Fuzzy multiple criteria base realignment and closure (BRAC) benchmarking system at the Department of Defense

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Abstract

Purpose – The US Government adopted the base realignment and closure (BRAC) to resolve the military, economic and political issue of excess base capacity. There have been five rounds of BRAC since 1988, and more are expected to come in the years ahead. The complexity of the closure and realignment decisions and the plethora of factors that are often involved necessitate the need for a sound theoretical framework to structure and model the decision-making process. This paper aims to address the issues.

Design/methodology/approach – The paper presents a multiple criteria benchmarking system that integrates the employment, environmental, financial, strategic, and tactical impacts of the closure and realignment decisions into a weighted-sum measure called the "survivability index." The proposed index is used to determine whether the returns generated by each military base on the Department of Defense (DoD) hit list meet a sufficient target benchmark.

Findings – There is a significant amount of evidence that intuitive decision making is far from optimal and it deteriorates exponentially with problem complexity. The benchmarking system presented in this study helps decision makers (DMs) crystallize their thoughts and reduce the environmental complexities inherent in the BRAC decisions. The presented model is intended to create an even playing field for benchmarking and pursuing consensus not to imply a deterministic approach to BRAC decisions.

Originality/value – An iterative process is used to consistently analyze the objective and subjective judgments of multiple DMs within a structured framework based on the analytic network process and fuzzy logic. This iterative and interactive preference modeling procedure is the basic distinguishing feature of the presented model as opposed to statistical and optimization decision-making approaches.

Keywords Benchmarking, Decision theory, Fuzzy logic, United States of America, Government agencies

Paper type Research paper

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1. Introduction

It is best, Sun Tzu said, to prepare for war in peace and to prepare for peace in war. The Department of Defense (DoD) adopted the base realignment and closure (BRAC) process as a national strategy to resolve the military, economic, and political issue of excess base capacity created by the collapse of the former Soviet Union. As forces were drawn down, excess base capacity was created. BRAC was brought together to evaluate the USA population of bases on certain criteria and set forth a recommendation to the Secretary of Defense to close some bases and realign others. The strategic and financial impacts of BRAC are immense. When bases are closed or realigned, the community is dramatically affected by losing/gaining jobs and environmental affects. Economic issues in terms of costs and savings are of great importance in BRAC. People, communities, and environmental impacts are direct consequences of the closure and realignment efforts. The immediate fears of base closures are the loss of jobs in the adjacent communities. Directly tied to the future reuse of closed military installations are the cleanup of known environmental contamination. Beginning in 1988, Congress authorized the DoD to conduct five rounds of BRC including the recent round in 2005. At the completion of all five rounds, the DoD had 130 fewer major bases, 84 major realignments and hundreds of other smaller facilities realigned (United States Government Accountability Office, 2007). Table I provides a general overview of BRAC Activities since its initiation in 1988.

Legislation authorizing BRAC has stipulated that closure and realignment decisions must be based upon selection criteria, a current force structure plan and infrastructure inventory developed by the Secretary of Defense. The criteria historically included employment, environmental, financial, strategic and tactical impacts. BRAC is essentially a multi-criteria capital budgeting problem where the Commission is charged to determine whether the military bases on the hit list should be left alone, realigned or closed. Ideally, the Commission should pursue those military bases that enhance shareholder (American public) value. A large body of intuitive and analytical multi-criteria capital budgeting models has evolved over the last several decades to assist decision makers (DMs) in strategic decision making. While these models have made great strides, the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences.

We present a structured multi-criteria benchmarking framework that processes objective and subjective estimates provided by a group of DMs with the analytic network process (ANP) and fuzzy logic. The proposed framework provides a set of performance measurements that could be utilized for benchmarking or BRAC decisions. The remainder of the paper is organized as follows. The next section presents the state of the art in multi-criteria decision analysis (MCDA) and

BRAC	Major base closures	Major base realignments	Minor closures and realignments	Costs ($billion)	Annual recurring savings ($billion)
1988	16	4	23	2.7	0.9
1991	26	17	32	5.2	2.0
1993	28	12	123	7.7	2.6
1995	27	22	57	6.5	1.7
2005	33	29	775	31.0	4.0

Table I. History of BRAC rounds

benchmarking followed by a description of the hierarchical and network model in Section 3. In Section 4, we demonstrate the procedural steps of the model along with the results of a study conducted by the US Navy. Section 6 presents the conclusions and future research directions.

2. State of the art in MCDA and benchmarking

The state of the art in multi-criteria capital budgeting contains hundreds of methods, including scoring methods, economic methods, portfolio methods, and decision analysis methods. Scoring methods use algebraic formulas to produce an overall score in capital budgeting (Osawa and Murakami, 2002; Osawa, 2003). Economic methods use financial models to calculate the monetary payoff of alternative projects (Graves and Ringuest, 1991; Huang, 2008; Kamrad and Ernst, 2001; Lotfi *et al.*, 1998). Portfolio methods evaluate the entire set of projects to identify the most attractive subset (Cooper *et al.*, 1999; Girotra *et al.*, 2007; Mojsilovi *et al.*, 2007; Wang and Hwang, 2007). Cluster analysis, a more specific portfolio method, groups projects according to their support of the strategic positioning of the firm (Mathieu and Gibson, 1993). Decision analysis methods compare various projects according to their expected value (Hazelrigg and Huband, 1985; Thomas, 1985). Finally, simulation, a more specific decision analysis method, uses random numbers and simulation to generate a large number of problems and pick the best outcome (Abacoumkin and Ballis, 2004; Mandakovic and Souder, 1985; Paisittanand and Olson, 2006).

Most of these methods are used to evaluate research and development projects (Coffin and Taylor, 1996; Girotra *et al.*, 2007; Osawa and Murakami, 2002; Osawa, 2003; Wang and Hwang, 2007), information systems projects (Mojsilovi *et al.*, 2007; Paisittanand and Olson, 2006) and capital budgeting projects (Graves and Ringuest, 1991; Mehrez, 1988). Recently, researchers working on project evaluation and selection have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Costa *et al.*, 2003; Hsieh *et al.*, 2004; Liesiö *et al.*, 2007; Tavana, 2006). MCDA has also been applied to important military applications involving complex alternatives, conflicting quantitative and qualitative objectives, and major uncertainties. Parnell (2006) compared 10 single-decision applications and 14 portfolio decision value model applications. Ewing *et al.* (2006) developed a similar model to determine the military value of 63 army installations. Additional multi-criteria portfolio decision models used by the military include Archer and Ghasemzadeh (1999); Stummer and Heidenberger (2003).

Finding the "best" MCDA framework is an elusive goal that may never be reached (Triantaphyllou, 2000). Pardalos and Hearn (2002) discuss the importance of exploring ways of combining criteria aggregation methodologies to enable the development of models that consider the DM's preferential system in complex problems. Belton and Stewart (2002) also argue the need for integrating frameworks in MCDA. We propose a multi-criteria BRAC model for benchmarking at the DoD. The model solves complex and judgmental multi-criteria problems by carefully combining a set of well-known and proven techniques in MCDA. This integration allows for the objective data and subjective judgments to be collected and used side-by-side in a weighted sum model (Triantaphyllou, 2000). The proposed MCDA model systematically considers a series of hierarchical and networked factors in a structured framework to develop a measure to determine whether the returns generated by each military base on DoD hit list meet a sufficient target benchmark.

Benchmarking is the systematic comparison of performance elements in an organization against those best practices of relevant organizations and obtaining information that will help the observing organization to identify and implement improvement (Lau *et al.*, 2001). While a number of benchmarking definitions can be found in the literature, they all essentially share the same theme. Benchmarking is a framework within which indicators and best practices are examined in order to identify areas where performance can be improved. Public sector benchmarking has been the subject of numerous studies (Magd and Curry, 2003; Tavana, 2004, 2008; Triantafillou, 2007; Vagnoni and Maran, 2008; Wynn-Williams, 2005). The benchmarking system developed in this study uses a numeric measure called the "survivability index" to help policy makers and the commanding officers of the military bases on the DoD hit list identify their strengths and weaknesses by learning from "best-in-class" and other competing bases on the list. The survivability index is used to identify each military base on the hit list as either efficient, with high benefits and low costs; active, with high benefits and high costs; inactive with low benefits and low costs; and inefficient with low benefits and high costs.

3. The hierarchical and network multi-criteria model

The US Congress has chartered the BRAC Commission to consider employment, environmental, financial, strategic, and tactical impacts of BRAC decisions.

Employment impacts are measured by three sub-factors: direct job changes, indirect job changes and total job changes as a percentage of area employment. Direct job changes are comprised of military, civilian and contractor jobs that are either gained or lost in a certain location due to the change recommended by the Commission. Indirect job changes are those jobs changes that would be indirectly affected (gained or lost) by the recommendation set forth by the Commission.

Environmental impacts are used to measure the impact of the military base on the surrounding environment. For example, the closure of a military chemical depot would require an extensive and costly cleanup. Other examples include the clean up required due to fuel spills on an air force base or weapons disposal at an army munitions depot. Conversely, there are also many instances such as a medical center or a guard station where there is minimal to no environmental impact. Several military bases have already begun an environmental restoration. The costs to complete the environmental restoration as well as the cost that have already been incurred are considered by the Commission. It should be noted that several bases do not have any environmental restoration costs.

