Robust efficiency measurement with common set of weights under varying degrees of conservatism and data uncertainty

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Abstract: The conventional paradigm in data envelopment analysis (DEA) is to develop an efficiency measurement model that assumes the input and output data are precise and equal to some nominal values. However, this paradigm does not take into consideration the inherent uncertainties in real-life performance measurement problems. As a result of these uncertainties, the input and output data may take non-nominal values and violate the basic assumptions in DEA. This phenomenon has motivated us to design a DEA model that is 'robust' and immune to uncertain data. We present a robust DEA model with a common set of weights (CSWs) under varying degrees of conservatism and data uncertainty. We use goal programming (GP) and compute the relative efficiencies of the decision making units (DMUs) by producing CSWs in one run. The proposed model uses a confidence criterion to produce a ranking of the DMUs and determine a set of efficient DMUs. We present a numerical example and a case study to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed method to a performance measurement problem in the banking industry.

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1 Introduction

Data envelopment analysis (DEA) is a widely used non-parametric mathematical programming approach for evaluating the relative efficiency of decision making units (DMUs) in organisations. Non-parametric frontier analysis was initially introduced by Farrell (1957) and later developed into DEA by Charnes et al. (1978). The Charnes et al.'s (1978) model, called the CCR model, is also known as a constant returns to scale (CRS) DEA model. DEA does not require the assignment of predetermined weights to the input and output indices. DEA has been used in evaluating DMUs and computing the efficiency score in various organisations and industries including the energy and power industry (Lo et al., 2001; Sueyoshi and Goto, 2001; Chien et al., 2003; Fallahi et al., 2011), education (Sarricl et al., 1997), research and development (Chen et al., 2004), healthcare (Bannick and Ozcan, 1995; De Nicola et al., 2012), banking (Schaffnit et al., 1997; Seiford and Zhu 1999; Ebrahimnejad et al., 2014), and military (Charnes et al., 1985; Sutton and Dimitrov, 2013) among others. Further reviews and additional applications are provided by Charnes et al. (1994), Seiford (1997), and Cook and Seiford (2009).

In conventional DEA models, each DMU is evaluated by its own weight. The advantage of this approach is that each DMU can augment its efficiency using suitable weights compared to the other DMUs, while the disadvantage is that solving different models provide different weights for the inputs and outputs. This may not be rational and acceptable by a decision maker because of the variation between the weights. To overcome this problem, several methods have been proposed in the DEA literature. Cook et al. (1990) and Roll et al. (1991) were among the first who introduced a common set of weights (CSWs) in DEA models for evaluating highway maintenance units. The role of multiplier bounds in calculating the efficiency score of DMUs was presented by Thompson et al. (1990). They described a special case of an assurance region to construct linear homogeneous conditions on the multipliers in efficiency analysis. Cook and Kress (1990, 1991) proposed a subjective ordinal preference ranking based on DEA in the presence of upper and lower bounds on the weights. Hosseinzadeh Lotfi et al. (2000) and Jahanshahloo et al. (2005) worked on two different DEA models with CSWs in which the efficiency of DMUs could be obtained by a nonlinear programming problem instead of solving n linear programming models. Hosseinzadeh Lotfi et al. (2000) utilised the concept of multi objective programming and obtained the efficiency score of all the DMUs using CSWs. Jahanshahloo et al. (2005) introduced an approach based on the concept of CSWs and ranked efficient DMUs in a two-step process to measure the efficiency scores. Amin and Toloo (2007) presented a model capable of finding the most efficient DMUs using CSWs. Davoodi and Zhiani Rezai (2011) suggested a CSWs method for solving a linear programming problem and calculating the efficiency scores and rankings of all the DMUs.

The uncertainty of data in traditional DEA models is ignored. These models consider the best estimation of the required data known as nominal data. However, these models may involve a perturbation in the nominal data, which in turn violates several constraints. Consequently, the optimal solution obtained using the nominal data may no longer be optimal or even feasible. To cope with this problem, researchers have focused on the theoretical developments of techniques using uncertain data. In many real-life applications with uncertain data, the distribution of data is not known and only two extreme points of a range may be available for inclusion in the model. Despotis and Smirlis (2002) presented two models using interval data for computing the upper and lower bounds of the efficiency scores of each DMU, which are known as the optimistic and pessimistic cases, respectively. Wang et al. (2005) proposed another pair of interval DEA models for measuring the efficiencies of DMUs by using a fixed production frontier. Jahanshahloo et al. (2004) determined the radius of stability for all DMUs using interval data and showed that the original classification remained unchanged under perturbations of data. Park (2007) considered the same classification as that of Despotis and Smirlis (2002) but in a more general structure of imprecise data consisting of any combination of bounded and ordinal data.

Another approach for dealing with uncertain data is known as the robust optimisation method. This approach uses an uncertain but bounded data model. Indeed, robust optimisation constructs a solution that is optimal for any realisation of the uncertainty in a given set. The first step in this approach was taken by Soyster (1973) who considered a linear optimisation model to obtain a solution that is feasible for all data lying in a convex set. Recent works using this general approach include Ben-Tal and Nemirovski (1998, 1999, 2000), El-Ghaoui and Lebret (1997), and El-Ghaoui et al. (1998). Bertsimas and Sim (2004) proposed a different approach of robust optimisation to control the level of conservatism in the robust solution in terms of probabilistic bounds of constraint violations. Sadjadi and Omrani (2008) proposed a robust DEA model assuming uncertainty for the output parameters. Based on a robust optimisation model, Shokouhi et al. (2010) proposed a robust DEA model in which the input and output parameters vary only in a certain range. It should be noted that this method completely covers the interval approach presented by Despotis and Smirlis (2002).

