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The effect of preference similarity on the formation of clusters and the connectivity of social networks

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ABSTRACT

The current paper analyzes the formation of social networks determined by the preferences of their users, who are endowed with incomplete information regarding the characteristics of other users from who they receive friendship requests. The acceptance or rejection decision is determined by the limited information available when receiving the requests, the expectations of the users regarding the remaining characteristics of the requesters and the resulting improvement in network capacity derived from accepting the friendship requests. We illustrate how the similarity in preferences among users leads to more concentrated clusters within the incomplete information scenario analyzed. At the same time, the emergence of disutility costs derived from a suboptimal decision when accepting to interact with other users increments the dispersion between clusters. In this regard, the inclusion of requesters endowed with average preferences relative to those of the standard users composing the network acts as a connectivity-enhancing mechanism designed to reduce the dispersion and differences existing between clusters.

1. Motivation and contribution

Social media provide a substantial amount of information regarding the set of potential friends with whom one may connect after joining as a user (Adamic & Adar, 2003; Zuo, Blackburn, Kourtellis, Skvoretz, & Iamnitchi, 2016). The strategic use of the information available in social media by other users and, in particular, by companies has been consistently analyzed in the literature on social networks (Hofstra, Corten, & Buskens, 2015; Stefanone, Hurley, Egnoto, & Covert, 2015). At the same time, this literature acknowledges the existing diversity of users determined by their networking capacities and their ability to influence other users (Guo, Pathak, & Cheng, 2015; Klein, Ahlf, & Sharma, 2015).

Contrary to the standard literature on social networks, which generally focuses on analyzing the main properties of networks that have already been built (Han, Wang, Crespi, Park, & Cuevas, 2015; Jackson, 2010), we concentrate on the formation of networks and clusters within them based on the characteristics of their users. That is, consider a social medium whose users are endowed with incomplete information regarding the characteristics of other users from who they may receive friendship requests. As a result, a decision maker (DM) has to decide whether to accept a given friendship request, generating a link that may allow him to expand his current network of connections further, or reject it, expecting to find a requester who aligns better with his preferences.

If the request is accepted, then additional information becomes available regarding the characteristics of the requester. However, the acceptance decision must be made while constrained by the initial amount of incomplete information. At the same time, this information must be used by a DM to define his expectations about

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the remaining characteristics of the requesters and the potential network improvements that may be achieved by accepting the request. Therefore, we will assume that the information initially available to the DM conditions the expected realizations of the remaining characteristics of the requesters, including their capacity to expand the network of connections of the DM. Consequently, our model will be designed following a decision theoretical approach based on the expected utility that could be achieved by the DMs composing a given social medium (Kahneman & Tversky, 2000; Tavana, Di Caprio & Santos-Arteaga, 2016; Tavana, Di Caprio, Santos-Arteaga, and Tierney, 2016).

The main objective of the current paper is to analyze the type of clustered structures generated within a (social) network by

- the preferences of the DMs composing the network;
- the subjective beliefs of the DMs regarding the networking capacity of the requesters;
- the disutility costs derived by the DMs from accepting the friendship of a requester whose tastes and characteristics differ significantly from their own.

We formalize the problem faced by a DM, define the expected utility tradeoffs that he faces when receiving a friendship request and simulate the resulting acceptance and rejection incentives numerically. These incentives determine the social behavior of the DM together with the structure of the resulting networks, which is based on the preferences of the DMs and their expectations regarding those of the requesters.

We build the corresponding social networks through self-organizing maps that cluster the DMs by their friendship acceptance behavior. This behavior is, at the same time, determined by the distribution of characteristics of the requesters relative to the preferences of the DMs. We illustrate how the similarity in preferences among users leads to more concentrated clusters within the incomplete information scenario analyzed. Moreover, the emergence of disutility costs derived from a suboptimal decision when accepting to interact with other users increments the dispersion between clusters. In this regard, the inclusion of requesters endowed with average preferences relative to those of the standard users composing the network acts as a connectivity-enhancing mechanism designed to reduce the dispersion and differences existing between clusters.

We should emphasize that, even though self-organizing maps do not quantify the connectivity of the resulting graphs, a visual examination will suffice to analyze their main clustering properties. In the current setting, we are interested in the concentration arising within clusters and the separation between them, both of which can be inferred from a visual examination of the weight planes and U-matrices provided by the self-organizing map algorithm.

The paper proceeds as follows. Sections 2 and 3 describe the basic structure on which the accept and reject functions are built. These functions are introduced in Section 4 and Section 5, respectively, and simulated numerically in Section 6. Section 7 analyzes the different clustered structures that arise after implementing a self-organizing map algorithm to classify the number of friendship requests accepted by the DMs. Section 8 describes the consequences from increasing the amount of information available to the DM before accepting a request. Section 9 presents some concluding remarks.

2. Basic assumptions

The choice made by the DM regarding the friendship request depends on the following variables, whose domains are also provided in the respective definitions:

- $X_1 = [x_1^m, x_1^M]$: represents the characteristics/preferences of the requester that are directly observable when receiving a friendship request. This variable accounts for the publicly available information describing the main preferences of the requester. That is, we assume that these preferences can be inferred from the interests (i.e. likes) displayed in the profile of a requester (Kosinski, Stillwell, & Graepel, 2013, 2015; Meshi, Tamir, & Heekeren, 2015). In this regard, the X_1 variable provides only part of the information required to fully infer the preferences of the requester. We will assume that the value of the realization of X_1 observed is related to the remaining information completing the profile of the requester. However, this information only becomes available after accepting the request, together with the list of friends and, therefore, the networking capacity of the requester.
- $X_2 = [x_2^m, x_2^M]$: accounts for the characteristics/preferences of the requester that become observable after accepting the friendship request. This variable provides additional information to the DM regarding the interests and tastes of the requester together with his potential networking capacity. Thus, the distribution defining the expected realization of this variable is conditioned by the observed realization of X_1. At the same time, both X_1 and X_2 will be used by the DM to infer the potential capacity of the requester to extend his network with friends whose preferences are similar to his own (Ding, Yan, Zhang, Dai, & Dong, 2016; Hu & Yang, 2015; Mislove, Viswanath, Gummadi, & Druschel, 2010).
- $X_3 = [0, 1]$: reflects the networking capacity of the requester. The shape of its associated probability function is subjectively determined by the DM based on the observed value of X_1 and the expected realization of X_2. It should be remarked that the requester can generally classify his friends in several categories, granting them access to different amounts of information depending on their degree of friendship with the requester. However, even if the DM is not granted access to the whole network, he can still benefit from the fact that other friends of the requester are actually able to observe him. That is, the DM can expand his network with friends of the requester even if they are classified in different categories.

The decision taken by the DM will be based on two incentive functions that define the expected utility derived from either accepting a given friendship request or rejecting it. Note that both these functions must be defined for the values of all the realizations of X_1 that may be initially observed by the DM. We describe these functions in detail through the following sections.

3. Utility and probability density functions

Throughout the paper, we will assume the DM to be endowed with the following utility functions and probability density functions.

- Utility function on X_i, $i = 1, 2$:

$$u_i(x_i) = x_i.$$

Hence, the first two characteristics are additively separable.

- Utility function on $X_1 \times X_2 \times X_3$:

$$u(x_1, x_2, x_3) = (x_1 + x_2)x_3.$$

The function u defines the DM's utility derived from accepting a requester with first and second characteristics given by x_1 and x_2, and networking capacity x_3. This utility plays a crucial role when constructing a decision function that allows the DM to evaluate his

expected payoff based on the networking capacities of new potential friends.

- Probability density function on X_i, $i = 1, 2$:

$$\mu_i(x_i) = \frac{1}{x_i^M - x_i^m}.$$

The density function on X_i is assumed to be uniform to better reflect the DM's complete uncertainty regarding the distribution of potential friends. Indeed, uniform density functions are well-known to be associated with the maximum information entropy value.

- Certainty equivalent values, ce_i, $i = 1, 2$:

$$ce_i = u_i^{-1}\left(\int_{X_i} \mu_i(x_i)u(x_i)dx_i\right)$$

$\int_{X_i} \mu_i(x_i)u(x_i)dx_i$ is the expected value of u_i. The value ce_i is known as the *certainty equivalent of* μ_i *and* u_i. It is a characteristic in X_i that the DM is indifferent to accept in place of the expected one induced by μ_i and u_i. The continuity and strictly increasingness of u_i imply that ce_i exists and it is unique. This characteristic is used as the reference point against which the DM compares both the observed and potential characteristics of a requester (Gilboa, 2009).

- Conditional probability density function on X_2:

$$\mu_2(x_2|x_1) = \begin{cases} \dfrac{1}{x_2^M - x_2^m} + \varphi\left(\dfrac{x_1 - ce_1}{x_1^M - ce_1}\right)\dfrac{1}{x_2^M - x_2^m}, & \text{if } x_2 > \dfrac{x_2^M + x_2^m}{2} \\ \dfrac{1}{x_2^M - x_2^m} - \varphi\left(\dfrac{x_1 - ce_1}{x_1^M - ce_1}\right)\dfrac{1}{x_2^M - x_2^m}, & \text{if } x_2 \le \dfrac{x_2^M + x_2^m}{2} \end{cases}$$

where $\varphi \in [0, 1]$.

