International Journal of Uncertainty,
Fuzziness and Knowledge-Based Systems
Vol. 25, No. 6 (2017) 1005–1018

DOI: 10.1142/S0218488517500441

A Novel Decision Support Framework for Computing Expected Utilities from Linguistic Evaluations

Debora Di Caprio
Department of Mathematics and Statistics, York University,
Toronto, M3J 1P3, Canada
dicaper@mathstat.yorku.ca

Francisco J. Santos-Arteaga
School of Economics and Management,
Free University of Bolzano,
Piazza Università 1, 39100, Bolzano, Italy
fsantosarteaga@unibz.it

Madjid Tavana*
Business Systems and Analytics Department,
Distinguished Chair of Business Analytics,
La Salle University, Philadelphia, PA 19141, USA
Business Information Systems Department,
Faculty of Business Administration and Economics,
University of Paderborn, D-33098 Paderborn, Germany
tavana@lasalle.edu
http://tavana.us/

Received 14 March 2016
Revised 11 March 2017

The increase in the amount and variety of evaluations provided by the users of different websites regarding the products displayed is becoming an increasingly familiar scenario. That is, decision makers (DMs) constantly receive linguistic evaluations (LEs) from unknown evaluators when considering different choice alternatives. The imprecision of the LEs and the fact that the evaluators may have biased interests when describing a product must be considered by the DMs when computing their expected utilities. We define a Bayesian-updated probability (BUP) function that accounts for the fuzziness inherent in the LEs and the reputation of the evaluator to represent the beliefs of DMs. The proposed BUP process allows the DMs to subjectively adjust the probability mass that is shifted across evaluation intervals when updating their beliefs and computing their corresponding expected utilities. We illustrate the behavior of the BUP function numerically and describe potential decision support applications.

Keywords: Linguistic evaluation; reputation; Bayesian update; expected utility; fuzzy number.

*Corresponding author.

1. Motivation

In the latter years, we have experienced a substantial increment in the amount of linguistic evaluations (LEs) received from our friends, other users or sellers describing online products and services. One just have to think of the users' evaluations and opinions provided by websites such as TripAdvisor, Amazon, eBay, etc. Moreover, besides the LEs reported by other users, we are also given information regarding the reputation of the reporters.[1]

Thus, given our initial subjective beliefs, we can form our own opinion regarding a product based on the information provided by the reporter and his reputation.[2] As a result, besides the recommendations of our friends, we have seen the emergence of recommender systems, formed by other users who employ linguistic adjectives defined within a given explicit or implicit rating scale to describe a potential choice object.[3,4]

The operations research and decision support literatures have focused on analyzing the consideration probabilities of decision makers (DMs), which determine the short-list of products selected for a detailed evaluation and potential choice after a given initial number of products is inspected.[5,6] This analysis is used to determine the choice probability of a product, defined as the chance of choosing a product from a consumer's consideration set.[7] However, these models do not account for the fact that most of these recommendations consist of LEs that must be assessed by the DMs when computing the expected utility derived from a potential choice object. For example, the fuzzy version of the consideration set developed by Ref. 8 is defined in terms of the degree of membership in the consideration set exhibited by different brands while leaving aside the imprecision inherent in LEs of the reporters.

Fuzzy numbers, characterized by their membership functions, are generally used to represent linguistic variables within a given evaluation interval. However, this characterization does not provide a valid probabilistic measure on which to base the behavior of DMs.[9] At the same time, the linguistic adjectives employed to describe a given object constitute signals about its characteristics that should be used to update the subjective beliefs of the DMs. In this regard, the reputation of the reporter providing the LE should be taken into account when updating the beliefs of the DM.

We define a novel Bayesian-updated probability (BUP) function that represents the beliefs of DMs when accounting for the fuzziness that characterizes the LEs received and the reputation of the reporter providing the evaluations. The proposed BUP process allows the DMs to subjectively adjust the probability mass that is shifted across evaluation subintervals when updating their beliefs and computing their corresponding expected utilities. We illustrate the behavior of the BUP function numerically and describe potential decision support applications.

In other words, we transform a fuzzy evaluation environment into a Bayesian updating process determined by the reports (signals) received, which introduce a strategic component in the analysis when accounting for the reputation of the reporter,[10] and the subjective beliefs of the DMs.[11] As a result, the updating process defined in this paper provides a novel decision support framework that can be used by DMs to compute their subjective expected utilities from any set of LEs.

2. Basic Bayesian Setting

We will consider a simplified version of the statistical decision model introduced by Ref. 12 and implemented by Ref. 13 within a fuzzy setting. In our model, the DM does not need to choose an experiment in order to obtain information about the actual state of nature, since the information is revealed to him by a reporter via LEs. Hence, we do not define a family of experiments. Moreover, the utility defined by the DM does not depend on the information received, that is, it is a function only of the act and the state of nature.

