Computers & Industrial Engineering 66 (2013) 10–23

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A fuzzy group data envelopment analysis model for high-technology project selection: A case study at NASA

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ARTICLE INFO

Article history:
Available online 10 June 2013

Keywords:
Data envelopment analysis
Project portfolio selection
NASA
Fuzzy sets
Ambiguity
Vagueness

ABSTRACT

The assessment and selection of high-technology projects is a difficult decision making process at the National Aeronautic and Space Administration (NASA). This difficulty is due to the multiple and often conflicting objectives in addition to the inherent technical complexities and valuation uncertainties involved in the assessment process. As such, a systematic and transparent decision making process is needed to guide the assessment process, shape the decision outcomes and enable confident choices to be made. Various methods have been proposed to assess and select high-technology projects. However, applying these methods has become increasingly difficult in the space industry because there are many emerging risks implying that decisions are subject to significant uncertainty. The source of uncertainty can be *vagueness* or *ambiguity*. While vague data are uncertain because they lack detail or precision, ambiguous data are uncertain because they are subject to multiple interpretations. We propose a data envelopment analysis (DEA) model with ambiguity and vagueness. The vagueness of the objective functions is modeled by means of multi-objective fuzzy linear programming. The ambiguity of the input and output data is modeled with fuzzy sets and a new α-cut based method. The proposed models are linear, independent of α-cut variables, and capable of maximizing the satisfaction level of the fuzzy objectives and efficiency scores, simultaneously. Moreover, these models are capable of generating a common set of multipliers for all projects in a single run. A case study involving high-technology project selection at NASA is used to demonstrate the applicability of the proposed models and the efficacy of the procedures and algorithms.

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1. Introduction

The recent global economic meltdown and the passage of austere budgets have focused critical attention on government agencies that support technology development. The public is concerned with the governance of these agencies and with obtaining the maximum return on public spending. Public pressure has forced Congress to mandate the National Aeronautic and Space Administration (NASA) to be more accountable in its evaluation of high-technology projects. The high-technology assessment process at NASA is intended to: (1) identify what technologies are needed and when they need to be available; (2) develop and implement a rigorous and objective technology prioritization process; and (3) develop technology investment recommendations about which existing projects should continue and which new projects should be established (NASA ESAS Final Report, 2005). The assessment process involves budget, schedule, safety, reliability, feasibility and reusability considerations needed to develop an optimal portfolio of high-technologies projects to facilitate more feasible future space missions. The role of the Ground System Working Team (GSWT) at the Kennedy Space Center (KSC) is to help determine the value of investing in a particular high-technology that will maintain NASA's current space science capabilities, and enable safe and successful future space exploration missions. The GSWT is a carefully balanced panel of senior technology and systems experts from five different divisions at the KSC.

The assessment and selection of projects is an important issue in technology management (Linton, Walsh, & Morabito, 2002; Shehabuddeen, Probert, & Phaal, 2006; Sun & Ma, 2005). The rapid development of technological changes, together with increasing

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http://dx.doi.org/10.1016/j.cie.2013.06.002

complexity, has made the task of technology assessment and selection a difficult one (Shehabuddeen et al., 2006; Solak, Clarke, Johnson, & Barnes, 2010). The literature on project selection contains hundreds of models, including: scoring methods, ad hoc methods, comparative methods, economic methods, portfolio methods, mathematical optimization methods and simulation methods.

Scoring methods use a relatively small number of quantitative criteria to specify project desirability. Using these methods, the merit of each project is determined with respect to each criterion, and then scores are combined to yield an overall performance score for each project (Coldrick, Longhurst, Ivey, & Hannis, 2005; DePiante & Jensen, 1999; Henriksen & Traynor, 1999; Oh, Yang, & Lee, 2012). Ad hoc methods are a special form of scoring methods. In these methods, limits are set for the various criteria levels, and then any projects which fail to meet these limits are eliminated.

Comparative methods consider both quantitative and qualitative attributes. In these methods, the weights of different attributes are determined and alternatives are compared on the basis of their contributions to these attributes, and then a set of project benefit measures is computed. Once the projects have been arranged on a comparative scale, the decision makers (DMs) proceed from the top of the list and select projects until available resources are exhausted (Huang, Chu, & Chiang, 2008; Khalili-Damghani & Sadi-Nezhad, 2013; Tiryaki & Ahlatcioglu, 2009).

Economic methods use financial metrics and models to calculate the monetary payoff of each project under consideration. In these methods, two dimensions such as the expected monetary value and the likelihood of success are selected, and then a representative mix of projects with respect to these dimensions are selected (Eilat, Golany, & Shtub, 2006; Ho & Liao, 2011; Zapata & Reklaitis, 2010).

Mathematical optimization methods optimize various objective functions within the constraints of resources, project logic and dynamics, technology, and project-related strategies. They include a wide range of methods, such as linear, non-linear, integer, dynamic, goal and stochastic mathematical programming methods (Beaujon, Marin, & McDonald, 2001; Dickinson, Thornton, & Graves, 2001; Elazouni & Abido, 2011; Kester, Hultink, & Lauche, 2009; Khalili-Damghani, Sadi-Nezhad, & Aryanezhad, 2011).

Simulation is the process of imitating the operation of a real-world process or system over time. Essentially, simulation consists of: (1) building a model that describes the behavior of a system; and (2) experimenting with this model to support decision making and problem solving. The purpose of simulation is to shed light on the underlying mechanisms that determine the behavior of a system. Simulation is also used when the real system cannot be engaged, because it may not be accessible, or it may be dangerous or unacceptable to engage, or it is being designed but not yet built, or it may simply not exist (Banks, Carson, Nelson, & Nicol, 2009). Simulation is a powerful tool for evaluating alternative designs, plans and/or policies without having to experiment on a real system, which may be prohibitively costly, time-consuming, or simply impractical.

Almost any system which can be modeled using equations and/or rules can be simulated. Computer simulation is often used as an addition to, or substitution for, modeling systems for which simple closed form analytic solutions are not possible. Computer simulations are generally developed to generate a random sample of representative scenarios for which a complete enumeration of all possible states would be prohibitive or impossible. However using high level computers, it is often feasible to generate a sufficient number of cases in a relatively short amount of processing time to accurately describe the behavior of the system under a variety of conditions. In many cases the problem is not the total number of enumerations, but the inability to completely validate the equations and the rules that are used to generate the simulations.

Multi-scenario optimization is used for problems where the number of alternatives and/or decision parameters is large and the system cannot be modeled with deterministic optimization methods. Multi-scenario optimization is a convenient way to formulate design optimization problems that are tolerant to uncertainties and/or need to operate under a variety of different conditions. The behavior of the parameters and the states of the system are estimated stochastically using probability distribution functions when the number of scenarios increases extensively. A stochastic process is one whose behavior is non-deterministic and is a sequence of random variables. By definition any system or process that can be analyzed using probability theory is stochastic. The stochastic component of the problem (uncertainty) is reproduced by simulation and the overall performance of the system is measured by calculating the expected values and variation of the output variables for a very large number of iterations. Statistical techniques can be used to test various hypotheses about the system performance. Sensitivity analysis can also be used as a major tool to support decision making and problem solving. Law and Kelton (2007) and Banks et al. (2009) provide excellent overviews of simulation modeling and techniques.

Optimization methods are also a special form of decision analysis. In these methods the DMs select from the list of candidate projects a set that provides maximum benefit (e.g., maximum net present value). These methods employ mathematical programming to facilitate the optimization process and to take into account project interactions such as resource dependencies and constraints, technical and market interactions, or program considerations (Araúzo, Pajares, & Lopez-Paredes, 2010; Stamelos & Angelis, 2001; Vithayasrichareon & MacGill, 2012).

In this study we use data envelopment analysis (DEA) for project portfolio selection at NASA. DEA is a non-parametric approach to performance evaluation. With DEA, the efficient frontier is the benchmark against which the relative performance of projects is measured. Given a group of high-technology projects, all projects should be able to operate at an optimal efficiency level which is determined by the efficient projects in the group. These efficient projects are usually referred to as the "peer projects" and determine the efficient frontier. The projects that form the efficient frontier use a minimum quantity of inputs to produce the same quantity of outputs. The distance to the efficiency efficient frontier provides a measure for the efficiency or its lack thereof. We use DEA for project portfolio selection at NASA because DEA: (1) accommodates a multiplicity of inputs and outputs; (2) does not require a priori weights on inputs and outputs or explicitly hypothesized forms of relations between the various inputs and outputs; (3) inputs and outputs can be quantified using different units of measurement including imprecise data represented by fuzzy sets; and (4) can determine possible sources of inefficiency in each project.

The remainder of this paper is organized as follows. In Section 2 we review the relevant literature on DEA. The details of the proposed DEA method are presented in Section 3. In Section 4 we illustrate a high-technology project portfolio selection study at NASA to demonstrate the applicability of the proposed framework and exhibit the efficacy of the procedures. We end the paper with our conclusions and future research directions in Section 5.