Financial impacts are measured by one-time costs, payback period, six year net savings, annual recurring savings and 20 year net present value (NPV) savings. One-time costs are those costs associated with closing a particular base. The Secretary of Defense initially submits an estimated cost, which is then reviewed by the BRAC Commission. Upon the Commissions approval, a final one-time cost is determined. Since federal cost savings is the main driver behind BRACs, the one-time cost plays a very important factor in determining whether to close a base or not. Payback period is the time period it would take to recuperate the one time closing costs through savings incurred by closing the base. The range of payback periods varies but the majority of the bases fall somewhere between 1 and 20 years. Twenty year NPV is the present value of 20 years worth of savings for closing a military base.

Strategic impacts are non-monetary impacts that usually cannot directly be assigned with a value but greatly sway the BRAC decisions. The post cold war era has changed the strategic significance of several bases located in the USA. During the cold war, military installations were placed to defend or attack against the Soviet Union. Depots were maintained at high levels in order to support any conflicts that would arise. With the end of the cold war, the primary purpose of several installations became obsolete. These bases were given new roles, and in some instances, these roles were just as important as their cold war era roles. Strategic impacts are measured using a sliding scale (0 = unimportant to 10 = extremely important).

Tactical impacts are measured by community support, commercial, and residential use of land. Community support for base closures or realignments has generally been low. Military installations have typically benefited the surrounding community by providing jobs, boosting local economies and attracting visitors who may not have otherwise come to town. A military presence also provides a sense of pride for the community, knowing that their community is playing a role in the defense of the USA. The ability for land to be re-used after a closure is also an important factor. The communities affected by a closure need to be able to re-claim the land for either commercial or residential purposes. Some bases are obviously more suited for commercial use. For example, an air base can easily be converted to a private or regional airfield. A naval base can be converted to a commercial ship yard, or due to it is proximity to water; it may be attractive to real estate development. Army bases, depending on location, may also be viable for other uses. As noted earlier, sites with higher environmental clean up costs may not be attractive for any future use as a high clean up cost would indicate some type of on-site contamination. Tactical impacts are also measured using a sliding scale (0 = unimportant to 10 = extremely important).

This study was conducted at a naval facility in the USA with seven naval logistic experts. The expert officers contributed their professional experience to identify factors and sub-factors that influence the BRAC decision and constructed the network presented in Figure 1 based on document reviews and stakeholder analysis. Numerous legal, strategy, policy and planning documents were used to define the military value of the installations on the DoD hit list. The solid lines in this diagram represent the hierarchical dependencies and the dotted arrows represent influence and interdependencies among the BRAC factors and sub-factors.

4. The procedure and results

We use a nine-step procedure to systematically evaluate the bases by plotting them in a 4D space based on their "survivability index." The survivability index is the Euclidean distance from the ideal alternative. Ideal alternative is an unattainable choice that serves as a norm or rationale facilitating a human choice problem. Using the theory of displaced ideal to grasp the extent of the emerging conflict between means and ends, the DM explores the limits attainable with each benefit and cost. As all alternatives are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance. The nine steps used in our model are:

(1) Consider a set of military bases for realignment and benchmarking.

(2) Identify the relevant objective and subjective factors and sub-factors and define their importance weights using the ANP.

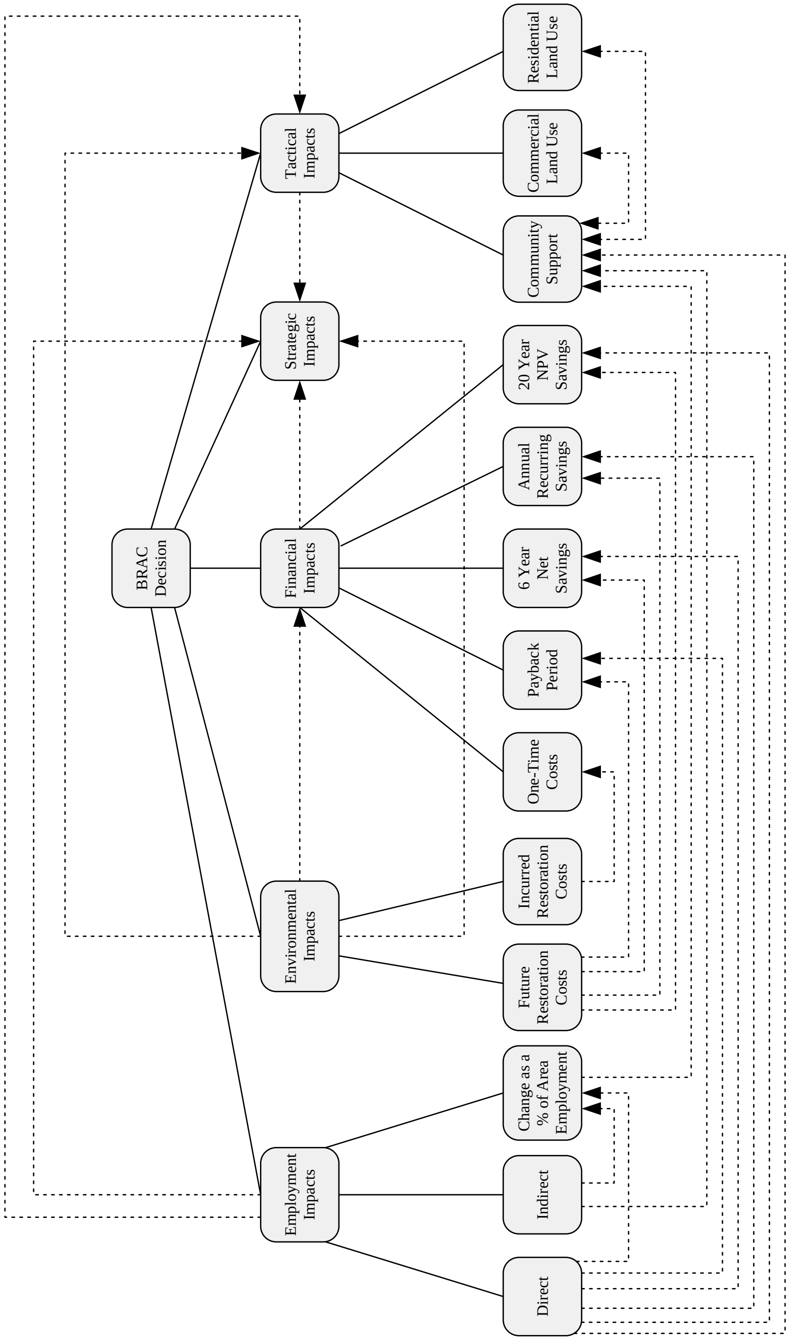

Figure 1.
BRAC hierarchical and network interdependencies

(3) Develop scores for the subjective factors and identify values of the objective factors on each alternative.
(4) Group all the factors into benefit and cost factors.
(5) Normalize all estimates to obtain identical units of measurement.
(6) Aggregate subjective and objective factor estimates for the costs and benefits on each alternative for each DM.
(7) Find combined fuzzy group ratings for the alternative benefits and costs.
(8) Identify the ideal alternative and calculate the total Euclidean distance of each base.
(9) Rank the bases using visual and numerical information, taking into consideration the level of uncertainty of their fuzzy characteristics.

4.1 Consider a set of military bases for realignment and benchmarking

Alternatives are the set of potential means by which the previously identified objectives may be attained. Assuming that there are m alternatives ($m = 1,2,\ldots, M$), there must be a minimum of two mutually exclusive alternatives in the set to permit a choice to be made (Zeleny, 1982). A total of 52 US military bases comprised of 16 Air Force, 19 Army, and 17 Navy bases from 27 states and the District of Columbia were assessed in this study.

4.2 Identify the relevant objective and subjective factors and sub-factors and define their importance weights using the analytic network process (ANP)

The ANP is a more general form of the analytic hierarchy process (AHP) used in MCDA. Saaty (1980) developed the AHP to capture the intuitive judgments in multi-criteria decision problems. AHP assumes unidirectional hierarchical relationships among the decision elements in a problem. However, in many real-life problems, there are dependencies among the elements in a hierarchy. ANP does not require independence and allows for decision elements to "influence" or "be influenced" by other elements in the model. Both processes have been widely used on a practical level and numerous applications have been published in literature (Saaty, 1996). The hierarchical model presented in Figure 1 was used in this study. There are two different kinds of dependencies in a hierarchy, within level or between levels dependencies. The directions of the arrows (or arcs) signify dependence (or influence). An example of a between level dependency (or outer dependency) is the dependency between direct employment impacts and community support and an example of a within level dependency (or inner dependency) is the interdependency between future restoration costs and payback period. With such interactions, the hierarchical structure becomes a network and a matrix manipulation approach developed by Saaty and Takizawa (1986) is used to measure the relative importance or strength of the impacts on a given element in the network using a ratio scale similar to AHP (Saaty, 1996).

According to Saaty (2005), the ANP comprises four main steps:

(1) problem structuring;
(2) pairwise comparisons;
(3) super-matrix formation; and
(4) selection of best alternatives.

In ANP, similar to AHP, DMs are asked to provide a series of pairwise comparisons of the elements at each level of the hierarchy with respect to a control element. The control element can be an element at the upper or lower levels of the hierarchy. This is the fundamental requirement for developing the super-matrix in the ANP (Saaty, 2001). The pairwise comparison for the elements at one level with respect to the control element at another level is expressed in a matrix form (A) with Saaty's 1-9 scale shown in Table II.