Omrani (2013) introduced a robust optimisation method for finding CSWs in DEA with uncertain input and output data. He first solved the proposed robust DEA model and found the ideal solution for each DMU. He then found the CSWs for all the DMUs by utilising a goal programming approach. Avkiran (2015) studied dynamic network DEA in commercial banking with emphasis on testing robustness. He conceptualised a bank network as comprised of two divisions and used robustness testing to discuss discrimination by the dimensionality of the performance model, efficiency estimates, stability of estimates through re-sampling, and sensitivity of results to divisional weights and returns-to-scale assumptions. Ghahtarani and Najafi (2013) used goal programming and proposed a robust optimisation model for the portfolio selection problem. They used robust optimisation to deal with uncertain parameters and guarantee the feasibility of the solutions. Mardani and Salarpour (2015) proposed a DEA model with uncertain data to analyse the technical and scale efficiency in agriculture. They also used a Monte Carlo simulation to compute the conformity of the rankings obtained from their robust DEA model. Hafezalkotob et al. (2015) proposed a robust DEA model to investigate the efficiencies of DMUs when there are discrete uncertain input and output data. Their method is based on the discrete robust optimisation approach proposed by Mulvey et al. (1995) that utilises probable scenarios for capturing the effects of uncertain data. Lu (2015) developed robust DEA models by representing uncertain outputs with uncertainty sets and maximising the DMUs' worst-case efficiencies with respect to their uncertainty set. A genetic algorithm and a set of parameter settings of a simulated annealing heuristic were used to evaluate the robustness of the models.

In this paper, we present a robust DEA model with CSWs under adjusting degrees of conservatism and data uncertainty. A new model is proposed to measure the efficiency of DMUs with CSWs, where inputs and outputs can vary in an interval. Initially, the relative efficiency of DMUs are computed by goal programming (GP) and a confidence criterion ranks the DMUs to determine the efficient units. Then, a novel model based on the adjusting degrees of conservatism is presented. DMUs are evaluated by the same production possibility set (PPS) using a conservatism level determined for each input and output. The proposed GP model is solved only once to determine the efficiency of all the DMUs. Moreover, the obtained efficiency can be considered as a method for ranking DMUs because of the availability of the CSWs. Because the data varies in an interval, DMUs can be divided into a set of efficient DMUs or inefficient DMUs for some values of the interval with a confidence criterion. Numerical examples demonstrate the efficacy and applicability of the proposed model.

The rest of the paper is organised as follows. In Section 2 we present some preliminary concepts. In Section 3 we present the details of the model proposed in this study. We present a numerical example and a case study in Section 4 to exhibit the efficacy of the procedures and to demonstrate the applicability of the proposed method to a performance measurement problem in the banking industry. Finally, Section 5 presents our conclusions and future research directions.

2 Preliminaries

Assume that there are n DMUs under consideration with m inputs and s outputs. Relative efficiency is defined as the ratio of total weighted outputs to total weighted inputs, i.e., the efficiency score of DMU_o is given as follows:

$$E_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}, \tag{1}$$

where u_r and v_i are the non-negative weight factors. Charnes et al. (1978) utilised the following model to compute the CCR-efficiency score of DMU_o.

$$\begin{aligned} & Max \ \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ & s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad \forall j, \\ & \qquad u_r \ge \varepsilon, v_i \ge \varepsilon, \qquad \forall r, \forall i. \end{aligned} \tag{2}$$

where ε is a non-Archimedean number. The optimal objective value of model (2) ranges between 0 and 1. DMU_o is called CCR-efficient if and only if the optimal objective value of model (2) obtains a score of 1 (Charnes et al., 1978). Note that the abovementioned model is used in a fractional programming problem. This model can be converted to a linear programming problem via the transformation approach proposed by Chanes and Cooper (1962) as follows:

$$
\begin{aligned}
& Max \sum_{r=1}^{s} u_r y_{ro} \\
& s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1, && && (3a) \\
& \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, && \forall j, && (3b) \\
& \quad u_r \geq \varepsilon, v_i \geq \varepsilon, && \forall r, \forall i. && (3c)
\end{aligned} \tag{3}
$$

Model (3) seeks input and output weights that maximise the efficiency score of DMU_o. constraint (3a) is a normalisation constraint and constraints (3b) guarantee DMUs place in PPS. Because there are no restrictions on the multipliers in model (3), a set of unbounded input and output weights is involved. One of the drawbacks associated with the CCR model is that it obtains a set of different weights for different DMUs. Consequently, the DMUs cannot be compared appropriately and an acceptable economic interpretation cannot be presented. Several methods have been proposed to deal with these drawbacks. The weights obtained by model (3) can be used in the cross-efficiency method to identify the rankings of the DMUs. The CSWs in DEA is one approach for ensuring that all the DMUs are evaluated with unique weights. Using this method, only one model is solved to calculate the efficiency score of the DMUs (in the most optimal fashion). In this case, the following multi-objective fractional programming (MOFP) problem can be applied to maximise the efficiency score of all the DMUs simultaneously:

$$
\begin{aligned}
& Max \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rn}}{\sum_{i=1}^{m} v_i x_{in}} \right\} \\
& s.t. \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad \forall j, \\
& \quad u_r \geq \varepsilon, v_i \geq \varepsilon, \qquad \forall r, \forall i.
\end{aligned} \tag{4}
$$

There are several methods for solving the above MOFP problem (e.g., Chankong and Haimes, 1983; Hwang and Masud, 1979; Marler and Arora, 2004; Sawaragi et al., 1985; Steuer, 1986). The following GP based model proposed by Liu and Peng (2008) is introduced here to simultaneously maximise the efficiency of the DMUs with CSWs:

$$
\begin{aligned}
& Min \sum_{j=1}^{n}\left(\delta_j^{+}+\delta_j^{-}\right) \\
& s.t.\quad \frac{\sum_{r=1}^{s} u_r y_{rj}+\delta_j^{+}}{\sum_{i=1}^{m} v_i x_{ij}-\delta_j^{-}}=1, \quad \forall j, \\
& \qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad \forall j, \\
& \qquad u_r \geq \varepsilon, v_i \geq \varepsilon, \qquad \forall r, \forall i, \\
& \qquad \delta_j^{+}, \delta_j^{-} \geq 0, \qquad \forall j,
\end{aligned} \tag{5}
$$

where δ_j^{-} and δ_j^{+} are the negative and positive deviations of the j^{th} goal, respectively, and the goals for the objective functions are considered to be one. Therefore, the DMUs should minimise the sum of the total virtual gaps to the benchmarking frontier by adding δ_j^{+} to $\sum_{r=1}^{s} u_r y_{rj}$ and subtracting δ_j^{-} from $\sum_{i=1}^{m} v_i x_{ij}$ In other words, model (5) minimises the value of δ_j^{-} 's and δ_j^{+} 's in the numerator and dominator of $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$ for each DMU such that the efficiency value of the DMUs approaches one. The following linear programming model is presented by converting model (5) to linear form using the cross-multiplication method:

$$
\begin{aligned}
& Min \sum_{j=1}^{n}\left(\delta_j^{+}+\delta_j^{-}\right) \\
& s.t.\quad \sum_{r=1}^{s} u_r y_{rj}-\sum_{i=1}^{m} v_i x_{ij}+\delta_j^{+}+\delta_j^{-}=0, \qquad \forall j, \\
& \qquad \sum_{r=1}^{s} u_r y_{rj}-\sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j, \\
& \qquad u_r \geq \varepsilon, v_i \geq \varepsilon, \qquad \forall r, \forall i, \\
& \qquad \delta_j^{+}, \delta_j^{-} \geq 0, \qquad \forall j.
\end{aligned} \tag{6}
$$