2. Literature review

DEA is a mathematical programming technique that uses multiple inputs and outputs to construct piece-wise linear convex production frontiers and measure relative efficiencies within a group of Decision Making Units (DMUs). The technique was first proposed by Charnes, Cooper, and Rhodes (1978) and later extended by Banker, Charnes, and Cooper (1984). The relative efficiency performance of a DMU is defined as the ratio of multiple weighted outputs to multiple weighted inputs. The two basic DEA models are named after the respective researchers who first introduced them: the Charnes Cooper Rhodes (CCR) and the Banker Charnes Cooper (BCC) models. The two models are generally distinguished by the type of their envelopment surfaces and orientations. The envelopment surfaces are depicted by either a constant-return-to-scale (CRS) or a variable return-to-scale (VRS) represented in the CCR and the BCC models, respectively. The DEA models are either input-oriented or output-oriented. Input orientation implies that an inefficient DMU may be made efficient by reducing the proportions of its inputs but keeping the output proportions constant. Output orientation implies that an inefficient DMU may be made efficient by increasing the proportions of its outputs while keeping the input proportions constant. DEA is used to: (1) identify the best alternative; (2) rank the alternatives; or (3) establish a shortlist of the better alternatives for detailed review (Cooper, Seiford, & Tone, 2006). Cook and Seiford (2009) provide a detailed review of major research directions in DEA.

2.1. Fuzzy DEA methods

One limitation of the conventional DEA methods is the need for accurate measurement of both the inputs and the output data. While crisp input and output data are fundamentally indispensable in the conventional DEA models, input and output data in real-world problems are often uncertain. Uncertain data may be the result of unquantifiable, incomplete, or non-obtainable information. In recent years, many researchers have formulated a wide range of DEA models to deal with the uncertain input and output data. The "Stochastic approach" and the "fuzzy approach" are two existing approaches for modeling uncertainty in the DEA literature. The stochastic approach involves specifying a probability distribution function (e.g., normal) for the error process (Sengupta, 1992). However, as pointed out by Sengupta (1992), the stochastic approach has two drawbacks associated with modeling the uncertainty in DEA problems:

(a) Small sample sizes in DEA make it difficult to use stochastic models, and
(b) In stochastic approaches, the Decision Maker (DM) is required to assume a specific error distribution (e.g., normal or exponential) to derive specific results. However, this assumption may not be realistic because on an a priori basis there is very little empirical evidence to choose one type of distribution over another.

Some researchers have proposed various fuzzy methods for dealing with uncertain data in DEA. Fuzzy sets algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of uncertain estimates in fuzzy DEA. In this study, we propose a new fuzzy DEA model for evaluating the efficiency of a set of high-technology projects with fuzzy inputs and outputs. In general, fuzzy DEA methods can be classified into four primary categories, namely, the tolerance approach (Kahraman & Tolga, 1998; Sengupta, 1992), the α-level based approach (Hatami-Marbini, Saati, & Tavana, 2010; Kao & Liu, 2000, 2003), the fuzzy ranking approach (Guo, 2009; Guo & Tanaka, 2001), and the possibility approach (Lertworasirikul, Fang, Joines, & Nuttle, 2003; Lertworasirikul, Fang, Nuttle, & Joines, 2003). An exhaustive review and taxonomy of various fuzzy DEA models can be found in Hatami-Marbini, Emrouznejad, and Tavana (2011).

Kao and Liu (2000) transformed fuzzy input and output data into intervals by using α-level sets. Entani, Maeda, and Tanaka (2002) extended the α-level set research by changing fuzzy input and output data into intervals. Dia (2004) proposed a fuzzy DEA model where a fuzzy aspiration level and a safety α-level were used to transform the fuzzy DEA model into a crisp DEA. Soleimani-damaneh, Jahanshahloo, and Abbasbandy (2006) addressed some of the limitations of the fuzzy DEA models proposed by Kao and Liu (2000), León, Liern, Ruiz, and Sirvent (2003) and Lertworasirikul, Fang et al. (2003) and suggested a fuzzy DEA model to produce crisp efficiencies. Liu (2008) and Liu and Chuang (2009) extended the α-level set approach by proposing the assurance region approach in the fuzzy DEA model.

Wang, Luo, and Liang (2009) developed two fuzzy DEA models using fuzzy arithmetic to handle fuzziness in input and output data in DEA. Soleimani-damaneh (2008) used the fuzzy signed distance and the fuzzy upper bound concepts to formulate a fuzzy additive model in DEA with fuzzy input–output data. Soleimani-damaneh (2009) put forward a theorem on the fuzzy DEA model which was proposed by Soleimani-damaneh (2008) in order to show the existence of a distance-based upper bound for the objective function of the model. Khodabakhshi, Gholami, and Kheirollahi (2010) formulated two alternative fuzzy and stochastic additive models to determine returns to scale in DEA. Tavana, Khanjani Shiraz, Hatami-Marbini, Agrell, and Paryab (2012) proposed three fuzzy DEA models with respect to probability–possibility, probability–necessity and probability–credibility constraints. In addition to addressing the possibility, necessity and credibility constraints in the DEA model they also considered probability constraints.

2.2. Project selection and evaluation with DEA

The DEA model, often used for relative efficiency analysis and productivity analysis, has also been successfully constructed for project selection. Cook and Green (2000) used DEA and the knapsack method to determine the overall ranking of a set of research and development projects. They combined the evaluation and selection processes in a single process by placing the DEA model within a mixed-binary linear programming framework. Sowlati, Paradi, and Suld (2005) proposed a new DEA model for prioritizing information system projects. They used the inputs and outputs of the model as the criteria for judging the importance of the projects. Their model provided a fair and equitable ranking and new projects could be prioritized at any time without affecting the priority of the previously assessed projects.

Vitner, Rozenes, and Spraggett (2006) investigated the possibility of using the DEA method for evaluating the performances of projects according to a hybrid earned value management system and a multidimensional control system. Eilat et al. (2006) proposed a methodology for constructing and analyzing efficient, effective and balanced portfolios of projects with interactions. Their methodology was based on an extended DEA model that quantified some of the qualitative concepts embedded in the balanced scorecard approach. Eilat, Golany, and Shtub (2008) developed a multi-criteria approach for evaluating research and development projects in different stages of their life cycle. They integrated the balanced scorecard with the DEA model through a hierarchical structure of constraints and proposed an extended DEA model.

More recently, Asosheh, Nalchigar, and Jamporazmey (2010) proposed a new project selection model by combining the balanced scorecard and DEA. They used the balanced scorecard as a comprehensive framework to define project selection criteria and employed DEA as a non-parametric technique for ranking the projects. Chang and Lee (2012) proposed a fuzzy DEA and knapsack formulation to deal with the problem of selecting a portfolio of projects in the engineering-procurement-construction industry. They applied three constraint handling techniques to transform a constrained optimization problem into an unconstrained problem and compared the performance of these three techniques.

3. The proposed fuzzy DEA model

A commonly cited definition of uncertainty given by Hunter and Goodchild (1993) is "the degree to which the lack of knowledge about the amount of error is responsible for hesitancy in accepting the results and observations without caution." The source of uncertainty can be *vagueness* or *ambiguity*. Vagueness refers to data with lack of clarity and ambiguity refers to data with several overlapping values. While vague data are uncertain because they lack detail or precision, ambiguous data are uncertain because they are subject to multiple interpretations.

The achievement level of the objective functions in our DEA model is assumed to have a considerable amount of vagueness. This vagueness is attributed to the multiple and conflicting objectives in the DEA problem. In this case, the DMs may have to settle for a satisficing or compromise solution (whose attributes all exceed certain minimum desired levels of aspiration) instead of an optimal solution. In addition, crisp input and output data are fundamentally indispensable in the traditional DEA evaluation process. However, the input and output data in real-world problems are often ambiguous. The ambiguity of the input and output data is represented with fuzzy sets in our model.

Many researchers have proposed various fuzzy methods for dealing with the imprecise data in DEA (i.e., the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach). In this study, we use the α-level based approach since it is the most popular fuzzy DEA model (Hatami-Marbini et al., 2011). The α-level based approach proposed in this study will address the shortcomings of the classical α-level based approach such as excessive computational efforts, conflicting rankings, and local optimal solutions.

3.1. Modeling DEA problems with multiple decision makers

Considering the conventional DEA notation in the literature, let us assume that each DMU_j $(j = 1, 2, \ldots, n)$ consumes m inputs $x_{ij}(i = 1, 2, \ldots, m)$ to produce s outputs $y_{rj}(r = 1, 2, \ldots, s)$. For a given DMU_o, the E_o is defined as the efficiency score. Charnes et al. (1978) have proposed the following DEA model:

$$\begin{aligned}
&Max\ E_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\
&s.t. \begin{cases} \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad j = 1, 2, \ldots, n, \\ v_i \geqslant \varepsilon, \quad i = 1, 2, \ldots, m, \quad u_r \geqslant \varepsilon, \quad r = 1, 2, \ldots, s. \end{cases}
\end{aligned} \tag{1}$$

Suppose that the values for the inputs and outputs of the DMUs are based on the subjective opinion of d Decision Makers (DMs), $k = 1, 2, \ldots, d$. Subsequently, DM_k estimates that each DMU_j $(j = 1, 2, \ldots, n)$ consumes m inputs $x_{ijd}(i = 1, 2, \ldots, m; k = 1, 2, \ldots, d)$ to produce s outputs $y_{rjk}(r = 1, 2, \ldots, s; k = 1, 2, \ldots, d)$. We use the geometric mean to aggregate the k different subjective opinions established for each criterion. Consequently, Model (1) is changed as follows:

$$\begin{aligned}
&Max\ E_o = \frac{\sum_{r=1}^{s} u_r \left(\prod_{k=1}^{d} y_{rok}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\prod_{k=1}^{d} x_{iok}\right)^{\frac{1}{d}}} \\
&s.t. \begin{cases} \frac{\sum_{r=1}^{s} u_r \left(\prod_{k=1}^{d} y_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\prod_{k=1}^{d} x_{ijk}\right)^{\frac{1}{d}}} \leqslant 1 \quad j = 1, 2, \ldots, n, \\ v_i \geqslant \varepsilon, \quad i = 1, 2, \ldots, m, \quad u_r \geqslant \varepsilon, \quad r = 1, 2, \ldots, s. \end{cases}
\end{aligned} \tag{2}$$

where $\left(\prod_{k=1}^{d} x_{ijk}\right)^{\frac{1}{d}}$ is the geometric mean of k DM's opinions with respect to the ith input for DMU_j and $\left(\prod_{k=1}^{d} y_{rjk}\right)^{\frac{1}{d}}$ is the geometric mean of k DM's opinions with respect to the rth output for DMU_j.