Therefore, the elements that allow us to formalize the decision problem of the DM are the following:

- Space of terminal decisions (acts): $A = \{a\}$.
- Sample space: $Z = \{z\}$.
- State space: $\Theta = \{\theta\}$.
- Utility function: $u(a,\theta)$ defined on $A \times \Theta$.

In the above description, we have adopted the notations of Ref. 13. Note that the sets A, Z and Θ can have any cardinality. The notation $A = \{a\}$ means that a is the generic element of the set A, and similarly for $Z = \{z\}$ and $\Theta = \{\theta\}$.

Before making a final decision, the DM needs to evaluate the expected utility that follows from making a decision $a \in A$ after observing a realization $z \in Z$ and assuming that the true state of nature is $\theta \in \Theta$. To this end, the DM is required to define a joint probability distribution on the Cartesian product $\Theta \times Z$. However, this requirement is equivalent to defining the following marginal and conditional density functions:

- $\pi(\theta)$: the marginal density function on the state space Θ. This function describes the *priors* (or *beliefs*) of the DM on the elements of Θ. That is, it accounts for the *prior* information of the DM relative to the potential states of nature.
- $f(z \mid \theta)$: the conditional density function on the sample space Z for a given state of nature θ.
- $g(\theta \mid z)$: the conditional density function on the state space Θ for a given observation (signal) z. This function accounts for the *posterior* information of the DM on the potential states of nature.

Once the DM has defined the above density functions, he can calculate the expected utility of a decision a subject to the observation of a signal z as follows:

$$E[u(a,\theta) \mid z] = \int_{\Theta} g(\theta \mid z) u(a,\theta) d\theta,$$

where

$$g(\theta \mid z) = \frac{f(z \mid \theta)\pi(\theta)}{\int_{\Theta} f(z \mid \theta)\pi(\theta)d\theta}.$$

3. Proposed Decision Support Framework

We assume that a given object, which can range from a product or service to a business project proposal, is evaluated using a fixed number of LEs and that these LEs are represented by triangular fuzzy numbers (TFNs).

A TFN (a,b,c) is a subset of the real numbers characterized by a membership function from a real interval $[m,M]$ into $[0,1]$. The membership function of a TFN (a,b,c) associated with a certain LE represents the degree of certainty that a given value in the set (a,b,c) belongs to the LE.[14] The membership functions of the TRNs relative to the LEs composing a linguistic variable must have the same domain $[m,M]$. This common domain is partitioned in a finite number of intervals whose extremes are the crossing points of the graphs of the membership functions. This partition will be denoted by $P([m,M])$.

Table 1 shows a standard representation of a linguistic variable consisting of five LEs and their corresponding TFNs defined on the interval [0, 1].

Table 1. Linguistic evaluations and triangular fuzzy numbers.

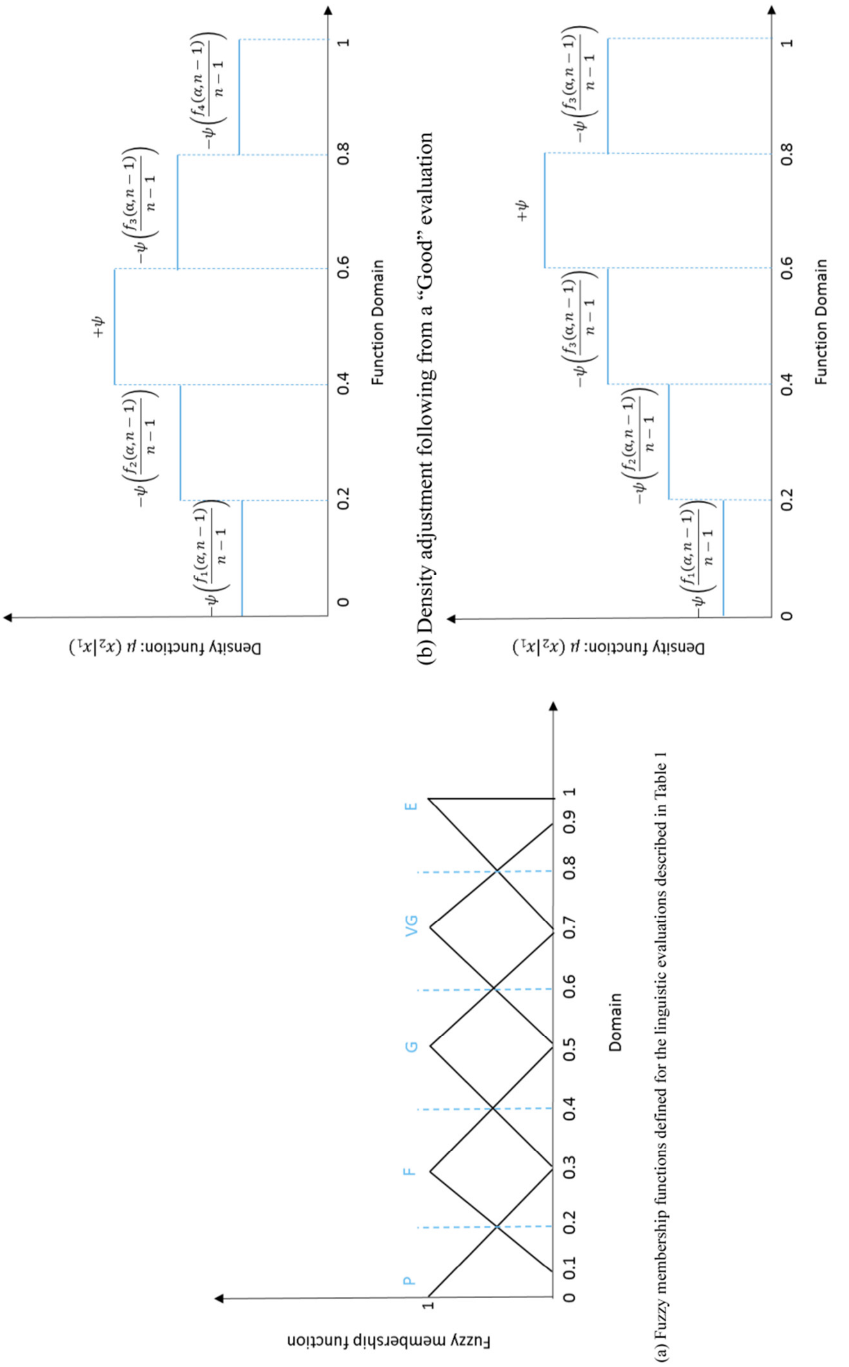
Linguistic Variable	**Linguistic Evaluations**				
Attribute Rating	Poor (P)	Fair (F)	Good (G)	Very Good (VG)	Excellent (E)
Triangular Fuzzy Number	(0, 0, 0.3)	(0.1, 0.3, 0.5)	(0.3, 0.5, 0.7)	(0.5, 0.7, 1.0)	(0.7, 1.0, 1.0)