A reciprocal value is assigned to the inverse comparison; that is, $a_{ij} = 1/a_{ji}$, where $a_{ij}(a_{ji})$ represents the importance weight of the ith (jth) element. Once the pairwise comparisons are completed, the local priority vector w is computed as the unique solution to $A \times w = \lambda_{\max} w$ where A is the matrix of pairwise comparison, w is the eigenvector, and $\lambda_{\max}$ is the largest eigenvalue of A. There are several algorithms available for approximating the vector w (Saaty and Takizawz, 1986). We use a two-stage algorithm proposed by Meade and Sarkis (1998) for averaging normalized columns and approximating the vector w:

$$w_i = \frac{\left(\sum_{j=1}^{n} \left(A_{ij} / \sum_{i=1}^{n} A_{ij} \right) \right)}{n} \quad \text{for } i = 1, \ldots, n \tag{1}$$

The deviation from consistency of the pairwise comparisons must be addressed in the assessment process. Saaty (1980) provides a consistency index (CI) defined as $\mathrm{CI} = (\lambda_{\max} - n)/(n-1)$ for this test in which λ_{max} is approximated by $\sum_{i=1}^{n}[(Aw_i)/w_i]/n$. The acceptable consistency index is $\mathrm{CI} \leq 0.10$.

Next, the super-matrix is formed. The super-matrix concept is similar to a Markov chain process (Saaty, 1996). The local priority vectors developed earlier are entered in the appropriate columns of a matrix to obtain global priorities in a problem with interdependencies. As a result, a partitioned matrix called a super-matrix is created,

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Table II. The fundamental scale used in AHP and ANP where each matrix segment represents a relationship between two elements in the model. When there is an interrelationship between the elements of a component or two components, zeros can be replaced by a matrix in the super-matrix. Let the components of a decision system be C_i, $i = 1, \ldots, N$; each component i is assumed to have u_i elements, denoted by $e_{i1}, e_{i2}, \ldots, e_{Nu_N}$. The standard form of a super-matrix and a W_{ij} component matrix proposed by Saaty (1996) are shown in Figures 2(a) and (b) (Saaty, 1996).

The super-matrix is then converged to obtain a long-term stable set of weights. For convergence to occur, the super-matrix needs to be column stochastic. In other words, the sum of each column of the super-matrix needs to be one. Saaty (1996) suggests raising the weighted super-matrix to the power of $2k + 1$, where k is an arbitrarily large number, to achieve a convergence on the importance weights. This new matrix is called the limit super-matrix. The limit super-matrix has the same form as the weighted super-matrix but all the columns of the limit super-matrix are the same. By normalizing each block of the limit super-matrix, the final importance weights of all the elements in the matrix can be obtained. The limit may not converge unless the matrix is column stochastic (i.e. each of its columns sums to one). Note that $\lambda_{\max}(T) = 1$ for the super-matrix. Since $\mathrm{Max}\sum_{j=1}^{n} a_{ij} \geq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max}$ for max w_i and

$$
\begin{array}{cc}
 & \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{N1}e_{N2}\cdots e_{Nn_N} & e_{N1}e_{N2}\cdots e_{Nn_N} & \cdots & e_{N1}e_{N2}\cdots e_{Nn_N} \end{array} \\
W = \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \cdots & \cdots \\ C_N & \begin{array}{c} e_{N1} \\ e_{N1} \\ \vdots \\ e_{Nu_N} \end{array} \end{array} & \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix}
\end{array}
$$

(a)

$$
W = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_1)} & \cdots & W_{i1}^{(jn_j)} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_1)} & \cdots & W_{i2}^{(jn_j)} \\ \cdots & \cdots & \cdots & \cdots \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_1)} & \cdots & W_{in_i}^{(jn_j)} \end{bmatrix}
$$

(b)

Figure 2.
Standard super-matrix and W_{ij} componenet (a) super-matrix and (b) W_{ij} component of supermatrix

$\text{Min}\sum_{j=1}^{n} a_{ij} \leq \sum_{j=1}^{n} a_{ij}\frac{w_j}{w_i} = \lambda_{\max}$ for min w_i, the eigenvalue of the matrix ($\lambda_{\max}$), lies between its largest and smallest column sums ($1 = \text{Min}\sum_{j=1}^{n} a_{ij} \leq \lambda_{\max} \leq \text{Max}\sum_{j=1}^{n} a_{ij} = 1$). When the eigenvalues of the matrix W are distinct then the power series expansion of $f(x)$ converges for all finite values of x with x replaced by W:

$$f(W) = \sum_{i=1}^{n} f(\lambda_i)Z(\lambda_i), Z(\lambda_i) = \frac{\prod_{j\neq i}(\lambda_j I - A)}{\prod_{j\neq i}(\lambda_j - \lambda_i)}, \quad \sum_{i=1}^{n} Z(\lambda_i) = I, Z(\lambda_i)Z(\lambda_j) = 0, \tag{2}$$

$$Z^2(\lambda_i) = Z(\lambda_i)$$

where I and 0 are the identity and the null matrices, respectively.

A similar expression is also available when some or all of the eigenvalues have multiplicities. When $f(W) = W^k$, then $f(\lambda_i) = \lambda_i^k$ and as $k \to \infty$, the only terms that give a finite nonzero value are those for which the modulus of λ_i is equal to one. The priorities of the clusters (or any set of elements in a cluster) are obtained by normalizing the corresponding values in the appropriate columns of the limit matrix. For complete treatment, see Saaty (2001) and Saaty and Ozdemir (2005). Let us further define:

Gn = the nth cluster of factors; ($n = 1, 2, \ldots, N,\ 2 \leq n \leq N$)

Gn^i = the ith sub-factor within the nth cluster of factors; ($n = 1, 2, \ldots, N$, $i = 1, 2, \ldots, I, 1 \leq i \leq N$)

W_{Gn} = the importance weight of the nth cluster; ($n = 1, 2, \ldots, N, 2 \leq n \leq N$)

W_{Gn^i} = the importance weight of the nth sub-factor; ($n = 1, 2, \ldots, N$, $i = 1, 2, \ldots, I, 1 \leq i \leq N$)

k = the kth DMs, ($k = 1, 2, \ldots, K, k \geq 0$)

W_{Gnk} = the kth DM weight for the nth cluster; ($n = 1, 2, \ldots, N,\ 2 \leq n \leq N$, $k = 1, 2, \ldots, K, k \geq 0$)

W_{Gn^ik} = the kth DM weight for the nth sub-factor; ($n = 1, 2, \ldots, N$, $i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0$)

W_{VGnk} = the kth DM weight for the nth cluster of objective criteria (V); ($n = 1, 2, \ldots, N, 2 \leq n \leq N, k = 1, 2, \ldots, K, k \geq 0$)

W_{VGn^ik} = the kth DM weight for the nth sub-factor of objective criteria (V); ($n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0$)

W_{UGnk} = the kth DM weight for the nth cluster of subjective criteria (U); ($n = 1, 2, \ldots, N, 2 \leq n \leq N\ k = 1, 2, \ldots, K\ k \geq 0$)

W_{UGn^ik} = the kth DM weight for the nth sub-factor of subjective criteria (U); ($n = 1, 2, \ldots, N, i = 1, 2, \ldots, I, k = 1, 2, \ldots, K, k \geq 0$).

The general views of the factor and sub-factor weights for the K DMs are given in Table III.

Table III.
Factor and sub-factor weight notations

	$k=1$	$k=2$	...	$k=K-1$	$k=K$
Factor weights					
G_1	$W_{G_1 1}$	$W_{G_1 2}$	...	$W_{G_1 (K-1)}$	$W_{G_1 K}$
G_2	$W_{G_2 1}$	$W_{G_2 2}$	...	$W_{G_2 (K-1)}$	$W_{G_2 K}$
...	...	...	...	...	...
$G_{N\text{-}1}$	$W_{G_{N-1} 1}$	$W_{G_{N-1} 2}$	...	$W_{G_{N-1} (K-1)}$	$W_{G_{N-1} K}$
G_N	$W_{G_N 1}$	$W_{G_N 2}$	...	$W_{G_N (K-1)}$	$W_{G_N K}$
Sub-factor weights					
G_1^1	$W_{G1^1 1}$	$W_{G1^1 2}$	...	$W_{G1^1 (K-1)}$	$W_{G1^1 K}$
G_1^2	$W_{G1^2 1}$	$W_{G1^2 2}$	...	$W_{G1^2 (K-1)}$	$W_{G1^2 K}$
...	...	...	...	...	...
G_1^{I1}	$W_{G1^{I1} 1}$	$W_{G1^{I1} 2}$	...	$W_{G1^{I1} (K-1)}$	$W_{G1^{I1} K}$
G_2^1	$W_{G2^1 1}$	$W_{G2^1 2}$	...	$W_{G2^1 (K-1)}$	$W_{G2^1 K}$
G_2^2	$W_{G2^2 1}$	$W_{G2^2 2}$	...	$W_{G2^2 (K-1)}$	$W_{G2^2 K}$
...	...	...	...	...	...
G_2^{I2}	$W_{G2^{I2} 1}$	$W_{G2^{I2} 2}$	...	$W_{G2^{I2} (K-1)}$	$W_{G2^{I2} K}$
...	...	...	...	...	...
G_N^1	$W_{GN^1 1}$	$W_{GN^1 2}$	...	$W_{GN^1 (K-1)}$	$W_{GN^1 K}$
G_N^2	$W_{GN^2 1}$	$W_{GN^2 2}$	...	$W_{GN^2 (K-1)}$	$W_{GN^2 K}$
...	...	...	...	...	...
G_N^{IN}	$W_{GN^{IN} 1}$	$W_{GN^{IN} 2}$	...	$W_{GN^{IN} (K-1)}$	$W_{GN^{IN} K}$

The expert DMs participating in this study provided their independent pairwise comparison matrices. The local priority vectors are then calculated and entered in the appropriate columns of a matrix for each DM to obtain global priorities in a problem with interdependencies. A super-matrix was created for each DM. Normalizing each block of the limit super-matrix resulted in the importance weights of the factors and sub-factors presented in Table IV.

