International Journal of Information Technology & Decision Making
Vol. 14, No. 2 (2015) 339–374

DOI: 10.1142/S0219622014500886

Choice Manipulation Through Comparability in Markets with Verifiable Multi-Attribute Products

Debora Di Caprio

Department of Mathematics and Statistics
York University, 4700 Keele Street, M3J 1P3, Toronto, Canada
dicaper@mathstat.yorku.ca

Francisco Javier Santos Arteaga*

ICEI, Universidad Complutense de Madrid
Campus de Somosaguas, 28223 Madrid, Spain
fransant@ucm.es

Madjid Tavana

Business Systems and Analytics Department
Lindback Distinguished Chair of Information Systems and Decision Sciences
La Salle University, Philadelphia, PA 19141, USA

Business Information Systems Department
Faculty of Business Administration and Economics
University of Paderborn, D-33098 Paderborn, Germany
tavana@lasalle.edu

Published 15 December 2014

We illustrate how an information sender may use unverifiable signals regarding a set of substitute products located in an alternative market to manipulate the choices made by uninformed but perfectly rational decision makers (DMs) within the *verifiable* market where the information sender operates. We do so by defining an optimal information gathering structure for rational DMs who acquire information sequentially from a set of multidimensional products. The resulting strategic signaling environment delivers two main results that are illustrated numerically. First, in order for the sender to successfully manipulate the information gathering and choice behavior of DMs, he should release signals on characteristics that differ from their most preferred ones. Second, the capacity of the sender to manipulate the behavior of DMs depends *negatively* on his reputation regarding the expected value of the unobserved characteristics guaranteed to DMs within the market where he operates. Normative applications to online search environments conditioned by the provision of strategic reviews in social media are presented.

Keywords: Choice manipulation; sequential information gathering; multi-criteria decision making; reasoning under uncertainty; rationality.

*Corresponding author.

1. Introduction

The main objective of the current paper is to illustrate how an information sender may use unverifiable signals regarding a set of substitute products located in an alternative market to manipulate the choices made by rational decision makers (DMs) from a set of verifiable products. In order to do so, the paper deals with and merges two different research lines linked by their respective analyses of the optimal information acquisition processes of rational DMs. In particular, the information gathering algorithmic environment defined in this paper relates to the consumer choice literature and the decision theoretical branch of economics and operations research.

1.1. *On sequential information acquisition*

Consider the problem faced by a rational DM regarding what information to gather given a limited capacity to do so. The consumer choice literature studies this problem mainly from a psychological perspective, in particular when dealing with the strategic side of the information transmission process defining the choices made by consumers. In this case, this research line focuses on how the information given to DMs can be strategically designed to affect their final choice in a way that some predetermined options appear more attractive than others. Particularly relevant in this respect are the framing mechanisms that may be employed by the information sender,[1] together with the overemphasis placed by DMs on the information acquired over that expected.[2]

Some empirical phenomena identified by this research line include, among many others, the existence of context effects allowing for modifications in the preference formation process of DMs,[3] guided search mechanisms implemented through screening tools, used, for example, in electronic shopping,[4] and the generation and transmission of superfluous information,[5] with its corresponding additional processing requirements on the DM. These effects, together with the limited cognitive ability of DMs to assimilate information, allow for choice modifications to be induced through their information gathering process.

The previous research line sets the empirical base for the development of the corresponding search theoretical economic models that analyze fads and herds as rational phenomena, following the seminal works of Refs. 6 and 7. These models deal with the influence that informative signals have on the optimal (and rational) behavior of the DMs receiving them. However, studying the influence that information transmission processes, and signals in particular, have on the optimal information gathering behavior and choice structures of DMs remains outside the main scope of this research line, refer to Ref. 8 for a comprehensive review of the literature.

In summary, the effect that information transmission processes [signals] have on the choice [strategic] behavior of DMs within a given equilibrium system has been empirically [formally] analyzed by the consumer choice [economic] research line. The design and study of algorithmic information acquisition processes remains outside

their scope but within that of the operations research literature, which, at the same time, tends to overlook the strategic implications that different signaling and preference manipulation strategies have for the information gathering and choice behavior of DMs.

As a matter of fact, the management/operations research literature has been considering the optimal information gathering problem of firm managers for quite some time, refer to the seminal models of Refs. 9 and 10. However, and despite the inclusion of Bayesian learning mechanisms into their algorithms, even the most recent models omit the strategic choice effects inherent to the information transmission process, see Refs. 11–13 for a comprehensive review of the literature. This research line remains focused on the importance that search costs have in limiting the information processing capacity of generally risk neutral DMs when deciding whether to continue or stop their search within settings defined by the adoption of a given technology.

Finally, a substantial amount of the information sciences literature dealing with decision-making processes has highlighted the imprecision existing in the evaluation capacities of DMs within uncertain environments with limited (and imprecise) information.[14,15] This branch of the literature provides alternatives to standard expected utility theory and its main variants, such as prospect theory, which constitute the dominant approach to the subject of decision making under uncertainty among economists.[16,17] For example, Ref. 18 use quantum probabilities to measure the effect that shared mutual information has on the decision-making process of socially interacting agents.

1.2. *Strategic transmission of information in the economics literature*

Consider the basic behavioral principle defining bounded rationality: DMs are driven by heuristic (satisficing) mechanisms that simplify their information acquisition and assimilation processes.[19] This principle applies due to the inability of DMs to acquire and process all the information required to make fully informed rational decisions when subject to environmental and time constraints. Thus, given the limited capacity of DMs to analyze all the information available, the process of information transmission remains open to manipulation by self-interested agents. This type of asymmetric information environment was considered by Ref. 20, with his analysis being performed on products defined by a unique characteristics. A related though more recent model is presented by Ref. 21.

The main problem arising in asymmetric information environments is that the knowledge of the uninformed DMs can be modified and their preferences altered by interested third parties. This is due to the fact that the priorities and preferences of the information senders may differ from those of the agents acquiring the information.[22–24] There exist several examples of this strategic preference alteration process in the literature. For example, Ref. 25 show that uninformed agents are especially susceptible to information from interested and third parties when deciding whether

or not to purchase genetically modified food. A compensation mechanism that follows from the resulting strategic interactions among agents consists of lowering the reputation of manipulative senders after the DMs verify the information and recommendations received. Reference 26 describe the optimal way to transmit reputation information while being constrained by one main product characteristic.

1.3. *Strategic transmission of information in social media*

The literature on consumer (information acquisition and choice) behavior and the strategic one regarding information transmission in economics interact when analyzing the strategic transmission of information in social media.

Social media constitute a highly important strategic device determining consumer decisions. Online reviews have become increasingly popular as a mechanism to judge the quality of various products and services.[27] The transmission of online opinions through electronic word-of-mouth on websites such as Yelp and Tripadvisor is generally motivated by the reputation and social status achieved by the reviewer together with the enjoyment derived from helping and influencing other consumers.[28–30] A similar approach applies when considering books, with readers generally checking book reviews written by others.[31]

However, recent work demonstrates that the absence of reporting incentives leads to a biased set of reviews that may not reflect the true quality of a given product.[32] Reference 27 study both reporting incentives and biases observed in the Tripadvisor website and conclude that the ratings given by users partly reflect the difference between the true quality of the product and their prior expectations as inferred from previous reviews. In this regard, Ref. 33 examine netnographic evidence to analyze the way in which rankings such as those of TripAdvisor generate trust. A review of the literature on the models of trust proposed to assess the credibility of peers in an open multi-agent system environment is presented by Ref. 34.

At the same time, as the paradox of choice states, the information and cognitive overload faced by DMs when considering all reviews and evaluations available may lead to worse decisions.[35] In such a situation, DM tend to rely more on heuristics than rationality to arrive at information acquisition and purchase decisions. For example, a common heuristic mechanism is defined in terms of satisficing criteria, with DMs comparing each attribute value of a given product with a predetermined cut-of level and rejecting alternatives that do not meet it. In this regard, Ref. 36 illustrate how positive information plays an important role in the choice process of DMs when a relatively large number of aspects is considered, while negative experiences become more important when products are defined by a limited number of relevant characteristics.

1.4. *Main results obtained*

Consider a situation where a DM is allowed to check a number of characteristics from a set of multidimensional products. The search process must therefore be

defined both "between-attributes" and "between-alternatives".[37] We analyze and compare the cases where the information acquisition process is based on the possibility of collecting two and three pieces of information. The evolution of the information acquisition process described in the paper depends directly on the values of *all the characteristics observed previously by a DM and their potential combinations with those expected to be observed. This property limits the dimension of the products that may be considered by a DM and prevents the use of standard* dynamic programming techniques in the design of the algorithm.

At the same time, the decision theoretical model is extended to account for the reception of publicly observable signals within a standard Bayesian learning setting. The introduction of signals within the current multi-attribute information gathering framework allows for possible generalizations of Ref. 6 "restaurant" herding model with sequential moves and publicly observable signals to start being considered. Though important differences exist with respect to Banerjee's model, in particular regarding the quality of the signals received, the introduction of multidimensional products and additional decision variables allows us to account explicitly for the effects that different types of signals and learning processes have on the optimal information gathering behavior of DMs. Moreover, allowing for subjective interpretations of the signals by DMs and accounting for the resulting effects on their information acquisition and choice behavior would link the current paper directly to the branch of the information sciences literature analyzing decision-making processes.

We illustrate how the willingness to search of DMs is influenced by the reception of *credible signals* defined on the distribution of unverifiable characteristics within an alternative market. That is, we show how this type of signals can be issued so as to manipulate the choices made by uninformed but perfectly rational DMs within the *verifiable* market where the information sender operates. In particular, we present several numeral simulations illustrating how signals should be released on characteristics different from the most preferred ones among DMs. Otherwise, the information sender would be unable to manipulate the information gathering process of DMs in the desired direction. Moreover, it follows from our formal and numerical analysis that the capacity of the sender to manipulate the information gathering and choice behavior of DMs depends *negatively* on his reputation regarding the expected value of the unobserved characteristics guaranteed to DMs within the market where he operates.

We have already dealt with the topic of preference manipulation by an informed sender in a previous paper.[38] However, our previous setting differs significantly from the current one, with the main difference between both consisting of the ability of the sender to impose *any* product on the DM independently of the value of the realization of the DM's most preferred characteristic. Clearly, in order for these normative results to be considered, positive support is required illustrating whether or not DMs do actually compare the characteristics of products located in different markets and choose according to their relative value. In this regard, Refs. 39 and 40

presents empirical evidence providing the required support for our comparability assumption.

Regarding its applicability, the current paper provides a normative approach to the behavior of consumers in online purchasing settings, where Google constitutes the main search engine used by DMs.[41] The prioritized page order in which search results are displayed gives place to different markets on which DMs define their information acquisition processes. In this case, the current model applies a selection mechanism determining the information acquisition behavior of DMs when shifting among different displayed alternatives both within and among result pages as different observations are acquired. Thus, the most popular options displayed within the first result pages constitute a market on which different negative signals may be issued in the form of initially unverifiable reviews on a subset of product characteristics. These signals affect the information acquisition and choice behavior of DMs while being issued by agents whose reputation is exogenously determined online.

The paper proceeds as follows. Section 2 deals with the standard notation and basic assumptions needed to develop the model. Section 3 defines the expected search utility functions, while Sec. 4 introduces the corresponding Bayesian learning processes. Section 5 illustrates numerically the main results obtained. Section 6 analyzes the information acquisition process of DMs when considering products defined by three characteristics. Section 7 summarizes the main findings and suggests possible extensions.

2. Basic Notations and Main Assumptions

The algorithmic structure described through the paper must be redefined after each observation is gathered by the DM and recalculated in terms of *all previously observed variables, their sets of potential combinations with newly acquired observations and the corresponding expected payoffs*. This requisite leads to the low dimensionality of the model and can be justified by the limited information as similation and memory capacities with which DMs are endowed when comparing products within and between different markets. In addition, the literature usually concentrates on a small number of attributes when describing the products available to consumers, i.e., performance and cheapness in the case of Ref. 42 and variety and quality in the case of Ref. 43.

