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A three-stage Data Envelopment Analysis model with application to banking industry

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ABSTRACT

The changing economic conditions have challenged many financial institutions to search for more efficient and effective ways to assess their operations. Data Envelopment Analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous Decision Making Units (DMUs) by evaluating their relative efficiency. The traditional DEA treats DMUs as black boxes and calculates their efficiencies by considering their initial inputs and their final outputs. As a result, some intermediate measures are lost in the process of changing the inputs to outputs. In this study we propose a three-stage DEA model with two independent parallel stages linking to a third final stage. We calculate the efficiency of this model by considering a series of intermediate measures and constraints. We present a case study in the banking industry to exhibit the efficacy of the procedures and demonstrate the applicability of the proposed model.

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric performance evaluation method that was originally developed by Charnes et al. [19] and later extended by Banker et al. [8] to include variable returns to scale. DEA generalizes the Farrell's [36] single-input single-output technical efficiency measure to the multiple-input multiple-output case to evaluate the relative efficiency of peer units with respect to multiple performance measures [18,26]. The units under evaluation in DEA are called Decision Making Units (DMUs). A DMU is considered efficient when no other DMU can produce more outputs using an equal or lesser amount of inputs. The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs [27]. Unlike parametric methods which require detailed knowledge of the process, DEA does not require an explicit functional form relating inputs and outputs (see Cooper et al. [27] and Cook and Seiford [25] for an appraisal of the theoretical foundations and developments in DEA).

Although DEA can evaluate the relative efficiency of a set of DMUs, it cannot identify the sources of inefficiency in the DMUs because conventional DEA models view DMUs as

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black boxes that consume a set of inputs to produce a set of outputs [4]. In such cases, using single-stage DEA may result in inaccurate efficiency evaluation [82]. In contrast, a two-stage DEA model allows one to further investigate the structure and processes inside the DMU, to identify the misallocation of inputs among sub-DMUs and generate insights about the sources of inefficiency within the DMU [31,65].

1.1. Multi-stage DEA models

The existing multi-stage DEA models in the literature can be classified into two categories: closed-system and open-system models. In the closed-system DEA models, the intermediate outputs remain unchanged from one stage to another. In contrast, in the open-system DEA models, the intermediate outputs in one stage are partial inputs in a subsequent stage.

1.1.1. Two-stage closed DEA system

In this type of systems, unlike the first stage, the second stage has inputs that are the intermediate variables since the outputs of the first stage are the inputs of the second stage. Fig. 1 presents a graphical representation of a closed two-stage DEA system.

Seiford and Zhu [88] used a two stage network model to measure the profitability and marketability of American commercial banks. In the first stage, they use labor and assets as inputs to produce profitability as output of the first-stage. In the second stage, they use profitability from the first stage and marketability as inputs in the second stage to produce market value and earnings per share as outputs of the second-stage. Zhu [99] also used this two-stage network for Fortune Global 500 companies. Chilingerian and Sherman [23] used a two-stage procedure to measure the physician care. This two-stage procedure has also been used to evaluate the performance of mental health care programs [86], the education sector [69], information technology [21,22], and purchasing and supply management [84].

These methods produce three separate efficiency measures for the first stage, second stage, and the DMU as a whole with no consideration of the interactions between these components. Kao and Hwang [59] showed that the performance of the DMU is a combination of the performance of two stages with a chain relation between them. The efficiencies estimated from this two-stage DEA approach was more meaningful than those estimated from the independent two-staged DEA approaches. Kao and Hwang [59] used this method in a Taiwanese insurance company and compared their results with the results from the independent stage performance measurement models. Chen et al. [20] also proposed a DEA model similar to Kao and Hwang's [59] two-stage model, but in additive format.

1.1.2. Two-stage open DEA system

In this type of system, unlike for the first stage, the second stage has other inputs in addition to the intermediate variables and the outputs of the first stage are not necessarily inputs of the second stage. Fig. 2 presents a graphical representation of an open two-stage DEA system.

There are many cases in the real-world systems in which some outputs of one stage (e.g., parts in an automobile manufacturing plant) might be delivered to customers and the rest of the output will proceed to the next stages in the manufacturing process. Most of these models are in the form of DEA network systems. In the open-system models, each stage operates as an open system and gets the inputs from outside just as it may get some from the previous stages. Golany et al. [45] designed a performance measurement system that comprised of two linked sub-systems. Each sub-system uses separate resources and produces outputs. These resources could be labor or capital. Their network DEA could calculate the performance in each sub-system as well the overall performance in the entire system.

There are a number of examples for these chained processes where each sub-process uses other resources than the outputs of the previous stage. For example, consider the *production* and *delivery* sub-systems in a manufacturing system. Labor and raw materials are the inputs in the production sub-process and the finished goods are the outputs of this sub-process. The finished goods are also considered as inputs of the delivery sub-process. Other inputs of the delivery sub-process could be drivers and trucks and the final delivered product could be the output of the delivery sub-process. Liang et al. [66] applied this network concept to the performance measurement in a supply chain using Stackelberg game strategy (or leader–follower). In their two-stage model, the second stage receives inputs other than outputs of the first stage. Another network DEA was created for systems with more than two processes based on this assumption. Castelli et al. [14] studied two-stage and two-layer DMUs. Other examples of open-system multi-stage DEA models include Färe Fare and Whirraker [35], Färe and Grosskopf [30] and Tone and Tsutsui [94,95].

1.2. DEA models with undesirable variables

Modeling and consideration of undesirable outputs in productivity and performance measurement date back to

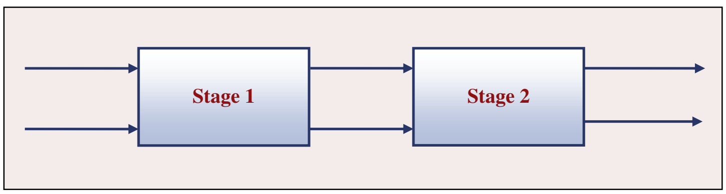

Fig. 1. A closed two-stage DEA system.

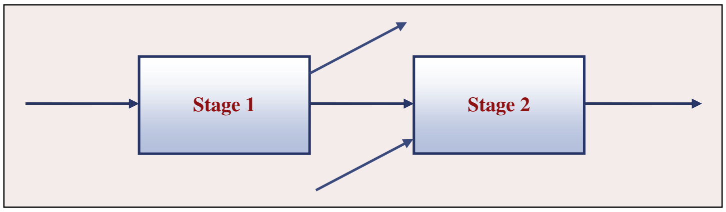

Fig. 2. An open two-stage DEA system.