The density function $\mu_2(\cdot|x_1)$ is a conditional density describing the direct (subjective) correlation existing between the realization of the requester's first characteristic observed by the DM and the realization of the requester's second characteristic expected by the DM. The definition proposed for $\mu_2(\cdot|x_1)$ reflects the fact that as the value of x_1 observed by the DM increases, his belief of observing a value of x_2 that provides him with a high utility improves. Indeed, $\mu_2(\cdot|x_1)$ is defined as a step function whose graph is decreasing if $x_1 \in [x_1^m, ce_1)$, increasing if $x_1 \in (ce_1, x_1^m]$ and coincides with the uniform distribution if $x_1 = ce_1$.

The parameter $\varphi \in [0, 1]$ weighs the strength of the shift in probability mass between the $[x_2^m, \frac{x_2^M + x_2^m}{2}]$ and the $(\frac{x_2^M + x_2^m}{2}, x_2^M]$ subintervals of X_2 as x_1 varies in X_1. Clearly, there is no shift in probability mass when $x_1 = ce_1$, which produces a uniform distribution on X_2.

Note that in the current paper we consider only the risk neutral case, which allows us to exploit the symmetry of X_1 with respect to ce_1 (ce_1 coincides with the middle point of X_1). If u_1 is not linear, the density function $\mu_2(\cdot|x_1)$ needs to be modified accordingly adjusting the shift in probability mass based on the location of the value ce_1.

- Probability density function on X_3:

$$\mu_3(x_3) = B_3\left(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 + ce_2\right) = \frac{x_3^{x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))) - 1}(1 - x_3)^{ce_1 + ce_2 - 1}}{\int_0^1 u^{x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))) - 1}(1 - u)^{ce_1 + ce_2 - 1}du}$$

where $E(x_2|\mu(x_2|x_1)) = \int_{X_2} \mu_2(x_2|x_1)u_2(x_2)dx_2$.

The value $u_2^{-1}(E(x_2|\mu(x_2|x_1)))$ expresses the expected realization of the requester's second characteristic given the one initially observed. The density function μ_3 measures the degree of optimism or pessimism of the DM regarding the requester's networking capacity. The definition proposed for this density function is the one of a standard Beta distribution with parameters $x_1 + u_2^{-1}(E(x_2|\mu(x_2|x_1)))$ and $ce_1 + ce_2$. That is, the DM's belief on the potential capacity of the requester to extend his network with friends whose preferences are similar to his own depends on both the first characteristic observed and the expected realization of the second characteristic. Moreover, the certainty equivalent values of both realizations are used as reference values for evaluating the information acquired about the requester.

4. Accepting the request

As stated in the previous sections, the information available to the DM when deciding whether to accept the friendship request or reject it is limited to the observation of a realization from X_1, namely, the requester's first characteristic.

To simplify notations, henceforth, we will denote by x_1^o the value initially observed by the DM, that is, the value of the requester's first characteristic, and by x_2^* the value in X_2 such that $x_2^* = ce_1 + ce_2 - x_1^o$.

Given the linearity of the utility functions u_1 and u_2, the value x_2^* is the minimum value required by the DM for the new friendship to deliver a level of utility not inferior to the expected one. It follows that accepting the friendship request would produce of a disutility for the DM whenever the requester's second characteristic turns out to be $x_2 < x_2^*$. We will denote this disutility by $c(x_1, x_2)$.

Note that the disutility costs faced by the DM are not limited to being directly connected with a user whose tastes and characteristics differ significantly from his own. The DM and the network to which he already belongs may suffer from undesired communication or further friendship requests from the network of the new connection, which may negatively affect the relationships among the users composing the DM's network.

Considering the above discussion on disutility costs and the interpretation of the utility and probability density functions introduced in Section 3, we define the following utility function for the DM.

$$\begin{aligned} Accept = &\int_0^1 \int_{x_2^*}^{x_2^M} B_3\Big(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 \\ &+ ce_2\Big)\mu_2(x_2|x_1)u(x_1, x_2, x_3)dx_2dx_3 \\ &+ \int_0^1 \int_{x_2^m}^{x_2^*} \Big[B_3\Big(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 \\ &+ ce_2\Big)\mu_2(x_2|x_1)\{u(x_1, x_2, x_3) - c(x_1, x_2)\}\Big]dx_2dx_3 \end{aligned} \tag{1}$$

This function allows the DM to calculate the utility, in

expected terms, that he derives from accepting the friendship request while knowing only the first characteristic of the requester.

5. Rejecting the request

To be able to methodically choose between accepting and rejecting a request, the DM needs to calculate the payoff that he would derive from rejecting the request and compare it with the utility expected from accepting it.

As in the acceptance case, the DM faces several costs when rejecting the requester's friendship and acquiring information about a new one.

- If $x_1 \in [x_1^o, x_1^M]$, the DM incurs the search costs that result from observing the first characteristic of a new requester. These costs will be denoted by $s(x_1)$. If, at the case time, $x_2 \in [x_2^m, x_2^*]$, the DM faces also the disutility costs from accepting the friendship of a requester whose tastes and characteristics differ from his own ones. These costs will be denoted again by $c(x_1, x_2)$.
- If $x_1 \in [x_1^m, x_1^o)$, the DM should reject the new requester, which implies that he will be incurring two different search costs: the ones derived from having performed a new search and the disutility costs due to the fact that the DM was not able to increase his network of connections. These costs will be denoted by $sc(x_1)$.

Considering the above discussion on rejection costs and the interpretation of the utility and probability density functions introduced in Section 3, we introduce the following utility function to model the rejection payoff of the DM.

$$\begin{aligned}
Reject = &\int_0^1 \int_{x_2^*}^{x_2^M} \int_{x_1^o}^{x_1^M} \Big[B_3\Big(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 \\
&+ ce_2\Big)\mu_2(x_2|x_1)\mu_1(x_1)\{u(x_1,x_2,x_3) - s(x_1)\}\Big]dx_1dx_2dx_3 \\
&+ \int_0^1 \int_{x_2^m}^{x_2^*} \int_{x_1^o}^{x_1^M} \Big[B_3\Big(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 \\
&+ ce_2\Big)\mu_2(x_2|x_1)\mu_1(x_1)\{u(x_1,x_2,x_3) - c(x_1,x_2) \\
&- s(x_1)\}\Big]dx_1dx_2dx_3 - \int_0^1 \int_{x_2^m}^{x_2^M} \int_{x_1^m}^{x_1^o} B_3\Big(x_3; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), \\
&ce_1 + ce_2\Big)\mu_2(x_2|x_1)\mu_1(x_1)sc(x_1)dx_1dx_2dx_3
\end{aligned} \tag{2}$$

In Equation (2):

- the first term defines the DM's expected payoff assuming that the new requester provides an expected utility higher than both the certainty equivalent sum $ce_1 + ce_2$ and the initial requester's utility;
- the second term defines the DM's expected payoff assuming that the new requester provides an expected utility higher than that of the initial requester but lower than the certainty equivalent sum $ce_1 + ce_2$;
- the last term accounts for the search costs $sc(x_1)$ that are incurred when the new requester's first characteristic x_1 is lower than x_1^o.

6. Simulating the accept and reject functions

To show the applicability of the proposed model we have simulated numerically the functions *Accept* and Reject. In these simulations, we have set $X_1 = [5, 10]$ and $X_2 = [0, 10]$. In particular, $ce_1 = 7,5$ and $ce_2 = 5$. Also, recall that the shape of the probability density functions $\mu_2(\cdot|x_1)$ and $\mu_3(\cdot) = B_3(\cdot; x_1 + u_2^{-1}(E(x_2|\mu_2(x_2|x_1))), ce_1 + ce_2)$ depends on the realization that the DM observes for the first characteristic x_1 of the requester. In other words, as x_1 varies in X_1, the DM considers different potential expansions of his network.

In order to simplify the computations and account for the limited capacity of DMs to assimilate and manage information (Simon, 1955, 1997), we will consider six different potential networking subintervals of X_1 defined by the DM in terms of the value of the first characteristic observed. Consequently, reasoning as in the case of a fuzzy variable whose realizations are characterized by membership functions, we can define a Beta density function per each of the identified subintervals. The partition of X_1 and the corresponding Beta density functions associated with each subinterval in the simulations are outlined in Table 1.

Fig. 1 represents the functions *Accept* and Reject generated for $c(x_1, x_2) = s(x_1) = 0$ and $sc(x_1) = 1$. The graphs of both *Accept* and Reject have been constructed by simulating Equations (1) and (2) in each of the six subintervals partitioning X_1 and using the corresponding Beta density functions as shown in Table 1. While the rejection function turns out to be continuous, the acceptance function is piecewise continuous. Thus, we have used the images of the middle points of the subintervals partitioning X_1 to define a continuous approximation to the acceptance function.

The graphs of *Accept* and Reject intersect at $x_1^* = 6.54$, which is the threshold value determining the DM's choice between accepting and rejecting the requester's friendship. That is, $[x_1^m, x_1^*)$ and $[x_1^*, x_1^M]$ define the friendship rejection and acceptance intervals of the DM, respectively.

