the literature.

Finally, a variety of research fields analyzes how different types of DMs make a considerable amount of decisions based on a relatively small amount of observations and constrained by limited information assimilation capacities. As a result, time constraints or any other type of cost should have a strategic effect on the information

acquisition process of DMs, see, for example, Lomax and Vadera (2011) and Masseglia *et al.* (2009), and should be accounted for in potential extensions of the current paper. In this regard, several papers have shown that as the time and effort required to complete a task increases, DMs tend to reduce information search at the expense of decision quality (see Ahituv *et al.*, 1998; Durrande-Moreau, 1999).

## Notes

1. The authors would like to thank an anonymous referee for suggesting this application.
2. It should be noted that we have actually used  $x_1^*$  as the reference value in the numerical simulations of the current scenario. The corresponding MATLAB codes are available from the authors upon request.

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## Appendix. Alternative scenarios

### A.1 Absent heuristics: the three-observations setting

This appendix section illustrates both formally and numerically the optimal information acquisition behavior of a rational DM when allowed to gather three pieces of information and absent any heuristic mechanism. That is, the DM must determine his optimal information acquisition behavior and the corresponding threshold values without relying on the heuristic mechanism derived from the two-observations setting's threshold. For consistency and comparability purposes and unless stated otherwise, the numerical values assumed through this section will be those used in the main text when simulating the three-observations risk neutral environment.

#### A.1.1 Intuition

Assume that the heuristic mechanism is not imposed and consider the environment where the DM acquires three observations. The DM faces the following options:

- (1) (CT): It is easy to see that the continuation option leads to an expected search utility identical to the one defining the heuristic continuation environment. This subsetting provides also important intuition regarding the development of the non-heuristic process. That is, when calculating the continuation payoff, the only reference points considered by the DM are the certainty equivalent values and the observed characteristic from the first product,  $x_1$ , relative to which the information acquisition improvements must be defined in terms of  $(x_1, x_2)$  and  $(x_1^n, ce_2)$ . If the DM decides to continue, he will be observing  $x_1, x_2$  and  $x_1^n$ . His information acquisition structure is therefore completely determined for any given set of realizations. That is, the DM has no other option than to observe  $x_1^n$  as third characteristic independently of the realizations of  $x_1$  and  $x_2$ . His final payoff will therefore be determined by  $x_1^n$  being either above or below  $ce_1$  and its relation to  $(x_1, x_2)$ . This final payoff must be used by the DM to calculate the continuation expected search utility before acquiring the second observation.
- (2) (ST): The calculation of the starting expected value would differ considerably from that of the heuristic framework. Note that the heuristic mechanism uses  $x_1^*$  as the reference point against which to define the behavior of the DM when acquiring the third and final piece of information after having observed  $x_1$  and  $x_1^n$  previously. This simplification allows the DM to define all his expected payoffs in relation to this final decision, which may initially seem not to be the correct one. To see why this is the case, consider what a fully rational DM should actually do. After observing the first characteristic,  $x_1$ , and knowing that he is going to observe  $x_1^n$ , the DM must account for four main possibilities given the values of  $x_1, x_1^n$  and the reference certainty equivalent value,  $ce_1$ . That is, for every  $x_1^n > x_1$  the DM should consider whether to continue with  $x_1^n$  or start with a third product,  $x_1^{n+1}$ , with a similar process taking place for  $x_1$  when  $x_1 > x_1^n$ . The resulting final

expected payoff must be used by the DM to calculate the starting expected search utility before acquiring the second observation. For example, assume that  $x_1^n > x_1$ . The DM must compute the value of the interim starting search utility given  $x_1^n$  as well as the value of the interim continuation search utility given both  $x_1^n$  and  $x_1$ . Then, the DM has to calculate the resulting set of optimal interim threshold values and the respective expected utilities located both above and below the threshold. These interim expected utilities should, at the same time, be inserted in the  $x_1^n > x_1$  subscenario multiplied by the corresponding occurrence probabilities relative to the certainty equivalent product,  $(ce_1, ce_2)$ , which serves as a reference value. The union of all potential subscenarios determines the value of the starting expected search utility before the DM acquires the second observation.

In summary, given the previous description it should be intuitively clear that the heuristic mechanism allows the DM to calculate the continuation and starting payoffs in the final decision stage for any dimension of the information acquisition problem, i.e. for any total number of observations considered. As we have already emphasized, this simplification does not necessarily deliver the optimal set of threshold values that should arise from a given n-dimensional space, but it allows the DM to compute a set of expected payoffs and choose accordingly. Otherwise, he should deal with an n-dimensional structure determining the final payoffs that must be brought back and considered before acquiring the second observation within his information acquisition process.

*A.1.2 Formalization: continuation option after acquiring  $x_1$*

The Continuation function is identical to the one introduced in the numerical simulations subsection within the three observations setting. Despite this fact, we describe here the entire information acquisition process of the DM when following the continuation option. The notation employed in the main text will be maintained through the appendix unless stated otherwise.

