



Modeling Sequential Information Acquisition Behavior in Rational Decision Making*

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ABSTRACT

Most real-life decisions are made with less than perfect information and there is often some opportunity to acquire additional information to increase the quality of the decision. In this article, we define and study the sequential information acquisition process of a rational decision maker (DM) when allowed to acquire any finite amount of information from a set of products defined by vectors of characteristics. The information acquisition process of the DM depends both on the values of the characteristics observed previously and the number and potential realizations of the remaining characteristics. Each time an observation is acquired, the DM modifies the probability of improving upon the products already observed with the number of observations available. We construct two real-valued functions whose crossing points determine the decision of how to allocate each available piece of information. We provide several numerical simulations to illustrate the information acquisition incentives defining the behavior of the DM. Applications to knowledge management and decision support systems follow immediately from our results, particularly when considering the introduction and acceptance of new

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INTRODUCTION

Consider a situation where a decision maker (DM) has to check a finite number of characteristics from a set of two dimensional products. In the words of Bearden and Connolly (2007), a DM must “continuously decide when to stop searching *within* an option—to get a better estimate of its value—and when to stop searching *between* options—to find one of high value. Striking a balance between *depth* (within-option) and *breadth* (between-option) search presents a complex problem.”

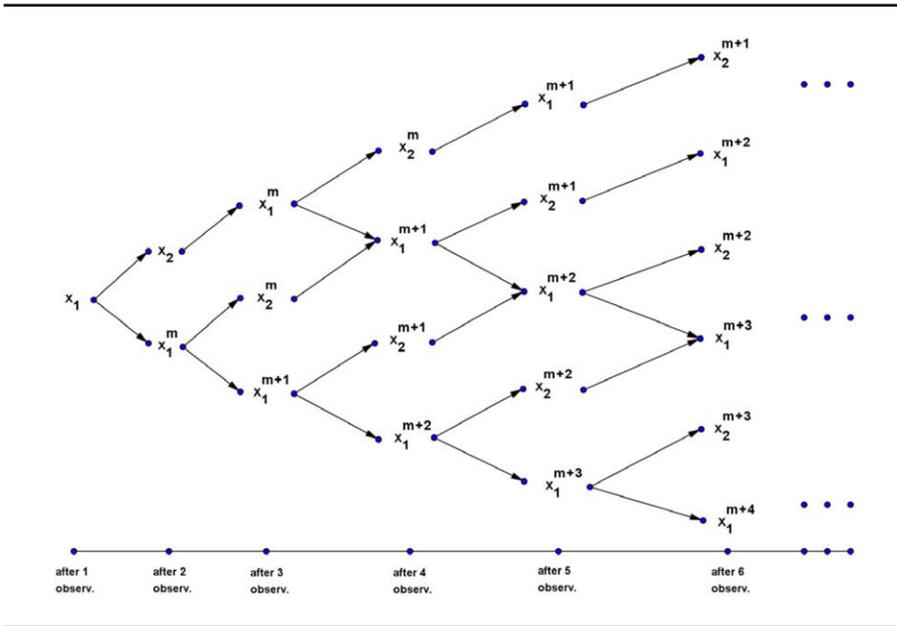
The standard information acquisition process generally considered by the operations research and management literatures is described in Figure 1. The subscripts of each observation denote the characteristic of the product, either the first or the second. Each superscript indicates the product being observed after the initial one. In particular, m refers to the second product observed, $m + 1$ to the third one, and so on. Whenever a DM acquires information sequentially he must choose between continuing with the latest product observed (upper arrow leaving from each node when a choice must be made) or start acquiring information on a new product (lower arrow leaving from each node).

The first characteristic of a product has been assumed to be more important than the second one, leading the DM to start acquiring information on the first characteristic of each new product instead of the second one. This is the case because acquiring information on the first characteristic provides a higher expected utility than doing so on the second, as follows trivially from the information acquisition context that will be described in the article.

The behavior of the DM and the resulting stopping rule depend on the expected value derived from the next characteristic to be observed and the information acquisition costs faced by the DM. The DM rarely considers recalling previous partially observed products and the information acquisition algorithm does not depend on the number of observations remaining to be acquired or the set of potential improvements that may be realized relative to the products previously observed. The inclusion of these search features is particularly important when analyzing online search environments, where the costs of recalling a previous alternative among those displayed by the search engine is considerably low. On the other hand, the cognitive costs required on the side of the DM increase substantially depending on the amount of information that must be acquired and the capacity of the DM to assimilate this information.

The increasing complexity of this type of approach is reflected in Figure 2, where the DM considers the values of all the realizations observed before acquiring the next observation. Each arrow represents this causality, with the number of arrows leaving from a given node defining the set of alternatives (product characteristics) that must be considered by a DM when acquiring the next observation.

Figure 1: Standard sequential information acquisition process—absent recall and independent from the number of observations remaining to be acquired.



At the same time, when defining his information acquisition process, the DM should consider the number and potential realizations of all the remaining observations, together with the probability that they lead to a product delivering a higher utility than the best among the observed ones. In order to illustrate the complexity and nonrecursivity of the information acquisition process that must be defined by the DM, consider the space of potential realizations that should be analyzed when a total of two observations are acquired. Figure 3 illustrates this scenario.

In this case, before deciding how to allocate his second (and final) observation, the DM must account for all the combinations of the first observation from an initial product, x_1 , with both the second characteristic of this product, x_2 , and the first characteristic from a new product, x_1^m .

A similar, though more complex, intuition follows from a setting where the DM acquires three observations. As illustrated in Figure 4, the space of potential realizations that must be analyzed when three observations are acquired becomes more complex because a larger amount of potential combinations must be considered by the DM.

Note how each new observation produces a set of possible combinations that must be accounted for by the DM when acquiring the first observation. Despite this fact, the setting with three observations can still be analyzed within a three dimensional space that determines the information acquisition incentives of the DM based on all the potential realizations of x_1 and x_1^m .

However, the set of potential realizations that must be analyzed when four observations are acquired requires a higher dimensional space to account for

Figure 2: Sequential information acquisition process with recall.

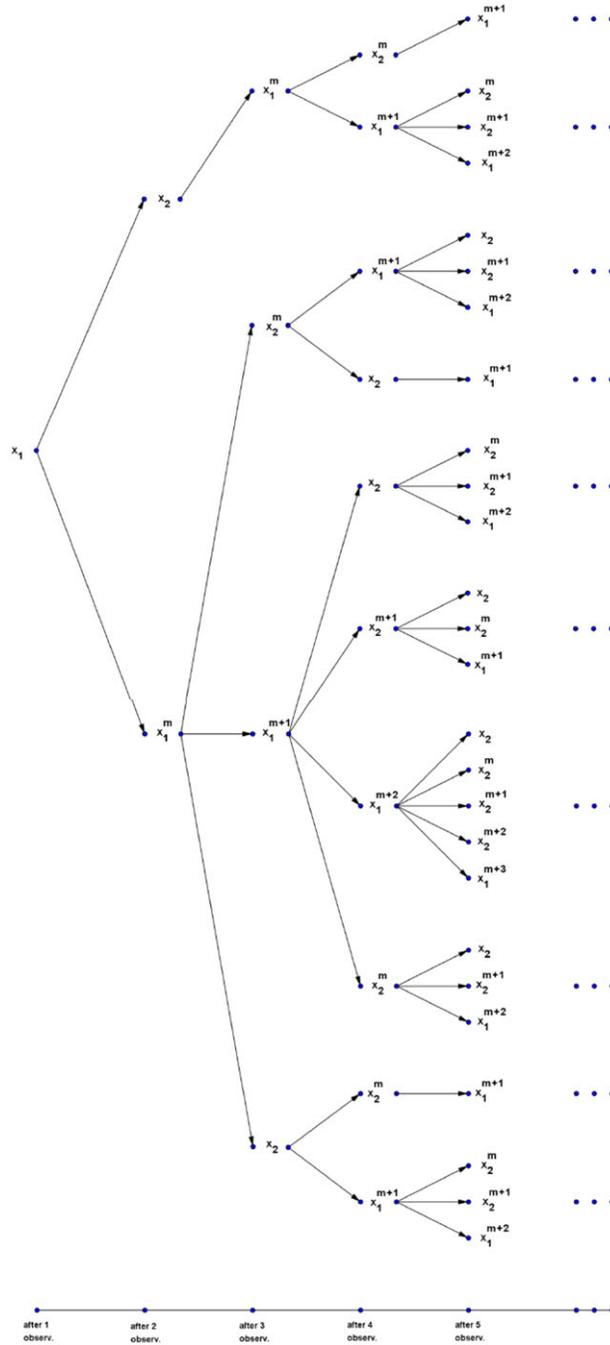


Figure 3: Space of potential realizations that must be considered with two observations.

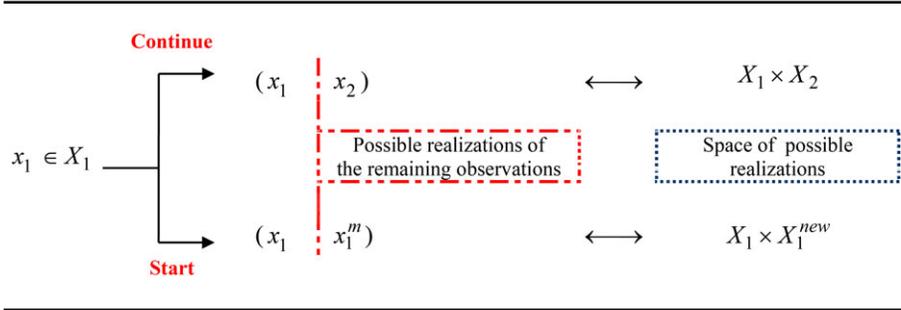
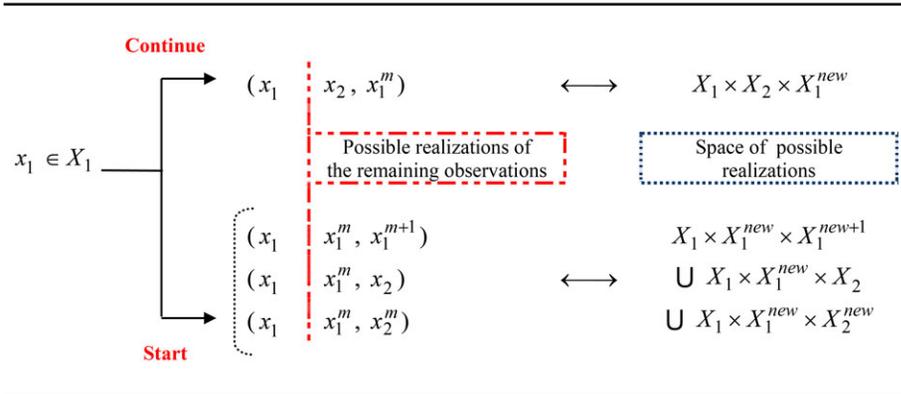


Figure 4: Space of potential realizations that must be considered with three observations.



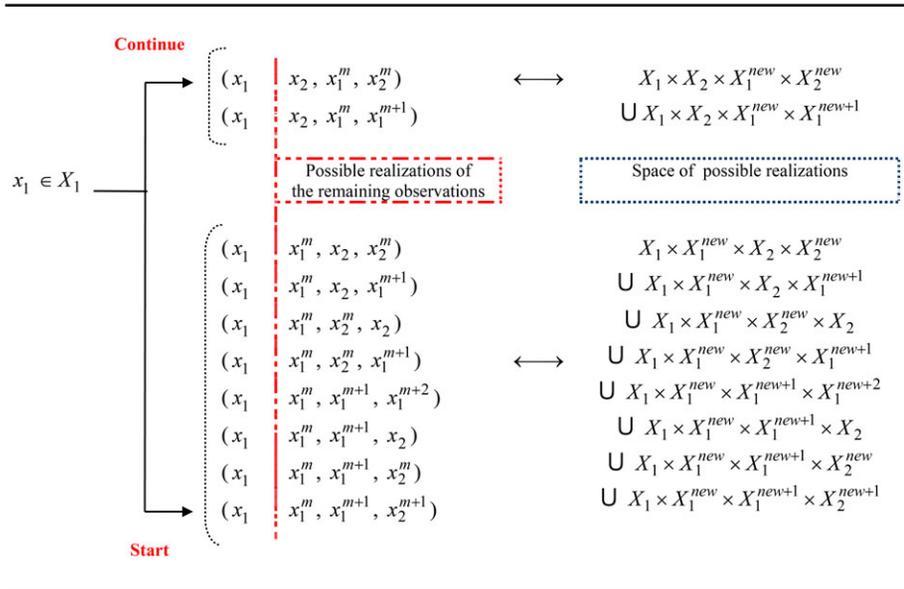
the combinations that must be considered by the DM. Figure 5 illustrates the corresponding scenario.

The nonrecursivity of this information acquisition environment, whose dimension is modified each time we add or observe a realization, constitutes a serious drawback when designing formal sequential search processes. In particular, the information acquisition incentives of DMs must be recalculated each time a new observation is acquired.

Thus, the evolution of the information acquisition process must depend directly on the values of *all the characteristics observed previously, the number and potential realizations of all the remaining observations, and all the possible combinations between the potential realizations of the remaining observations and the characteristics observed.* These requirements prevent the use of standard dynamic programming techniques in the design of the algorithm.