The five membership functions associated to the LEs of Table 1 and defined within the domain [0, 1] are presented in Figure 1(a). At the same time, Figures 1(b) and 1(c) illustrate the partition $P([0,1])$ of the common domain $[0,1]$ and the adjustment process for the report-based (conditional) densities induced by the "Good" and "Very Good" LEs, respectively. The conditional density functions together with the other elements necessary to define our BUP function are formally introduced below.

Following the basic Bayesian approach described in Section 2, we need to identify the main elements characterizing the decision problem of the DM.

- ***Space of terminal decisions (acts):*** $A = \{purchase\}$.

  The space A consists of only one act, $purchase$. The DM must decide whether or not to purchase the object described by the reporter. Clearly, the act must be adapted to the type of object being considered, i.e. $A = \{select\}$ when dealing with business proposals.



(a) Fuzzy membership functions defined for the linguistic evaluations described in Table 1

(b) Density adjustment following from a "Good" evaluation

(c) Density adjustment following from a "Very Good" evaluation

Fig. 1. Fuzzy membership function and the corresponding density adjustments following from a "Good" and a "Very Good" evaluation report.

- ***Sample space:*** $X_1 = [x_1^m, x_1^M]$ with $0 < x_1^m < x_1^M$.

  X_1 represents numerically the quality of the characteristics of the object on which a report is received by the DMs. That is, X_1 is the domain of the membership functions of the TFNs representing the LEs used by the reporter to describe the object.
- ***State space:*** $X_2 = [x_2^m, x_2^M]$ with $0 < x_2^m < x_2^M$.

  X_2 represents numerically the quality of the characteristics of the object expected by the DMs. The BUP function associated with X_2 is determined by the $\text{LE} \in X_1$ received, the reputation of the reporter, and the subjective beliefs of the DM.
- ***Utility function:*** a utility function $u(purchase, x_2)$ defined on $A \times X_2$.

  Since there is only one act, in the following, we will use $u(x_2)$ in place of $u(purchase, x_2)$.
- ***Marginal density function:*** uniform density function $\mu(x_2)$ defined on X_2.
- ***Conditional density functions on the sample space:*** for every given x_2, the conditional density function $f(x_1 | x_2)$ is defined by cases as follows:

  $f(x_1 | x_2) = \mu_j(x_1 | x_2)$ if x_1 belongs to the *j*-th interval of the partition $P(X_1)$.

  The functions $\mu_j(x_1 | x_2),\ j = 1,\dots,n$ are determined by the shift in probability mass across the intervals composing the partition $P(X_1)$ that the DM subjectively applies if the state of nature is x_2.
- ***Conditional density functions on the state space:*** for every given x_1, the conditional density function $g(x_2 | x_1)$ is defined by cases as follows:

  $g(x_2 | x_1) = \mu_j(x_2 | x_1)$ if x_2 belongs to the j-th interval of the partition $P(X_2)$, where the partition $P(X_2)$ is induced by the partition $P(X_1)$. The functions $\mu_j(x_2 | x_1),\ j = 1,\dots,n$ are the conditional updated densities defining the BUP function.

