A dynamic multi-stage slacks-based measure data envelopment analysis model with knowledge accumulation and technological evolution

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**Abstract**

Dynamic data envelopment analysis (DEA) models are built on the idea that single period optimization is not fully appropriate to evaluate the performance of decision making units (DMUs) through time. As a result, these models provide a suitable framework to incorporate the different cumulative processes determining the evolution and strategic behavior of firms in the economics and business literatures. In the current paper, we incorporate two distinct complementary types of sequentially cumulative processes within a dynamic slacks-based measure DEA model. In particular, human capital and knowledge, constituting fundamental intangible inputs, exhibit a cumulative effect that goes beyond the corresponding factor endowment per period. At the same time, carry-over activities between consecutive periods will be used to define the pervasive effect that technology and infrastructures have on the productive capacity and efficiency of DMUs. The resulting dynamic DEA model accounts for the evolution of the knowledge accumulation and technological development processes of DMUs when evaluating both their overall and per period efficiency. Several numerical examples and a case study are included to demonstrate the applicability and efficacy of the proposed method.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric method based on linear programming and applied to evaluate the relative efficiency of homogenous decision making units (DMUs) that use multiple inputs to produce different sets of outputs (Cooper, Seiford, & Tone, 2006). Classic DEA models have been adapted to a variety of research environments, ranging from the inclusion of different production stages (Zhu, 2014) to the use of fuzzy data (Emrouznejad & Tavana, 2014; Hatami-Marbini, Emrouznejad, & Tavana, 2011).

However, classic DEA models are not designed to evaluate DMUs whose efficiency must be measured through a planning horizon composed by multiple time periods. In this case, a particular configuration modifies the inputs and outputs displayed by a DMU through the planning horizon, resulting in a dynamic process. Dynamic DEA models are based on the idea that single period optimization is not completely appropriate to evaluate the performance of DMUs through time. In the current paper, we will define a DEA model designed to evaluate the dynamic efficiency of DMUs.

The dynamic DEA model on which we will build our diffusion and structural accumulation processes within and across DMUs is that of Tone and Tsutsui (2010). TTs henceforth. These authors incorporated carry-over activities to measure the efficiency of DMUs per time period over a long term optimization framework. In particular, TTs extended the dynamic DEA model of Fare and Grosskopf (1996) using a slacks-based measure setting (Tone, 2001). Their model was non-radial and designed to deal with inputs and outputs individually without requiring proportional changes. Tone and Tsutsui (2009) illustrated the direct connections existing between dynamic and network DEA models. According to both authors, their 2010 paper could be considered an extension of their network slacks-based measure model of 2009 to a dynamic environment. In this regard, Tone and Tsutsui (2014) defined a dynamic DEA model that extended their slacks-based measure setting...
of 2010 to evaluate the efficiency of organizational network structures per time period.

It should be highlighted that DEA models have been only recently adapted to consider the network of internal production processes taking place within a DMU (Chen, 2009; Kao, 2017). Chen (2009) argued that if these dynamic effects are not accounted for, then the measure of efficiency obtained would be biased and provides misleading information to the decision-makers. Network DEA models have been mainly defined within deterministic environments. However, in real-life problems, processes taking place at internal levels within a DMU are generally based on stochastic structures exhibiting different degrees of efficiency (Wen, 2015). Our dynamic DEA model will account for the stochastic features determining the evolution of the different diffusion and cumulative processes taking place within a DMU.

1.1. Contribution

Dynamic DEA models provide a suitable framework to incorporate the different cumulative processes determining the evolution and strategic behavior of firms in the economics and business literatures. In this regard, the formal structure introduced in the current paper has been designed to widen the scope of potential DEA applications and increase its impact among branches of the literature that remain dominated by parametric approaches.

We incorporate two distinct complementary types of sequentially cumulative processes within a dynamic slacks-based measure DEA model. More precisely, we integrate the dynamic non-radial slacks-based measure framework of TTs with cumulative knowledge diffusion processes and a stochastic extension of the multi-stage DEA setting of Khalili-Damghani, Tavana, Santos-Arteaga, and Mohtasham (2015). Both processes imply that inputs sequentially accumulate through the different time periods composing the planning horizon. Two main types of input processes will be considered:

- Human capital and knowledge: constitute fundamental intangible inputs exhibiting a cumulative effect that goes beyond the corresponding factor endowment per period.
- Carry-over activities between consecutive periods: account for the pervasive effects that technology and infrastructures have on the productive capacity and efficiency of DMUs.

The resulting dynamic DEA model accounts for the behavior and evolution of the knowledge accumulation and technological development processes of DMUs when evaluating both their overall and per period efficiency. In this regard, our DEA model depends on the type of dynamic consistency requirement imposed to measure the changes in efficiency that take place over different periods of time.

It should be emphasized that stochastic versions of DEA such as the super-efficiency (Andersen & Petersen, 1993) and chance-constrained ones (Land, Lovell, & Thore, 1993) impose normal distributions when defining the constraints of the corresponding efficiency models. However, technological accumulation and the resulting innovations are generally modeled using Poisson probability functions and knowledge is assumed to be transmitted through logistic diffusion processes, requiring the incorporation of these types of dynamic processes within the constraints of the corresponding DEA models. Note also that two- and three-stage DEA models do not generally consider the knowledge diffusion processes taking place across the different levels composing a DMU or the cumulative capital structures that can be developed through time (Zhu, 2014).

One of the main features of our dynamic DEA model, which provides a substantial advantage over the currently existing ones, is its flexibility. That is, our model can be easily adapted to the type of learning or innovation process being analyzed as well as to the potential interactions taking place across the different levels composing a DMU or among DMUs. The incorporation of the different diffusion and cumulative processes considered within our formal DEA framework allows for direct extensions into the innovation environments studied by the business and economics literatures.

In this regard, the existing differences between the Japanese and American industrial structures defining the case study of TTs should play a significant role when determining the efficiency of the corresponding DMUs. More precisely, and from an economic viewpoint, the efficiency analysis of TTs should consider the effect of the national innovation systems defining the technological infrastructures of both countries. Integrating this effect within the dynamic structure defined by TTs constitutes the intuitive basis on which the current model is built. Learning and innovation processes are cumulative, implying that the dynamic consistency requirement introduced by TTs must be maintained when incorporating the diffusion of knowledge taking place across agents and the development of technological infrastructures.

The current paper illustrates the main effects derived from these processes when considering input modifications, which implies that technologically developed countries have a structural advantage relative to the laggards. In other words, consider two DMUs that use the same amounts of inputs to produce identical amounts of outputs while being located in differently developed countries such as Germany and Lithuania. Clearly, the German DMU has access to a more skilled labor force than the Lithuanian one. Moreover, the level of technological development of the German infrastructures should be superior to that of the Lithuanian ones. Both differences imply that German DMUs, endowed with access to technologically developed infrastructures and highly skilled human capital but exhibiting identical production results, are less efficient than the Lithuanian ones.

However, the main objective of the current paper goes beyond the numerical examples designed to illustrate the importance of cumulative technological and learning processes. That is, the stochastic structure introduced can be directly applied to evaluate the strategic implications derived from mergers, acquisitions and other types of interactions taking place among DMUs endowed with different learning capacities and levels of technological development. Thus, the main contribution of the current manuscript consists of extending the standard dynamic DEA environment into the realms defined by the economics and business literatures.