Defining an information acquisition structure that acknowledges and accounts (to a certain extent) for the complexity of this problem requires introducing a heuristic mechanism to improve the tractability of the problem. As a result, the information acquisition incentives of the DMs will be based on the

Figure 5: Space of potential realizations that must be considered with four observations.



potential improvements upon the product being observed (or upon the best among the previously observed ones) that may be achieved with the number of observations available. Given this heuristic, the current article formalizes and studies the information acquisition behavior of a rational DM when acquiring n pieces of information from a set of products defined by vectors of two characteristics.

We show that the decision of how to allocate the information depends on two well-defined real-valued functions. One function describes the utility that the DM expects to derive from continuing acquiring information on a given product, while the other defines the expected utility obtained from checking the characteristics of a new product. We also illustrate how the DMs' willingness to search depends on their attitude towards risk. Moreover, even though we do not consider information acquisition costs, they can be trivially incorporated into our model.

Applications to knowledge management and decision support systems follow immediately from the proposed framework, particularly when considering the introduction and acceptance of new technological products and when formalizing online search environments. Our formal setting also allows firms to forecast the information acquisition behavior of DMs and the probability of having their products checked within the DMs' search process.

The remainder of this article is organized as follows. In "Literature Review" section, we review the information acquisition literature. "Basic Notations and Main Assumptions" section deals with the standard notation and basic assumptions needed to develop the model. "Expected Search Utilities" section defines the expected search utility functions within a reference two-observations setting. "Acquiring More Observations" section extends the analysis to the three and five

observations environments while “The Proposed Information Acquisition Structure with n Observations” section defines the information acquisition structure when any finite number of observations can be acquired by the DM. “Framing and Reference Products” section illustrates the consequences from modifying the reference product employed in the design of the algorithm. “The Information Acquisition Process” section describes the sequential information acquisition process of the DM. “Conclusions and Managerial Significance” section summarizes the main findings and highlights their managerial significance. An elaborate example based on the results displayed by an online search engine when the DM acquires a total of five observations is presented in the Online Appendix.

LITERATURE REVIEW

The elimination of uncertainty and its transformation into risk provides the main motivation for the development of information acquisition algorithms (Di Caprio, Santos Arteaga, & Tavana, 2014b). The design and study of algorithmic information acquisition processes constitutes one of the main focuses of the operations research literature (MacQueen & Miller, 1960; MacQueen, 1964). Indeed, the management and operations research literatures have been considering the optimal information gathering problem of DMs for quite some time, in particular when analyzing the acquisition of a new technology (McCardle, 1985; Lippman & McCardle, 1991). These authors build rational information gathering algorithms where the decision of which information to acquire is based on the utility gains expected to be obtained by the DM. The use of dynamic programming techniques to illustrate the existence of optimal decision threshold values requires imposing several formal restrictions on the corresponding return functions. These constraints help understand the structural complexities involved in designing optimal information acquisition and evaluation processes.

This research line remains focused on the importance that search costs have in limiting the information processing capacity of generally risk neutral DMs when deciding whether to continue or to stop their search within settings defined by the acceptance of a given investment opportunity or the adoption of a technology (Levesque & Maillart, 2008; Ulu & Smith, 2009; Smith & Ulu, 2012). Extensions of this framework are provided by Shepherd and Levesque (2002), who endow the DM with basic memory capacities when computing the evolution of the expected profits derived from a given business opportunity. At the same time, the decision theoretical branch of operations research has also extended this type of models to allow for comparisons between different technologies (Cho & McCardle, 2009; Kwon, 2010).

Consider now the potential applications of these information acquisition models within the decision support and consumer choice literatures. As was the case with their theoretical counterparts, applications of multi-attribute information acquisition algorithms to these branches of the literature rely on simplifying mechanisms that allow for the use of dynamic programming techniques to obtain optimal sequential choice policies. For example, Feinberg and Huber (1996) implemented a screening heuristic limiting the number of alternatives evaluated by the DM.

Lim, Bearden, and Smith (2006) eliminated recall and concentrated on linear additive value functions. Bearden and Connolly (2007) accounted for the sequential information acquisition structure determining the search process of DMs but did not derive the corresponding optimal thresholds or analyze their behavior. This is also the case in Wilde (1980) when defining the optimal choice behavior of a DM for the search and experience characteristics of two dimensional products and in Ratchford (1982) when considering deviations from an unidentified optimum choice value.

A considerable amount of research relates to the applicability of information acquisition algorithms in the design of tools that customize the online shopping environment to the individual preferences of DMs (Haubl & Trifts, 2000). The main lines of research opened by management scholars find their way into the decision support literature dealing with online search environments. In this regard, this literature concentrates on creating decision support tools that allow the DM to compare attributes among different online alternatives (Abrahams & Barkhi, 2013). At the same time, Browne, Pitts, and Wetherbe (2007) illustrate how DMs stop searching for information online depending on the search task being performed. From a supply perspective, Wang, Wei, and Chen (2013) proposed a method to estimate the probability that a product is considered for purchase after being inspected by a consumer, while Dou, Lim, Su, Zhou, and Cui (2010) analyzed how firms can employ their ranking position in the result pages of online search engines to differentiate their products from those of the competitors.

BASIC NOTATIONS AND MAIN ASSUMPTIONS

The main assumptions on which the expected search utilities are built correspond to those described by Di Di Caprio, Santos Arteaga, and Tavana (2014a). In order to keep the current article self-contained, we restate them below.

Let X be a nonempty set and \succsim a *preference relation* defined on X . A *utility function representing a preference relation* \succsim on X is a function $u : X \rightarrow \Re$ such that

$$\forall x, y \in X, \quad x \succsim y \quad \Leftrightarrow \quad u(x) \geq u(y). \quad (1)$$

We use the symbol \geq to denote the standard partial order on the reals. When $X \subseteq \Re$ and \succsim coincides with \geq , we say that u is a *utility function on X* .

Henceforth, G will denote the set of all products. We let X_1 and X_2 , respectively, represent the sets of all possible variants for the first and second characteristics of a product in G . Also, $X = X_1 \times X_2$, so that every product in G can be described by a pair $\langle x_1, x_2 \rangle$ in X . Note that, X_k is called the k th *characteristic factor space*, with $k = 1, 2$, while X stands for the *characteristic space*.

We work under the following assumptions.

Assumption 1: For every $k = 1, 2$, there exist $\alpha_k, \beta_k > 0$, with $\alpha_k \neq \beta_k$, such that $X_k = [\alpha_k, \beta_k]$, where α_k and β_k are the minimum and maximum of X_k .

Assumption 2: The characteristic space X is endowed with a strict preference relation \succ .

This assumption guarantees the rationality of the DM. The model introduced in this article requires the DM to be endowed only with a standard strict preference relation (complete and transitive, thus, rational; Mas-Colell, Whinston, & Green, 1995) that allows him to order the set of products and choose between two of them.

Assumption 3: There exist a continuous additive utility function u representing \succ on X such that each one of its components $u_k : X_k \rightarrow \mathfrak{R}$, where $k = 1, 2$ is a continuous utility function on X_k . A utility function $u : X \rightarrow \mathfrak{R}$ representing \succ on $X = X_1 \times X_2$ is called *additive* (Wakker, 1989) if there exist $u_k : X_k \rightarrow \mathfrak{R}$, where $k = 1, 2$, such that $\forall \langle x_1, x_2 \rangle \in X_1 \times X_2, u(\langle x_1, x_2 \rangle) = u_1(x_1) + u_2(x_2)$.

The sequential property of the information acquisition process provides an intuitive justification for the additive separability assumption imposed on the utility function. However, it seems perfectly reasonable to consider products whose complementarity among characteristics requires a nonseparable utility representation. An immediate example could relate to the purchase of a house, whose evaluation does not only depend on features related to the house itself but also to external location type factors, both of which are usually interrelated. In this sense, the relationship between both types of characteristics could be interpreted such that the second dimension represents the portion of the second characteristic that is not explained by the first one. In this regard, as illustrated by Tavana, Di Caprio, and Santos Arteaga (2014), each characteristic can also be interpreted as a category accounting for different related properties of a product. As a result, search processes would be defined by observable (X_1) and experience (X_2) components, the latter requiring a more detailed inspection of the product to be verified (Nelson, 1970).

Assumption 4: For every $k = 1, 2, \mu_k : X_k \rightarrow [0, 1]$ is a continuous probability density on X_k , whose support, the set $\{x_k \in X_k : \mu_k(x_k) \neq 0\}$, will be denoted by $\text{Supp}(\mu_k)$.

The probability densities μ_1 and μ_2 represent the subjective “beliefs” of the DM. That is, for $k = 1, 2, \mu_k(Y_k)$ is the subjective probability that a randomly observed product from G displays an element $x_k \in Y_k \subseteq X_k$ as its k th characteristic. The probability densities μ_1 and μ_2 are assumed to be independent. However, the information acquisition structure described in the article allows for subjective correlations to be defined between different characteristics within a given product. Considering a correlated environment would not modify the main theoretical structure built in the article though it would lead to different quantitative results through the numerical simulations.

Note that the characteristics and probabilities associated with different products depend on the type of product under consideration. For example, the characteristics that are important to a DM when choosing a laptop computer may be quite different from those considered when choosing a desktop one. It should be emphasized that the DM will not solve a classic optimization problem but searches for a sufficiently good product given his subjective preferences and beliefs. In this sense, our model is in line with the empirical psychological literature on consumer choice based on bounded rationality and the limited capacity of DMs to assimilate all the information available (Samiee, Shimp, & Sharma, 2005; Diab, Gillespie, & Highhouse, 2008)

Finally, following the standard economic theory of choice under uncertainty, we assume that the DM elicits the k th certainty equivalent (CE) value induced by μ_k and u_k as the reference point against which to compare the information collected on the k th characteristic of a given product. Given $k = 1, 2$, the *certainty equivalent* of μ_k and u_k , denoted by ce_k , is a characteristic in X_k that the DM is indifferent to accept in place of the expected one to be obtained through μ_k and u_k . That is, for every $k = 1, 2$, $ce_k = u_k^{-1}(E_k)$, where E_k denotes the expected value of u_k . The existence and uniqueness of the k th CE value ce_k are guaranteed by the continuity and strict increasingness of u_k , respectively.

EXPECTED SEARCH UTILITIES

Two Observations Reference Setting

The set of all products, G , is identified with a compact and convex subset of the 2-dimensional real space \Re^2 . In the current setting, after observing the value of the first characteristic from an initial product, the DM has to decide whether to check the second characteristic from the same product, or to check the first characteristic from a different new product. In this regard, DMs are tacitly assumed to have a well-defined preference order both *within* and *between* characteristics. That is, the first characteristic will be assumed to be more important and, as a result, provide a higher expected utility to the DM than the second one.

We show below that the decision of how to allocate the second available piece of information depends on two real-valued functions defined on X_1 . The DM considers the sum $E_1 + E_2$, corresponding to the expected utility values of the pairs $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$, as the main reference value when calculating both these functions.

Assume that the DM has already checked the first characteristic from an initial product, x_1 , and that he uses his remaining piece of information to observe the second characteristic from this product, x_2 . Clearly, the expected utility gain over $E_1 + E_2$ varies with the value of x_1 observed. That is, for every $x_1 \in X_1$, let

$$P^+(x_1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \tag{2}$$

and

$$P^-(x_1) = \{x_2 \in X_2 \cap \text{Supp}(\mu_2) : u_2(x_2) \leq E_1 + E_2 - u_1(x_1)\}, \tag{3}$$

where $P^+(x_1)$ and $P^-(x_1)$ define the set of x_2 values from the initial product such that their combination with x_1 delivers a higher or lower equal utility than a randomly chosen product from G , respectively.

Let $F : X_1 \rightarrow \Re$ be defined by

$$F(x_1) \stackrel{\text{def}}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2, \tag{4}$$

where $F(x_1)$ describes the DM's expected utility derived from checking the second characteristic of the initial product after observing that the value of the first characteristic is given by x_1 . Note that, if $u_1(x_1) + u_2(x_2) \leq E_1 + E_2$, then choosing a

product from G randomly delivers an expected utility of $E_1 + E_2$ to the DM, which is higher than the utility obtained from choosing the initially observed product, that is, $u_1(x_1) + u_2(x_2)$.

Consider now the expected utility that the DM could gain over the initial (partially observed) product if the second piece of information is used to observe the first characteristic from a different product, x_1^m . For every $x_1 \in X_1$, define $H : X_1 \rightarrow \Re$ as follows:

$$H(x_1) \stackrel{def}{=} (u_1(x_1) + E_2) + C, \quad (5)$$

where

$$C \stackrel{def}{=} \mu_1(x_1^m \leq x_1)(E_1 + E_2) + \mu_1(x_1^m > x_1)(u_1(x_1^m) + E_2). \quad (6)$$

$H(x_1)$ describes the expected utility derived from checking the first characteristic of a new product after observing x_1 . If $x_1^m \leq x_1$, then the new product observed by the DM does not deliver a higher utility than the initial one and the payoff obtained from such an event is set equal to $E_1 + E_2$. This reference value has been chosen to generate a similar payoff environment to the one defined by the function $F(x_1)$.

Note however that the payoffs assigned to the $x_1 > x_1^m \geq ce_1$ and the $ce_1 > x_1^m > x_1$ outcomes within C may seem counterintuitive. In the former case, the new observation is located above the CE value but does not improve upon the initial product, leading to a payoff of $E_1 + E_2$. A similar intuition applies when considering the latter case, with the new observation located below the CE value but its expected utility accounted for as a payoff (because $x_1^m > x_1$). We will analyze the main consequences derived from modifying the reference values considered by the DM in “Framing and Reference Products” section.