Model (6) can be written as model (7) by setting $\delta_j=\delta_j^{+}+\delta_j^{-}, \forall j$ as follows:

$$
\begin{aligned}
& Min \sum_{j=1}^{n} \delta_j \\
& s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \delta_j = 0, \qquad \forall j, \qquad (7a) \\
& \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad \forall j, \qquad (7b) \\
& \qquad u_r \ge \varepsilon, v_i \ge \varepsilon, \qquad \forall r, \forall i, \quad (7c) \\
& \qquad \delta_j \ge 0, \qquad \forall j. \qquad (7d)
\end{aligned}
\tag{7}
$$

It is clear that constraints (7b) are redundant because of constraints (7a).

Definition 1: DMU_j is efficient if and only if $\delta_j^* = 0$ in model (7). We can also calculate the efficiency scores of DMU_j as follows:

Let $(u_r^*, v_i^*, \delta_j^*)$ be the optimal solution of model (7), then the efficiency scores of DMU_j can be obtained as follows:

$$
e_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = 1 - \frac{\delta_j^*}{\sum_{i=1}^{m} v_i x_{ij}}, \qquad \forall j. \tag{8}
$$

Definition 2: DMU_j is efficient if and only if in equation (8), $e_j = 1$.

3 Proposed performance measurement model

In this section we formulate a model to measure the efficiency of DMUs with CSWs where the input and output data belong to intervals. Assume that there are n DMUs under consideration with m inputs and s outputs. Let the observed input and output vectors of DMU_o be $x_j = (x_{1j}, x_{2j},\ldots,x_{mj})$ and $y_j = (y_{1j}, y_{2j},\ldots, y_{sj})$, respectively, where all components of vectors x_j and y_j for all DMUs are non-negative, and each DMU has at least one strictly positive input and output. It is assumed that the input and output values of each DMU_j is located in a certain interval, where x_{ij}^L and x_{ij}^U are the lower and upper bounds of the i^{th} input, respectively, and y_{rj}^L and y_{rj}^U are the lower and upper bounds of the r^{th} output, respectively, that is to say, $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ and $y_{rj} \in [y_{rj}^L, y_{rj}^U]$.

The interval of the uncertain input and output data are used for sensitivity analysis and historical records. The sensitivity analysis explores and quantifies the impact of possible errors in the data on the system performance indices. On the other hand, the historical records of the system can determine an interval of uncertain data by applying an expert system or calculating uncertainty as a confidence interval. In addition, the non-negative weights u_r and v_i are associated with output r and input i, respectively. Let J_j^x and J_j^y denote the set of indices of the interval input and output parameters for

DMU_j, respectively. In real-life problems, it is unlikely for all uncertain data to be equal to their nominal value or values in their intervals. Therefore, it is desirable to be conservative and measure the efficiency score of the DMUs with interval data. Hence, values $\gamma_j^x \in [0, |J_j^x|], j = 1, \ldots, n$ and $\gamma_j^y \in [0, |J_j^y|], j = 1, \ldots, n$ are defined such that J_j^x and J_j^y are the set of indices of the interval input and output parameters for DMU_j, respectively. In addition, γ_j^x and γ_j^y are not necessarily integer-valued.

The parameters γ_j^x and γ_j^y are used to adjust the robustness of the proposed method against the level of conservatism of the solution. Indeed, they impose a budget of uncertainty in the sense that the total (scaled) variation of the parameters cannot exceed some thresholds $|J_j^x|$ and $|J_j^y|$. The variables p_{rj}, q_{ij}, z_j^x and z_j^y are auxiliary variables used to ensure the inputs and outputs can take fixed values in their intervals. The deviations from the benchmarking frontier is defined by ρ_j.

Then, model (7) can be written as follows:

$$
\begin{aligned}
&\eta(\gamma_j^x, \gamma_j^y) = \min \sum_{j=1}^{n} \rho_j \\
&s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \rho_j = 0, && \forall j, && \text{(9a)} \\
&\quad y_{rj}^U - y_{rj} - z_{rj}^y \gamma_j^y - p_{rj} \geq 0, && \forall r, j && \text{(9b)} \\
&\quad y_{rj}^L - y_{rj} \leq 0, && \forall r, j && \text{(9c)} \\
&\quad x_{ij}^L - x_{ij} + z_{ij}^x \gamma_j^x + q_{ij} \leq 0, && \forall i, j && \text{(9d)} \\
&\quad x_{ij}^U - x_{ij} \geq 0, && \forall i, j && \text{(9e)} \qquad \text{(9)} \\
&\quad z_j^y = \sum_{r=1}^{s} z_{rj}^y, && \forall j, && \text{(9f)} \\
&\quad z_j^x = \sum_{i=1}^{m} z_{ij}^x, && \forall j, && \text{(9g)} \\
&\quad z_j^y + p_{rj} \geq y_{rj}^U - y_{rj}^L, && \forall r, j, && \text{(9h)} \\
&\quad z_j^x + q_{ij} \geq x_{ij}^U - x_{ij}^L, && \forall r, j, && \text{(9i)} \\
&\quad x_{ij}, y_{rj}, z_{ij}^x, z_{rj}^y, z_j^x, z_j^y, q_{ij}, p_{rj}, \delta_j \geq 0, && \forall i, r, j, && \text{(9j)} \\
&\quad v_i, u_r \geq \varepsilon, && \forall i, r. && \text{(9k)}
\end{aligned}
$$

