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A common-weights DEA model for centralized resource reduction and target setting ☆

Adel Hatami-Marbini [a,*], Madjid Tavana [b,c], Per J. Agrell [a], Farhad Hosseinzadeh Lotfi [d], Zahra Ghelej Beigi [e]

[a] Louvain School of Management, Center of Operations Research and Econometrics (CORE), Université catholique de Louvain, L1.03.01, B-1348 Louvain-la-Neuve, Belgium
[b] Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA
[c] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany
[d] Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
[e] Department of Mathematics, Mabarakeh Branch, Islamic Azad University, Isfahan, Iran

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ABSTRACT

Data Envelopment Analysis (DEA) is a powerful tool for measuring the relative efficiency for a set of Decision Making Units (DMUs) that transform multiple inputs into multiple outputs. In centralized decision-making systems, management normally imposes common resource constraints to maximize operating revenues and minimize operating expenses. In this study, we propose an alternative DEA model for centrally imposed resource or output reduction across the reference set. We determine the amount of input and output reduction needed for each DMU to increase the efficiency score of all the DMUs. The contribution of the proposed model is fourfold: (1) we take into consideration the performance evaluation of the centralized budgeting in hierarchical organizations; (2) we use a Common Set of Weights (CSW) method based on the Goal Programming (GP) concept to control the total weight flexibility in the conventional DEA models; (3) we propose a comprehensive approach for optimizing the inputs and/or outputs contractions and improving the final efficiencies of the DMUs while reducing the computational complexities; (4) we compare the proposed method with an approach in the literature; and (5) we demonstrate the applicability of the proposed method and exhibit the efficacy of the procedure with a numerical example.

1. Introduction

Changing economic conditions have challenged many financial institutions and banks to search for more efficient ways to assess their operations. Non-parametric frontier analysis was first introduced by Farrell (1957) and later developed into Data Envelopment Analysis (DEA) by Charnes, Cooper, and Rhodes (1978) as a linear programming based technique for efficiency assessment. DEA is a powerful mathematical method for determining the relative efficiency of a set of functionally similar Decision Making Units (DMUs) (e.g., banks, brokerage firms, and insurance companies) that use multiple inputs to produce multiple outputs. Although in theory the conventional DEA assumes that all DMUs enjoy complete autonomy in accessing available resources, the DMUs are in practice often subject to common resources and market constraints imposed by a central decision maker. Hence, in many real-world situations, one may have to consider retrenchment programs requiring curtailment of some of the inputs and outputs for a variety of exogenous reasons. However, a good retrenchment program should not diminish any DMU efficiencies. For example, consider a public agency in charge of staffing and supplying a school district with special resources and assigning students to different schools. A budget reduction in the district will result in budget cuts in the schools while demographic changes may lead to reductions in the number of students across the board in the district. In both cases, it is desirable to maintain or improve technical efficiency of the schools after resource reallocation. Several researchers have applied the input and/or output deterioration to DEA models in the literature. Activity planning in DEA was proposed by Banker, Charnes, Cooper, and Clarke (1989), Bogetoft (1993, 1994, 2000) and Golany and Tamir (1995).

Cook and Kress (1999) were the first to introduce the idea of resource or cost allocation in DEA by characterizing an equitable way for allocating the shared costs. However, their approach cannot provide the cost allocation for the DMUs in a straightforward

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* Corresponding author. Tel.: +32 10 47 43 19.
E-mail addresses: adel.hatamimarbini@uclouvain.be (A. Hatami-Marbini), tavana@lasalle.edu (M. Tavana), per.agrell@uclouvain.be (P.J. Agrell), farhad@hosseinzadeh.ir (F. Hosseinzadeh Lotfi), zahra.ghlichbegi@yahoo.com (Z.G. Beigi).
URL: http://tavana.us/ (M. Tavana).

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way and requires a huge computational burden. Jahanshahloo, Hosseinzadeh Lotfi, Shoja, and Sanei (2004) identified the shortcoming of Cook and Kress's (1999) approach and devised a simple method for costs allocation without solving any linear program. Cook and Zhu (2005) extended Cook and Kress's (1999) approach to an equitable direct cost allocation method. Lin (2011) extended Cook and Zhu's (2005) method for allocating fixed resources with some additional constraints by eliminating the occasional infeasibility problem in their method. Athanassopoulos (1995) proposed a method for target setting and resource allocation in multi-level planning problems using Goal Programming (GP) and DEA. Athanassopoulos (1998) later proposed a resource allocation model which consisted of two steps: (1) determining the optimal weights using a multiplier DEA model; and (2) defining the feasible trade-offs in allocation. Athanassopoulos, Lambroukos, and Seiford (1999) imposed upper and lower bounds on inputs for each individual DMU that had to be satisfied after reallocation.

Ito, Namatame, and Yamaguchi (1999) reallocated the management resources to provide the maximum outputs using the concept of the production possibility set of the DEA-BCC (Banker, Charnes, & Cooper, 1984) model. Wei, Zhang, and Zhang (2000) introduced the inverse DEA model using the concept of an inverse optimization problem in which the efficiency remains unchanged in the presence of data changes (interested readers for more details on the inverse optimization are referred to Ahuja & Orlin (2001)). The authors tried to answer the question: If some of the inputs of the DMU are changed by a certain amount, by how much should the outputs be changed in order to preserve the original level of efficiency. Their inverse DEA model was solved as a Multi-Objective Linear Programming (MOLP) problem. Yan, Wei, and Hao (2002) developed an inverse generalized DEA model and then discussed the application of the extended model to resource reallocation problems. Hadi-Vencheh, Foroughi, and Soleimani-damaneh (2008) presented a counterexample to show that Wei et al. (2000)'s method does not always produce useful results when using a weakly efficient solution of the MOLP problems. Cook and Zhu (2003) developed a DEA model for the maximally achievable efficiency measurement of highway maintenance crews by reduction in the inputs without impacting the outputs. Beasley (2003) used a non-linear program to maximize the average efficiency scores of the DMUs to simultaneously allocate fixed costs, input resources, and output targets. Amirteimoori and Kordrostami (2005) modified the constraints of Beasley's (2003) model to minimize cases of infeasibility. Korhonen and Syrjänen (2004) developed a resource-allocation model for finding an equitable allocation plan using DEA and MOLP. Jahanshahloo, Hosseinzadeh Lotfi, and Moradi (2005a) presented a method for fairly allocating a fixed output among DMUs without solving any linear program while keeping the efficiency scores unchanged. Amirteimoori and Shafiei (2006) proposed a DEA-based method for equitably removing a fix resource from all the DMUs and ensuring that the efficiency of units before and after reduction remains unchanged. Li and Cui (2008) presented a resource allocation framework consisting of a various returns to scale model, an inverse DEA model, a common weight analysis model, and an extra resource allocation algorithm. Li, Yang, Liang, and Hua (2009) first considered the linkage between the efficiency scores and the cost allocation and then developed a DEA approach to allocate the fixed cost between DMUs.