*A.1.2.1 The interim subdecision stage.* After observing the first characteristic from an initial product,  $x_1$ , the DM must consider all the possible realizations of  $x_1^n$  and  $x_2$  before deciding whether to continue with the same product or to start over with a new one. That is, the DM must calculate all the possible combinations of  $(x_1, x_2)$  and  $(x_1^n, ce_2)$  relative to  $(ce_1, ce_2)$  when deciding what to do after observing  $x_1$ . The continuation scenario is simpler than the starting one, as continuing determines the whole information acquisition behavior within the three observations setting. Note that the payoff received by the DM after having acquired  $x_1$  and  $x_1^n$  depends on the relative values of these variables together with that of  $x_2$ . Thus, the interim expected continuation payoff obtained after having acquired two observations is given by:

- (1) If  $x_1^n \geq ce_1$ :

$$\int_{P^+(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1^n) + E_2)dx_2 \quad (A1)$$

- (2) If  $x_1^n < ce_1$ :

$$\int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2 \quad (A2)$$

These functions represent the expected payoffs derived from the potential set of combinations of all three realizations,  $x_1, x_2$  and  $x_1^n$ . The expected utility attained by the DM after observing  $x_2$ , but before acquiring the third observation,  $x_1^n$ , in

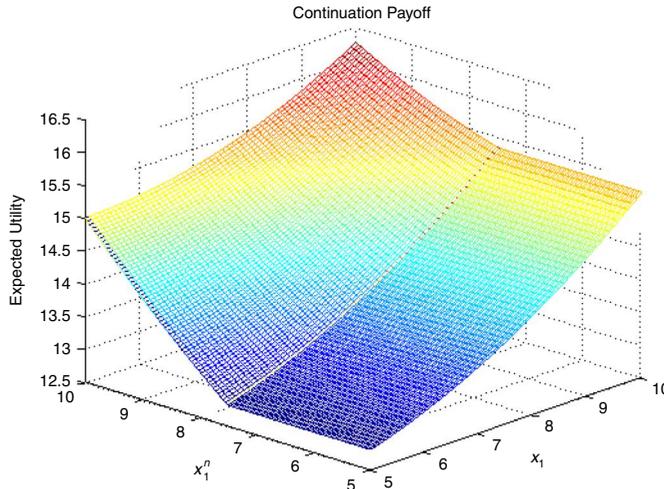
$(x_1, x_1^n, F(x))$ -space and based on the set of all possible  $x_2$  realizations is illustrated in Figure A1. Note that, in this case,  $F(x)$  stands for the continuation expected search utility resulting from all the potential combinations of  $x_1, x_2$  and  $x_1^n$ .

Note also that, when calculating this expected search utility  $x_2$  has not yet been observed and enters the equations in expected terms. The resulting figure is divided in two differentiated parts depending on  $x_1^n$  being above or below  $ce_1$ . That is, the potential improvements relative to  $(x_1, x_2)$  based on the realization of  $x_1^n$  depend on whether this latter observation is above the certainty equivalent value  $ce_1$ . This is particularly relevant when the product observed initially does not deliver a utility higher than the certainty equivalent one.

*A.1.2.2 The initial subdecision.* Given the expected utility values obtained in Figure A1, the DM must still compute the resulting expected search utility derived from continuing acquiring information on the first product observed. This brings us to the expected search utility equation described in the paper and given by:

$$\begin{aligned} \int_{ce_1}^{x_1^{nM}} \mu_1(x_1^n) & \left[ \int_{P^+(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 \right. \\ & \left. + \int_{P^-(x_1|x_1^n)} \mu_2(x_2)(u_1(x_1^n) + E_2)dx_2 \right] dx_1^n \\ & + \int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) \left[ \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))dx_2 \right. \\ & \left. + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)dx_2 \right] dx_1^n \end{aligned} \tag{A3}$$

Note that this expression corresponds to the expected Continuation payoff obtained when the uncertainty regarding  $x_1$  has been resolved. That is, this expression



**Figure A1.**  
Three-observation setting (Continuation option chosen after acquiring  $x_1$ ): acquiring the third observation for full range of  $x_1$  and  $x_1^n$

determines the expected continuation payoff obtained after observing  $x_1$  and given the expectation on the realizations of  $x_1^n$  applied to the equations defining Figure A1. Therefore this equation translates the three-dimensional setting of Figure A1 into a two-dimensional structure represented by the continuation function described within Figure 6 in the main text.

Finally, note that we have not imposed any heuristic mechanism, since the reference value remains the certainty equivalent product and improvements are defined relative to the certainty equivalent for the initial product (fully) observed and relative to the initial product and the certainty equivalent for the second (partially) observed product.

### A.1.3 Formalization: starting option after acquiring $x_1$

The starting option implies observing  $x_1$  and  $x_1^n$ , and then deciding whether to continue acquiring information on any of these two products or starting with a new (third) one,  $x_1^{n+1}$ . The starting option opens a new product and a new (third) dimension to be accounted for by the DM in the interim stage. Consequently, if four observations are considered, there must be a four dimensional figure determining the threshold values when the DM has to decide how to use his fourth and final piece of information.

*A.1.3.1 The three-dimensional interim subdecision stage.* Define by  $\bar{x}_1 = \max\{x_1, x_1^n\}$  and by  $\bar{x}_1$  the remaining element of the set, i.e. the lowest of both elements. We will need both these reference points together with  $ce_1$  as the main values determining the behavior of the DM when acquiring the third and final observation.

The decision of how to use the last piece of information depends on the realizations of both  $x_1$  and  $x_1^n$ . Given these realizations, the DM must calculate the expected payoffs derived from both available options: starting with a new product,  $x_1^{n+1}$ , and continuing with the highest of the previously observed ones, either  $x_1$  or  $x_1^n$ . These latter values, together with  $ce_1$ , divide the three-dimensional  $(x_1, x_1^n, E[U(x)])$ -space in four different octants, which define a symmetric structure along the plane that halves the space in two symmetric subspaces. In this case,  $E[U(x)]$  refers to the corresponding expected search utilities  $F(x)$  and  $H(x)$  resulting from all the potential combinations of  $x_1, x_2, x_1^n$ , and  $x_1^{n+1}$ .