1.2. Literature review: knowledge diffusion and technological infrastructures

Economists have verified the fact that countries require substantial amounts of physical and human capital to assimilate and utilize the knowledge inherent to the most technological advanced capital (Acemoglu, 2008; Aghion & Howitt, 1999, 2005). From an empirical viewpoint, the formation and accumulation of human capital combined with the institutional and technological infrastructures of a country determine its capacity to grow via innovations (Chen, Hu, & Yang, 2011; Fagerberg et al., 2007; Fagerberg & Srholec, 2008; Varsakelis, 2006). Technology has been consistently illustrated to grow cumulatively (Mukoyama, 2003), implying that substantial costs must be incurred to learn and assimilate a novel advanced technology (Engel & Kleine, 2015; Jovanovic, 1997).

At the same time, the technological development level of countries conditions the assimilation of knowledge inherent to a given technology and their capacity to developed innovations (Ballot & Taymaz, 1997). The literature has highlighted the direct relationship existing between the development of a country’s technological infrastructure and both the productivity derived from
implementing a novel technology and its innovation probability (Castellacci & Natera, 2016; López, Molero, & Santos Artega, 2011; Teixeira, Silva, & Mamede, 2014). As a result, the main efficiency and productivity differences arising across firms and countries are generally associated to the level of development of their corresponding national innovation systems (Hardeman, Frenken, Nomaner, & Ter Wal, 2015; Lundvall, 2007).

Farmer, Porter, and Stern (2002) and Castellacci and Natera (2013) have illustrated empirically that the technological infrastructure of a country must be simultaneously developed with the knowledge of its workers. Otherwise, its capacity to assimilate a given technology, learn from it and innovate becomes highly limited. Two self-reinforcing constraints are identified as the main determinants of the behavior of knowledge diffusion and innovation processes. First, workers have the capacity to update and improve any previous technological knowledge. However, they are constrained by the differences existing between their own knowledge, delimited by the technological development level of the country, and the advanced one they are trying to assimilate. In other words, absent an adequate technological infrastructure, human capital becomes redundant. Second, the development of the technological infrastructure of a country, including industrial districts and technoeconomic webs, conditions its innovation and learning capacities.

It follows from the above analysis that the capacity of a DMU to efficiently implement a given technology should be determined by the skills exhibited by its labor force, which, if limited, would constrain the assimilation and diffusion of knowledge and its innovation capacity (Di Caprio, Santos Artega, & Tavana, 2015; Murphy & Topel, 2016; Pargianas, 2016; Santos-Artega et al., 2017). Similarly to López et al. (2011) and Alvarez, Di Caprio, and Santos Artega (2016), the learning process taking place across different skilled workers is generally determined by a sigmoid function defining their capacity to assimilate new knowledge. Moreover, the technological infrastructure of a country also determines the capacity of the workers to learn and is built cumulatively, while limited by the technological knowledge accumulated by the workers.

Therefore, within the formal setting introduced in the current paper, knowledge diffusion as well as technological infrastructures will be assumed to follow a dynamic cumulative process, whose prevalence should be reflected in the set of clusters and infrastructures constituting the national innovation system of a country.

2. Basic environment: dynamic slacks-based framework

We build on the dynamic slacks-based measure framework of TTs given its capacity to define an interconnected dynamic environment that allows for modifications (in the slack-based constraints) designed to account for a variety of interactions across inputs, outputs as well as their potential combinations. The inclusion of a knowledge diffusion and learning structure within their dynamic setting as well as a complementary stochastic cumulative innovation framework provide additional flexibility to the model and enhance its capacity to combine and incorporate different processes and types of capital, both physical and human. It should be noted that the latter framework builds on a stochastic version of the dynamic cumulative environment introduced by Khalili-Damghani et al. (2015).

The dynamic structure defined by TTs is presented in Fig. 1. These authors consider n DMUs (i = 1, . . . , n) over T periods of time (t = 1, . . . , T). At time t, every DMU is endowed with a set m of period t inputs (i = 1, . . . , m) and carry-over activities, called links, from the previous period t − 1. Both of them are used to produce a given set s of period t outputs (i = 1, . . . , s) while sending carry-overs to the next period t + 1. The observed set of inputs and outputs for DMU i at period time t are denoted by xjt (i = 1, . . . , m) and yjt (i = 1, . . . , s), respectively.

The temporal lag involved in the applicability of the carry-over activities within the production process of the DMU is explicitly reflected in Fig. 1. Their prevalence, resulting from the dynamic structure of the process, implies that DMUs must continuously deal with the consequences, both positive and negative, derived from their productive and structural capabilities.

In particular, the main difference between the dynamic DEA model of TTs and the standard static ones is the existence of carry-overs connecting consecutive time periods. We will identify these carry-overs with structural capital endowments determining the productive capability of DMUs. In TTs, carry-over activities are classified in four different categories:

1. Desirable (good) links, zgood, treated as outputs such that a shortage of links is considered to be inefficient.
2. Undesirable (bad) links, zbad, treated as inputs such that an excess of links is considered inefficient.
3. Discretionary (free) links, zfree, that DMUs can increase or decrease freely. These links affect the efficiency of DMUs through the intertemporal continuity condition described below.
4. Non-discretionary (fixed) links, zfix, that are beyond the control of the DMUs. These links also affect the efficiency of DMUs through the intertemporal continuity condition.

The different links are identified by time period, t, DMU, j, and item, i, through the notation zijt (i = 1, . . . , ngood; j = 1, . . . , n; t = 1, . . . , T), with ngood referring to the number of good links. In the current paper, we will follow the model of TTs while focusing on input accumulation and diffusion through time, i.e. we will only consider undesirable links zijt (i = 1, . . . , nbad). However, it should be emphasized that our cumulative stochastic structure can be easily implemented in the output-oriented and non-oriented versions of TTs’s framework. Note that input-oriented models focus on the reduction of inputs while preserving the observed levels of output. On the other hand, output-oriented models aim at increasing outputs while using the same or a lower amount of inputs. Non-oriented models combine both previous scenarios within a single framework where inputs are reduced and outputs increased simultaneously.

TTs define the production constraints associated to DMUs (o = 1, . . . , n) when considering only desirable and undesirable links as follows

\[ \begin{align*}
    x_{ot} &= \sum_{j=1}^{n} \lambda_j^t x_{jt} + s^-_{ot} (i = 1, \ldots, m; \quad t = 1, \ldots, T) \\
    y_{ot} &= \sum_{j=1}^{n} \lambda_j^t y_{jt} - s^+_{ot} (i = 1, \ldots, s; \quad t = 1, \ldots, T) \\
    z^{\text{good}}_{int} &= \sum_{j=1}^{n} \lambda_j^t z_{jt}^{\text{good}} - z^{\text{good}}_{it} (i = 1, \ldots, n\text{good}; \quad t = 1, \ldots, T) \\
    z^{\text{bad}}_{int} &= \sum_{j=1}^{n} \lambda_j^t z_{jt}^{\text{bad}} + z^{\text{bad}}_{it} (i = 1, \ldots, n\text{bad}; \quad t = 1, \ldots, T) \\
    \sum_{j=1}^{n} \lambda_j^t &= 1 (t = 1, \ldots, T) \\
    \lambda_j^t &\geq 0, \quad s^-_{ot} \geq 0, \quad s^+_{ot} \geq 0, \quad z^{\text{good}}_{it} \geq 0, \quad z^{\text{bad}}_{it} \geq 0 (\forall \lambda_j^t) 
\end{align*} \]

where \( \lambda_j^t \in \mathbb{R}^+ \) (t = 1, . . . , T) is the intensity vector defined at time period t, \( s^-_{it}, s^+_{it}, z^{\text{good}}_{it}, \) and \( z^{\text{bad}}_{it} \) are slack variables denoting input excess, output shortfall, link shortfall, and link excess, respectively. The continuity of carry-overs between two consecutive time periods, t and t + 1, is guaranteed by imposing the following set of
Intuitively, Eq. (2) guarantees the dynamic consistency of the production possibilities defined for the different types of carry-overs. TTs compute the overall efficiency of $DMU_o (o = 1, \ldots, n)$ taking \{[$\lambda^+_1$, [$s^+_1$], [$s^-_1$], [$s^{mod}_1$]}, as variables within input-, output- and non-oriented environments. Input-oriented models focus on maximizing the relative slack of inputs and undesirable links. On the other hand, output-oriented models focus on maximizing the relative slack of outputs and desirable links. Non-oriented models focus on simultaneously reducing input-related factors and enlarging the output-related ones, encompassing input- and output-oriented models within a unique framework.