The current approach to the information acquisition behavior of DMs considers only potential improvements relative to the initial reference product, whose value is determined by x_1 . In this case, the function $F(x_1)$ completely eliminates any uncertainty regarding the initial product, while DMs only observe the initial product partially within $H(x_1)$. However, the function $H(\cdot)$ allows for an additional product to be observed relative to $F(\cdot)$. This observational advantage distorts the incentives of DMs when a small number of observations is considered, with larger amounts eliminating the resulting effect, as intuition prescribes and the numerical simulations will illustrate.

In order to account for the set of potential improvements based on the number of remaining observations, we rewrite the expression for C within $H(x_1)$ in the following way:

$$C = \psi_1(1, 0, \mu_1)(E_1 + E_2) + \psi_1(1, 1, \mu_1)(u_1(x_1^m) + E_2). \quad (7)$$

We will motivate this notational modification in the following section.

Finally, note that the expected search utilities F and H determine the information acquisition behavior of the DM. That is, after observing x_1 , the DM will either continue acquiring information on the initial product or start acquiring information on a new one depending on whether it is F or H the function taking the highest value at x_1 . It may also happen that $F(x_1^*) = H(x_1^*)$ at a given $x_1^* \in X_1$, making the DM indifferent between continuing with the initial product and

starting with a new one. These x_1^* values behave as information acquisition thresholds that partition X_1 in subintervals whose values induce the DM to either continue acquiring information on the initial product or to switch and start observing a new one.

Numerical Simulations

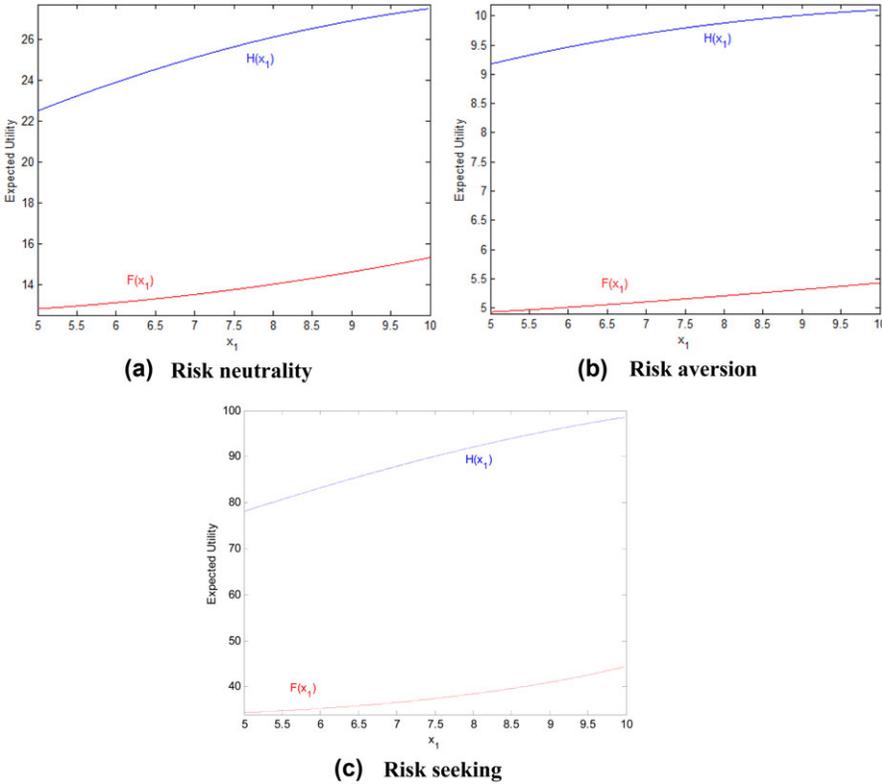
This section presents numerical simulations that illustrate the information acquisition behavior of DMs when acquiring two observations. Simulations will be provided for risk neutral and risk averse DMs, while keeping in mind that a large set of potential scenarios can be studied numerically following the theoretical setting introduced in the article.

Consider, as the basic reference cases, the two observations acquisition behavior that follows from a standard risk neutral and a risk averse utility function when uniform probabilities are assumed on both X_1 and X_2 . The following parameter values will be used in all the numerical simulations presented in the article:

- (i) Characteristic spaces: $X_1 = [5, 10]$, $X_2 = [0, 10]$;
- (ii) Risk neutral utility functions: $\forall x_1 \in X_1, u_1(x_1) = x_1$; $\forall x_2 \in X_2, u_2(x_2) = x_2$;
- (iii) Risk averse utility functions: $\forall x_1 \in X_1, u_1(x_1) = \sqrt{x_1}$; $\forall x_2 \in X_2, u_2(x_2) = \sqrt{x_2}$;
- (iv) Risk seeking utility functions: $\forall x_1 \in X_1, u_1(x_1) = (x_1)^{3/2}$; $\forall x_2 \in X_2, u_2(x_2) = (x_2)^{3/2}$;
- (v) Probability densities: $\forall x_1 \in X_1, \mu_1(x_1) = \frac{1}{5}$; $\forall x_2 \in X_2, \mu_2(x_2) = \frac{1}{10}$.

The basic reference risk neutral, risk averse and risk seeking settings are represented in Figures 6a–c, respectively. In all figures, the horizontal axis represents the set of x_1 realizations that may be observed by the DM, with the corresponding subjective expected utilities defined on the vertical axis. Observe the complete dominance of the function $H(x_1)$ over $F(x_1)$ when two observations are considered. As we will see through the rest of the numerical simulations, this initial effect wears off as a higher number of observations become available to the DM. Clearly, when only two observations are acquired, continuing with the initial product observed implies ending up with a unique product: either the initial one if it provides a utility higher than $\langle ce_1, ce_2 \rangle$ or a randomly chosen product if the utility derived from the initial one is below that of $\langle ce_1, ce_2 \rangle$.

However, when acquiring information on a new product, the DM will have two products to choose from, the initial and a new one, both partially observed. The new product is added to the total utility of the DM, biasing his incentives towards observing a product other than the initial one. This effect is eliminated in Di Caprio and Santos Arteaga (2009), where a unique final product is accounted for in the choice set of DMs through both functions F and H . The current framework is designed to accommodate a large amount of potential observations and it is to be treated as such, with a large bias towards diversification arising when a small number of observations are acquired.

Figure 6: Information acquisition incentives with two observations.

ACQUIRING MORE OBSERVATIONS

The information acquisition setting with bidimensional products and two observations described in the previous section has been extensively analyzed by Di Caprio and Santos Arteaga (2009), Di Caprio et al. (2014a), and Tavana et al. (2014). It provides a useful set of guidelines in environments with information constrained DMs as well as the basic framework on which to add sophistication to the assimilation capacities of DMs. We remain constrained within a bidimensional product environment but require DMs to be sophisticated enough to forecast the expected improvements that may arise from any of the remaining observations available. These improvements are relative to a given reference product whose value may change as additional information is acquired. A general formulation will be provided after introducing two intuitive settings with three and five observations, respectively.

Before describing these settings, we must define the forecasting capacities of DMs. A binomial distribution determined by the number of observations remaining to be acquired by the DM will be used to define his sequential dynamic behavior.

The probability that l among the remaining n observations improve upon the observed characteristic x_1 and deliver an expected product better than the one that has been partially observed is given by the following binomial distribution:

$$\psi_1(n, l, \mu_1(x_1^m > x_1)) = \binom{n}{l} \mu_1(x_1^m > x_1)^l (1 - \mu_1(x_1^m > x_1))^{n-l}, \quad (8)$$

where $\mu_1(x_1^m > x_1)$ is the probability that a new randomly selected product is endowed with a better first characteristic than the currently observed one, which is endowed with x_1 . Similarly, when the second characteristic is considered, all possible combinations delivering an improvement over the initial partially observed product should be accounted for

$$\psi_2(n, l, \mu_2(x_2^m \in P^+(x_1))) = \binom{n}{l} \mu_2(x_2^m \in P^+(x_1))^l (1 - \mu_2(x_2^m \in P^+(x_1)))^{n-l} \quad (9)$$

where $\mu_2(x_2^m \in P^+(x_1))$ is the probability that a new randomly selected product has a second characteristic belonging to the set $P^+(x_1)$ determined by the observed x_1 realization.

The combination of $\psi_1(n, l, \mu_1(x_1^m > x_1))$ and $\psi_2(n, l, \mu_2(x_2^m \in P^+(x_1)))$ determines the probability that a randomly selected product is endowed with an expected set of characteristics from X_1 and X_2 better than the partially observed product defined by $(x_1, P^+(x_1))$. In order to simplify notation, we will refer to both these binomials by $\psi_1(n, l, \mu_1)$ and $\psi_2(n, l, \mu_2)$, respectively, while accounting for the corresponding values of n and l . Moreover, we will allow for the initial reference points to be modified in “Framing and Reference Products” section, where the notation will be adjusted accordingly.

The Proposed Information Acquisition Structure with Three Observations

The information acquisition scenario with three observations is introduced to provide some basic intuition on the transition from the two observations setting to a general one with a total of n observations.

Consider the information acquisition problem faced by a DM after having gathered the first observation from an initial product, given by x_1 , when a total of three observations can be acquired. In this case, the DM must calculate two enhanced versions of the original functions $F(x_1)$ and $H(x_1)$. These new functions must account for the two observations left to be acquired by the DM and the probability that such observations provide a product better than the initially observed one, which will be used through this section and the following one as the main reference value. When calculating the enhanced function $F(x_1)$, we must account for the fact that the DM uses the second piece of information available to acquire x_2 and observe the initial product fully, that is, the DM observes (x_1, x_2) . As a result, the payoff obtained by the DM depends on the expected realization of x_2 and that of x_1^m , that is, the first characteristic from the new product observed, with the following combinations being considered regarding the third and final observation:

- (1-0)** This option corresponds to the subcase defined by the binomial probability $\psi_1(1, 0, \mu_1)$. It implies that after fully observing the initial product, the final observation x_1^m does not provide a characteristic higher than x_1 . As a result, because the final observation does not improve upon the initial one *and the DM has not yet observed x_2* , the expected outcome following from this event is defined by the CE product: $\psi_1(1, 0, \mu_1)(E_1 + E_2)$. That is, the default payoff derived from an unsuccessful search is assumed to be given by the CE product.
- (1-1)** This option corresponds to the subcase defined by the binomial probability $\psi_1(1, 1, \mu_1)$. It implies that after fully observing the initial product, the final observation x_1^m provides a characteristic higher than x_1 . As a result, because the final observation improves upon the initial one *and the DM has not yet observed x_2* , the expected outcome following from this event is given by $\psi_1(1, 1, \mu_1)(u_1(x_1^m) + E_2)$.

When defining the enhanced version of function $H(x_1)$, we must account for the fact that the DM has one more observation left to acquire than in the $F(x_1)$ setting, because the second observation has not been used to acquire x_2 . Thus, the sets of possible combinations that must be considered when defining the enhanced versions of function $H(x_1)$ always include one observation more than those defining the enhanced $F(x_1)$ setting. The following combinations arise from the set of two observations that the DM has left to acquire within the current setting, with the notation describing the same type of sequential pattern as the one introduced in the subcases above.

- (2-0)** This option implies that none of the two observations left provides a $x_1^m > x_1$. Therefore, the expected outcome following from this event is defined by the CE product: $\psi_1(2, 0, \mu_1)(E_1 + E_2)$.
- (2-1)** This option implies that only one of the two observations left provides a $x_1^m > x_1$. It also implies that the observation leading to $x_1^m > x_1$ must be the second one. That is, if the observation providing $x_1^m > x_1$ would have been the first one, then the second observation would have been used by the DM to acquire x_2^m , leading to the (1-1) subcase described below. Thus, the DM ends up with a partially observed product $\langle x_1^m, ce_2 \rangle$. The resulting expected payoff is therefore given by: $\psi_1(2, 1, \mu_1)(u_1(x_1^m) + E_2)$.
- (1-1)** This option implies that the first observation acquired leads to a characteristic $x_1^m > x_1$, which allows the DM to use the remaining observation to acquire x_2^m , that is, the second characteristic from the new product observed. This is trivially the optimal way to proceed because $x_1^m > x_1$ and all second characteristics are equally distributed. Note that the set of possible outcomes derived from observing x_2^m is defined by (1-0) and (1-1). The resulting expected payoff is therefore given by $\psi_1(1, 1, \mu_1)[\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)]$.

Even though we will modify the reference points defining the corresponding enhanced functions $F(x_1)$ and $H(x_1)$ and allow for alternative payoff scenarios

based on the expected realizations of x_2 in “Framing and Reference Products” section, several comments are due now. The basic reference product upon which the DM is expected to improve through the information acquisition process is given by $(x_1, P^+(x_1))$. That is, the characteristic initially observed determines the reference point on which the information acquisition process is based. The consequences for the information acquisition process of DMs will become evident below, as we describe the different sections composing the algorithm. The intuitive explanation for this assumption may range from framing and context effects together with optimism or pessimism on the side of the DMs (Kahneman & Tversky, 2000; Novemsky, Dhar, Schwarz, & Simonson, 2007) to subjective motivations based on the value of the information being acquired (Diehl, 2005; Santos Arteaga, Di Caprio, & Tavana, 2014). Moreover, when acquiring three or more observations, shifting the reference points to the CE product implies forcing the DM to ignore combinations of products that could potentially improve upon the CE one. We will however provide numerical simulations accounting for different reference-based settings. The rationale for the current environment will become clearer through the next section.