It may initially seem that allowing DMs to acquire only two or three observations per product imposes a considerable constraint on the set of information available online, despite the paradox of choice described in Introduction. It should be however noted that observations do not necessarily account for a unique property of the product, but a series of them whose combination defines a characteristic element to which a numerical evaluation and a subjective probability function have been assigned. For example, when considering the choice of a restaurant the first

characteristic could be defined as category and include location, number of stars, and menu complexity, while the second characteristic set may include the range of wines in the list together with prices and be defined as affordability, see, for example, Chapter 2 in Ref. 44.

The basic model described through Secs. 2 and 3 corresponds to the one introduced by Ref. 45, whose main components have been restated here.

Let X be a nonempty set and $\succsim$ a *preference relation* defined on X. A *utility function representing a preference relation* $\succsim$ *on* X is a function $u : X \to \mathbb{R}$ such that:

$$\forall\, x, y \in X, \quad x \succsim y \Longleftrightarrow u(x) \geq u(y). \tag{2.1}$$

The symbol $\geq$ denotes the standard partial order on the reals. When $X \subseteq \mathbb{R}$ and $\succsim$ coincides with $\geq$, we say that *u is a utility function on X.*

Let $\mathcal{G}$ denote the set of all products and fix $n \in N$. For every $i \leq n$, let X_i represent the set of all possible variants for the ith characteristic of any product in $\mathcal{G}$ and X stand for the Cartesian product $\prod_{i \leq n} X_i$. Thus, every product in $\mathcal{G}$ is described by an n-tuple $\langle x_1, \ldots, x_n \rangle$ in X. X_i is called the *ith characteristic factor space*, while X stands for the *characteristic space.*

Following the classical approach to information demand by economic agents,[46] we restrict our attention to the case where each X_i is identified with a compact and connected nondegenerate real subinterval of $[0, +\infty)$. The topology and the preference relation on each X_i are those induced by the standard Euclidean topology and the standard linear order $>$, respectively.

Without loss of generality, we work under the following assumptions.

Assumption 1. For every $i \leq n$, there exist $x_i^m, x_i^M > 0$, with $x_i^m \neq x_i^M$, such that $X_i = [x_i^m, x_i^M]$, where x_i^m and x_i^M are the minimum and maximum of X_i.

Assumption 2. The characteristic space X is endowed with the product topology τp and a strict preference relation $\succ$.

Assumption 3. There exist a continuous additive utility function u representing on X such that each one of its components $u_i : X_i \to \mathbb{R}$, where $i \leq n$, is a continuous utility function on X_i.[a]

Assumption 4. For every $i \leq n$, $\mu_i : X_i \to [0, 1]$ is a continuous probability density on X_i, whose support, the set $\{x_i \in X_i : \mu_i(x_i) \neq 0\}$, will be denoted by $\mathrm{supp}(\mu_i)$.[b]

[a] Let $\succsim$ be a preference relation on $\prod_{i \leq n} X_i$. A utility function $u : \prod_{i \leq n} X_i \to \mathbb{R}$ representing $\succsim$ on $\prod_{i \leq n} X_i$ is called *additive*[47] if there exist $u_i : X_i \to \mathbb{R}$, where $i \leq n$, such that $\forall \langle x_1, \ldots, x_n \rangle \in \prod_{i \leq n} X_i$, $u(\langle x_1, \ldots, x_n \rangle) = u_1(x_1) + \cdots + u_n(x_n)$.

[b] The results introduced through the paper are derived for continuous μ_1 and μ_2 probability densities. The remaining cases, quite similar to the continuous one, are left to the reader.

The probability densities $\mu_1, \ldots, \mu_n$ must be interpreted as the subjective "beliefs" of the DM. For $i \leq n$, $\mu_i(Y_i)$ is the subjective probability that a randomly observed product from $\mathcal{G}$ displays an element $x_i \in Y_i \subseteq X_i$ as its ith characteristic.[c]

Following the standard economic theory of choice under uncertainty, we assume that the DM elicits the ith certainty equivalent value induced by the subjective probability density μ_i and the utility function u_i as the reference point against which to compare the information collected on the ith characteristic of a certain product.

Given $i \leq n$, the *certainty equivalent of* μ_i *and* u_i, denoted by ce_i, is a characteristic in X_i that the DM is indifferent to accept in place of the expected one to be obtained through μ_i and u_i. That is, for every $i \leq n$, $\mathrm{ce}_i = u_i^{-1}(E_i)$, where E_i denotes the expected value of u_i. The existence and uniqueness of the ith certainty equivalent value ce_i are guaranteed by the continuity and strict increasingness of u_i, respectively.

3. Expected Search Utilities

The set of all products, $\mathcal{G}$, is identified with a compact and convex subset of the n-dimensional real space $\mathbb{R}^n$. In the simplest nontrivial scenario, $\mathcal{G}$ consists of at least two products and the DM is allowed to collect two pieces of information, not necessarily from the same product. That is, once the value of the first characteristic from one of the products becomes known to the DM, she has to decide whether to check the second characteristic from the same product, or to check the first characteristic from a different product. Henceforth, we denote by A and B the two products that can be randomly checked by the DM.

We show below that the decision of how to allocate the second available piece of information depends on two real-valued functions defined on X_1. The DM considers the sum $E_1 + E_2$, corresponding to the expected utility values of the pairs $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$, as the main reference value when calculating both these functions.

Assume that the DM has already checked the first characteristic from product A and that she uses her remaining information piece to observe the second characteristic from A. In this case, the expected utility gain over $E_1 + E_2$ varies with the value x_1 observed for the first characteristic. For every $x_1 \in X_1$, let

$$P^+(x_1) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \tag{3.1}$$

and

$$P^-(x_1) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}. \tag{3.2}$$

[c]The probability densities $\mu_1, \ldots, \mu_n$ are assumed to be independent. However, the current model allows for subjective correlations to be defined among different characteristic within a given product.

$P^+(x_1)$ and $P^-(x_1)$ define the set of values for the second x_2 characteristic from product A such that their combination with the observed first x_1 characteristic delivers a respectively higher or lower-equal utility than a randomly chosen product from $\mathcal{G}$.

Let $F : X_1 \to \mathbb{R}$ be defined by:

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)\,dx_2. \tag{3.3}$$

$F(x_1)$ describes the DM's expected utility derived from checking the second characteristic x_2 of product A after observing that the value of the first characteristic is given by x_1. Note that, if $u_2(x_2) + u_1(x_1) \leq E_1 + E_2$, then choosing a product from $\mathcal{G}$ randomly delivers an expected utility of $E_1 + E_2$ to the DM, which is higher than the expected utility obtained from choosing product A, that is, $u_2(x_2) + u_1(x_1)$.

Consider now the expected utility that the DM could gain over $E_1 + E_2$ if the second available piece of information is employed to observe the first characteristic from product B. For every $x_1 \in X_1$, let

$$Q^+(x_1) = \{y_1 \in X_1 \cap \operatorname{supp}(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \tag{3.4}$$

and

$$Q^-(x_1) = \{y_1 \in X_1 \cap \operatorname{supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}. \tag{3.5}$$

$Q^+(x_1)$ and $Q^-(x_1)$ define the set of values for the first y_1 characteristic from product B such that they deliver a respectively higher or lower-equal utility than the maximum between the observed first x_1 characteristic from product A and a randomly chosen product from $\mathcal{G}$.

Define $H : X_1 \to \mathbb{R}$ as follows:

$$H(x_1) \stackrel{\text{def}}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)\,dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)\,dy_1. \tag{3.6}$$

$H(x_1)$ describes the expected utility obtained from checking the first characteristic y_1 of product B after having already observed the value of the first characteristic x_1 from product A. If $u_1(y_1) \leq \max\{u_1(x_1), E_1\}$, then the DM must choose between A and a randomly chosen product from $\mathcal{G}$, delivering an expected utility of E_1.

Finally, note that the domain of both F and H is the support of μ_1.

3.1. *Existence of optimal thresholds*

Clearly, the expected utility functions $F(x_1)$ and $H(x_1)$ guide the DM's optimal information gathering process. Assume that the information search on product A has produced x_1 as first result. Then, the DM will choose to continue checking product A or switching to product B depending on which function, either $F(x_1)$ or $H(x_1)$, takes the highest value at x_1. It may also happen that she is indifferent between continuing

with A and switching to B. It is reasonable to think of these indifference values as optimal information gathering thresholds. Thus, X_1 turns out to be partitioned in subintervals whose values induce the DM either to continue checking the initial product A or to switch and start checking B.[d]

4. Signals and Learning Processes

The current section defines the optimal information gathering behavior of DMs when an information sender issues credible signals regarding changes in the expected characteristics of substitute products existing in markets that cannot be directly observed by the DM. The [information] sender is unable to manipulate the choices made by DMs directly due to the verifiability of the characteristics being displayed. The sender may however affect the information gathering and choice processes of DMs by releasing unverifiable signals regarding substitute products located in alternative markets. This would actually be the case if DMs allow for the reference points of the observable characteristics to be compared with those of unobservable products when gathering both observations.

We will study four different scenarios depending on the type of signal issued by the sender, either positive or negative, and the characteristic to which signals relate.

First, we consider the release of positive and negative signals by the information sender. We start by analyzing the effects that positive signals regarding the distribution of characteristics on X_i, $i = 1, 2$, and the resulting learning process have on the optimal information gathering behavior of rational DMs. Consider, as the basic reference case and without loss of generality, the optimal information gathering behavior of the DM when uniform probabilities are assumed on both X_1 and X_2.[e]

We will assume that receiving a credible positive signal, θ, regarding the distribution of characteristics on X_i implies that the probability mass accumulated on the lower half of the distribution halves. At the same time, the probability mass eliminated from the lower half of the distribution is shifted to the upper one. Thus, given the distribution of X_i characteristics defined by $\mu_i(x_i) = \frac{1}{\beta-\alpha}$ for $x_i \in [\alpha, \beta]$, with $\alpha, \beta \geq 0$ and $\alpha < \beta$, the corresponding conditional density function is given by

$$\pi(\overline{\theta|x_i}) = \begin{cases} \dfrac{3}{2(\beta-\alpha)} & \text{if } x_i \in \left(\dfrac{\alpha+\beta}{2}, \beta\right], \\ \dfrac{1}{2(\beta-\alpha)} & \text{if } x_i \in \left[\alpha, \dfrac{\alpha+\beta}{2}\right], \end{cases} \quad i = 1, 2. \tag{4.1}$$

[d]Reference 45 illustrate how the existence of optimal threshold values, or reversing points, in the DM's information gathering process can be guaranteed under common nonpathological assumptions. For example, it can be easily shown that $H(x_1^M) \leq F(x_1^M)$, with $H(x_1^M) = F(x_1^M)$ if and only if $u_1(x_1^M) + u_2(x_2^m) \geq E_1 + E_2$. Therefore, $u_1(x_2^M) + u_2(x_2^m) < E_1 + E_2$ suffices to guarantee the existence of at least one threshold value whenever $P^+(x_1^m) = \emptyset$.

[e]Even though we will only study the effects resulting from signals for the uniform density case defined in the paper, the analysis could be generalized to any other density function whose probability mass is redistributed to generate either higher or lower E_1 and E_2 values.