1983 when Pittman [79] enhanced the multilateral productivity index of Caves et al. [16,17]. Following Pittman, Färe et al. [34] integrated this concept into Farrell's [36] technical efficiency measurement framework by introducing the new concept of "weak" versus "strong" disposability of undesirable outputs. DEA assumes that either making more output with the same input or making the same output with less input is a criterion for efficiency. In the presence of undesirable outputs, DMUs with more good (desirable) outputs and less bad (undesirable) outputs (relative to less input resources) should be categorized as efficient units. Färe et al. [34] modeled the weak disposability of undesirable outputs by changing the inequality constraint of the undesirable outputs to an equality constraint in the envelopment model. They expanded desirable outputs (by a linear constraint) and contracted undesirable outputs (by a non-linear constraint).

Färe et al.'s [34] work was the turning point in undesirable output modeling in DEA and other researchers joined him in studying undesirable outputs a decade later [50,32,49,64]. Recent DEA studies on performance measurement in the presence of undesirable outputs include Fukuyama and Weber [41], Avkiran [5], Barros et al. [9], and Hwang et al. [55] among others. Liu et al. [68] has given a systematic classification of DEA models which consider undesirable factors, including radial models, non-radial models, and slacks-based models.

Scheel [85] divided modeling of undesirable outputs into two groups: direct and indirect approaches. The direct approaches use the original values and perform the transformation by: (i) multiplying its value by -1 [62]; (ii) using the inverse of the undesirable outputs [46]; and (iii) multiplying the undesirable output by -1 and adding a positive scalar that is large enough to make the transformed value greater than zero [89]. In contrast, the indirect approaches transform the values of undesirable outputs by a monotone decreasing function to desirable outputs. As noted by Gomes and Lins [47] and Mahdiloo et al. [73], Seiford and Zhu's [89] approach is only applicable to models with the translation invariance property. Färe and Grosskopf [32] argued that undesirable outputs should be modeled under the assumption of weak disposability to be consistent with physical laws. Färe and Grosskopf [33] modeled the undesirable outputs with the directional distance function approach since Seiford and Zhu's [89] approach did not consider the disposability assumption for the undesirable outputs.

On the other hand, two of the most famous direct approaches deal with the undesirable outputs as inputs [52,80] and nonlinear abatement of undesirable outputs [34]. The idea of considering undesirable outputs as inputs adopted in the model proposed in this study is also discussed and justified in detail by Hailu and Veeman [50] and Korhonen and Luptacik [63].

We investigate the efficiency decomposition in a three-stage performance measurement system that has two independent parallel stages linking to a third final stage in series. In this three-stage performance measurement system, the outputs of the two parallel stages are the inputs in the third stage. We consider both desirable and undesirable variables and use the Chen and Zhu's [22] two-stage model to develop the three-stage DEA model proposed in this study.

The remainder of this paper is organized as follows. In Section 2, we review the DEA literature in banking. In Section 3, we show the structure of the proposed DEA model. In Section 4, we use a real-world case study in the banking industry to demonstrate the applicability of the proposed models and exhibit the efficacy of the procedures and algorithms. In Section 5, we conclude with our conclusions and future research direction.

2. DEA banking literature review

The field of DEA has grown immensely since the pioneering papers of Farrell [36] and Charnes et al. [19]. Numerous applications in recent years have been accompanied by new extensions and developments in expanding the concept and methodology of DEA (see Seiford [87] and Emrouznejad et al. [29] for an extensive bibliography of DEA). Evaluating the overall performance and monitoring the financial condition of commercial banks has been the focus of numerous research studies since the early works of Greenbaum [48] and Benston [11]. DEA has been widely used to measure the relative efficiency of a set of bank branches that possess shared functional goals with disproportionate inputs and outputs [39,3,91,24,82,4,67,6,72,90,5,38,42,58,61,74,96].

Two fundamental input–output systems are used to calculate bank efficiencies [53]: the production approach [81,28,44] and the intermediation approach [56,10,43,92,93,71,7,54]. There is no commonly accepted approach for measuring efficiency in the banking industry, which is why different efficiency scores are obtained using similar data [12].

Rho and An [82] extended two-stage DEA models by considering input and output slacks. They applied their model to the data from the banking industry and compared the results with those of the previous two-stage DEA models. Akther et al. [1] proposed a two-stage DEA model with a slacks-based inefficiency measure and directional technology distance function for evaluating the performance of private commercial banks in Bangladesh. Paradi et al. [77] proposed a two-stage DEA model for simultaneously benchmarking the performance of bank branches along different dimensions. Favero and Papi [37] used a sample of banks and determined which of the two DEA models are more appropriate for describing the efficiency level in banking: constant returns to scale (CRS) or variable returns to scale (VRS). Pastor et al. [78] used DEA and a non-parametric approach to estimate the efficiency in the European and U.S. banking systems. Yıldırım [97] used the DEA methodology to study the technical and scale efficiencies of the Turkish commercial banks. Casu and Girardone [15] used DEA to study the efficiency of the Italian banking system.

Isık and Hassan [57] showed that DEA methodology could be utilized to analyze the performance of banks in transition countries. Özkan-Günay and Tektas [76] used nonparametric DEA methodology to conduct a similar study. Yıldırım and Philippatos [98] evaluated the efficiency level of commercial banks in central and East-European countries by employing the stochastic frontier approach (SFA) and DFA techniques. Halkos and Tzeremes [51] proposed a bootstrapped DEA-based procedure to precalculate and pre-evaluate the short-run operating efficiency gains of a potential bank merger or acquisition.

A few studies have used DEA models with undesirable variables in the banking industry. Barros et al. [9] analyzed the technical efficiency of the Japanese banks based on the Russell directional distance function that takes into consideration not only desirable outputs but also an undesirable output represented by non-performing loans. Fujii et al. [40] examined the technical efficiency and productivity growth in the Indian banking sector by further modifying and extending the methodological approach introduced by Barros et al. [9]. Assaf et al. [2] analyzed the productivity and efficiency of Turkish banks. They obtained estimates of efficiency, productivity growth and efficiency growth using a Bayesian stochastic frontier approach. Lozano et al. [70] proposed a directional distance approach to deal with network DEA problems in which the processes may generate not only desirable final outputs but also undesirable outputs. They applied the proposed approach to the problem of modelling and benchmarking Spanish airport operations. Their network DEA model considered two processes (Aircraft Movement and Aircraft Loading) with two final outputs (Annual Passenger Movement and Annual Cargo handled), one intermediate product (Aircraft Traffic Movements) and two undesirable outputs (Number of Delayed Flights and Accumulated Flight' Delays).

3. The proposed DEA model

As we argued in the previous section, in order to properly evaluate the performance of multi-stage DMUs, we need to change the conventional approach and consider the intermediate measures in addition to the initial inputs and the final outputs. In this section we first present the Chen and Zhu's [22] two-stage model and we then describe the conventional closed system for a two-stage performance evaluation system with two parallel sub-DMUs and a third sub-DMU in series. Finally, we describe an open system DEA model and extend Chen and Zhu's [22] two-stage model to a two-stage performance evaluation system with parallel and series sub-DMUs proposed in this study.