Clearly, Fig. 1 shows one of the many possible implementations of the proposed friendship acceptance model. For example, we can analyze the effects from an increase in the disutility costs that result from accepting the friendship of a requester whose tastes and characteristics differ significantly from those of the DM. Fig. 2 represents the functions *Accept* and Reject generated for $c(x_1, x_2) = 2$, $s(x_1) = 0$ and $sc(x_1) = 1$. In this case, the graphs of *Accept* and Reject intersect at $x_1^* = 6.89$.

In order to provide additional intuition regarding the behavior and computation of the acceptance and rejection functions, we describe both these functions explicitly for the $x_1^o \in [5, 5.5]$ subcase in the appendix section.

7. Clustering requesters through self-organizing maps

7.1. Basic intuition

As already stated, the main objective of the paper is to analyze the type of clustered structures generated within a network by the preferences of the DM, his subjective beliefs regarding the networking capacity of the requesters and the disutility costs from accepting the friendship of a requester whose tastes and interests differ significantly from his own ones.

The corresponding network structures and the clustering of DMs into different friendship acceptance groups, determined by the distribution of requesters relative to the preferences of the DMs, are generated using a self-organizing map (Kohonen (2001) and Sulkava, Sepponen, Yli-Heikkilä, and Latukka (2015) provide a

Table 1
Partition of X_1 and corresponding Beta density functions.

Subinterval of $X_1 = [5,\ 10]$ to which the realization x_1 belongs	Beta density function $B_3(x_3; x_1 + u_2^{-1}(E(x_2\|\mu_2(x_2\|x_1))), ce_1 + ce_2)$ considered by the DM
$[5, 5.5]$	$B_3(x_3; 5 + u_2^{-1}(E(x_2\|\mu_2(x_2\|5))), ce_1 + ce_2)$
$[5.5, 6.5]$	$B_3(x_3; 6 + u_2^{-1}(E(x_2\|\mu_2(x_2\|6))), ce_1 + ce_2)$
$[6.5, 7.5]$	$B_3(x_3; 7 + u_2^{-1}(E(x_2\|\mu_2(x_2\|7))), ce_1 + ce_2)$
$[7.5, 8.5]$	$B_3(x_3; 8 + u_2^{-1}(E(x_2\|\mu_2(x_2\|8))), ce_1 + ce_2)$
$[8.5, 9.5]$	$B_3(x_3; 9 + u_2^{-1}(E(x_2\|\mu_2(x_2\|9))), ce_1 + ce_2)$
$[9.5, 10]$	$B_3(x_3; 10 + u_2^{-1}(E(x_2\|\mu_2(x_2\|10))), ce_1 + ce_2)$

detailed description of the main components of this type of neural network).

We introduce several distributions of friendship requesters defined in terms of the initial characteristic observed by the DM. In particular, we define four different Beta functions on the realizations of X_1, describing four types of requesters relative to the preferences of the DM. These functions are represented in Fig. 3 and correspond to a Beta$(x_1; 2, 4)$, a Beta$(x_1; 4, 4)$, a Beta$(x_1; 4, 2)$ and a Beta$(x_1; 0.6, 0.6)$ density.

Then, we consider the number of friendship acceptances when 25, 50, 75 and 100 randomly generated requests drawn from each distribution are received by 100 different DMs. As a result, we end up with a total of 400 DMs categorized in four different groups determined by the Beta distribution used to generate their respective requesters. A plausible interpretation of this environment would consist of considering four groups of requesters whose preferences differ to a certain extent with respect to the average ones prevailing among the users composing the social medium. That is, the preferences of the DMs correspond to those of the standard users composing a given social networking site. Note that, the threshold values $x_1^* = 6.54$ and $x_1^* = 6.89$ defined over the $[5, 10]$ domain have been transformed into $x_1^* = 0.308$ and $x_1^* = 0.378$, respectively, when operating within the [0, 1] domain on which the Beta distributions are defined.

For illustrative purposes, consider the scenario without disutility costs, i.e. $c(x_1, x_2) = 0$. Given the location of the threshold value, the DMs receiving requests following from a Beta$(x_1; 2, 4)$ density will reject a larger number of them than those DMs receiving requests that follow from either a Beta$(x_1; 4, 4)$ or a Beta$(x_1; 4, 2)$. Moreover, as illustrated in Fig. 4, the dispersion observed in the acceptance behavior of the DMs should be smaller in the Beta$(x_1; 4, 2)$ case, with the DMs receiving requests from a Beta$(x_1; 2, 4)$ and a Beta$(x_1; 4, 4)$ density exhibiting a larger dispersion in their behavior.

7.2. The cohesive role of the average requester

Fig. 5 presents the output obtained when applying a self-organizing map algorithm to the number of friendship acceptances from requesters who are drawn from a Beta$(x_1; 2, 4)$ and a Beta$(x_1; 4, 2)$ distribution. We observe two differentiated clusters of DMs, one of which is surrounded by several isolated nodes. Note that the isolated nodes obtained among the Beta$(x_1; 2, 4)$ requesters correspond to those DMs accepting either a larger or a smaller number of requests than the others. Moreover, following the intuition provided in Fig. 4, the SOM weights obtained from these requesters should exhibit a larger dispersion than those obtained from the requesters who are drawn from a Beta$(x_1; 4, 2)$. In this latter case, isolated nodes describe those DMs accepting a lower number of requests. Thus, as expected, requesters whose preferences are highly similar to those of the standard users composing a network receive high acceptance rates, while those with dissimilar preferences will be generally rejected.

We illustrate now how average requesters, relative to the preferences of the DMs, can be used to build social bridges between the two clusters, as can observed in Fig. 6. In order to do so, we introduce a third set of requesters following a Beta$(x_1; 4, 4)$ distribution into the previous setting. That is, we introduce a set of requesters with average preferences relative to those of the standard users composing the network. As can be observed in Fig. 6, the average Beta$(x_1; 4, 4)$ DMs approach the acceptance behavior of the friendly Beta$(x_1; 4, 2)$ DMs composing the compact group on the upper right

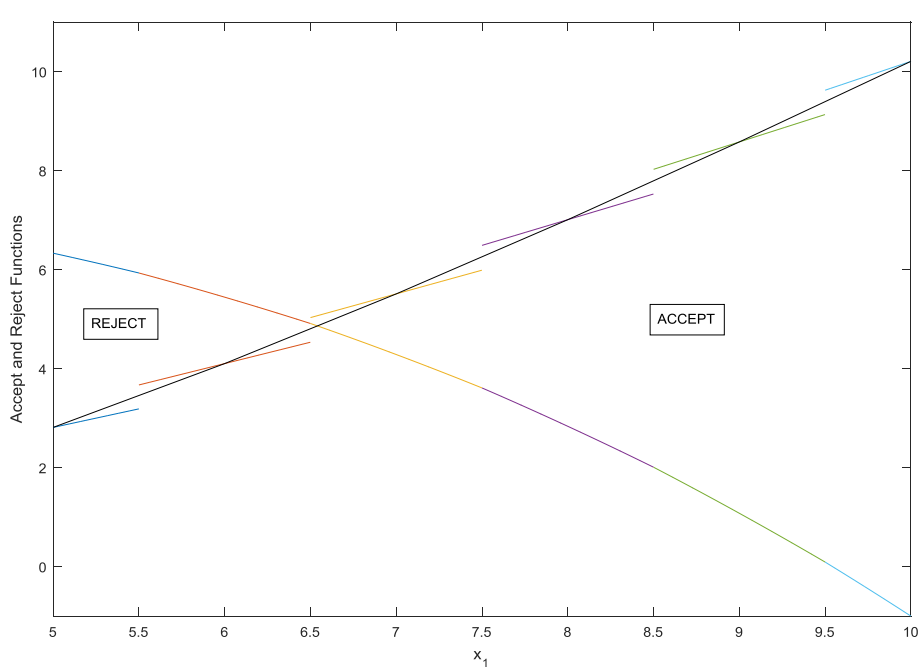

Fig. 1. Accept and reject functions generated for $c(x_1, x_2) = s(x_1) = 0$ and $sc(x_1) = 1$ with a threshold value of $x_1^* = 6.54$.

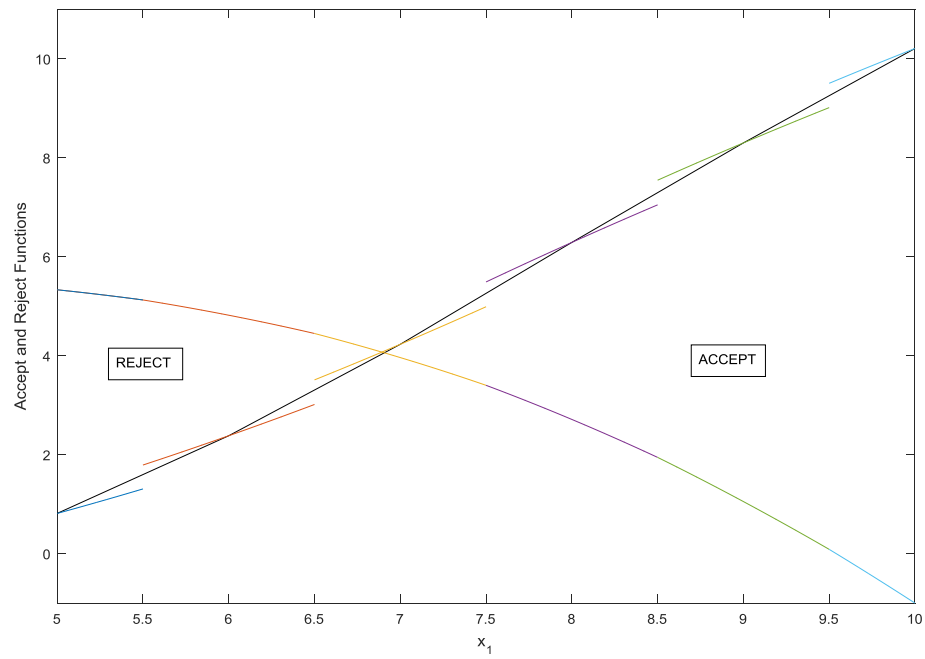

Fig. 2. Accept and reject functions generated for $c(x_1, x_2) = 2$, $s(x_1) = 0$ and $sc(x_1) = 1$ with a threshold value of $\boldsymbol{x}_1^* = 6.89$.

corner of the weight plane. The divide between the two limit groups of DMs' acceptances is still evident, but the capacity of the average ones to act as intermediaries increases the connectivity of the network, whose clusters, while still dispersed, approach between themselves. This tendency is also observed in the U-Matrix representing the hits, where a connection is established between the previously separated clusters.