After receiving a positive signal, rational DMs update their initial beliefs, given by $\mu_i(x_i)$, following Bayes' rule. Therefore, if a signal is received, i.e., $\theta = 1$, the updated beliefs of DMs will be given by[f]

$$\mu_i(\overline{x_i|\theta = 1}) = \frac{\pi(\overline{\theta|x_i})\mu_i(x_i)}{\int_{X_i} \pi(\overline{\theta|x_i})\mu_i(x_i)\,dx_i}, \quad i = 1, 2. \tag{4.2}$$

Negative signals shift the probability mass in the opposite direction, that is, towards the lower end of the distribution. This case follows trivially from the positive setting, with the only modification taking place through the conditional density function, which in the negative signal case would be given by

$$\pi(\underline{\theta|x_i}) = \begin{cases} \dfrac{1}{2(\beta-\alpha)} & \text{if } x_i \in \left(\dfrac{\alpha+\beta}{2}, \beta\right], \\ \dfrac{3}{2(\beta-\alpha)} & \text{if } x_i \in \left[\alpha, \dfrac{\alpha+\beta}{2}\right], \end{cases} \quad i = 1, 2 \tag{4.3}$$

with the same Bayesian updating process following as in the positive case. The updated $\mu_i(x_i)$ density generated by the signal(s) modifies the $F(x_1)$ and $H(x_1)$ functions through the new induced values of the corresponding E_i variables, with $i = 1, 2$.

The second type of scenario under consideration relates to the variable on which signals are issued. There are two possibilities, signals may be released on either X_1 or X_2. In both cases, DMs are unable to verify the validity of the signals, but these may affect their information gathering processes. This scenario is designed to differentiate between the characteristics that are immediately observable by the DM, such as X_1, and the experience type quality of X_2, which can only be verified if the DM sticks to the initially observed product.

The classification described above allows us to define new $F(\cdot)$ and $H(\cdot)$ functions based on the type of signal observed and the characteristic on which it is defined. The following definitions will be used through the next section when introducing numerical simulations to analyze the effects of the signals on the behavior of DMs.

The characteristics conditioned by the reception of positive and negative signals will be denoted by $\overline{x_i}$ and $\underline{x_i}$, $i = 1, 2$, respectively. Similarly, the corresponding expected utilities will be denoted by $\overline{E_1}$ and $\overline{E_2}$. It should be emphasized that these utilities correspond to the products located in an alternative [unverifiable] market, not to the verifiable ones located within the market on which the sender operates.

[f]This process can be assumed to continue as rational DMs keep on updating their beliefs using Bayes' rule after receiving further signals. For example, a second signal, providing DMs with the same qualitative information, i.e., $\theta = 2$, would lead to a second Bayesian updating process and the following distribution of beliefs on X_i

$$\mu_i(\overline{x_i|\theta = 2}) = \frac{\pi(\overline{\theta|x_i})\mu_i(\overline{x_i|\theta = 1})}{\int_{X_i} \pi(\overline{\theta|x_i})\mu_i(\overline{x_i|\theta = 1})\,dx_i}, \quad i = 1, 2.$$

Receiving a positive signal on the first characteristic of a set of products located in an alternative market leads to the following updated $P^+(\cdot)$ and $P^-(\cdot)$ sets

$$P^+(\overline{x_1}) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) > \overline{E_1} + E_2 - u_1(x_1)\} \tag{4.4}$$

and

$$P^-(\overline{x_1}) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) \leq \overline{E_1} + E_2 - u_1(x_1)\}. \tag{4.5}$$

An identical logic applies when accounting for the reception of a negative signal, $\underline{x_1}$, or a signal on X_2. Clearly, the exact same effect is induced through the $P^+(\cdot)$ and $P^-(\cdot)$ sets on the $F(\cdot)$ function after a signal is observed on either X_1 or X_2

$$F(\overline{x_1}) \stackrel{\text{def}}{=} \int_{P^+(\overline{x_1})} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(\overline{x_1})} \mu_2(x_2)(E_1 + E_2)\,dx_2. \tag{4.6}$$

Note that, even though one may intuitively expect positive and negative signals to have opposite effects on the $F(\cdot)$ function, this is actually *not* the case, as we will illustrate numerically in the next section. However, when considering the $H(\cdot)$ function, we will observe that positive and negative signals on X_1 have opposite effects through the corresponding $Q^+(\cdot)$ and $Q^-(\cdot)$ sets

$$Q^+(\overline{x_1}) = \{y_1 \in X_1 \cap \mathrm{supp}(\mu_1) : u_1(y_1) > \max\{u_1(x_1), \overline{E_1}\}\} \tag{4.7}$$

and

$$Q^-(\overline{x_1}) = \{y_1 \in X_1 \cap \mathrm{supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), \overline{E_1}\}\}, \tag{4.8}$$

with

$$H(\overline{x_1}) \stackrel{\text{def}}{=} \int_{Q^+(\overline{x_1})} \mu_1(y_1)(u_1(y_1) + E_2)\,dy_1 + \int_{Q^-(\overline{x_1})} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)\,dy_1, \tag{4.9}$$

and similarly for $\underline{x_1}$. We will illustrate and elaborate on these results through the numerical simulations of the next section. As already stated, E_1 remains unmodified within the corresponding definitions of the $F(\cdot)$ and $H(\cdot)$ functions, as it is part of the unsignaled market where the sender operates. Only the integration limits, which are determined by the expectations about the alternative market, are modified by the signals issued by the sender.

Consider now the reception of either a positive or a negative signal on the second characteristic of a set of products located in an alternative market. As stated above, the effects on the $F(\cdot)$ function that take place through modifications of the $P^+(\cdot)$ and $P^-(\cdot)$ sets are identical to those derived from observing signals on X_1. We will make use of the following notation to distinguish between both cases, though it should be clear that the variable defining the information gathering behavior of the DM remains the observed X_1 characteristic within the so-called unsignaled market

$$P^+(x_1|\overline{x_2}) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) > E_1 + \overline{E_2} - u_1(x_1)\} \tag{4.10}$$

and

$$P^{-}(x_1|\overline{x_2}) = \{x_2 \in X_2 \cap \mathrm{supp}(\mu_2) : u_2(x_2) \leq E_1 + \overline{E_2} - u_1(x_1)\}, \tag{4.11}$$

with

$$F(x_1|\overline{x_2}) \stackrel{\mathrm{def}}{=} \int_{P^{+}(x_1|\overline{x_2})} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^{-}(x_1|\overline{x_2})} \mu_2(x_2)(E_1 + E_2)\,dx_2, \tag{4.12}$$

and similarly for $(x_1|\underline{x_2})$. As was the case in the X_1 scenario, the value of E_2 defined within $F(\cdot)$ that determines the corresponding expected utility when searching within the unsignaled market is not modified.

Finally, it should be evident that signals issued on the value of X_2 expected to be obtained in an alternative market do not have any effect on the $Q^{+}(\cdot)$ and $Q^{-}(\cdot)$ sets and, therefore, on the corresponding $H(\cdot)$ functions.

Even though we could formally analyze the effect that signals have on the $F(\cdot)$ and $H(\cdot)$ functions in the current section, we will provide a more intuitive presentation via numerical simulations in the following one. Besides providing some intuition regarding the evaluation of the shifts in the $F(\cdot)$ and $H(\cdot)$ functions, we will study the effect that signals have on the behavior of the optimal thresholds, which cannot be derived analytically as it depends on the relative strength of the shifts taking place in the corresponding functions.

5. Manipulability and Market Comparability

This section presents a numerical analysis of the comparative signaling scenario described in the previous one. We will analyze the effects on the optimal information gathering and choice behavior of DMs derived from the unverifiable signals issued by the sender regarding the probability distributions defined on the characteristics of products located in alternative markets. Given the analysis presented in the previous section, we will assume that DMs update their expectations after observing a credible signal and modify their original $F(x_1)$ and $H(x_1)$ functions accordingly.

That is, signals issued regarding the characteristics of products whose expected utility is modified in an alternative market affect the information acquisition behavior of DMs within the current verifiable market. The expected utility of the products located in the market where the information sender operates remains unchanged, but the comparability between products located in different markets modifies the behavior of DMs.[5,28] We have studied formally the framework where credible signals are issued on the market where the DMs operate in Refs. 48 and 49. However, the models described in these papers adapt better to an environment dealing with the introduction of technologically superior products by competing firms, such as that of Ref. 50. In this case, the resulting submarkets arising as a result of the signals define the information acquisition behavior of DMs based on the characteristics of the products expected to be obtained within them.

On the other hand, when relying on social media, DMs do not only consider the characteristics of the products located within the market where the information sender operates, but also alternative ones on which information is issued. It is the change in the expected utility derived from shifting to an alternative market what generates modifications in the threshold values of DMs within the market where they are performing their search.

For example, when choosing a restaurant within a given area of a city that the DM is visiting, we are assuming that the DM does not only consider the social media reviews of the restaurant where he could have lunch right away, but that opinions on alternative ones will have an effect on the resulting choice. While positive opinions will generally lead to stricter continuation criteria on the initial alternative, the effect of negative initially unverifiable ones may be considered when DMs face a small number of characteristics per alternative.[36] Note that the information sender wants the DM to acquire a second piece of information on its restaurant as opposed to directly rejecting it and continuing with another option within the same or a close by area to where the restaurant is located.

Note also that the initial (sub)market faced by DMs consists of the results obtained from the corresponding online search engine used to retrieve information. Online rating websites and other social media provide additional information not only about the option being considered by the DM but also regarding other alternatives. It could also be assumed that the initial market faced by DMs is composed by alternatives that have received a minimum number of reviews. In this regard, all alternatives are initially equivalent within the market and therefore, unless a subjective bias is imposed on the information acquisition behavior of DMs, all the alternatives within a given market have the same probability of being chosen to start acquiring information on.

Consider, as the basic reference case, the optimal information acquisition behavior following from a standard risk neutral utility function when uniform probabilities are assumed on both X_1 and X_2. DMs will be assumed to have a well-defined preference order between the characteristics composing the products. As a result, the first characteristic will be more important for DMs and, therefore, lead to a higher expected utility than the second one. The reference risk neutral case is described by the following parameter values

(i) Characteristic spaces: $X_1 = [5, 10]$, $X_2 = [0, 10]$,
(ii) Utility functions: $u_1(x_1) = x_1$, $u_2(x_2) = x_2$,
(iii) Probability densities: $\forall\, x_1 \in X_1$, $\mu_1(x_1) = \frac{1}{5}$, $\forall\, x_2 \in X_2$, $\mu_2(x_2) = \frac{1}{10}$.

This case is represented in Fig. 1 by the functions H and $F(ns)$. In all figures, the horizontal axis represents the set of possible x_1 realizations that may be observed by the DM, with the corresponding subjective expected utility values defined on the vertical axis. Positive and negative signals will be respectively denoted by gs and bs within the corresponding $F(\cdot)$ and $H(\cdot)$ functions, while the number of signals

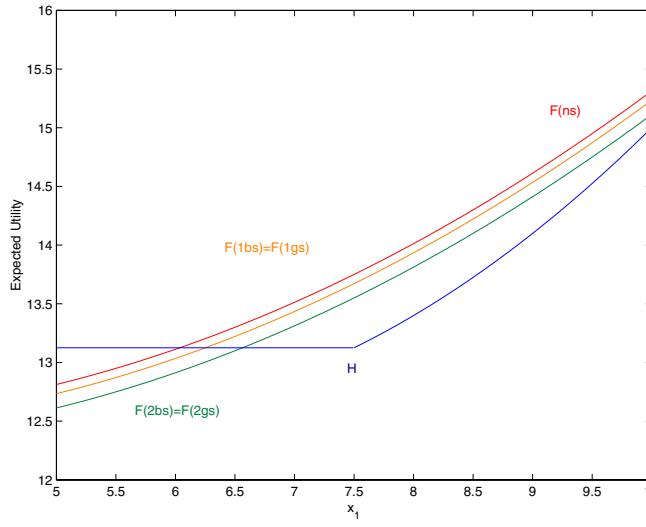

Fig. 1. Positive and negative signals on X_2 with full credibility and reputation.

observed is indicated before their type. In order to facilitate comparisons among the threshold values generated by different types and number of signals the support of the corresponding signal-induced probability functions has been kept unchanged through all the simulations.

It should be highlighted that assuming risk averse DMs would modify the threshold values derived through the numerical analysis, though the main results obtained would remain qualitatively unchanged. The effects that different degrees of risk aversion have on the threshold values computed by the corresponding DMs are illustrated numerically in Ref. 45.