Given the variable returns to scale (VRS) assumption imposed on Eq. (1), the input-oriented model defined to compute the efficiency score of $DMU_o$ is given by:

$$\theta_o^* = \min \left\{ \frac{1}{T} \sum_{t=1}^{T} w^t \left[ 1 - \frac{1}{1 + m + nbad} \left( \sum_{i=1}^{m} \frac{w^t_i s^+_i}{\lambda^+_i} + \sum_{i=1}^{nbad} \frac{s^-_i}{\lambda^-_i} \right) \right] \right\}$$ (3)

subject to (1) and (2), where $w^t$ and $w^-_i$ are exogenous weights assigned to time period $t$ and input $i$, respectively, depending on their relative importance. These weights satisfy the following conditions:

$$\sum_{t=1}^{T} w^t = T \quad \text{and} \quad \sum_{i=1}^{m} w^-_i = m$$ (4)

In the current paper, we will assume that all periods and inputs are equally important, i.e. all weights are assigned identical $w^t = 1 (\forall t)$ and $w^-_i = 1 (\forall i)$ values. Note that the objective function accounts for excess in undesirable links and inputs in the same way. That is, even though undesirable links are not inputs per se, DMUs aim at reducing their amount across different time periods. Note also that the terms within the square brackets of Eq. (3) define the period $t$ efficiency of $DMU_o$ based on the relative slack of inputs and links. This expression is units-invariant and its value is located within the [0, 1] interval, reaching a maximum of one if all the slacks are equal to zero. Thus, Eq. (3) represents the weighted average of term efficiencies over the whole sample period and will be defined as the input-oriented overall efficiency of $DMU_o$, whose value is also located within the [0, 1] interval.

Formally, the input-oriented term efficiency of $DMU_o$, $\theta^*_o$, is given by

$$\theta^*_o = 1 - \frac{1}{m + nbad} \left( \sum_{i=1}^{m} \frac{w^t_i s^+_i}{\lambda^+_i} + \sum_{i=1}^{nbad} \frac{s^-_i}{\lambda^-_i} \right), \quad (t = 1, \ldots, T)$$ (5)

for an optimal set of values obtained by minimizing (3) subject to (1) and (2), \{[$\lambda^+_o$, [$s^+_o$], [$s^-_o$], [$s^{mod}_o$]}, [$s^{bad}_o$]}. Clearly, $\theta^*_o$ represents the input-oriented efficiency of $DMU_o$ at time $t$, while its overall efficiency defined over the whole sample period, $\theta^*_o$, is given by

$$\theta^*_o = \frac{1}{T} \sum_{t=1}^{T} \theta^*_o$$ (6)

Following TTs, $DMU_o$ is input-oriented term efficient at time $t$ if the optimal solutions of Eq. (3) satisfy $\theta^*_o = 1$, which implies that $s^+_o = 0 (\forall i)$ and $s^{bad}_o = 0 (\forall i)$ at time $t$. Similarly, $DMU_o$ is input-oriented overall efficient if $\theta^*_o = 1$, implying that $s^+_o = 0 (\forall i, t)$ and $s^{bad}_o = 0 (\forall i, t)$. Finally, $DMU_o$ is input-oriented overall efficient if and only if it is input-oriented term efficient for all terms.

Output- and non-oriented DEA models can be defined in a similar way to obtain the efficiency score of $DMU_o$. The choice of model is determined based on the research and managerial objectives considered by the decision makers. In this regard, we must emphasize that even though we will be focusing on the input-oriented version of the model, immediate extensions to the output and non-oriented environments follow from the current paper. That is, our model can be easily adapted depending on the type of learning processes that the decision maker wants to account for and the potential interactions taking place among firms or within the sectors of a given firm.

3. Extended framework

Given the intuition derived from the economics and business literatures and provided in the introductory sections, we will consider different types of inputs classified by their knowledge endowment as well as technological infrastructures acting as cumulative links that determine the production capacity of DMUs. For illustrative purposes, three main types of human capital inputs will be incorporated within the formal framework of TTs, namely, high, $x^{high}_{ijr}$ ($i = 1, 2, \ldots, mhigh$), middle, $x^{mid}_{ijr}$ ($i = 1, 2, \ldots, mmid$), and low, $x^{low}_{ijr}$ ($i = 1, 2, \ldots, mlow$), skilled workers can be used as inputs by $DMU_j$ at time $t$. That is, DMUs categorize – and remunerate – workers according to their relative technical skills. For example, senior engineers constitute high skilled human capital due to their experience and accumulated knowledge that allows them to teach and supervise junior engineers, who can be
considered middle skilled human capital. At the same time, junior engineers collaborate with and pass their knowledge to different technical operators, who can be defined as low skilled workers.

Similar categorization processes can be identified across the multiple hierarchical structures based on knowledge differences that are defined within an organization. In this regard, the analysis extends directly to any input classifiable in terms of its technological content and with potential spillover effects over other inputs being used by the DMU. As explained in the contribution section, infrastructures will be considered as a bad link that accumulates through time, since more developed infrastructures provide a technological advantage to the DMUs located within the corresponding country.

We extend the model of TIs focusing on the input-oriented overall efficiency $\theta^*_n$ of DMUs as defined through the following optimization problem

$$\theta^*_n = \min \frac{1}{T} \sum_{t=1}^{T} w^t \left[ 1 - \left( \frac{m_{\text{mmid}} + m_{\text{mlow}} + n_{\text{bad}}}{\sum_{j=1}^{m_{\text{high}}} w_{j}^t + \sum_{j=1}^{m_{\text{mid}}} w_{j}^t + \sum_{j=1}^{n_{\text{bad}}} w_{j}^t} \right) \right]$$

s.t.