Pachkova (2009) proposed a DEA model to reallocate inputs based on the trade-off between the maximum allowed reallocation cost and the highest possible aggregate efficiencies of all the DMUs. Vaz, Camanho, and Guimarães (2010) first assessed the efficiency of retail stores with several selling sections in a network DEA model under Variable Returns to Scale (VRS) and showed how resource reallocation and target setting in Färe, Grabowski, Grosskopf, and Kraft's (1997) method improves the efficiency scores. Bi, Ding, Luo, and Liang (2011) suggested a resource allocation and target setting model for a parallel production system based on Kao's (2009) parallel DEA model. In their proposed model, the sub-DMUs were evaluated using the common weights without deteriorating the efficiency. Amirteimoori and Mohaghegh Tabar (2010) proposed a DEA approach for resource allocation and target setting problems. In their setting, the decision maker(s) could decide to allocate additional resources equitably among all DMUs and, in exchange, demand additional aggregate output from them. Amirteimoori and Emrouznejad (2011) presented a DEA-based approach to determine the highest possible input reduction and lowest possible output deterioration without reducing the efficiency score for each DMU. We demonstrate the advantages of the method proposed in this study by comparing it with the approach in Amirteimoori and Emrouznejad (2011). Similar to Amirteimoori and Emrouznejad (2011, 2012) presented an alternative DEA-based approach involving an additional assumption that the sum of the efficiencies of the DMUs is improved with respect to their prior performance. Recently, Lozano, Villa, and Canca (2011) introduced a number of non-radial, output-oriented and centralized DEA models for resource allocation and target setting for inputs with integer constraints.

Lertworasirikul, Charnsethikul, and Fang (2011) extended the inverse DEA model to VRS by using preserved efficiency for all DMUs in a resource allocation problem. This inverse DEA study considered the efficiency scores of all DMUs while the previous studies on the inverse DEA, c.f., Wei et al. (2000) and Yan et al. (2002), take the efficiency of the considered DMU into consideration. They proposed a MOLP model for the inverse DEA model and transformed into a linear programming model to obtain an optimal solution. Their method was applied to a case study at a motorcycle-part company. Wei and Chang (2011) introduced the optimal system design DEA model to optimally implement a DMU's resource allocation. Their model helps DMUs discover an optimal design or configuration given some cost or effort constraints. Hosseinzadeh Lotfi, Hatami-Marbini, Agrell, Aghayi, and Gholami (2013) recently proposed an allocation mechanism using a common dual weights approach for allocating the fixed resources to the units and equitably setting the expected common increase of the targets to the DMUs.

In the original DEA model, Charnes et al. (1978) proposed that the efficiency of a DMU can be obtained as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that the same ratio for all the DMUs must be less than or equal to one. In fact, there are no restrictions on how much weight (multiplier) can be placed on each input or output relative to the others. Thus, the endogenous weights for each individual DMU are chosen uniquely to maximize its own efficiency. This characteristic of DEA is called "total weights flexibility". Obviously, it is possible that a particular DMU only takes into account weights on a few variables. Moreover, it is highly implausible and overly conservative to assume that each DMU faces unique marginal costs and benefits when evaluating a set of structurally comparable units. Consequently, many applications involve decision makers providing a priori preferred weights in efficiency evaluation.

Many researchers have focused on the problem of unacceptable weighting schemes. Dyson and Thanassoulis (1988) proposed a method for absolute weight restrictions. Charnes, Cooper, Huang, and Sun (1990) demonstrated that undesirable weighting plans are unavoidable in many DEA applications and proposed cone ratio restrictions models to provide more realistic weights. Thompson, Dharmapala, and Thrall (1995) used Charnes et al.'s (1990) models and introduced the "assurance region" as a special case of the cone ratio concept (Thompson, Langemeier, Lee, Lee, & Thrall, 1990). There are some extensions of the assurance region concept in the DEA literature (see Allen, Athanasopoulos, Dyson, & Thanassoulis (1997) and Cook & Seiford (2009) for a comprehensive overview).

Bessent, Bessent, Elam, and Clark (1988) presented the constrained facet analysis to deal with the inherent problem involving the occurrence of zero weights. Lang, Yolalan, and Kettani (1995) improved this latter approach by adopting a two-stage approach. Similar methods have been suggested by Green, Doyle, and Cook (1996) and Olesen and Petersen (1996).

The Common Set of Weights (CSW) approach in DEA was initially introduced by Cook, Roll, and Kazakov (1990) and developed by Roll, Cook, and Golany (1991). Jahanshahloo, Memariani, Hosseinzadeh Lotfi, and Rezaei (2005b) used a multi-objective model to specify a CSW for all DMUs using a non-linear transformation. Amin and Toloo (2007) proposed a CSW integrated DEA model to obtain the most efficient DMUs. Agrell and Bogetoft (2010) proposed a game-theoretical approach to determine a set of CSWs in a setting where the DMUs must agree upon a common endogenous evaluation. In a recent study, Saati, Hatami-Marbini, Agrell, and Tavana (2012) proposed a two-phase CSW approach using an ideal virtual unit that is computationally efficient. Their method was applied in energy regulation using panel data from Danish district heating plants. Omrani (2013) incorporated the uncertainty into the Zohrehbandian, Makui, and Alinezhad's (2010) common weight DEA model. He first extended a robust DEA model based on the robust optimization approach proposed by Bertsimas and Sim (2004) to calculate the efficiency of each DMU as the ideal solution. He then applied the GP approach to find the common weights by minimizing the amount of deviation from the ideal solution.