5.1. *Signals on X_2*

We proceed now to illustrate the effect that signals, either positive or negative, regarding the distribution of characteristics on X_2 in an alternative market and the resulting learning process have on the optimal information gathering behavior of rational DMs. Through this section, we will denote the value of the E_i variable in the alternative market by $\hat{E}_i$, with $i = 1, 2$.

The first immediate result that follows from the numerical simulations presented in Fig. 1 is the identical effect that positive and negative signals have on the expected search utility payofs received by DMs.[g] In both cases, the effect on the expected utility defined by $F(\cdot)$ is negative, resulting in what we will refer to as a decrease in

[g] The identical change induced by positive and negative signals on $F(\cdot)$ is due to the linearity of the utilities defined on the characteristic spaces as well as to the symmetry of the domain and the uniformity of the probability function defined on X_2.

search aversion. That is, the area where the $H(\cdot)$ function remains above the $F(\cdot)$ one increases for relatively low x_1 realizations. As a result, DMs would require relatively higher realizations from the first characteristic space in order to continue gathering information on the observed product. Clearly, changes in the value of $\hat{E}_2$ do not have any effect on the $H(\cdot)$ function, which remains unaltered.

The main implication derived from this basic result is quite a strong one: *a fully credible sender cannot manipulate the DM into accepting whatever product is initially displayed.* In the current setting, credibility determines the ability of the sender to convince the DM to expect the certainty equivalent product from a random purchase. The formal intuition behind this result is indeed quite simple. A positive signal released on the distribution of X_2 within an alternative market, i.e., an increase in $\hat{E}_2$, decreases the cardinality [size] of $P^+(x_1)$ and increases that of $P^-(x_1)$. That is, $P^+(x_1|\overline{x_2}) \subset P^+(x_1)$ and $P^-(x_1) \subset P^+(x_1|\overline{x_2})$. This is due to the higher expected utility derived from acquiring information on the alternative market, given by $E_1 + \hat{E}_2$.

Thus, a higher realization of X_2 is now required to accept a product within the unsignaled verifiable market operated by the sender. Otherwise, *the certainty equivalent product defined by $E_1 + E_2$ is expected to be received.* As a result, the expected search utility from continuing gathering information on the observed product decreases. It may seem, following the same type of reasoning, that a negative signal on $\hat{E}_2$ should increase the expected search utility derived from $F(\cdot)$. However, this is actually *not* the case. A decrease in $\hat{E}_2$ leads the DM to accept products whose expected utility is below $E_1 + E_2$ but above $E_1 + \hat{E}_2$. In this case, the size of the $P^+(x_1)$ set increases but to include products leading to a lower expected utility than that derived from a random choice within the unsignaled observable market. Consequently, the $F(\cdot)$ function shifts downwards.

Given the previous result, the obvious direction to follow consists of modifying the credibility of the sender. In this respect, we will eliminate all credibility from the sender *when the DM must decide whether or not to purchase a product randomly from the unsignaled market within which the sender operates.* That is, the DM will be reluctant to purchase a product unless he can observe at least one characteristic leading to a higher than the certainty equivalent utility. Thus, if the characteristics observed do not lead to a product satisfying this requirement, no purchase will take place, which results in an expected utility of zero.

The lack of credibility on the side of the information sender leads to the following definitions of the expected search utilities. $F : X_1 \to \mathbb{R}$ will be defined by

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2, \tag{5.1}$$

emphasizing the fact that any realizations contained within the set $P^-(x_1)$ result in an expected utility of zero. $H : X_1 \to \mathbb{R}$ must be divided in two different parts, introducing a discontinuity in the resulting function. That is, whenever $x_1 \geq \mathrm{ce}_1$

we have

$$H(x_1) \stackrel{\text{def}}{=} \int_{x_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_2)\,dy_1 + \int_{x_1^m}^{x_1} \mu_1(y_1)(u_1(x_1) + E_2)\,dy_1. \tag{5.2}$$

In this case, the DM has observed a realization of x_1 located above the certainty equivalent value. This fact guarantees the purchase of the initial (partially observed) product if the realization from the new product does not provide a suffciently high utility. However, for all $x_1 < \text{ce}_1$ we have

$$H(x_1) \stackrel{\text{def}}{=} \int_{\text{ce}_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_2)\,dy_1. \tag{5.3}$$

In this case, the DM will only consider the purchase of the new partially observed product if its first characteristic provides a higher utility than the certainty equivalent one.

We will also be considering the *reputation* of the sender, which will be assumed to modify both the $E_1 + E_2$ payoffs within $F(\cdot)$ and E_2 within $H(\cdot)$. The reputation of a sender is a variable $\gamma \in [0, 1]$ that weighs the values of E_1 and E_2 expected to be obtained in the unsignaled market. The corresponding function $F : X_1 \to \mathbb{R}$ defined for a given reputation level γ is therefore given by

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)\gamma(E_1 + E_2)\,dx_2. \tag{5.4}$$

Similarly to the lack of credibility case, modifying the reputation of the information sender generates a discontinuity in the function $H : X_1 \to \mathbb{R}$, which must be again defined in terms of the initial realization observed by the DM. In this case, when $x_1 \geq \text{ce}_1$ we have

$$H(x_1) \stackrel{\text{def}}{=} \int_{x_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + \gamma E_2)\,dy_1 + \int_{x_1^m}^{x_1} \mu_1(y_1)(u_1(x_1) + \gamma E_2)\,dy_1, \tag{5.5}$$

while for $x_1 < \text{ce}_1$ we have

$$H(x_1) \stackrel{\text{def}}{=} \int_{\text{ce}_1}^{x_1^M} \mu_1(y_1)(u_1(y_1) + \gamma E_2)\,dy_1 + \int_{x_1^m}^{\text{ce}_1} \mu_1(y_1)\gamma(E_1 + E_2)\,dy_1. \tag{5.6}$$

The intuition is similar to that defining the functions $F(\cdot)$ and $H(\cdot)$ under full credibility and reputation. Note however that reputation and credibility represent different concepts. That is, a well-reputed sender is not necessarily credible when purchasing a product randomly.[h] Reputation affects *all* the expected values defined on the individual characteristics of the product guaranteed by the sender. Thus,

[h] Refer, for example, to the *Managerial Decision Making and Disruptive Technological Change* section in Ref. 51 for several examples illustrating this point. The author shows how verifiability is required by the consumer base of different firms when purchasing technologically updated versions of a product. Thus, even though a given firm may have a good reputation among its consumer base, verifiability may still be required before purchasing a product, i.e., credibility is not necessarily guaranteed *ad hoc*.

reputation affects *all* unobserved characteristics while the lack of credibility prevents DMs from purchasing products randomly.

All potential scenarios are compared in Fig. 2, with the lack of credibility setting being described by the nc functions while the halved reputation one that we will analyze below is denoted by rp. Note how the absence of credibility forces DMs to acquire as many initial observations as possible to try guaranteeing a product with a relatively high first characteristic. At the same time, halving the reputation of the information sender leads DMs to try guaranteeing a sufciently high product on both characteristics. In this case, the incentives of DMs to continue acquiring information on the initial product observed are even higher than under full credibility, where a relatively better product is expected to be obtained from a random purchase.

It should be noted that all these scenarios emphasize the reference role played by the expectations of DMs when determining their information acquisition process. The information sender concentrates on his own capacity to impose the product initially observed to the DM independently of the realization of its first characteristic. As we highlight in the conclusion, the DM may still require both characteristics to provide a utility higher than that of the certainty equivalent product before purchasing the product observed. However, issuing signals on products located in alternative markets would allow the information sender to manipulate the DM into continuing observing products that would have otherwise been immediately rejected.

Consider now the previous risk neutral environment with negative signals issued on $\hat{E}_2$ and a limit case where $\hat{E}_2 = 0$ [denoted by $F(\lim)$], which is presented in Fig. 3. As in the fully credible case, $H(\cdot)$ remains unaltered independently of the signals received on $\hat{E}_2$. However, within the current setting, an unobservable random

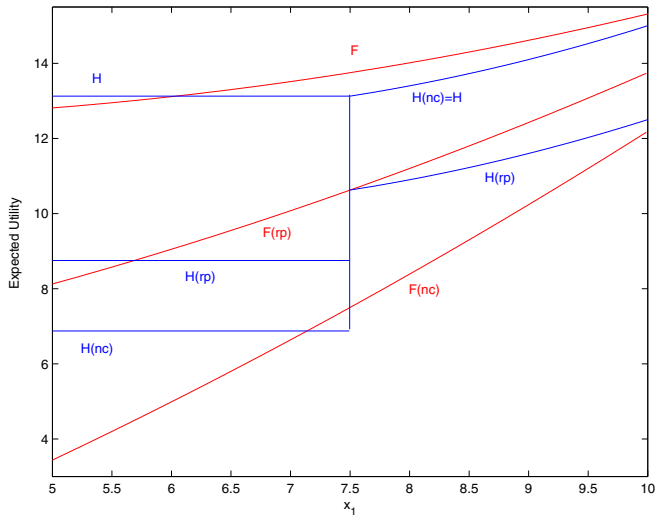

Fig. 2. Full credibility and reputation, lack of credibility (nc) and halved reputation (rp) scenarios absent signals.

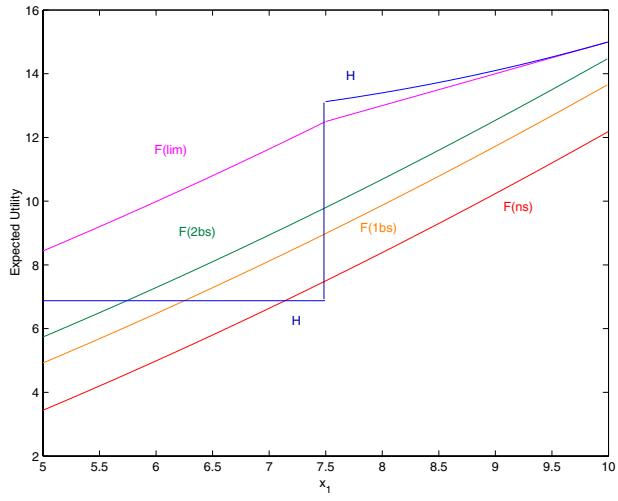

Fig. 3. Negative signals on X_2 with zero credibility and full reputation.

purchase assigns a value of zero to $E_1 + E_2$ within both $F(\cdot)$ and $H(\cdot)$.[i] The simulations illustrate how refraining from purchasing a product randomly has a larger negative impact on the $F(\cdot)$ function when moving through the domain of X_1 located above the certainty equivalent value, i.e., from ce_1 to x_1^M. Clearly, as illustrated in Fig. 2, all the X_1 realizations within this domain guarantee a value of $H(\cdot)$ identical to that of the full reputation environment, while $F(\cdot)$ still faces realizations below the certainty equivalent value, due to the yet unobserved X_2 characteristic, leading to a payoff of zero.

The main conclusion to be derived from these simulations is that a decrease in credibility is required for the sender to manipulate the information gathering process of the DM in the desired direction. That is, if the sender wants the DM to consider only the product observed initially within the unsignaled market while ignoring any other search possibilities, then unverifiable signals released on $\hat{E}_2$ become increasingly efficient as his credibility decreases. This is due to the fact that, within the current setting, the increase in the number of realizations contained in $P^+(x_1|\overline{x_2}) \supset P^+(x_1)$ does actually lead to an upper shift of $F(\cdot)$, since the corresponding values deliver a higher than zero expected utility.

Finally, we halve the value of the sender's reputation from an initial $\gamma = 1$. In this case, if the sender has full credibility but lacks reputation, DMs may purchase a product randomly but expect a lower utility derived from such a purchase than the one guaranteed by the sender. This is the case represented in the set of simulations within Fig. 4. Clearly, and due to the exact same reasons outlined in the previous setting, a decrease in $\hat{E}_2$ leads to an upper shift of the $F(\cdot)$ function while leaving $H(\cdot)$ unaffected. At the same time, the ability of DMs to eliminate all uncertainty

[i]It should be noted that the vertical lines joining the discontinuous pieces of the corresponding $H(\cdot)$ functions have been added to allow for a more intuitive graphical presentation and to simplify comparisons between markets.