$$x_{i,t} = \sum_{j=1}^{n} \lambda_{j}^{t} x_{ij,t} + s_{g}^{t} \quad (i = 1, ..., m; \ t = 1, ..., T)$$

$$y_{i,t} = \sum_{j=1}^{n} \lambda_{j}^{t} y_{ij,t} - s_{l}^{t} \quad (i = 1, ..., m; \ t = 1, ..., T)$$

$$\left[ \psi_{i,t}^{\text{high}} - \left( 1 - \varphi_{i,t}^{\text{high}} \right) \right] x_{ij,t_{\text{low}}}^{\text{high}} = \sum_{j=1}^{n} \lambda_{j}^{t} \left[ \psi_{i,j,t_{\text{low}}}^{\text{high}} - \left( 1 - \varphi_{i,j,t_{\text{low}}}^{\text{high}} \right) \right] x_{ij,t_{\text{low}}}^{\text{low}} + s_{l}^{t} \quad (i = 1, 2, ..., m_{\text{high}}; \ t = 1, 2, ..., T)$$

$$\left[ \psi_{i,t}^{\text{mid}} - \left( 1 - \varphi_{i,t}^{\text{mid}} \right) \right] x_{ij,t_{\text{low}}}^{\text{mid}} = \sum_{j=1}^{n} \lambda_{j}^{t} \left[ \psi_{i,j,t_{\text{low}}}^{\text{mid}} - \left( 1 - \varphi_{i,j,t_{\text{low}}}^{\text{mid}} \right) \right] x_{ij,t_{\text{low}}}^{\text{low}} + s_{l}^{t} \quad (i = 1, 2, ..., m_{\text{mid}}; \ t = 1, 2, ..., T)$$

$$\left[ \left( 1 - \eta_{i,t} \right) + \eta_{i,t} \right] y_{ij,t_{\text{low}}}^{\text{bad}} = \sum_{j=1}^{n} \lambda_{j}^{t} \left[ \left( 1 - \eta_{i,j,t_{\text{low}}} \right) + \eta_{i,j,t_{\text{low}}} \right] y_{ij,t_{\text{low}}}^{\text{bad}} + s_{l}^{t} \quad (i = 1, 2, ..., m_{\text{low}}; \ t = 1, 2, ..., T)$$

$$\sum_{j=1}^{n} \lambda_{j}^{t} \varphi_{i,j,t_{\text{low}}}^{\text{bad}} = \sum_{j=1}^{n} \lambda_{j}^{t} \varphi_{i,j,t_{\text{low}}}^{\text{bad}} \quad (\forall i; \ t = 1, ..., T; \ \alpha = \text{bad})$$

$$\sum_{j=1}^{n} \lambda_{j}^{t} \psi_{i,j,t_{\text{low}}}^{\text{bad}} = \sum_{j=1}^{n} \lambda_{j}^{t} \psi_{i,j,t_{\text{low}}}^{\text{bad}} \quad (\forall i, t)$$

$$\lambda_{j}^{t} \geq 0, \ s_{g}^{t} \geq 0, \ s_{l}^{t} \geq 0, \ \psi_{i,j,t_{\text{low}}}^{\text{bad}} \geq 0 \quad (\forall i, t) \quad (7)$$

Note that we have extended the set of constraints defined in (1) and (2) to include three new types of inputs and a novel expression for the undesirable links. As already stated, knowledge diffusion and structural capital accumulation will be treated as inputs, whose comparative excess will be accounted for as inefficiency. However, as will be emphasized through the empirical section, these cumulative processes could be used to determine the potential outputs that may result from different DMUs interacting within a given market configuration. As such, the accumulation of human capital and infrastructures would lead to increments in output or define desirable links when determining the relative efficiency of the corresponding DMUs.

3.1. Knowledge diffusion and learning

Consider the input-oriented DEA model defined in Eq. (7). Its dynamic structure, which builds directly on that of TIs described in Fig. 1, is presented in Fig. 2. The main difference between the current dynamic DEA model and the one defined by TIs is the existence of interconnected input sets and carry-overs through the different time periods. Input sets exhibit a knowledge diffusion process that affects its components both within and across periods, while carry-overs account for structural capital endowments determining the production capacity of DMUs through time. These latter endowments are included in Fig. 2 both within $l_{t-1}$ and $C_{t-1}$, $t = 1, ..., T$, in order to emphasize the dynamic consistency inherent to the structural constraints.

Note how the cumulative character of the diffusion and structural processes implies that any modification implemented at any point through their dynamic evolution would have both direct consequences for the output produced in that time period as well as the inputs, carry-overs and corresponding outputs obtained in all future periods. The complex set of interdependencies illustrated in Fig. 2 highlights the importance of accounting for the whole dynamic process when determining the efficiency of a DMU. Intuitively, this becomes particularly important when selecting the distribution of inputs or investments in infrastructures while considering their potential dynamic cumulative effects through the subsequent periods of time.

In order to provide additional intuition, assume that the input sets defined in Fig. 2 are composed by high, medium and low skilled workers and that knowledge flows within the set from the more towards the less skilled human capital. The following cumulative diffusion process will be used to account for the knowledge spillovers taking place across the different levels of human capital within an organization

$$\varphi_{i,j,t} = \frac{1}{1 + \left( \frac{1}{\varphi_{i,j,t_{\text{low}}}^{\text{bad}}} - 1 \right) e^{-\Gamma \left( \sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}} \right) \sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}}}} \quad (8)$$

where $\sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}}$ refers to the average accumulation of type $i$ input by DMU $j$ until time period $t - 1$. $f(\sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}})$ defines the proportion of type $i$ input relative to the total amount of inputs across which knowledge can be diffused and with which DMU $j$ is endowed. $\Gamma(\sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}})$ accounts for the relative strength of the diffusion process across less skilled workers. This value is determined by the amount of skilled human capital accumulated by DMU $j$, relative to that of the DMU endowed with the largest amount. Thus, depending on the initial proportion of high skilled human capital, $f(\sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}})$, and the relative strength of the diffusion process, $\Gamma(\sum_{j=1}^{n} \psi_{i,j,t_{\text{low}}}^{\text{bad}})$, a proportion $\varphi_{i,j,t}$ of high skilled workers will be available by the end of period $t$.

From a formal perspective, $\varphi_{i,j,t}$ is a logistic learning function generally used by economists and business scholars to study
technological diffusion processes (Aghion & Howitt, 1999; Geroski, 2000; Iwai, 2000). In this regard, \( \varphi_{ijt} \) defines a cumulative diffusion process that takes place across different types of skilled workers and highlights their capacity to learn as they acquire knowledge from their more skilled colleagues.

### 3.1.1. Cumulative input processes across different levels of knowledge

The cumulative learning process that takes place across the three hierarchical levels of knowledge diffusion illustrated in Fig. 2 can be formally described as follows.

#### 3.1.2. High skilled human capital

The cumulative diffusion process triggered by the high skilled human capital owned by DMU\(_j\) can be defined as follows

\[
\varphi_{ijt} = \frac{1}{1 + \left( \frac{\sum_{t=1}^{T-1} x_{ijt}^{\text{high}}}{\sum_{t=1}^{T-1} x_{ijt}^{\text{Total}}} - 1 \right) e^{-\left( \frac{\sum_{t=1}^{T-1} x_{ijt}^{\text{high}}}{\sum_{t=1}^{T-1} x_{ijt}^{\text{Total}}} \right)}} \tag{9}
\]

where

- \( \sum_{t=1}^{T-1} x_{ijt}^{\text{high}} / t - 1 \) defines the average accumulation of high skilled human capital by DMU\(_j\) until time period \( t - 1 \);
- \( \sum_{t=1}^{T-1} x_{ijt}^{\text{Total}} / t - 1 \) accounts for the average total amount of inputs across which knowledge can be diffused within DMU\(_j\);
- \( \max_{i} \left( \frac{\sum_{t=1}^{T-1} x_{ijt}^{\text{high}}}{t - 1} \right) \) refers to the highest amount of skilled capital available across the different DMUs, which constrains the relative strength of the knowledge diffusion process through \( \Gamma \left( \sum x_{ijt} \right) \).