Note that in order to simplify notations, we will use $\mu_j(x_1 | x_2)$ and $\mu_j(x_2 | x_1)$ in place of $f(x_1 | x_2)$ and $g(x_2 | x_1)$.

Moreover, we introduce a variable accounting for the credibility of the reporter and a family of functions allowing us to formalize the subjectively induced shifts in probability mass across the intervals of the partition $P(X_1)$.

- $\psi \in [0, 1]$: This variable represents the credibility of the reporter. The higher its value, the higher the probability mass assigned to the interval within which the corresponding LE is located. The value taken by this variable could be either assigned subjectively by the DMs or determined by the information available in online environments.

- $f_i(\alpha, n-1), i = 1, \ldots, n-1$: The DM is allowed to subjectively modify the shift of probability mass across the partition $P(X_1)$ based on the LE received by the reporter. This shift in probability depends on the number of intervals from which the probability mass has to be gathered and the subjective characteristics of the DMs. These shifts are described by n – 1 functions, $f_i(\alpha, n-1), i = 1, \ldots, n-1$, determined by both these variables, with α accounting for the subjective retrieval of mass across intervals based on the relative distance from a given interval to the one within which the LE is located.

In order to simplify the presentation, we have assumed that both X_1 and X_2 coincide with the domain [0, 1]. The n intervals composing the partition of X_2 have been assumed to have the same width, allowing for a proportional shift of probability mass across them, that is:

$$
\begin{aligned}
& x_2 < x_2^m + \frac{x_2^M - x_2^m}{n}; \\
& x_2^m + \frac{x_2^M - x_2^m}{n} \le x_2 < x_2^m + 2\frac{x_2^M - x_2^m}{n}; \\
& \ldots \\
& x_2^m + (n-1)\frac{x_2^M - x_2^m}{n} \le x_2 < x_2^m + n\frac{x_2^M - x_2^m}{n}.
\end{aligned} \tag{1}
$$

The resulting BUP density is composed by the conditional updated density functions $\mu_1(x_2 \mid x_1)$, $\mu_2(x_2 \mid x_1)$, …, $\mu_n(x_2 \mid x_1)$ defined through the different intervals in which X_2 is divided. We describe below the density functions $\mu_1(x_2 \mid x_1)$ and $\mu_2(x_2 \mid x_1)$ corresponding to the first and second interval composing the domain of the BUP density, respectively. Note that the interval within which the corresponding LE is located is $\left(x_2^m + \left\lfloor \frac{n}{2} \right\rfloor \frac{x_2^M - x_2^m}{n}, x_2^m + \left\lceil \frac{n}{2} \right\rceil \frac{x_2^M - x_2^m}{n} \right)$.

The density function $\mu_1(x_2 \mid x_1)$ when $x_2 < x_2^m + \frac{x_2^M - x_2^m}{n}$ (i.e., in the first interval) is given by:

$$
\mu_1(x_2 \mid x_1) = \frac{\mu_1(x_1 \mid x_2)\mu(x_2)}{\int_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2} = \frac{\left[\frac{1}{x_1^M - x_1^m} - \psi\left(\frac{f_1(\alpha, n-1)}{n-1} \right) \frac{1}{x_1^M - x_1^m} \right] \left(\frac{1}{x_2^M - x_2^m} \right)}{\int_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2}, \tag{2}
$$

while the density function $\mu_2(x_2 \mid x_1)$ when $x_2^m + \frac{x_2^M - x_2^m}{n} \le x_2 < x_2^m + 2\frac{x_2^M - x_2^m}{n}$ (i.e., in the second interval) is given by:

$$\mu_2(x_2 \mid x_1) = \frac{\mu_2(x_1 \mid x_2)\mu(x_2)}{\int\limits_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2} = \frac{\left[\frac{1}{x_1^M - x_1^m} - \psi\left(\frac{f_2(\alpha, n-1)}{n-1}\right)\frac{1}{x_1^M - x_1^m}\right]\left(\frac{1}{x_2^M - x_2^m}\right)}{\int\limits_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2}, \quad (3)$$

where:

$$\int\limits_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2 = \int\limits_{x_2 \in \left[x_2^m,\, x_2^m + \frac{x_2^M - x_2^m}{n}\right]} \mu_1(x_1 \mid x_2)\mu(x_2)dx_2 + \ldots + \int\limits_{x_2 \in \left[x_2^m + (n-1)\frac{x_2^M - x_2^m}{n},\, x_2^M\right]} \mu_n(x_1 \mid x_2)\mu(x_2)dx_2, \quad (4)$$

that is:

$$\int\limits_{x_2 \in X_2} \mu_j(x_1 \mid x_2)\mu(x_2)dx_2 = \int_{x_2^m}^{x_2^m + \frac{x_2^M - x_2^m}{n}} \left[\frac{1}{x_1^M - x_1^m} - \psi\left(\frac{f_1(\alpha, n-1)}{n-1}\right)\frac{1}{x_1^M - x_1^m}\right]\left(\frac{1}{x_2^M - x_2^m}\right)dx_2 + \ldots$$
$$+ \int_{x_2^m + \left\lfloor \frac{n}{2} \right\rfloor \frac{x_2^M - x_2^m}{n}}^{x_2^m + \left\lceil \frac{n}{2} \right\rceil \frac{x_2^M - x_2^m}{n}} \left[\frac{1}{x_1^M - x_1^m} + \psi\frac{1}{x_1^M - x_1^m}\right]\left(\frac{1}{x_2^M - x_2^m}\right)dx_2 + \ldots \quad (5)$$
$$+ \int_{x_2^m + (n-1)\frac{x_2^M - x_2^m}{n}}^{x_2^M} \left[\frac{1}{x_1^M - x_1^m} - \psi\left(\frac{f_5(\alpha, n-1)}{n-1}\right)\frac{1}{x_1^M - x_1^m}\right]\left(\frac{1}{x_2^M - x_2^m}\right)dx_2.$$

Several remarks must be emphasized regarding the definition of the density functions $\mu_j(x_2 \mid x_1)$, $j = 1, \ldots, n$.

- Note that all the intervals of the partitions we used for X_1 and X_2 have all the same length. However, asymmetries can be introduced across the membership functions and the resulting conditional updated density functions adjusted accordingly.[15]
- In this regard, the intervals defined within the domain of our BUP function or the subjective weights assigned to each interval composing $P(X_1)$ could be defined in

terms of the relative dominance arising among the different membership functions (consider, for example, the preference degree approach for comparing and ranking fuzzy numbers introduced by Ref. 16). Consequently, even though we have considered TFNs, our BUP function can be modified to account for trapezoidal ones.

- We have used uniform densities, which endow each interval with the highest information entropy,[17] to illustrate the uncertainty faced by the DM. However, triangular distributions could be assumed when defining $\mu_i(x_1 \mid x_2)$, $i = 1,\ldots,n$, and $\mu(x_2)$ within the different intervals composing X_1 and X_2, respectively.

4. Numerical Simulations

Figures 2 and 3 present the BUP density functions defined on the different intervals composing $P(X_2)$ and the corresponding expected utilities computed after receiving a "Good" and a "Very Good" LE, respectively. The expected utility obtained by the DM, $E[u(x_2) \mid x_1]$, is defined as follows:

$$E[u(x_2) \mid x_1] = \int_{x_2 \in X_2} \mu(x_2 \mid x_1) u(x_2) dx_2 =$$

$$\int_{x_2^m}^{x_2^m + \frac{x_2^M - x_2^m}{n}} \mu_1(x_2 \mid x_1) u(x_2) dx_2 + \ldots + \int_{x_2^m + \left\lfloor \frac{n}{2} \right\rfloor \frac{x_2^M - x_2^m}{n}}^{x_2^m + \left\lceil \frac{n}{2} \right\rceil \frac{x_2^M - x_2^m}{n}} \mu_{\left\lceil \frac{n}{2} \right\rceil}(x_2 \mid x_1) u(x_2) dx_2 + \ldots \tag{6}$$

$$+ \int_{x_2^m + (n-1)\frac{x_2^M - x_2^m}{n}}^{x_2^M} \mu_n(x_2 \mid x_1) u(x_2) dx_2$$

For illustrative purposes, we have assumed that $u(x_2) = x_2^2$. Figures 2 and 3 describe the shape of the density function within each interval as ψ and α vary within their respective domains. When the report received is "Good", we consider the following mass shifting functions for each interval: $f_1(\alpha, n-1) = (2-\alpha) = f_5(\alpha, n-1)$ and $f_2(\alpha, n-1) = \alpha = f_4(\alpha, n-1)$, with $\alpha \in [0,1]$. Note that, when $\alpha = 0$, we are shifting probability from the extreme intervals into the central one, within which the LE is located. As the value of α increases, probability mass is also shifted from the intervals neighboring the central one, with $\alpha = 1$ implying that the same mass is shifted from each of the four intervals to the central one. Similarly, when the report received is "Very Good", we consider the following mass shifting functions: $f_1(\alpha, n-1) = (2.5-\alpha)$, $f_2(\alpha, n-1) = (1.5-\alpha)$ and $f_3(\alpha, n-1) = \alpha = f_4(\alpha, n-1)$. However, in this case, $\alpha \in [0, 0.75]$.

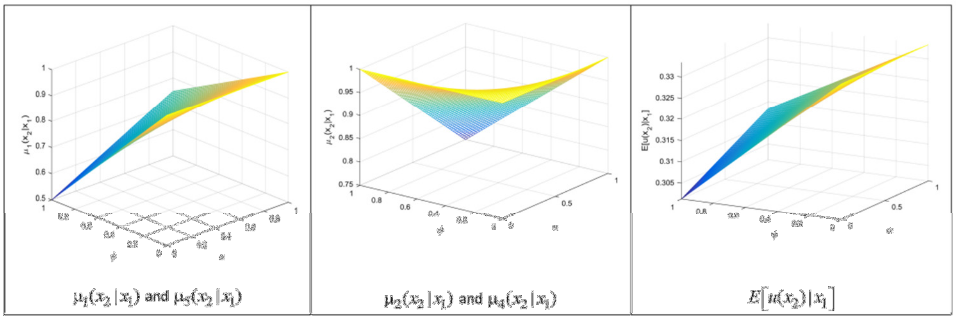


Fig. 2. Bayesian updated probability and expected utility when the evaluation of the reporter is “Good”.