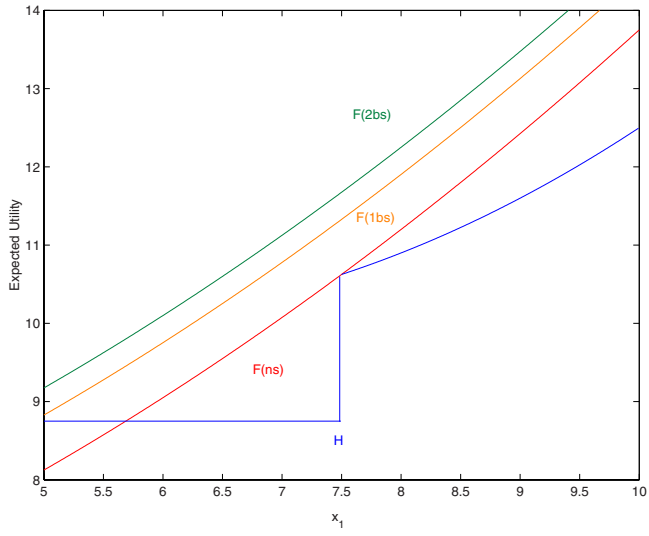

Fig. 4. Negative signals on X_2 with full credibility and halved reputation.

from $F(\cdot)$ by gathering a second observation leads to an increase in search aversion as $H(\cdot)$ shifts downwards due to the decrease in the value of E_2 caused by the loss of reputation. Thus, an immediate conclusion derived from our normative information gathering framework is that *the ability of a sender to impose any initially displayed product as a plausible choice among rational DMs decreases in his reputation.* That is, a decrease in the sender's reputation allows for negative signals about $\hat{E}_2$ to have a positive effect on the $F(\cdot)$ function defined within the unsignaled market. It should be emphasized that we are assuming the signals issued by a sender to be fully credible, an assumption that, if relaxed, would weaken the strength of the shift in $F(\cdot)$, leading to DMs that are harder to manipulate.

The previous result may initially seem counterintuitive, a thought that vanishes upon further reflection. Note that, for the sender to increase the expected search utility derived from continuing gathering information on the product observed within the unsignaled market, the certainty equivalent product expected to be obtained from the signaled one must allow for an improvement upon the certainty equivalent product offered by the sender. In the full credibility case, this requirement cannot be met. That is, negative signals released on the alternative market make the DM eager to accept products that would otherwise be located below the certainty equivalent offered by the sender, which decreases his expected search utility from continuing gathering information on an initially observed product. However, if the reputation of the sender decreases to the point where such signals induce the DM to accept products that improve upon the certainty equivalent value expected to be obtained within the unsignaled market, then verifiability leads to an increase in the expected search utility derived from $F(\cdot)$. Note that the fundamental force behind this effect is the ability of the DM to verify the second characteristic of the product

under consideration, while being unable to observe any characteristic from the products located within the signaled market. Thus, despite his ability to verify the characteristics of the product being purchased, we have seen how the DM may be induced to consider purchasing any product initially ofered by the sender, independently of the value of its X_1 realization.[j]

We are not the first ones to consider the potential positive effects derived from negative reviews. Reference 52 show how negative publicity may increase the likelihood of purchase by increasing awareness about the corresponding product. Thus, as argued by these authors, negative publicity has different effects depending on whether the product being considered is an already established or a relatively unknown one. In our setting, as illustrated above, the capacity of information senders to impose their respective products through negative signals follows directly from their (lower) reputation.

5.2. *Information acquisition costs*

Through the previous analysis we have omitted the information acquisition costs that are generally assumed on the side of DMs within the operations research and economics literatures. However, information acquisition costs can be easily accounted for within the current formal environment. The search costs incurred when observing the second characteristic of a given product are generally assumed to be lower than those derived from starting acquiring information on the first characteristic of a new product. Thus, given this difference in search costs between characteristics and among products, we analyze how different information acquisition costs affect the information gathering and choice behavior of DMs.

In order to illustrate this point, we denote by c_1 and c_2 the costs of acquiring information on the first and second characteristics from a product, respectively. We assume that $c_1 > c_2$, in order to account for the lower costs incurred when observing the second characteristic of a product relative to the first one. The resulting functions $F(\cdot)$ and $H(\cdot)$ absent signals and given full credibility and reputation on the side of the information sender are given by

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2) - c_2)\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2 - c_2)\,dx_2 \tag{5.7}$$

and

$$\begin{aligned} H(x_1) \stackrel{\text{def}}{=} & \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2 - c_1)\,dy_1 \\ & + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2 - c_1)\,dy_1. \end{aligned} \tag{5.8}$$

[j] Of course, the final choice depends on whether or not the product observed provides a higher utility than the certainty equivalent one to the DM.

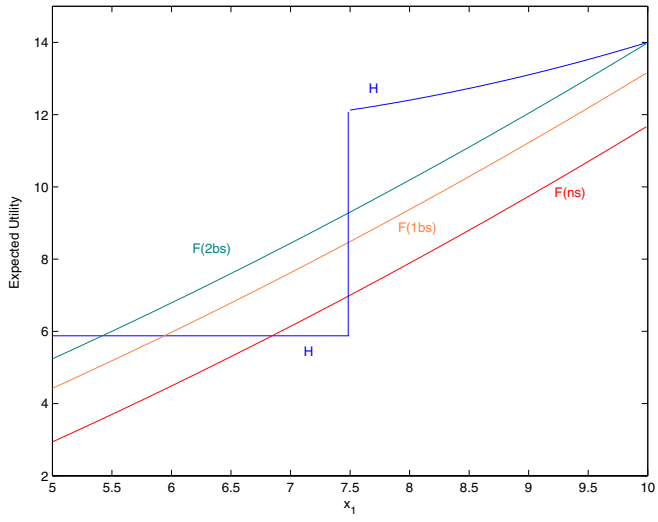

Fig. 5. Negative signals on X_2 with zero credibility, full reputation and search costs: $c_1 = 1$ and $c_2 = 0.5$.

The immediate intuition derived from these equations in search theoretical terms is the emergence of a market captivity effect that favors the product observed initially. This effect follows from the relatively larger cost of shifting between market products when compared to the cost from continuing acquiring information on the initial product. This is illustrated in Fig. 5, where search costs given by $c_1 = 1$, $c_2 = 0.5$ have been introduced within the setting described in Fig. 3, i.e., negative signals issued on X_2 with zero credibility and full reputation on the side of the information sender. Figure 5 shows the downward shift in the corresponding functions $F(\cdot)$ and $H(\cdot)$ with respect to those described in Fig. 3. Note the relatively larger shift of $H(\cdot)$ due to the higher costs incurred when observing a new product. Thus, the larger search costs being incurred when shifting among products will amplify the effect of negative signals on the information acquisition behavior of DMs and simplify the manipulation process of the information sender. A similar effect would be obtained when considering products composed of three characteristics, as can be intuitively inferred from the analysis presented in Sec. 6.

5.3. *Signals on X_1*

Consider the decision theoretical model described in the previous section but with signals released on the value of E_1 in an alternative market. This scenario is introduced to study how should the DM behave when signals are observed on his most preferred characteristic, which affects both expected search utilities directly through $\hat{E}_1$.

The basic effects derived from positive and negative signals issued on X_1 are similar to those described in the X_2 scenario. Therefore, the corresponding analysis is

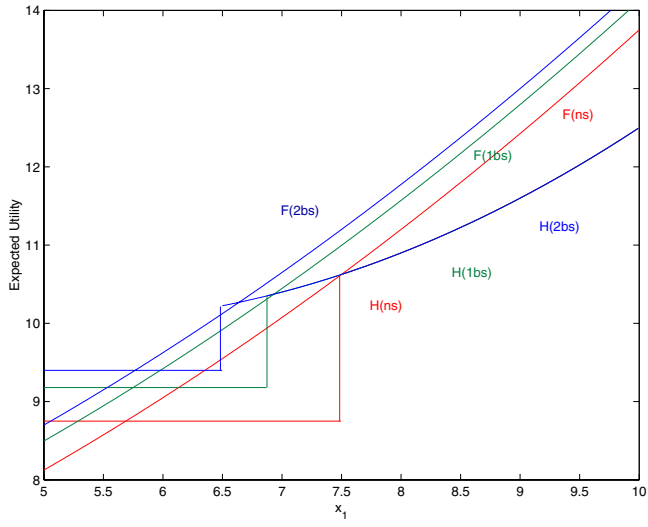

Fig. 6. Negative signals on X_1 with full credibility and halved reputation.

omitted in order to concentrate on the differences between both settings arising from changes in the reputation and credibility of the sender.

Figure 6 represents a normative risk neutral environment where negative signals are released on $\hat{E}_1$ and $\gamma = 1/2$. As in the X_2 case, this decrement in the reputation of the sender leads the DM to expect half the value of E_1 and E_2 when these characteristics cannot be directly verified. However, when comparing the current setting with the previous one on X_2, we observe how verifiability prevents full manipulability. This result follows from the differences in search aversion between the previous X_2 case, where search aversion was induced $\forall\, x_1 \in X_1$, and the current one on X_1, where the respective information gathering areas remain almost unchanged. Clearly, the changes induced by $\hat{E}_1$ on the $H(\cdot)$ function are what prevents the signaling process from fully manipulating the DM into accepting the product initially displayed by the sender. It is also obvious that the strength of the signals required to manipulate the choice and search processes of DMs is higher than in the X_2 case. This is due to the fact that the $\hat{E}_1$ value induced by the signals is much closer to the original [unsignaled] one than that induced on X_2, whose wider domain allows for larger expected value improvements.

The main conclusion following from these simulations is, however, the fact that the changes induced by the signals on $H(\cdot)$ decrease the ability of the sender to manipulate the choices made by DMs. This result is particularly evident in the zero credibility case presented in Fig. 7, where the sender is unable to shift the information gathering process of the DM in the desired direction.

Finally, note that, in both these previous settings, negative signals shift the $H(\cdot)$ function upwards. This is due to the same reason why identical signals would not

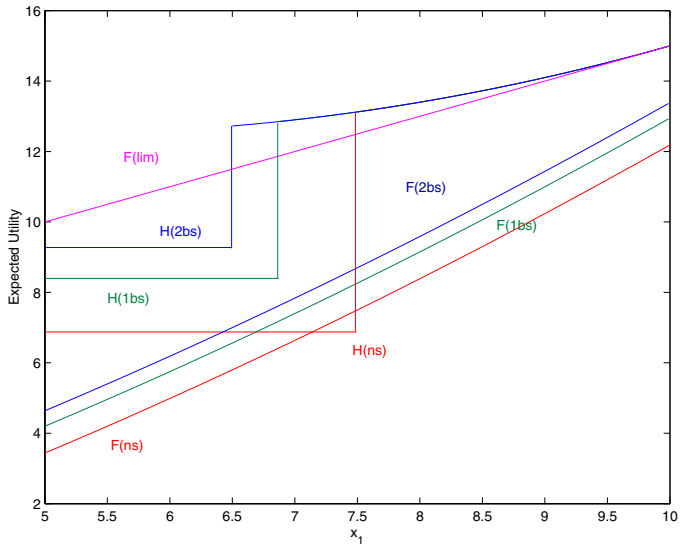

Fig. 7. Negative signals on X_1 with zero credibility and full reputation.

increase the value of $H(\cdot)$ in the full credibility case. Negative signals on $\hat{E}_1$ lead, in both these settings, to an increase in the size of the $Q^+(x_1)$ set, i.e., $Q^+(x_1) \subset Q^+(\overline{x_1})$. Thus, as long as the products contained within this set provide the DM with an expected utility higher than $E_1 + E_2$, after reputation is accounted for, the expected search utility delivered by $H(\cdot)$ increases. It is indeed this value improvement, due to the decrease in the credibility and reputation of the sender, what prevents complete manipulation from taking place within the current setting.