4.3 Develop scores for the subjective factors and identify values of the objective factors on each alternative

The decision criteria in this study were divided into two groups: objective (such as monetary, physical or statistical) and subjective (such as beliefs, likeliness or judgments). Data on objective factors were obtained from financial, statistical, and economic reports. Subjective judgments were obtained from our seven expert DMs who considered five groups of factors divided into 14 sub-factors. The sub-factors were further grouped into three clusters. One cluster included 10 objective factors (employment, environmental and financial impacts) and the other two clusters included subjective strategic and tactical factors. The objective factors and their respective uncertainty levels (distribution) are presented in Table V.

Objective factors are treated as fuzzy numbers and their values are defined as:

$$\tilde{v}_{m_n^i} = \left\{(x, \mu_{m_n^i}(x)) | x \in R\right\} \tag{3}$$

where $\tilde{v}_{m_n^i}$ is the set of fuzzy objective values for the ith objective sub-factor within the nth cluster on alternative m represented by pairs $(x, \mu_{m_n^i}(x))$ with membership functions of LR-type; $(n = 1, 2, \ldots, N,\ i = 1, 2, \ldots, I,\ 1 \le i \le N,\ m = 1, 2, \ldots, M)$. $\mu_{m_n^i}(x) \in [0, 1]$ represents the interval from which the membership functions take on

	DM-1	DM-2	DM-3	DM-4	DM-5	DM-6	DM-7
Factor weights							
Criteria							
1 Employment impacts	0.291	0.243	0.346	0.256	0.32	0.258	0.27
2 Environmental impacts	0.122	0.095	0.087	0.102	0.111	0.166	0.15
3 Financial impacts	0.412	0.352	0.372	0.365	0.312	0.42	0.291
4 Strategic impacts	0.144	0.212	0.110	0.228	0.180	0.090	0.179
5 Tactical impacts	0.031	0.098	0.085	0.049	0.077	0.066	0.11
Sub-factor weights							
Sub-criteria							
1.1 Direct	0.063	0.072	0.054	0.090	0.108	0.029	0.078
1.2 Indirect	0.054	0.044	0.065	0.034	0.033	0.044	0.035
1.3 Changes as a percent of area employment	0.092	0.107	0.112	0.097	0.104	0.088	0.123
2.1 Future environmental restoration costs	0.073	0.067	0.078	0.082	0.071	0.068	0.054
2.2 Incurred environmental restoration costs	0.056	0.032	0.044	0.056	0.049	0.093	0.042
3.1 One-time costs	0.112	0.131	0.091	0.121	0.122	0.104	0.128
3.2 Payback period (years)	0.084	0.064	0.077	0.053	0.091	0.043	0.054
3.3 Six year net savings	0.074	0.087	0.069	0.065	0.056	0.077	0.021
3.4 Annual recurring savings	0.143	0.142	0.126	0.178	0.189	0.168	0.176
3.5 20 year NPV savings	0.055	0.034	0.066	0.051	0.032	0.055	0.065
5.1 Community support for closure	0.064	0.072	0.066	0.054	0.071	0.032	0.045
5.2 Commercial land use	0.073	0.063	0.073	0.076	0.045	0.044	0.066
5.3 Residential land use	0.056	0.085	0.079	0.043	0.029	0.156	0.113

Table IV. Factor and sub-factor weights for the seven DMs in this study

Factor	Uncertainty level (percent)	Normalized uncertanty level
Direct	± 1.42	0.0142
Indirect	± 3.69	0.0369
Changes as a percent of area employment	± 0.45	0.0045
Future environmental restoration costs	± 5.75	0.0575
Incurred environmental restoration costs	No deviation	0.0000
One-time costs	± 0.75	0.0075
Payback period (years)	± 7.78	0.0778
Six year net savings	± 2.65	0.0265
Annual recurring savings	± 4.25	0.0425
20 year NPV savings	± 12.50	0.1250

Table V. Objective criteria and their uncertainty levels

their values and $\alpha \in [0,1]$ is a representation of $\tilde{v}_{m_n^i}$ by the set of α-levels (α-cuts). Interval representations of the fuzzy objective values $\tilde{v}_{m_n^i}$ on α-levels are:

$$\tilde{v}_{m_n^i} = \left\{ v_{m_n^i}^{\alpha} = \left[v_{m_n^i}^{\alpha L}, v_{m_n^i}^{\alpha R} \right] \right\} \tag{4}$$

where $v_{m_n^i}^{\alpha L}$ and $v_{m_n^i}^{\alpha R}$ are the left (L) and right (R) bounds on α-cuts of fuzzy value $\tilde{v}_{m_n^i}$. The graphical representation of a fuzzy set and its characteristics are depicted in Figure 3.

In this study, the values of the objective factors are considered as triangular fuzzy numbers on two α-levels of 0 and 1. According to Zadeh (1996), triangular fuzzy numbers are characterized by a triple $x = (x_1, x_2, x_3)$ in which x_1, x_2, and x_3 are the

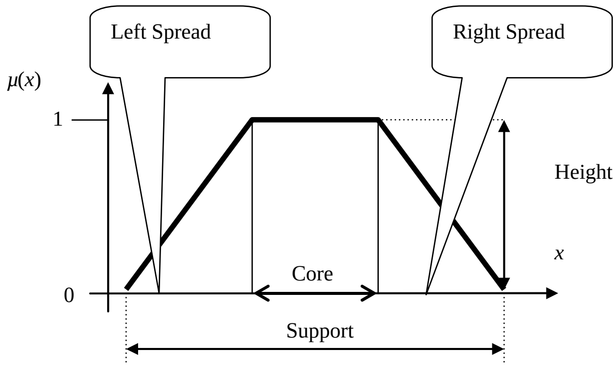

Figure 3.
Fuzzy set and its characteristics

abscissae of the three vertices of the triangle [i.e. $\mu(x_1) = \mu(x_3) = 0, \mu(x_2) = 1$]. The graphical representation of a triangular fuzzy number is shown in Figure 4.

A triangular fuzzy number with center x_2 is a fuzzy quantity where "x is approximately equal to x_2." The deviations given in Table V showed maximal and minimal possible spreads for the most reliable values given on $\alpha = 1$. The closer the objective value is to the left-hand side or the right-hand side boundaries (defined by the uncertainty level), the less reliable the value. Consequently, those values smaller than the left-hand side boundary or larger than the right-hand side boundary are considered impossible (unreliable). On α-cut $= 1$, the left bound value will coincide with the right bound value and on α-cut $= 0$, the left and right bounds will be calculated as shown in the following equations:

$$v_{m_n^i}^{\alpha=1L} = v_{m_n^i}^{\alpha=1R} = v_{m_n^i}^{\alpha=1} \tag{5a}$$

$$v_{m_n^i}^{\alpha=0L} = v_{m_n^i}^{\alpha=1} - \left(v_{m_n^i}^{\alpha=1} \cdot s_{G_n^i} \right) \tag{5b}$$

$$v_{m_n^i}^{\alpha=0R} = v_{m_n^i}^{\alpha=1} + \left(v_{m_n^i}^{\alpha=1} \cdot s_{G_n^i} \right) \tag{5c}$$

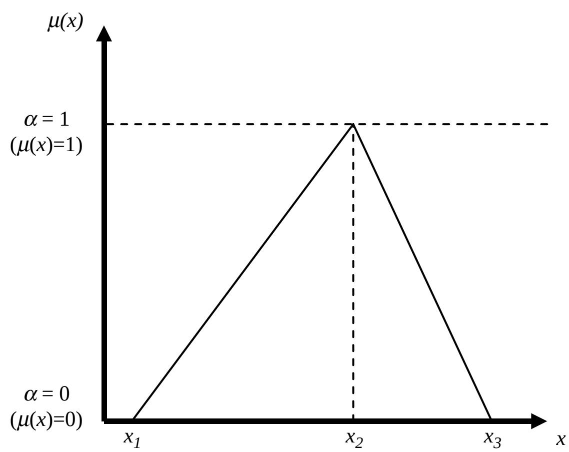

Figure 4.
Triangular fuzzy number

where $s_{G_n^i}$ is the normalized spread (deviation) of the objective value G characterizing the ith sub-factor within the nth cluster; $(n = 1, 2, \ldots, N,\ \ i = 1, 2, \ldots, I,\ \ 1 \leq i \leq N)$.

An equal scoring scale of 0-10 is used for all subjective factors. Seven DMs ($K = 7$) evaluated the bases independently on the subjective factors. The scores of the subjective factors are represented by $u_{mk_n^i}$, the intensity of the ith subjective sub-factor within the nth cluster on alternative m for kth DM; $(m = 1, 2, \ldots, M,\ \ n = 1, 2, \ldots, N,\ \ i = 1, 2, \ldots, I,\ \ k = 1, 2, \ldots, K,\ \ k \geq 0)$.