We are now able to write an expression for the functions $F(x_1)$ and $H(x_1)$ in a setting with three observations. We will denote these functions by $F(x_1|3)$ and $H(x_1|3)$, respectively.

$$\begin{aligned}
 F(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + A(x_1|3) \\
 &+ \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 + B(x_1|3), \tag{10}
 \end{aligned}$$

where

$$\begin{aligned}
 A(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1)(E_1 + E_2) \\
 &+ \psi_1(1, 1, \mu_1)(u_1(x_1^m) + E_2)]dx_2, \tag{11}
 \end{aligned}$$

$$\begin{aligned}
 B(x_1|3) &\stackrel{def}{=} \int_{P^-(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1)(E_1 + E_2) \\
 &+ \psi_1(1, 1, \mu_1)(u_1(x_1^m) + E_2)]dx_2. \tag{12}
 \end{aligned}$$

The reference value framework assumed in the current section implies that the set of acceptable expected outcomes within B relates directly to the potential realizations of $x_2 \in P^+(x_1)$. That is, the realizations of the second characteristic from the initial product have a reference effect on the resulting expected payoffs derived from the posterior observations calculated within both A and B . As we will see later, the current framework makes more intuitive sense as the number of observations increases. This is the case even though it may however seem intuitively correct to define the expected improvements to be achieved relative

to the CE product. That is, with the new characteristic x_1^m being located above the value ce_1 and the corresponding set $P^+(x_1^m)$ requiring that $x_2^m > ce_2$ when added to x_1^m . These requirements, $x_1 > ce_1$ and $x_2^m > ce_2$, would also imply that a substantial amount of potential combinations of characteristics leading to expected products with a higher utility than the CE one would be eliminated. Clearly, in a sequential information acquisition setting, as the DM observes products leading to a higher utility than the CE one, the highest among these products will be used as the reference one on which improvements must be defined. We return to this topic later in the article.

Consider now the function $H(x_1|3)$.

$$H(x_1|3) \stackrel{def}{=} (u_1(x_1) + E_2) + C(x_1|3), \quad (13)$$

where

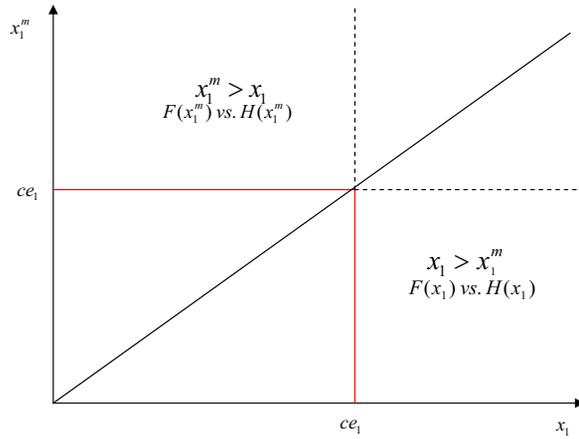
$$\begin{aligned} C(x_1|3) &\stackrel{def}{=} \psi_1(2, 0, \mu_1)(E_1 + E_2) + \psi_1(2, 1, \mu_1)(u_1(x_1^m) + E_2) \\ &\quad + \psi_1(1, 1, \mu_1)[\psi_2(1, 1, \mu_2)(u_1(x_1^m) \\ &\quad + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)]. \end{aligned} \quad (14)$$

Note that in the function $H(x_1|3)$ case, the improvements must be calculated with respect to the partially observed product defined by x_1 . The additional observation available to the DM modifies the set of potential improvements when compared to the function $F(x_1|3)$. This latter one is based on the weighted average that follows from the expected realizations of x_2 and the corresponding sets $P^+(x_1)$. Here, improvements are based on a new observed product calculated with respect to the set $P^+(x_1)$ defined by the partially observed first product absent any potential x_2 realization.

We should emphasize that the heuristic constraint imposed on the information assimilation and cognitive capacities of the DM could be relaxed. That is, the information acquisition environment described in the current article takes a given observed product as a reference and assumes that the DM tries to improve upon this product using the remaining information available. The resulting sequential structure allows us to condense this process into a two dimensional setting determined by the realizations of the first characteristic of the product that is currently being observed by the DM.

However, the information assimilation capacities of the DM could be stretched. That is, the information acquisition structure considered by the DM could be defined in terms of the first characteristic from both the current product being observed and the next (new) product that will be observed. In other words, the DM should be aware of the fact that he can define the continuation and starting payoffs in terms of the realizations of x_1 and x_1^m (a characteristic that he will be surely observing either before or after x_2) and the set of potential realizations of x_2 , x_2^m , and x_1^{m+1} . The implementation of this type of information acquisition setting requires imposing a different type of heuristic mechanism that has been explored by Di Caprio, Santos Arteaga, and Tavana (in press). As Figure 7 illustrates, the analysis of the continuation and starting payoffs requires a three dimensional space defined in terms of x_1 and x_1^m .

Figure 7: Projection of the three-dimensional space when DMs consider three observations simultaneously.



In particular, the corresponding expected search utilities are defined on a symmetric space delimited by the 45° line. This symmetric division highlights the fact that the product considered by the DM must be the highest one (in utility terms) between the two that have been partially observed. Depending on which realization is the highest one, the DM must compute the potential combinations of either x_1 or x_1^m with x_2 or x_2^m , respectively, when defining the corresponding expected search utilities. At the same time, whenever the realizations of both x_1 and x_1^m are located above ce_1 , the default payoff accounted for by the DM will not consist of the CE product. A similar intuition applies, though in terms of x_1 and x_2 , when considering products defined by vectors of three characteristics.

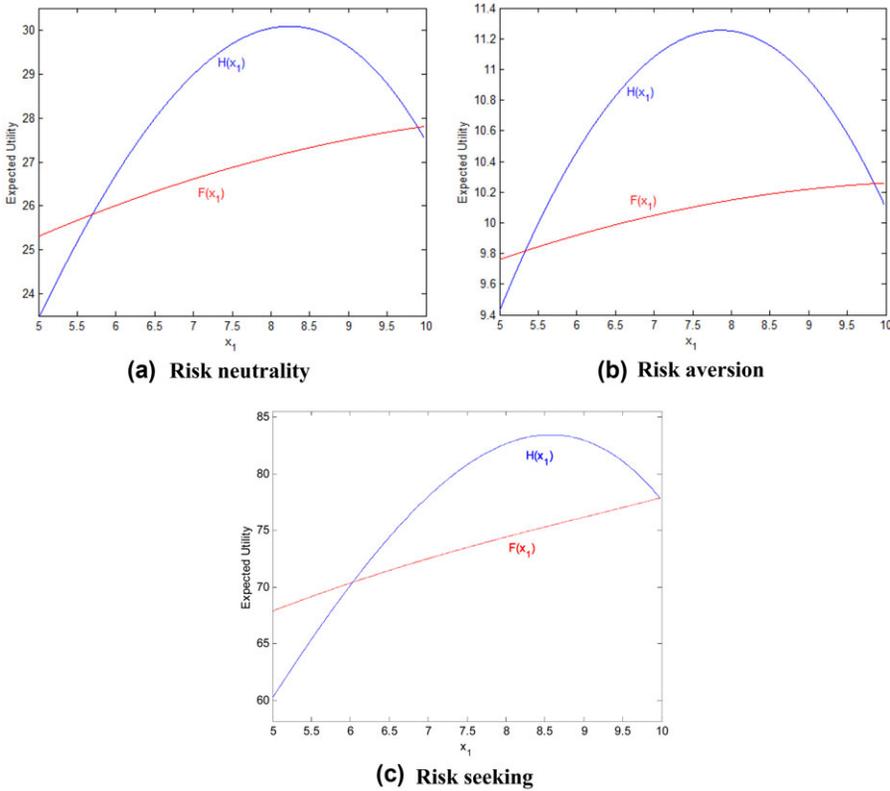
This information acquisition structure exploits the capacity of the DM to consider a total of three observations simultaneously. As can be intuitively inferred from the above description, the three dimensional space required to analyze this setting could be incorporated into the current one when defining the functions F and H . However, doing so would complicate the analysis unnecessarily and shift focus away from the n -observations structure that constitutes the main result of the current article. Moreover, the inclusion of this type of information acquisition structure within the current setting would require additional modifications that must be implemented depending on the realizations of the fully and partially observed products.

The current three observations setting has provided some intuition that will prove useful when analyzing the general n observations environment. We will however reinforce it through the five observations framework described below.

Numerical Simulations

Figures 8a–c illustrate how the three observations setting leads to a set of information acquisition incentives that differs substantially from the one obtained when two observations are acquired. All the cases (risk neutral [Figure 8a], risk averse [Figure 8b], and risk seeking [Figure 8c]) present a decrement in the *Starting*

Figure 8: Information acquisition incentives with three observations.



dominance pattern obtained within the two observations environment. The *Starting* option still dominates the *Continuing* one through most of the domain X_1 but to a much lesser extent than in the two observations case.

Thus, when having a third observation to acquire, DMs will favor the acquisition of information on a new product over continuing with the one initially observed. At the same time, note how the dominant tendency extends over a wider interval in the risk averse case, followed by the risk neutral and the risk seeking one, respectively. The exact interval values are presented in Table 1.

Finally, as will become explicit in the setting with five observations, we have assumed the DM to account for the possibility of selecting the best product among the potentially acceptable ones expected to be observed.

The Proposed Information Acquisition Structure with Five Observations

Similarly to the three observations case, when calculating the function $F(x_1|5)$ we must consider the following set of combinations based on the three observations that remain to be acquired after gathering two observations on the first product: (3-0), (3-1), (2-1), and (2-2). If the realization required (to be higher than x_1) is

Table 1: Threshold values and continuation intervals

	Observations			
	2	3	4	5
Risk seeking	–	[5, 6.0276] [9.9837, 10]	[5, 7.9345] [9.9865, 10]	[5, 7.4863] [9.9825, 10]
Risk neutrality	–	[5, 5.6966] [9.9030, 10]	[5, 7.9294] [9.8990, 10]	[5, 7.0851] [9.8968, 10]
Risk aversion	–	[5, 5.3309] [9.8574, 10]	[5, 7.8542] [9.8480, 10]	[5, 5.5956] [9.8429, 10]
Risk neutrality absent framing	–	[9.8528, 10]	[5, 7.9800] [9.4595, 10]	[5, 5.4132] [9.5253, 10]

either never observed or is the last observation to be acquired, we would be facing the (3-0) and (3-1) cases, respectively.

- (3-0) This option implies that none of the three observations left provides a $x_1^m > x_1$. The resulting expected payoff is given by $\psi_1(3, 0, \mu_1)(E_1 + E_2)$.
- (3-1) This option implies that only one of the three observations left provides $x_1^m > x_1$. It also implies that the observation delivering $x_1^m > x_1$ must be the third one because, otherwise, one of the two observations remaining would have been used to acquire x_2^m , leading to the (2-1) case described below. Thus, the DM will be left with a partially observed product and the following expected payoff: $\psi_1(3, 1, \mu_1)(u_1(x_1^m) + E_2)$.

Moreover, out of the three pieces of information left, the DM may observe a realization above the required reference value before reaching the third observation, leading to the (2-1) and (2-2) cases. Within each case, the DM will face the (1-0) and (1-1) subcases corresponding to the second characteristic of the improved product.

- (2-1) This option implies that either the first (but not the third) or the second (but not the first) observation acquired leads to a characteristic $x_1^m > x_1$, which allows the DM to use either the second or the third observation, respectively, to acquire x_2^m . This is trivially the optimal way to proceed because $x_1^m > x_1$ and all the second characteristics are equally distributed. In addition, the set of possible outcomes derived from observing x_2^m is defined by (1-0) and (1-1). Thus, the resulting expected payoff is given by

$$\psi_1(2, 1, \mu_1)[\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)].$$

- (2-2) This option implies that the first and the third observations acquired lead to characteristics $x_1^m > x_1$, which allows the DM to use the second

observation to acquire x_2^m , that is, the second characteristic from the second product observed, while a third product with $x_1^{m+1} > x_1$ remains partially observed, its second characteristic defined by the CE value. Due to the partial observation of the third and final product, the set of potential outcomes derived from observing x_2^m is given by (1-0) and (1-1). Therefore, the corresponding expected payoff is given by

$$\psi_1(2, 2, \mu_1) [\psi_2(1, 1, \mu_2) \max \{ (u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + E_2) \} + \psi_2(1, 0, \mu_2) (u_1(x_1^{m+1}) + E_2)].$$

Note that the two possibilities regarding the potential realizations of x_2^m must be accounted for separately. That is, $\psi_2(1, 1, \mu_2)$ implies that the realization of x_2^m delivers an observation located within $P^+(x_1)$, leading to an expected payoff defined by the fully observed new product m and the partially observed new product $m + 1$. Both these products are the result of the search process and the DM must choose the product providing the highest (expected) utility. Similarly, the probability $\psi_2(1, 0, \mu_2)$ indicates that x_2^m does not deliver a product located within $P^+(x_1)$ and the DM is therefore left with the partially observed product $m + 1$.