Thus, the DM must calculate for each octant the expected search utilities derived from using the last piece of information to either start acquiring information on a new product or continue with any of the previously observed ones. These functions will determine the optimal behavior of the DM when acquiring the last observation for each possible value of  $x_1$  and  $x_1^n$ . Once the DM has determined the optimal behavior to follow when acquiring the last piece of information, he must consider the resulting expected search utilities from each octant and build a three-dimensional expected search function comparable to the one described in the continuation case above. After doing this, the DM must consider all the possible realizations of  $x_1^n$  with their corresponding probabilities and, given the optimal behavior derived from the last observation, calculate the expected value from continuing based on  $x_1^n$  and its associated density. That is, after computing both the continuation and starting expected search utilities based on the potential realizations of  $x_1^n$  and  $x_2$ , the DM faces a two-dimensional scenario where the starting and continuation functions determine his optimal behavior for any observed realization of  $x_1$ . The resulting starting expected search utility defines the corresponding two-dimensional starting function described in Figure 6, which will be the exact same figure that we will be obtaining through this section.

The possible subcases taking place after observing  $x_1$  and  $x_1^n$  relative to  $ce_1$  are described in the following subsections.

*A.1.3.2 First interim octant:  $\bar{x}_1 \geq ce_1$  and  $\bar{x}_1 \geq ce_1$ .* Consider first the subcase where both initial realizations are above the certainty equivalent value. The threshold values within the current subcase, if any, must therefore be determined by the highest of both realizations,  $\bar{x}_1$ , with  $\bar{x}_1$  serving as a substitute together with  $ce_2$  in case the product fully observed does not deliver a utility higher than the certainty equivalent one. The corresponding continuation (with  $\bar{x}_1$ ) function is given by:

$$F(\bar{x}_1) \stackrel{def}{=} \int_{P^+(\bar{x}_1)} \mu_2(x_2)(u_1(\bar{x}_1) + u_2(x_2)) dx_2 + \int_{P^-(\bar{x}_1)} \mu_2(x_2)(u_1(\bar{x}_1) + E_2) dx_2 \quad (A4)$$

with:

$$P^+(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > u_1(\bar{x}_1) + E_2 - u_1(\bar{x}_1)\} \quad (A5)$$

and:

$$P^-(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) \leq u_1(\bar{x}_1) + E_2 - u_1(\bar{x}_1)\} \quad (A6)$$

while the starting (with a new third product  $x_1^{n+1}$ ) function is given by:

$$H(\bar{x}_1) \stackrel{def}{=} \int_{Q^+(\bar{x}_1)} \mu_1(x_1^{n+1})(u_1(x_1^{n+1}) + E_2) dx_1^{n+1} + \int_{Q^-(\bar{x}_1)} \mu_1(x_1^{n+1})(u_1(\bar{x}_1) + E_2) dx_1^{n+1} \quad (A7)$$

with:

$$Q^+(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap Supp(\mu_1) : u_1(x_1^{n+1}) > u_1(\bar{x}_1)\} \quad (A8)$$

and:

$$Q^-(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap Supp(\mu_1) : u_1(x_1^{n+1}) \leq u_1(\bar{x}_1)\}. \quad (A9)$$

Both these functions are represented in Figure A2. It should be noted that the intuition describing these functions is identical to the one provided through the main body of the paper when accounting for changes in the corresponding reference values.

Note also that this figure is symmetric along the  $x_1 = x_1^n$  plane. Clearly, given the numerical values chosen, the continuation function (denoted by  $F(x_1)$  in all the figures of this section) is higher than the starting function (denoted by  $H(x_1)$  in all the figures of this section) for all the  $x_1$  and  $x_1^n$  values within the current octant.

Thus, given the current numerical setting, if the DM chooses to start acquiring information on a new product after observing  $x_1$ , and both  $x_1$  and  $x_1^n$  are above  $ce_1$ , then the DM will continue acquiring information on the product observed with the highest first characteristic, either  $x_1$  or  $x_1^n$ .

*A.1.3.3 Second interim octant:*  $\bar{x}_1 < ce_1$  and  $\bar{x}_1 < ce_1$ . In the second octant, with both  $x_1$  and  $x_1^n$  below  $ce_1$ , the continuation (with  $\bar{x}_1$ ) function is given by:

$$F(\bar{x}_1) \stackrel{def}{=} \int_{P^+(\bar{x}_1)} \mu_2(x_2)(u_1(\bar{x}_1) + u_2(x_2)) dx_2 + \int_{P^-(\bar{x}_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \quad (A10)$$

with:

$$P^+(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(\bar{x}_1)\} \quad (A11)$$

and:

$$P^-(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(\bar{x}_1)\} \quad (A12)$$

while the starting (with a new third product  $x_1^{n+1}$ ) function is given by:

$$H(\bar{x}_1) \stackrel{def}{=} \int_{Q^+(\bar{x}_1)} \mu_1(x_1^{n+1})(u_1(x_1^{n+1}) + E_2) dx_1^{n+1} + \int_{Q^-(\bar{x}_1)} \mu_1(x_1^{n+1})(E_1 + E_2) dx_1^{n+1} \quad (A13)$$

with:

$$Q^+(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap Supp(\mu_1) : u_1(x_1^{n+1}) > E_1\} \quad (A14)$$

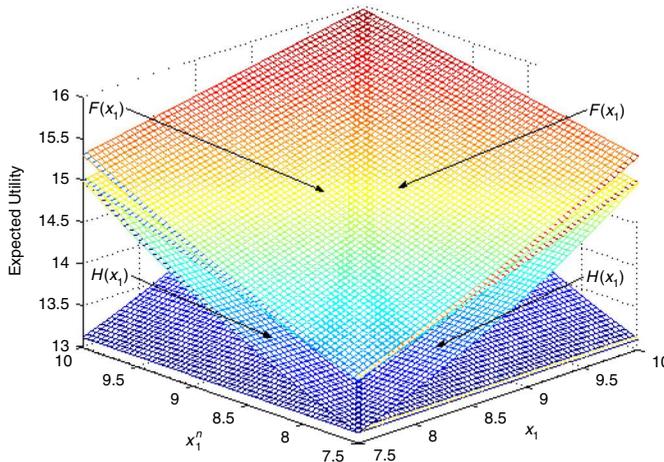
and:

$$Q^-(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap Supp(\mu_1) : u_1(x_1^{n+1}) \leq E_1\} \quad (A15)$$