In comparison with model (7), model (9) is obtained by considering the variables p_{rj}, q_{ij}, z_j^x, z_j^y and the parameters γ_j^x and γ_j^y. Consequently, the inputs and outputs can take fixed values in their intervals by varying γ_j^x and γ_j^y from zero to $|J_j^x|$ and $|J_j^y|$, respectively. In fact, the efficiency score of DMUs is computed using a CSW and the conservatism level. The objective function of the model (9) minimises the sum of deviations from the efficiencies for all the DMUs. Constraints (9a) guarantee that all the DMUs lie on the same side of the hyperplane and calculate the CSWs in the model in relation to output y. Constraints (9b), (9c), (9f) and (9h) control the level of conservatism in the solution and guarantee that the input and output data are within their interval. According to constraints (9b) and (9c), we have $y_{rj}^L \le y_{rj} \le y_{rj}^U - z_{rj}^y \gamma_j^y - p_{rj}$. By setting $\gamma_j^y = 0$, this relation becomes $y_{rj}^L \le y_{rj} \le y_{rj}^U - p_{rj}$. Because p_{rj} can be equal to zero due to constraints (9f) and (9h), the relation $y_{rj}^L \le y_{rj} \le y_{rj}^U$ can be obtained (i.e., y_{rj}^U can be the maximum value of y_{rj}). Moreover, if $\gamma_j^y = |J_j^y|$, then by using constraints (9i) and (9d), the value of $z_{rj}^y \gamma_j^y + p_{rj}$ increases and becomes equal to y_{rj}^L. In additional, if $\gamma_j^y = |J_j^y| = 1$, then we have $z_{rj}^y + p_{rj} = y_{rj}^U - y_{rj}^L$. Assuming $0 < \gamma_j^y < |J_j^y|$, then model (9) can be used to determine y_{rj}^U as the maximum value of y_{rj} using constraints (9f) and (9h). If there exists some t such that $z_{tj}^y = p_{tj} = 0$ and constraints (9h) are satisfied, then, $z_j^y \neq 0$ and there exists an output of DMU_j, z_{lj}^y, such that $z_{lj}^y \neq 0$ because of constraints (9f). Therefore, we have $z_{lj}^y \gamma_j^y + p_{lj} \neq 0$ and $y_{lj} < y_{lj}^U$ using constraints (9b). Note that the non-zero value of the parameters γ_j^y is imposed on the l^{th} output of DMU_j. Constraints (9d), (9e), (9g), and (9i) have the same interpretation as constraints (9b), (9c), (9f) and (9h).

Theorem 1: Model (9) is always feasible.

Proof: Assuming $z_{ij}^x = z_{rj}^y = z_j^x = z_j^y = 0, q_{ij} = x_{ij}^U - x_{ij}^L, p_{rj} = y_{rj}^U - y_{rj}^L, \rho_j = 0,$ we can obtain $x_{ij} \ge x_{ij}^U$ and $x_{ij} \le x_{ij}^U$ using constraints (9d) and (9e), respectively. Similarly, can obtain $y_{rj} \le y_{rj}^L$ and $y_{rj} \ge y_{rj}^L$ using constraints (9b) and (9c), respectively. Therefore, $x_{ij} = x_{ij}^U$ and $y_{rj} = y_{rj}^L$. By setting $v_{ij} = \dfrac{1}{x_{ij}^U}$ and $u_{rj} = \dfrac{1}{y_{rj}^L}$, the proof is completed. □

Theorem 2: If $\bar{\gamma}_j^x < \tilde{\gamma}_j^x$ and $\bar{\gamma}_j^y < \tilde{\gamma}_j^y$ then $\eta(\bar{\gamma}_j^x, \bar{\gamma}_j^y) \le \eta(\tilde{\gamma}_j^x, \tilde{\gamma}_j^y)$.

Proof: Since constraints (9a), (9c), (9e), (9f), (9g), (9h) and (9i) are independent of $(\bar{\gamma}_j^x, \bar{\gamma}_j^y)$ and $(\tilde{\gamma}_j^x, \tilde{\gamma}_j^y)$, it is sufficient that constraints (9b) and (9c) are considered.

Let $(\bar{u}_{rj}, \bar{v}_{ij}, \bar{z}_j^x, \bar{z}_j^y, \bar{z}_{ij}^x, \bar{z}_{rj}^x, \bar{q}_{ij}, \bar{p}_{rj}, \bar{x}_{ij}, \bar{y}_{rj}, \bar{\rho}_j)$ and $(\tilde{u}_{rj}, \tilde{v}_{ij}, \tilde{z}_j^x, \tilde{z}_j^y, \tilde{z}_{ij}^x, \tilde{z}_{rj}^x, \tilde{q}_{ij}, \tilde{p}_{rj}, \tilde{x}_{ij}, \tilde{y}_{rj}, \tilde{\rho}_j)$ be optimal solutions for $(\bar{\gamma}_j^x, \bar{\gamma}_j^y)$ and $(\tilde{\gamma}_j^x, \tilde{\gamma}_j^y)$, respectively. Then, the relation $y_{rj}^U - \tilde{y}_{rj} - \tilde{z}_{rj}^y \bar{\gamma}_j^y - \tilde{p}_{rj} \ge y_{rj}^U - \tilde{y}_{rj} - \tilde{z}_{rj}^y \tilde{\gamma}_j^y - \tilde{p}_{rj}$ is established because of (9j) and (9k), and $\bar{\gamma}_j^y < \tilde{\gamma}_j^y$. Similarly, we can obtain $x_{ij}^L - \tilde{x}_{ij} + \tilde{z}_{ij}^x \bar{\gamma}_j^x + \tilde{q}_{ij} \le x_{ij}^L - \tilde{x}_{ij} + \tilde{z}_{ij}^x \tilde{\gamma}_j^x + \tilde{q}_{ij}$. Hence, $(\tilde{u}_{rj}, \tilde{v}_{ij}, \tilde{z}_j^x, \tilde{z}_j^y, \tilde{z}_{ij}^x, \tilde{z}_{rj}^x, \tilde{q}_{ij}, \tilde{p}_{rj}, \tilde{x}_{ij}, \tilde{y}_{rj}, \tilde{\rho}_j)$ is a feasible solution for the case $(\bar{\gamma}_j^x, \bar{\gamma}_j^y)$ and $(\tilde{\gamma}_j^x, \tilde{\gamma}_j^y)$. Thus, we can obtain the inequality $\eta(\bar{\gamma}_j^x, \bar{\gamma}_j^y) \le \eta(\tilde{\gamma}_j^x, \tilde{\gamma}_j^y)$. □

Corollary 1: $\eta(\gamma_j^x = |J_j^x|, \gamma_j^y = |J_j^y|) \ge \eta(\gamma_j^x = 0, \gamma_j^y = 0)$

Theorem 3: Let $(u_{rj}^*, v_{ij}^*, z_j^{x^*}, z_j^{y^*}, z_{ij}^{x^*}, z_{rj}^{x^*}, q_{ij}^*, p_{rj}^*, x_{ij}^*, y_{rj}^*, \rho_j^*)$ be the optimal solution of models (9). Then, there is $j \in \{1,\ldots,n\}$ such that $\rho_j^* = 0$.