3.2. Fuzzy DEA modeling with multiple decision makers

There are two different types of uncertainties in real-world problems: *ambiguity* and *vagueness* (Dubois & Prade, 1982; Klir, 1987). While *ambiguity* is associated with situations in which the choice between two or more alternatives is left unspecified, *vagueness* is associated with situations in which some domain of interest is vague and cannot be delimited by sharp boundaries (Inuiguchi & Ramík, 2000). Assume that d Decision Makers (DMs), $k = 1, 2, \ldots, d$, have provided their judgements on some inputs and outputs using linguistic terms associated with fuzzy numbers. DM_k has estimated that each DMU_j $(j = 1, 2, \ldots, n)$ consumes m fuzzy inputs $\tilde{x}_{ijk}(i = 1, 2, \ldots, m; k = 1, 2, \ldots, d)$ to produce s fuzzy outputs $\tilde{y}_{rjk}(r = 1, 2, \ldots, s; k = 1, 2, \ldots, d)$. This assumption reflects *ambiguity* since it refers to an unspecified choice between two or more alternatives.

Buckley (1985) extended the geometric mean technique to fuzzy numbers. We use Buckley's (1985) approach to determine the geometric means of the fuzzy inputs and fuzzy outputs as follows:

$$\begin{aligned}
&(\tilde{x}_{ij1} \otimes \tilde{x}_{ij2} \otimes \tilde{x}_{ij3} \otimes \ldots \otimes \tilde{x}_{ijd})^{\frac{1}{d}} = \left(\otimes \prod_{k=1}^{d} \tilde{x}_{ijk}\right)^{\frac{1}{d}}, \ i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \\
&(\tilde{y}_{rj1} \otimes \tilde{y}_{rj2} \otimes \tilde{y}_{rj3} \otimes \ldots \otimes \tilde{y}_{rjd})^{\frac{1}{d}} = \left(\otimes \prod_{k=1}^{d} \tilde{y}_{rjk}\right)^{\frac{1}{d}}, \ r = 1, 2, \ldots, s; \ j = 1, 2, \ldots, n
\end{aligned} \tag{3}$$

where $\left(\otimes \prod_{k=1}^{d} \tilde{x}_{ijk}\right)^{\frac{1}{d}}$ is the fuzzy geometric mean of k DM's opinions about the ith input for DMU_j and $\left(\otimes \prod_{k=1}^{d} \tilde{y}_{rjk}\right)^{\frac{1}{d}}$ is the fuzzy geometric mean of k DM's opinions about the rth output of DMU.

Fuzzy mathematical programming, developed for treating uncertainties in optimization problems, can be classified into three categories with respect to vagueness and ambiguity (Inuiguchi, Sakawa, & Kume, 1994): fuzzy mathematical programming with vagueness (Bellman & Zadeh, 1970), (2) fuzzy mathematical programming with ambiguity (Dubois & Prade, 1980), and (3) fuzzy mathematical programming with vagueness and ambiguity (Negoita, Minoiu, & Stan, 1976). Inuiguchi and Ramík (2000) refer to the mathematical programming models with ambiguity and vagueness as *robust programming*. Werners (1987) has argued that the objective functions of a mathematical programming model with fuzzy constraints should be fuzzy. Consider a DEA model with fuzzy objective functions and constraints. The robust version of the DEA model with fuzzy objective functions and fuzzy coefficients which resolves the vagueness and ambiguity in Model (3) can be written as follows:

$$\begin{aligned}
&Max\ \tilde{E}_o = \frac{\sum_{r=1}^{s} u_r \left(\otimes \prod_{k=1}^{d} \tilde{y}_{rok}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\otimes \prod_{k=1}^{d} \tilde{x}_{iok}\right)^{\frac{1}{d}}} \\
&s.t. \begin{cases} \frac{\sum_{r=1}^{s} u_r \left(\otimes \prod_{k=1}^{d} \tilde{y}_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\otimes \prod_{k=1}^{d} \tilde{x}_{ijk}\right)^{\frac{1}{d}}} \leqslant 1 \quad j = 1, 2, \ldots, n, \\ v_i \geqslant \varepsilon, \quad i = 1, 2, \ldots, m, \quad u_r \geqslant \varepsilon, \quad r = 1, 2, \ldots, s. \end{cases}
\end{aligned} \tag{4}$$

3.3. Resolving the Vagueness of the DEA model

Model (4) can be re-written as the following fuzzy multiple objective programming problem:

$$\begin{aligned}
&Max\ \tilde{E}_o = \frac{\sum_{r=1}^{s} u_r \left(\otimes \prod_{k=1}^{d} \tilde{y}_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\otimes \prod_{k=1}^{d} \tilde{x}_{ijk}\right)^{\frac{1}{d}}}, \quad j = 1, 2, \ldots, n, \\
&s.t. \begin{cases} \frac{\sum_{r=1}^{s} u_r \left(\otimes \prod_{k=1}^{d} \tilde{y}_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i \left(\otimes \prod_{k=1}^{d} \tilde{x}_{ijk}\right)^{\frac{1}{d}}} \leqslant 1 \quad j = 1, 2, \ldots, n, \\ v_i \geqslant \varepsilon, \quad i = 1, 2, \ldots, m, \quad u_r \geqslant \varepsilon, \quad r = 1, 2, \ldots, s. \end{cases}
\end{aligned} \tag{5}$$

Model (5) can be converted into a multiple objective linear programming problem with fuzzy objectives and fuzzy coefficients. Fig. 1 shows the linear membership functions of the objective functions in Model (5). The lower bound and the upper bound of the objective functions are respectively set to zero and one in the DEA.

In order to resolve the vagueness in Model (5), we use the fuzzy mathematical programming approach proposed by Zimmermann (1996) as follows:

$$\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\mu(E_j) \geqslant \lambda \;\; j=1,2,\ldots,n,\\ \left(\sum_{r=1}^{s} u_r\left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}}\right)\Big/\left(\sum_{i=1}^{m} v_i\left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}}\right) \leqslant 1 \;\; j=1,2,\ldots,n,\\ v_i \geqslant \varepsilon,\;\; i=1,2,\ldots,m;\;\; u_r \geqslant \varepsilon,\;\; r=1,2,\ldots,s,\\ 0 \leqslant \lambda \leqslant 1.\end{cases}\end{aligned} \tag{6}$$

Replacing the membership function of the objectives in Model (6) will result in the following model:

$$\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\dfrac{\sum_{r=1}^{s} u_r\left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i\left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}}} \geqslant \lambda, & j=1,2,\ldots,n\\ \dfrac{\sum_{r=1}^{s} u_r\left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}}}{\sum_{i=1}^{m} v_i\left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}}} \leqslant 1, & j=1,2,\ldots,n,\\ v_i \geqslant \varepsilon, & i=1,2,\ldots,m,\\ u_r \geqslant \varepsilon, & r=1,2,\ldots,s,\\ \lambda \leqslant 1,\\ 0 \leqslant \lambda.\end{cases}\end{aligned} \tag{7}$$

Model (7) can be converted into a linear form as follows:

$$\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\sum_{r=1}^{s} u_r\left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}} - \lambda\left(\sum_{i=1}^{m} v_i\left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}}\right) \geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s} u_r\left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}} - \sum_{i=1}^{m} v_i\left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}} \leqslant 0, & j=1,2,\ldots,n\\ v_i \geqslant \varepsilon, & i=1,2,\ldots,m,\\ u_r \geqslant \varepsilon, & r=1,2,\ldots,s,\\ \lambda \leqslant 1,\\ 0 \leqslant \lambda.\end{cases}\end{aligned} \tag{8}$$

3.4. Resolving the ambiguity in the DEA model

The vagueness in Model (8) was resolved in the previous section. However, this model contains fuzzy coefficients (ambiguity). For simplicity's sake, let us assume that $\tilde{x}_{ij} = \left(\otimes\prod_{k=1}^{d}\tilde{x}_{ijk}\right)^{\frac{1}{d}}$ and $\tilde{y}_{rj} = \left(\otimes\prod_{k=1}^{d}\tilde{y}_{rjk}\right)^{\frac{1}{d}}$ are the fuzzy geometric means of the fuzzy inputs and outputs, respectively. Let us further assume that $(x^L_{ij})_{\alpha_i}/(y^L_{rj})_{\alpha_r}$ and $(x^U_{ij})_{\alpha_i}/(y^U_{rj})_{\alpha_r}$ are the lower and upper bounds of the inputs/outputs, respectively. The lower bound of the satisfaction level can be attained using the following two-level optimization model:

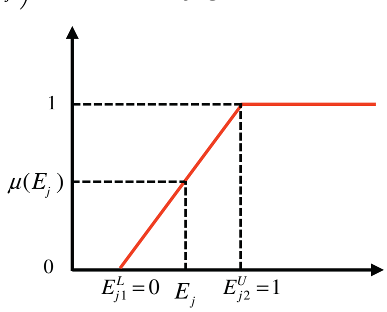


Fig. 1. The linear membership function of the objective function.