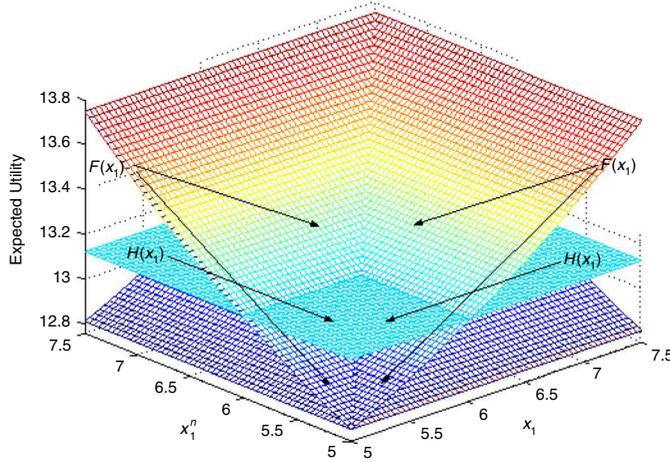
Both these functions are represented in Figure A3.

Note that this figure is also symmetric along the  $x_1 = x_1^n$  plane. Clearly, given the numerical values chosen, the continuation function is higher than the starting one whenever  $x_1$ ,  $x_1^n$  or both are higher than  $x_1^* = 6.0355$ .

Thus, within the current numerical setting, if the DM chooses to start acquiring information on a new product after observing  $x_1$ , and both  $x_1$  and  $x_1^n$  are below  $ce_1$ , then



**Figure A2.** Three-observation setting (Starting option chosen after acquiring  $x_1$ ): acquiring the third observation when  $x_1$  and  $x_1^n$  are both higher than  $ce_1 = 7.5$



**Notes:** The functions cross at  $(x_1=6.0355, x_1^n \leq 6.0355)$  and  $(x_1 \leq 6.0355, x_1^n=6.0355)$

**Figure A3.** Three-observation setting (Starting option chosen after acquiring  $x_1$ ): acquiring the third observation when  $x_1$  and  $x_1^n$  are both lower than  $ce_1=7.5$

the DM will continue acquiring information on the product observed with the highest first characteristic, either  $x_1$  or  $x_1^n$ , if this is higher than  $x_1^*$ . Otherwise, the DM will start acquiring information on a new third product,  $x_1^{n+1}$ . This result is identical to the one obtained using the heuristic mechanism. However, this equivalence does not necessarily hold when four observations are considered, as the discontinuity in Figure 9 illustrates intuitively.

*A.1.3.4 Third and Fourth interim octants:  $\bar{x}_1 \geq ce_1$  and  $\bar{x}_1 < ce_1$ .* Consider the last two subcases, where one realization is above the certainty equivalent value while the other one is below. The subcase threshold values, if any, must be based on the highest of both realizations,  $\bar{x}_1$ , while the other one,  $\bar{x}_1$ , cannot be used as a substitute since, together with  $ce_2$ , it would deliver an expected utility lower than the certainty equivalent product. The corresponding continuation (with  $\bar{x}_1$ ) function is therefore given by:

$$F(\bar{x}_1) \stackrel{def}{=} \int_{P^+(\bar{x}_1)} \mu_2(x_2)(u_1(\bar{x}_1) + u_2(x_2)) dx_2 + \int_{P^-(\bar{x}_1)} \mu_2(x_2)(E_1 + E_2) dx_2 \quad (A16)$$

with:

$$P^+(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(\bar{x}_1)\} \quad (A17)$$

and:

$$P^-(\bar{x}_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(\bar{x}_1)\} \quad (A18)$$

while the starting (with a new third product  $x_1^{n+1}$ ) function is given by:

$$H(\bar{x}_1) \stackrel{def}{=} \int_{Q^+(\bar{x}_1)} \mu_1(x_1^{n+1})(u_1(x_1^{n+1}) + E_2) dx_1^{n+1} + \int_{Q^-(\bar{x}_1)} \mu_1(x_1^{n+1})(u_1(\bar{x}_1) + E_2) dx_1^{n+1} \quad (A19)$$

with:

$$Q^+(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap \text{Supp}(\mu_1) : u_1(x_1^{n+1}) > u_1(\bar{x}_1)\}. \quad (\text{A20})$$

and:

$$Q^-(\bar{x}_1) = \{x_1^{n+1} \in X_1 \cap \text{Supp}(\mu_1) : u_1(x_1^{n+1}) \leq u_1(\bar{x}_1)\}. \quad (\text{A21})$$

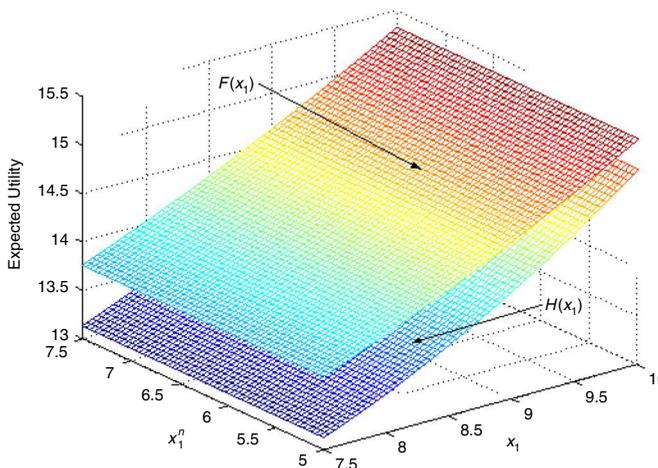
Both these functions are represented in Figure A4, which describes the case where  $x_1$  is above and  $x_1^n$  below  $ce_1 = 7.5$ , i.e. the lower right octant. Note that the same type of figure would be obtained in the upper left octant, where  $x_1$  is below and  $x_1^n$  above  $ce_1 = 7.5$ .