4.4 Group all the factors into benefit and cost factors

Next, the DMs analyzed all relevant objective and subjective factors and classified them into benefits and costs. While most factors were either a benefit or a cost, some were classified into both groups, depending on their values. The employment impact values for direct, indirect and changes as a per cent of area employment were considered costs if negative and benefits if positive. The environmental impact values for future and incurred environmental restoration costs were considered costs. The financial impact values for one-time costs, six year net savings, annual recurring savings and 20 year NPV savings were considered benefits if negative and costs if positive. However, the financial impact values for the payback period were considered costs since a shorter payback period was more desirable than a longer payback period. The strategic and tactical impact values were all considered benefits.

Variables defined in steps (ii) and (iii) can be rewritten for benefits and costs as follows:

Gn_b^i = the ith benefit sub-factor within the nth cluster of factors; $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, b = 1, 2, \ldots, B, b \leq i)$.

Gn_C^i = the ith cost sub-factor within the nth cluster of factors; $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, c = 1, 2, \ldots, C, c \leq i)$.

Subsequently, equation (3) can be rewritten for benefits and costs as:

$$\tilde{v}_{m_{nb}^i} = \left\{ \left((x, \mu_{m_{nb}^i}(x) \right) | x \in R \right\} \tag{6a}$$

$$\tilde{v}_{m_{nc}^i} = \left\{ \left((x, \mu_{m_{nc}^i}(x) \right) | x \in R \right\} \tag{6b}$$

Equations (6a) and (6b) define sets of fuzzy objective values of the ith benefit (cost) sub-factors within the nth clusters of objective factors on alternative m which are represented by pairs $(x, \mu_{m_{nb}^i}(x))$ and $(x, \mu_{m_{nc}^i}(x))$ with membership functions of LR-type, $\mu_{m_{nb}^i}(x) \in [0, 1]$ and $\mu_{m_{nc}^i}(x) \in [0, 1]$ are the intervals from which the membership functions for benefits and costs take on their values $(n = 1, 2, \ldots, N,$ $i = 1, 2, \ldots, I, 1 \leq i \leq N, m = 1, 2, \ldots, M)$.

According to equation (4), the interval representations of the fuzzy objective values $\tilde{v}_{m_{nb}^i}(\tilde{v}_{m_{nc}^i})$ on α-levels are:

$$\tilde{v}_{m_{nb}^i} = \left\{ v_{m_{nb}^i}^{\alpha} = \left[v_{m_{nb}^i}^{\alpha L}, v_{m_{nb}^i}^{\alpha R} \right] \right\} \tag{7a}$$

$$\tilde{v}_{m_{nc}^i} = \left\{ v_{m_{nc}^i}^{\alpha} = \left[v_{m_{nc}^i}^{\alpha L}, v_{m_{nc}^i}^{\alpha R} \right] \right\} \tag{7b}$$

where $v^{\alpha L}_{m^i_{nb}}$ and $v^{\alpha R}_{m^i_{nb}}$ ($v^{\alpha L}_{m^i_{nc}}$ and $v^{\alpha R}_{m^i_{nc}}$) are the left (L) and right (R) bounds on α-cuts of fuzzy value $\tilde{v}_{m^i_{nb}}$ ($\tilde{v}_{m^i_{nc}}$).

Analogous to equations (5a)-(5c), we derive function values on the left and right bounds of α-levels for benefits and costs. Values on $\alpha = 1$ and $\alpha = 0$ for benefits are:

$$v^{\alpha=1L}_{m^i_{nb}} = v^{\alpha=1R}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} \tag{8a}$$

$$v^{\alpha=0L}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} - \left(v^{\alpha=1}_{m^i_{nb}} \cdot s_{G^i_n}\right) \tag{8b}$$

$$v^{\alpha=0R}_{m^i_{nb}} = v^{\alpha=1}_{m^i_{nb}} + \left(v^{\alpha=1}_{m^i_{nb}} \cdot s_{G^i_n}\right) \tag{8c}$$

Values on $\alpha = 1$ and $\alpha = 0$ for costs are:

$$v^{\alpha=1L}_{m^i_{nc}} = v^{\alpha=1R}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} \tag{9a}$$

$$v^{\alpha=0L}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} - \left(v^{\alpha=1}_{m^i_{nc}} \cdot s_{G^i_n}\right) \tag{9b}$$

$$v^{\alpha=0R}_{m^i_{nc}} = v^{\alpha=1}_{m^i_{nc}} + \left(v^{\alpha=1}_{m^i_{nc}} \cdot s_{G^i_n}\right) \tag{9c}$$

The scores of the subjective factors for benefits and costs are represented by $u_{mk^i_{nb}}$ ($u_{mk^i_{nc}}$), the intensity of the ith sub-factor within the nth cluster of subjective benefits (costs) factors on alternative m for the kth DM.

4.5 Normalize all estimates to obtain identical units of measurement

Next, we normalize variables with multiple measurement scales to assure uniformity. The literature reports on several normalization methods. The selection of a specific normalization method must be based on the problem characteristics and model requirements. In this study, we use the approach where the normalized value is the quotient of the initial value divided by the sum of the values of all alternatives on that criterion:

$$d_i' = \frac{d_i}{\sum_{i=1}^{n} d_i} \tag{10}$$

Using the above normalization procedure, the normalized values for the objective benefits are:

$$\tilde{v}'_{m^i_{nb}} = \left\{ v'^{\alpha}_{m^i_{nb}} = \left[v'^{\alpha L}_{m^i_{nb}}, v'^{\alpha R}_{m^i_{nb}} \right] \right\} \tag{11}$$

where:

$$\nu'^{\alpha=1}_{m^i_{nb}} = \frac{\nu'^{\alpha=1}_{m^i_{nb}}}{\sum_{m=1}^{M} \nu'^{\alpha=1}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on α-level $= 1$;

$$v'^{\alpha=0L}_{m^i_{nb}} = \frac{v'^{\alpha=0L}_{m^i_{nb}}}{\sum_{m=1}^{M} v'^{\alpha=0L}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on the left bound of α-level $= 0$; and:

$$v'^{\alpha=0R}_{m^i_{nb}} = \frac{v'^{\alpha=0R}_{m^i_{nb}}}{\sum_{m=1}^{M} v'^{\alpha=0R}_{m^i_{nb}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of benefit factors n on the right bound of α-level $= 0$.

Using equation (10), we obtain the normalized values of the objective costs as:

$$\tilde{v}'_{m^i_{nc}} = \left\{ v'^{\alpha}_{m^i_{nc}} = \left[v'^{\alpha L}_{m^i_{nc}}, v'^{\alpha R}_{m^i_{nc}} \right] \right\} \tag{12}$$

where:

$$v'^{\alpha=1}_{mc} = \frac{v'^{\alpha=1}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=1}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on α-level $= 1$;

$$v'^{\alpha=0L}_{m^i_{nc}} = \frac{v'^{\alpha=0L}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=0L}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on the left bound of α-level $= 0$; and:

$$v'^{\alpha=0R}_{m^i_{nc}} = \frac{v'^{\alpha=0R}_{m^i_{nc}}}{\sum_{m=1}^{M} v'^{\alpha=0R}_{m^i_{nc}}}$$

is the normalized fuzzy value of alternative m on sub-criterion i from the group of cost factors n on the right bound of α-level $= 0$.

The normalized scores for the subjective benefits and costs are:

$$u'_{mk^i_{nb}} = \frac{u_{mk^i_{nb}}}{\sum_{m=1}^{M} u_{mk^i_{nb}}} \tag{13a}$$

$$u'_{mk^i_{nc}} = \frac{u_{mk^i_{nc}}}{\sum_{m=1}^{M} u_{mk^i_{nc}}} \tag{13b}$$

4.6 Aggregate subjective and objective factor estimates for the costs and benefits on each alternative for each DM

After the normalization process, we calculate the fuzzy characteristics of each alternative military base for K DMs. Zadeh's Extension Principle (1965, 1975) is widely used technique to perform arithmetic operations with fuzzy values represented by functions having pointwise arguments on level-cuts. The main interest of the level-cut representation is to be very handy when extending set-theoretic notations of fuzzy sets. Any usual point-to-point function can be lifted to a fuzzy-set-to-fuzzy-set function on this basis. See DeBaets and Kerre (1994) for a survey of fuzzy concepts defined via cuts – the main application of the Extension Principle is fuzzy interval analysis. In order to apply an operation f to fuzzy values A and B, it is necessary to apply f to the values $a \in A_\alpha$ and $b \in B_\alpha$ of fuzzy sets A and B on all α-levels. Since we treat our objective values as fuzzy triangular numbers, we can apply arithmetic operations to them on the given (0 and 1) α-cuts in accordance with the Extension Principle. Weighted objective benefits and costs values on the mth alternative for the kth DM on $\alpha = 1$ are calculated as follows:

$$V_{mkb}^{\alpha=1} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=1}_{m^i_{nb}} \quad (14a)$$

$$V_{mkc}^{\alpha=1} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=1}_{m^i_{nc}} \quad (14b)$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

Using the Extension Principle, we calculate the weighted objective benefits and costs values on the mth alternative for kth DM on the left and right bounds of a zero-α-level using equations (15a), (15b), (16a), and (16b), respectively:

$$V_{mkb}^{\alpha=0L} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0L}_{m^i_{nb}} \quad (15a)$$

$$V_{mkb}^{\alpha=0R} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0R}_{m^i_{nb}} \quad (15b)$$

$$V_{mkc}^{\alpha=0L} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0L}_{m^i_{nc}} \quad (16a)$$

$$V_{mkc}^{\alpha=0R} = \sum_{n=1}^{N} \sum_{i=1}^{I} W_{VGnk} \cdot W_{VGn^ik} \cdot v'^{\alpha=0R}_{m^i_{nc}} \quad (16b)$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