Therefore, introducing requesters whose preferences and tastes approximate those of the DMs helps filling the gap between both clusters, which results in a more cohesive network. However, as follows from Figs. 1 and 2, the same type of result can be achieved by decreasing the disutility costs derived from accepting a suboptimal friendship, i.e. $c(x_1, x_2)$. This result follows from the location of the threshold value relative to the center of the Beta$(x_1; 4, 4)$ distribution. Thus, considering the location of the threshold value and the set of potential distributions of requesters, shifting the

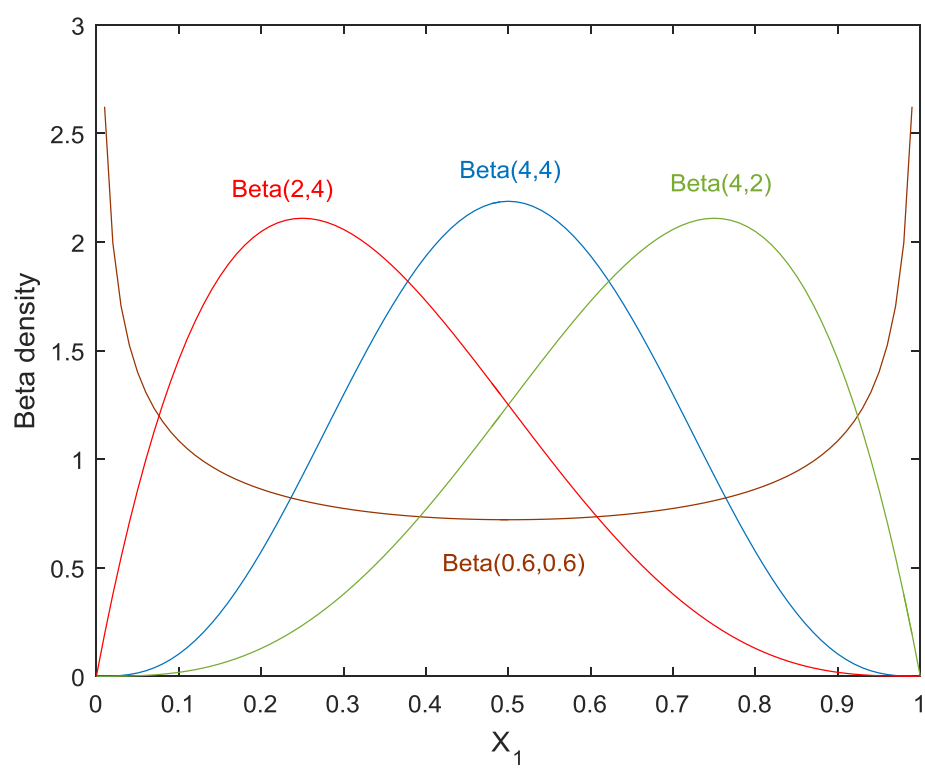

Fig. 3. Density functions used to generate requesters relative to the preferences of the DM.

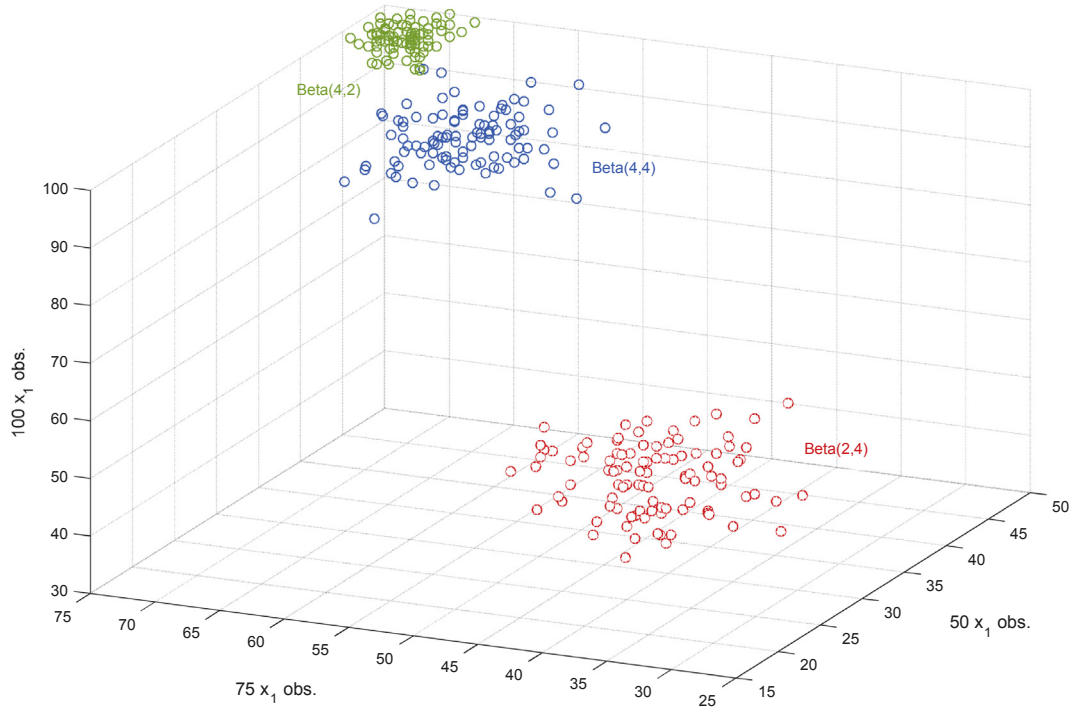

Fig. 4. Dispersion in the acceptance behavior of DMs based on the Beta density considered.

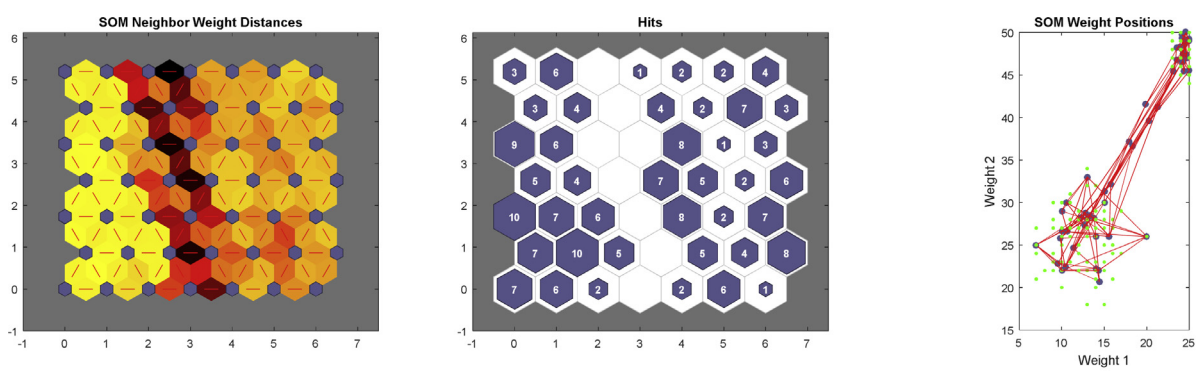

Fig. 5. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4) and a Beta($\boldsymbol{x}_1$; 4,2) distribution of requesters (totalling 200 DMs).