Clearly, continuing acquiring information on  $\bar{x}_1$  constitutes the optimal behavior for the DM in both octants.

*A.1.3.5 Summing up the starting option.* The union of the expected search utilities obtained through the previous octants defines the continuation and starting expected search utilities derived from the third observation for all possible  $x_1$  and  $x_1^n$  realizations. Figure A5 summarizes the starting and continuation regions within a two-dimensional diagram based on the potential realizations of  $x_1$  and  $x_1^n$ .

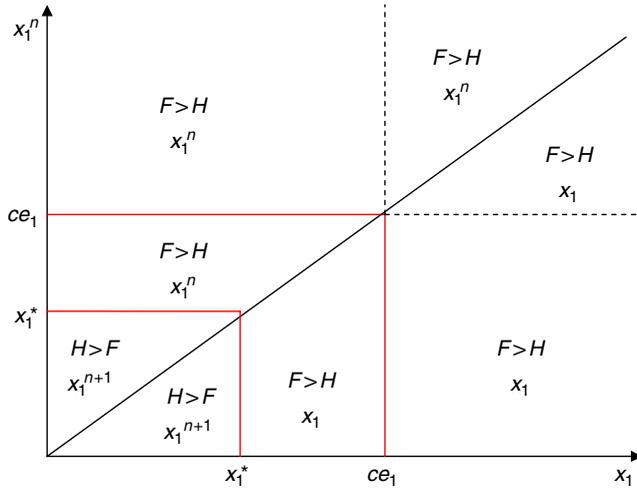
This figure illustrates the optimal information acquisition behavior when acquiring the third and final observation, after the DM decides to start acquiring information on a new product,  $x_1^n$ , when observing  $x_1$ . The resulting behavior follows from the subdivision of the  $(x_1, x_1^n)$ -space in four quadrants. The product on which to optimally acquire the final piece of information is indicated in each subsection of each quadrant. Note that the DM must consider the optimal behavior to follow when acquiring the third observation when calculating the expected search value from starting after having only acquired the initial observation,  $x_1$ .

Clearly, given any possible realization of  $x_1$ , the starting payoff depends on the expected realizations of  $x_1^n$  together with the expected payoffs resulting from the optimal behavior that takes place after considering all the potential combinations of



**Figure A4.** Three-observation setting (Starting option chosen after acquiring  $x_1$ ): acquiring the third observation when  $x_1$  is above and  $x_1^n$  below  $ce_1 = 7.5$

**Figure A5.** Three-observation setting (Starting option chosen after acquiring  $x_1$ ): continuation and starting regions when acquiring the third observation



$x_1$  and  $x_1^n$ . This calculation (based on the expected realizations of  $x_1^n$ ) accounts for the  $x_1^n$ -induced dimension and allows for a two-dimensional representation that, as we will show below, corresponds to the starting function described in Figure 6.

Consider now the two main subcases composing the Starting option after observing  $x_1$ . Except where indicated, the notation corresponds to the one used in Sections 3 and 4 when analyzing the three observations setting.

First, we describe the Starting above  $ce_1$  scenario for  $x_1$ . In this case, there are two possible subcases depending on the value taken by  $x_1^n$  relative to  $ce_1$ , with their corresponding expected search utilities based on the optimal information acquisition behavior of the DM when acquiring the final piece of information.

If ( $x_1^n \geq ce_1$ ), the expected search utility is defined relative to the  $x_1 = x_1^n$  plane:

$$\left[ \int_{x_1}^{x_1^{nm}} \mu_1(x_1^n) F(x_1^n | x_1) dx_1^n \right] + \left[ \int_{ce_1}^{x_1} \mu_1(x_1^n) F(x_1 | x_1^n) dx_1^n \right] \tag{A22}$$

If ( $x_1^n < ce_1$ ), the expected search utility is determined by  $x_1 \geq ce_1$  and  $x_2$ :

$$\left[ \int_{x_1^{nm}}^{ce_1} \mu_1(x_1^n) F(x_1) dx_1^n \right] \tag{A23}$$

We describe now the Starting below  $ce_1$  scenario for  $x_1$ . In this case, we have to subdivide the scenario depending on whether  $x_1$  is located above or below  $x_1^*$ .