Note, however, that it was indeed the lack of credibility and reputation what allowed for manipulation in the X_2 case. The existing differences in the final result between both scenarios follow from the effect of the X_1 variable on the $H(\cdot)$ function, which was absent in the X_2 setting.[k]

6. On Products Composed of Three Characteristics

This section provides an analysis and intuition regarding the extension of the framework introduced in the paper to products composed of three characteristics. As it will become evident through the analysis, considering products composed of more than three characteristics would require operating within a space of a dimension equal to the number of characteristics being considered. In this case, heuristic mechanisms should be introduced to reduce the dimension of the resulting model and improve its

[k]It should be emphasized that similar results would also be obtained within a risk averse frame-work. In this case, however, search aversion would increase relative to the linear risk neutral case, an effect already described in Ref. 45.

operability. This is also the case when analyzing the acquisition of more than two pieces of information by DMs within the current theoretical setting.[53]

When acquiring information sequentially on products composed of three characteristics, DMs have to compute their optimal gathering behavior through the different stages of the process on which they are located. That is, the different information acquisition trade-offs faced by DMs depend on the characteristics already observed from a given subset of products. These potential combinations can be described as follows:

(0-0) Initial acquisition of information stage: no information has been acquired from any product.

(1-0) The DM has observed the first characteristic from an initial product and must decide whether to continue acquiring information on the product initially observed or to start acquiring information on a new product.

(1-1) The DM has observed the first characteristic from two products and must decide whether to continue acquiring information on any of these products or to start acquiring information on a new product.

(2-0) The DM has observed the first and second characteristics from an initial product and must decide whether to acquire information on the final characteristic of the product initially observed or to start acquiring information on a new product.

(2-1) The DM has observed the first and second characteristics from an initial product and the first characteristic from a different product. The DM must decide among the following options:

  (i) acquiring information on the final characteristic of the product initially observed,
  (ii) observing the second characteristic from the product whose first characteristic has already been observed,
  (iii) start acquiring information on a third new product.

(2-2) The DM has observed the first and second characteristics from two products and must decide whether to acquire information on the final characteristic of any of these products or to start acquiring information on a new product.

Note that in all of these potential information acquisition stages, starting to observe a new product constitutes an alternative to continuing with any of the products observed previously. We analyze the (2-0) and (1-1) subcases formally below. The intuition is similar to that of the bidimensional products setting, but the set of potential combinations of characteristics that must be considered by DMs increases considerably. We will illustrate numerically the behavior of the resulting functions $F(\cdot)$ and $H(\cdot)$ in two of the quadrants composing the (2-0) subcase. The remaining subcases can be easily derived from combining the two described below, but a formal description of the entire information acquisition process would require an analysis of its own beyond the scope of the current paper.

6.1. *The* (2-0) *subcase*

The (2-0) subcase studies the trade-off faced by a DM after having acquired information on the first and second characteristics from a given product. The DM must decide between acquiring information on the third characteristic of the product observed and starting acquiring information on a new product. The notation follows directly from that employed through the paper, with the subindex "three" referring to the third characteristic of the product under consideration. This subindex will also be used to differentiate the functions $F(\cdot)$ and $H(\cdot)$ corresponding to the current setting from those defined for bidimensional products in the previous sections of the paper.

The integration intervals required to define the corresponding function $F(\cdot)$ follow from

$$P^{+}(x_1, x_2) = \{x_3 \in X_3 \cap \operatorname{supp}(\mu_3) : u_3(x_3) > E_1 + E_2 + E_3 - u_1(x_1) - u_2(x_2)\} \tag{6.1}$$

and

$$P^{-}(x_1, x_2) = \{x_3 \in X_3 \cap \operatorname{supp}(\mu_3) : u_3(x_3) \leq E_1 + E_2 + E_3 - u_1(x_1) - u_2(x_2)\} \tag{6.2}$$

for any possible x_1 and x_2 values observed by the DM. Thus, $F : \prod_{i \leq 2} X_i \to \mathbb{R}$ is defined by

$$\begin{aligned} F_3(x_1, x_2) \stackrel{\text{def}}{=} & \int_{u_3(x_3) > E_1 + E_2 + E_3 - u_1(x_1) - u_2(x_2)}^{x_3^M} \mu_3(x_3)(u_1(x_1) + u_2(x_2) + u_3(x_3))\, dx_3 \\ & + \int_{x_3^m}^{u_3(x_3) \leq E_1 + E_2 + E_3 - u_1(x_1) - u_2(x_2)} \mu_3(x_3)(E_1 + E_2 + E_3)\, dx_3. \end{aligned} \tag{6.3}$$

This function must be represented on a three-dimensional space defined for all possible realizations of x_1 and x_2. It will be used to determine the information acquisition behavior of the DM when considering all the potential realizations of x_3. The intuition is identical to that defining the function $F(\cdot)$ within the bidimensional product setting.

Consider now the corresponding function $H(\cdot)$, whose integration intervals follow from

$$Q^{+}(x_1, x_2) = \{y_1 \in X_1 \cap \operatorname{supp}(\mu_1) : u_1(y_1) + E_2 > \max\{u_1(x_1) + u_2(x_2), E_1 + E_2\}\} \tag{6.4}$$

and

$$Q^{-}(x_1, x_2) = \{y_1 \in X_1 \cap \operatorname{supp}(\mu_1) : u_1(y_1) + E_2 \leq \max\{u_1(x_1) + u_2(x_2), E_1 + E_2\}\}. \tag{6.5}$$

Note that in the bidimensional product case described in Sec. 3 the integration intervals were based on

$$Q^{+}(x_1) = \{y_1 \in X_1 \cap \mathrm{supp}(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \tag{6.6}$$

and

$$Q^{-}(x_1) = \{y_1 \in X_1 \cap \mathrm{supp}(\mu_1) : u_1(y_1) \leq \max\{u_1(x_1), E_1\}\}. \tag{6.7}$$

In that case, the DM had to consider the possibility of improving upon a unique observed characteristic from a given product. That is, the only comparable characteristics were defined on X_1. Thus, the resulting integration intervals were based on the maximum value between the observed x_1 realization and the certainty equivalent ce_1.

In the current setting, the DM must compare the characteristic that may be observed from a new product with those two already observed from the initial product. Thus, all potential combinations leading to improvements over both the partially observed initial product and the certainty equivalent one must be considered. In this case, the new product observed must improve upon ce_1, with the DM computing whether or not its expected combination with the ce_2 value improves also upon the initially observed product.

The resulting function $H : \prod_{i \leq 2} X_i \to \mathbb{R}$ is therefore defined as follows

$$\begin{aligned} H_3(x_1, x_2) \stackrel{\mathrm{def}}{=} & \int_{\mathrm{ce}_1}^{x_1^M} \mu_1(y_1)(\max\{u_1(x_1) + u_2(x_2) + E_3, u_1(y_1) + E_2 + E_3\})\, dy_1 \\ & + \int_{x_1^m}^{\mathrm{ce}_1} \mu_1(y_1)(\max\{u_1(x_1) + u_2(x_2) + E_3, E_1 + E_2 + E_3\})\, dy_1. \end{aligned} \tag{6.8}$$

Figures 8 and 9 illustrate the numerical integration areas on which the functions $F(\cdot)$ and $H(\cdot)$ are respectively built based on the potential realizations of x_1 and x_2.

The reference parameter values describing X_1 and X_2 are those used in the numerical computation of the bidimensional setting. The third characteristic is described by the following parameter values:

(i) Characteristic space: $X_3 = [0, 5]$,
(ii) Utility function: $u_3(x_3) = x_3$,
(iii) Probability density: $\forall\, x_3 \in X_3$, $\mu_3(x_3) = \frac{1}{5}$.

As a result, when defining the corresponding function $F(\cdot)$, Area 1 in Fig. 8 is conditioned by the fact that $P^{+}(x_1, x_2) = \emptyset$ for all realizations of x_1 and x_2 within it. Similarly, Area 3 is conditioned by $P^{-}(x_1, x_2) = \emptyset$. Trivially, $P^{+}(x_1, x_2) \neq \emptyset$ and $P^{-}(x_1, x_2) \neq \emptyset$ within Area 2. The same type of reasoning applies when defining the function $H(\cdot)$. That is, Area 1 in Fig. 9 is conditioned by the fact that $u_1(x_1) + u_2(x_2) \leq E_1 + E_2$, while within Area 3 we have $u_1(x_1) + u_2(x_2) + E_3 \geq u_1(y_1^M) + E_2 + E_3$. Finally, within Area 2 we have that $u_1(y_1^M) + E_2 + E_3 > u_1(x_1) + u_2(x_2) > E_1 + E_2$.

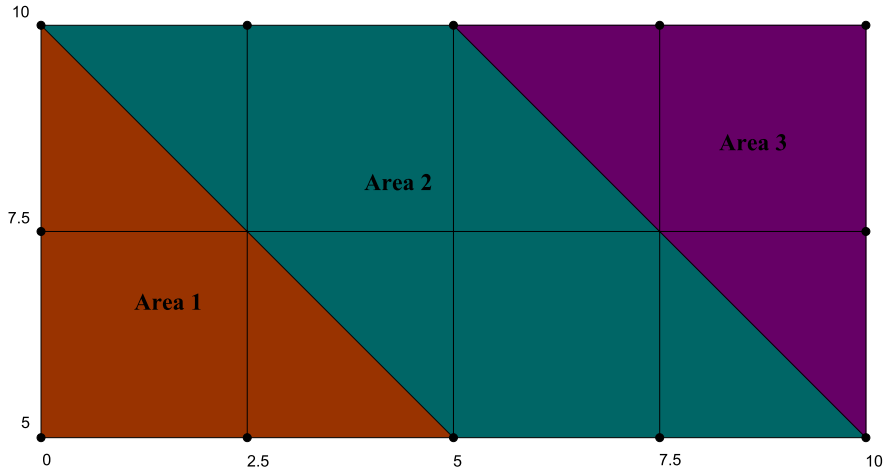

Fig. 8. Numerical integration areas on which the function F is built.

The corresponding starting and continuation regions are the result of the values taken by the functions $F(\cdot)$ and $H(\cdot)$ within each quadrant. Figures 10 and 11 illustrate the quadrant delimited by $x_1 \in [5, 7.5]$ and $x_2 \in [2.5, 5]$, while Figs. 12 and 13 present the quadrant delimited by $x_1 \in [7.5, 10]$ and $x_2 \in [2.5, 5]$. At the same time, Figs. 10 and 12 represent the functions $F(\cdot)$ and $H(\cdot)$ within their respective quadrants absent any signal, while Figures 11 and 13 assume that a negative signal has been issued decreasing the value of E_2 from 5 to 2.5. An immediate effect derived from the negative signal is the shift in the area limits on which both functions are built. We have only illustrated the shift in the area limits of function $F(\cdot)$ in Fig. 14, with the shift in the area limits of function $H(\cdot)$ being similar (and therefore

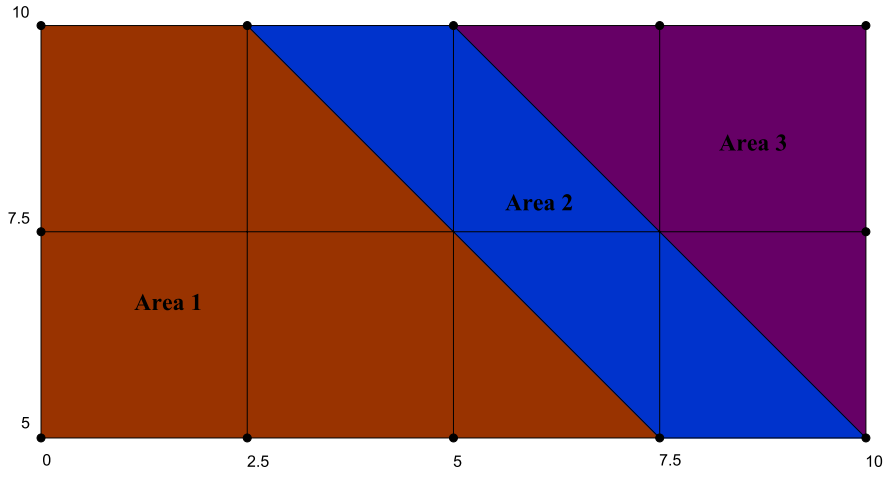

Fig. 9. Numerical integration areas on which the function H is built.