3.1. Two-stage DEA model

Chen and Zhu [22] developed an efficiency model that identified the efficient frontier of a two-stage production process linked by intermediate measures. They used a set of firms in the banking industry to illustrate how the new model could be utilized to: (i) characterize the indirect impact of information technology on firm performance, (ii) identify the efficient frontier of two principal value-added stages related to information technology investment and profit generation, and (iii) highlight those firms that could be further analyzed for best practice benchmarking.

We consider the two-stage performance measurement model proposed by Chen and Zhu [22]. As shown in Fig. 3, the first stage uses x_i $(d = 1, 2, \ldots, m)$ inputs to produce z_d $(d = 1, 2, \ldots, D)$ outputs. The outputs of the first stage (z_d) are intermediary variables that are used as inputs in the second stage to produce the final outputs y_r $(r = 1, 2, \ldots, s)$.

Chen and Zhu [22] proposed the following two-stage DEA model for measuring the indirect effect of information technology on the firm's performance:

$$\begin{aligned}
&\max \quad w_1\alpha - w_2\beta\\
&s.t.\\
&(\text{Stage } 1)\\
&\sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \alpha x_{io} \quad i = 1,2,\ldots,m,\\
&\sum_{j=1}^{n}\lambda_j z_{dj} \geqslant \tilde{z}_{do} \quad d = 1,2,\ldots,D,\\
&\sum_{j=1}^{n}\lambda_j = 1,\\
&\lambda_j \geqslant 0 \quad j = 1,2,\ldots,n,\\
&(\text{Stage } 2)\\
&\sum_{j=1}^{n}\mu_j z_{dj} \leqslant \tilde{z}_{do} \quad d = 1,2,\ldots,D,\\
&\sum_{j=1}^{n}\mu_j y_{rj} \geqslant \beta y_{ro} \quad r = 1,2,\ldots,s,\\
&\sum_{j=1}^{n}\mu_j = 1,\\
&\mu_j \geqslant 0 \quad j = 1,2,\ldots,n.
\end{aligned} \tag{1}$$

Model (1) defines the efficient frontier of a two stage process, where x_{ij} is the ith input, y_{ij} is the rth output of DMU$_j$ $(i = 1, 2, \ldots, m,\ r = 1, 2, \ldots, s,\ j = 1, 2, \ldots, n)$, x_{io} and y_{ro} are the ith input and the rth output of the DMU under evaluation, respectively. In addition α and β are the efficiency scores corresponding to Stage 1 and Stage 2,

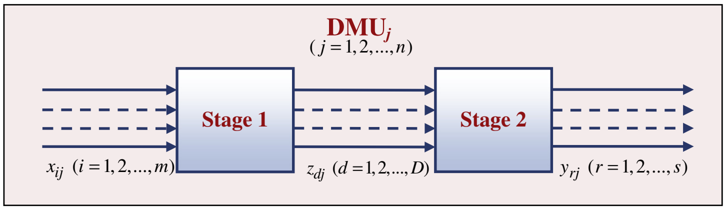

Fig. 3. Two-stage DMU [22].

respectively; and values of z_{dj} are the intermediary inputs which are outputs of Stage 1 and inputs of Stage 2 simultaneously. Symbol ~ represents unknown decision variables; so values of $\tilde{z}_{do}$ are unknown. Moreover, w_1 and w_2 are the weights reflecting the total preference over the two stages. When two Stages 1 and 2 have the same importance, the values of w_1 and w_2 will be equal and they add up to 1. The Chen and Zhu's [22] two-stage model proposes to minimize the inputs x_{ij} and maximize the final outputs y_{rj}, simultaneously. In addition, their model computes the intermediary variables in the most optimistic case. According to their model, firms that achieve $\alpha = \beta = 1$ in both stages are considered efficient.

3.2. Conventional closed-system BCC model

In this study we consider a three-stage open-system DEA model. The proposed model considers a three-stage performance measurement system with two independent parallel stages linking to a third final stage as shown in Fig. 4.

Let us further assume that the two parallel stages in Fig. 4 are independent and that the sub-DMUs A and B each have two independent input sets x^1 and x^2, respectively. In this DMU, the outputs w and v generated by sub-DMUs A and B, respectively in Stages 1 and 2 are consumed as inputs by sub-DMU C in Stage 3 to produce output y. There are many real-world DEA networks that match the structure depicted in Fig. 4. For example, a professional sports team typically implements a recruiting and budgeting process that works in parallel to recruit and budget for new players. The outputs of these two processes are then used as an input in the selection process where the coaches and team management decide which recruited players should be awarded a contract based on their performance in the trial rounds.

Now let us consider a network with n homogeneous DMUs and a structure similar to Fig. 4. Suppose DMU$_j$ $(j = 1, 2, \ldots, n)$ is the jth DMU under evaluation, x^1_{ij} $(i = 1, 2, \ldots, m)$ is the ith input of system A, w_{tj} $(t = 1, 2, \ldots, k)$ is the tth intermediary variable relative to sub-DMU A and sub-DMU C, x^2_{ej} $(e = 1, 2, \ldots, c)$ is the eth input of sub-DMU B, v_{rj} $(r = 1, 2, \ldots, h)$ is the rth intermediary variable relative to sub-DMU B and sub-DMU C, and y_{sj} $(s = 1, 2, \ldots, q)$ as sth output of sub-DMU C.

Next, we develop a three-stage performance evaluation system with two parallel sub-DMUs and a third sub-DMU in series similar to the structure depicted in Fig. 4. The conventional closed-system BBC models use a black box approach similar to the one presented in Fig. 5 to evaluate the performance of multi-level DMUs.

However, this approach does not take into account the internal tradeoffs among the sub-DMUs and the intermediary variables. The system is viewed as one black box system with a set of inputs and a set of outputs. In this structure, the efficiency of DMU$_o$ is measured by the BCC model (2) proposed by Banker et al. [8]:

$$
\begin{aligned}
&\max \frac{\sum_{r=1}^{s} u_r y_{ro} + u_o}{\sum_{i=1}^{m} v_i x_{io}} \\
&s.t. \\
&\frac{\sum_{r=1}^{s} u_r y_{rj} + u_j}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1, 2, \ldots, n, \\
&v_i \geqslant 0, \quad i = 1, 2, \ldots, m, \\
&u_r \geqslant 0, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{2}
$$

Model (2) is an input-oriented model and the optimal value of its efficiency is less than or equal to one. If the efficiency index is equal to one, DMU$_o$ is considered efficient and inefficient otherwise. However, the black box approach does not consider the internal mechanism of a DMU and its sub-DMUs.

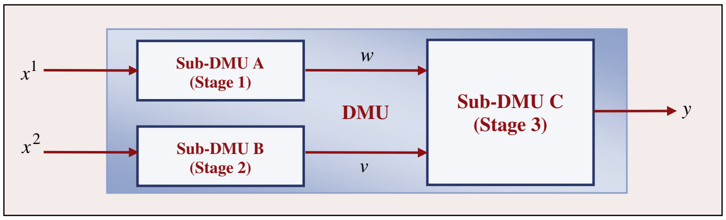

Fig. 4. Open system DMU with two parallel and one series sub-DMU.