We will interpret the value of \( \varphi_{ijt} \) not as the proportion of high skilled human capital available by the end of period \( t \), but as the probability of improving the high skilled input capacity of the DMU triggered by the knowledge diffused across the remaining workers. The corresponding improvement in the input capacity of the DMU will be determined by the value of the exogenous parameter \( \varepsilon > 1 \) as follows

\[
\varepsilon \left( \varphi_{ijt} x_{ijt}^{\text{high}} \right) = \sum_{j=1}^{n} \lambda_{ij} \left( \varphi_{ijt} (1 - \varepsilon) x_{ijt}^{\text{high}} + \theta^{h} \right) \quad (i = 1, 2, ..., m_{\text{high}}, t = 1, 2, ..., T) \tag{10}
\]

which defines the set of capacity constraints described in Eq. (7) when accounting for high skilled human capital inputs. Note that the parameter \( \varepsilon \) has been assumed to be identical across all DMUs, though differences based on their relative level of technological development could be used to modify its value. A similar comment applies to the weighted value of the relative input improvement defined for the set of DMUs, i.e. \( \left[ \varphi_{ijt} x_{ijt}^{\text{high}} + \theta^{h} \right] \), \( j = 1, 2, ..., n \). It should be highlighted that the stochastic structure defined in Eq. (10) becomes strategically relevant when accounting for potential output capacity improvements resulting from the cooperation of different DMUs. That is, when the intensity of cooperation between DMUs exhibiting different levels of technological development determines the value of the potential output obtained.

Eq. (10) considers the direct effect that the accumulation of high skilled human capital has on the subset of high skilled workers itself. However, high skilled input variables have also a direct effect on lower skilled ones, that is, high skilled engineers help less skilled ones, who, at the same time, help less skilled workers, and so on, through the different hierarchical levels within an organization.

In this regard, the cumulative learning process taking place across different organizational levels can be easily introduced within the dynamic DEA framework defined in Eq. (7) by slightly modifying Eq. (9) as described in Eqs. (11) and (13). That is, the structures defined below reflect the sequential cumulative evolution of knowledge at different levels within an organization and its transmission, i.e. spillovers, across them.
3.1.3. Middle skilled human capital

The cumulative diffusion process taking place from the high, $\phi_{ij}^{\text{high}}$, to the middle skilled human capital, $x_{ijt}^{\text{mid}}$, owned by DMU$_j$ can be defined as follows:

$$\phi_{ij}^{\text{high}} = \frac{1}{1 + \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}} \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}.$$  \hspace{1cm} (11)

The corresponding input capacity improvement will be determined by both the value of the exogenous parameter $\varepsilon > 1$ and the accumulation of high skilled human capital $[\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}]$. Therefore, the middle skilled human capital $x_{ijt}^{\text{mid}}$ has an exogenous knowledge spillover structure defined by $\phi_{ij}^{\text{high}}$. In this regard, note how a persistent increment in the high skilled capital owned by a DMU has a direct cumulative effect over the middle skilled one, $x_{ijt}^{\text{mid}}$, through the $\phi_{ij}^{\text{high}}$ variable.

3.1.4. Low skilled labor

Similarly to the above settings, the cumulative diffusion process taking place from the middle, $x_{ijt}^{\text{mid}}$, to the low skilled human capital, $x_{ijt}^{\text{low}}$, owned by DMU$_j$ can be defined as follows:

$$\phi_{ij}^{\text{low}} = \frac{1}{1 + \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}} \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}.$$  \hspace{1cm} (13)

The corresponding input capacity improvement will be determined by both the value of the exogenous parameter $\varepsilon > 1$ and the accumulation of high skilled human capital $[\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}]$. Therefore, the low skilled human capital $x_{ijt}^{\text{low}}$ has an exogenous knowledge spillover structure defined by $\phi_{ij}^{\text{low}}$. In this regard, note how a persistent increment in the high skilled capital owned by a DMU has a direct cumulative effect over the middle skilled one, $x_{ijt}^{\text{low}}$, through the $\phi_{ij}^{\text{low}}$ variable.

The immediate effect derived from this cumulative process on the resulting input constraints can be observed in Fig. 3. For purely illustrative purposes, this figure describes the value of $\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}})$ for different ranges of the $\varepsilon$ and $\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}})$ terms when $\phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}}) = 50$ and $\max_{j} \phi_{ij}^{\text{low}} e (1 - \phi_{ij}^{\text{mid}})/\varepsilon_{ijt} = 20$. Thus, we are assuming that the DMU under assessment has a total of 50 workers among which knowledge can be diffused while the highest amount of skilled capital across DMUs, limiting the strength of the diffusion process, equals 20 workers.

Fig. 3 shows how the resulting diffusion process, $\phi_{ij}$, leads to an expected improvement in the input capacity of a given DMU as its amount of high skilled human capital increases relatively to that of the other DMUs. Once again, we can intuitively infer the strategic environment that follows from this figure if improvements in the output obtained by a DMU, resulting from different potential collaborations with other DMUs, are considered. Clearly, the intuition remains valid independently of the numerical values selected as a benchmark across the different hierarchical levels of the DMU.

3.1.5. Alternative diffusion frameworks

We propose two alternative diffusion settings that could be implemented within the formal environment described through the previous sections, while noting that several other alternative scenarios could be easily defined and incorporated to the model described in Eq. (7).

First, note that we could consider a subset of carry over activities, $\text{nbad}^a$, affecting only a particular subset of inputs and reflecting, for instance, the interactions taking place between high skilled labor and technologically advanced capital. This type of interactions can be used to highlight the interconnections existing between both factors of production and required for firms (and countries) to grow and evolve (López et al., 2011).

The resulting cumulative diffusion process triggered by the interactions between a given subset of carry-overs and the high skilled human capital owned by DMU$_j$ can be defined as follows:

$$\phi_{ij}^{\text{high}} = \frac{1}{1 + \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}} \left(\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ij}^{\text{high}} e (1 - \phi_{ij}^{\text{mid}}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}.$$  \hspace{1cm} (14)

Note that we are dealing with expected terms of evolution across the different organizational levels. These stochastic components become more complex as we move further down the chain of learning effects, as can be inferred from the definitions of the $\phi_{ijt}^{\text{high}}$ and $\phi_{ijt}^{\text{mid}}$ terms. That is, the model incorporates a sequential cumulative knowledge transmission process across the different hierarchical levels composing an organization. The interactions taking place across these organizational levels are reflected in the increasing cumulative complexity inherent to the $\phi_{ijt}^{\text{high}}$ and $\phi_{ijt}^{\text{mid}}$ terms.

$$\phi_{ijt}^{\text{high}} = \frac{1}{1 + \left(\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{high}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{high}} + \varepsilon_{ijt}}}} \left(\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{high}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{high}} + \varepsilon_{ijt}}}.$$  \hspace{1cm} (15)

with $\phi_{ijt}^{c} < \phi_{ijt}^{\text{high}}, \forall r$, i.e. high skilled workers have a limited amount of technologically developed capital with which to interact and improve their skills. The corresponding input capacity improvement would therefore be given by

$$\phi_{ijt}^{\text{mid}} = \frac{1}{1 + \left(\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}} \left(\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}} - 1\right) e^{-\sum_{i=1}^{n} \frac{1}{\phi_{ijt}^{c} e (1 - \phi_{ijt}^{c}) x_{ijt}^{\text{mid}} + \varepsilon_{ijt}}}.$$  \hspace{1cm} (16)
We have implicitly defined Eq. (16) in terms of the number of high skilled workers, \( i = 1, 2, \ldots, m_{\text{high}}, \) interacting with or making use of a limited amount of technologically advanced capital, \( i = 1, 2, \ldots, n_{\text{bad}} \). It should be noted that the equation could be easily modified to allow for different types of interactions taking place between workers and capital.