Consider now the four remaining observations that define the function $H(x_1|5)$. As already explained in the previous subsection, the DM has an extra unit of information available to acquire in this setting relative to the $F(x_1|5)$ one. The DM has to account for the following set of potential outcomes: (4-0), (4-1), (3-1), (3-2), and (2-2). If the realization required (to be higher than x_1) is either never observed or is the last observation to be acquired, we would be facing the (4-0) and (4-1) cases, respectively.

- (4-0) This option implies that none of the four observations left provides a $x_1^m > x_1$. The resulting expected payoff is given by $\psi_1(4, 0, \mu_1)(E_1 + E_2)$.
- (4-1) This option implies that only one of the four observations left provides a $x_1^m > x_1$. It also implies that the observation providing a $x_1^m > x_1$ must be the fourth one. Otherwise, any of the other three observations available would have been used to acquire x_2^m , leading to the (3-1) subcase described below. In other words, if the observation delivering $x_1^m > x_1$ was not the last one but, for example, the third one, the last observation available would have been used to acquire information on x_2^m , an event described by the (3-1) subcase. Thus, in the (4-1) case the DM observes a partially improved product, leading to an expected payoff given by: $\psi_1(4, 1, \mu_1)(u_1(x_1^m) + E_2)$.

Moreover, out of the four pieces of information left, the DM may observe only one realization above the required reference value before reaching the fourth observation, leading to the (3-1) case. Within this case, the DM will face the (1-0) and (1-1) subcases corresponding to the second characteristic of the improved product.

- (3-1) This option implies that only one of the first three observations provides a $x_1^m > x_1$, which allows the DM to use one of the remaining

observations to acquire x_2^m , that is, the second characteristic from the new observed product. Clearly, the set of possible outcomes derived from observing x_2^m is defined by (1-0) and (1-1). The corresponding expected payoff must therefore be based on combinations of x_1^m and x_2^m leading to products located above the reference one ($x_1, P^+(x_1)$): $\psi_1(3, 1, \mu_1)[\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)]$.

Finally, out of the four pieces of information left, the DM may observe two realizations located above the required reference value, leading to the (3-2) and (2-2) cases. Within each case, the DM will face different subcases corresponding to the second characteristic of the improved product(s).

- (3-2)** This option implies that either the first and the fourth or the second and the fourth observations lead to a characteristic $x_1^m > x_1$, which allows the DM to use one of the remaining observations (the second or the third one, respectively) to acquire x_2^m , that is, the second characteristic from the new observed product, while a second new product with $x_1^{m+1} > x_1$ remains partially observed, its second characteristic defined by the CE value. As in the previous subcase, the set of possible outcomes derived from observing x_2^m is defined by (1-0) and (1-1). The resulting expected payoff is therefore given by

$$\psi_1(3, 2, \mu_1)[\psi_2(1, 1, \mu_2) \max \{ (u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + E_2) \} + \psi_2(1, 0, \mu_2)(u_1(x_1^{m+1}) + E_2)].$$

- (2-2)** This option implies that the first and the third observations acquired lead to a characteristic $x_1^m > x_1$, which allows the DM to use the second and fourth observations to acquire x_2^m and x_2^{m+1} , respectively, that is, the second characteristic from the first and second new observed products. Similarly to the previous subcases, the set of possible outcomes derived from observing x_2^m and x_2^{m+1} is defined by (2-0), (2-1), and (2-2). The corresponding expected payoff is given by

$$\psi_1(2, 2, \mu_1)[\psi_2(2, 2, \mu_2) \max \{ (u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1})) \} + \psi_2(2, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(2, 0, \mu_2)(E_1 + E_2)].$$

Note that the DM must account for the three potential search outcomes separately. First, $\psi_2(2, 2, \mu_2)$ implies that both observations x_2^m and x_2^{m+1} deliver realizations located within $P^+(x_1)$, leading to an expected payoff defined by the new products, m and $m + 1$, being fully observed. These products are the result of the search process and the DM must choose the one providing the highest utility. Second, $\psi_2(2, 1, \mu_2)$ implies that only one of the observations x_2^m and x_2^{m+1} delivers a realization located within $P^+(x_1)$, leading to an expected payoff defined by either the new product m or the new product $m + 1$ being fully observed. Without loss of generality, we have used the m notation to refer to the fully observed product. Finally, $\psi_2(2, 0, \mu_2)$ indicates that none of the second observations from

the new products delivers a realization located within $P^+(x_1)$ and the DM is therefore left with the random CE outcome.

We are now able to write an expression for the functions $F(x_1|5)$ and $H(x_1|5)$. The former reads as follows:

$$\begin{aligned}
 F(x_1|5) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2) (u_1(x_1) + u_2(x_2)) dx_2 + A(x_1|5) \\
 &+ \int_{P^-(x_1)} \mu_2(x_2) (E_1 + E_2) dx_2 + B(x_1|5), \tag{15}
 \end{aligned}$$

where

$$\begin{aligned}
 A(x_1|5) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2) [\psi_1(3, 0, \mu_1)(E_1 + E_2) + \psi_1(3, 1, \mu_1)(u_1(x_1^m) + E_2) \\
 &+ \psi_1(2, 1, \mu_1) [\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)] \\
 &+ \psi_1(2, 2, \mu_1) [\psi_2(1, 1, \mu_2) \max\{u_1(x_1^m) + u_2(x_2^m), (u_1(x_1^{m+1}) + E_2)\} \\
 &+ \psi_2(1, 0, \mu_2)(u_1(x_1^{m+1}) + E_2)] dx_2, \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 B(x_1|5) &\stackrel{def}{=} \int_{P^-(x_1)} \mu_2(x_2) [\psi_1(3, 0, \mu_1)(E_1 + E_2) + \psi_1(3, 1, \mu_1)(u_1(x_1^m) + E_2) \\
 &+ \psi_1(2, 1, \mu_1) [\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)] \\
 &+ \psi_1(2, 2, \mu_1) [\psi_2(1, 1, \mu_2) \max\{u_1(x_1^m) + u_2(x_2^m), (u_1(x_1^{m+1}) + E_2)\} \\
 &+ \psi_2(1, 0, \mu_2)(u_1(x_1^{m+1}) + E_2)] dx_2. \tag{17}
 \end{aligned}$$

Note that the integration sets, $P^+(x_1)$ and $P^-(x_1)$, depend on the initial observation acquired, which defines the (expected) product that must be improved upon through the information acquisition process. In the function F case, potential improvements of the second characteristic are defined with respect to $P^+(x_1)$. If we were to use $P^+(x_1^m)$, with $x_1^m > x_1$, then some acceptable characteristics of X_2 would be lower than those defined within $P^+(x_1)$. However, we are assuming through this section that DMs only consider full potential improvements relative to the reference (partially observed) product given by $(x_1, P^+(x_1))$. As a result, x_1^m must be located above x_1 and x_2^m must be potentially at least as good as $x_2 \in P^+(x_1)$. Defining improvements on the function $H(x_1|5)$ is subject to the same type of [subjective] constraint.

$$H(x_1|5) \stackrel{def}{=} (u_1(x_1) + E_2) + C(x_1|5) \tag{18}$$

with

$$\begin{aligned}
 C(x_1|5) \stackrel{def}{=} & \psi_1(4, 0, \mu_1)(E_1 + E_2) + \psi_1(4, 1, \mu_1)(u_1(x_1^m) + E_2) \\
 & + \psi_1(3, 1, \mu_1)[\psi_2(1, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2)(E_1 + E_2)] \\
 & + \psi_1(3, 2, \mu_1)[\psi_2(1, 1, \mu_2) \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + E_2)\} \\
 & + \psi_2(1, 0, \mu_2)(u_1(x_1^{m+1}) + E_2)] \\
 & + \psi_1(2, 2, \mu_1)[\psi_2(2, 2, \mu_2) \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1}))\} \\
 & + \psi_2(2, 1, \mu_2)(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(2, 0, \mu_2)(E_1 + E_2)]. \tag{19}
 \end{aligned}$$

The improvements in the function H case are clearly based on the x_1 realization acquired, which defines the partially observed product that must be improved upon. In this regard, the reference value determining the product to improve is given by $P^+(x_1)$. However, we may also assume that the set of potential improvements is determined either by $\langle ce_1, ce_2 \rangle$, whenever $x_1 \leq ce_1$, or by $\langle x_1, ce_2 \rangle$, whenever $x_1 > ce_1$. In this case, the new product must improve upon both, the observed variable and the expected utility from the unobserved one. We consider this possibility in “Framing and Reference Products” section.

Numerical Simulations

When computing the functions $F(x_1|5)$ and $H(x_1|5)$ numerically, we have made a distinction among the expected utility levels that may be achieved from different potential final choice sets such as, for example,

$$\begin{aligned}
 & \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1}))\}; \\
 & \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + E_2)\}; \tag{20} \\
 & (u_1(x_1^m) + u_2(x_2^m)).
 \end{aligned}$$

The DM does not know the exact products that he will be observing, just the intervals containing them and the number of improved products he will get to observe. Clearly, observing ten products from the intervals (x_1, β_1) and $P^+(x_1)$ should provide a different expected payoff from observing only one product. In other words, the probability of obtaining a better product from a given subset should increase in the number of products observed, with the DM subjectively accounting for this fact when computing his expected search payoffs. While the consumer psychology literature has extensively analyzed the effects of optimism and pessimism on expectations (Kahneman & Tversky, 2000), we opt for a relatively simple rule to determine the expected utility obtained when several products contained within the intervals (x_1, β_1) and $P^+(x_1)$ are observed.

For example, given $x_1^m > x_1$ and $x_2^m \in P^+(x_1)$, assume that each $X_k, k = 1, 2$ is uniformly distributed on its domain $[\alpha_k, \beta_k]$. We have applied the following rule when calculating the expected utility obtained by the DM after observing n

products within the intervals (x_1, β_1) and $P^+(x_1)$:

$$\int_{x_1}^{\frac{\beta_1+x_1}{2}} \left(\frac{1 - \frac{n-1}{100}}{\beta_1 - x_1} \right) u_1(x_1^m) dx_1^m + \int_{\frac{\beta_1+x_1}{2}}^{\beta_1} \left(\frac{1 + \frac{n-1}{100}}{\beta_1 - x_1} \right) u_1(x_1^m) dx_1^m$$

$$+ \int_{u_2^{-1}(E_1+E_2-u_1(x_1))}^{\frac{u_2^{-1}(u_2(\beta_2)+E_1+E_2-u_1(x_1))}{2}} \left(\frac{1 - \frac{n-1}{100}}{\beta_2 - u_2^{-1}(E_1 + E_2 - u_1(x_1))} \right) u_2(x_2^m) dx_2^m$$

$$+ \int_{\frac{u_2^{-1}(u_2(\beta_2)+E_1+E_2-u_1(x_1))}{2}}^{\beta_2} \left(\frac{1 + \frac{n-1}{100}}{\beta_2 - u_2^{-1}(E_1 + E_2 - u_1(x_1))} \right) u_2(x_2^m) dx_2^m \tag{21}$$

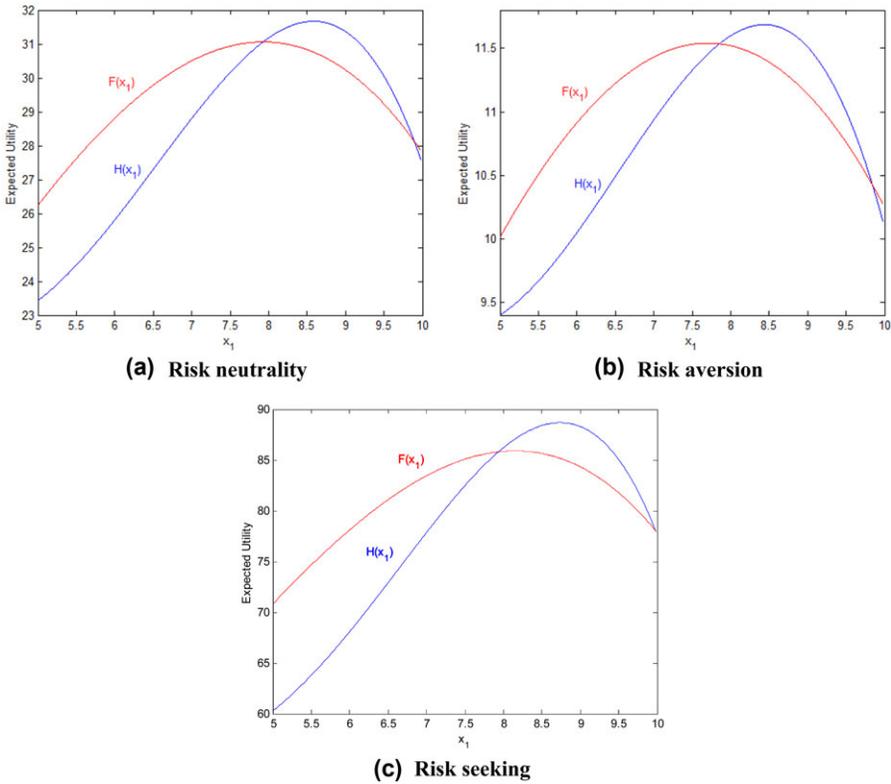
That is, after expecting to observe n products providing a higher utility than the initial [reference] one, we have assumed that the DM shifts probability mass from the lower half of the uniform distribution to the upper one at a rate of $\frac{1}{100}$. Here n accounts for the number of *fully observed products* providing a higher expected utility than the reference [initially observed] one. In other words, if the DM expects to fully observe one product better than the reference one, then $n = 1$; if he expects to fully observe one product and another one partially, then $n = 1.5$, that is, a partially observed product adds 0.5 to the probability shift, while expecting to fully observe two products leads to $n = 2$. We have assumed that after expecting to fully observe $n = 101$ products, the DM shifts the whole mass from the lower half of the uniform distribution *defined by* x_1 to the upper one.