If ( $x_1^* \leq x_1 \leq ce_1$ ), the expected search utility is defined relative to the  $x_1 = x_1^n$  plane:

$$\left[ \int_{x_1}^{x_1^{nm}} \mu_1(x_1^n) F(x_1^n) dx_1^n \right] + \left[ \int_{x_1^{nm}}^{x_1} \mu_1(x_1^n) F(x_1) dx_1^n \right] \tag{A24}$$

If  $(x_1 \leq x_1^*)$ , the expected search utility is defined relative to  $x_1^*$  and the  $x_1 = x_1^n$  plane:

$$\left[ \int_{x_1^*}^{x_1^M} \mu_1(x_1^n) F(x_1^n) dx_1^n \right] + \left[ \int_{x_1}^{x_1^*} \mu_1(x_1^n) H(x_1^n) dx_1^n \right] + \left[ \int_{x_1^m}^{x_1} \mu_1(x_1^n) H(x_1) dx_1^n \right] \quad (\text{A25})$$

with:

$$H(x_1) = \int_{Q^+(x_1)} \mu_1(x_1^{n+1}) (u_1(x_1^{n+1}) + E_2) dx_1^{n+1} + \int_{Q^-(x_1)} \mu_1(x_1^{n+1}) (E_1 + E_2) dx_1^{n+1} \quad (\text{A26})$$

and:

$$Q^+(x_1) = \{x_1^{n+1} \in X_1 \cap \text{Supp}(\mu_1) : u_1(x_1^{n+1}) > E_1\}. \quad (\text{A27})$$

A visual comparison with the expected search utilities calculated in the numerical simulations Subsection 4.2 shows that the functions are actually identical. That is, the heuristic mechanism does not impose any exogenous distortion on the information acquisition behavior of the DM, while, as explained in the text, it does when considering four (or more) observations. Thus, the optimal information acquisition process of the DM in the current numerical setting is given by Figure 6 and, afterwards, by Figure A5. Both of them define the optimal information acquisition behavior of the DM when acquiring three observations given the numerical values assumed. Clearly, the notation and presentation of the process provided in the body of the paper favors a tree-like structure, which is particularly helpful when considering the four and more observations settings.

We extend the three observations model below in two potential directions. First, we analyze the effect that shifting the reference value from  $x_1^*$  to  $ce_1$  has on the information acquisition process. Second, we consider alternative parameter values, given by  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$ , allowing for the complete dominance of the first characteristic over the second one.

#### A.1.4 Shifting the reference value from $x_1^*$ to $ce_1$

We impose now  $ce_1$  as the reference value instead of  $x_1^*$ . That is, assume that the DM only considers products whose first characteristic is above  $ce_1$ , with the entire product providing an expected utility above  $(ce_1, ce_2)$ . The only formal modification applied to the  $x_1^*$ -based setting takes place at the Starting below  $ce_1$  scenario, which simplifies to:

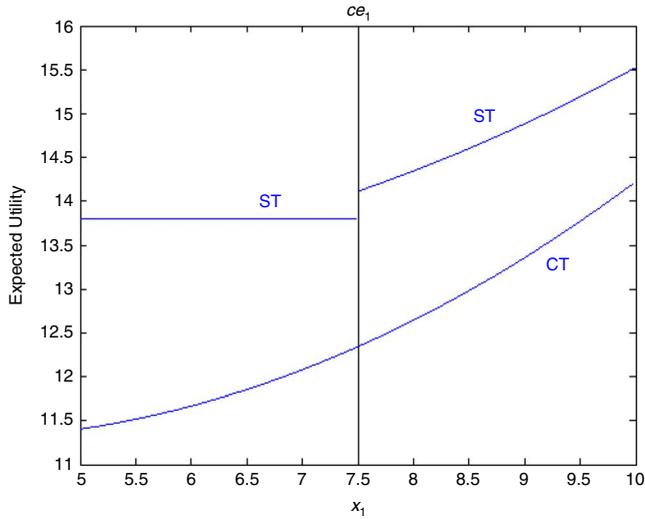
If  $(x_1 \leq ce_1)$ , the expected search utility is defined relative to the  $x_1 = x_1^n$  plane:

$$\left[ \int_{ce_1}^{x_1^M} \mu_1(x_1^n) F(x_1^n) dx_1^n \right] + \left[ \int_{x_1}^{ce_1} \mu_1(x_1^n) H(x_1^n) dx_1^n \right] + \left[ \int_{x_1^m}^{x_1} \mu_1(x_1^n) H(x_1) dx_1^n \right] \quad (\text{A28})$$

While the difference in expected search utilities is not substantial within the three observations environment, the distortion introduced is cumulative and the differences with respect to the  $x_1^*$ -based setting increase as the number of observations considered is incremented.

As Figure A6 illustrates, the main difference between the  $ce_1$ -based setting and the optimal  $x_1^*$ -based one when three observations are considered is located in the  $[x_1^*, ce_1]$  interval. In the

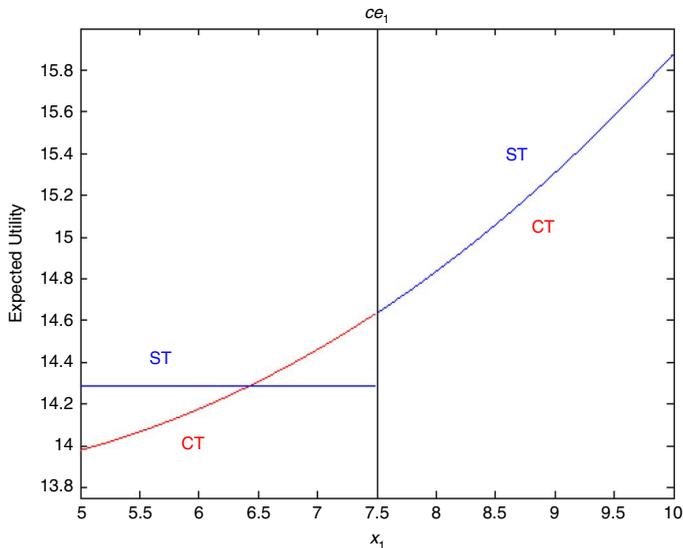
**Figure A6.**  
Three-observation  
setting (initial  
decision after  
observing  $x_1$ ) when  
 $ce_1$  is used as the  
reference threshold  
value instead of  $x_1^*$



optimal case, the expected search utility when  $x_1 \in [x_1^{opt}, x_1^*]$  equals 13.88838. In the suboptimal  $ce_1$  case the expected search utility when  $x_1 \in [x_1^{opt}, x_1^*]$  is lower and equals 13.80208. This difference is due to the missing opportunities of finding an acceptable product despite  $x_1$  being initially located below  $ce_1$ . This inefficiency causes a discontinuity in the Starting function. A similar conclusion follows when  $ce_1$  is used as the reference value in the four observations environment.