The objective function described in Eq. (7) can be easily modified as follows

\[
\theta^* = \min \left\{ \frac{1}{T} \sum_{t=1}^{T} W_t \left[ 1 - \frac{1}{m_{\text{high}} + n_{\text{bad}} + n_{\text{bad}}} \left( \sum_{i=1}^{m} \frac{w_i}{h_{\text{high}}} + \sum_{i=1}^{n_{\text{bad}}} \frac{s_{ij}}{z_{\text{bad}}} + \sum_{i=1}^{n_{\text{bad}}} \frac{s_{ij}}{z_{\text{bad}}} \right) \right] \right\}
\]

with

\[
[(1 - \eta_{\text{total}})E + \eta_{\text{total}}]e_{\text{bad}} = \sum_{i=1}^{n} \lambda_i (1 - \eta_{ij})E + \eta_{ij} \Gamma_{ij} e_{\text{bad}} + s_{ij}^{\text{bad}}
\]

\[(i = 1, 2, \ldots, n_{\text{bad}}; t = 1, 2, \ldots, T)\]

Second, note that a discount term could be included when accounting for the knowledge diffused through previous time periods. That is, the effectiveness of the knowledge obtained from the more skilled human capital could be assumed to decrease through time, which would transform the expression \( \sum_{t=1}^{T} x_{ij} \) into \( \sum_{t=1}^{T} x_{ij} \) with \( \delta < 1 \) and \( \gamma = \text{high, mid, low} \), when defining the corresponding \( \psi \) variables.

3.2. Structural capital accumulation

We focus now on the dynamic accumulation and evolution of structural capital and technology within a given DMU. Following standard results from the economic literature (Aghion & Howitt, 1999), we will assume that technological innovations -- involving either products or production processes -- are governed by a Poisson process. The average number of events occurring in a given time interval, i.e. the event rate of the process, is given by \( \gamma \). As a result, the probability of observing \( k \) events in a given time interval is given by

\[
P(k) = e^{-\lambda} \frac{\lambda^k}{k!}
\]

Thus, when \( k = 1 \) and decision makers focus on a single event per time interval we have

\[
P(1) = e^{-\lambda} \frac{\lambda^1}{1!} = \lambda e^{-\lambda}
\]

In order to account for the capacity of a given DMU to innovate within a given time period, we will focus on the case where \( k = 0 \)

\[
P(0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda}
\]

Note that \( P(0) \) defines the probability of not observing any event, namely, any innovation, within a given time period. That is, we will be defining the dynamic evolution of carry-over activities in terms of the probability of developing at least one innovation through a given time period, i.e. \( (1 - \eta_{ij}) \). In this regard, Eq. (21) leads to the following sequentially cumulative probability of DMU, not developing any (product or process) innovation for input \( i \) within a given time period \( t \)

\[
\eta_{ij} = e^{-\lambda t}
\]

It should be emphasized that the dynamic cumulative model introduced through this section can differentiate DMUs by the number of innovations expected to be developed within a given time period, though such a setting would complicate the presentation considerably without providing any additional intuition.

The sequential cumulative process of technological development of DMU

\[
\lambda(t) = \sum_{s=1}^{t} \left( 1 + \frac{\beta_{st}}{\max_i \{\beta_{ij}\}} \right) \frac{\max_j \{\delta_{ij}\}}{\max_j \{\delta_{ij}\}}
\]

accounts for the effect of two distinct structural capital variables.
(1) The relative technological development level of the DMU described by the dynamic evolution of the previous carry-over activities: \( z_{j,t-1}^\text{bad} \), \( \forall j \) and \( t = 1, \ldots, t - 1 \);

(2) The direct effect that the national system of innovation of a country, including cluster infrastructures and external capital available to fund firms projects, has on the production capacities of domestic DMUs: \( \beta_{jt} \), \( \forall j \) and \( t = 1, \ldots, t - 1 \).

Note that, as we did when defining the set of \( \phi \) variables, the values of \( z_{j,t-1}^\text{bad} \) and \( \beta_{jt} \) are compared to those of the most technologically developed DMU per time period so as to focus on the relative strength of the process.

A similar cumulative process is defined by Khalili-Damghani et al. (2015) to describe inputs that are partially consumed each period and a percentage of which accumulates through time exponentially, as compounded interest rates do in finance.

In order to provide additional intuition and abusing notation, we develop Eq. (23) over a total of three time periods so the cumulative sequence of capital can be explicitly observed while noting that the \( \beta_{jt-1} \) variables could differ across time periods within a DMU,

\[
\lambda(t) = \sum_{s=1}^{t-1} \left(1 + \frac{\beta_{j,t-1}}{\max\{\beta_{j,t-1}\}}\right)^s \frac{z_{j,t-1}^\text{bad}}{\max\{z_{j,t-1}\}}
\]

\[
= \left(1 + \frac{\beta_{j,t-1}}{\max\{\beta_{j,t-1}\}}\right)^{t-1} \frac{z_{j,t-1}^\text{bad}}{\max\{z_{j,t-1}\}}
\]

\[
+ \left(1 + \frac{\beta_{j,t-2}}{\max\{\beta_{j,t-2}\}}\right)^{t-2} \frac{z_{j,t-2}^\text{bad}}{\max\{z_{j,t-2}\}}
\]

\[
+ \left(1 + \frac{\beta_{j,t-3}}{\max\{\beta_{j,t-3}\}}\right)^{t-3} \frac{z_{j,t-3}^\text{bad}}{\max\{z_{j,t-3}\}}
\]

\[
\mathbf{\text{(24)}}
\]

We can define now the dynamic evolution of the carry-over activities described in Eq. (7), which is also determined by the value of the exogenous parameter \( \beta \) as follows.

\[
(1 - \eta_{l,t})e + \eta_{l,t} \varepsilon = \sum_{i=1}^{n} \lambda_i \left(1 - \eta_{i,j} \varepsilon + \eta_{i,j} \right) z_{i,j,t} \varepsilon^\text{bad} + s_{l,t}^\text{bad}
\]

\[
(i = 1, 2, \ldots, n_{bad}, \ t = 1, 2, \ldots, T)
\]

As in the human capital input case, Fig. 4 represents the immediate effect derived from the structural capital accumulation process on the corresponding carry-over constraints. More precisely, this figure describes the value of \( (1 - \eta_{l,t})e + \eta_{l,t} \varepsilon \) for different ranges of \( \varepsilon \) and \( \eta_{l,t} \), while assuming, for simplicity, that \( \frac{\beta_{j,t-1}}{\min\{\beta_{j,t-1}\}} = \frac{z_{j,t-1}^\text{bad}}{\max\{z_{j,t-1}\}} \). \( \forall i, j, t \). The intuition derived from this figure is identical to the one used to describe Fig. 3. Note, in particular, the upward shift exhibited by the function as the number of time periods considered increases, enhancing the effect derived from the accumulation of structural capital.

3.3. Integrating DMUs: knowledge diffusion paired with technological evolution

Among the potential extensions of the current setting, we emphasize the complementarities arising from the integration of human and physical capital. For instance, consider the value \( \phi_{ij,t} \) of knowledge transmitted from a group of high skilled human capital.