Clearly, the expected improvement defined on the first characteristic is much simpler to compute than the improvement on the second one, because the shape of the utility function determines the lower limit point defining the set $P^+(x_1)$. Thus, as intuition suggests, risk taking, risk neutral and risk averse DMs will have different information acquisition incentives and behavior. Note that the functional form assumed above generates a relatively small distortion on the expected utility derived from the set of potential improved products. In this regard, DMs could be assumed to be either more optimistic or pessimistic or completely unaware of this adjustment.

Figures 9a–c illustrate the risk neutral, risk averse and risk seeking environments, respectively, when four observations are acquired by the DMs. Similarly, Figures 10a–c account for these respective environments within the five observations setting. The diversity of scenarios obtained is considerable, with all the settings being almost identical in information acquisition incentives when four observations are considered, while differing significantly in the five observations case, see the corresponding entries in Table 1.

We should highlight the decrease in the width of the continuation intervals as we move from a risk seeking environment towards a risk averse one. That is, risk seekers are more prone to continue acquiring information on the product being

Figure 9: Information acquisition incentives with four observations.



observed, while risk averse DMs will tend to acquire partial information on a larger amount of products. Note also the reversals in dominance between the functions F and H through the different scenarios considered, incrementing the dependence of the information acquisition process on the initial number of observations that must be acquired by a DM.

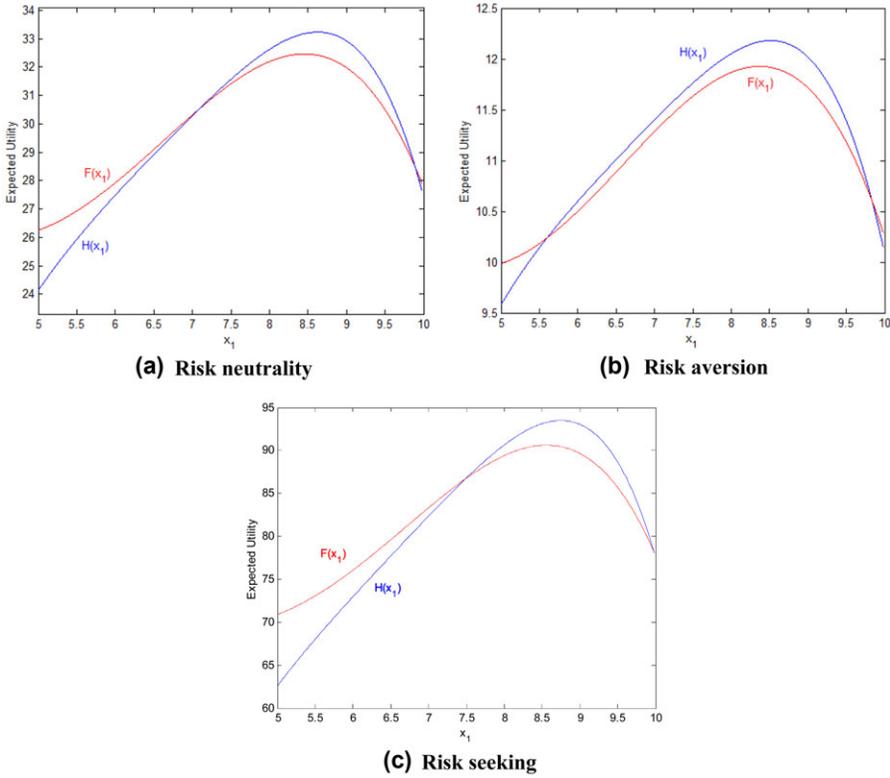
The Proposed Information Acquisition Structure with N Observations

We are ready now to generalize the information acquisition process of DMs following the intuition provided by the cases described through the previous section. The final form of the information acquisition structure will depend on whether the DM has an odd or an even number of observations to acquire. This is the case because the number of fully and partially observed products composing the potential final choice sets obtained from the search is determined by this fact.

Odd Number of Observations

Through this and the following subsection, $\lceil \cdot \rceil$ will denote the ceiling function of the corresponding variable. When considering an odd number of total observations,

Figure 10: Information acquisition incentives with five observations.



as in the cases described through the previous section, we have the following expressions for the functions F and H .

Function F

The total that must be considered by the DM equals $\lceil \frac{n-1}{2} \rceil$, with n consisting of the total number of observations that will be gathered by the DM. In this case, the DM must account for the acquisition of the remaining $n - 2$ observations (i.e., 9 in the case with a total of 11, 3 in the case with a total of 4) following the pattern described below for the potential combinations $(n - j - j - 2)$, $(n - j - j - 1)$, where $j = 2, \dots, \lceil \frac{n}{2} \rceil$.

$$\begin{aligned}
 & \int_{P^+(x_1)} \mu_2(x_2) [\psi_1(n - j, j - 2, \mu_1(x_1^m) > x_1)) \\
 & [\psi_2(j - 2, 0, \mu_2(x_2^m) \in P^+(x_1))](E_1 + E_2) \\
 & +
 \end{aligned}$$

$$\begin{aligned}
 & \psi_2(j - 2, 1, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^m) + u_2(x_2^m)) \\
 & + \\
 & \psi_2(j - 2, 2, \mu_2(x_2^m \in P^+(x_1))) \\
 & \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1}))\} \\
 & + \dots + \\
 & \psi_2(j - 2, j - 2, \mu_2(x_2^m \in P^+(x_1))) \\
 & \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-3}) + u_2(x_2^{m+j-3}))\}]dx_2, \tag{22} \\
 & + \\
 & \int_{P^+(x_1)} \mu_2(x_2)[\psi_1(n - j, j - 1, \mu_1(x_1^m > x_1)) \\
 & [\psi_2(j - 2, 0, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^{m+j-2}) + E_2) \\
 & + \\
 & \psi_2(j - 2, 1, \mu_2(x_2^m \in P^+(x_1)))\max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+j-2}) + E_2)\} \\
 & + \\
 & \psi_2(j - 2, 2, \mu_2(x_2^m \in P^+(x_1))) \\
 & \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1})), (u_1(x_1^{m+j-2}) + E_2)\} \\
 & + \dots + \\
 & \psi_2(j - 2, j - 2, \mu_2(x_2^m \in P^+(x_1)))\max(u_1(x_1^m)\{u_2(x_2^m) \\
 & , \dots, (u_1(x_1^{m+j-3}) + u_2(x_2^{m+j-3})), (u_1(x_1^{m+j-2}) + E_2)\}]dx_2. \tag{23}
 \end{aligned}$$

Note that we have only considered the section of the function F defined within the set $P^+(x_1)$, because the complementary section based on $P^-(x_1)$ has the same structure. The notation and intuition describing the function follow from the ones used to describe the scenarios with three and five observations. The main difference arising in the general case is the division of the function in two separate blocks, which follows from the potential combinations of fully and partially observed products located above the reference one. A similar description applies to the function H , whose extra observation leads to a third block of potential combinations having to be considered. Clearly, the above blocks compose part $A(x_1|n)$ of the function $F(x_1|n)$, while the three blocks introduced below compose part $C(x_1|n)$ of the function $H(x_1|n)$.

Function H

The total number of combinations that must be considered by the DM equals $\lceil \frac{n}{2} \rceil$, with n consisting of the total number of observations that will be gathered by the

DM. The DM must account for the acquisition of the remaining $n - 1$ observations (i.e., 10 in the case with a total of 11, 4 in the case with a total of 5) following the pattern described below for the potential combinations $(n - j - j - 1)$, $(n - j - j)$, where $j = 1, \dots, \lceil \frac{n}{2} \rceil - 1$.

$$\begin{aligned}
& \psi_1(n - j, j - 1, \mu_1(x_1^m > x_1)) \\
& \quad [\psi_2(j - 1, 0, \mu_2(x_2^m \in P^+(x_1)))(E_1 + E_2) \\
& \quad + \psi_2(j - 1, 1, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^m) + u_2(x_2^m)) \\
& \quad + \psi_2(j - 1, 2, \mu_2(x_2^m \in P^+(x_1)))] \\
& \quad \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1}))\} \\
& \quad + \dots + \psi_2(j - 1, j - 1, \mu_2(x_2^m \in P^+(x_1))) \\
& \quad \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-2}) + u_2(x_2^{m+j-2}))\} +
\end{aligned} \tag{24}$$

$$\begin{aligned}
& \psi_1(n - j, j, \mu_1(x_1^m > x_1)) \\
& \quad [\psi_2(j - 1, 0, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^{m+j-1}) + E_2) \\
& \quad + \psi_2(j - 1, 1, \mu_2(x_2^m \in P^+(x_1))) \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+j-1}) + E_2)\} \\
& \quad + \psi_2(j - 1, 2, \mu_2(x_2^m \in P^+(x_1))) \\
& \quad \max\{(u_1(x_1^m) + u_2(x_2^m)), (u_1(x_1^{m+1}) + u_2(x_2^{m+1})), (u_1(x_1^{m+j-1}) + E_2)\} \\
& \quad + \dots + \psi_2(j - 1, j - 1, \mu_2(x_2^m \in P^+(x_1))) \\
& \quad \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-2}) + u_2(x_2^{m+j-2})), (u_1(x_1^{m+j-1}) + E_2)\} +
\end{aligned} \tag{25}$$

$$\begin{aligned}
& \psi_1\left(n - \lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil - 1, \mu_1(x_1^m > x_1)\right) [\psi_2\left(\lceil \frac{n}{2} \rceil - 1, 0, \mu_2(x_2^m \in P^+(x_1))\right)(E_1 + E_2) \\
& \quad + \psi_2\left(\lceil \frac{n}{2} \rceil - 1, 1, \mu_2(x_2^m \in P^+(x_1))\right)(u_1(x_1^m) + u_2(x_2^m)) \\
& \quad + \psi_2\left(\lceil \frac{n}{2} \rceil - 1, 2, \mu_2(x_2^m \in P^+(x_1))\right) \max\{(u_1(x_1^m) + u_2(x_2^m)), \\
& \quad (u_1(x_1^{m+1}) + u_2(x_2^{m+1}))\} + \dots + \psi_2\left(\lceil \frac{n}{2} \rceil - 1, \lceil \frac{n}{2} \rceil - 1, \mu_2(x_2^m \in P^+(x_1))\right) \\
& \quad \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+\lceil \frac{n}{2} \rceil - 2}) + u_2(x_2^{m+\lceil \frac{n}{2} \rceil - 2}))\}.
\end{aligned} \tag{26}$$

As in the function F case, the division of the function H in three separate blocks follows from the potential combinations of fully and partially observed products located above the initial reference one.

Even Number of Observations

Consider now an even number of total observations. For illustrative purposes, assume that the DM may acquire a total of four or six observations. The following combinations, leading to the corresponding expressions for F and H , must be accounted for.

The potential combinations faced by the DM when computing the function F in the setting with four observations are (2-0), (2-1), and (1-1), while the function H requires (3-0), (3-1), (2-1), and (2-2). Similarly, in the setting with six observations, the potential combinations faced by the DM when computing the function F are (4-0), (4-1), (3-1), (3-2), and (2-2), while the function H requires (5-0), (5-1), (4-1), (4-2), (3-2), and (3-3). The intuition in all these cases is identical to the

one employed to describe the environments with an odd number of observations. However, in the current setting, the function F will be the one composed by three different blocks, while H remains composed by only two.

In order to simplify the presentation, we will only describe the $\psi_2(\cdot, 0, \mu_2(x_2^m \in P^+(x_1)))$ and $\psi_2(\cdot, \cdot, \mu_2(x_2^m \in P^+(x_1)))$ potential outcomes, because the remaining ones can be easily inferred from the context. Moreover, we have already provided additional potential outcomes for each block composing the functions F and H when describing the odd number of observations environment above.