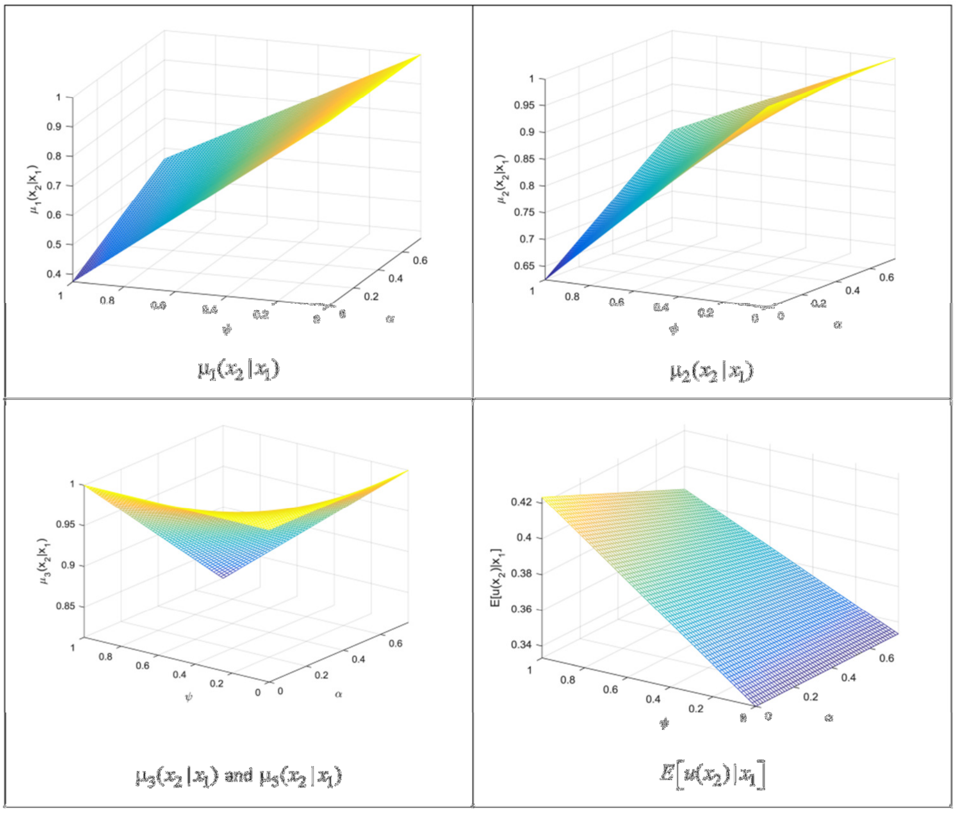


Fig. 3. Bayesian updated probability and expected utility when the evaluation of the reporter is “Very Good”.

As intuition suggests, the expected utility obtained when the report received is “Very Good” is higher than the one derived from a “Good” report. However, we should emphasize that, depending on the type of report received, there is a significant difference in the behavior exhibited by the expected utility as the variables ψ and α vary. The intuition behind these results reads as follows. As ψ increases, so does the probability weight assigned to the interval determined by the LE issued by the reporter. Thus, given the convex shape of the utility function, as ψ increases in the "Good" report case, we shift probability from the sides and extremes of the distribution towards the central interval $[0.4, 0.6]$, leading to a lower expected utility. The same type of reasoning applies as we shift probability mass into the interval $[0.6, 0.8]$ when the report received is "Very Good" – though in this case the corresponding relation is increasing.

Consider now the α variable, whose effect on the expected utility is also reversed in both report cases. When the report received is "Good", the expected utility obtained by the DM increases in α. In this case, when $\alpha = 0$, all the probability mass shifted to the interval within which the LE is located is taken from the extreme intervals of the function. As α increases, we start shifting probability away from the intervals neighboring the central one, while increasing the probability mass of the extremes. Thus, given the shift of mass towards the extreme intervals of the probability function ($[0, 0.2]$ and $[0.8, 1]$) and the convexity of the utility function, we have an increase in the expected utility obtained by the DM. This tendency is reversed when the report received is "Very Good", since, in this case, the extreme intervals of the probability function are given by $[0, 0.2]$ and $[0.2, 0.4]$, while those neighboring the interval within which the LE is located are $[0.4, 0.6]$ and $[0.8, 1]$.