In this paper, we propose an alternative DEA model for a centrally imposed resource or output reduction across all DMUs. We show how much the inputs and outputs of each DMU should be reduced without decreasing the efficiency scores of the DMUs. A scaling formula is used to decrease the inputs and the outputs of the DMUs proportional to their input utilizations and output productions. The remainder of this paper is organized into five sections. In Section 2 we provide a brief review of the conventional DEA model and in Section 3 we present the common-weights DEA model. Section 4 presents the details of the method proposed in this study. In section 5 we present the approach in Amirteimoori and Emrouznejad (2011) and compare this approach with the method proposed in this study. In Section 6 we show a numerical example to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedure. In Section 7 we summarize our conclusions and future research directions.

2. The traditional DEA model

DEA estimates a convex hull covering a set of DMUs and radially projects them against the hull in a specified direction. Suppose that there are n DMUs to be evaluated where every DMU_j, $j = 1,2,\ldots,n$, produces s outputs $y_{rj} \in R^+$, $O = \{1, 2,\ldots,r,\ldots,s\}$, using m inputs, $x_{ij} \in R^+$, $I = \{1, 2,\ldots,i,\ldots,m\}$. The input-oriented model (CCR or CRS for constant returns to scale) for evaluating the relative efficiency of a given DMU_o is as follows (Charnes et al., 1978):

$$\begin{aligned} \max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1,2,\ldots,n, \\ & u_r, v_i \geqslant \varepsilon, \quad r = 1,2,\ldots,s; \quad i = 1,2,\ldots,m. \end{aligned} \tag{1}$$

where ε is a positive non-Archimedean infinitesimal number. In this paper, we employ the method proposed by Amin and Toloo (2004) to calculate ε in terms of input and output data as shown below:

$$\varepsilon = \min\{b^{-1}, q\} \tag{2}$$

where $b = \max\left\{\sum_{i=1}^{m} x_{ij} : j = 1,2,\ldots,n\right\}$ and $q = \min\left\{\sum_{i=1}^{m} x_{ij} / \sum_{r=1}^{s} y_{rj} : j = 1,2,\ldots,n\right\}$.

Model (1) is called the multiplier model.

Definition 1. DMU_o is efficient if there exists at least one optimal (u_r^*, v_i^*) of model (1) with $u_r^* \geqslant \varepsilon, v_i^* \geqslant \varepsilon$ and $\sum_{r=1}^{s} u_r^* y_{ro} / \sum_{i=1}^{m} v_i^* x_{io} = 1$. Otherwise, DMU_o is inefficient (Charnes et al., 1978).

3. The common-weights DEA model

The relative efficiency using the multiplier DEA model (1) is determined by assigning weights to the inputs and outputs of a DMU to maximize its own ratio of the weighted sum of outputs to the weighted sum of inputs. The only underlying assumption for the weights on the inputs and outputs is positive (called "total weights flexibility"). The calculation of DEA scores obtains an individual set of endogenous weights by solving the linear program (1) for each DMU. We recall that the differences among the individual weights may be unacceptable for management or market reasons, technical or economic necessities. Charnes, Cooper, Lewin, and Seiford (1994) have noted the necessity to control the weights in a DEA model when: (i) some inputs and/or outputs may be entirely disregarded in performance analysis since the weights associated with these inputs and/or outputs are zero (or epsilon); (ii) the model does not take the opinions of the decision maker into consideration; (iii) the decision maker has strong preferences on the relative importance of given factors; or (iv) the number of factors is proportionately large in comparison with the number of the DMUs. The DEA model (1) developed in this study leads to a weak discrimination due to the large number of DMUs under consideration. To cope with aforementioned issues, the CSW model can be used to generate weights for all the DMUs that produce the highest efficiency scores. In the following section, we examine a CSW model based on the Multi-Objective Program (MOP). Many researchers have investigated the relationships between DEA and MOP from different points of view (e.g., see Hosseinzadeh Lotfi, Jahanshaloo, Ebrahimnejad, Soltanifar, & Manosourzadeh, 2010; Yang, Wong, Xu, & Stewart, 2009). To simultaneously maximize the efficiencies of all DMUs we take the same approach as (Hosseinzadeh Lotfi et al., 2013; Chiang, Hwang, & Liu, 2011; Jahanshahloo, Memariani, Hosseinzadeh Lotfi, & Rezai, 2005) and consider the Multi-Objective Fractional Program (MOFP) below:

$$\begin{aligned} \max \quad & \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rn}}{\sum_{i=1}^{m} v_i x_{in}} \right\} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1,2,\ldots,n, \\ & u_r, v_i \geqslant \varepsilon, \quad r = 1,2,\ldots,s; \ i = 1,2,\ldots,m. \end{aligned} \tag{3}$$

Several methods have been developed in the optimization literature to solve the multi-objective problems (see e.g., Hwang & Masud, 1979; Steuer, 1986).

In the maximization MOFP (3), $\sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}$ is the jth objective function which should be as close to unity (full technical efficiency) from below as possible. Let s_j denote the "virtual gap" in the weighted output i.e., $\sum v_i x_{ij} - \sum u_r y_{rj}$. The solution of (4) then assures that the combined efficiencies of the DMUs are as close to unity as possible.