The weighted subjective benefit values on the mth alternative for the kth DM on α-cuts of 1 and 0 are:

$$U_{mkb}^{\alpha=1} = U_{mkb}^{\alpha=0L} = U_{mkb}^{\alpha=0R} = \sum_{n=1}^{N}\sum_{i=1}^{I} W_{UGnk} \cdot W_{UGn^ik} \cdot u'_{mk^i_{nb}} \tag{17a}$$

$$U_{mkc}^{\alpha=1} = U_{mkc}^{\alpha=0L} = U_{mkc}^{\alpha=0R} = \sum_{n=1}^{N}\sum_{i=1}^{I} W_{UGnk} \cdot W_{UGn^ik} \cdot u'_{mk^i_{nc}} \tag{17b}$$

$$(n = 1, 2, \ldots, N, \;\; k = 1, 2, \ldots, K, \;\; k \geq 0, \;\; i = 1, 2, \ldots, I, \;\; m = 1, 2, \ldots, M).$$

The overall (aggregated) fuzzy benefit characteristic for the mth alternative and the kth DM is:

$$B_{mk} = \left\{B_{mk}^{\alpha=0L}, B_{mk}^{\alpha=1}, B_{mk}^{\alpha=0R}\right\} \tag{18}$$

where $B_{mk}^{\alpha=0L} = V_{mkb}^{\alpha=0L} + U_{mkb}^{\alpha=0L}$, $B_{mk}^{\alpha=0R} = V_{mkb}^{\alpha=0R} + U_{mkb}^{\alpha=0R}$, and $B_{mk}^{\alpha=1} = V_{mkb}^{\alpha=1} + U_{mkb}^{\alpha=1}$. The overall aggregated fuzzy cost characteristic for the mth alternative and the kth DM is:

$$C_{mk} = \left\{C_{mk}^{\alpha=0L}, C_{mk}^{\alpha=1}, C_{mk}^{\alpha=0R}\right\} \tag{19}$$

where $C_{mk}^{\alpha=0L} = V_{mkc}^{\alpha=0L} + U_{mkc}^{\alpha=0L}$, $C_{mk}^{\alpha=0R} = V_{mkc}^{\alpha=0R} + U_{mkc}^{\alpha=0R}$, and $C_{mk}^{\alpha=1} = V_{mkc}^{\alpha=1} + U_{mkc}^{\alpha=1}$.

4.7 Find combined fuzzy group ratings for the alternative benefits and costs

We use arithmetic mean to collapse the fuzzy values obtained for multiple DMs on the previous step and find a single fuzzy rating for each alternative in the benefit and cost groups. Lets define $B_m = \{B_m^{\alpha=0L}, B_m^{\alpha=1}, B_m^{\alpha=0R}\}$ as the fuzzy rating of alternative m in the group of benefits, ($m = 1,2, \ldots, M$), and $C_m = \left\{C_m^{\alpha=0L}, C_m^{\alpha=1}, C_m^{\alpha=0R}\right\}$ as the fuzzy rating of alternative m in the group of costs ($m = 1, 2, \ldots, M$).

Equations (20a), (20b), (21a), and (21b) are used to calculate the spreads of the above fuzzy ratings. The left and right spreads of the fuzzy number characterizing benefits for the mth alternative are:

$$B_m^L = B_m^{\alpha=1} - B_m^{\alpha=0L} \tag{20a}$$

$$B_m^R = B_m^{\alpha=0R} - B_m^{\alpha=1} \tag{20b}$$

Analogously, the left and right spreads of fuzzy number characterizing costs for the mth alternative are:

$$C_m^L = C_m^{\alpha=1} - C_m^{\alpha=0L} \tag{21a}$$

$$C_m^R = C_m^{\alpha=0R} - C_m^{\alpha=1} \tag{21b}$$

where:

$$B_m^{\alpha=1} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=1}}{K},$$

$$B_m^{\alpha=0L} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=0L}}{K},$$

$$B_m^{\alpha=0R} = \frac{\sum_{k=1}^{K} B_{mk}^{\alpha=0R}}{K},$$

$$C_m^{\alpha=1} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=1}}{K},$$

$$C_m^{\alpha=0L} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=0L}}{K}, \quad \text{and}$$

$$C_m^{\alpha=0R} = \frac{\sum_{k=1}^{K} C_{mk}^{\alpha=0R}}{K}.$$

4.8 Identify the ideal alternative and calculate the total Euclidean distance of each base

The weighted-sum fuzzy values in this study are used to compare potential military bases among themselves and with the ideal base. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana, 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p. 144) shows that the Euclidean measure can be used as a proxy measure of distance.

Using the Euclidean measure suggested by Zeleny (1982), we synthesize the results by determining the ideal benefits and costs values. The ideal benefit (B^*) is the highest value among the set B_m on $\alpha = 1$, and the ideal cost (C^*) is the lowest value among the set C_m on $\alpha = 1$. We then find the Euclidean distance of each military base from the ideal base. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the mth indices of the benefits and costs. To formulate the described model algebraically, let us assume:

D_B^m = total Euclidean distance from the ideal benefit for the mth alternative military base; ($m = 1,2,\ldots, M$)

D_C^m = total Euclidean distance from the ideal cost for the mth alternative military base; ($m = 1,2,\ldots, M$)

D^m = overall, Euclidean distance of the mth alternative military base; ($m = 1,2,\ldots, M$)

$$D^m = \sqrt{(D_B^m)^2 + (D_C^m)^2} \tag{22}$$

where:

$$B^* = \text{Max}\left\{B_m^{\alpha=1}\right\}$$

$$C^* = \text{Min}\left\{C_m^{\alpha=1}\right\}$$

and:

$$D_B^m = B* - U_m^{\alpha=1}$$

$$D_C^m = C_m^{\alpha=1} - C*.$$

Alternative military bases with smaller D^m are closer to the ideal base and are preferred to alternative bases with larger D^m which are further away from the ideal base.

Fuzzy relations play an important role in the theory of fuzzy sets. A fuzzy relation is a fuzzy subset R of a Cartesian product of sets. Fuzzy relations obtained by combining fuzzy sets offer a general setting for multi-factorial evaluation. A particular case of fuzzy relation is a fuzzy Cartesian product. It is presumed that R has projections on all the axes. An example of Cartesian product of two triangular fuzzy sets A and B is shown on Figure 5.

Let us further define triangular fuzzy benefits ($\tilde{B}_m$) and costs ($\tilde{C}_m$) estimates for the mth alternative that compose the survivability index:

$$\tilde{B}_m = \left\{D_B^{\alpha=0L}, D_B^m, D_B^{\alpha=0R}\right\}$$
$$\tilde{C}_m = \left\{D_C^{\alpha=0L}, D_C^m, D_C^{\alpha=0R}\right\} \quad (23)$$

where $D_B^{\alpha=0L} = D_B^m - B_m^L$ and $D_B^{\alpha=0R} = D_B^m + B_m^R$ are the left and right boundaries of the fuzzy benefits component of the survivability index for the m −th alternative, and; $D_C^{\alpha=0L} = D_C^m - C_m^L$ and $D_C^{\alpha=0R} = D_C^m + C_m^R$ are the left and right boundaries of the fuzzy costs component of survivability index for the mth alternative.

Next, we evaluate the Cartesian product of the benefit and cost components of the fuzzy survivability index for each of the 52 alternative military bases. A general view of the Cartesian product for the mth alternative is given in Figure 6.

The numerical designations of the alternative military bases and the survivability indices and their components are presented in Tables VI and VII.

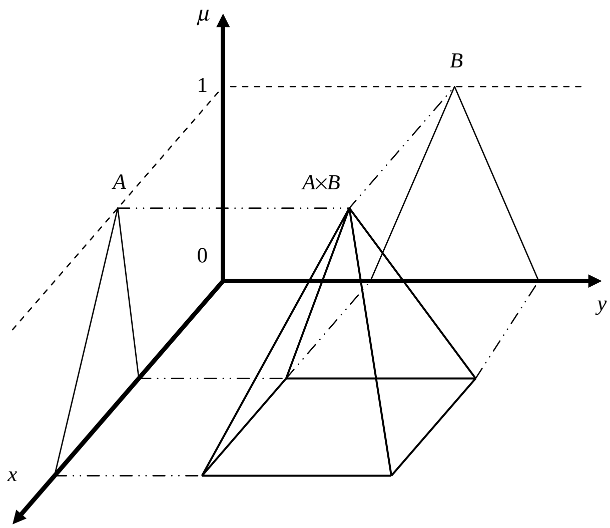

Figure 5.
The Cartesian product of two triangular fuzzy sets

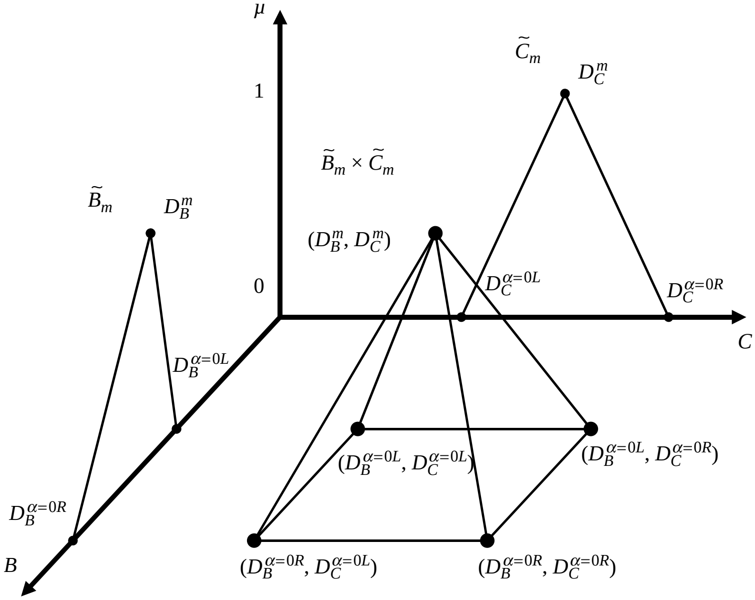

Figure 6.
The Cartesian product of fuzzy survivability index components

4.9 Rank the bases using visual and numerical information, taking into consideration the level of uncertainty of their fuzzy characteristics

The computations described earlier result in 52 pyramids for our set of the alternative military bases. In order to compare the results, we first consider the most reliable values given on α-level $= 1$. Table VIII provides the ranking of each alternative military base according to it Euclidean distance from the *ideal base*.