$$\begin{aligned}&\text{Min } \phi = \left\{\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\sum_{r=1}^{s} u_r\tilde{y}_{rj} - \lambda\left(\sum_{i=1}^{m} v_i\tilde{x}_{ij}\right) \geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s} u_r\tilde{y}_{rj} - \left(\sum_{i=1}^{m} v_i\tilde{x}_{ij}\right) \leqslant 0, & j=1,2,\ldots,n\\ v_i \geqslant \varepsilon, & i=1,2,\ldots,m,\\ u_r \geqslant \varepsilon, & r=1,2,\ldots,s.\end{cases}\end{aligned}\right\}\\ &s.t.\begin{cases}(x^L_{ij})_{\alpha_i} \leqslant \tilde{x}_{ij} \leqslant (x^U_{ij})_{\alpha_i}, & i=1,2,\ldots,m,\\ (y^L_{rj})_{\alpha_r} \leqslant \tilde{y}_{rj} \leqslant (y^U_{rj})_{\alpha_r}, & r=1,2,\ldots,s,\\ \alpha_i \in [0,1], & i=1,2,\ldots,m,\\ \alpha_r \in [0,1], & r=1,2,\ldots,s,\\ \lambda \leqslant 1,\\ 0 \leqslant \lambda.\end{cases}\end{aligned} \tag{9}$$

The upper bound of the satisfaction level can be attained using the following two-stage optimization model:

$$\begin{aligned}&\text{Max } \phi = \left\{\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\sum_{r=1}^{s} u_r\tilde{y}_{rj} - \lambda\left(\sum_{i=1}^{m} v_i\tilde{x}_{ij}\right) \geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s} u_r\tilde{y}_{rj} - \left(\sum_{i=1}^{m} v_i\tilde{x}_{ij}\right) \leqslant 0, & j=1,2,\ldots,n\\ v_i \geqslant \varepsilon, & i=1,2,\ldots,m,\\ u_r \geqslant \varepsilon, & r=1,2,\ldots,s.\end{cases}\end{aligned}\right\}\\ &s.t.\begin{cases}(x^L_{ij})_{\alpha_i} \leqslant \tilde{x}_{ij} \leqslant (x^U_{ij})_{\alpha_i}, & i=1,2,\ldots,m,\\ (y^L_{rj})_{\alpha_r} \leqslant \tilde{y}_{rj} \leqslant (y^U_{rj})_{\alpha_r}, & r=1,2,\ldots,s,\\ \alpha_i \in [0,1], & i=1,2,\ldots,m,\\ \alpha_r \in [0,1], & r=1,2,\ldots,s,\\ \lambda \leqslant 1,\\ 0 \leqslant \lambda.\end{cases}\end{aligned} \tag{10}$$

The two-stage optimization Models (9) and (10) involves four extreme cases where: (1) all inputs and outputs take the upper bounds; (2) all inputs take the lower bound and all outputs take the upper bound; (3) all inputs take the upper bound and all outputs take the lower bound; and (4) all inputs and outputs take the lower bound. Model (10) can be applied for the aforementioned four cases as follows:

Case (1): All inputs and outputs take the upper bounds

$$\begin{aligned}&\text{Max } \lambda\\ &s.t.\begin{cases}\sum_{r=1}^{s} u_r(y^U_{rj})_{\alpha_r} - \lambda\left(\sum_{i=1}^{m} v_i(x^U_{ij})_{\alpha_i}\right) \geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s} u_r(y^U_{rj})_{\alpha_r} - \left(\sum_{i=1}^{m} v_i(x^U_{ij})_{\alpha_i}\right) \leqslant 0, & j=1,2,\ldots,n\\ v_i \geqslant \varepsilon, & i=1,2,\ldots,m,\\ u_r \geqslant \varepsilon, & r=1,2,\ldots,s,\\ \alpha_i \in [0,1], & i=1,2,\ldots,m,\\ \alpha_r \in [0,1], & r=1,2,\ldots,s,\\ 0 \leqslant \lambda,\\ \lambda \leqslant 1.\end{cases}\end{aligned} \tag{11}$$

Case (2): All inputs take the lower bound and all outputs take the upper bound

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^U)_{\alpha_r} - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^L)_{\alpha_i} \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^U)_{\alpha_r} - \left(\sum_{i=1}^{m} v_i (x_{ij}^L)_{\alpha_i} \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{12}$$

Case (3): All inputs take the upper bound and all outputs take the lower bound

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^U)_{\alpha_i} \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \left(\sum_{i=1}^{m} v_i (x_{ij}^U)_{\alpha_i} \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{13}$$

Case (4): All inputs and outputs take the lower bound

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^L)_{\alpha_i} \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \left(\sum_{i=1}^{m} v_i (x_{ij}^L)_{\alpha_i} \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{14}$$

3.5. Applicable DEA models

In order to present a practical model for n DMUs, we assume that all fuzzy parameters are Trapezoidal Fuzzy Numbers (TrFNs) with left and right spread. Each DMU$_j$ ($j = 1, 2, \ldots, n$) consumes m fuzzy inputs $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4), i = 1,2,\ldots,m$ to produce s fuzzy outputs $\tilde{y}_{rj} = (y_{rj}^1, y_{rj}^2, y_{rj}^3, y_{rj}^4)$, $r = 1,2,\ldots,s$. Using an arbitrary α-cut for each TrFN, the lower and upper bound of the membership functions for the inputs and outputs are calculated as follows:

$$\begin{aligned} (x_{ij}^L)_{\alpha_i} &= x_{ij}^1 + \alpha_i (x_{ij}^2 - x_{ij}^1), & \alpha_i \in [0,1], \quad i = 1,\ldots,m; \quad j = 1,\ldots,n \\ (x_{ij}^U)_{\alpha_i} &= x_{ij}^4 - \alpha_i (x_{ij}^4 - x_{ij}^3), & \alpha_i \in [0,1], \quad i = 1,\ldots,m; \quad j = 1,\ldots,n \\ (y_{rj}^L)_{\alpha_r} &= y_{rj}^1 + \alpha_r (y_{rj}^2 - y_{rj}^1), & \alpha_r \in [0,1], \quad r = 1,\ldots,s; \quad j = 1,\ldots,n \\ (y_{rj}^U)_{\alpha_r} &= y_{rj}^4 - \alpha_r (y_{rj}^4 - y_{rj}^3), & \alpha_r \in [0,1], \quad r = 1,\ldots,s; \quad j = 1,\ldots,n \end{aligned} \tag{15}$$

Replacing (15) in Models (11) and (14) will result in following models, respectively:

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^4 - \alpha_r (y_{rj}^4 - y_{rj}^3)) - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^4 - \alpha_i (x_{ij}^4 - x_{ij}^3)) \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^4 - \alpha_r (y_{rj}^4 - y_{rj}^3)) - \left(\sum_{i=1}^{m} v_i (x_{ij}^4 - \alpha_i (x_{ij}^4 - x_{ij}^3)) \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{16}$$

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^4 - \alpha_r (y_{rj}^4 - y_{rj}^3)) - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^1 + \alpha_i (x_{ij}^2 - x_{ij}^1)) \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^4 - \alpha_r (y_{rj}^4 - y_{rj}^3)) - \left(\sum_{i=1}^{m} v_i (x_{ij}^1 + \alpha_i (x_{ij}^2 - x_{ij}^1)) \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{17}$$

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^1 + \alpha_r (y_{rj}^2 - y_{rj}^1)) - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^4 - \alpha_i (x_{ij}^4 - x_{ij}^3)) \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^1 + \alpha_r (y_{rj}^2 - y_{rj}^1)) - \left(\sum_{i=1}^{m} v_i (x_{ij}^4 - \alpha_i (x_{ij}^4 - x_{ij}^3)) \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{18}$$

$$\text{Max } \lambda$$
$$s.t.\begin{cases} \sum_{r=1}^{s} u_r (y_{rj}^1 + \alpha_r (y_{rj}^2 - y_{rj}^1)) - \lambda \left(\sum_{i=1}^{m} v_i (x_{ij}^1 + \alpha_i (x_{ij}^2 - x_{ij}^1)) \right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r (y_{rj}^1 + \alpha_r (y_{rj}^2 - y_{rj}^1)) - \left(\sum_{i=1}^{m} v_i (x_{ij}^1 + \alpha_i (x_{ij}^2 - x_{ij}^1)) \right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ \alpha_i \in [0,1], & i = 1,2,\ldots,m, \\ \alpha_r \in [0,1], & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, \\ \lambda \leqslant 1. \end{cases} \tag{19}$$

3.6. Resolving the non-linearity in the DEA model

Solving Models (16)–(19) for several α-cut levels is time consuming. In addition, there is no predefined step-size for the α-cut levels and conflicting rankings may result for a given DMU because of different α-cut levels. Finally, these models are non-linear and it is hard to find a global optimal solution for them. We propose two models to resolve the aforementioned issues in the α-cut based approaches.