As Figure A7 illustrates, there is one threshold value in the  $ce_1$ -based setting when four observations are acquired. The threshold value equals 6.4193, which is higher than  $x_1^* = 6.0355$ . The overlapping of both functions for realizations located above  $ce_1$  may generate, together with the threshold value, a maximum of three different information acquisition intervals. Moreover, it

**Figure A7.**  
Four-observation  
setting (initial  
decision after  
observing  $x_1$ ) when  
 $ce_1$  is used as the  
reference threshold  
value instead of  $x_1^*$



should be noted that the expected search continuation and starting utilities are higher for all  $x_1$  realizations in the  $x_1^*$ -based setting. Thus, the  $x_1^*$ -based setting tends to provide a higher expected search utility, except for a small interval when approaching  $x_1^*$  from the right. It should also be noted that  $ce_1$  is implicitly present in the  $x_1^*$ -based setting, since the DM always requires a product whose expected utility is above the certainty equivalent one. However, if we impose  $ce_1$  also as a threshold value, we would be eliminating potentially acceptable products that are initially located below  $ce_1$  but above  $x_1^*$ .

*A.1.5 Alternative parameter values:  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$*

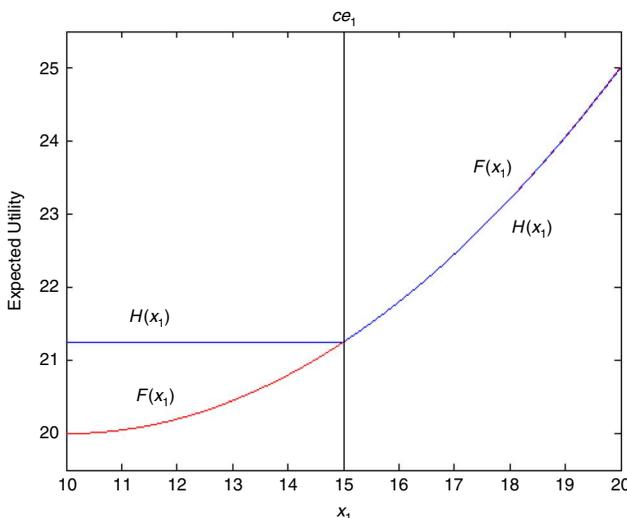
Finally, we shift the domain on which the first characteristic is defined away from the second order stochastic dominance setting described in the paper, with  $x_1 \in [5, 10]$  and  $x_2 \in [0, 10]$ , toward a complete dominance one with  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$ . That is, we increase the relative importance of the first characteristic when determining the behavior of the DM. The resulting Continuation and Starting functions within a two observations setting are presented in Figure A8.

It is immediately evident that a considerable amount of uncertainty is generated regarding the behavior of the DM. The increment in the relative importance of  $x_1$  leads the DM to start acquiring information on a new product whenever  $x_1 < ce_1$ , but does not provide a clear guidance on how to proceed when  $x_1 \geq ce_1$ . In this case, we are forced to impose an ad hoc mechanism regarding the option that the DM follows when the Continuation and Starting functions overlap. As Figure A9 illustrates, the initial uncertainty remains if the DM decides to continue by default after observing  $x_1 \geq ce_1$ , but vanishes completely if the DM decides to start by default after observing  $x_1 \geq ce_1$ , a decision environment described in Figure A10.

These simulations open the way for the analysis of a wide range of possible scenarios and also illustrate the complexity of the model and the impossibility of describing formally the properties of the information acquisition process, which are determined by the value of the parameters, the number of observations being acquired and the type of utility function under consideration.

**A.2 Moving forward: the four observations environment revisited**

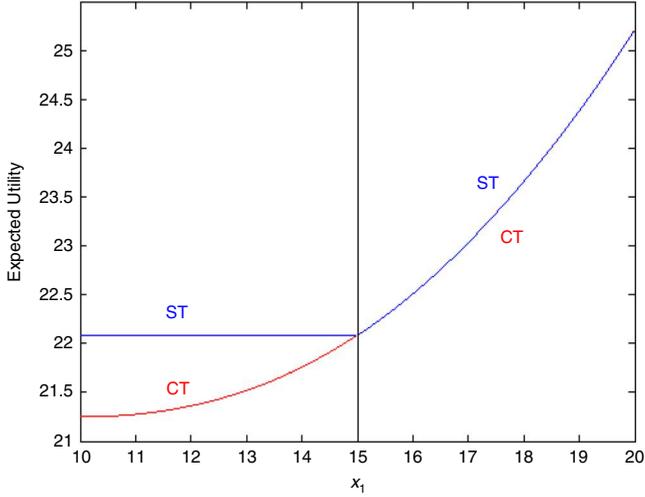
Consider now the Starting information acquisition sequence but in the four observations setting. This implies that the DM has observed  $x_1$  and  $x_1^*$  and must therefore decide whether to continue