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} + s_j}{\sum_{i=1}^{m} v_i x_{ij}} = 1, \quad j = 1,\ldots,n, \\ & u_r, v_i, s_j \geqslant \varepsilon, \quad r = 1,\ldots,s; \ i = 1,\ldots, \ m; \ j = 1, \ \ldots, \ n. \end{aligned} \tag{4}$$

This program (4) can be simply transformed into the following linear program:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} s_j \\ s.t. \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j = 1, \ j = 1, \ldots, n, \\ & u_r, v_i, s_j \geqslant \varepsilon, \quad r = 1, \ldots, s; \ i = 1, \ldots, m; \ j = 1, \ldots, n. \end{aligned} \tag{5}$$

Given an optimal solution $\left(u_r^*, v_i^*, s_j^*\right)$ $\forall r, i, j$ to (5), the efficiency scores for DMU_j, $j = 1, 2, \ldots, n$, are calculated as follows:

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{s_j^*}{\sum_{i=1}^{m} v_i^* x_{ij}}, \quad j = 1, 2, \ldots, n. \tag{6}$$

Definition 2. *DMU_j, $j = 1, 2, \ldots, n$, is non-dominated if and only if $s_j^* = 0, j = 1, 2, \ldots, n$, in the model (5).*

Li and Reeves (1999) considered the problem of ranking the DMUs as well as unrealistic weight distribution in MOLP. They introduced a deviation variable, denoted by d_j, where d_j represents the deviation of the jth DMU from its efficiency (i.e., $d_j = 1 - \theta_j$). They utilized three objective functions (i.e., minimizing deviation, minimizing the maximum deviation, and minimizing the sum of the deviations). The first objective function and constraints are equivalent to the linear form of model (1) for a DMU under evaluation. Nonetheless, it is possible to disregard the first objective function and the first constraint in their model and evaluate the efficiency of all the DMUs simultaneously with respect to the second and third objectives. Li and Reeves (1999, p.516) presented the detailed formulation for evaluating DMU_A in Example 1. They solved their model n times to calculate the efficiency score of DMU_o where our model calculates the efficiency scores of all DMUs simultaneously. There are special CSW models stemming from Li and Reeves (1999) model for calculating the efficiency score for all DMUs simultaneously. However, these CSW models are different in comparison with the model proposed in this study. For example, Toloo, Sohrabi, and Nalchigar (2009), Toloo (2012), and Toloo and Nalchigar (2009, 2011) only considered the second objective of Li and Reeves (1999)'s model (i.e., minimizing the maximum deviation) and those models have very little in common with model (5). Similarly, Toloo and Ertay (2014) only considered the first objective of Li and Reeves (1999)'s model (i.e., minimizing d_0). A comparison between Toloo and Ertay (2014)'s model with model (5) proposed in this study reveals that model (4) of Toloo and Ertay (2014, p.138) contains n more constraints, $\sum_{i=1}^{m} v_i x_{ij} \leqslant 1$, $j = 1, \ldots, n$ while the model proposed in this study, based on the fractional DEA model (1), does not consider similar constraints.

4. The proposed model

In performance evaluation of centralized organizations with some common control, the objective to maintain optimal technical efficiency goes hand-in-hand with the necessity to comply with the resource and market constraints. Take for example the case of a public authority staffing and supplying schools with special resources, as well as assigning students to regions and districts. A budget reduction to the sector that must be implemented across the schools, likewise demographic changes may lead to reductions in the number of students both admitted and graduated. In both cases, it is imperative to maintain or improve technical efficiency of its units after the resource and target reallocation. In this section, we propose an alternative approach for determining the optimum input and attendant output reduction without impairing the efficiency scores of the DMUs derived from the CSW approach.

Let $I_1 = \{i_1, i_2, \ldots, i_k\} (k \leqslant m)$ and $O_1 = \{r_1, r_2, \ldots, r_t\} (t \leqslant s)$ be the subsets of inputs (I) and outputs (O) to be restricted. The rest of the members of sets I and O are denoted by I_2 and O_2 e.g., $I_2 = I - I_1$ and $O_2 = O - O_1$. If we denote by $\bar{c}_{ij}$ and $\bar{p}_{rj}$ the reductions of these inputs and outputs, respectively, the system-wide reductions are then $C_i = \sum_{j=1}^{n} \bar{c}_{ij}, i \in I_1$, and $P_r = \sum_{j=1}^{n} \bar{p}_{rj}, \ r \in O_1$. Let θ_j^* be the efficiency score of the jth DMU obtained from Eq. (6) before carrying out the input and output reduction. A feasible reduction program that does not cause any efficiency deterioration entails determining a feasible solution in u_r, v_i, $\bar{c}_{ij}$ and $\bar{p}_{rj}$ to the following set of constraints:

$$\begin{aligned} & \theta_j^* \leqslant \frac{\sum_{r \in O_2} u_r y_{rj} + \sum_{r \in O_1} u_r (y_{rj} - \bar{p}_{rj})}{\sum_{i \in I_2} v_i x_{ij} + \sum_{i \in I_1} v_i (x_{ij} - \bar{c}_{ij})} \leqslant 1, \ j = 1, 2, \ldots, n, && \text{(7i)} \\ & \sum_{j=1}^{n} \bar{c}_{ij} = C_i, \ i \in I_1, && \text{(7ii)} \\ & \sum_{j=1}^{n} \bar{p}_{rj} = P_r, \ r \in O_1, && \text{(7iii)} \\ & \bar{c}_{ij} \leqslant x_{ij}, \ i \in I_1, \ j = 1, 2, \ldots, n, && \text{(7iv)} \\ & \bar{p}_{rj} \leqslant y_{rj}, \ r \in O_1, \ j = 1, 2, \ldots, n, && \text{(7v)} \\ & u_r, \ v_i \geqslant \varepsilon, \ \bar{c}_{ij}, \bar{p}_{rj} \geqslant 0, \ r = 1, 2, \ldots, s, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n. \end{aligned} \tag{7}$$

Constraints (7i) prohibit efficiency deterioration of any DMU after input/output reduction and, as in the general DEA model, limit them to at most 1. Constraints (7ii) and (7iii) ensure that required aggregate input and output reductions are achieved. Constraints (7iv) and (7v) prevent inputs and outputs to be negative by the reductions. A feasible solution for (7) can be obtained by various methods such as the Jordan Gaussian method (see Datta (1994) for further details).