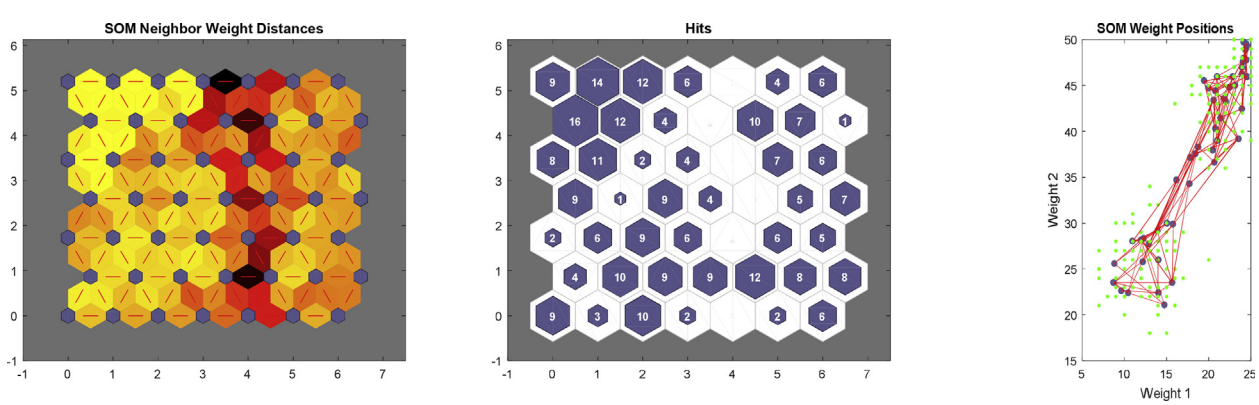

Fig. 6. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4), a Beta($\boldsymbol{x}_1$; 4,2) and a Beta($\boldsymbol{x}_1$; 4,4) distribution of requesters (totalling 300 DMs).

threshold further to the left while adding more average-type requesters to the population will increase the connectivity of the network. In this regard, the effect that differences in the size of their networks of friends have on the self-esteem of the users

composing a social networking site is described in Subsection 7.4.

Note that, in the current setting, a more cohesive network does not necessarily imply a larger number of friendship acceptances among the DMs. To illustrate this point, consider the scenarios presented in Figs. 7 and 8. The former figure describes the clusters that result when requesters are drawn from a $\text{Beta}(x_1; 4, 4)$ and a $\text{Beta}(x_1; 0.6, 0.6)$ distribution absent disutility costs. Fig. 8 represents the same scenario but with disutility costs increasing from zero to $c(x_1, x_2) = 2$. Given the symmetry of the density functions being considered, the rightward shift of the threshold value increases the cohesion in the friendship acceptance behavior of DMs and the connectivity of the resulting network. However, given the increase in disutility costs, the DMs will be rejecting a higher number of friendship requests.

7.3. Disutility costs and cluster dispersion

Consider the scenario with positive disutility costs, i.e. $c(x_1, x_2) = 2$. Intuitively, disutility costs increase the separation between the clusters generated by the $\text{Beta}(x_1; 2, 4)$ and the $\text{Beta}(x_1; 4, 2)$ requesters. This is the case since the corresponding threshold value would be approaching the center of the $\text{Beta}(x_1; 4, 4)$ distribution. If the center is reached, adding a set of requesters with average preferences would lead the DMs to display a symmetric acceptance-rejection behavior equidistant from the extreme cases represented by the $\text{Beta}(x_1; 2, 4)$ and $\text{Beta}(x_1; 4, 2)$ distributions.

The wider dispersion of clusters obtained in the setting with positive disutility costs together with the moderate influence of the $\text{Beta}(x_1; 4, 4)$ requesters on the dispersion of the corresponding network can be observed in Figs. 9 and 10. It should be highlighted that these effects prevail when adding even further amounts of requesters with average preferences, such as those drawn from a $\text{Beta}(x_1; 0.6, 0.6)$ distribution, to the scenario with disutility costs. Figs. 11 and 12 illustrate how the dispersion between the clusters generated by the different types of requesters becomes more evident when increasing the disutility costs faced by the DMs.

Finally, it should be emphasized that, even though not considered explicitly, the current model could be extended so as to complement the sociological approach to the structural cohesion and embeddedness of networks (Moody & White, 2003). That is, the behavior of the network depends on the location of the threshold value, with friendly cohesive networks requiring a relatively low threshold while unfriendly but cohesive ones require a high threshold value. Thus, increments in the disutility costs of friendship can be used either to isolate requesters further or to build exclusive groups of friends, depending on the side of the $\text{Beta}(x_1; 4, 4)$ distribution where the threshold is located:

- If to the left, as in the case presented in the paper, higher costs imply more separability between the different groups of requesters. In this regard, note that an increase in search costs, i.e. $s(x_1)$ and $sc(x_1)$, would lead to an increase in cohesion within the current setting, since it induces a downward shift of the reject function.
- If to the right, then higher costs imply more cohesion in the behavior of DMs, though leading also to more isolation, i.e. users of the social medium would belong to exclusive groups composed by very few friends.

7.4. On users' self-esteem and the size of their network of friends

The model introduced in this paper studies the effects that the preferences of the DMs together with their subjective beliefs and expectations have on the size of their networks of friends. The current section emphasizes the importance that the size of the network of friends has on the self-esteem of the users of a social networking site.

There is a growing interest in the literature regarding the effect that the size of the group of friends composing the network of the DM has on his subjective well-being. DMs with many online friends are considered to be more popular than those with few friends (Utz, 2010). This is the case despite the fact that the cognitive constraint defined by Dunbar (1992) limiting the size of social networks prevails when endowing the DMs with the communication capacity available in online media (Dunbar, 2016).

However, the positive relation established between popularity and social attractiveness becomes negative after a given number of friends is reached, since a large number of connections raises doubts about the popularity and desirability of joining the network of a given requester (Tong, Van Der Heide, Langwell, & Walther, 2008). A similar effect has been identified when analyzing the relationship between the number of online friends and subjective well-being, with very high numbers of friends constituting a source of negative experiences (Best, Taylor, & Manktelow, 2015). Lönnqvist and Deters (2016) obtained a positive relation between network size and the subjective well-being of DMs, which was mainly associated to the extraverted personality of the DMs. In this regard, genetic traits have been reported to determine social connectivity (Fowler, Dawes, & Christakis, 2009; Jackson, 2009).

Modifications in his network of friends have a psychological impact on the DM, ranging from proudness when a friendship request is accepted (Lewis & West, 2009), to emotional numbness

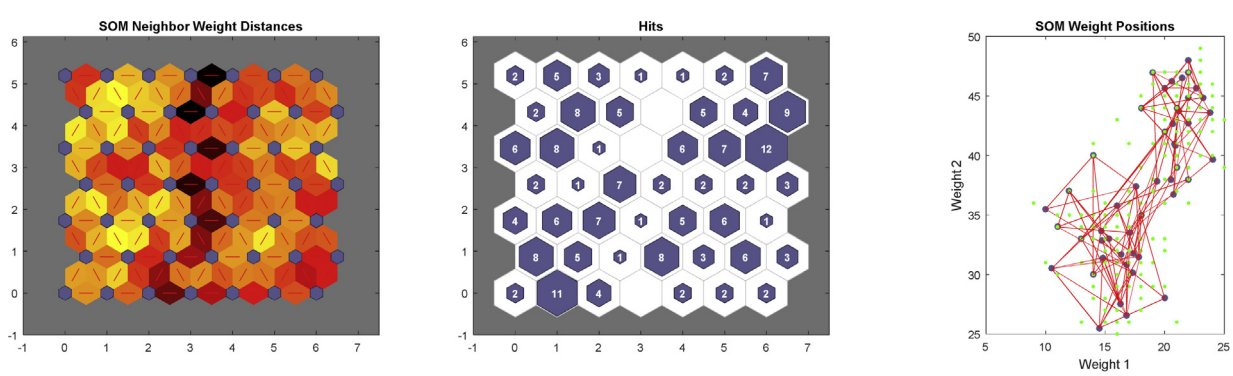

Fig. 7. Self-organizing map clusters following from a $\text{Beta}(\boldsymbol{x}_1; 4, 4)$ and a $\text{Beta}(\boldsymbol{x}_1; 0.6, 0.6)$ distribution of requesters (totalling 200 DMs).

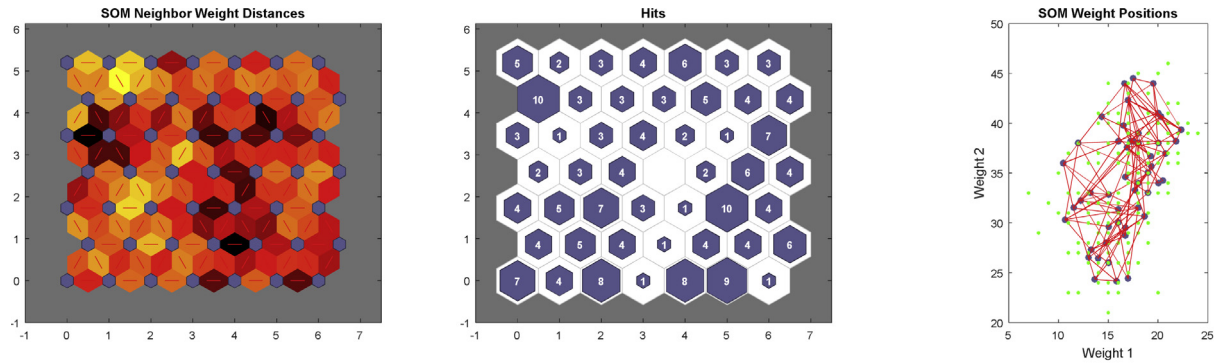

Fig. 8. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 4, 4) and a Beta($\boldsymbol{x}_1$; 0.6, 0.6) distribution of requesters (totalling 200 DMs) with the $\boldsymbol{c}(\boldsymbol{x}_1, \boldsymbol{x}_2) = 2$ case.