Proof: By contradiction, suppose $\rho_j^* \neq 0$. Therefore, there is a feasible solution $(\overline{u}_{rj}, v_{ij}^*, z_j^{x^*}, z_j^{y^*}, z_{ij}^{x^*}, z_{rj}^{x^*}, q_{ij}^*, p_{rj}^*, x_{ij}^*, y_{rj}^*, \overline{\rho}_j)$ such that $\overline{u}_{rj} > u_{rj}^*$. Consequently, the relation $\sum_{r=1}^{s} \overline{u}_r y_{rj}^* - \sum_{i=1}^{m} v_i^* x_{ij}^* > \sum_{r=1}^{s} u_r^* y_{rj}^* - \sum_{i=1}^{m} v_i^* x_{ij}^*$ is obtained. Therefore $\overline{\rho}_j$ can be decreased as $(\overline{\rho}_j = 0) < \rho_j^*$. This is a contradiction and completes the proof. □

Theorem 4: The hyperplane obtained from model (9) is supporting.

Proof: We have to demonstrate that all the DMUs lie on the same side of the hyperplane and there is at least one DMU on the hyperplane. Constraints (9a) guarantee that all DMUs lie on the same side of the hyperplane. According to Theorem 3, there is at least one optimal solution $(u_{rj}^*, v_{ij}^*, z_j^{x^*}, z_j^{y^*}, z_{ij}^{x^*}, z_{rj}^{x^*}, q_{ij}^*, p_{rj}^*, x_{ij}^*, y_{rj}^*, \rho_j^*)$ with $\rho_j^* = 0$, ensuring the existence of a DMU on the hyperplane. □

Theorem 5: DMU_j is efficient if and only if $\rho_j^* = 0$ in model (9).

Proof: (If only part): Suppose that DMU_j is efficient. Because of Theorem 4, there is a binding hyperplane. In other words, DMU_j is a boundary point and therefore $\rho_j^* = 0$.

(If part): We prove that if DMU_j is efficient, then $\rho_j^* = 0$ in model (9).

In order to prove this, we can prove its contrapositive, that if DMU_j is inefficient then $\rho_j^* \neq 0$ in model (9).

Assume that DMU_j is inefficient and therefore an interior point. Thus there is no binding hyperplane in DMU_j implying that $\rho_j^* \neq 0$. □

We can calculate the efficiency scores of DMU_j. Assuming $(u_r^*, v_i^*, z_j^{x^*}, z_j^{y^*}, z_{ij}^{x^*}, z_{rj}^{x^*}, q_{ij}^*, p_{rj}^*, x_{ij}^*, y_{rj}^*, \rho_j^*)$ to be the optimal solution of model (9), then the efficiency scores of DMU_j can be obtained as follows:

$$E_j = \frac{\sum_{r=1}^{s} u_r y_r^*}{\sum_{i=1}^{m} v_i x_i^*} = 1 - \frac{\rho_j^*}{\sum_{i=1}^{m} v_i x_i^*}, \qquad \forall j. \tag{10}$$

Corollary 2: DMU_j is efficient if and only if $E_j = 1$ in equation (10).

Each DMU can be categorised as either efficient or inefficient because model (9) presents different values for all inputs and outputs using Γ variation, $\Gamma(= \gamma^x + \gamma^y)$. Hence, a set of efficient DMUs are introduced to identify the confidence criterion of inefficient DMUs belonging to the efficient set. Note that some DMUs may be efficient for all conservatism levels and the others may be inefficient. Moreover, there may exist DMUs that are efficient and inefficient corresponding to their conservatism level. Therefore, the definition of this set is important. Let Γ_e be a conservatism level in which a DMU assumes an efficiency score less than unity. When a DMU is inefficient for Γ_e, it cannot be efficient for Γ greater than Γ because of Theorem 2. All DMUs that are efficient for all Γ's have a confidence criterion of 100%. It is evident that the confidence criterion of DMUs is decreased when their efficiency scores are reduced from unity. The confidence criterion of the DMUs can be calculated by the relation $\frac{\Gamma_e}{|J|} \times 100$.

4 Numerical results

In this section, we demonstrate the practical aspects of the proposed model with an example and a real-life problem. In the first example, a problem is considered with five DMUs, one interval input, and one interval output. The interval input and output data are given in Table 1 and Figure 1.

Figure 1 Input and output data for five DMUS (see online version for colours)