5. Potential Applications and Extensions

Given the subjective variability allowed for when constructing our BUP function, the elicitation of the preferences and beliefs of DMs could become a difficult task. As the numerical simulations illustrate, different degrees of credibility and subjective probability mass shifts across intervals may lead to the same or very similar expected utility values. It could therefore be difficult to differentiate between the effect due to the subjective probability shifts of DMs and the one following from the credibility assigned to the reporter. This task could be further complicated when dealing with asymmetric fuzzy numbers and the corresponding assignment of interval weights, given the variety of ranking methods available in the literature.[16,18,19] Thus, the results obtained in such a setting would be sensitive to the method chosen to rank fuzzy numbers.

However, we should emphasize that these drawbacks are compensated by the capacity of the BUP function to accommodate any type of LE within a standard statistical decision making structure. This is done while accounting for potential subjective evaluations of the DMs together with the reputation of the reporter, a property that becomes particularly useful in online recommender environments. That is, the BUP function introduced in this paper can be used to generate rankings of products based on the recommendations obtained by the DM from other online agents. In other words, the BUP function serves as a complement to the literature on recommender systems, providing an alternative approach to define the choices made by the DM.

The formalization of recommender systems leads to rankings of products determined by their predicted ratings. The predictions made by the DMs are generally subject to uncertainty, which can be accounted for using the confidence level of the predictions determining the ranking.[20] In this regard, the BUP function formalizes the existence of uncertainty inherent to the evaluations and predictions of the DM, which affect his expected utility and resulting behavior.[21]

Among the potential decision support applications that go beyond the recommender systems research area,[6] we should highlight the elaboration of rankings determined by the entropy of the evaluations received when the reports consist of subjective linguistic evaluations instead of crisp numerical values.[22] An immediate extension to group decision-making would allow us to account for the entropy inherent to the subjective evaluations of the reporters as well as their credibility.

Indeed, several additional applications of the BUP function arise when considering multi-criteria group decision-making problems, where the preferences and beliefs of DMs play a fundamental role in the outcome obtained.

Among the recent developments presented in the literature, we should highlight the use of the qualitative flexible multiple criteria method as a basis on which to incorporate either semantic evaluation differences together with the risk preferences of DMs[23] or hesitant fuzzy linguistic information.[24] Hesitant fuzzy linguistic numbers have been recently introduced in the literature. They consist of a linguistic term, a set of membership degrees and a set of non-membership degrees, with the DMs determining subjectively the respective membership and non-membership degrees depending on their preferences. These numbers have been used to describe the preferences of DMs and reflect the uncertainty faced when evaluating choice objects[25] or dealing with multi-criteria decision-making (MCDM)[26] and group decision-making problems.[27] In this regard, multi-criteria group decision-making problems have been defined in which the criteria values considered are given by interval linguistic variables and the weights of the DMs are unknown.[28]

We conclude by noting that the subjective reallocations of probability allowed for on the side of the DMs are similar to those taking place in prospect and cumulative prospect theory, where the subjective evaluations of DMs determine the shift in their probabilities.[29,30] The literature has already considered the inclusion of prospect theory and cumulative prospect theory in formal models of decision making under uncertainty[31] and MCDM methods such as TODIM,[32] respectively. Consequently, the BUP function introduced in this paper could also be used to formalize the effects that the subjective evaluations of DMs and the credibility of the reporters have on MCDM techniques such as TODIM and TOPSIS.

6. Conclusions

Given the imprecision inherent in the LEs, a fuzzy set is generally used to quantify the corresponding terms. However, this characterization does not provide a valid probabilistic measure on which to base the behavior of DMs. We have formalized the problem of defining a probability function based on LEs when these constitute a signal regarding the quality or the characteristic of a given object. The resulting probability has been designed to account for the reputation of the reporter providing the evaluations and the subjective beliefs of the DM.