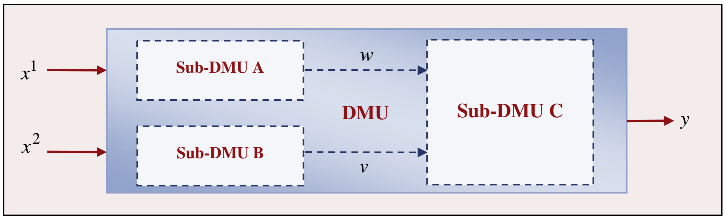

Fig. 5. Conventional closed-system DMU.

3.3. Open-system DEA model

Let us consider the open-system depicted in Fig. 4 and use Chen and Zhu's [22] two stage model to present a mathematical model (3) for this system as follows:

$$\begin{aligned}
&\max \quad w_1\theta + w_2\theta' - w_3\beta \\
&s.t. \\
&\sum_{j=1}^{n}\lambda_j x_{ij}^1 \leqslant \theta x_{io}^1 \quad i = 1,2,\ldots,m, \\
&\sum_{j=1}^{n}\lambda_j w_{tj} \geqslant \tilde{w}_{to} \quad t = 1,2,\ldots,k, \\
&\sum_{j=1}^{n}\lambda_j' x_{ej}^2 \leqslant \theta' x_{eo}^2 \quad e = 1,2,\ldots,c, \\
&\sum_{j=1}^{n}\lambda_j' v_{rj} \geqslant \tilde{v}_{ro} \quad r = 1,2,\ldots,h, \\
&\sum_{j=1}^{n}\pi_j w_{tj} \leqslant \tilde{w}_{to} \quad t = 1,2,\ldots,k, \\
&\sum_{j=1}^{n}\pi_j v_{rj} \leqslant \tilde{v}_{ro} \quad r = 1,2,\ldots,h, \\
&\sum_{j=1}^{n}\pi_j y_{sj} \geqslant \beta y_{so} \quad s = 1,2,\ldots,q, \\
&\sum_{j=1}^{n}\lambda_j = 1, \\
&\sum_{j=1}^{n}\lambda_j' = 1, \\
&\sum_{j=1}^{n}\pi_j = 1, \\
&\lambda_j, \lambda_j', \pi_j \geqslant 0 \quad j = 1,2,\ldots,n.
\end{aligned} \tag{3}$$

where θ, θ' and β are the efficiency scores of sub-DMUs A, B and C, respectively. With regard to unknown intermediate variables (i.e., w and v), model (3) is used to minimize the inputs (x^1, x^2) and simultaneously maximize the final output y.

3.4. Extended DEA model

Once again consider the Chen and Zhu's [22] two-stage DEA model. It can be seen that in addition to the efficiency scores, we have to obtain a set of optimal intermediate measures. As a result, generally additional constraints are added to the model to account for these intermediate variables. This modification helps in properly considering the indirect impact of the input variables on the final output variables. Consequently, intermediate variables are used to consider the limited trade-offs among the sub-DMUs and they are not really considered as the outputs of the first stage and the inputs in the second stage. In this study, we do not consider the intermediate variables as unknown decision variables and add additional constraints to the model. Instead, we minimize the input variables in the first stage and simultaneously maximize the output variables of the second stage. We consider this interpretation and apply the following changes to model (3):

- Constraints related to w_{tj}:

$$\sum_{j=1}^{n}\lambda_j w_{tj} \geqslant \tilde{w}_{to} \quad t = 1,2,\ldots,k, \tag{4}$$

$$\sum_{j=1}^{n}\pi_j w_{tj} \leqslant \tilde{w}_{to}, \quad t = 1,2,\ldots,k, \tag{5}$$

- Constraints related to v_{rj}:

$$\sum_{j=1}^{n}\lambda_j' v_{rj} \geqslant \tilde{v}_{ro} \quad r = 1,2,\ldots,h, \tag{6}$$

$$\sum_{j=1}^{n}\pi_j v_{rj} \leqslant \tilde{v}_{ro} \quad r = 1,2,\ldots,h, \tag{7}$$

By comparing relation (4) with relation (5) and relation (6) with relation (7), it is clear that we can aggregate the constraints related to intermediary variables and reduce the number of them by half as follows:

$$\sum_{j=1}^{n}\lambda_j w_{tj} \geqslant \sum_{j=1}^{n}\pi_j w_{tj} \quad t = 1,2,\ldots,k, \tag{8}$$

$$\sum_{j=1}^{n}\lambda_j' v_{rj} \geqslant \sum_{j=1}^{n}\pi_j v_{rj} \quad r = 1,2,\ldots,h, \tag{9}$$

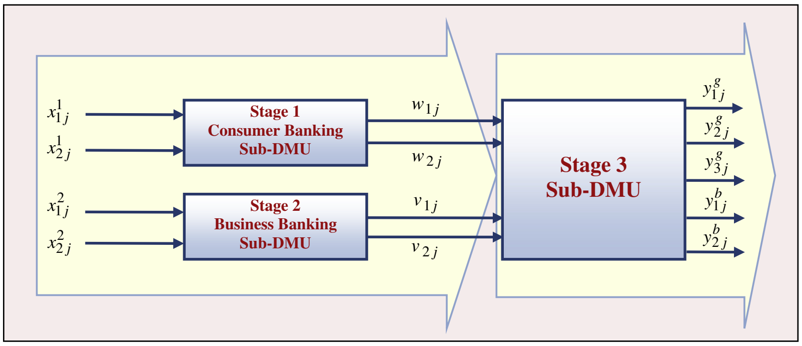

Fig. 6. Three-stage performance measurement system at Peoples Bank.