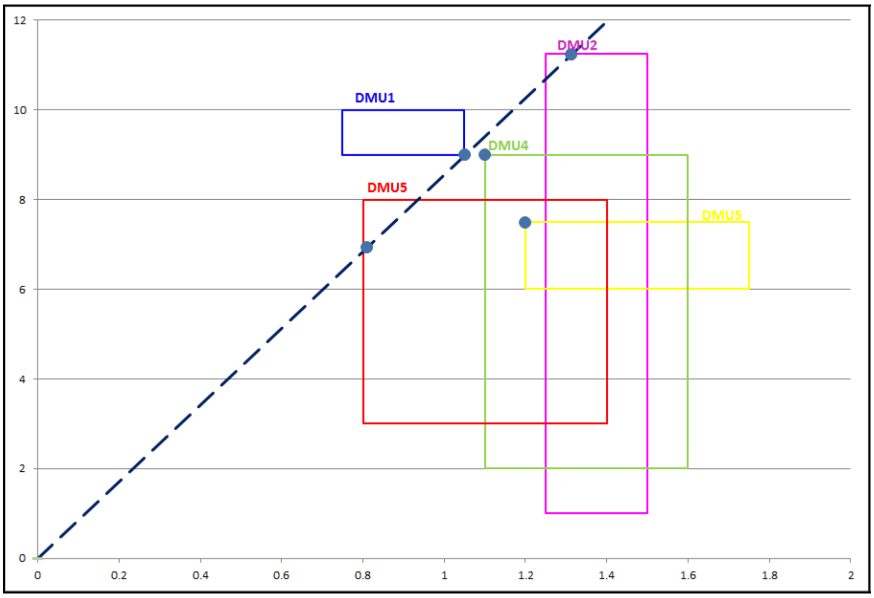


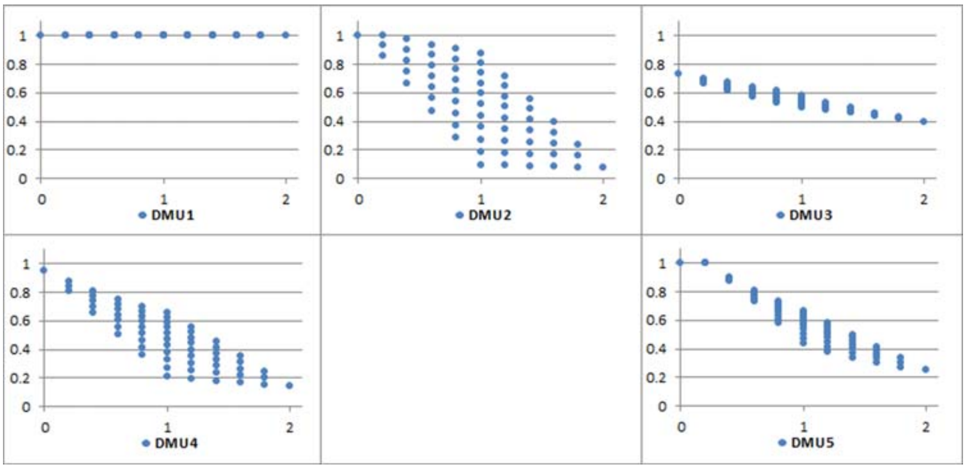
Table 1 Input and output data for five DMUs

DMU_j	x	y
1	[0.75, 1.05]	[9.00, 10.00]
2	[1.25, 1.50]	[1.00, 11.30]
3	[1.20, 1.75]	[6.00, 7.50]
4	[1.10, 1.60]	[2.00, 9.00]
5	[0.80, 1.40]	[3.00, 8.00]

As previously mentioned, when $\Gamma = 0$, model (9) can take the input and output values for each DMU in the associated intervals in such a way that the sum of efficiency scores of the DMUs is maximised. As seen in Figure 1, DMUs 1, 2, and 5 obtain the input and output values in such a way that the sum of the efficiency scores is maximised and these DMUs remain efficient. All inputs and outputs take their lowest boundary value when has Γ its minimum value. However, as the value of Γ is between 0 and 2, the model can compute the corresponding input and output values.

Model (9) was run for different combinations of $\gamma_j^x (= \gamma^x)$ and $\gamma_j^y (= \gamma^y)$, and a fixed $\Gamma (= \gamma^x + \gamma^y)$, using the generalised algebraic modelling system (GAMS) and $\varepsilon = 10^{-6}$. As mentioned before, the parameters γ_j^x and γ_j^y (and consequently parameter $\Gamma = \gamma^x + \gamma^y$) are intended to adjust the robustness of the proposed method against the level of conservatism of the solution. Bertsimas and Sim (2004) have shown that it is sufficient to choose γ_j^x and γ_j^y at least equal to $1 + \Phi^{-1}(1-\theta)\sqrt{n_i}$, where Φ is the cumulative distribution function of the standard normal variable and n_i is the number of uncertain data. In each case, the efficiency scores were obtained for the five DMUs. The efficiency scores of these DMUs are shown in Figure 2 for all possible γ^x and γ^y such that $\Gamma = \gamma^x + \gamma^y$. It is clear that there are many γ^x and γ^y so that $\Gamma = \gamma^x + \gamma^y$ where $0 < \Gamma < 2$. Therefore, the efficiency scores of each DMU were calculated for all possible cases.

Figure 2 Efficiency scores of five DMUS (see online version for colours)



In the second example, the model proposed in this study was used to solve the real-life performance measurement problem in Jahanshahloo et al. (2009). Jahanshahloo et al. (2009) studied the performance of 14 commercial bank branches in Iran. Each branch uses three inputs in order to produce five outputs. The inputs and outputs data for these DMUs are given in Tables 2 and 3, respectively.

Table 2 Input data for 14 bank branches

DMU_j	*1*		*2*		*3*	
	Lower	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
1	587.69	1,205.47	27.29	27.48	27,827	29,005
2	4,646.39	9,559.61	24.52	25.07	9,070	9,983
3	1,554.29	3,427.89	20.47	21.59	412,036	413,902
4	17,528.31	36,297.54	14.84	15.05	8,638	10,229
5	2,444.34	4,955.78	20.42	20.54	500	937
6	7,303.27	14,178.11	22.87	23.19	16,148	21,353
7	9,852.15	19,742.89	18.47	21.83	17,163	17,290
8	4,540.75	9,312.24	22.83	23.96	17,918	17,964
9	3,039.58	6,304.01	39.32	39.86	51,582	55,136
10	6,585.81	13,453.58	25.57	26.52	20,975	23,992
11	4,209.18	8,603.79	27.59	27.95	41,960	43,103
12	1,015.52	2,037.82	13.63	13.93	18,641	19,354
13	5,800.38	11,875.39	27.12	27.26	19,500	19,569
14	1,445.68	2,922.15	28.96	28.96	31,700	32,061

Model (9) was run for different combinations of γ_j^x and γ_j^y, assuming a fixed Γ value and $\varepsilon = 10^{-8}$. In each case, the efficiency scores were obtained for the 14 DMUs. The efficiency scores of all the DMUs are shown in Figure 3 for all possible γ_j^x and γ_j^y. The value of Γ lies between [0,8] and increases by 0.2 in each step. The efficiency score of the branches is computed for all γ^x and γ^y such that $\Gamma = \gamma^x + \gamma^y$. It is evident that DMUs 1, 3, 4 and 11 are efficient for all Γ's; however, there are also DMUs that are inefficient for all Γ's (i.e., DMUs 6, 7, and 11). In addition, some DMUs have become inefficient when Γ is increased. DMUs 1, 3, 4, and 11 belong to the efficient set with the confidence criterion that is equal to 100%. Note that the confidence criterion of the inefficient DMUs belonging to the efficient set was increased when Γ_e becomes larger.