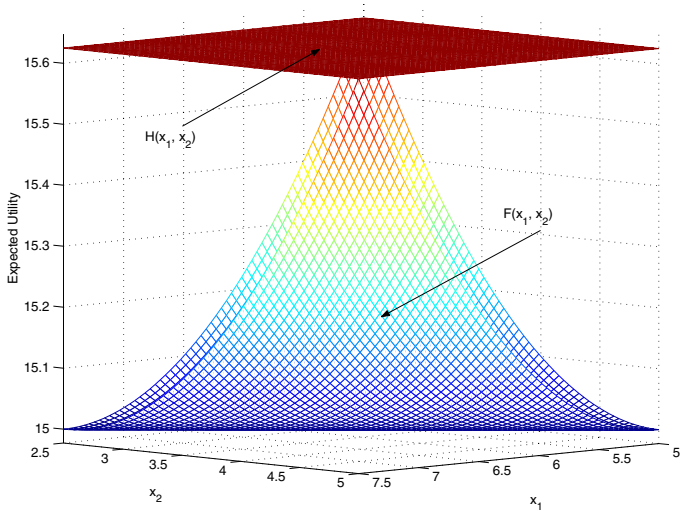

Fig. 10. Three characteristics products and absence of signals: $x_1 \in [5, 7.5]$ and $x_2 \in [2.5, 5]$.

omitted). Moreover, the negative signal leads to a decrease in the value of the function $F(\cdot)$ for all x_1 and x_2 realizations. Note that the function $F(\cdot)$ shifts down and remains below $H(\cdot)$ in both quadrants. As expected, this effect is identical to the one obtained in the bidimensional case under full credibility and reputation.

It should be noted that, when considering the quadrant delimited by $x_1 \in [7.5, 10]$ and $x_2 \in [2.5, 5]$, the functions $F(\cdot)$ and $H(\cdot)$ overlap for values of x_1 and x_2 located above the limit equation $x_2 = 12.5 - x_1$ that divides the quadrant in two identical

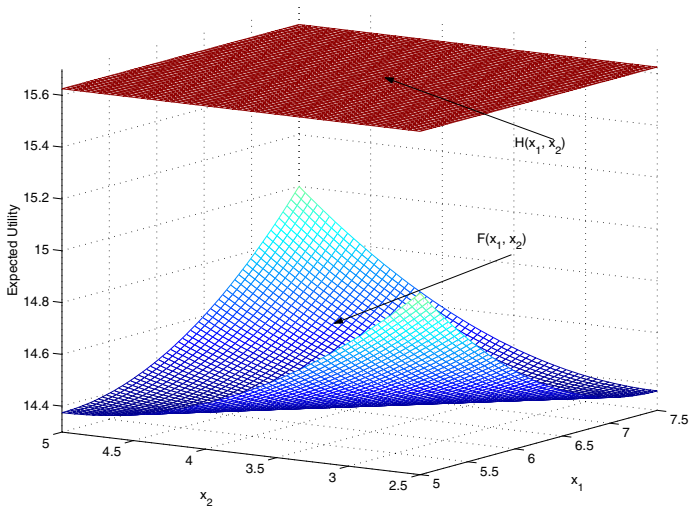

Fig. 11. Three characteristics products and one negative signal: E_2 decreases from 5 to 2.5, with $x_1 \in [5, 7.5]$ and $x_2 \in [2.5, 5]$.

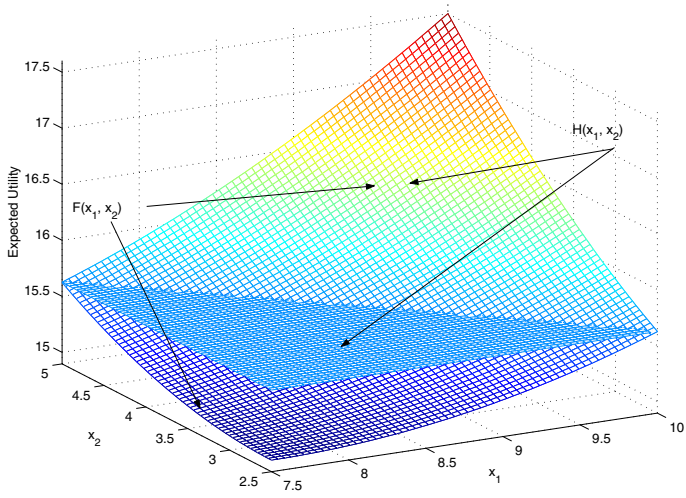

Fig. 12. Three characteristics products and absence of signals: $x_1 \in [7.5, 10]$ and $x_2 \in [2.5, 5]$.

triangles. This overlapping generates uncertainty regarding the information acquisition and choice behavior of DMs. This uncertainty is completely eliminated by the signal through the corresponding quadrant. It should however be highlighted that the initial overlapping between both functions described in Fig. 12 opens the way for further strategic scenarios to be considered together with a link to the literature on fuzzy sets and decision making.

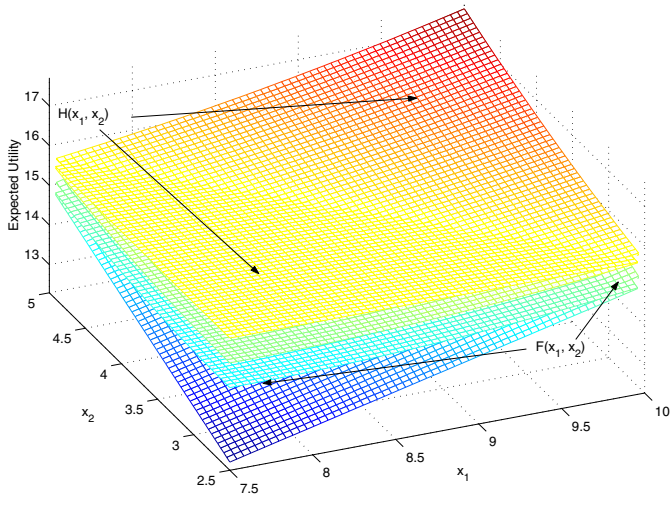

Fig. 13. Three characteristics products and one negative signal: E_2 decreases from 5 to 2.5, with $x_1 \in [7.5, 10]$ and $x_2 \in [2.5, 5]$.

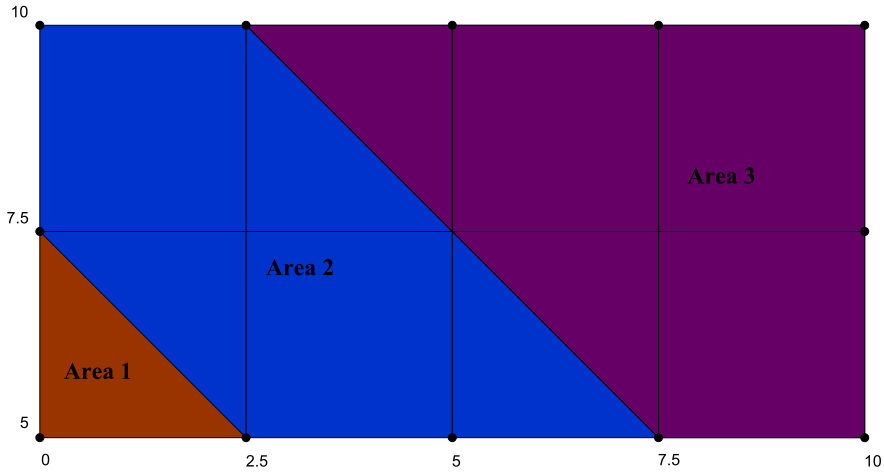

Fig. 14. Shift in the numerical integration areas on which F is built as E_2 decreases from 5 to 2.5.

6.2. *The* (1-1) *subcase*

The (1-1) subcase studies the trade-off faced by a DM after having acquired information on the first characteristic from two different products. The DM must decide between acquiring information on the second characteristic from one of the partially observed products and starting acquiring information on a new (third) product.

Define the highest utility derived from the first characteristics of the two products observed as $u_1(\widehat{x_1}) = \max\{u_1(x_{11}), u_1(x_{21})\}$, with x_{11} and x_{21} being the first characteristic from the first and the second product observed, respectively. The function $F : X_1 \to \mathbb{R}$ is therefore given by

$$\begin{aligned} F_3(x_1) \stackrel{\text{def}}{=} & \int_{u_2(x_2)>E_1+E_2-u_1(\widehat{x_1})}^{x_2^M} \mu_2(x_2)(u_1(\widehat{x_1}) + u_2(x_2) + E_3)\,dx_2 \\ & + \int_{x_2^m}^{u_2(x_2)\le E_1+E_2-u_1(\widehat{x_1})} \mu_2(x_2)\max\{\min\{u_1(x_{11}), u_1(x_{21})\} \\ & + E_2 + E_3, E_1 + E_2 + E_3\}\,dx_2. \end{aligned} \tag{6.9}$$

Note that in the $F_3(x_1)$ case the DM either finds a product whose second characteristic combined with $\widehat{x_1}$ provides a higher utility than the certainty equivalent one or, if this were not the case, then the highest between the remaining partially observed product and the certainty equivalent one would be chosen.

An alternative definition would require the product whose second characteristic is observed to improve also upon $\min\{u_1(x_{11}), u_1(x_{21})\} + E_2$ whenever this remaining product delivers an expected utility higher than $E_1 + E_2$. That is, the function

$F : X_1 \to \mathbb{R}$ could alternatively be defined as follows

$$\begin{aligned} F_3(x_1) \stackrel{\text{def}}{=} & \int_{u_2(x_2)>\max\{E_1+E_2-u_1(\widehat{x_1}),\min\{u_1(x_{11}),u_1(x_{21})\}+E_2-u_1(\widehat{x_1})\}}^{x_2^M} \\ & \times\ \mu_2(x_2)(u_1(\widehat{x_1}) + u_2(x_2) + E_3)\,dx_2 \\ & + \int_{x_2^m}^{u_2(x_2)\le\max\{E_1+E_2-u_1(\widehat{x_1}),\min\{u_1(x_{11}),u_1(x_{21})\}+E_2-u_1(\widehat{x_1})\}} \\ & \times\ \mu_2(x_2)\max\{\min\{u_1(x_{11}), u_1(x_{21})\} + E_2 + E_3, E_1 + E_2 + E_3\}\,dx_2. \end{aligned} \tag{6.10}$$

The choice between both $F_3(x_1)$ alternatives depends on the degree of sophistication we want to endow the DM with.

Similarly, given $\bar{x}_1 = \max\{\widehat{x_1}, E_1\}$, the function $H : X_1 \to \mathbb{R}$ is defined as follows

$$H_3(x_1) \stackrel{\text{def}}{=} \int_{u^{-1}(\bar{x}_1)}^{x_1^M} \mu_1(y_1)(u_1(y_1) + E_2 + E_3)\,dy_1 + \int_{x_1^m}^{u^{-1}(\bar{x}_1)} \mu_1(y_1)(\bar{x}_1 + E_2 + E_3)\,dy_1. \tag{6.11}$$

Both functions will determine the optimal information acquisition behavior of DMs within this stage, which, as it is clear from the respective definitions, follows an almost identical intuition as the one described in the bidimensional product setting.

7. Conclusions and Extensions

The main conclusion to be derived from this paper is that an information sender can issue (unverifiable) signals on products located in alternative markets so as to manipulate the choice of uninformed but perfectly rational DMs, even when the latter are able to verify the characteristics of the products offered by the sender. We have illustrated how, in order for the sender to be able to manipulate the choice and information gathering processes of DMs in the desired direction, he should release signals on characteristics that differ from the most preferred ones.