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

References

1. F. Fouss, Y. Achbany and M. Saerens, A probabilistic reputation model based on transaction ratings, *Information Sciences* **180** (2010) 2095–2123.
2. R. A. Peterson and M. C. Merino, Consumer information search behavior and the internet, *Psychology & Marketing* **20** (2003) 99–121.
3. B. Xiao and I. Benbasat, E-commerce product recommendation agents: Use, characteristics, and Impact, *MIS Quarterly* **31** (2007) 137–209.
4. D. Fleder and K. Hosanagar, Blockbuster culture's next rise or fall: The impact of recommender systems on sales diversity, *Management Science* **55** (2009) 697–712.
5. M. Kukar-Kinney and A. G. Close, The determinants of consumers' online shopping cart abandonment, *Journal of the Academy of Marketing Science* **38** (2010) 240–250.
6. H. Wang, Q. Wei and G. Chen, From clicking to consideration: A business intelligence approach to estimating consumers' consideration probabilities, *Decision Support Systems* **56** (2013) 397–405.
7. W. W. Moe, An empirical two-stage choice model with varying decision rules applied to internet clickstream data, *Journal of Marketing Research* **43** (2006) 680–692.
8. J. Wu and A. Rangaswamy, A fuzzy set model of search and consideration with an application to an online market, *Marketing Science* **22** (2003) 411–434.
9. W. Pedrycz and F. Gomide, *An Introduction to Fuzzy Sets: Analysis and Design* (MIT Press, (1998).
10. D. Fudenberg and J. Tirole, *Game Theory* (MIT Press, 1991).
11. D. Kahneman and A. Tversky, *Choices, Values, and Frames* (Cambridge University Press, 2000).
12. H. Raiffa and R. Schlaifer, *Applied Statistical Decision Theory* (Wiley, New York, 2000).
13. O. Hryniewicz, Possibilistic approach to Bayes statistical decisions, *Soft Methods in Probability, Statistics and Data Analysis*, eds. in P. Grzegorzewski, O. Hryniewicz and M. A. Gil (Physica-Verlag, Heidelberg, 2002), pp. 207–218.
14. H. J. Zimmermann, *Fuzzy Set Theory and Its Applications* (Kluwer Academic Publishers, 2001).
15. M. Tavana, D. Di Caprio, F. J. Santos-Arteaga and K. Tierney, Modeling signal-based decisions in online search environments: A non-recursive forward-looking approach, *Information & Management* **53** (2016) 207–226.
16. Y. M. Wang, Y. Luo and L. Liang, Fuzzy data envelopment analysis based upon fuzzy arithmetic with an application to performance assessment of manufacturing enterprise, *Expert Systems with Applications* **36** (2009) 5205–5211.
17. M. Tavana, A subjective assessment of alternative mission architectures for the human exploration of Mars at NASA using multicriteria decision making, *Computers & Operations Research* **31** (2004) 1147–1164.

18. F. Zhang, J. Ignatius, C. P. Lim and Y. Zhao, A new method for ranking fuzzy numbers and its application to group decision making, *Applied Mathematical Modelling* **38** (2014) 1563−1582.
19. K. C. Chai, K. M. Tay and C. P. Lim, A new method to rank fuzzy numbers using Dempster–Shafer theory with fuzzy targets, *Information Sciences* **346–347** (2016) 302–317.
20. M. Zhang, X. Guo and G. Chen, Prediction uncertainty in collaborative filtering: Enhancing personalized online product ranking, *Decision Support Systems* **83** (2016) 10–21.
21. K. Yao and X. Ji, Uncertain decision making and its application to portfolio selection problem, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **22** (2014) 113, DOI: http://dx.doi.org/10.1142/S0218488514500056.
22. M. Tavana, D. Di Caprio, F. J. Santos-Arteaga and A. O'Connor, A novel entropy-based decision support framework for uncertainty resolution in the initial subjective evaluations of experts: The NATO enlargement problem, *Decision Support Systems* **74** (2015) 135−149.
23. Z.-P. Tian, J. Wang, J.-Q. Wang and H.-Y. Zhang, Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development, *Group Decision and Negotiation* (2016a), DOI: 10.1007/s10726-016-9479-5
24. Z.-P. Tian, J. Wang, J.-Q. Wang and H.-Y. Zhang, A likelihood-based qualitative flexible approach with hesitant fuzzy linguistic information, *Cognitive Computation* (2016b), DOI: 10.1007/s12559-016-9400-1.
25. H. Zhou, J. Wang, X.-E. Li and J.-Q. Wang, Intuitionistic hesitant linguistic sets and their application in multi-criteria decision-making problems, *Operational Research* **16** (2016) 131−160.
26. J.-Q. Wang, J.-T. Wu, J. Wang, H-Y. Zhang and X.-H. Chen, Multi-criteria decision-making methods based on the Hausdorff distance of hesitant fuzzy linguistic numbers, *Soft Computing* **20** (2016) 1621−1633.
27. C. Wei, N. Zhao and X. Tang, A novel linguistic group decision-making model based on extended hesitant fuzzy linguistic term sets, *Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems* **23** (2015) 379, DOI: http://dx.doi.org/10.1142/s0218488515500166.
28. J.-Q. Wang, J.-J. Peng, H.-Y. Zhang, T. Liu and X.-H. Chen, An uncertain linguistic multi-criteria group decision-making method based on a cloud model, *Group Decision and Negotiation* **24** (2015) 171−192.
29. D. Kahneman and A. Tversky, An analysis of decision under risk, *Econometrica* **47** (1979) 263−292.
30. A. Tversky and D. Kahneman, Advances in prospect theory, cumulative representation of uncertainty, *Journal of Risk and Uncertainty* **5** (1992) 297−323.
31. E. Nusrat and K. Yamada, A descriptive decision-making model under uncertainty: combination of Dempster-Shafer theory and prospect theory, *Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems* **21** (2013) 79, DOI: http://dx.doi.org/10.1142/S0218488513500050.
32. L. F. A. M. Gomes and X. I. González, Behavioral multi-criteria decision analysis: Further elaborations on the TODIM method, *Foundations of Computing and Decision Sciences* **37** (2012) 3–8.