Function F

The total number of combinations that must be considered by the DM equals $\lceil \frac{n}{2} \rceil$, with n consisting of the total number of observations that will be gathered by the DM. In this case, the DM must account for the acquisition of the remaining $n - 2$ observations (i.e., 8 in the case with a total of 10, 4 in the case with a total of 6) following the pattern described below for the potential combinations $(n - j - j - 2), (n - j - j - 1)$, where $j = 2, \dots, \lceil \frac{n+1}{2} \rceil - 1$.

$$\int_{P^+(x_1)} \mu_2(x_2)[\psi_1(n - j, j - 2, \mu_1(x_1^m > x_1)) \psi_2(j - 2, 0, \mu_2(x_2^m \in P^+(x_1)))(E_1 + E_2) + \dots + \psi_2(j - 2, j - 2, \mu_2(x_2^m \in P^+(x_1))) \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-3}) + u_2(x_2^{m+j-3}))\}] dx_2 + \tag{27}$$

$$\int_{P^+(x_1)} \mu_2(x_2)[\psi_1(n - j, j - 1, \mu_1(x_1^m > x_1)) \psi_2(j - 2, 0, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^{m+j-2}) + E_2) + \dots + \psi_2(j - 2, j - 2, \mu_2(x_2^m \in P^+(x_1))) \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-3}) + u_2(x_2^{m+j-3})), (u_1(x_1^{m+j-2}) + E_2)\}] dx_2 + \tag{28}$$

$$\int_{P^+(x_1)} \mu_2(x_2)[\psi_1\left(n - \left\lceil \frac{n+1}{2} \right\rceil, \left\lceil \frac{n}{2} \right\rceil - 1, \mu_1(x_1^m > x_1)\right) \psi_2\left(\left\lceil \frac{n}{2} \right\rceil - 1, 0, \mu_2(x_2^m \in P^+(x_1))\right)(E_1 + E_2) + \dots + \psi_2\left(\left\lceil \frac{n}{2} \right\rceil - 1, \left\lceil \frac{n}{2} \right\rceil - 1, \mu_2(x_2^m \in P^+(x_1))\right) \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+\lceil \frac{n}{2} \rceil - 2}) + u_2(x_2^{m+\lceil \frac{n}{2} \rceil - 2}))\}] dx_2. \tag{29}$$

Function H

The total number of combinations that must be considered by the DM equals $\lceil \frac{n}{2} \rceil$, with n consisting of the total number of observations that will be gathered by the DM. In this case, the DM must account for the acquisition of the remaining $n - 1$ observations (i.e., 9 in the case with a total of 10, 5 in the case with a total of

6) following the pattern described below for the potential combinations $(n - j - j - 1)$, $(n - j - j)$, where $j = 1, \dots, \lceil \frac{n}{2} \rceil$.

$$\begin{aligned} & \psi_1(n - j, j - 1, \mu_1(x_1^m > x_1)) \\ & [\psi_2(j - 1, 0, \mu_2(x_2^m \in P^+(x_1)))(E_1 + E_2) \\ & + \dots + \psi_2(j - 1, j - 1, \mu_2(x_2^m \in P^+(x_1)))] \\ & \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-2}) + u_2(x_2^{m+j-2}))\} + \end{aligned} \quad (30)$$

$$\begin{aligned} & \psi_1(n - j, j, \mu_1(x_1^m > x_1)) \\ & [\psi_2(j - 1, 0, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^{m+j-1}) + E_2) \\ & + \dots + \psi_2(j - 1, j - 1, \mu_2(x_2^m \in P^+(x_1)))] \\ & \max\{(u_1(x_1^m) + u_2(x_2^m)), \dots, (u_1(x_1^{m+j-2}) + u_2(x_2^{m+j-2})), (u_1(x_1^{m+j-1}) + E_2)\}. \end{aligned} \quad (31)$$

FRAMING AND REFERENCE PRODUCTS

Until now, the set of potential improvements upon the reference realization has been defined with respect to the pair $(x_1, P^+(x_1))$. This assumption can be justified as follows. First, it accounts for any framing effect that may result from the (partial) observation of the initial product. Second, it conditions improvements on the information available to the DM, who does not know the outcome from the second observation but is able to compute the set of acceptable realizations. In this regard, all potential improvements are consistently based on the first characteristic observed, x_1 , even those defining section B within the corresponding function F . We refer to this scenario as the framing environment. It assumes that initial and realized observations condition the information acquisition behavior of the DM by defining the set of acceptable expected ones.

We can however redefine this environment by modifying the acceptance requirements of DMs regarding the expectations relative to the first and second characteristics whenever $x_1 \leq ce_1$. That is, acceptable improvements could be defined relative to the pair of expected values $\langle ce_1, ce_2 \rangle$. This would clearly modify the information acquisition incentives of DMs. In this case, two types of improvement would be required on the initial product before having actually observed it fully. If $x_1 > ce_1$, we will assume that the DM considers the reference pair $(x_1, P^+(x_1))$. However, if $x_1 \leq ce_1$, then the DM would shift his reference requirements to the pair $\langle ce_1, ce_2 \rangle$, due to the absence of an acceptable observed product.

The framing approach focuses on the potential improvements derived from the information acquisition process while the elimination of framing shifts the focus of the search process to the set of acceptable products expected to be obtained. Though relatively small, the effect of framing [or its absence] is noticeable on the information acquisition behavior of DMs even when only three observations are acquired. However, its main consequences are better understood when using, as a basic example, an explicit formulation of this setting, which is described by the

following set of equations:

$$\begin{aligned}
 F(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + A(x_1|3) \\
 &+ \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 + B(x_1|3), \tag{32}
 \end{aligned}$$

$$\begin{aligned}
 A(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1(x_1^m > x_1))(E_1 + E_2) \\
 &+ \psi_1(1, 1, \mu_1(x_1^m > x_1))(u_1(x_1^m) + E_2)]dx_2, \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 B(x_1|3) &\stackrel{def}{=} \int_{P^-(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1(x_1^m > x_1))(E_1 + E_2) \\
 &+ \psi_1(1, 1, \mu_1(x_1^m > x_1))(u_1(x_1^m) + E_2)]dx_2. \tag{34}
 \end{aligned}$$

Note how we have emphasized explicitly within $\mu_1(\cdot)$ that the reference point considered by the DM when defining potential improvements is given by x_1 . This reference choice constitutes the framing effect that determines improvements with respect to the pair $(x_1, P^+(x_1))$. Clearly, part *A* remains unaffected by the framing effect, as expected improvements are defined on the fully observed products providing a higher utility than the CE one. However, the framing effect becomes evident within part *B*, which allows for potential improvements over x_1 , the initial observation serving as a reference point, even if they deliver products that will not be chosen by the DM. To see why this is the case, consider the alternative setting without framing, where the DM accounts explicitly for the realization of the second characteristic (whether either about to be actually observed, as in *A* and *B*, or expected, as in *C*) when determining his optimal information acquisition strategy

$$\begin{aligned}
 F(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2)) dx_2 + A(x_1|3) \\
 &+ \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2) dx_2 + B(x_1|3), \tag{35}
 \end{aligned}$$

$$\begin{aligned}
 A(x_1|3) &\stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1(x_1^m > x_1))(E_1 + E_2) \\
 &+ \psi_1(1, 1, \mu_1(x_1^m > x_1))(u_1(x_1^m) + E_2)]dx_2, \tag{36}
 \end{aligned}$$

$$B(x_1|3) \stackrel{def}{=} \int_{P^-(x_1)} \mu_2(x_2)[\psi_1(1, 0, \mu_1(x_1^m > ce_1))(E_1 + E_2) + \psi_1(1, 1, \mu_1(x_1^m > ce_1))(u_1(x_1^m) + E_2)]dx_2. \quad (37)$$

Note the immediate change in part *B*, where the reference point has been shifted within $\mu_1(\cdot)$ to the ce_1 -based product. That is, after the fully observed initial product delivers a utility below the CE one, the DM has to start acquiring information without a reference product to improve upon. As a result, the DM is assumed to use the coordinates of the CE product, that is, $\langle ce_1, ce_2 \rangle$, as the corresponding reference values.

Thus, the new first characteristic x_1^m must be located above the value ce_1 and the corresponding set $P^+(x_1^m)$ should require that $x_2^m > ce_2$. These differences become much more evident when analyzing part *C* within function *H*. The framing environment considers the product partially observed as the main reference point defining the improvements expected to be obtained from the information acquisition process.

$$H(x_1|3) \stackrel{def}{=} (u_1(x_1) + E_2) + C(x_1|3), \quad (38)$$

$$C(x_1|3) \stackrel{def}{=} \psi_1(2, 0, \mu_1(x_1^m > x_1))(E_1 + E_2) + \psi_1(2, 1, \mu_1(x_1^m > x_1))(u_1(x_1^m) + E_2) + \psi_1(1, 1, \mu_1(x_1^m > x_1))[\psi_2(1, 1, \mu_2(x_2^m \in P^+(x_1)))(u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2(x_2^m \in P^+(x_1)))(E_1 + E_2)]. \quad (39)$$

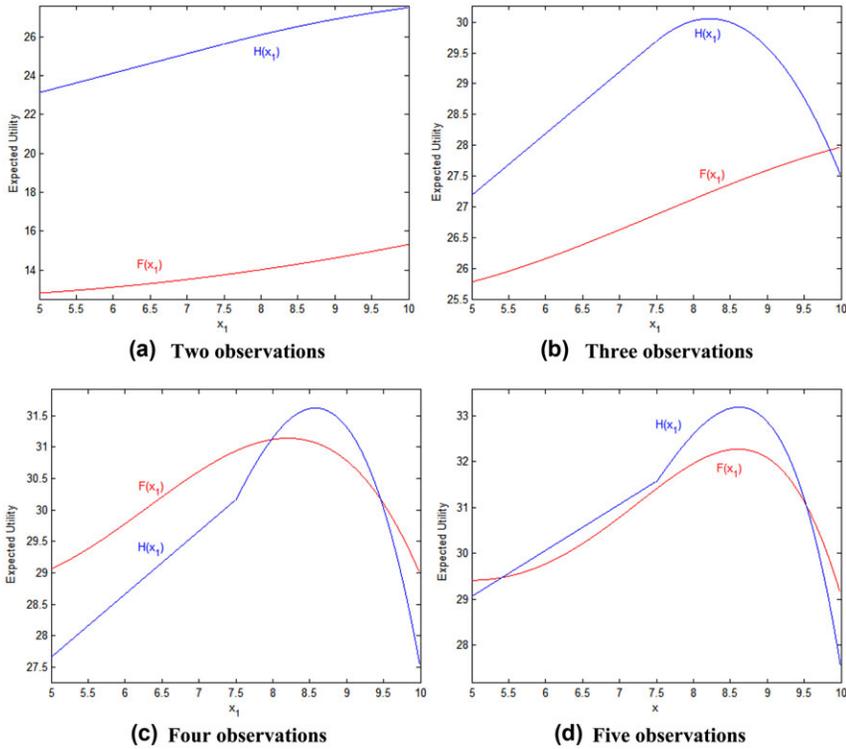
When eliminating the framing effect, part *C* must be divided in two differentiated subcases. The first one is defined for $x_1 \in [\alpha_1, ce_1]$. Observing an initial realization within this interval implies that x_1^m must improve upon the value ce_1 and x_2^m should be based on the corresponding potential realizations that improve upon ce_2 , that is, $\mu_2(x_2^m \in P^+(x_1^m)|x_2^m > ce_2)$.

$$C(x_1|3) \stackrel{def}{=} \psi_1(2, 0, \mu_1(x_1^m > ce_1))(E_1 + E_2) + \psi_1(2, 1, \mu_1(x_1^m > ce_1))(u_1(x_1^m) + E_2) + \psi_1(1, 1, \mu_1(x_1^m > ce_1))[\psi_2(1, 1, \mu_2(x_2^m \in P^+(x_1^m)|x_2^m > ce_2)) \times (u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2(x_2^m \in P^+(x_1^m)|x_2^m > ce_2)) \times (E_1 + E_2)]. \quad (40)$$

Note that both x_1^m and x_2^m must constitute improvements upon the CE reference product defined by ce_1 and ce_2 , respectively. This required improvement is based on the fact that the initial product observed, $(x_1 \leq ce_1, E_2)$, does not provide a higher utility than the CE one.

The second subcase is defined for $x_1 \in [ce_1, \beta_1]$, which implies that x_1^m must improve upon the initial value x_1 and x_2^m should be based on the corresponding potential realizations improving upon the CE product while constrained by $\mu_2(x_2^m \in P^+(x_1)|x_2^m > ce_2)$. That is, because x_2 remains unobserved through *H*, though it is expected to be observed when defining *F*, the resulting reference value will be given by ce_2 . In other words, potential improvements are defined with respect to the initial product observed, which is given by the first characteristic

Figure 11: Information acquisition incentives under risk neutrality absent framing.



observed—located above ce_1 —and the second unobserved one, given by ce_2 .

$$\begin{aligned}
 C(x_1|3) \stackrel{def}{=} & \psi_1(2, 0, \mu_1(x_1^m > x_1))(E_1 + E_2) + \psi_1(2, 1, \mu_1(x_1^m > x_1))(u_1(x_1^m) + E_2) \\
 & + \psi_1(1, 1, \mu_1(x_1^m > x_1))[\psi_2(1, 1, \mu_2(x_2^m \in P^+(x_1)|x_2^m > ce_2)) \\
 & \times (u_1(x_1^m) + u_2(x_2^m)) + \psi_2(1, 0, \mu_2(x_2^m \in P^+(x_1)|x_2^m > ce_2)) \\
 & (E_1 + E_2)]. \tag{41}
 \end{aligned}$$

Clearly, the DM will observe products leading to a higher utility than the CE one through his sequential information acquisition process. Those products will, at some point within the process, be used as the reference ones upon which improvements must be defined. This would eliminate the framing effect, which is however substantial in the initial stages of the information acquisition process, as the numerical simulations will illustrate.

Numerical Simulations

Figures 11a–d illustrate the absence of framing environment under risk neutrality for the two to five observations cases, respectively. The main differences in the

width of the continuation intervals with respect to the framing environment are described in Table 1. Trivially, there is no difference between both settings when only two observations are acquired. However, the differences become already evident within the three observations case. Note that without framing the continuation interval widens in the upper part of the distribution. The behavior of the lower part depends on the number of observations that must be acquired.

The steeper descend of the function H observed in Figures 11b–d for high realizations of the initial variable is due to the stronger requirements of the DM to start acquiring information. Absent framing, any second observation acquired must be higher than ce_2 and not simply contained within $P^+(x_1)$, which, for $x_1 > ce_1$, provides a larger set of potential improvements over the CE reference product. Note the expected (due to its division in two differentiated subcases) abrupt change in the behavior of the function H taking place at the value $ce_1 = 7.5$, particularly in Figures 11c and d, as the framing effect disappears.