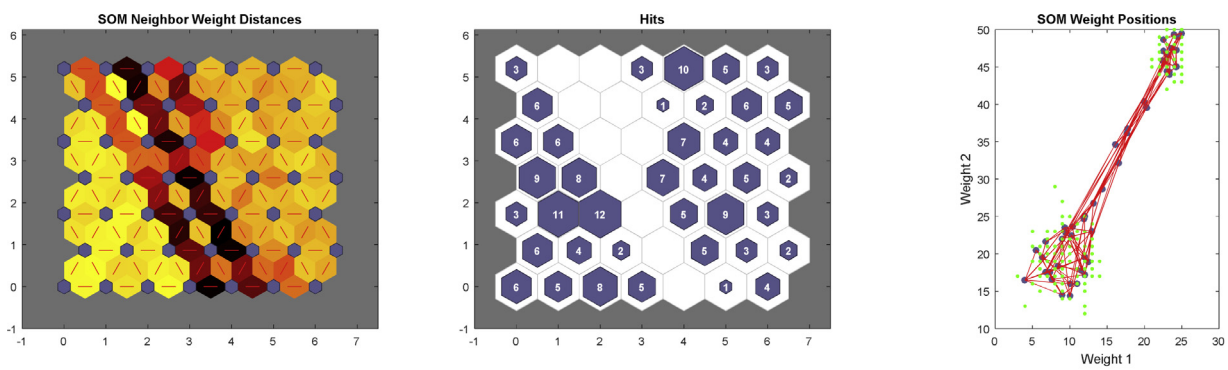

Fig. 9. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4) and a Beta($\boldsymbol{x}_1$; 4,2) distribution (totalling 200 DMs) with the $\boldsymbol{c}(\boldsymbol{x}_1, \boldsymbol{x}_2) = 2$ case.

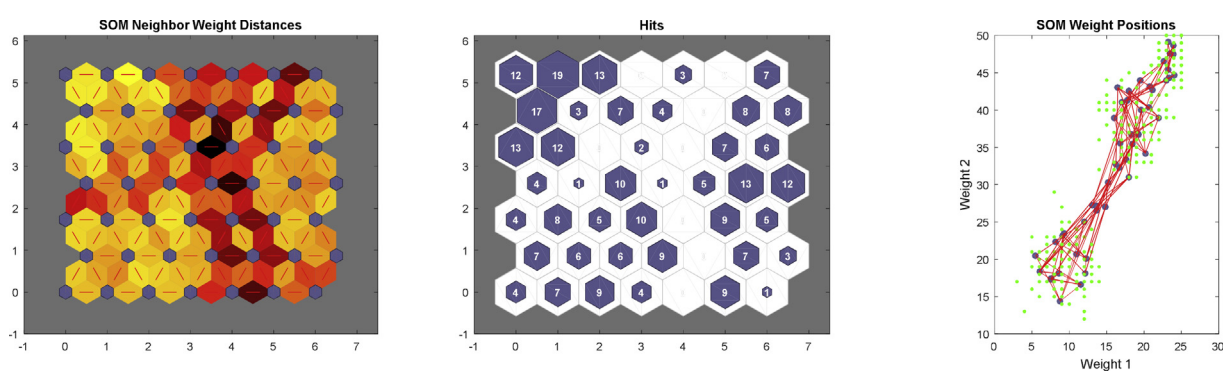

Fig. 10. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4), a Beta($\boldsymbol{x}_1$; 4,2) and a Beta($\boldsymbol{x}_1$; 4,4) distribution (totalling 300 DMs) with the $\boldsymbol{c}(\boldsymbol{x}_1, \boldsymbol{x}_2) = 2$ case.

when rejected (Filipkowski & Smyth, 2012), and rumination and negative emotions when the DM is unfriended (Bevan, Pfyl, & Barclay, 2012). When considering a social networking site such as Facebook, users have been observed to engage in self-enhancing comparisons with other users to manage their mood (Johnson & Knobloch-Westerwick, 2014). In particular, self-esteem has been shown to be negatively related to the social comparison frequency of Facebook users (Lee, 2014). At the same time, a positive relation has been identified between the number friends composing the Facebook network of a DM and his subjective well-being (Kim & Lee, 2011).

Therefore, following the findings of social comparison theory, the self-esteem of a DM should be negatively affected when exposed to profiles with a larger number of friends while being positively affected when coming across profiles with fewer friends (Gibbons & Gerrard, 1989). That is, it could be hypothesized that the relative number of friends composing the network of a DM affects his self-esteem.

Greitemeyer (2016) conducted two experiments to examine whether or not the number of friends displayed by other Facebook users had any influence on the self-esteem of DMs. He did not find any significant impact caused by this type of comparisons on the self-esteem of DMs. However, as emphasized by Greitemeyer (2016), his experiments displayed the profiles of users unknown to the DMs, while it has been empirically verified that comparisons and emotions have a significant effect particularly when considering known friends (Lim & Yang, 2015).

8. Increasing the amount information available to the DM

The decision environment described in the previous sections corresponds to the one faced by a standard user of a social network

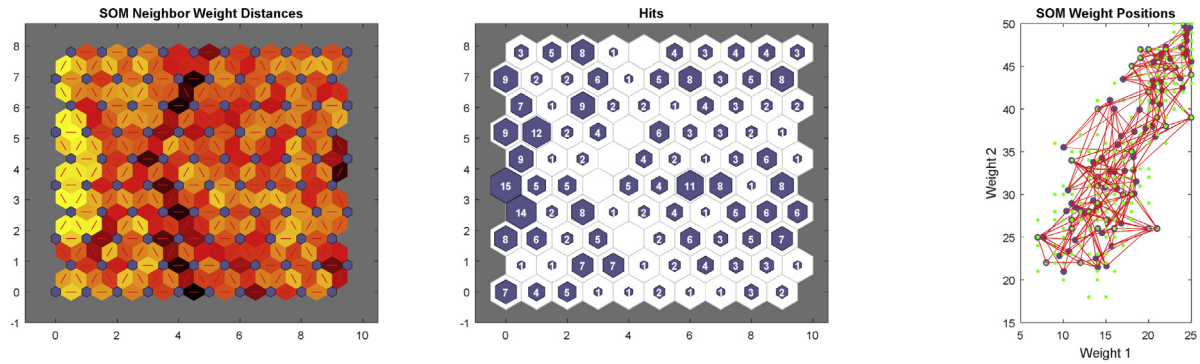

Fig. 11. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4), a Beta($\boldsymbol{x}_1$; 4,2), a Beta($\boldsymbol{x}_1$; 4,4) and a Beta($\boldsymbol{x}_1$; 0.6, 0.6) distribution (totalling 400 DMs).

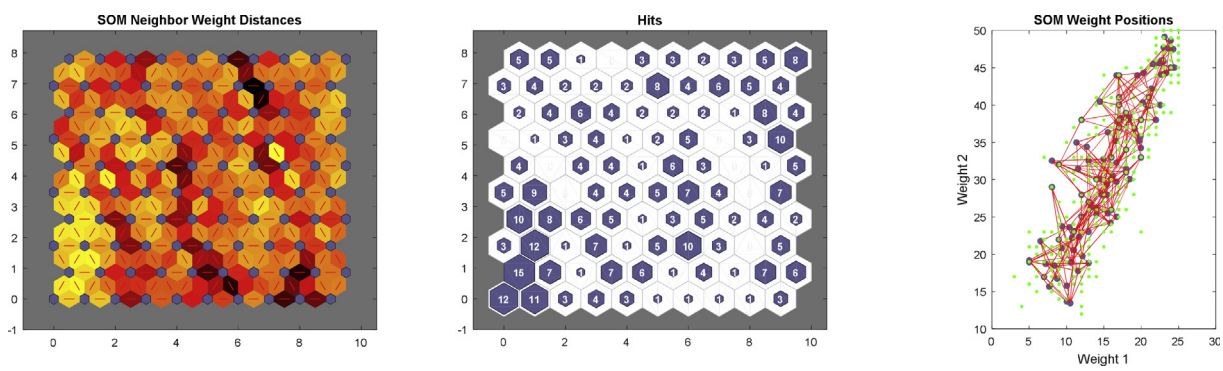

Fig. 12. Self-organizing map clusters following from a Beta($\boldsymbol{x}_1$; 2,4), a Beta($\boldsymbol{x}_1$; 4,2), a Beta($\boldsymbol{x}_1$; 4,4) and a Beta($\boldsymbol{x}_1$; 0.6, 0.6) distribution (totalling 400 DMs) with the $\boldsymbol{c}(\boldsymbol{x}_1, \boldsymbol{x}_2) = 2$ case.

site such as Facebook where only partial information is available, requiring the acceptance of the request to obtain additional information about the requester and his network of connections. However, the users of other social network sites such as LinkedIn are interested in displaying as much information as possible. That is, hiring firms try to elicit as much information as possible from their potential employees before deciding whether to accept a given request or continue searching. The information displayed (either directly or indirectly) by the requesters has to be evaluated by the DM while allowing for a certain degree of uncertainty regarding their potential performance. In other words, firms are aware of the fact that uncertainty prevails when trying to account for the behavior of potential employees once their requests are accepted.

We consider now a decision scenario where DMs are allowed to observe all the characteristics defining the users of a social network, i.e. skills, competences, interests, before accepting or rejecting a given request. That is, the scenario analyzed in this section assumes that the information retrieved by the DM provides a full description of both X_1 and X_2 for all requesters. Thus, we start by assuming that these characteristics can be elicited from the profiles of different users via recommender systems (Greenhow & Robelia, 2009).