Constraints (8) ensure that the inputs of the sub-DMU C are not greater than the outputs of sub-DMU A and similarly constraints (9) ensure that the inputs of sub-DMU C do not exceed the outputs of sub-DMU B. Next, we generate the following new linear DEA model (1) by applying this modification to model (3):

$$\begin{aligned}
&\max\ w_1\theta + w_2\theta' - w_3\beta\\
&s.t.\\
&\sum_{j=1}^{n}\lambda_j x^1_{ij} \leqslant \theta x^1_{io} \quad i = 1,2,\ldots,m,\\
&\sum_{j=1}^{n}\lambda_j w_{tj} \geqslant \sum_{j=1}^{n}\pi_j w_{tj} \quad t = 1,2,\ldots,k,\\
&\sum_{j=1}^{n}\lambda'_j x^2_{ej} \leqslant \theta' x^2_{eo} \quad e = 1,2,\ldots,c,\\
&\sum_{j=1}^{n}\lambda'_j v_{rj} \geqslant \sum_{j=1}^{n}\pi_j v_{rj} \quad r = 1,2,\ldots,h,\\
&\sum_{j=1}^{n}\pi_j y_{sj} \geqslant \beta y_{so} \quad s = 1,2,\ldots,q,\\
&\sum_{j=1}^{n}\lambda_j = 1,\\
&\sum_{j=1}^{n}\lambda'_j = 1,\\
&\sum_{j=1}^{n}\pi_j = 1,\\
&\theta,\theta' \leqslant 1,\\
&\beta \geqslant 1,\\
&\lambda_j,\lambda'_j,\pi_j \geqslant 0, \quad j = 1,2,\ldots,n.
\end{aligned} \tag{10}$$

The extended model is proposed to minimize the inputs $X_j = \left(x^1_{ij}, x^2_{ej}\right)$ and simultaneously maximize the final outputs $Y_j = (y_{sj})$. In addition, w_1, w_2 and w_3 are the subjective importance weights assigned to the efficiency scores in sub-DMUs A, B and C, respectively. Several approaches such as point allocation, paired comparisons, trade-off analysis and regression estimates could be used to specify these importance weights [60]. Alternatively, the pairwise comparisons and eigenvalue theory proposed by Saaty [83] could be used to determine a suitable weight for the efficiency score the sub-DMUs. We also add constraints θ, $\theta' \leqslant 1$ to the proposed model to ensure that the inputs do not exceed a maximum pre-specified level as well as constraint $\beta \geqslant 1$ to ensure that the outputs meet a minimum pre-specified level. Note that $\theta = \theta' = \beta = 1$ and $\lambda_o = \lambda'_o = \pi_o = 1$, $\lambda_j = \lambda'_j = \pi_j = 0$ $(j\ ne\ 0)$ are feasible solutions in model (10).

4. Peoples Bank[1] case study

The primary role of a bank is to efficiently transform savings into investments. Successful investments build up the capital in the economy and foster future growth. Although banks are not the only financial institutions, they play a dominant role in the local and regional economy.

4.1. Extended model of a banking system

Peoples Bank, the largest local bank in the State of East Virginia, is feeling the crunch due to under-evaluation. As shown in Fig. 6, the three-stage performance measurement system Peoples Bank is comprised of three stages. Stage 1 and Stage 2 represent two sub-DMUs of consumer banking and business banking, respectively. We assume that these two sub-DMUs operate independently and there are no trade-offs between them. The consumer banking sub-DMU consumes 2 inputs and produces 2 outputs. Similarly, the business banking sub-DMU consumes 2 inputs and produces 2 outputs. The four outputs of the first and second stages are inputs in the third stage. Stage 3 consumes these 4 inputs and produces 3 good outputs and two bad outputs.

More specifically, the consumer banking sub-DMU uses operational costs $\left(x^1_{1j}\right)$ and capital costs $\left(x^1_{2j}\right)$, and the business banking sub-DMU uses operational costs $\left(x^2_{1j}\right)$ and capital costs $\left(x^2_{2j}\right)$ as inputs to generate consumer checking deposits (w_{1j}), consumer saving deposits (w_{2j}), business checking deposits (v_{1j}), and business saving deposits as the intermediate measures (Stage 1 and Stage 2 outputs/Stage 3 inputs). In the third stage, the intermediate measures are used to produce a mix of good (desirable)

[1] The names are changed to protect the anonymity of the bank.

and bad (undesirable) outputs. Return on assets $\left(y^g_{1j}\right)$, user fees income $\left(y^g_{2j}\right)$ and interest income $\left(y^g_{3j}\right)$ are the good (desirable) outputs and consumer loan delinquencies $\left(y^b_{1j}\right)$ and business loan delinquencies $\left(y^b_{2j}\right)$ are the bad (undesirable) outputs (see Table 1).

It can be seen that the performance measurement system in Fig. 6 is similar to the performance measurement system in Fig. 4. Therefore, we apply the proposed model (10) to the DEA system at Peoples Bank. The data set consisted of 49 branches throughout the State of East Virginia. One important point to note is that we have bad outputs in Stage 3 (final stage). To address this problem, we should apply a few alterations into the objective function and the constraints in model (10).

In this study, we treat bad outputs similar to the inputs and try to reduce their values because of their undesirability. Hence, we add a new variables, δ, to minimize the bad outputs in model (10). Also we use different weight values based on ε to prioritize our goals ($\varepsilon = 0.01$). The modified model can be expressed as follows:

$$
\begin{aligned}
&\max \quad \varepsilon\theta + \varepsilon\theta' + \varepsilon^2\delta - \varepsilon^3\beta \\
&s.t. \\
&\sum_{j=1}^{n}\lambda_j x^1_{ij} \leqslant \theta x^1_{io} \quad i = 1,2,\ldots,m, \\
&\sum_{j=1}^{n}\lambda_j w_{tj} \geqslant \sum_{j=1}^{n}\pi_j w_{tj} \quad t = 1,2,\ldots,k, \\
&\sum_{j=1}^{n}\lambda'_j x^2_{ej} \leqslant \theta' x^2_{eo} \quad e = 1,2,\ldots,c, \\
&\sum_{j=1}^{n}\lambda'_j v_{rj} \geqslant \sum_{j=1}^{n}\pi_j v_{rj} \quad r = 1,2,\ldots,h, \\
&\sum_{j=1}^{n}\pi_j y^g_{sj} \geqslant \beta y^g_{so} \quad s = 1,2,\ldots,q, \\
&\sum_{j=1}^{n}\pi_j y^b_{lj} \leqslant \beta y^b_{lo} \quad l = 1,2,\ldots,p, \\
&\sum_{j=1}^{n}\lambda_j = 1, \\
&\sum_{j=1}^{n}\lambda'_j = 1, \\
&\sum_{j=1}^{n}\pi_j = 1, \\
&\theta, \theta', \delta \leqslant 1, \\
&\beta \geqslant 1, \\
&\lambda_j, \lambda'_j, \pi_j \geqslant 0 \quad j = 1,2,\ldots,n.
\end{aligned}
\tag{11}
$$

The weights assigned to the efficiency values in the objective function of model (11) indicate that the reduction in the input of the parallel sub-systems is most important. Reduction of the bad outputs and increasing the good outputs are the third and fourth priorities, respectively. We note that θ, θ' and β are the efficiency values of the consumer banking unit (Stage 1), the business banking unit (Stage 2), and the Stage 2 sub-DMUs. Next, we analyze the performance of the 49 Peoples Bank branches in the State of East Virginia.

Table 1
Peoples Bank inputs, intermediate measures, and outputs.