For equitability concerns, it is desirable to carry out the DMUs' input and output reduction in proportion to their share of the restricted inputs or outputs. After all, if a DMU is using 10% of an input which is to be restricted, it stands to reason that it should share 10% of the required reduction. Hence, an equitably ideal reduction program follows the following rules:

$$c_{ij}^* = \rho_{ij} C_i$$
$$p_{rj}^* = \mu_{rj} P_r$$

where $\rho_{ij} = x_{ij} / \sum_{g=1}^{n} x_{fg}, \ j = 1, \ldots, n, \ i \in I_1, f \in I$ and $\mu_{rj} = y_{rj} / \sum_{g=1}^{n} y_{zg}, \ j = 1, \ldots, n, \ r \in O_1, z \in O$. However, these rules may result in infeasibility in model (7). Because of this we propose a Goal Programming (GP) approach to extract a feasible solution from model (7) which deviates as little as possible from the ideal reductions. Let $u_r \bar{p}_{rj} = p_{rj}, \ v_i \bar{c}_{ij} = c_{ij}$.

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} \sum_{i \in I_1} \left(\alpha_{ij}^- + \alpha_{ij}^+\right) + \sum_{j=1}^{n} \sum_{r \in O_1} \left(\beta_{rj}^- + \beta_{rj}^+\right) \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^{m} v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \geqslant \theta_j^*, \qquad j = 1, \ldots, n, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj} - \sum_{r \in O_1} p_{rj}}{\sum_{i=1}^{m} v_i x_{ij} - \sum_{i \in I_1} c_{ij}} \leqslant 1, \quad j = 1, 2, \ldots, n, \\ & c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i(\rho_{ij} C_i), \quad i \in I_1, \quad j = 1, 2, \ldots, n, \\ & p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r(\mu_{rj} P_r), \quad r \in O_1, \quad j = 1, 2, \ldots, n, \\ & c_{ij} \leqslant v_i x_{ij}, \quad i \in I_1, \quad j = 1, 2, \ldots, n, \\ & p_{rj} \leqslant u_r y_{rj}, \quad r \in O_1, \quad j = 1, 2, \ldots, n, \\ & \sum_{j=1}^{n} c_{ij} = v_i C_i, \qquad i \in I_1, \\ & \sum_{j=1}^{n} p_{rj} = u_r P_r, \quad r \in O_1, \\ & u_r, v_i \geqslant \varepsilon; \quad c_{ij}, p_{rj}, \alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-, \beta_{rj}^+ \geqslant 0, \ r = 1, 2, \ldots, s; \\ & i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n. \end{aligned} \tag{8}$$

Allowing negative and positive deviations α_{ij}^- and α_{ij}^+ for $c_{ij} (i \in I_1)$, and β_{rj}^- and β_{rj}^+ for p_{rj} prevents infeasibility. We simply convert (9) into the following linear program problem:

$$\begin{aligned}
\zeta = \min \ & \sum_{j=1}^{n}\sum_{i\in I_1}\left(\alpha_{ij}^- + \alpha_{ij}^+\right) + \sum_{j=1}^{n}\sum_{r\in O_1}\left(\beta_{rj}^- + \beta_{rj}^+\right) \\
s.t. \quad & \left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{r\in O_1} p_{rj}\right) - \theta_j^*\left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{i\in I_1} c_{ij}\right) \geqslant 0,\ j=1,2,\ldots,n, \\
& \left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{r\in O_1} p_{rj}\right) - \left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{i\in I_1} c_{ij}\right) \leqslant 0,\ j=1,2,\ldots,n, \\
& c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i(\rho_{ij} C_i),\ i\in I_1,\ j=1,2,\ldots,n, \quad (9i) \\
& p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r(\mu_{rj} P_r),\ r\in O_1,\ j=1,2,\ldots,n, \quad (9ii) \\
& c_{ij} \leqslant v_i x_{ij},\ i\in I_1,\ j=1,2,\ldots,n, \\
& p_{rj} \leqslant u_r y_{rj},\ r\in O_1,\ j=1,2,\ldots,n, \\
& \sum_{j=1}^{n} c_{ij} = v_i C_i,\ i\in I_1, \\
& \sum_{j=1}^{n} p_{rj} = u_r P_r,\ r\in O_1, \\
& u_r, v_i \geqslant \varepsilon;\ c_{ij}, p_{rj}, \alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-, \beta_{rj}^+ \geqslant 0,\ r=1,2,\ldots,s; \\
& i=1,2,\ldots,m;\ j=1,2,\ldots,n.
\end{aligned} \tag{9}$$

Theorem 1. *There always exists a feasible solution to model (9).*

Proof. We have the following feasible solution to model (9):

$$v_i = \frac{\sum_{j=1}^{n} c_{ij}}{C_i} \quad i\in I_1; \quad u_r = \frac{\sum_{j=1}^{n} p_{rj}}{P_r}, \quad r\in O_1,$$

$$v_i = 1, \quad i\in I_2, \quad u_r = \frac{\gamma_j}{k y_{rj}} r\in O_2, \quad j=1,2,\ldots,n$$

where $\gamma_j = \theta_j^* \sum_{i\in I_2} x_{ij}$ and $|O_2| = k$.[1]

$$\sum_{r\in O_2} u_r y_{rj} = \sum_{r\in O_2} \frac{\gamma_j}{k y_{rj}} y_{rj} = \gamma_j, \quad j=1,2,\ldots,n$$

In addition, we have

$$c_{ij} = v_i x_{ij}, \quad i\in I_1, j=1,2,\ldots,n, \Rightarrow c_{ij} = \frac{x_{ij}\sum_{j=1}^{n} c_{ij}}{C_i} \qquad i\in I_1,\ j=1,2,\ldots,n$$

$$p_{rj} = u_r y_{rj}, \quad r\in O_1, \quad j=1,2,\ldots,n, \Rightarrow p_{rj} = \frac{y_{rj}\sum_{j=1}^{n} p_{rj}}{P_r} \quad i\in I_1,\ j=1,2,\ldots,n$$

$$\sum_{r\in O_1} u_r y_{rj} = \sum_{r\in O_1} p_{rj}$$

$$\sum_{i\in I_1} v_i x_{ij} = \sum_{i\in I_1} c_{ij}$$

Then, the optimal solutions of $\alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-$ and β_{rj}^+ can be directly obtained from constrains (9i) and (9ii) of model (9). Thus, the above solution satisfies all the constraints of (9). The proof is complete. □