Note that, as illustrated in Eq. (15), such value could be determined by a subset of carry-over activities from the DMU. The learning environment introduced in the current dynamic DEA framework allows us to incorporate this value into the \( \lambda(t) \) equation as follows.

\[
\mathbf{\text{(26)}}
\]

\[
\lambda(t) = \sum_{s=1}^{t-1} \left(1 + \frac{\phi_{ij,t-s}}{\max\{\phi_{ij,t-s}\}}\right)^s \frac{z_{ij,t-s}^\text{bad}}{\max\{z_{ij,t-s}\}}, \ \forall i, j, t
\]

with \( \Xi_{tot} = e^{-\lambda(t)} \), leading to

\[
\left[ (1 - \Xi_{tot})e + \Xi_{tot} \right] \varepsilon^{\text{high}} + \Xi_{tot} z_{ijt}^\text{bad}
\]

\[
= \sum_{j=1}^{n} \lambda_j \left[ \Xi_{ijt} e + (1 - \Xi_{ijt}) \right] \varepsilon^{\text{high}} + \sum_{j=1}^{n} \Xi_{ijt} z_{ijt}^\text{bad} + \sum_{j=1}^{n} z_{ijt}^\text{bad}
\]

\[
t = 1, 2, \ldots, T
\]

and with \( i \leq \max\{m_{\text{high}}, n_{bad}\} \), or \( i \leq m_{\text{high}} \times n_{bad} \) if all the potential combinations of high skilled labor and structural capital are considered, defining the number of resources shared across \( x_{ijt}^\text{high} \) and \( z_{ijt}^\text{bad} \).

That is, the complementarities arising between the high skilled human capital and the technological development level of a DMU determine its input endowment capacity, leading to

\[
\varepsilon_c = \min \sum_{t=1}^{T} \sum_{i=1}^{m} \left[ \frac{1}{m + n_{bad} + \sum_{j=1}^{n} \left( \sum_{t=1}^{T} \sum_{i=1}^{m} \phi_{ij,t}^c z_{ijt}^\text{bad} + s_{l,t}^\text{bad} \right) \right]
\]

\[
\mathbf{\text{(29)}}
\]

\[
\mathbf{\text{(27)}}
\]

\[
\mathbf{\text{(28)}}
\]

\[
\mathbf{\text{(25)}}
\]

In other words, different diffusion and structural processes can be combined so as to define integrated input environments determining the innovation capacity of DMUs. In this regard, inputs from two different DMUs, determining their respective innovation probabilities with different intensities (i.e. through different diffusion processes), can be combined so as to define the evolution of the carry over activities of both DMUs in expected terms. Thus, strategic considerations can also be accounted for by the current formal setting, as well as network interactions taking place across organizational levels with different intensities.

4. Numerical examples

We provide two different numerical examples to illustrate the effects that the modification of the relative inputs and carry-overs with which DMUs are endowed has on their efficiency. The first example corresponds to a dynamic sample of purposely generated DMUs, while the second one considers panel data from a set of different European Union countries so as to measure their relative dynamic efficiency. The results obtained validate the intuition that follows from the input-oriented structure defined throughout the paper, which highlights the capacity of the model to account for the strategic interactions that would arise within the output and non-oriented settings.
4.1. Rank modifications caused by knowledge diffusion and structural accumulation

We have created a dynamic data set to illustrate the effects derived from knowledge diffusion and structural accumulation on the relative efficiency of DMUs. The set of initial numerical values is presented in Table 1(a). We generate five DMUs (A–E) endowed with two inputs and two links and producing two outputs over a total of two time periods (T = 1 and T = 2).

We assume that the first link (B1) affects the first input through a knowledge diffusion process as the one described in Eq. (15). On the other hand, the second link (B2) is assumed to affect the second input through the structural accumulation framework defined in Eqs. (22), (23) and (25). Also, in order to simplify the
presentation, it has been assumed that \( \frac{\beta_{jk,-t}}{\max_i(\beta_{jk,-t})} \) Note that both dynamic processes have been defined on input variables so as to focus on the effects of cumulative input differentials across DMUs.

Input variables can be assumed to represent skilled labor and basic capital inputs complemented by different types of technologically advanced capital accumulated and carried over to the next period. Note that input amounts and carry overs increase in the second period for all DMUs. \( \text{DMU}_{jk} \) has been designed to be efficient in all numerical settings, while \( \text{DMU}_{jk} \) and \( \text{DMU}_{lk} \) constitute inefficient benchmarks. We will therefore focus on the endowments and ranking behavior of \( \text{DMU}_{lk} \) and \( \text{DMU}_{jk} \). A variable returns to scale framework with \( \varepsilon = 2 \) is assumed in all the numerical settings of this section.

Table 1(a) describes the results obtained from the (input-oriented) dynamic DEA model of TTs, i.e. when the diffusion and structural development effects on inputs are not considered. The results generated by the dynamic DEA model after accounting for learning and infrastructures are presented in Table 1(b). Note that the links available to \( \text{DMU}_{lk} \) are higher than those available to \( \text{DMU}_{jk} \) in both period. However, it is the endowment of links in period one what determines the input capacity of the DMUs in the second period. In this regard, Table 1(b) illustrates how the larger amount of links available to \( \text{DMU}_{lk} \) decreases its efficiency when considering the cumulative effect of learning and infrastructures. The same pattern applies to D and E, which remain as the most inefficient DMUs.

Consider now the second set of tables, where the first period links and second period inputs of \( \text{DMU}_{jk} \) have both been increased. The intuition behind this modification is twofold. On one hand, given the efficiency displayed by \( \text{DMU}_{lk} \) in Table 1(a), incrementing its links an inputs should lead to an increase in the relative efficiency of the other DMUs. That is, as illustrated in Table 2(a), the first three DMUs are now considered efficient by the model of TTs and the total efficiency of the last two DMUs has increased when compared to the values displayed in Table 1(a). On the other hand, despite the increase in the links and inputs of \( \text{DMU}_{lk} \), \( \text{DMU}_{jk} \) remains endowed with a relatively larger amount of links and a similar amount of inputs. Thus, when learning and infrastructures are introduced in Table 2(b), we observe how the cumulative effect of the links of \( \text{DMU}_{jk} \) on its second period inputs renders this DMU inefficient. At the same time, the total efficiency of DMUs B, D and E increases when compared to the values displayed in Table 1(b) due to the increase in the corresponding links and inputs of \( \text{DMU}_{lk} \).

Finally, consider the third set of simulations presented in Table 3, where the second period inputs of \( \text{DMU}_{jk} \) have been increased further. In this case, when diffusion and structural development are not considered, all first three DMUs are efficient. However, as was the case in the second set of tables, the effects of learning and structural development render \( \text{DMU}_{lk} \) inefficient, though the relative efficiency of DMUs B, D and E increases with respect to the previous scenarios.

These numerical results illustrate how the effects of learning and structural development operate within a dynamic input-oriented DEA setting, an intuition that will be validated when considering the case study presented in the following section.

### 4.2. A European efficiency comparison

We apply the dynamic cumulative input-oriented DEA setting implemented in the previous section to measure the efficiency of several European Union countries, categorized as either technologically advanced (Denmark, Finland, Germany and Sweden) or laggards (Greece, Italy, Portugal and Spain). We use the variables defined in Table 4 to account for the different inputs, carry-overs and outputs describing the relative performance of each country. Constant returns to scale with variable \( \varepsilon \) values will be assumed across the different settings analyzed through this section for a total of five periods of time.
Table 3
Increasing further the second period inputs of DMUs.