Stage 1 Consumer banking inputs	x^1_{1j}	Consumer banking operational costs
	x^1_{2j}	Consumer banking capital costs
Stage 2 Business banking inputs	x^2_{1j}	Business banking operational costs
	x^2_{2j}	Business banking capital costs
Intermediate measures (Stage 1 and Stage 2 outputs) (Stage 3 inputs)	w_{1j}	Consumer checking deposits
	w_{2j}	Consumer saving deposits
	v_{1j}	Business checking deposits
	v_{2j}	Business saving deposits
Stage 3 outputs	y^g_{1j}	Return on assets
	y^g_{2j}	User fees income
	y^g_{3j}	Interest income
	y^b_{1j}	Consumer loan delinquencies
	y^b_{2j}	Business loan delinquencies

4.2. Analysis and results

The input and output values for the 49 branches of Peoples Bank are presented in Appendix A. The extended model report for the 49 branches of Peoples Bank is presented in Table 2.

As shown in Table 2, the second to fifth columns illustrate the values of the sub systems' efficiencies. The numbers in the parentheses represents the ranking of each bank branch with regards to its efficiency. The sixth column is created on order to easily compare the branches. This column presents the average of the sub systems' efficiencies. As shown in this table, bank branches 7 and 49 possess the value of 1 in all four columns indicating that these branches are efficient in all their sub-systems. In other words, it is suggested that most branches are efficient in Stage 2 and are inefficient in Stage 1. The last column in Table 2 presents the average efficiency value for each sub-system. Branches 7, 30, 42 and 49 showed 100% efficiency in consumer banking while the least efficiency was shown by branch 2, 24, 35 and 48. Branches 10, 21, 42, 30, 23, 24, 45 were 100% efficient in three efficiency measures while they were inefficient only in one part. Branches 1, 2, 6, 8, 14, 12, 11, 9, 15, 16, 17, 18, 19, 20, 22, 25, 26, 27, 31, 32, 35, 36, 37, 38, 40, 41, 43, 44, 46, 47, 48 or in other words 63% of the branches were efficient in Stage 2 and inefficient in Stage 1 of their operations.

A comparison of the sub-system rankings reveals that most branches have similar rankings in components and the overall average which supports the parallelization of the entire performance measurement system at Peoples Bank. In Stage 3, most branches follow the same scenario but in Stages 1 and 2, branches 25 and 45 have a noticeable gap in consumer banking and business banking sub-sys-

Table 2
Extended model results for Peoples Bank.

Branches	θ	θ'	δ	β	Average
1	0.342(26)	0.358(29)	1.000(1)	1.000(2)	0.657(27)
2	0.102(46)	0.126(44)	1.000(1)	1.000(2)	0.557(46)
3	0.365(24)	0.783(12)	0.923(46)	1.000(2)	0.768(20)
4	0.583(13)	0.937(9)	0.976(45)	1.000(2)	0.874(11)
5	0.211(33)	0.257(34)	0.999(40)	1.000(2)	0.616(34)
6	0.795(9)	0.441(27)	1.000(1)	1.000(2)	0.809(14)
7	1.000(1)	1.000(1)	1.000(1)	1.000(2)	1.000(1)
8	0.156(39)	0.080(49)	1.000(1)	1.000(2)	0.559(45)
9	0.312(27)	0.262(33)	1.000(1)	1.000(2)	0.643(32)
10	0.692(11)	1.000(1)	1.000(1)	1.000(2)	0.923(9)
11	0.158(38)	0.147(40)	1.000(1)	1.000(2)	0.576(41)
12	0.147(41)	0.160(38)	1.000(1)	1.000(2)	0.577(39)
13	0.423(20)	0.266(32)	0.987(42)	1.000(2)	0.669(28)
14	0.209(34)	0.289)31)	1.000(1)	1.000(2)	0.624(33)
15	0.968(5)	0.814(10)	1.000(1)	1.000(2)	0.945(5)
16	0.185(35)	0.138(43)	0.984(44)	1.000(2)	0.577(39)
17	0.136(44)	0.109(47)	1.000(1)	1.000(2)	0.561(44)
18	0.152(40)	0.115(46)	1.000(1)	1.000(2)	0.566(43)
19	0.304(29)	0.806(11)	1.000(1)	1.000(2)	0.777(18)
20	0.429(18)	0.494(26)	1.000(1)	1.000(2)	0.730(24)
21	0.858(8)	1.000(1)	1.000(1)	1.000(2)	0.964(3)
22	0.382(23)	0.222(35)	1.000(1)	1.000(2)	0.651(31)
23	0.642(12)	1.000(1)	1.000(1)	1.000(2)	0.910(10)
24	0.100(47)	1.000(1)	1.000(1)	1.000(2)	0.775(19)
25	0.492(15)	0.689(20)	1.000(1)	1.000(2)	0.795(16)
26	0.251(30)	0.189(37)	1.000(1)	1.000(2)	0.610(36)
27	0.139(43)	0.142(42)	1.000(1)	1.000(2)	0.570(42)
28	0.147(41)	0.095(48)	0.837(48)	1.000(2)	0.519(49)
29	0.463(16)	0.712(16)	0.991(41)	1.000(2)	0.791(17)
30	1.000(1)	0.722(14)	1.000(1)	1.000(2)	0.930(6)
31	0.421(21)	0.543(23)	1.000(1)	1.000(2)	0.741(23)
32	0.946(6)	0.768(13)	1.000(1)	1.000(2)	0.928(8)
33	0.311(28)	0.147(40)	0.713(49)	1.000(2)	0.542(48)
34	0.894(7)	0.971(8)	0.986(43)	1.0362(1)	0.954(4)
35	0.072(48)	0.534(24)	1.000(1)	1.000(2)	0.651(30)
36	0.245(31)	0.155(39)	1.000(1)	1.000(2)	0.600(37)
37	0.168(36)	0.500(25)	1.000(1)	1.000(2)	0.667(29)
38	0.580(14)	0.700(17)	1.000(1)	1.000(2)	0.802(15)
39	0.226(32)	0.351(30)	0.874(47)	1.000(2)	0.612(35)
40	0.429(18)	0.580(22)	1.000(1)	1.000(2)	0.752(22)
41	0.440(17)	0.429(28)	1.000(1)	1.000(2)	0.717(25)
42	1.000(1)	0.715(15)	1.000(1)	1.000(2)	0.928(7)
43	0.163(37)	0.210(36)	1.000(1)	1.000(2)	0.593(38)
44	0.749(10	0.700(17)	1.000(1)	1.000(2)	0.862(12)
45	0.363(25)	1.000(1)	1.000(1)	1.000(2)	0.840(13)
46	0.125(45)	0.694(19)	1.000(1)	1.000(2)	0.704(26)
47	0.402(22)	0.640(21)	1.000(1)	1.000(2)	0.760(21)
48	0.064(49)	0.123(45)	1.000(1)	1.000(2)	0.546(47)
49	1.000(1)	1.000(1)	1.000(1)	1.000(2)	1.000(1)
Average	0.429	0.512	0.985	1	0.729

tems. The proposed model was able to pinpoint the efficient and inefficient sub-systems for Peoples Bank branches throughout the State of East Virginia.