Corollary 1. *Model (9) is unit-invariant.*

Proof. Let us assume $\left(u_r^*,\ v_i^*,\ c_{ij}^*,\ p_{rj}^*,\ \alpha_{ij}^{-*},\ \alpha_{ij}^{+*},\ \beta_{rj}^{-*},\ \beta_{rj}^{+*}\right)$ and ζ^* be the optimal solution and optimal value of model (9). In addition, we assume τx_{ij} and κy_{rj} for model (9) for positive scale factors. Then, we have (10):

$$\begin{aligned}
\bar{\zeta} = \min \ & \sum_{j=1}^{n}\sum_{i\in I_1}\left(\alpha_{ij}^- + \alpha_{ij}^+\right) + \sum_{j=1}^{n}\sum_{r\in O_1}\left(\beta_{rj}^- + \beta_{rj}^+\right) \\
s.t. \quad & \left(\sum_{r=1}^{s} u_r \kappa y_{rj} - \sum_{r\in O_1} p_{rj}\right) - \theta_j^*\left(\sum_{i=1}^{m} v_i \tau x_{ij} - \sum_{i\in I_1} c_{ij}\right) \geqslant 0, \\
& j=1,2,\ldots,n, \\
& \left(\sum_{r=1}^{s} u_r \kappa y_{rj} - \sum_{r\in O_1} p_{rj}\right) - \left(\sum_{i=1}^{m} v_i \tau x_{ij} - \sum_{i\in I_1} c_{ij}\right) \leqslant 0, \\
& j=1,2,\ldots,n, \\
& c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i(\rho_{ij} \kappa C_i), \quad i\in I_1, \quad j=1,2,\ldots,n, \\
& p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r(\mu_{rj} \tau P_r), \quad r\in O_1, \quad j=1,2,\ldots,n, \\
& c_{ij} \leqslant v_i \tau x_{ij}, \quad i\in I_1, \quad j=1,2,\ldots,n, \\
& p_{rj} \leqslant u_r \tau y_{rj}, \quad r\in O_1, \quad j=1,2,\ldots,n, \\
& \sum_{j=1}^{n} c_{ij} = v_i \kappa C_i, \quad i\in I_1, \\
& \sum_{j=1}^{n} p_{rj} = u_r \tau P_r, \quad r\in O_1, \\
& u_r, v_i \geqslant \varepsilon; \quad c_{ij}, p_{rj}, \alpha_{ij}^-, \alpha_{ij}^+, \beta_{rj}^-, \beta_{rj}^+ \geqslant 0, \quad r=1,2,\ldots,s; \\
& i=1,2,\ldots,m;\ j=1,2,\ldots,n.
\end{aligned} \tag{10}$$

The optimal solution of model (10) is $\left(\bar{u}_r = \frac{u_r^*}{\tau_r}, \bar{v}_i = \frac{v_i^*}{\kappa_i}, c_{ij}^*, p_{rj}^*, \alpha_{ij}^{-*}, \alpha_{ij}^{+*}, \beta_{rj}^{-*}, \beta_{rj}^{+*}\right)$. Therefore, the objective function of models (9) and (10) are the same, $\bar{\zeta}^* = \zeta^*$, and the proof is complete. □

Our approach, which is composed of a common reduction of resources without any effects on the individual efficiency scores, used GP as an extension of the Amirteimoori and Emrouznejad's models (2011, 2012) to solve this problem.

5. Comparison with the AE approach

In this section we compare our method with Amirteimoori and Emrouznejad's (2011) method (hereinafter named as the "AE approach") to demonstrate the advantages of the proposed method by comparing it to an alternative method. The AE approach proposed by Amirteimoori and Emrouznejad (2011) assesses the efficiency of a set of DMUs after reducing the values of a given input and output.

The AE approach aims is proposed to maintain the efficiency score of each DMU calculated from the standard DEA (CCR or CRS) model before reducing the input and output values. The comparison between the AE approach and the proposed method in this study can be summarized in the following three steps:

Step 1. The AE approach requires n repetitions of the multiplier CCR model where a CCR model typically involves $n + 1$ constraints and $m + s$ variables. However, we measure the efficiency scores of the DMUs by solving a single common-weights DEA model (5) which consists of n constraints and $n + m + s$ variables.

Step 2. This step specifies the given values of the input and output reduction that are characterized by the decision makers. The AE approach determines the input and output reduction for each DMU while maintaining their efficiency score obtained in Step 1. However, we determine the reduction values for the inputs and outputs and ensure that the efficiency scores of the DMUs remain greater than or equal to the efficiency scores of the DMUs in the previous step. The AE approach contains $2n \times (k + h + 1) + (n + 1)(k + h)$ constraints where k and h are the number of reduction indexes for the inputs and outputs, respectively, whereas our proposed model (9) includes $2n \times (k + h + 1) + k + h + m + s$ constraints where $m + s$ is the number of $u_r, v_i \geqslant \varepsilon$ constraints.

[1] $|q|$ represents the cardinality of q.

Moreover, the AE approach has $2n \times (k+h) + n \times (k+h+1) + m + s$ variables, respectively, while our model contains $m + s + 3n(k+h)$ variables. In summary, the method proposed in this study decreases the number of constraints and variables, which results in reducing the computational requirements.

Step 3. In this step, the efficiency scores of the DMUs are re-calculated according to the input and output reduction in Step 2. Similar to step 1, the AE approach solves the multiplier CCR model n times to measure the efficiency of the DMUs, whereas our method solves a single common-weights DEA model. As a result, our method solves $(n-1)$ less linear programs compared to the AE approach and this constitutes a computational advantage.

Naturally, the reduction in the computational burden is primarily linked to the adoption of the common-weights concept in all the steps while the AE approach uses a different method. Fig. 1 summarizes the comparisons between the AE approach and the method proposed in this study.

6. A numerical example

In this section, we use panel data from a banking application proposed by Kao and Hwang (2009) and also used by Amirteimoori and Emrouznejad (2011) to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedure. To assess the impact of information technology (IT) on bank performance, we take into account three inputs and two outputs as follows:

- The input 1 (x_1): IT budget (USD)
- The input 2 (x_2): Fixed assets (USD)
- The input 3 (x_3): Staff (headcount)
- The output 1 (y_1): Deposits (USD)
- The output 2 (y_2): Profit (USD)

The inputs and outputs data for 27 banks are reported in Table 1.