Next, we plot the alternative military bases on a graph where the x-axis is represented by the benefits (B) and the y-axis is represented by the costs (C). Figure 7 shows the alternative arrangement on $\alpha = 1$. The position of the point corresponding to alternative military base m has the Cartesian coordinates (D_B^m, D_C^m). We have excluded military bases 3, 12, 17, 44, 48, and 52 from this figure to zoom on the area where the majority of the bases are located. The 52 military bases fell into efficient, active, inactive or inefficient quadrants. The efficient zone with $D_B^m < \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m < \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or high benefits and low costs included military base 2, 4, 7-9, 11, 14, 20, 22, 28, 32, 36, 38, and 40 (plus 12 and 44 excluded from the graph). The active zone with $D_B^m < \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m > \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or high benefits and high cost included military base 10 (plus 3 excluded for the graph). The inactive zone with $D_B^m > \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m < \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or low benefits and low costs included military bases 1, 5, 6, 13, 15, 16, 18, 19, 21, 23-27, 29, 30, 31, 35, 37, 39, 41-43, 45-47, 49, 50, and 51. Finally, the inefficient zone with $D_B^m > \underset{m}{\mathrm{MAX}}\{D_B^m\}/2$ and $D_C^m > \underset{m}{\mathrm{MAX}}\{D_C^m\}/2$ or low benefits and high costs included military base 33 and 34 (plus 17, 48, and 52 excluded from the graph).

In the final step of the process, we compare the fuzzy attributes of the M alternatives to see if the uncertainty levels could influence the rankings. In general, alternatives with close Euclidean distance and varying uncertainty levels could swap their rankings.

Code	Military base
1	Army Reserve Personnel Center St. Louis
2	Brooks City Base
3	Cannon Air Force Base
4	Desecret Chemical Depot
5	Eielson, AFB
6	Elmendorf AFB
7	Fort Gillem
8	Fort Knox
9	Fort McPherson
10	Fort Monmouth
11	Fort Monroe
12	Ft Eustis
13	Gen Mitchell International Airport ARS
14	Grand Forks AFB
15	Kansas Amunition Plant
16	Kulis Air Guard Station
17	Lackland AFB
18	Lone Star Army Ammunition Plant
19	Marine Corps Logistics Barstow
20	McChord AFB
21	Mississippi Army Ammunition Plant
22	Mountain Home AFB
23	NAS Corpus Chisti
24	NAS Oceana
25	NAS Pensacola
26	Naval Air Station Atlanta
27	Naval Air Station Brunswick
28	Naval Air Station Williow Grove
29	Naval Base Coranado
30	Naval Base Ventura City
31	Naval District Washington DC
32	Naval Medical Center Portsmouth
33	Naval Medical Center San Diego
34	Naval Station Great Lakes
35	Naval Station Ingleside
36	Naval Station Pascagoula
37	Naval Suport Activity, New Orleans
38	Naval Support Activity Crane
39	Naval Weapons Stations Seal Beach Concord Detachment
40	Newport Chemical Depot
41	Niagara Falls International Airport Air Guard Station
42	Onizuka Air Force Station
43	Otis Air National Guard Base
44	Pope AFB
45	Red River Army Depot
46	Riverbank Army Ammunition Plant
47	Rock Island Arsenal
48	Selfridge Army Activity
49	Sheppard AFB
50	Umatilla Army Depot
51	W.K. Kellogg Air Force Guard Station
52	Walter Reed National Military Medical Center

Table VI. Alternative military bases and their numerical designations

Table VII.
The survivability indices and their components

Base m	Benefits ($\tilde{B}_m$) $D_B^{\alpha=0L}$	D_B^m	$D_B^{\alpha=0R}$	Costs ($\tilde{C}_m$) $D_C^{\alpha=0L}$	D_C^m	$D_C^{\alpha=0R}$
1	0.05897	0.05905	0.05913	0.00099	0.00102	0.00105
2	0.05464	0.05487	0.05510	0.00461	0.00468	0.00476
3	0.04617	0.04688	0.04760	0.00857	0.00863	0.00869
4	0.05680	0.05690	0.05700	0.00141	0.00149	0.00156
5	0.06036	0.06039	0.06043	0.00113	0.00115	0.00118
6	0.06089	0.06092	0.06095	0.00313	0.00323	0.00333
7	0.05799	0.05810	0.05821	0.00114	0.00118	0.00121
8	0.05757	0.05763	0.05769	0.00296	0.00303	0.00309
9	0.05472	0.05494	0.05516	0.00301	0.00307	0.00312
10	0.05603	0.05634	0.05664	0.01262	0.01286	0.01311
11	0.05542	0.05561	0.05579	0.00213	0.00218	0.00223
12	0.00000	0.00000	0.00000	0.00019	0.00021	0.00023
13	0.05886	0.05887	0.05888	0.00097	0.00102	0.00107
14	0.05617	0.05633	0.05649	0.00246	0.00249	0.00252
15	0.05823	0.05825	0.05828	0.00156	0.00161	0.00166
16	0.06076	0.06079	0.06082	0.00282	0.00291	0.00301
17	0.06161	0.06162	0.06162	0.03332	0.03479	0.03626
18	0.06134	0.06138	0.06142	0.00044	0.00045	0.00046
19	0.06041	0.06047	0.06053	0.00001	0.00002	0.00002
20	0.05783	0.05787	0.05792	0.00000	0.00000	0.00000
21	0.05840	0.05841	0.05842	0.00066	0.00071	0.00075
22	0.05799	0.05804	0.05809	0.00407	0.00412	0.00417
23	0.05867	0.05882	0.05898	0.00183	0.00187	0.00190
24	0.06004	0.06008	0.06012	0.00649	0.00664	0.00678
25	0.05966	0.05966	0.05967	0.00099	0.00104	0.00108
26	0.05893	0.05905	0.05917	0.00045	0.00047	0.00048
27	0.05855	0.05875	0.05895	0.00544	0.00555	0.00567
28	0.05621	0.05640	0.05659	0.00232	0.00236	0.00241
29	0.06099	0.06100	0.06101	0.00000	0.00002	0.00005
30	0.06098	0.06100	0.06102	0.00115	0.00119	0.00123
31	0.06036	0.06041	0.06045	0.00062	0.00064	0.00066
32	0.05403	0.05433	0.05464	0.00321	0.00325	0.00330
33	0.05910	0.05910	0.05910	0.01211	0.01337	0.01464
34	0.06077	0.06077	0.06077	0.00869	0.00941	0.01012
35	0.05805	0.05820	0.05835	0.00268	0.00273	0.00278
36	0.05496	0.05514	0.05533	0.00102	0.00103	0.00104
37	0.06048	0.06050	0.06051	0.00195	0.00202	0.00210
38	0.05686	0.05692	0.05698	0.00306	0.00308	0.00310
39	0.05995	0.06000	0.06006	0.00120	0.00125	0.00130
40	0.05800	0.05804	0.05807	0.00019	0.00020	0.00020
41	0.06189	0.06189	0.06189	0.00444	0.00472	0.00500
42	0.05927	0.05933	0.05938	0.00151	0.00156	0.00161
43	0.05860	0.05868	0.05876	0.00407	0.00424	0.00441
44	0.04291	0.04365	0.04438	0.00389	0.00394	0.00399
45	0.06003	0.06008	0.06013	0.00301	0.00312	0.00323
46	0.06019	0.06021	0.06022	0.00097	0.00100	0.00103
47	0.06085	0.06086	0.06087	0.00157	0.00164	0.00171
48	0.05919	0.05919	0.05919	0.03008	0.03233	0.03458
49	0.05980	0.05984	0.05987	0.00417	0.00426	0.00435
50	0.06025	0.06034	0.06042	0.00115	0.00117	0.00118
51	0.05991	0.05991	0.05991	0.00043	0.00047	0.00050
52	0.05603	0.05631	0.05658	0.01773	0.01809	0.01846

Survivability Index (spanning header over all Benefits and Costs columns)