We determine the optimal values of the satisfaction levels for, the α-cut level, the multiplier of inputs, and the multiplier of outputs using the single-stage optimization models (16)–(19). Let us represent the optimal values of the satisfaction levels, the α-cut level, the multiplier of inputs, and the multiplier of outputs as $\lambda*$, $\alpha*$, $v*$, and $u*$, respectively. The following variable exchanges help us linearize Models (16)–(19). Assuming that $\gamma_i = \alpha_i v_i (i = 1, \ldots, m$ and $0 \leqslant \gamma_i \leqslant v_i)$ and $\eta_r = \alpha_r u_r (r = 1, \ldots, s$ and $0 \leqslant \eta_r \leqslant u_r)$ for all inputs and outputs, respectively; Models (20)–(23) are proposed in place of Models (16)–(19), respectively:

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \lambda\left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, & \\ \lambda \leqslant 1. & \end{cases} \tag{20}$$

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r (y_{rj}^2 - y_{rj}^1) - \lambda\left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r (y_{rj}^2 - y_{rj}^1) - \left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, & \\ \lambda \leqslant 1. & \end{cases} \tag{21}$$

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \lambda\left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, & \\ \lambda \leqslant 1. & \end{cases} \tag{22}$$

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r (y_{rj}^2 - y_{rj}^1) - \lambda\left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r (y_{rj}^2 - y_{rj}^1) - \left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ 0 \leqslant \lambda, & \\ \lambda \leqslant 1. & \end{cases} \tag{23}$$

λ, v_i, u_r, η_r and γ_i are assumed to be the decision variables in Models (20)–(23). Therefore, Models (20) and (22) are non-linear because of the term $\lambda\left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right)$. Similarly, Models (21) and (23) are non-linear because of the term $\lambda\left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right)$. Considering $\phi_i = \gamma_i \lambda, i = 1, \ldots, m$ where $0 \leqslant \phi_i \leqslant \gamma_i$ and $\theta_i = v_i \lambda, i = 1, \ldots, m$ where $0 \leqslant \theta_i \leqslant v_i$, Models (20)–(23) can be converted into the following models, respectively:

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} \theta_i x_{ij}^4 - \phi_i (x_{ij}^4 - x_{ij}^3)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} v_i x_{ij}^4 - \gamma_i (x_{ij}^4 - x_{ij}^3)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ \gamma_i \geqslant \phi_i \geqslant 0, & i = 1,2,\ldots,m, \\ v_i \geqslant \theta_i \geqslant 0, & i = 1,2,\ldots,m, \\ 0 \leqslant \lambda^L, & \\ \lambda^L \leqslant 1. & \end{cases} \tag{24}$$

$$\text{Max } \lambda$$
$$\text{s.t.}\begin{cases} \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} \theta_i x_{ij}^1 + \phi_i (x_{ij}^2 - x_{ij}^1)\right) \geqslant 0, & j = 1,2,\ldots,n \\ \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \left(\sum_{i=1}^{m} v_i x_{ij}^1 + \gamma_i (x_{ij}^2 - x_{ij}^1)\right) \leqslant 0, & j = 1,2,\ldots,n \\ v_i \geqslant \varepsilon, & i = 1,2,\ldots,m, \\ u_r \geqslant \varepsilon, & r = 1,2,\ldots,s, \\ v_i \geqslant \gamma_i \geqslant 0, & i = 1,2,\ldots,m, \\ u_r \geqslant \eta_r \geqslant 0, & r = 1,2,\ldots,s, \\ \gamma_i \geqslant \phi_i \geqslant 0, & i = 1,2,\ldots,m, \\ v_i \geqslant \theta_i \geqslant 0, & i = 1,2,\ldots,m, \\ 0 \leqslant \lambda^U, & \\ \lambda^U \leqslant 1. & \end{cases} \tag{25}$$

$$Max\ \lambda$$
$$s.t.\begin{cases}\sum_{r=1}^{s}u_r y_{rj}^1+\eta_r(y_{rj}^2-y_{rj}^1)-\left(\sum_{i=1}^{m}\theta_i x_{ij}^4-\phi_i(x_{ij}^4-x_{ij}^3)\right)\geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s}u_r y_{rj}^1+\eta_r(y_{rj}^2-y_{rj}^1)-\left(\sum_{i=1}^{m}\upsilon_i x_{ij}^4-\gamma_i(x_{ij}^4-x_{ij}^3)\right)\leqslant 0, & j=1,2,\ldots,n\\ \upsilon_i\geqslant\varepsilon, & i=1,2,\ldots,m,\\ u_r\geqslant\varepsilon, & r=1,2,\ldots,s,\\ \upsilon_i\geqslant\gamma_i\geqslant 0, & i=1,2,\ldots,m,\\ u_r\geqslant\eta_r\geqslant 0, & r=1,2,\ldots,s,\\ \gamma_i\geqslant\phi_i\geqslant 0, & i=1,2,\ldots,m,\\ \upsilon_i\geqslant\theta_i\geqslant 0, & i=1,2,\ldots,m,\\ 0\leqslant\lambda^L, & \\ \lambda^L\leqslant 1. & \end{cases}\tag{26}$$

$$Max\ \lambda$$
$$s.t.\begin{cases}\sum_{r=1}^{s}u_r y_{rj}^1+\eta_r(y_{rj}^2-y_{rj}^1)-\left(\sum_{i=1}^{m}\theta_i x_{ij}^1+\phi_i(x_{ij}^2-x_{ij}^1)\right)\geqslant 0, & j=1,2,\ldots,n\\ \sum_{r=1}^{s}u_r y_{rj}^1+\eta_r(y_{rj}^2-y_{rj}^1)-\left(\sum_{i=1}^{m}\upsilon_i x_{ij}^1+\gamma_i(x_{ij}^2-x_{ij}^1)\right)\leqslant 0, & j=1,2,\ldots,n\\ \upsilon_i\geqslant\varepsilon, & i=1,2,\ldots,m,\\ u_r\geqslant\varepsilon, & r=1,2,\ldots,s,\\ \upsilon_i\geqslant\gamma_i\geqslant 0, & i=1,2,\ldots,m,\\ u_r\geqslant\eta_r\geqslant 0, & r=1,2,\ldots,s,\\ \gamma_i\geqslant\phi_i\geqslant 0, & i=1,2,\ldots,m,\\ \upsilon_i\geqslant\theta_i\geqslant 0, & i=1,2,\ldots,m,\\ 0\leqslant\lambda^U, & \\ \lambda^U\leqslant 1. & \end{cases}\tag{27}$$

Assuming that $\lambda_{24}^*, \lambda_{25}^*, \lambda_{26}^*$, and λ_{27}^* are the optimal values of the objective functions in Models (24)–(27), respectively, $\varphi_9^* = Min\{\lambda_{24}^*, \lambda_{25}^*, \lambda_{26}^*, \lambda_{27}^*\}$ and $\varphi_{10}^* = Max\{\lambda_{24}^*, \lambda_{25}^*, \lambda_{26}^*, \lambda_{27}^*\}$ will be the optimal values of Models (9) and (10), respectively.

3.7. Advantages of the proposed models

Models (24)–(27) have the following advantages in comparison with the existing procedures in the fuzzy DEA literature:

- The *ambiguity* and *vagueness* are considered concurrently,
- Uncertain judgments concerning the input and output data are considered,
- The ranking conflict is avoided since common multipliers are determined for all DMUs in a single run,
- The models are linear and can determine the global optimal values for the lower and upper bounds of the satisfaction levels and efficiency scores in a single run,
- The models are computationally agile. They are independent of the α-cut levels and do not have to be solved for different α-cut levels. As a result, there is no need for defining a proper step-size for the α-cut levels,

We should note that the final results from the fuzzy DEA model proposed in this study do not require any additional interpretations (i.e., defuzzification, ranking the fuzzy numbers, or averaging the efficiency scores) because the vagueness and ambiguity are considered simultaneously in the model based on the α-level based approach and the possibility approach, respectively. The α-level based approach is used to model the vague data while the possibility approach is used to model the ambiguous interpretation of the criteria and the objectives. Fuzzy sets are used to model both vagueness and ambiguity. Most α-level based methods require some form of defuzzification phase in which fuzzy sets are abandoned and the model is solved according to specific α-level. Although defuzzification reduces the complexity of the model and facilitates the interpretation of the results, it also causes some loss of data and requires additional computational efforts. The following simple example further explores this issue. Let us add two simple fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ and call the addition $\widetilde{C}$ (i.e., $\widetilde{A}+\widetilde{B}=\widetilde{C}$). In this example, if we subtract $\widetilde{B}$ from $\widetilde{C}$, the outcome will not be $\widetilde{A}$. The vagueness of $\widetilde{A}$ and $\widetilde{B}$ are increasingly and unknowingly mixed in $\widetilde{C}$. The fuzzification and defuzzification processes have similar effects on the results and cause loss of data. In addition, α-level based approaches generally require additional computational efforts. Several DEA models are run for different α-levels and it is often difficult to find the optimal step-size for the α-levels. In addition, inconsistent results may be obtained for different α-levels and a ranking procedure is required to make the final decision. In contrast, we determine the best α-levels for each vague input and output and there is no loss of data or additional computational efforts in our model because there is no need for the defuzzification of the inputs and the outputs.

4. Case study: High-technology project assessment at NASA[3]

The project engineering office at the Kennedy Space Center (KSC) currently uses the Consensus Ranking Organizational Support System (CROSS) proposed by Tavana (2003) to assess high-technology projects initiated by the contractors or divisions within the KSC. Project evaluation is the primary responsibility of the Ground System Working Team (GSWT).

The contractors and divisions within KSC submit approximately 30–50 proposals for evaluation and possible funding annually. The GSWT uses CROSS to assess the importance of each project relative to the longevity of the space program and selects the most suitable projects for funding depending on the available budget for that fiscal year. One of the shortfalls of CROSS is its inability to handle *ambiguous* and *vague* data. Ambiguous or vague data may be the result of unquantifiable, incomplete and non-obtainable information. Ambiguous or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers.

A high-technology project can be defined as a DMU with several inputs and outputs. The GSWT identified two inputs (total cost and production time) for high-technology project portfolio selection at the KSC:

- ***Total cost***: The total cost of designing, prototyping, manufacturing and testing a high-technology equipment or concept. Total cost reflects the probability of staying within the budget.
- ***Production time***: The total time of producing a high-technology equipment or concept. Production time reflects the probability of meeting the schedule.