To evaluate the performance of the proposed model, the results obtained from the proposed method are compared with models presented by Kao and Hung (2005), Liu and Peng (2008), and Omrani (2013). Kao and Hung (2005) proposed the concept of compromise solution and obtained an ideal solution by using standard DEA models and generating CSWs based on the vector of efficiency score closet to the ideal solution. Omrani (2013) introduced a robust optimisation approach to find the CSWs in DEA with uncertain data. In this study, we consider a numerical example used by Kao and Hung (2005). In this example, there are 17 forest districts; four inputs: budget (in US dollars), initial stocking (in cubic metres), labour (in number of employees), land (in hectares); and three outputs: main product (in cubic metres), soil conservation (in cubic metres), and recreation (in number of visits). The data are shown in Table 4.

Table 3 Output data for 14 bank branches

DMU_j	1		2		3		4		5	
	Lower	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*	*Lower*	*Upper*
1	144,906	165,818	180,530	180,617	288,513	323,721	40,507.97	45,847.48	176.58	370.81
2	408,163	416,416	405,396	486,431	1,044,221	1,071,812	56,260.09	73,948.09	4,654.71	5,882.53
3	335,070	410,427	337,971	449,336	1,584,722	1,802,942	176,436.8	189,006.1	560.26	2,506.67
4	700,842	768,593	14,378	15,192	2,290,745	2,573,512	662,725.2	791,463.1	58.89	86.86
5	641,680	696,338	114,183	241,081	1,579,961	2,285,079	17,527.58	20,773.91	1,070.81	2,283.08
6	453,170	481,943	27,196	29,553	245,726	275,717	35,757.83	42,790.14	375.07	559.85
7	553,167	574,989	21,298	23,043	425,886	431,815	45,652.24	50,255.75	438.43	836.82
8	309,670	342,598	20,168	26,172	124,188	126,930	8,143.79	11,948.04	936.62	1,468.45
9	286,149	317,186	149,183	270,708	787,959	810,088	106,798.6	111,962.3	1,203.79	4,335.24
10	321,435	347,848	66,169	80,453	360,880	379,488	89,971.47	165,524.2	200.36	399.8
11	618,105	835,839	244,250	404,579	9,136,507	9,136,507	33,036.79	41,826.51	2,781.24	4,555.42
12	248,125	320,974	3,063	6,330	26,687	29,173	9,525.6	10,877.78	240.04	274.7
13	640,890	679,916	490,508	684,372	2,946,797	3,985,900	66,097.16	95,329.87	961.56	1,914.25
14	119,948	120,208	14,943	17,495	297,674	308,012	21,991.53	27,934.19	282.73	471.22

Figure 3 Efficiency scores of 14 DMUS (see online version for colours)

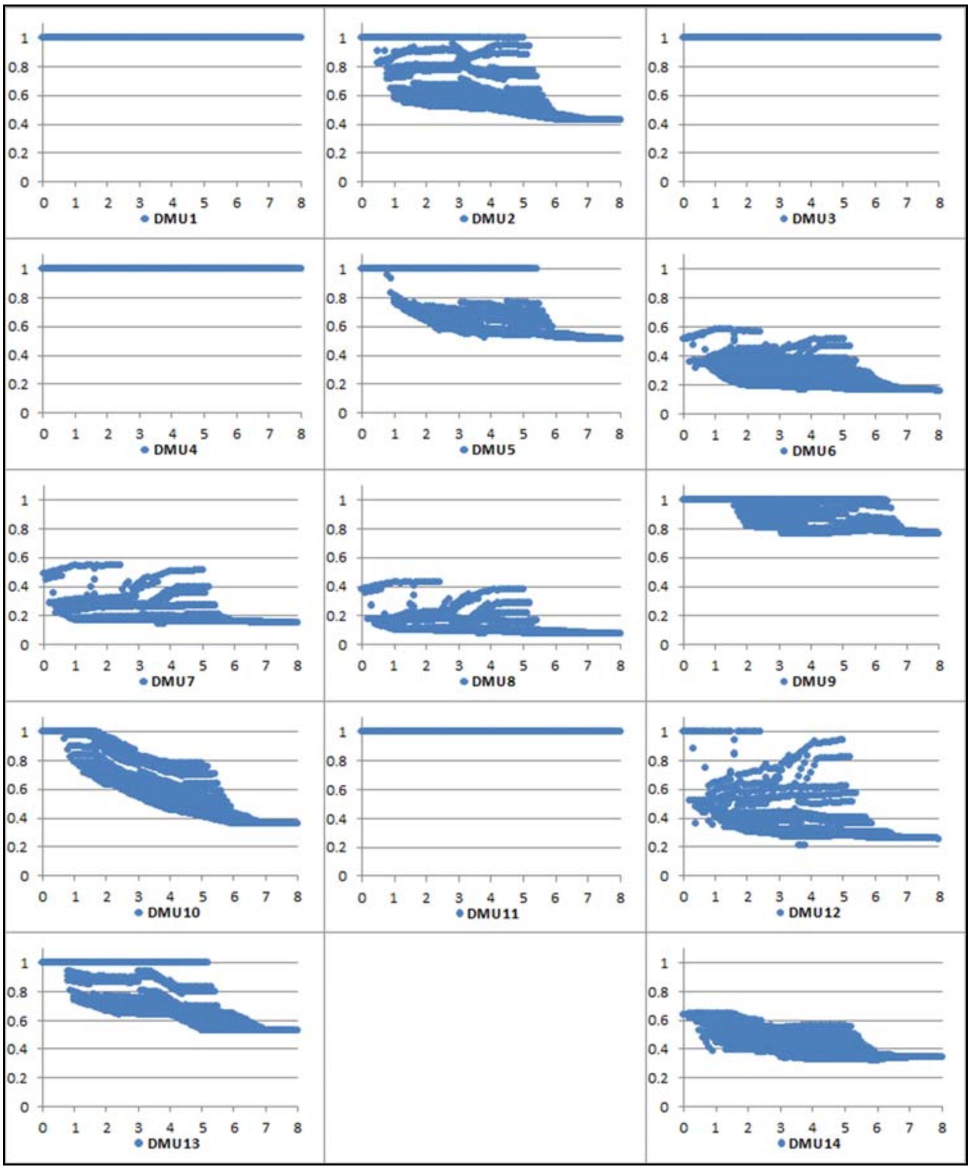


Table 4 Kao and Hung's (2005) data

DMU	*Input*				*Output*		
	Budget (US dollars)	*Initial stocking (m^3)*	*Labour (employees)*	*Land (hectares)*	*Main product (m^3)*	*Soil conservation (m^3)*	*Recreation (visits)*
1	51.62	11.23	49.22	33.52	40.49	14.89	3,166.71
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
3	66.65	104.18	257.09	13.65	139.74	115.96	0