5. Conclusion and future research directions

The conventional DEA models view DMUs as black boxes that consume a set of inputs to produce a set of outputs and do not take into consideration the intermediate measures within a DMU. As a result, some intermediate measures are lost in the process of changing the inputs to outputs. In this study, we investigated the efficiency decomposition in a three-stage performance measurement system with two independent parallel stages linking to a third final stage in series. Herewith, we have extended Chen and Zhu's [22] two-stage model to a three-stage model where the outputs of the parallel stages were the inputs in the final stage. We presented a case study and exhibited the efficacy of the procedures and demonstrated the applicability of the proposed method to a three-stage performance evaluation problem in the banking industry. The DEA system in the case study was comprised of three stages. Stages 1 and 2 were the two sub-DMUs of consumer banking and business banking. We assumed that these two sub-DMUs were independent and there was no trade-offs between them. The outputs of the first and second stages were inputs in the third stage.

Performance measurement in the banking industry is most beneficial to management who monitor performance and to regulators who monitor financial stability while attempting to detect distress. In addition, investors and market analysts are also interested in ranking financial institutions and banks for inclusion in their investment portfolios. The additional benefits of performance measurement studies include ranking bank branches on efficiency estimates, monitoring performance of bank branches, reallocating resources from one bank to another, measuring the impact of internal and external factors on bank branch efficiency, and understanding regional and geographical differences in efficient frontiers and so on. Clearly, DEA has some assumptions that represents the core weaknesses of the technique. For example, DEA assumes no measurement error and measurement error can cause significant problems. In addition, DEA does not measure "absolute" efficiency, statistical tests are not applicable in DEA, and large problems can be computationally intensive.

The capacity of multinational firms to assess the efficiency of their subsidiaries within a value-chain environment plays a central role in their international expansion decisions [13,75]. While this study considered similar bank branches at the regional level, the application of the proposed method at the international level would constitute an interesting extension. Additional future research directions could include a more sophisticated performance measurement system where there are some interdependencies and trade-offs among the inputs and outputs of the sub-DMUs. Another extension of this research is to build a model where some intermediate outputs of the two parallel sub-DMUs exit the system without ever entering the third stage in the performance evaluation system.