In the first step we apply the proposed model (5) to obtain the optimal common weights. Then we measure the efficiency score $\left(\theta_j^*\right)$ of the banks using Eq. (6) presented in Table 2. In the second step the banking system was forced to reduce the IT budget and the profit values owing to some exogenous financial constraints. Therefore, the present budget, 5.8916 billion dollars, must be reduced by 3 billion dollars (i.e., $C_1 = 3$). In such case, management expects that the bank's profits will shrink from 11.948 billion dollars to 6.948 billion dollars (i.e., $P_2 = 5$). To determine the adequate values of IT budget-profit reductions (denoted by $c_{1j} - p_{2j}$) for each bank, we first apply model (9) to get the optimal solutions of c_{1j} and p_{2j}, $j = 1,\ldots,27$. We then use alteration variables $u_r\bar{p}_{rj} = p_{rj}$ and $v_i\bar{c}_{ij} = c_{ij}$ to obtain the amount of IT budget and profit reduction denoted by $\bar{c}_{1j}$ and $\bar{p}_{2j}$, $j = 1,\ldots,27$, respectively. The optimal values of $\bar{c}_{1j}$ and $\bar{p}_{2j}$ are presented in Table 2. In the last step, to re-gauge the efficiency scores $\left(\theta_j^{*new}\right)$ of the banks we use the proposed common-weights DEA model (5) and Eq. (6) in the presence of the new values for IT budget and profit determined from the preceding step. The result is presented in Table 2. Note that the value of ε in model (5) is calculated by Eq. (2) which is 0.004582 before alteration and is 0.004622 after alteration.

The purpose of this example is to obtain the proper reduction in the input (IT budget) and output (profit) such that the efficiency score of each bank is maintained. As shown in Table 2, when we apply the proposed model, the new efficiency scores of the banks are never lower than the efficiency scores before decreasing the

Table 1
The input–output data for 27 banks.

DMU_j	x_1	x_2	x_3	y_1	y_2
1	0.150	0.713	13.300	14.478	0.232
2	0.170	1.071	16.900	19.502	0.340
3	0.235	1.224	24.000	20.952	0.363
4	0.211	0.363	15.600	13.902	0.211
5	0.133	0.409	18.485	15.206	0.237
6	0.497	5.846	56.420	81.186	1.103
7	0.060	0.918	56.420	81.186	1.103
8	0.071	1.235	12.000	11.441	0.199
9	1.500	18.12	89.510	124.072	1.858
10	0.120	1.821	19.800	17.425	0.274
11	0.120	1.915	19.800	17.425	0.274
12	0.050	0.874	13.100	14.342	0.177
13	0.370	6.918	12.500	32.491	0.648
14	0.440	4.432	41.900	47.653	0.636
15	0.431	4.504	41.100	52.63	0.741
16	0.110	1.241	14.400	17.493	0.243
17	0.053	0.450	7.6000	9.512	0.067
18	0.345	5.892	15.500	42.469	1.002
19	0.128	0.973	12.600	18.987	0.243
20	0.055	0.444	5.6000	7.546	0.153
21	0.057	0.508	5.7000	7.595	0.123
22	0.098	0.370	14.100	16.906	0.233
23	0.104	0.395	14.600	17.264	0.263
24	0.206	2.680	19.600	36.43	0.601
25	0.067	0.781	10.500	11.581	0.120
26	0.100	0.872	12.100	22.207	0.248
27	0.0106	1.757	12.700	20.67	0.253

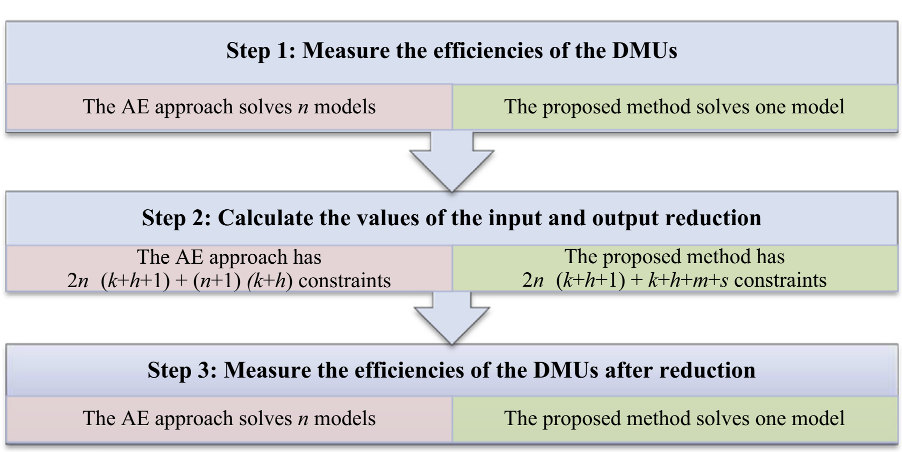

Fig. 1. A comparison between the proposed method and the AE approach.

Table 2
A comparison between the results of the AE and the proposed methods.