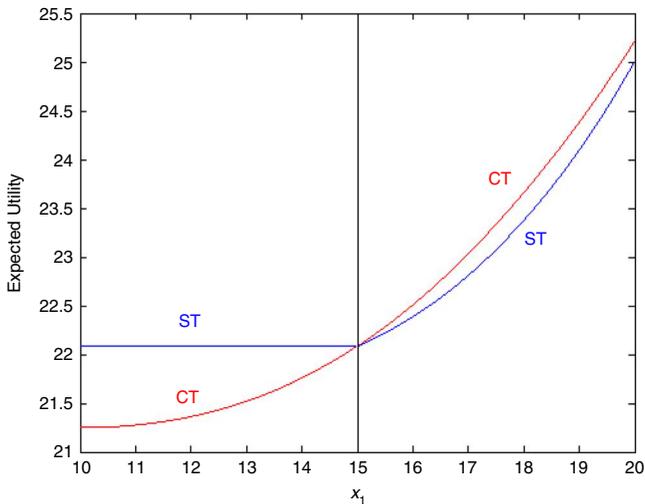
**Figure A8.**  
Continuation and starting regions within a two-observation setting when  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$

with the highest of both realizations or to start with a new product,  $x_1^{n+1}$ . This situation opens the way for two distinct information acquisition paths leading to two different scenarios. In each scenario the DM must decide whether to start or continue acquiring information depending on the potential realizations observed from the different products.

The first scenario is defined by  $x_1, x_1^n$  and  $x_2$  (we have assumed without loss of generality that  $x_1 > x_1^n$ ). The second scenario is defined by  $x_1, x_1^n$  and  $x_1^{n+1}$ . Now, in each case, the DM must decide whether to start or continue. This decision must be based on the realizations of the three observations collected until that decision moment is reached. We should use the



**Note:** Continuing by default above  $x_1 = 15$



**Note:** Starting by default above  $x_1 = 15$

**Figure A9.**  
Continuation and starting regions within a two-observation setting when  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$

**Figure A10.**  
Continuation and starting regions within a two-observation setting when  $x_1 \in [10, 20]$  and  $x_2 \in [0, 10]$

optimal choices made in this last stage (either continue or start and within which subsets of realizations) as we have done through the three observations example above and define the expected search utilities derived from each possible path. Given both these options and their corresponding payoffs, we know what the DM would do after acquiring three observations in the first and second potential scenarios. Each scenario would have an expected search utility function assigned based on the potential realizations of the four variables defining each path. These potential payoffs must be brought back to the second decision stage in order for the DM to decide whether to follow the first path (under which subset of  $x_1$  and  $x_1^n$  realizations) or the second one.

Then, given the optimal behavior followed after observing  $x_1$  and  $x_1^n$ , we can calculate the optimal choice to be made after observing  $x_1$  based on the expected realizations of  $x_1^n$ . In this case, we will obtain a two-dimensional figure similar in spirit to Figure 9. However, note that the final choice option to be calculated and brought back from the end of the decision tree requires a four dimensional payoff setting based on either  $(x_1, x_1^n, x_2)$  or  $(x_1, x_1^n, x_1^{n+1})$ , together with their corresponding expected search utilities. This is the main reason for the implementation of the heuristic mechanism since, otherwise, we would be unable to provide the DM with an information acquisition process to follow.

### A.3 Alternative information acquisition structure with $n$ observations

In order to generate sufficiently sophisticated DMs and a manageable framework of analysis when  $n$  observations are considered within a bi-dimensional product environment, we would need DMs to forecast the expected improvements that may arise from any of the remaining observations available given an initial reference product whose value may change as additional information is acquired. A general formulation is outside the scope of the current paper but we provide here some intuitive guidelines on how to proceed when building this alternative structure.

For example, the forecasting capacities of DMs could be based on a binomial distribution determined by the number of observations that remain to be acquired. These binomial-based potential improvements can be used to define the sequential dynamic behavior of DMs. In this regard, the probability that  $l$  among the remaining  $n$  observations improve upon an initially observed characteristic  $x_1$  and deliver a better product than the partially observed one would be given by the following binomial distribution:

$$\Psi_1(n, l, \mu_1(x_1^n > x_1)) = \binom{n}{l} \mu_1(x_1^n > x_1)^l (1 - \mu_1(x_1^n > x_1))^{n-l}, \quad (\text{A29})$$

where  $\mu_1(x_1^n > x_1)$  should represent the probability that a new randomly selected product is endowed with a better first characteristic than the currently partially observed one defined by  $x_1$ . Similarly, when considering the second characteristic, all possible combinations leading to an improvement over the partially observed initial product should be accounted for as follows:

$$\Psi_2(n, l, \mu_2(x_2^n \in P^+(x_1))) = \binom{n}{l} \mu_2(x_2^n \in P^+(x_1))^l (1 - \mu_2(x_2^n \in P^+(x_1)))^{n-l}, \quad (\text{A30})$$

where  $\mu_2(x_2^n \in P^+(x_1))$  should be the probability that a new randomly selected product has a second characteristic belonging to the  $P^+(x_1)$  set determined by the observed  $x_1$  realization.

The combination of  $\Psi_1(n, l, \mu_1(x_1^n > x_1))$  and  $\Psi_2(n, l, \mu_2(x_2^n \in P^+(x_1)))$  represents the probability that a randomly selected product is endowed with a better expected set of  $X_1$  and  $X_2$  characteristics than the partially observed product defined by  $(x_1, P^+(x_1))$ . These probabilities and the corresponding utilities could be used to define a general formulation in expected search utility terms accounting for the  $n$  observations setting in a manageable way.

This alternative information acquisition structure would also allow DMs to estimate the value of acquiring sets of additional observations of different sizes and compare the expected returns obtained with the cost of information. The current setting allows for this operation to be performed to a limited extent by comparing the expected search utilities obtained when either continuing or starting in different observations-based environments. Thus, while we can observe a positive value derived from the acquisition of additional information when comparing Figures 3, 6 and 9, the width of the scope is limited. Selecting an initial reference value and proceeding with the information acquisition structure described within the current section would widen the scope of the paper and allow for additional potential extensions.