Table 4 Kao and Hung's (2005) data (continued)

DMU	Input				Output		
	Budget (US dollars)	*Initial stocking (m^3)*	*Labour (employees)*	*Land (hectares)*	*Main product (m^3)*	*Soil conservation (m^3)*	*Recreation (visits)*
4	27.87	107.6	14	146.43	25.47	131.79	0
5	51.28	117.51	32.07	84.5	46.2	144.99	0
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
7	25.83	105.8	9.51	227.2	19.4	120.09	0
8	123.02	82.44	87.35	98.8	43.33	125.84	404.69
9	61.95	99.77	33	86.37	45.43	79.6	1252.6
10	80.33	104.65	53.3	79.06	27.28	132.49	42.67
11	205.92	183.49	144.16	59.66	14.09	196.29	16.15
12	82.09	104.94	46.51	127.28	44.87	108.53	0
13	202.21	187.74	149.39	93.65	44.97	184.77	0
14	67.55	82.83	44.37	60.85	26.04	85	23.95
15	72.6	132.73	44.67	173.48	5.55	135.65	24.13
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

It is supposed that the input and output data values lie within 5% of the nominal value. For example, the value 4 is in the interval [3.80–4.20]. To analyse the sensitivity of the solution, the values of γ_j^x and γ_j^y are set to 1 and 3. The results of the models CCR (Mosel 4), Kao and Hung's (2005) model, Liu and Peng's (2008) model, Omrani's (2013) model and our model (9) are shown in Table 5. The CCR-efficiency scores of DMUs calculated from the CCR ratio model (4) are shown in the first column of Table 5. These scores are considered as the ideal solutions because of their highest attainable values. There are nine efficient units, which cannot be differentiated. The second column of Table 5 shows the efficiency scores calculated from Kao and Hung's (2005) model. Here, the Euclidean distance is taken into consideration in Kao and Hung's (2005) model. The third column of Table 5 presents the results obtained from Omrani's (2013) model in which Γ is the total of the uncertain parameters. The results obtained from the proposed model are shown in the last two columns in Table 5.

As expected, the CCR model has the largest average efficiency score of 0.91, while other models have smaller values. The proposed model also has the smallest variance for the efficiency deviations of the DMUs. The calculated efficiency scores of the proposed method are suitable for ranking of the DMUs. In general, the rankings of the DMUs in model (9), as shown in Table 5, are consistent with those of the CCR model, indicating that the results are reasonable. Moreover, the efficiency scores reported by model (9) are more informative because they not only differentiate the efficient units, but also identify some abnormal efficiency scores calculated from the CCR model. As shown in Table 5, when the values of the parameters γ_j^x and γ_j^y increase, the values of efficiency scores will decrease because by increasing the parameters γ_j^x and γ_j^y, the protection level against violation is increased and less value is obtained from the objective function. These results confirm conclusions obtained by Bertsimas and Sim (2004) and Omrani (2013). Moreover, the average efficiency score of Omrani's (2013) model is no larger than the proposed model, while the variance efficiency score of our model is less that Omrani's (2013) model.

Table 5 Comparative CSWs results

DMU	*CCR model*	*Kao and Hung's (2005) model*	*Liu and Peng's (2008) model*	*Omrani's (2013) model* Γ = 4	*Model (9)*	
					$\gamma_j^x = \gamma_j^y = 1$	$\gamma_j^x = \gamma_j^y = 3$
1	1	1	1	0.9637	0.9857	0.9138
2	1	1	1	0.9482	0.9640	0.9138
3	1	0.9989	1	0.9797	0.9900	0.9229
4	1	0.9927	1	0.9362	0.9636	0.8904
5	1	0.9866	0.9876	0.9578	0.9838	0.9048
6	1	0.9123	0.9305	0.9499	0.9404	0.9229
7	1	0.8849	0.8937	0.8072	0.8421	0.7564
8	1	0.8707	0.8794	0.8262	0.8415	0.7722
9	1	0.669	0.6824	0.6495	0.8629	0.8081
10	0.9403	0.8768	0.8856	0.8517	0.6560	0.6238
11	0.9346	0.6518	0.6583	0.6463	0.6470	0.6150
12	0.829	0.7282	0.7428	0.6820	0.6926	0.6513
13	0.7997	0.626	0.6323	0.6074	0.6122	0.5793
14	0.7733	0.7142	0.7285	0.6973	0.7008	0.6608
15	0.7627	0.721	0.7282	0.6866	0.7013	0.6523
16	0.7435	0.6811	0.6879	0.6289	0.6449	0.6006
17	0.6873	0.6068	0.6189	0.5604	0.5728	0.5315
Average	0.9100	0.8189	0.8268	0.7870	0.8001	0.7482
Variance	0.0132	0.0222	0.0215	0.0222	0.0232	0.0201

5 Conclusions and future research directions

In this study we analysed the efficiency score of the DMUs in performance measurement problems with interval inputs and outputs using the CSW approach. We used GP and constructed a feasible model where exact values for the inputs and outputs were selected from their interval and the CSW and conservatism level concept were used to maximise the efficiency score of the DMUs. The proposed method uses a finite number of points that are representatives of the intervals. The GP method and the varying conservatism levels concept are used to rank order the DMUs. There are obviously DMUs that achieve the efficiency score of unity for all conservatism level values. In contrast, there are DMUs that are inefficient for all conservatism level values. There are also DMUs that have both efficient and inefficient performance for various conservatism level values. These circumstances indicate the necessity of identifying a set of efficient DMUs through a confidence criterion. As for future research directions, we are considering two extensions of the method proposed in this study. In the first case, we are studying a robust DEA model with uncertain data where the profit Malmquist index is used to rank the DMUs with respect to the overall profit efficiency. In the second case, we are considering the ellipsoidal uncertainty set to represent the uncertain parameters in the proposed model.

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