The GSWT also identified four outputs (safety, reliability, feasibility and reusability) for high-technology project portfolio selection:

- ***Safety***: The primary purpose of this factor is to enhance space safety by significantly constraining and eliminating the potential for future accidents involving loss of crew or vehicle. System safety reflects the probability of crew or vehicle survival.

[3] All names and data in the case study are changed to protect the anonymity of the projects.

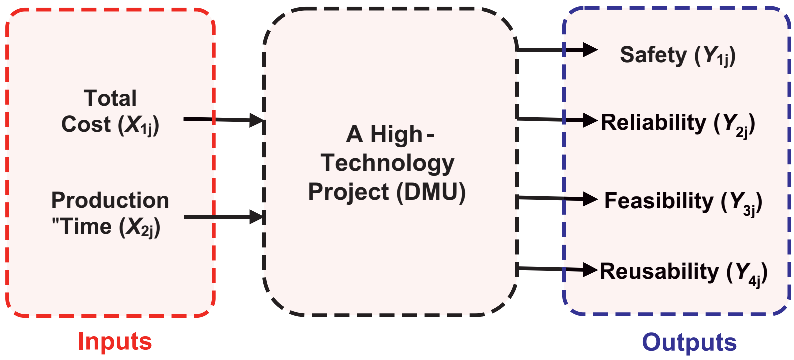


Fig. 2. A high-technology project as a DMU.

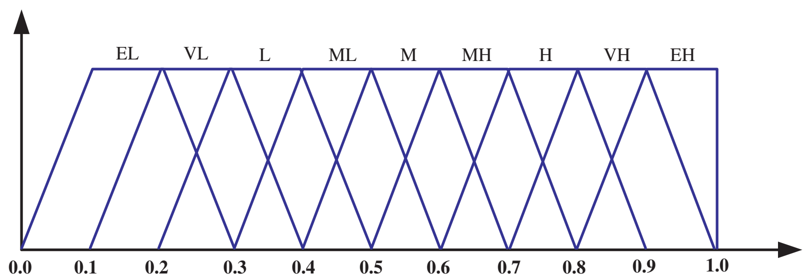


Fig. 3. The linguistic variables and their associated TrFNs for the input and output data.

- ***Reliability***: The ability to avoid fault by providing generous design margins for structure and for mechanical, thermal and electrical subsystems. In general, high reliability requires a combination of quality components, redundancy, good diagnostics, and frequent maintenance. System reliability reflects the probability of the system completing its designed mission.
- ***Feasibility***: The feasibility requirement ensures the readiness of selected technologies for the implementation phase. This includes not only the technical feasibility and concept but also the congruence of the technical aspects with the cost and schedule aspects. Feasibility reflects the probability of achieving the overall design parameters.
- ***Reusability***: The reusability criterion measures the likelihood of reusing a high-technology equipment or concept to add new functionalities. Reusable technologies reduce the implementation time and increase the likelihood that prior testing has eliminated the bugs and modifications when a change in implementation is required. Reusability reflects the probability of achieving additional functionalities.

In general, high-technology projects with lower cost and production time are preferred to projects with higher cost and production time. Similarly, high-technology projects with higher safety, reliability, feasibility, and reusability are preferred. Fig. 2 presents a schematic view of a high-technology project as a DMU in DEA at NASA.

The high-technology projects at NASA involve a great deal of uncertainties. The GSWT believed that the input (total cost and production time) and the output (safety, reliability, feasibility and reusability) variables for the high-technology projects were uncertain in nature and could not be estimated precisely. Therefore, fuzzy set theory was used to handle such an uncertainty and imprecision by generalizing the notion of membership in a set. Essentially, the GSWT members collectively developed the linguistic variables and their associated TrFNs presented in Table 1 and depicted in Fig. 3.

Table 1
The linguistic variables and the TrFNs used for rating the inputs and outputs.

Linguistic variable	TrFN scale
Extreme low (EL)	(0.0, 0.1, 0.2, 0.3)
Very low (VL)	(0.1, 0.2, 0.3, 0.4)
Low (L)	(0.2, 0.3, 0.4, 0.5)
Medium low (ML)	(0.3, 0.4, 0.5, 0.6)
Medium (M)	(0.4, 0.5, 0.6, 0.7)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.6, 0.7, 0.8, 0.9)
Very high (VH)	(0.7, 0.8, 0.9, 1)
Extreme high (EH)	(0.8, 0.9, 1, 1)

Twenty high-technology projects were considered in this study. The five members of the GSWT independently assessed the input (total cost and production time) and the output (safety, reliability, feasibility and reusability) variables associated with the twenty high-technology projects based on the fuzzy rating scheme presented in Table 1. Tables 2 and 3 present the fuzzy ratings for the inputs and outputs, respectively.

The optimal values of the objective functions and the main coefficients of Models (24)–(27) are presented in Table 4. The optimal values of the α-cuts for the inputs and the outputs are presented in Table 5.

Based on the optimal values of the coefficients and the α-cut variables of Models (24)–(27), four efficiency scores were calculated for each DMU as follows:

$$E_j^k = \frac{\sum_{r=1}^{s} u_r^{k^*} \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i^{k^*} \tilde{x}_{ij}}, \quad j = 1, 2, \ldots n; \quad k = 24, 25, 26, 27. \tag{28}$$

where $u_r^{k^*}$, $v_i^{k^*}$, $u_r^{k^*}$, and $v_i^{k^*}$ are the associated optimal values of Models (24)–(27). In Eq. (28), the values of $\tilde{y}_{rj}$ and $\tilde{x}_{ij}$ are determined from the associated α-cut variables in the model where they originated. We can categorize the DUMs based on the values of E_j^L and E_j^U as follows:

Table 2
The ratings of the inputs for the high-technology projects.

Project	Inputs									
	Cost					Production time (schedule)				
	DM_1	DM_2	DM_3	DM_4	DM_5	DM_1	DM_2	DM_3	DM_4	DM_5
Hubble	VH	MH	H	H	MH	VL	ML	ML	L	VL
Pioneer	ML	VL	L	ML	L	VH	VH	H	VH	H
Photo-Voltaic	EH	H	H	VH	VH	MH	H	VH	VH	H
Mariner	VL	L	ML	ML	L	L	VL	L	VL	EL
Airlock	ML	VL	ML	L	L	VL	L	VL	VL	L
Stardust	VL	L	ML	ML	L	MH	H	MH	H	H
Babaloon	VH	H	VH	H	MH	ML	L	ML	VL	VL
Voyager	VL	VL	L	EL	L	ML	VL	L	VL	ML
Planet-Finder	L	VL	ML	L	ML	H	VH	VH	EH	VH
Magellan	H	VH	MH	VH	H	L	VL	VL	ML	VL
Nebula	H	VH	H	VH	MH	H	VH	MH	MH	H
Genesis	MH	MH	H	H	VH	ML	VL	L	VL	ML
Solar	H	VH	H	VH	EH	ML	VL	ML	L	VL
Surveyor	VL	L	ML	ML	L	H	H	VH	MH	H
Truss	MH	VH	H	VH	H	L	VL	L	VL	M
Ranger	MH	H	MH	H	H	M	L	ML	ML	L
Centrifuge	VH	H	VH	VH	MH	VL	VL	L	ML	VL
Cassini	MH	H	MH	H	M	VL	L	L	ML	VL
Juno	L	VL	L	VL	EL	M	L	ML	ML	L
Tether	ML	L	M	M	ML	MH	ML	ML	M	MH

Table 3
The ratings of the outputs for the high-technology projects.

Project	Outputs																			
	Safety					Reliability					Feasibility					Reusability				
	DM_1	DM_2	DM_3	DM_4	DM_5	DM_1	DM_2	DM_3	DM_4	DM_5	DM_1	DM_2	DM_3	DM_4	DM_5	DM_1	DM_2	DM_3	DM_4	DM_5
Hubble	H	MH	H	MH	H	MH	H	VH	H	MH	EH	H	VH	H	H	H	MH	H	VH	H
Pioneer	H	MH	H	H	MH	MH	H	MH	H	H	H	H	MH	H	MH	H	MH	H	H	MH
Photo-Voltaic	MH	MH	H	H	VH	H	VH	MH	VH	H	ML	VL	L	VL	ML	MH	H	MH	H	M
Mariner	MH	H	VH	H	MH	H	VH	MH	MH	H	VL	ML	ML	L	VL	EH	H	H	VH	VH
Airlock	ML	L	ML	VL	L	MH	VH	H	H	MH	VH	VH	H	H	MH	H	H	VH	H	H
Stardust	MH	H	MH	H	M	H	MH	H	H	MH	H	VH	VH	EH	VH	ML	VL	L	ML	L
Babaloon	ML	ML	L	ML	L	H	H	MH	H	MH	M	L	ML	ML	L	VH	VH	H	VH	H
Voyager	MH	H	MH	H	M	MH	VH	H	H	MH	H	H	H	MH	VH	H	VH	MH	VH	H
Planet-Finder	ML	L	L	ML	M	VL	L	ML	ML	L	VL	VL	L	EL	L	VL	L	ML	L	ML
Magellan	VH	VH	H	VH	H	EH	H	H	VH	VH	MH	MH	H	H	VH	H	VH	VH	EH	VH
Nebula	H	MH	H	H	MH	L	ML	L	ML	VL	ML	VL	L	ML	L	H	H	MH	M	H
Genesis	EH	H	H	VH	VH	MH	VH	H	VH	H	MH	H	VH	VH	H	MH	VH	H	VH	H
Solar	H	H	H	MH	VH	VL	L	VL	ML	VL	VL	VL	L	ML	L	MH	H	MH	H	H
Surveyor	VH	H	VH	H	H	H	MH	H	VH	H	MH	VH	H	H	MH	ML	VL	L	VL	ML
Truss	H	MH	H	M	H	H	H	VH	MH	H	VH	H	VH	H	H	L	VL	L	VL	EL
Ranger	VL	L	VL	ML	VL	VL	L	VL	VL	L	ML	L	ML	VL	L	H	VH	MH	MH	H
Centrifuge	MH	H	H	H	MH	MH	VH	H	H	MH	VL	L	L	ML	VL	VL	VL	L	ML	L
Cassini	VL	L	ML	ML	L	VL	VL	L	ML	L	ML	L	L	ML	M	L	ML	L	ML	VL
Juno	L	ML	L	ML	VL	VL	ML	ML	L	VL	H	VH	MH	VH	H	M	L	ML	ML	L
Tether	ML	ML	M	M	ML	MH	H	MH	MH	VH	L	VL	VL	ML	VL	H	MH	H	H	MH

Table 4
The optimal objective values and coefficients.