For example, Stantchev, Prieto-González, and Tamm (2015) developed a cloud-computing-based tool to infer knowledge and interests from users through the data linked to them in different social network profiles. Their tool assesses the knowledge and skills of a given user in different topics with a certain degree of confidence. Similarly, Álvarez-Rodríguez, Colomo-Palacios, and Stantchev (2015) defined a hybrid algorithm to rank a list of experts based on an analysis of their profiles and activities in social networks. Potential experts were identified and ranked for every skill and interest according to a given trust metric.

It should be highlighted that the type of uncertainty described by these authors (and faced by the DM) is reflected in our formal decision environment through the variable X_3. In this regard, both X_1 and X_2 will be used to infer the capacity of the requesters to either perform according to the standards required by the DM or expand his network of connections with other candidates endowed with similar characteristics. Note that we have extended the interpretation of X_3 beyond the networking capacity of the requester to reflect also his potential performance.

As in the previous sections, the decision taken by the DM will be based on two incentive functions that define the expected utility derived from either accepting a given request or rejecting it. However, in the current setting, these functions are defined for the values of all the realizations of X_1 and X_2 that may be observed by the DM, increasing the information available to the user before accepting a request.

The corresponding *Accept* function must be modified as follows

$$Accept = \int_0^1 B_3(x_3; x_1 + x_2, ce_1 + ce_2) u(x_1, x_2, x_3) dx_3, \tag{3}$$

for $x_1 \in [x_1^m, x_1^M]$ and $x_2 \in [x_2^m, x_2^M]$. Given the larger amount of information available, the level of uncertainty corresponding to the potential realizations of X_2 has been eliminated. Two main consequences derived from this modification are described below

1. The Beta density function has to be redefined as follows

$$B_3(x_3;x_1+x_2,ce_1+ce_2)=\frac{x_3^{x_1+x_2-1}(1-x_3)^{ce_1+ce_2-1}}{\int_0^1 u^{x_1+x_2-1}(1-u)^{ce_1+ce_2-1}du}$$

In this case, the parameters defining the shape of the density are directly observable and the function measures the beliefs of the DM regarding the potential capacity of the requester to perform according to the standards required or to expand his network of connections.

2. Since the DM does not face the uncertainty derived from X_2, we consider a scenario without disutility costs, i.e. $c(x_1,x_2)=0$. Clearly, the DM could still accept the request from a user and regret this decision after interacting with him or observing his network of connections. However, in the previous setting, a request could be accepted only to realize about its suboptimality after observing $x_2<x_2^*$. In the current setting, the information required to prevent a suboptimal choice is available before the acceptance takes place and only the performance or networking capacity of the requester remains unknown. Note that $c(x_1,x_2)$ is not determined by the network of connections of the requester but the realizations of X_1 and X_2.

Consider now the modified version of the Reject function, where potential improvements are defined relative to the initial pair of realizations observed by the DM, (x_1^o,x_2^o).

$$\begin{aligned}Reject=&\int_0^1\int_{x_2^o}^{x_2^M}\int_{x_1^o}^{x_1^M}[B_3(x_3;x_1+x_2,ce_1+ce_2)\mu_2(x_2|x_1)\mu_1(x_1)\\&\{u(x_1,x_2,x_3)-s(x_1,x_2)\}]dx_1dx_2dx_3+\int_0^1\int_{x_2^m}^{x_2^o}\int_{x_1^o}^{x_1^M}[B_3(x_3;x_1\\&+x_2,ce_1+ce_2)\mu_2(x_2|x_1)\mu_1(x_1)\{u(x_1,x_2,x_3)\\&-s(x_1,x_2)\}]dx_1dx_2dx_3+\int_0^1\int_{x_2^o}^{x_2^M}\int_{x_1^m}^{x_1^o}[B_3(x_3;x_1+x_2,ce_1\\&+ce_2)\mu_2(x_2|x_1)\mu_1(x_1)\{u(x_1,x_2,x_3)-s(x_1,x_2)\}]dx_1dx_2dx_3\\&-\int_0^1\int_{x_2^m}^{x_2^o}\int_{x_1^m}^{x_1^o}B_3(x_3;x_1+x_2,ce_1+ce_2)\mu_2(x_2|x_1)\mu_1\\&(x_1)sc(x_1,x_2)dx_1dx_2dx_3\end{aligned}\tag{4}$$

Note, once again, that we have removed the uncertainty inherent to X_2 from the function since the DM observes x_2 before accepting a new request, eliminating the suboptimal choice associated to $c(x_1,x_2)$. Therefore, the only costs associated to the *Reject* alternative are

- the search cost $s(x_1,x_2)$ derived from observing both variables;
- the cost from receiving a request inferior to the one rejected, $sc(x_1,x_2)$, though adapted to account for both characteristics. This latter cost is assumed to be incurred only when both $x_1<x_1^o$ and $x_2<x_2^o$, though different assumptions could be imposed depending on the quadrant of potential realizations being considered.

As in the previous sections, the shape of the probability density function $\mu_2(\cdot|x_1)$ depends on the realization of x_1 that the DM expects to observe from the next requester. That is, the DM expects to observe a pair (x_1,x_2) from the next requester whose utility must be computed while maintaining the relationship defined between both variables through $\mu_2(x_2|x_1)$, i.e. relative to the ce_1 value in the linear utility setting analyzed.

As already stated, the DM must consider all the potential realizations of X_1 and X_2 when defining the *Accept* and Reject functions. The $X_1\times X_2$ space of potential realizations is described in Fig. 13. Similarly to Section 6, we simplify the computations by dividing the space in four quadrants and associating a Beta density function to each one of them. Note that additional partitions could be defined in order to generate a more concise approximation, though the main qualitative results obtained would remain unchanged. The resulting partition of $X_1\times X_2$ with respect to ce_1, ce_2 and the Beta and $\mu_2(\cdot|x_1)$ densities associated with each subinterval in the simulations is outlined in Fig. 13.

Note that different densities have to be considered when computing the *Accept* and Reject functions depending on the quadrant where the potential observations are located. We have kept a formal structure as similar to that of the previous sections as possible, though it should be noted that several alternative frameworks can be defined depending on the relative value assigned by the DM to the realizations located within the $(x_1\in[x_1^o,x_1^M],x_2\in[x_2^m,x_2^o])$ and $(x_1\in[x_1^m,x_1^o],x_2\in[x_2^o,x_2^M])$ quadrants and the subjective importance given to each characteristic.

Fig. 14 represents the *Accept* and Reject functions defined in Equations (3) and (4). These functions have been simulated using the numerical values described in Section 6, with $s(x_1,x_2)=0$ and $sc(x_1,x_2)=1$. Note that the basic patterns exhibited by these functions are identical to those observed in Fig. 1. The *Accept* function increases in the values of both X_1 and X_2 while the Reject function exhibits a decreasing pattern.

However, a novel and immediate result can be derived from Fig. 14(c) and (d). The capacity to verify the value of both characteristics before deciding leads DMs to constrain their acceptance to users delivering a utility higher than ce_1+ce_2. Note, for example, that the extreme threshold values defined by both functions are given by $(x_1=10,x_2=3.7975)$ and $(x_1=5.5944,x_2=10)$, while the certainty equivalent user delivers a utility of 12.5. A visual analysis of Fig. 14(d) illustrates how the potential combinations of X_1 and X_2 required for acceptance remain above the $ce_1+ce_2=12.5$ value. This constraint is considerable given the less restrictive one imposed when only the X_1 characteristic was being observed, i.e. $x_1^*=6.54<ce_1=7.5$.

Thus, an increment in the amount of information available to the DM restricts his acceptance behavior, as well as the creation of more inclusive clusters, when compared to a partial information environment. Verification allows the DM to become stricter in his selection procedure, while partial information leads to a more lenient acceptance behavior. In the latter case, DMs account for potentially suboptimal decisions as probabilistic events affecting their final utilities, while verification allows them to directly disregard the requests from those users that do not deliver a sufficiently high utility. Finally, note that search costs could be modified so as to increase the acceptance rate within the current setting, since higher search costs would induce a downward shift of the Reject function while keeping the *Accept* one unchanged.