Rank	Alternative	Military base	Euclidean Distance
1	12	Ft Eustis	0.000210
2	44	Pope AFB	0.043825
3	3	Cannon Air Force Base	0.047671
4	32	Naval Medical Center Portsmouth	0.054431
5	9	Fort McPherson	0.055027
6	2	Brooks City Base	0.055071
7	36	Naval Station Pascagoula	0.055153
8	11	Fort Monroe	0.055649
9	14	Grand Forks AFB	0.056383
10	28	Naval Air Station Williow Grove	0.056451
11	4	Desecret Chemical Depot	0.056920
12	38	Naval Support Activity Crane	0.057007
13	8	Fort Knox	0.057708
14	10	Fort Monmouth	0.057786
15	20	McChord AFB	0.057871
16	40	Newport Chemical Depot	0.058040
17	7	Fort Gillem	0.058114
18	22	Mountain Home AFB	0.058187
19	35	Naval Station Ingleside	0.058265
20	15	Kansas Amunition Plant	0.058273
21	21	Mississippi Army Ammunition Plant	0.058416
22	43	Otis Air National Guard Base	0.058829
23	23	NAS Corpus Chisti	0.058853
24	13	Gen Mitchell International Airport ARS	0.058878
25	27	Naval Air Station Brunswick	0.059009
26	26	Naval Air Station Atlanta	0.059050
27	1	Army Reserve Personnel Center St. Louis	0.059058
28	52	Walter Reed National Military Medical Center	0.059142
29	42	Onizuka Air Force Station	0.059347
30	25	NAS Pensacola	0.059673
31	51	W.K. Kellogg Air Force Guard Station	0.059911
32	49	Sheppard AFB	0.059987
33	39	Naval Weapons Stations Seal Beach Concord Detachment	0.060016
34	45	Red River Army Depot	0.060163
35	46	Riverbank Army Ammunition Plant	0.060214
36	50	Umatilla Army Depot	0.060348
37	5	Eielson, AFB	0.060404
38	31	Naval District Washington DC	0.060409
39	24	NAS Oceana	0.060445
40	19	Marine Corps Logistics Barstow	0.060470
41	37	Naval Suport Activity, New Orleans	0.060531
42	33	Naval Medical Center San Diego	0.060595
43	16	Kulis Air Guard Station	0.060857
44	47	Rock Island Arsenal	0.060881
45	29	Naval Base Coranado	0.061000
46	6	Elmendorf AFB	0.061007
47	30	Naval Base Ventura City	0.061014
48	18	Lone Star Army Ammunition Plant	0.061378
49	34	Naval Station Great Lakes	0.061498
50	41	Niagara Falls International Airport Air Guard Station	0.062067
51	48	Selfridge Army Activity	0.067442
52	17	Lackland AFB	0.070764

Table VIII. The rankings according to the Euclidean distance from the ideal military base

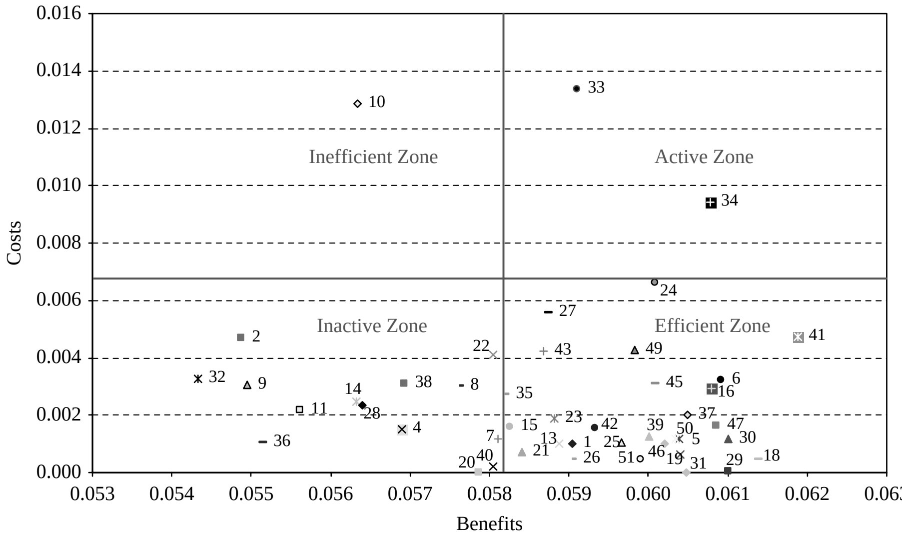

Figure 7.
Alternatives arrangement on α-level $= 1$

Given two sets A and B, we were interested in the following questions: Is the intersection between A and B empty or not? Is A a subset of B? Are A and B equal? These questions can be answered using a fuzzy extension of the Boolean inclusion index proposed by Bandler and Kohout (1980). Dubois and Prade (1982) have proposed a framework for building fuzzy comparison indices where three types of indices are considered: overlap indices (called partial matching), inclusion indices and similarity indices (evaluating equality between fuzzy sets). The comparison of fuzzy sets could also be described by means of a fuzzy-valued compatibility index introduced by Zadeh (1978). However, there is a gap in the literature on the comparison methods for multidimensional fuzzy relations. To compare our alternative military bases we must take into consideration the Cartesian product of their benefits and costs constituents. Consequently, we considered calculating the volume of each resulting pyramid as the characteristics of an alternative fuzziness degree. The volume of a pyramid is equal to one third of the product of the area of pyramid basis and the length of its height:

$$V = \frac{1}{3} S_{\mathrm{b}} \cdot H \tag{24}$$

where S_{b} is the area of pyramid basis and H is the length of the pyramid height.

In our case, $H = 1$ and S_{b} is a rectangle. Using our variables, equation (24) can be reformulated as:

$$V_m = \frac{1}{3}\left(D_{Bm}^{\alpha=0R} - D_{Bm}^{\alpha=0L}\right) \cdot \left(D_{Cm}^{\alpha=0R} - D_{Cm}^{\alpha=0L}\right) \tag{25}$$

On the basis of these uncertainty levels, it is reasonable to swap the rankings for some of the alternative military bases, namely 2 and 36, 4 and 38, 10 and 20, 35 and 15, 43 and 23, 27 and 26, 45 and 46, 24 and 19, 16 and 47, 6 and 30, 18 and 34. The alternative rankings are presented in Table IX.

Rank	Alternative	Base
1	12	Ft Eustis
2	44	Pope AFB
3	3	Cannon Air Force Base
4	32	Naval Medical Center Portsmouth
5	9	Fort McPherson
6	36	Naval Station Pascagoula
7	2	Brooks City Base
8	11	Fort Monroe
9	14	Grand Forks AFB
10	28	Naval Air Station Williow Grove
11	38	Naval Support Activity Crane
12	4	Desecret Chemical Depot
13	8	Fort Knox
14	20	McChord AFB
15	10	Fort Monmouth
16	40	Newport Chemical Depot
17	7	Fort Gillem
18	22	Mountain Home AFB
19	15	Kansas Amunition Plant
20	35	Naval Station Ingleside
21	21	Mississippi Army Ammunition Plant
22	23	NAS Corpus Chisti
23	43	Otis Air National Guard Base
24	13	Gen Mitchell International Airport ARS
25	26	Naval Air Station Atlanta
26	27	Naval Air Station Brunswick
27	1	Army Reserve Personnel Center St. Louis
28	52	Walter Reed National Military Medical Center
29	42	Onizuka Air Force Station
30	25	NAS Pensacola
31	51	W.K. Kellogg Air Force Guard Station
32	49	Sheppard AFB
33	39	Naval Weapons Stations Seal Beach Concord Detachment
34	46	Riverbank Army Ammunition Plant
35	45	Red River Army Depot
36	50	Umatilla Army Depot
37	5	Eielson, AFB
38	31	Naval District Washington DC
39	19	Marine Corps Logistics Barstow
40	24	NAS Oceana
41	37	Naval Suport Activity, New Orleans
42	33	Naval Medical Center San Diego
43	47	Rock Island Arsenal
44	16	Kulis Air Guard Station
45	29	Naval Base Coranado
46	30	Naval Base Ventura City
47	6	Elmendorf AFB
48	34	Naval Station Great Lakes
49	18	Lone Star Army Ammunition Plant
50	41	Niagara Falls International Airport Air Guard Station
51	48	Selfridge Army Activity
52	17	Lackland AFB

Table IX.
The revised rankings

5. Conclusions and future research directions

The benchmarking system presented in this study helps DMs crystallize their thoughts and reduce the environmental complexities inherent in the BRAC decisions. The BRAC Commission can utilize the survivability indices to arrive at a ranking of the military bases on the DoD hit list. Moreover, the commanding officers of the military bases can use the four-quadrant classification approach to identify their strengths and weaknesses by learning from "best-in-class" and other competing bases.

Our model is intended to create an even playing field for benchmarking and pursuing consensus not to imply a deterministic approach to BRAC decisions. The BRAC is a very complex problem requiring compromise and negotiation within various branches of government and public. The analytical methods in the proposed benchmarking system help DMs decompose complex MCDA problems into manageable steps, making this model accessible to a wide variety of situations. These methods are not developed through a straightforward sequential process where the DM's role is passive. On the contrary, the iterative process is used to analyze the objective and subjective judgments of multiple DMs and represent them as consistently as possible in an appropriate structured framework. This iterative and interactive preference modeling procedure is the basic distinguishing feature of our model as opposed to statistical and optimization decision making approaches.

MCDA and fuzzy sets are useful tools for handling inherent uncertainty and imprecision in rapidly changing environments. There are many facets of MCDA in fuzzy environment which require more thorough investigation. The model developed in this study can be extended to a multi-stage model with probabilistic outcomes. BRAC decisions are generally long-term and could be considered in stages over a period of time. A multi-stage model under fuzziness, involving objective and subjective aspects, could assess potential impact on different stakeholders over a period of time. The model could focus not only on which bases should be closed, but how closure and realignment should take place in stages.

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