Models	Optimum values						
	Objective	Coefficient of inputs		Coefficient of outputs			
	λ^*	V_1	V_2	U_1	U_2	U_3	U_4
Model (24)	1	0.00496	0.01400	0.002	0.002	0.002	0.002
Model (25)	1	0.00728	0.01650	0.002	0.002	0.002	0.002
Model (26)	1	0.00200	0.00768	0.002	0.002	0.002	0.002
Model (27)	1	0.00437	0.01300	0.002	0.002	0.002	0.002

$$
\begin{aligned}
&E^{++} = \{j \in J | E_j^L = 1\},\\
&E^{+} = \{j \in J | E_j^L < 1 \text{ and } E_j^U = 1\},\\
&E^{-} = \{j \in J | E_j^U < 1\}.
\end{aligned}
\tag{29}
$$

where J is the set of all DMUs with the cardinality of n(i.e., $|J| = n$) and $E_j^U = Max\ (E_j^{24}, E_j^{25}, E_j^{26}, E_j^{27})$ and $E_j^L = Min\ (E_j^{24}, E_j^{25}, E_j^{26}, E_j^{27})$.

Given that the efficiency score calculated using Eq. (28) is based on all extreme possible situations for a DMU, if a DMU is not

Table 5
The optimal values for the α-cut of the inputs and the outputs.

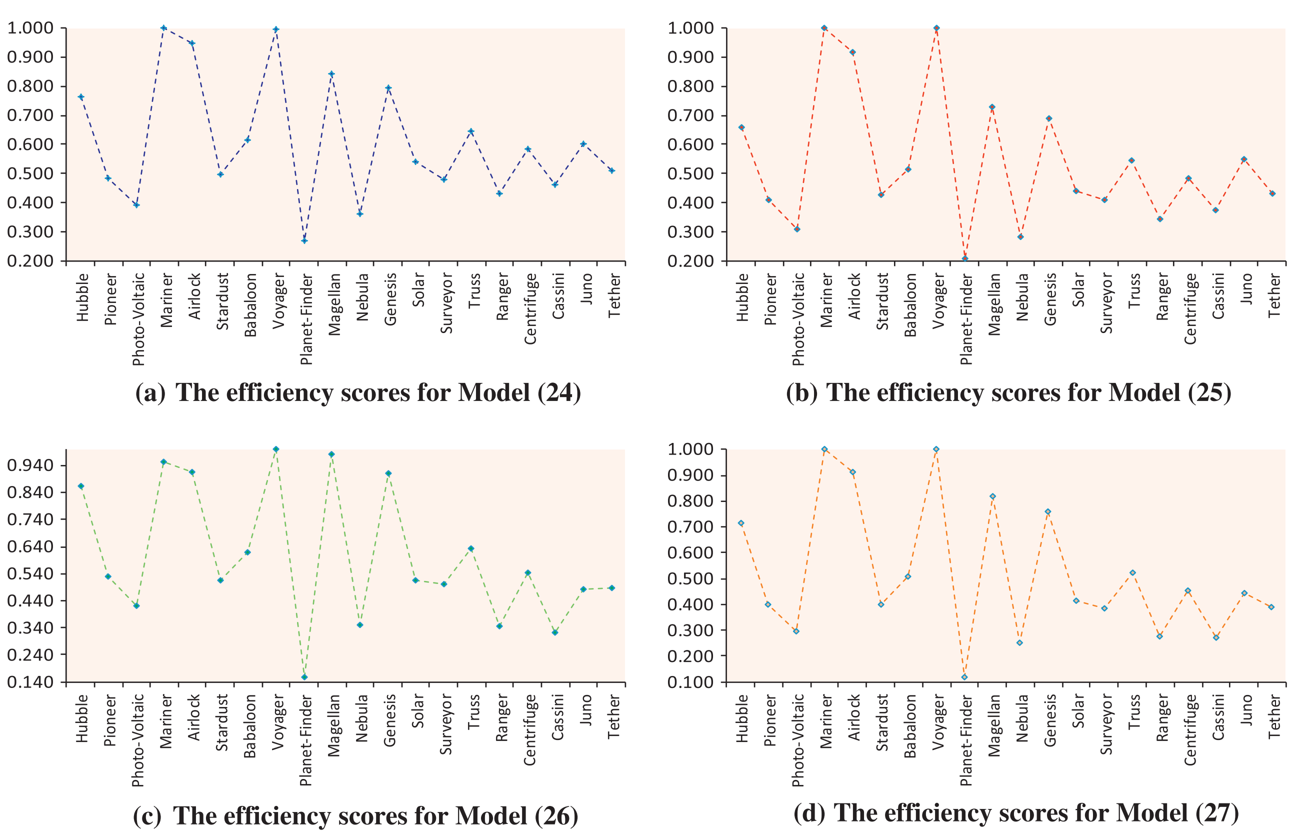
Models	Optimum values					
	α-cut of Inputs		α-cut of Outputs			
	α_1	α_2	α_1	α_2	α_1	α_2
Model (24)	1	1	0	0	0	0
Model (25)	1	1	1	1	1	1
Model (26)	0	0	0	0	0	0
Model (27)	1	1	0	0	0	0

recognized as an efficient DMU, it cannot be assumed efficient under any other circumstances. The recognized efficiency scores, standard deviations, means, and the ranges of the efficiency scores for the DMUs are presented in Table 6. Furthermore, the efficiency scores for each model are presented in Fig. 4.

In order to investigate the robustness of the DMUs, we plotted the standard deviation and the ranges of the efficiency scores for all the DMUs in Models (24)–(27). Fig. 5 presents these standard deviations and ranges of the efficiency scores.

Table 6
The efficiency scores.

Project	E_j^{24}	E_j^{25}	E_j^{26}	E_j^{27}	E_j^{L}	E_j^{U}	Standard deviation	Mean	Range	Category
Hubble	0.764	0.661	0.862	0.716	0.661	0.862	0.074	0.751	0.201	E^-
Pioneer	0.485	0.411	0.532	0.399	0.399	0.532	0.055	0.457	0.133	E^-
Photo-Voltaic	0.389	0.309	0.420	0.297	0.297	0.420	0.052	0.354	0.123	E^-
Mariner	1.000	1.000	0.955	1.000	0.955	1.000	0.020	0.989	0.045	E^+
Airlock	0.947	0.920	0.918	0.915	0.915	0.947	0.013	0.925	0.033	E^-
Stardust	0.496	0.425	0.518	0.399	0.399	0.518	0.049	0.460	0.119	E^-
Babaloon	0.614	0.513	0.621	0.510	0.510	0.621	0.053	0.565	0.111	E^-
Voyager	0.997	1.000	1.000	1.000	0.997	1.000	0.001	0.999	0.003	E^+
Planet-Finder	0.268	0.209	0.159	0.116	0.116	0.268	0.057	0.188	0.152	E^-
Magellan	0.842	0.731	0.980	0.820	0.731	0.980	0.089	0.843	0.249	E^-
Nebula	0.360	0.283	0.350	0.252	0.252	0.360	0.045	0.311	0.107	E^-
Genesis	0.794	0.690	0.912	0.757	0.690	0.912	0.081	0.788	0.222	E^-
Solar	0.540	0.440	0.513	0.412	0.412	0.540	0.052	0.476	0.128	E^-
Surveyor	0.481	0.407	0.502	0.383	0.383	0.502	0.050	0.443	0.119	E^-
Truss	0.645	0.543	0.633	0.521	0.521	0.645	0.054	0.586	0.124	E^-
Ranger	0.429	0.343	0.344	0.274	0.274	0.429	0.055	0.348	0.156	E^-
Centrifuge	0.583	0.482	0.546	0.454	0.454	0.583	0.051	0.516	0.129	E^-
Cassini	0.461	0.374	0.320	0.273	0.273	0.461	0.070	0.357	0.189	E^-
Juno	0.600	0.548	0.484	0.443	0.443	0.600	0.060	0.519	0.157	E^-
Tether	0.510	0.432	0.485	0.389	0.389	0.510	0.047	0.454	0.121	E^-



(a) The efficiency scores for Model (24)

(b) The efficiency scores for Model (25)

(c) The efficiency scores for Model (26)

(d) The efficiency scores for Model (27)

Fig. 4. The efficiency scores for the proposed models.