(a) Absent learning and structural development

<table>
<thead>
<tr>
<th>DMU</th>
<th>T=1</th>
<th>Link</th>
<th>T=2</th>
<th>Link</th>
<th>T=1</th>
<th>T=2</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>14</td>
<td>16</td>
<td>1.0000</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>16</td>
<td>1.0000</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>12</td>
<td>0</td>
<td>8</td>
<td>20</td>
<td>24</td>
<td>1.0000</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
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<td>15</td>
<td>15</td>
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<td>20</td>
<td>0.4117</td>
</tr>
<tr>
<td>E</td>
<td>20</td>
<td>25</td>
<td>20</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>0.4075</td>
</tr>
</tbody>
</table>

(b) Introducing learning and infrastructures

<table>
<thead>
<tr>
<th>DMU</th>
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<th>Link</th>
<th>T=2</th>
<th>Link</th>
<th>T=1</th>
<th>T=2</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>12</td>
<td>2</td>
<td>2</td>
<td>17</td>
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</tr>
<tr>
<td>B</td>
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<td>12</td>
<td>10</td>
<td>10</td>
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<td>24</td>
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</tr>
<tr>
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<tr>
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<td>15</td>
<td>15</td>
<td>48</td>
<td>48</td>
<td>0.4117</td>
</tr>
<tr>
<td>E</td>
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<td>20</td>
<td>20</td>
<td>48</td>
<td>52</td>
<td>0.4075</td>
</tr>
</tbody>
</table>

Table 4
On European divergence and efficiency: countries and variables.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Variables (Time period: 2011-2015) [Eurostat data matrix code]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>Input 1 Labor Force [lfsi_act_a]</td>
</tr>
<tr>
<td></td>
<td>Input 2 Gross Fixed Capital [tec00011]</td>
</tr>
<tr>
<td></td>
<td>Link 1 #R&amp;D [rd_p_persocc]</td>
</tr>
<tr>
<td></td>
<td>Link 2 $R&amp;D [rd_e_gerdito]</td>
</tr>
<tr>
<td></td>
<td>Output 1 GDP per capita [sdg_08_10]</td>
</tr>
<tr>
<td></td>
<td>Output 2 Summary Innovation Index [Operational Union Scoreboard]</td>
</tr>
<tr>
<td>Finland</td>
<td>Germany</td>
</tr>
<tr>
<td>Sweden</td>
<td>Greece</td>
</tr>
<tr>
<td>Italy</td>
<td>Portugal</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
</tr>
</tbody>
</table>

As we did in the previous section, it has been assumed that

\[
\frac{\beta_{1,t}}{\max_{t} \beta_{1,t}} = \frac{\max_{t} \beta_{1,t}}{\max_{t} \beta_{1,t}}. \forall j, t, \text{ Note, however, that the relative values of } \frac{\beta_{1,t}}{\max_{t} \beta_{1,t}} \text{ allow the structural capital and technology accumulation process to be modified through time depending, for instance, on the yearly GDP percentage invested in R&D activities or the Summary Innovation Index (SII) of a given country, which provides a general comparison across different systems of innovation. In particular, the SII has been designed by the European Commission to measure the technological development and innovation capacities of countries. A detailed description of the main variables composing the index can be found at } \text{http://ec.europa.eu/growth/industry/innovation/facts-figures/scoreboards_it.}

Fig. 5 describes the main results obtained when considering both GDP and the SII as outputs of the model, with Denmark, Finland, Greece and Portugal being input-oriented overall efficient. Similarly, Fig. 6 presents the overall efficiency of the countries when focusing only on the structural SII variable as the output of the model. In both cases, we observe a consistent decrease in efficiency as the value of the } \varepsilon \text{ parameter increases. Moreover, in accordance with the numerical results described through the previous section, Tables 5 and 6 highlight the decrease in term efficiency that arises when comparing the dynamic evolution derived from the TIs setting with that of the cumulative model introduced in the current paper (for an } \varepsilon = 2 \text{ scenario). The relatively low efficiencies displayed by technologically developed countries such as Sweden and Germany may seem surprising. However, this result has a very intuitive explanation. The Swedish economy has a highly developed national innovation system, a fact reflected in the largest value of the SII variable within the group of countries being analyzed. On the other hand, its input variables display similar values to those of the Finnish and Danish economies to which comparisons are generally drawn. Thus, the Swedish input capacity does not suffice to justify the considerable larger value of its SII variable. Due to its size, Germany exhibits considerable larger values of the Gross Fixed Capital and #R&D input variables than any of the other countries. At the same time,}
the value of the German SII variable is similar to those displayed by Denmark and Finland. The resulting cumulative effects condition the results obtained and provide important intuition regarding the critical role played by the variables selected and their inherent diffusion processes on the relative efficiency of countries.

In particular, the results obtained illustrate the dynamic behavior implied by the model, where a decrease in efficiency follows from an increase in the input cumulative capacity of the DMUs. Note, however, that we have designed and implemented a process that considers the accumulation of inputs as a negative outcome, while it could be considered a positive one. That is, DMUs should aim at increasing the amount of knowledge acquired, which, at the same time, enhances their capacity to interact and merge with other DMUs. These latter features increase the competitiveness of DMUs within industrial cluster environments, leading to the emergence of potential alliances with other DMUs.

These strategic network extensions, characterizing scenarios common to the international business and industrial organization literatures, remain unstudied in the standard DEA models. The current formal framework has been designed to incorporate them and aims at bridging the gap between the DEA literature and other research disciplines currently dominated by parametric approaches.

5. Conclusion and extensions

The model introduced in this paper aims at bridging the existing gap between the DEA literature and its business and economics counterparts. The diffusion of knowledge taking place...
within DMUs and their capacity to accumulate structural capital through time, together with the exogenous effects derived from industrial clusters or national innovation systems, should be accounted for when measuring their efficiency. In this regard, the current model provides a set of guidelines to account for these dynamic cumulative processes and should open the way for the development of novel models considering the capacity of DMUs to merge or potential interactions arising among different types of processes.

We have indeed illustrated how to combine both dynamic structures, i.e. the information spillovers taking place across different groups of workers within an organization and the sequential accumulation of technology based on its innovation capacity (or that of the country where the organization is located). It should be emphasized that we have used the numerical simulations and the case study to validate the intuition regarding the effects of learning and capital accumulation on the input-oriented DEA model. However, the effects of both processes on outputs as a result of the potential interactions taking place among different DMUs are more relevant from a strategic point of view. Thus, immediate extensions considering output- and non-oriented environments should be developed so as to analyze the different types of strategic interactions and objectives arising both within and across DMUs.

Among the potential extensions of the model, we should note that can be defined as a variable measuring the incremental dynamic efficiency required from a merger or the potential cooperation between two DMUs to reach the efficient frontier. Moreover, the model can be easily adapted and implemented within two- and three-stage DEA frameworks, incorporating the current set of diffusion and cumulative processes within the formal structure of models where knowledge and infrastructures are generally shared among sub-DMUs. In particular, the dynamic structure introduced in the current paper can be adapted to account for the optimal distribution of freely available resources through the different stages of a multi-stage DEA or network model. The inclusion of freely distributed inputs within a two-stage DEA framework was initially considered by Zha and Liang (2010) and recently extended to allow for shared intermediate outputs by Izadikhah, Tavani, Di Caprio, and Santos-Arteaga (2018).

In this regard, human capital could be assumed to be freely distributed across stages, with technological knowledge becoming an intermediate output applicable by the sub-DMUs located in subsequent stages and the efficiency of the whole system being determined by the infrastructure within which the set of sub-DMUs is located. Incorporating freely distributed inputs and shared intermediate outputs within our dynamic framework would also allow us to analyze further strategic incentives derived from the output and non-oriented versions of the current model.

Acknowledgment

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