DMU_j	AE model				Proposed method			
	$\bar{c}_{ij}$	$\bar{p}_{rj}$	$\bar{\theta}_j^{*AE}$	$\bar{\theta}_j^{*newAE}$	$\bar{c}_{ij}$	$\bar{p}_{rj}$	θ_j^*	θ_j^{*new}
1	0.0763	0.0971	0.721	0.721	0.0764	0.0971	0.567	0.569
2	0.0866	0.1423	0.792	0.792	0.0866	0.1423	0.613	0.614
3	0.1197	0.1519	0.634	0.634	0.1197	0.1520	0.470	0.471
4	0.1074	0.0883	0.662	0.662	0.1074	0.0883	0.452	0.456
5	0.0677	0.0992	0.632	0.632	0.0677	0.0959	0.478	0.478
6	0.2531	0.4617	0.763	0.763	0.2531	0.4617	0.760	0.760
7	0.0305	0.4617	1.000	1.000	0.0305	0.4617	0.972	0.972
8	0.0361	0.0833	0.555	0.555	0.0361	0.0833	0.538	0.538
9	0.7638	0.7777	0.625	0.625	0.7638	0.7777	0.596	0.599
10	0.0611	0.1147	0.505	0.505	0.0611	0.1147	0.498	0.498
11	0.0611	0.1147	0.503	0.503	0.0611	0.1147	0.496	0.496
12	0.0255	0.0741	0.669	0.669	0.0255	0.0741	0.662	0.662
13	0.1884	0.2712	0.949	0.949	0.1884	0.2712	0.799	0.801
14	0.2240	0.2662	0.591	0.591	0.2240	0.2662	0.580	0.581
15	0.2195	0.3102	0.670	0.670	0.2195	0.3102	0.652	0.652
16	0.0560	0.1017	0.676	0.676	0.0560	0.1017	0.666	0.666
17	0.0270	0.0280	0.718	0.718	0.0270	0.0281	0.705	0.705
18	0.1757	0.4194	1.000	1.000	0.1757	0.4194	1.000	1.000
19	0.0652	0.1017	0.840	0.840	0.0652	0.1017	0.788	0.790
20	0.0280	0.0640	0.999	0.999	0.0280	0.0636	0.714	0.714
21	0.0290	0.0515	0.774	0.774	0.0290	0.0513	0.696	0.696
22	0.0499	0.0975	0.807	0.807	0.0499	0.0975	0.696	0.696
23	0.0530	0.1101	0.862	0.862	0.0530	0.1101	0.685	0.685
24	0.1049	0.2516	0.954	0.954	0.1049	0.2516	0.932	0.932
25	0.0341	0.0502	0.627	0.627	0.0341	0.0502	0.624	0.624
26	0.0509	0.1038	1.000	1.000	0.0509	0.1078	1.000	1.000
27	0.0509	0.1059	1.000	1.000	0.0054	0.1059	1.000	1.000
Sum	3.0454	4.9997	20.528	20.528	3.0000	5.0000	18.639	18.655

values of the IT budget and profit. It shows that this banking system is able to improve the operating efficiency of each bank branch.

Here we make a comparison between the results of the proposed method and the AE approach. Table 2 shows the efficiency score $\left(\bar{\theta}_j^{*AE}\right)$ of the banks using the CCR DEA model (1) as well as the optimal solutions of $\bar{c}_{ij}$ and $\bar{p}_{rj}$ using the AE approach. In addition, the renewal efficiency score $\left(\bar{\theta}_j^{*newAE}\right)$ of the banks is calculated using model (1) in the presence of $\bar{c}_{ij}$ and $\bar{p}_{rj}$. As shown in Table 2, the reduction values of our model is almost similar to the AE approach but our model involves less computational complexity. The AE efficiency score before reduction $\left(\bar{\theta}_j^{*AE}\right)$ and after reduction $\left(\bar{\theta}_j^{*newAE}\right)$ for each DMU is exactly identical while in our method the efficiency score is improved for some units, including DMU_1, DMU_2, DMU_3, DMU_4, DMU_9, DMU_{13}, DMU_{14} and DMU_{19}.

In this example $n = 27$, $m = 3$, $s = 2$, $k = 1$ and $h = 1$, and the AE approach solves 27 models where every model has 33 constraints and 5 variables. However, we solve only one model with 32 constraints and 32 variables. Furthermore, in this example, the AE approach includes 218 constraints and 194 variables while the model proposed in this study contains 169 constraints and 172 variables. Therefore, the proposed method is computationally more feasible than the AE approach because it uses 49 fewer constraints than the AE approach. In summary, the proposed method decreases the number of constraints and variables, which leads to a large reduction in the computational requirements.

7. Conclusions and future research directions

Allocative vs. technical efficiency, where the aggregate or average unit may not be cost efficient unless the input and output mix correspond to the optimal relations, is a well-known problem in the productivity literature. This can be implemented through reallocations of resources among the evaluated units in fully-integrated organizations (e.g., large firms or public organizations). A related problem known as the restricted reallocation is treated in Pachkova (2009). In real situations, the transfer of resources from one plant or department to another, perhaps across large distances and in a different organizational culture, may be associated with high costs or direct feasibility constraints. We consider another practical case, where transfers cannot be made but reductions or output increases have to be implemented. Amirteimoori and Emrouznejad (2011) have discussed the case of the budget cuts in higher education in England. However, this scenario is often found in large private firms with global coverage, where transfers of staff are infeasible in the short or medium run. Regularly, private firms face financial and market constraints, e.g., in business cycle downturns and in-between product generations, where central management is forced to implement resource reduction across the units to maintain economic viability. Our model could assist in such projects, since it maintains the participation of the units through a common evaluation system.

The integration of activity planning, resource allocation, and performance managements are current challenges in both the theory and the practice of DEA. In this paper, we propose a new model to improve the efficiency of the DMUs when some given inputs and/or outputs are reduced in the evaluation process. Our goal is to optimize the resource contraction such that the efficiency of all DMUs is improved or equal to the efficiency prior to the change. We first introduce a common-weights method for measuring the efficiency of the DMUs before and after the data change. Thus, we achieve the efficiencies by solving a linear program which is computationally economical. In addition, in comparison with the total weights flexibility in the traditional DEA models, the common-weights DEA model takes into account the common weights. Then, based on the GP concept we propose a new model to find an adequate assignment for the reduction amount of the inputs and outputs in the presence of the effect of the current data in the evaluation system. We used a scaling formula for the inputs and the outputs to achieve an equitable reduction in them. The proposed model is not only consistent with the outlined managerial objectives; it also significantly reduces the computational burden for the analysis.

The developed framework in this paper can potentially lend itself to many practical applications. However, there are a number of challenges involved in the proposed research that provide a great deal of fruitful scope for future research. For example, there is no mechanism in the proposed model to determine the adequate reduction values for integer inputs and outputs. Another potential for future research is to identify the upper and lower bounds for C_i and P_r, respectively, such that the assignment system for reducing input/output remains feasible.

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