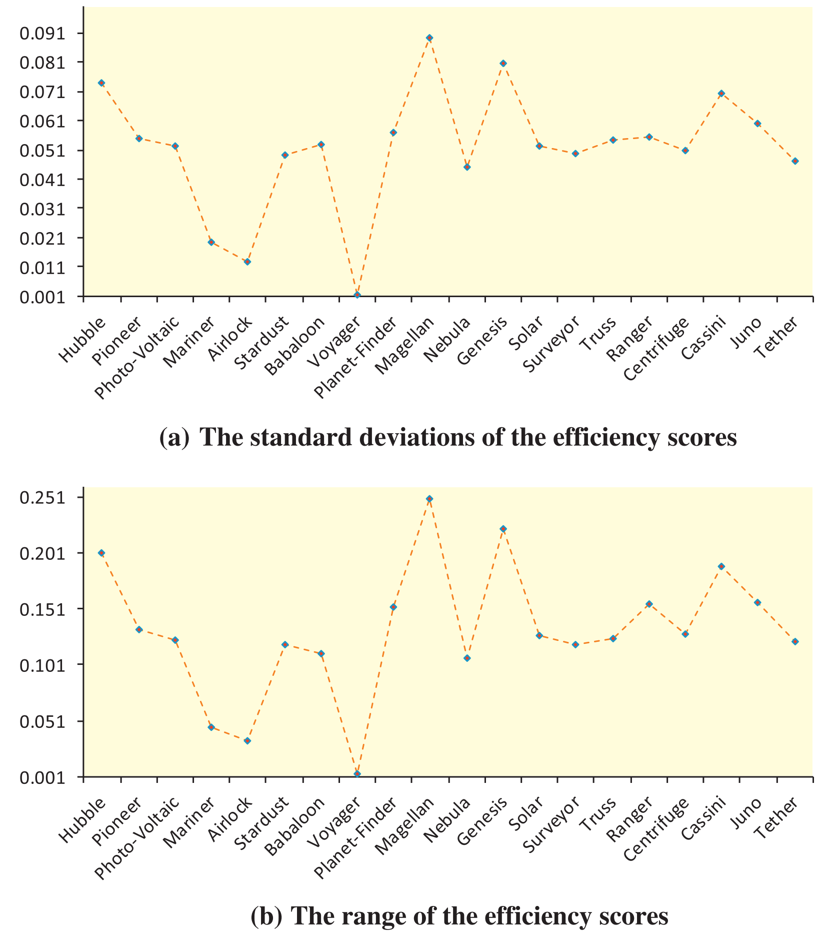


Fig. 5. The standard deviation and range of the efficiency scores for the DMUs.

Table 7
The project rankings based on various investment strategies.

Priority	Strategy #1(Pessimistic Strategy)	Strategy #2(Optimistic Strategy)	Strategy #3 (Neutral Strategy)	Strategy #4 (Robust Strategy)
1	Voyager	Voyager	Voyager	Voyager
2	Mariner	Mariner	Mariner	Mariner
3	Airlock	Magellan	Airlock	Airlock
4	Magellan	Airlock	Magellan	Magellan
5	Genesis	Genesis	Genesis	Genesis
6	Hubble	Hubble	Hubble	Hubble
7	Truss	Truss	Truss	Truss
8	Babaloon	Babaloon	Babaloon	Babaloon
9	Centrifuge	Juno	Juno	Juno
10	Juno	Centrifuge	Centrifuge	Centrifuge
11	Solar	Solar	Solar	Solar
12	Pioneer	Pioneer	Stardust	Pioneer
13	Stardust	Stardust	Pioneer	Stardust
14	Tether	Tether	Tether	Tether
15	Surveyor	Surveyor	Surveyor	Surveyor
16	Photo-Voltaic	Cassini	Cassini	Cassini
17	Ranger	Ranger	Photo-Voltaic	Photo-Voltaic
18	Cassini	Photo-Voltaic	Ranger	Ranger
19	Nebula	Nebula	Nebula	Nebula
20	Planet-Finder	Planet-Finder	Planet-Finder	Planet-Finder

Next, we arrange the project priorities according to the categories presented in Table 6. The Mariner and Voyager projects which were categorized in class E^{+} were highly recommended for investment. Among the remaining projects which were categorized as E^{-}, the remaining projects were ranked according to the following strategies:

- ***Strategy #1 (Pessimistic)***. This strategy assumes a pessimistic viewpoint where the GSWT is assumed to be risk-averse. The remaining projects were ranked according to the descending order of the E_j^L values. The ties were broken based on the descending order of the E_j^U.

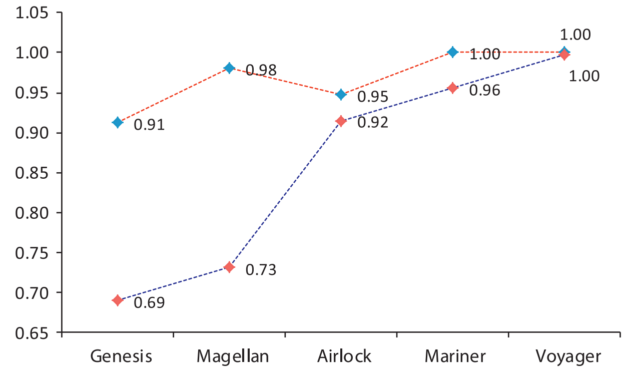


Fig. 6. The lower and upper bounds of the top five projects in all strategies.

- ***Strategy #2 (Optimistic)***. This strategy assumes an optimistic viewpoint where the GSWT is assumed to be a risk-seeker. The remaining projects were ranked according to the descending order of the E_j^U values. The ties were broken based on the descending order of the E_j^L.
- ***Strategy #3 (Neutral)***. This strategy assumes a neutral viewpoint where the GSWT is assumed to be risk-neutral. The remaining projects were ranked according to the descending order of the average values of the efficiency scores for $E_j^{24}, E_j^{25}, E_j^{26}$, and E_j^{27}.
- ***Strategy #4 (Robust)***. This strategy assumes a robust viewpoint where the GSWT is assumed to be risk-robust. The remaining projects were first ranked according to the descending order of the average of the E_j^U and E_j^L values. We then ranked the remaining projects according to the ascending order of the standard deviations of their efficiency scores. The ties were broken based on the ascending order of the range of the efficiency scores.

Table 7 presents the final priority of the high-technology projects based on the aforementioned strategies.

The selected projects have the least possibility of inefficiency in completely uncertain conditions. The top five projects in all strategies were plotted in Fig. 6 based on their E_j^U and E_j^L values.

5. Conclusions and future research directions

Austere economic times and major funding cuts in NASA's budget have forced the agency to obtain the maximum return on public spending. In response, NASA is trying to improve its national space posture during times of budget austerity by carefully evaluating high-technology projects and getting the most return on public spending. However, the assessment and selection of high-technology projects is often a difficult decision making process due to the multiple and often conflicting objectives in addition to the inherent technical complexities and valuation uncertainties involved in the assessment process.

We proposed a DEA model with ambiguity and vagueness. The vagueness of the objective functions was modeled with a multi-objective fuzzy linear programming method. The ambiguity of the input and output data was modeled with fuzzy sets and a new α-cut based method. The emerging models were linear, independent of α-cut variables, and able to maximize the satisfaction level of the fuzzy objectives and efficiency scores, simultaneously. Moreover, these models were capable of generating a common set of multipliers for all projects in a single run. We also demonstrated the applicability of the proposed models and the efficacy of the procedures through a case study involving high-technology project selection at NASA.

Our model was intended to assist DMs in their judgment when evaluating and selecting high-technology projects. In fact, human judgment is the core input in the decision making process. Our approach helped the DMs to think systematically about complex problems and improved the quality of their decisions. We decomposed the high-technology project selection process at NASA into manageable steps and integrated the results to arrive at a solution consistent with managerial goals and objectives. This decomposition encouraged the DMs to carefully consider the elements of uncertainty. The proposed structured framework does not imply a deterministic approach in project evaluation. While our approach enabled DMs to assimilate the information and organize their beliefs in a formal systematic approach, it should be used in conjunction with management experience. Managerial judgment is an integral component of high-technology project evaluation and selection decisions; therefore, the effectiveness of the model relies heavily on the DM's cognitive capabilities.

The methodology and the data analysis technique adopted in this paper have provided mitigations against the drawback of the previous project evaluation and selection studies. These include: (1) the use of financial measures, leading to difficulties in quantifying non-financial variables; (2) the use of regression techniques for the data analysis requiring a prior linear functional relationship between the variables and an arbitrary assignment of weights to the variables. The contribution of the proposed project assessment and selection method is fivefold: (1) it addresses the abovementioned drawback of the previous project evaluation and selection studies; (2) it is grounded in the DEA method that accommodates a multiplicity of inputs and outputs, does not require a priori weights on inputs and outputs, allows inputs and outputs to be quantified using different units of measurement, and can determine possible sources of inefficiency in each project; (3) it considers ambiguity in decision making by using verbal expressions and linguistic variables for subjective judgments (Poyhonen, Hamalainen, & Salo, 1997); (4) it considers vagueness in decision making by using imprecise data due to lack of expertise, unavailability of data, or time constraints (Kim & Ahn, 1999); and (5) it considers meaningful and robust aggregation of subjective and objective judgments that affect the evaluation process (Valls & Torra, 2000).

There are a number of challenges involved in the proposed research. These challenges provide a great deal of fruitful scope for future research. The practicality of this model can be further enhanced by developing the proposed framework into a decision support system to reduce the computation time and effort. Another future research direction, which could be an area of theoretical study, is investigating the similarities and differences between the model proposed in this study and other multi-criteria decision making models. Finally, a systematic investigation of different defuzzification and ranking methods can be carried out to see the effects on the final ranking of the high-technology projects. We hope that our study can inspire others to pursue further research.

Acknowledgements

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

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