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Appendix A

Input and output values for the Peoples Bank branches

Branch	Consumer banking (Stage 1)				Business banking (Stage 2)				Stage 3				
	x^1_{1j}	x^1_{2j}	w_{1j}	w_{2j}	x^2_{1j}	x^2_{2j}	v_{1j}	v_{2j}	y^g_{1j}	y^g_{2j}	y^g_{3j}	y^b_{1j}	y^b_{2j}
1	1.4E+11	8.6E+10	5E+13	4.8E+13	3.28E+12	4.03E+11	4.5E+14	9.8E+12	1.8E+12	6.2E+11	1.8E+12	2.6E+12	4.6E+13
2	2.6E+11	8E+10	3.2E+13	2.3E+13	7.87E+12	4E+11	2.7E+14	6.5E+12	4.6E+12	9E+09	4.6E+12	2E+12	4.6E+13
3	3.2E+11	1E+11	3.1E+13	4.8E+13	2.84E+12	6.48E+11	4.8E+14	1.2E+13	4.3E+12	2E+11	4.3E+12	3.2E+12	5.1E+13
4	3.7E+11	9E+10	2.8E+13	6.1E+13	4.05E+12	5.14E+11	4.9E+14	1.1E+13	5.2E+12	1.1E+12	5.2E+12	3.2E+12	5.3E+13
5	1.9E+11	2.3E+11	5E+13	4.8E+13	3.31E+12	1.09E+12	4.6E+14	1E+13	4.7E+12	4.4E+11	4.7E+12	2.6E+12	4.4E+13
6	2.1E+10	1.7E+10	1.9E+13	2.2E+13	1.65E+12	9.37E+10	2.2E+14	3.4E+12	1.2E+12	4.7E+11	1.2E+12	2.3E+12	5.4E+13
7	3.2E+10	2E+10	5.2E+13	3.7E+13	1.42E+11	1.07E+11	4.7E+14	9.1E+12	4.2E+12	1.1E+12	4.2E+12	2.6E+12	4.1E+13
8	4.4E+11	2.6E+11	6E+13	3.7E+13	5.47E+12	1.26E+12	4.6E+14	8.9E+12	6.1E+12	1.8E+12	6.1E+12	2.6E+12	3.9E+13
9	7.5E+10	4.5E+10	4.8E+13	2.7E+13	2.41E+12	2.26E+11	3.7E+14	7.7E+12	2.8E+12	8.2E+11	2.8E+12	2.6E+12	5.1E+13
10	4.6E+11	1.2E+11	3.1E+13	5.7E+13	3.56E+12	6.79E+11	5E+14	1.1E+13	7.2E+12	9E+11	7.2E+12	3.1E+12	5.8E+13
11	1.5E+11	5.1E+10	2.3E+13	2.3E+13	5.49E+12	2.75E+11	2.5E+14	4.8E+12	5.5E+12	1.9E+11	5.5E+12	2.9E+12	5E+13
12	1.7E+11	5.6E+10	2.7E+13	2.4E+13	3.63E+12	2.88E+11	2.6E+14	4.6E+12	5.3E+11	1.8E+11	5.3E+11	2.5E+12	4.7E+13
13	1.1E+11	8.8E+10	6.3E+13	4.2E+13	3.29E+12	4.03E+11	4.8E+14	9.6E+12	4.4E+12	8.4E+11	4.4E+12	2.7E+12	4.3E+13
14	3.9E+11	4.1E+10	4.4E+13	2.7E+13	6.29E+12	2.03E+11	3.5E+14	7.3E+12	8.5E+12	2.8E+12	8.5E+12	2.5E+12	4.9E+13
15	3.8E+10	8.6E+09	4.9E+13	2.8E+13	2.73E+12	4.17E+10	3.7E+14	7E+12	2.6E+12	6.5E+11	2.6E+12	2.6E+12	5.1E+13
16	9.1E+10	5.9E+10	2.7E+13	2.1E+13	5.03E+12	3E+11	2.4E+14	3.6E+12	5.9E+12	9.1E+11	5.9E+12	2.9E+12	5.1E+13
17	2E+11	6E+10	2.5E+13	2.2E+13	1.29E+13	3.32E+11	2.6E+14	4.9E+12	6.9E+12	2.4E+12	6.9E+12	2.5E+12	4.7E+13
18	1.3E+11	7.8E+10	3.4E+13	2.2E+13	9.04E+12	3.48E+11	2.5E+14	4.5E+12	9.9E+12	7.6E+11	9.9E+12	2.8E+12	5.1E+13
19	2.6E+11	8.9E+10	4.4E+13	3.9E+13	6.32E+12	5.12E+11	4.7E+14	1.1E+13	4.2E+12	4.2E+11	4.2E+12	2.5E+12	4.5E+13
20	7.4E+10	5.8E+10	6.6E+13	3.8E+13	3.32E+12	2.75E+11	4.8E+14	9E+12	4.5E+12	4.6E+11	4.5E+12	2.6E+12	4.3E+13
21	4E+11	7.1E+10	2.8E+13	5.1E+13	3.47E+12	4.6E+11	5E+14	1.1E+13	1.2E+12	3.9E+11	1.2E+12	2.8E+12	5.6E+13
22	4.5E+10	3.6E+10	1.9E+13	2.2E+13	1.71E+12	1.96E+11	2.2E+14	3.5E+12	1.5E+12	2.3E+11	1.5E+12	2.3E+12	5.4E+13
23	5.4E+11	2.5E+11	2.2E+13	6E+13	3.32E+12	1.58E+12	5.1E+14	1.2E+13	5.4E+12	2.8E+12	5.4E+12	4.5E+12	5.4E+13
24	2.9E+11	9.5E+10	4.1E+13	2.2E+13	1.07E+11	4.3E+11	2.7E+14	7E+12	1.9E+12	2.9E+11	1.9E+12	1.9E+12	4.6E+13
25	2.3E+11	1E+11	4.8E+13	4.4E+13	3.25E+12	5.47E+11	4.8E+14	1.1E+13	8.6E+12	1.7E+12	8.6E+12	2.5E+12	4.2E+13
26	6.5E+10	5.7E+10	2.6E+13	2.2E+13	1.68E+12	2.78E+11	2.3E+14	3.5E+12	3.2E+12	2.5E+11	3.2E+12	2.2E+12	5.5E+13
27	4.1E+11	5.1E+10	2.1E+13	2.3E+13	7.4E+12	3.05E+11	2.6E+14	5E+12	1E+13	1.5E+12	1E+13	3E+12	5.5E+13
28	1.1E+11	1.4E+11	2.9E+13	2.1E+13	2.74E+12	7.13E+11	2.4E+14	4.2E+12	2.8E+12	3.9E+11	2.8E+12	2.9E+12	5.2E+13
29	2.5E+09	3.3E+09	4.6E+12	3.5E+12	5.78E+11	2.99E+10	7.3E+13	1.1E+12	3.2E+11	7.2E+10	3.2E+11	1.6E+12	1.8E+13
30	3E+09	2.5E+09	7.2E+12	5.7E+12	6.23E+11	1.48E+10	7.5E+13	1.3E+12	5.1E+11	6.8E+10	5.1E+11	1.2E+12	5.5E+12
31	1.6E+10	6.3E+09	5.9E+12	7.7E+12	7.77E+11	3.42E+10	7.2E+13	1.6E+12	6.1E+11	6.2E+11	6.1E+11	1.2E+12	7.4E+12
32	1.3E+09	2.3E+09	4.3E+12	3.2E+12	5.55E+11	2.26E+10	7.2E+13	1.1E+12	2.3E+11	1.8E+11	2.3E+11	1.6E+12	1.8E+13
33	7.7E+09	9.7E+09	6.2E+12	5E+12	3.05E+12	7.33E+10	8.3E+13	1.6E+12	2.9E+12	1.7E+11	2.9E+12	1.9E+12	1.8E+13
34	1.1E+09	2.4E+09	8.9E+12	2.7E+12	4.58E+11	1.23E+10	5.8E+13	1.1E+12	3E+11	1.3E+11	3E+11	1.2E+12	5.7E+12
35	2.9E+10	1.6E+10	5.1E+12	8E+12	6.14E+11	8.86E+10	7.1E+13	1.7E+12	5.8E+11	8.9E+11	5.8E+11	1.6E+12	8.4E+12
36	1.2E+10	7.1E+09	5.1E+12	5E+12	4.6E+12	6.04E+10	8.5E+13	1.9E+12	1.9E+12	1.7E+12	1.9E+12	1.5E+12	1.7E+13
37	1.3E+10	9.8E+09	6.5E+12	4.5E+12	7E+11	8.13E+10	8.9E+13	3E+12	2.4E+11	6.4E+10	2.4E+11	1.3E+12	1.6E+13
38	9.9E+09	6.5E+09	7.3E+12	7.6E+12	5.87E+11	3.11E+10	7E+13	1.6E+12	1E+12	6.2E+11	1E+12	1.2E+12	5.6E+12
39	6.4E+09	1.6E+10	6.4E+12	4.4E+12	9.74E+11	1.22E+11	8E+13	1.6E+12	8.5E+11	7.6E+10	8.5E+11	2E+12	1.9E+13
40	6.7E+09	1.4E+10	7.7E+12	7.6E+12	6.32E+11	6.03E+10	6.7E+13	1.4E+12	5.5E+11	3.5E+10	5.5E+11	1.2E+12	5.9E+12
41	1.1E+10	4.7E+09	6.6E+12	6.3E+12	1.05E+12	2.45E+10	6.6E+13	1.6E+12	4.8E+11	8E+10	4.8E+11	1.2E+12	5.7E+12
42	5.1E+09	5.8E+09	8.3E+12	8.5E+12	5.98E+11	2.36E+10	6.7E+13	1.4E+12	1.3E+11	1.2E+11	1.3E+11	1.2E+12	6.1E+12
43	1.3E+10	1.1E+10	8.2E+12	4.9E+12	2.06E+12	7.78E+10	8.8E+13	2.8E+12	6.6E+11	2.8E+10	6.6E+11	1.4E+12	1.7E+13
44	4.2E+09	4.2E+09	8.4E+12	5.8E+12	6.18E+11	2.22E+10	7.3E+13	1.4E+12	5.2E+11	5.5E+11	5.2E+11	1.2E+12	5.8E+12
45	1.3E+10	5.4E+09	6.2E+12	7.5E+12	3.89E+11	2.77E+10	6.9E+13	1.6E+12	4.7E+11	3.8E+10	4.7E+11	1.2E+12	6.8E+12
46	2.3E+10	6.8E+09	6.6E+12	8.9E+12	5.87E+11	3.23E+10	7.2E+13	1.6E+12	8.4E+11	1.1E+11	8.4E+11	1.3E+12	8.5E+12
47	3.8E+09	6.1E+09	5.5E+12	3.6E+12	5.88E+11	4.94E+10	7.4E+13	1.1E+12	8E+11	6.8E+10	8E+11	1.5E+12	1.9E+13
48	1.5E+10	5.4E+10	3.6E+12	2.7E+12	2.52E+12	4.29E+11	4.9E+13	1.1E+12	2.1E+12	1.2E+12	2.1E+12	1.1E+12	5.6E+12
49	9.4E+08	5.1E+08	7.1E+12	2.9E+12	4.77E+11	2.89E+09	5.5E+13	1E+12	2.7E+11	1.1E+11	2.7E+11	1.2E+12	5.7E+12

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