The results obtained provide a strategic dimension to the information acquisition process of DMs. Thus, the strategic nature of information transmission processes should be explicitly analyzed in fields that do not currently account for it, such as knowledge management.[54] This is particularly important at the organizational level, where relatively small sets of decision variables are generally considered,[55] and trust constitutes an essential element of the organizations performance.[56] In addition, the results obtained allow for a formal treatment of the optimal acquisition of information and choice processes studied by the consumer choice literature, where the strategic side of information transmission is rarely formalized.[3–5,37]

Moreover, the set of potential strategic scenarios arising from the inability of DMs to initially differentiate information senders based on their reputation should be studied.[57] This theoretical possibility should reflect recent empirical phenomena such

as the Amazon sock puppet scandal, where book authors issued their own reviews to the firm's website.[58]

An immediate extension of the current paper should aim at analyzing the implications resulting from strategic information transmission processes within oligopolistic scenarios, spreading into the game theoretical branches of economics and operations research developed by Refs. 59 and 60. For example, it seems plausible to assume that after gathering both pieces of information, DMs decide to purchase (one of) the product(s) observed if and only if it provides them with an expected utility higher than $E_1 + E_2$ absent reputation frictions. However, if several senders located in different markets are considered, then the resulting set of signaling games would require further threshold modifications based on the reputation and credibility of the corresponding senders and the signals they release.

References

1. D. Kahneman and A. Tversky, *Choices, Values, and Frames* (Cambridge University Press, 2000).
2. D. M. Sanbonmatsu, F. R. Kardes, D. C. Houghton, E. A. Ho and S. S. Posavac, Overestimating the importance of the given information in multiattribute consumer judgment, *Journal of Consumer Psychology* **13**(3) (2003) 289–300.
3. N. Novemsky, R. Dhar, N. Schwarz and I. Simonson, Preference fluency in choice, *Journal of Marketing Research* **44**(3) (2007) 347–356.
4. K. Diehl, When two rights make a wrong: Searching too much in ordered environments, *Journal of Marketing Research* **42**(3) (2005) 313–322.
5. D. Ariely, Controlling the information flow: Effects on consumers' decision making and preferences, *Journal of Consumer Research* **27** (2000) 233–248.
6. A. V. Banerjee, A simple model of herd behavior, *Quarterly Journal of Economics* **107**(3) (1992) 797–817.
7. S. Bikhchandani, D. Hishleifer and I. Welch, A theory of fads, fashion, custom, and cultural change as informational cascades, *Journal of Political Economy* **100**(5) (1992) 992–1026.
8. C. P. Chamley, *Rational Herds* (Cambridge University Press, 2004).
9. K. F. McCardle, Information acquisition and the adoption of new technology, *Management Science* **31**(11) (1985) 1372–1389.
10. S. A. Lippman and K. F. McCardle, Uncertain search: A model of search among technologies of uncertain values, *Management Science* **37**(11) (1991) 1474–1490.
11. D. Ergu, G. Kou, Y. Shi and Y. Shi, Analytic network process in risk assessment and decision analysis, *Computers Operations Research* **42** (2014) 58–74.
12. G. Kou, Y. Lu, Y. Peng and Y. Shi, Evaluation of classification algorithms using MCDM and rank correlation, *International Journal of Information Technology & Decision Making* **11**(1) (2012) 197–225.
13. C. Ulu and J. E. Smith, Uncertainty, information acquisition, and technology adoption, *Operations Research* **57**(3) (2009) 740–752.
14. R. A. Aliev, W. Pedrycz and O. H. Huseynov, Behavioral decision making with combined states under imperfect information, *International Journal of Information Technology & Decision Making* **12**(3) (2013) 619–645.
15. M. Danielson, Handling imperfect user statements in real-life decision analysis, *International Journal of Information Technology & Decision Making* **3**(3) (2004) 513–534.

16. R. A. Aliev, W. Pedrycz and O. H. Huseynov, Decision theory with imprecise probabilities, *International Journal of Information Technology & Decision Making* **11**(2) (2012) 271–306.
17. J. Ni and D. Khazanchi, Information technology investment decisions under asymmetric information: A modified rational expectation model, *International Journal of Information Technology & Decision Making* **8**(1) (2009) 55–72.
18. V. I. Yukalov and D. Sornette, Role of information in decision making of social agents, *International Journal of Information Technology & Decision Making* (2014), in press, doi: 10.1142/S0219622014500564.
19. H. Simon, A behavioral model of rational choice, *Quarterly Journal of Economics* **69**(1) (1955) 99–118.
20. G. Akerlof, The market for "lemons": Quality uncertainty and the market mechanism, *Quarterly Journal of Economics* **84**(3) (1970) 488–500.
21. R. Spiegler, The market for quacks, *Review Economic Studies* **73**(4) (2006) 1113–1131.
22. V. Crawford and J. Sobel, Strategic information transmission, *Econometrica* **50**(6) (1982) 1431–1451.
23. R. Dur and O. Swank, Producing and manipulating information, *Economic Journal* **115** (500) (2005) 185–199.
24. A. Wolinsky, Eliciting information from multiple experts, *Games and Economic Behavior* **41**(1) (2002) 141–160.
25. W. Hufman, M. Rousu, J. Shogren and A. Tegene, The effects of prior beliefs and learning on consumers' acceptance of genetically modified foods, *Journal of Economic Behavioral & Organization* **63**(1) (2007) 193–206.
26. J. Kennes and A. Schif, Simple reputation systems, *Scandinavian Journal of Economic* **109**(1) (2007) 71–91.
27. R. Jurca, F. Garcin, A. Talwar and B. Faltings, Reporting incentives and biases in online review forums, *ACM Transactions on Web* **4**(2) (2010) Article No. 5.
28. J. Arenas-Gaitan, F. J. Rondan-Cataluna and P. E. Ramirez-Correa, Social identity, electronic word-of-mouth and referrals in social network services, *Kybernetes* **42**(8) (2013) 1149–1165.
29. B. Brown, Beyond recommendations: Local review web sites and their impact, *ACM Transactions on Computer-Human Interaction* **19**(4) (2012) 1–24.
30. C. M. K. Cheung and M. K. O. Lee, What drives consumers to spread electronic word of mouth in online consumer-opinion platforms, *Decision Support Systems* **53**(1) (2012) 218–225.
31. D. R. Liu, N. H. Chen and P. H. Chiu, Recommending quality book reviews from heterogeneous websites, *Internet Research* **23**(1) (2013) 27–46.
32. Y. C. Ku, C. P. Wei and H. W. Hsiao, To whom should I listen? Finding reputable reviewers in opinion-sharing communities, *Decision Support Systems* **53**(3) (2012) 534–542.
33. I. Jeacle and C. Carter, In TripAdvisor we trust: Rankings, calculative regimes and abstract systems, *Accounting Organizations Society* **36**(4–5) (2011) 293–309.
34. B. Khosravifar, J. Bentahar, M. Gomrokchi and R. Alam, CRM: An efficient trust and reputation model for agent computing, *Knowledge Based Systems* **30** (2012) 1–16.
35. E. D'Avanzo and T. Kuflik, E-Commerce websites services versus buyers expectations: An empirical analysis of the online marketplace, *International Journal of Information Technology & Decision Making* **12**(4) (2013) 651–677.
36. F. Bronner and R. de Hoog, Social media and consumer choice, *International Journal of Market Research* **56**(1) (2014) 51–71.
37. J. N. Bearden and T. Connolly, Multi-attribute sequential search, *Organization Behavior and Human Decision Processes* **103**(1) (2007) 147–158.

38. D. Di Caprio and F. J. Santos Arteaga, Strategic difusion of information and preference manipulation, *International Journal of Strategic Decision Sciences* **2**(2) (2011) 1–19.
39. O. H. Azar, Relative thinking in consumer choice between differentiated goods and services, *Judgment and Decision Making* **6**(2) (2011) 176–185.
40. O. H. Azar, Do people think about absolute or relative price differences when choosing between substitute goods? *Journal of Economic Psychology* **32**(3) (2011) 450–457.
41. The Nielsen Company, Global trends in online shopping: a Nielsen global consumer report (2010), Available at http://hk.nielsen.com/documents/Q12010OnlineShopping-TrendsReport.pdf.
42. F. Malerba, R. Nelson, L. Orsenigo and S. Winter, Demand, innovation, and the dynamics of market structure: The role of experimental users and diverse preferences, *Journal of Evoluation Economics* **17**(4) (2007) 371–399.
43. J. D. Bohlmann, P. N. Golder and D. Mitra, Deconstructing the pioneer's advantage: Examining vintage effects and consumer valuations of quality and variety, *Management Science* **48**(9) (2002) 1175–1195.
44. R. T. Gordon and M. H. Brezinski, *The Complete Restaurant Management Guide* (M. E. Sharpe, 1999).
45. D. Di Caprio and F. J. Santos Arteaga, An optimal information gathering algorithm, *International Journal of Applied Decision Sciences* **2**(2) (2009) 105–150.
46. L. L. Wilde, On the formal theory of inspection and evaluation in product markets, *Econometrica* **48**(5) (1980) 1265–1279.
47. P. Wakker, *Additive Representations of Preferences. A New Foundation of Decision Analysis* (Kluwer Academic Publishers, Dordrecht, 1989).
48. D. Di Caprio, F. J. Santos Arteaga and M. Tavana, The optimal sequential information acquisition structure: A rational utility-maximizing perspective, *Applied Mathematical Modelling* **38**(14) (2014) 3419–3435.
49. M. Tavana, D. Di Caprio and F. J. Santos Arteaga, An optimal information acquisition model for competitive advantage in complex multiperspective environments, *Applied Mathematics and Computation* **240** (2014) 175–199.
50. T. Xiao, G. Yu and Z. Sheng, Reputation, utility and technological innovation strategies, *International Journal of Information Technology & Decision Making* **3**(1) (2004) 81–100.
51. C. M. Christensen, *The Innovator's Dilemma: When New Technologies Cause Great Firms to Fail* (Harvard Business School Press, 1997).
52. J. Berger, A. T. Sorensen and S. J. Rasmussen, Positive effects of negative publicity: When negative reviews increase sales, *Marketing Science* **29**(5) (2010) 815–827.
53. D. Di Caprio, F. J. Santos Arteaga and M. Tavana, An optimal sequential information acquisition model subject to a heuristic assimilation constraint, *Benchmarking: An International Journal*, in press.
54. C. W. Holsapple (ed.), *Handbook of Knowledge Management 2. Knowledge Directions* (Springer-Verlag, Berlin, 2003).
55. B. R. Gaines, Organizational knowledge acquisition, in *Handbook of Knowledge Management 1. Knowledge Matters*, ed. C. W. Holsapple (Springer-Verlag, Berlin, 2003), pp. 317–348.
56. L.-A. Ho, T.-H. Kuo, C. Lin and B. Lin, The mediate effect of trust on organizational online knowledge sharing: An empirical study, *International Journal of Information Technology & Decision Making* **9**(4) (2010) 625–644.
57. A. Dimoka, Y. L. Hong and P. A. Pavlou, On product uncertainty in online markets: Theory and evidence, *MIS Quarterly* **36**(2) (2012) 395–426.

58. D. Vinjamuri, Do consumer reviews have a future? Why Amazon's sock puppet scandal is bigger than it appears, *Forbes* (2012), Available at http://www.forbes.com/sites/davidvinjamuri/2012/09/12/do-consumer-reviews-have-a-future-why-amazons-sock-puppet-scandal-is-bigger-than-it-appears/.
59. J. Reinganum, On the difusion of new technology: A game theoretic approach, *Review Economic Studies* **48**(3) (1981) 395–405.
60. J. W. Mamer and K. F. McCardle, Uncertainty, competition and the adoption of new technology, *Management Science* **33**(2) (1987) 161–177.