THE INFORMATION ACQUISITION PROCESS

The analysis introduced through the article describes the information acquisition incentives of the DM either when he starts acquiring information or after fully observing a given number of products. In this case, the observation about to be acquired on a new product constitutes the variable defined on the x axis of the figures. Moreover, the reference payoffs to be improved upon are given by the product providing the highest utility within the set of previously observed products. These assumptions determine the behavior of the corresponding functions F and H .

The information acquisition process must be adjusted when the set of products observed by the DM contains partially observed ones. In this case, the functions F and H will be conditioned by \bar{x}_1 , the first characteristic from the highest utility providing product that has been *fully observed*. Both functions must be used by the DM to determine whether he continues acquiring information on *one of the partially observed products* or starts acquiring information on a new product.

Note that the DM performs pairwise comparisons between continuing with the highest utility providing product partially observed, that is, the one with the highest x_1 realization, and the starting with a new product payoff. It is trivial to show why the DM considers only the product with the highest x_1 observed over any other partially observed product. Intuitively, the higher the value of x_1 , the higher the number of x_2 combinations leading to an expected utility above that of the CE product or, if a different product is used as the reference one, above the utility provided by this reference product. We will denote by $P^+(\bar{x}_1, \bar{x}_2)$ the set of second characteristics leading to an improvement over the highest utility-providing product that has been fully observed. With this in mind, we can introduce the functions F and H defining the continuing with the partially observed product and the starting with a new product payoffs.

Example with Two Remaining Observations

For comparability purposes with the three observations environment, consider the following example where two observations are left to be acquired. A more elaborate example based on the results displayed by an online search engine when

the DM acquires a total of five observations is presented in the Online Appendix. It should be emphasized that the DM always performs pairwise comparisons between two products and does not consider additional combinations with any of the remaining partially observed ones. This restriction should however be relaxed when accounting for products defined by more than two characteristics.

Assume, to simplify the presentation, that $u_1(\bar{x}_1) + u_2(\bar{x}_2) > E_1 + E_2$. The expressions for part *A* and part *B* within the corresponding function *F* would be respectively given by

$$\begin{aligned} &\mu_2(x_2^m \in P^+(\bar{x}_1, \bar{x}_2))[u_1(x_1^m) + u_2(x_2^m)] + \psi_1(1, 0, \mu_1(x_1^{m+1} > x_1^m))(E_1 + E_2) \\ &+ \psi_1(1, 1, \mu_1(x_1^{m+1} > x_1^m))(u_1(x_1^{m+1}) + E_2) \end{aligned} \tag{42}$$

with $\bar{x}_1 = x_1^m$, because $u_1(x_1^m) + u_2(x_2^m) > u_1(\bar{x}_1) + u_2(\bar{x}_2) > E_1 + E_2$ within *A*, and

$$\begin{aligned} &[1 - \mu_2(x_2^m \in P^+(\bar{x}_1, \bar{x}_2))][(E_1 + E_2) + \psi_1(1, 0, \mu_1(x_1^{m+1} > \bar{x}_1))(E_1 + E_2) \\ &+ \psi_1(1, 1, \mu_1(x_1^{m+1} > \bar{x}_1))(u_1(x_1^{m+1}) + E_2)] \end{aligned} \tag{43}$$

with \bar{x}_1 being the first characteristic from the fully observed $u_1(\bar{x}_1) + u_2(\bar{x}_2) > E_1 + E_2$ product, which remains as the reference one within *B*.

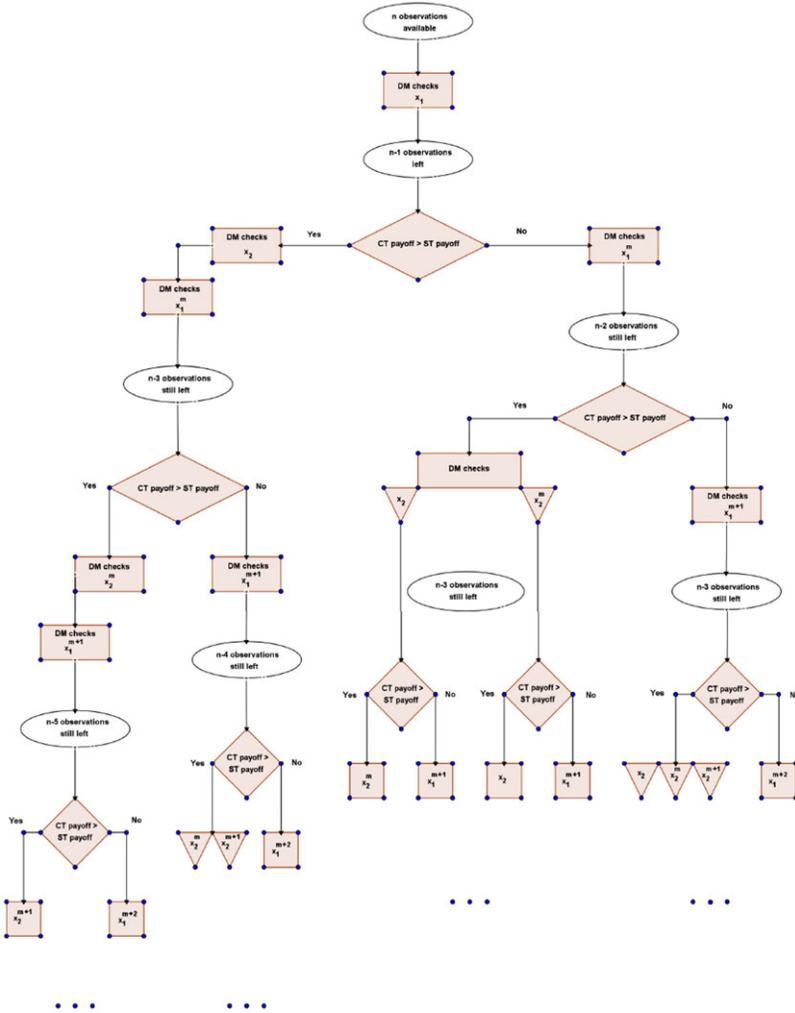
Consider the expression for part *B*. Note that observing a product that does not belong to $P^+(\bar{x}_1, \bar{x}_2)$ does not imply that it provides a utility lower than $E_1 + E_2$. However, we have equated the search outcome to $E_1 + E_2$ because it leads to a lower utility than the reference product determining the set $P^+(\bar{x}_1, \bar{x}_2)$. Moreover, acquiring the last piece of information may lead to $u_1(x_1^{m+1}) + E_2 < E_1 + E_2$. This would be the case for a relatively low (below ce_1) value \bar{x}_1 compensated by a high \bar{x}_2 . In this case, the DM would not accept the resulting product because $u_1(\bar{x}_1) + u_2(\bar{x}_2) > E_1 + E_2 > u_1(x_1^{m+1}) + E_2$.

We could justify this valuation structure through the remnant uncertainty when observing $u_1(x_1^{m+1}) + E_2$. Indeed, this last partially observed product could be located either above or below the reference (and, therefore, the CE) one. In this case, because the uncertainty is not completely resolved, there is still valuable information that could modify the reference product considered by the DM. However, when continuing with x_2^m , all uncertainty regarding the currently observed product is resolved. Note that the reference values could be constrained to be located above the CE ones, which would lead to a discussion similar to the one described in the previous section.

A similar set of comments applies to the function *H*, whose expression for part *C* is described below. It should be emphasized that, similarly to the function *F* case, the value of the reference product would depend on whether or not $x_1^m > \bar{x}_1$.

$$\begin{aligned} &\psi_1(2, 0, \mu_1(x_1^{m+1} > \bar{x}_1))(E_1 + E_2) + \psi_1(2, 1, \mu_1(x_1^{m+1} > \bar{x}_1))(u_1(x_1^{m+1}) + E_2) \\ &+ \psi_1(1, 1, \mu_1(x_1^{m+1} > \bar{x}_1))[\psi_2(1, 1, \mu_2(x_2^{m+1} \in P^+(\bar{x}_1, \bar{x}_2))) \\ &(u_1(x_1^{m+1}) + u_2(x_2^{m+1})) + \psi_2(1, 0, \mu_2(x_2^{m+1} \in P^+(\bar{x}_1, \bar{x}_2)))(E_1 + E_2)]. \end{aligned} \tag{44}$$

Figure 12: Initial stages of the sequential information acquisition process with n observations.



Note that a general form can be provided for these equations, as we have done in “The Proposed Information Acquisition Structure with n Observations” section for the ones based on a set of fully observed products.

Sequential Information Acquisition

Finally, we describe how the information acquisition process evolves sequentially. After acquiring the first observation, x_1 , the DM must choose between continuing (CT) acquiring information on the product observed and starting (ST) acquiring information on a new product.

CT: Assume that the DM acquires the first observation x_1 and decides to continue with the initial product observed. After this, he will have observed the initial product fully, (x_1, x_2) , and must start acquiring information on a new second product, whose first characteristic, x_1^m , will be treated as we have done in the general formulation section of the article but with the reference values defined relative to the initial *fully observed product* if it provides a utility higher than the CE one. The same continuing versus starting decision must be made by the DM after observing x_1^m .

ST: Consider now the case where, instead of continuing with the initial product, the DM decides to start acquiring information on a new one. In this case, he will have partially observed two products, whose first characteristics are given by x_1 and x_1^m . It should be clear that even if the DM prefers to start acquiring information on a new product instead of continuing with x_1 , this decision may differ as the number of observations left to be acquired decreases and as the products observed modify the reference values considered by the DM. The DM must decide now between continuing with either x_1 or x_1^m or starting with a new product. As already stated, he will consider the highest value between x_1 and x_1^m to define the corresponding functions F and H .

CT: If the DM continues with a partially observed product, he will observe it fully and then face once again the problem of either continuing with the remaining partially observed product or starting with a new one.

ST: If, on the other hand, he decides to start with a new product, then he will observe x_1^{m+1} from a third product. Given the potential realizations of this characteristic, the DM must define again the corresponding functions F and H based on the highest value among the partially observed products and the number of remaining observations.

The information acquisition process of the DM proceeds according to these guidelines until he runs out of information to acquire. Figure 12 provides an intuitive description of the initial decision stages of the information acquisition algorithm.

CONCLUSIONS AND MANAGERIAL SIGNIFICANCE

The current article has formalized the optimal information acquisition behavior of a rational DM when acquiring n pieces of information from a set of products defined by vectors of two characteristics. Our model requires the DM to be endowed only with a standard strict preference relation that allows him to order the set of products and choose between two of them. The model allows to investigate empirically the effect of changes in the degree of risk aversion of the DM, which are directly reflected on the shape of the utility function. As we have illustrated, different risk attitudes on the side of the DM lead to very different information acquisition incentives. A similar implication follows when considering the information assimilation capacity assumed on the DM, with different incentives obtained depending on the number of observations being acquired.

We have also shown how our model allows to account for and identify framing effects when different reference products are considered by the DM to

be improved upon. Moreover, the model can be used to analyze the effects that modifications in the subjective probabilities assigned by the DM to the expected payoffs from the search have on his information acquisition incentives. That is, a direct link to the consumer psychology literature, dealing with the effects of optimism and pessimism on expectations, can be established.

As already emphasized, in line with the bounded rationality hypothesis and the empirical psychological literature on consumer choice, the DM does not solve a classic optimization problem but searches for a sufficiently good product given his subjective preferences and beliefs. Therefore, in addition to the management and operations research literatures, the current model can be applied to the consumer choice and economics ones.

For example, the model could be used to analyze the equilibrium implications derived from the strategic transmission of information within oligopolistic scenarios, extending into the game theoretical branch of operations research (Reinganum, 1981; Mamer & McCardle, 1987; Erat & Kavadias, 2006). Furthermore, as emphasized in the Online Appendix, the results displayed by an online search engine can be easily interpreted as two dimensional objects, composed by a brief descriptive link and additional information that requires accessing the corresponding page to be acquired. Our model provides a formal reference framework to study online search processes and, for example, compute the probability of accessing a given link based on its relative position within the ranking displayed by the search engine.

Clearly, from a formal point of view, the main extension of the current model should consist of increasing the dimension of the products on which observations are acquired. As the dimension of the product increases, the set of possible combinations and expected payoffs that must be considered by the DMs also increases, which increments the sophistication requirements on the assimilation capacities of DMs. Moreover, in this case, framing should have an even stronger effect on the information acquisition incentives of DMs, allowing for multiple possibilities leading to very different results depending on the assumptions made.

Finally, the model can be easily extended to account for information acquisition tradeoffs among products and characteristics when search costs, frictions and signals modifying the beliefs of the DM are explicitly considered. For example, the introduction of information acquisition costs within the sequential search process described in the article is straightforward. In particular, different possibilities could be considered depending on the level of sophistication required from the DM. The simplest possibility would consist of assigning a fixed cost incurred by the DM when acquiring the next observation and comparing it to the expected payoff derived from the search. A more demanding possibility would consist of assigning a given cost to continuing acquiring information on a given product and a different one (either lower or higher, depending on the case being considered) to starting with a new product. These costs should then be subtracted from the corresponding functions F and H , leading to different downward shifts of both functions that would modify the corresponding continuation regions obtained when zero costs are assumed.

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