9. Concluding remarks

In the current paper, we have built different types of social networks based on the preferences and beliefs determining the friendship acceptance behavior of the users composing them. Differences in preferences between the friendship requesters and the

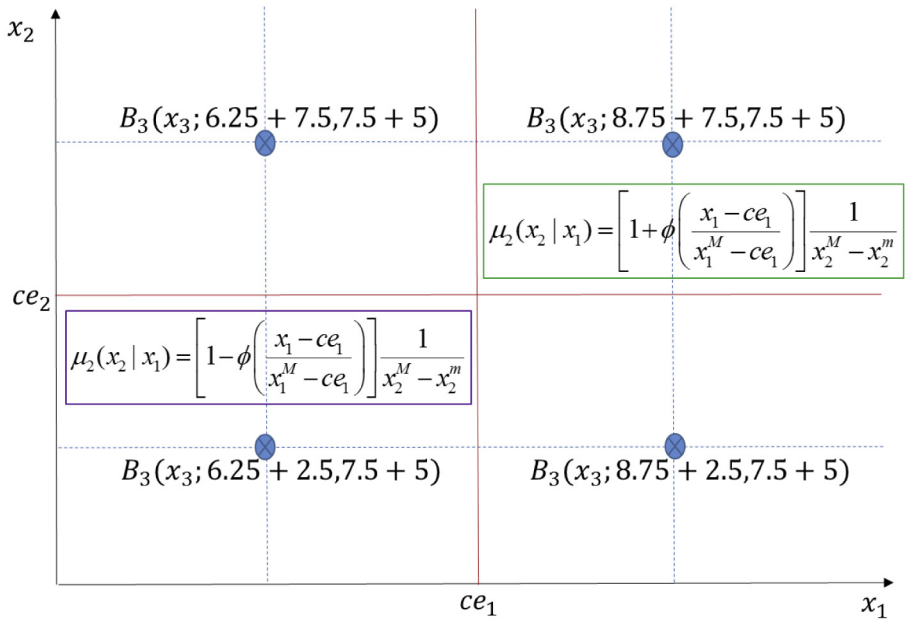

Fig. 13. Quadrant partition of $\boldsymbol{X}_1 \times \boldsymbol{X}_2$ when the DM observes both $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$.

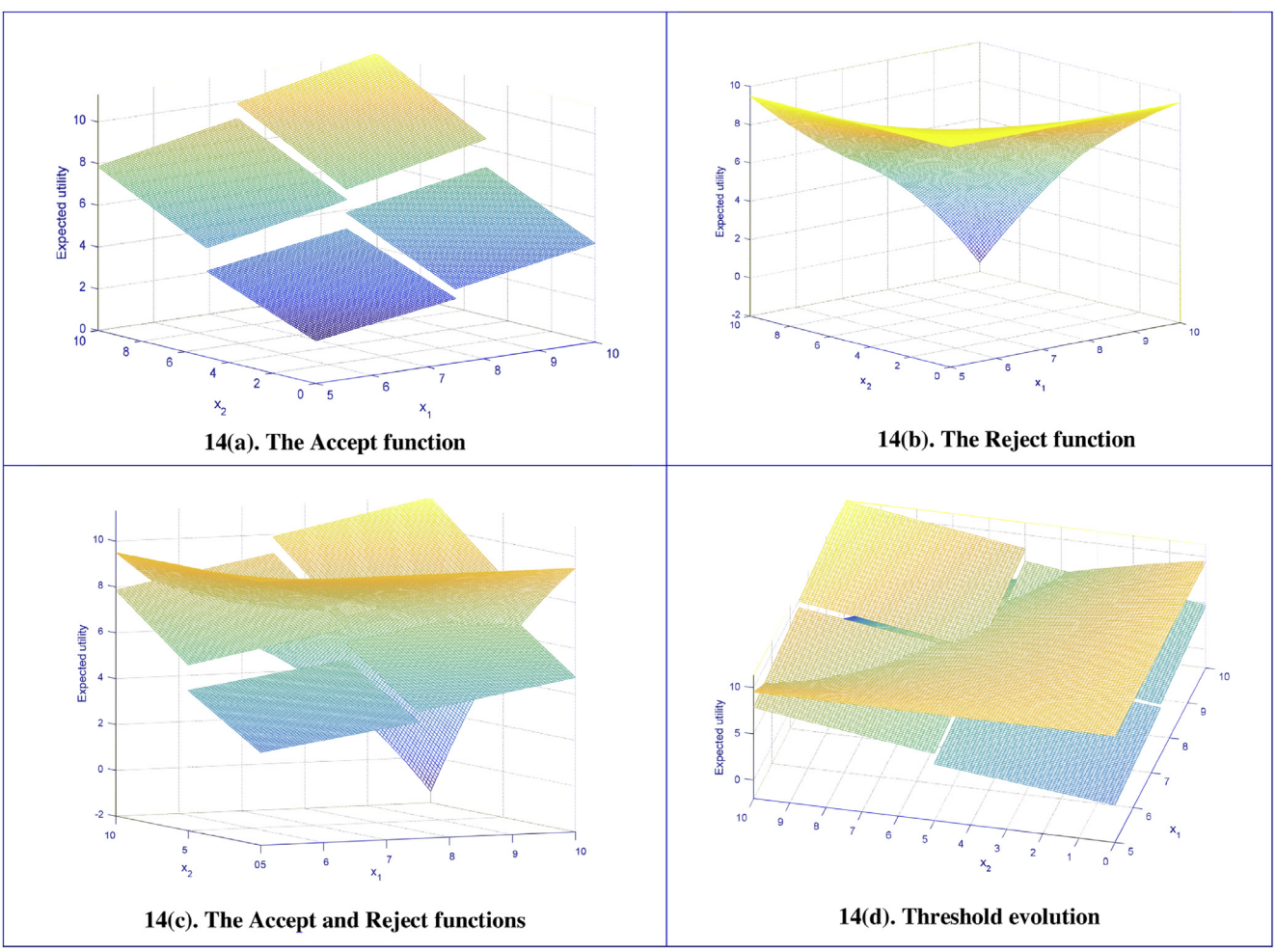

Fig. 14. The Accept and Reject functions when the DM observes both $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ with $s(x_1, x_2) = 0$ and $sc(x_1, x_2) = 1$.

DMs have been shown to lead to two main types of (dispersed) clusters, one of them composed by users facing multiple rejections and the other formed by requesters receiving multiple acceptances.

We have also provided a framework of analysis that allows to

account for the effect of multiple factors, such as the disutility costs derived from accepting a suboptimal friendship request or the formation process of the subjective beliefs of DMs, on the connectivity and cluster structure of the resulting network. In particular, we have illustrated how the inclusion of requesters with average preferences relative to those of the standard users acts as a connectivity-enhancing mechanism that reduces the dispersion and differences existing between the clusters.

We conclude by emphasizing that our friendship acceptance model can be modified and extended to account for segregation and isolation in social networks (Bojanowski & Corten, 2014) together with trust considerations in the selection of partners, which determine the cooperation expected to be achieved within the resulting network (Bravo, Squazzoni, & Boero, 2012).

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

Appendix

Accept and reject functions for $x_1^o \in [5, 5.5]$

$$
\begin{aligned}
Accept = {} & \int_0^1 \int_{12.5-x_1^o}^{10} B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left[\frac{1}{10}+\left(\frac{x_1^o-7.5}{10-7.5}\right)\frac{1}{10}\right](x_1^o+x_2)x_3\,dx_2dx_3 + \int_0^1 \int_5^{12.5-x_1^o} \Big[B_3(5+E(x_2|\mu_2(x_2|5)), 7.5 \\
& +5)\left[\frac{1}{10}+\left(\frac{x_1^o-7.5}{10-7.5}\right)\frac{1}{10}\right]\{(x_1^o+x_2)x_3-c(x_1,x_2)\}\Big]dx_2dx_3 + \int_0^1\int_0^5 \Big[B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1^o-7.5}{10-7.5}\right)\frac{1}{10}\right]\{(x_1^o \\
& +x_2)x_3-c(x_1,x_2)\}\Big]dx_2dx_3
\end{aligned}
$$

$$
\begin{aligned}
Reject = {} & \int_0^1\int_5^{10}\int_{9.5}^{10}\Big[B_3(10+E(x_2|\mu_2(x_2|10)), 7.5+5)\left[\frac{1}{10}+\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_{12.5-x_1}^{5}\int_{9.5}^{10}\Big[B_3(10+E(x_2|\mu_2(x_2|10)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_0^{12.5-x_1}\int_{9.5}^{10}\Big[B_3(10+E(x_2|\mu_2(x_2|10)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-c(x_1,x_2)-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_5^{10}\int_{8.5}^{9.5}\Big[B_3(9+E(x_2|\mu_2(x_2|9)), 7.5+5)\left[\frac{1}{10}+\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_{12.5-x_1}^{5}\int_{8.5}^{9.5}\Big[B_3(9+E(x_2|\mu_2(x_2|9)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_0^{12.5-x_1}\int_{8.5}^{9.5}\Big[B_3(9+E(x_2|\mu_2(x_2|9)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-c(x_1,x_2)-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \ldots \\
& \int_0^1\int_{12.5-x_1}^{10}\int_{x_1^o}^{5.5}\Big[B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left[\frac{1}{10}+\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-s(x_1,x_2)\}\Big]dx_1dx_2dx_3
\end{aligned}
$$

$$
\begin{aligned}
& + \int_0^1\int_5^{12.5-x_1}\int_{x_1^o}^{5.5}\Big[B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left[\frac{1}{10}+\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-c(x_1,x_2)-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& + \int_0^1\int_0^5\int_{x_1^o}^{5.5}\Big[B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left[\frac{1}{10}-\left(\frac{x_1-7.5}{10-7.5}\right)\frac{1}{10}\right]\left(\frac{1}{5}\right)\{(x_1+x_2)x_3-c(x_1,x_2)-s(x_1,x_2)\}\Big]dx_1dx_2dx_3 \\
& - \int_0^1\int_0^{10}\int_5^{x_1^o} B_3(5+E(x_2|\mu_2(x_2|5)), 7.5+5)\left(\frac{1}{10}\right)\left(\frac{1}{5}\right)sc(x_1)\,dx_1dx_2dx_3
\end{aligned}
$$

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