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A knowledge-driven pricing model for supply chain coordination under correlated demand

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ABSTRACT

The growing competition between national brands and store brands has intensified the need for decision-support systems to guide pricing and coordination strategies in retail supply chains. This study develops an analytical game-theoretic framework that integrates stochastic demand modeling, behavioral customer preferences, and computational optimization for dual-brand supply chains. The system models a two-echelon structure consisting of a national-brand manufacturer and a retailer who also produces a substitutable store brand, acting simultaneously as a collaborator and a competitor. Customer demand for both products is correlated and influenced by relative price and quality, introducing substantial uncertainty. A Stackelberg game framework is employed to capture the hierarchical decision-making process, in which the manufacturer sets the wholesale price, and the retailer optimizes retail prices for both products. Analytical and simulation-based evaluations reveal that conventional revenue-sharing contracts fail to coordinate the supply chain when demand correlation and substitution effects are present. To address this limitation, a two-part tariff contract is designed within the analytical framework to align incentives and achieve coordination. The results demonstrate that the proposed approach enhances profitability, system efficiency, and brand competitiveness, providing actionable insights for decision-makers in retail, apparel, and fast-food industries.

1. Introduction

The rapid growth of store brands (SBs), also known as private labels, has significantly reshaped the competitive dynamics faced by national brands (NBs) in retail markets. Leading retailers such as Walmart, Kroger, and Costco have steadily expanded their SB offerings across a broad range of product categories, driven by the pursuit of higher profit margins, enhanced bargaining power, and increased consumer loyalty (Ailawadi et al., 2008; Kuo & Yang, 2013). Recent evidence indicates that SB penetration continues to rise globally. A comprehensive study covering over 50 countries and 2,000 product categories reveals consistent private-label growth, with market convergence indicating further expansion even in mature markets and a concurrent decline in NB market share, intensifying competitive pressures on established brands (Van Cuneo et al., 2023).

The success of SB products is driven not only by cost advantages but also by shifting consumer preferences. Contemporary consumers increasingly perceive SB products as viable substitutes for NB, particularly when SB offerings offer competitive quality at substantially lower prices (Lamey et al., 2012; Ailawadi & Keller, 2004). As a result, the competition between SB and NB products has emerged as a central topic in operations and marketing research, with particular emphasis on pricing strategies, supply chain coordination, and consumer behavior under varying market conditions. However, much of the existing literature relies on assumptions of deterministic or independent demand for competing products. In practice, the demands for SB and NB products are often correlated and subject to considerable uncertainty. Variations in customer preferences, price sensitivity, and perceived product quality can significantly alter market dynamics (Pauwels & Srinivasan, 2004; Chen et al., 2015). These correlations suggest that an increase in demand

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for one product often leads to a decrease in demand for the other, thereby intensifying the strategic interplay between supply chain participants.

In this context, retailers face complex strategic decisions. They may opt to (i) exclusively sell the NB product, (ii) exclusively produce and sell the SB product, or (iii) offer both products simultaneously. Each option involves distinct decision variables, such as order quantities, production levels, and retail prices, all of which are further complicated by uncertainty in correlated market demand. Moreover, when retailers assume a dual role as both collaborators and competitors to NB manufacturers, conflicts may emerge that hinder effective supply chain coordination and reduce overall profitability (Zhang, 2008; Geylani et al., 2007). Existing studies have rarely examined how demand uncertainty and brand substitution jointly influence retailers' strategic choices in such dual-brand supply chains.

While revenue-sharing contracts (RSCs) are widely studied as mechanisms for aligning incentives within supply chains (Cachon & Lariviere, 2005), they often fall short in contexts where retailers also produce competing SB products. There is a need for more sophisticated contractual designs that can accommodate correlated demand uncertainty and retailer competition with NB manufacturers.

This paper aims to address the following research questions:

- What is the retailer's optimal strategy in a competitive environment marked by correlated demand uncertainty and heterogeneous customer preferences?
- How do the pricing policies of the NB manufacturer affect retailer decisions and shape customer preferences?
- How can the NB manufacturer design contractual agreements to influence retailer behavior and achieve effective supply chain coordination?
- What impact does a two-part tariff revenue-sharing contract have on supply chain performance and coordination?

To answer these questions, we develop a Stackelberg game-theoretic model for a two-echelon supply chain comprising an NB manufacturer and a retailer that also produces an SB product. We explicitly model stochastic, correlated demand influenced by customer preferences and product substitutability. Our analysis demonstrates that a simple revenue-sharing contract cannot coordinate the supply chain under these conditions. Consequently, we propose a two-part tariff revenue-sharing contract that effectively aligns incentives, enhances supply chain coordination, and improves overall profitability.

This study contributes to the literature by integrating customer preference dynamics, correlated demand uncertainty, and contract design into the analysis of dual-brand competition, providing new insights for both academics and practitioners in supply chain management.

2. Literature review

The literature review is structured into three subsections: (2.1) Pricing Competition, (2.2) Stochastic and Correlated Demand, and (2.3) Supply Chain Coordination, as outlined below.

2.1. Pricing competition

Pricing strategies in NB–SB competition are critical given the products' substitutability, where demand for one brand is often influenced by the price and perceived quality of the other (Abedsoltan et al., 2022). An initial body of work has employed experimental and conceptual approaches to examine how pricing and quality affect competition between NB manufacturers and SB retailers (Choi & Fredj, 2013). Early contributions include Connor and Peterson (1992), Hoch (1996), Grewal et al. (1998), and Hoch and Banerjee (1993), who investigated the pricing dynamics and consumer perception of SB and NB products.

Grewal et al. (1998) developed a conceptual pricing model for both SB and NB products, analyzing customer behavior and purchasing policies in competitive settings. Additional studies, such as those by Kurata et al. (2007), Karray and Martín-Herrán (2009), Choi and Fredj (2013), Kotani and Sumita (2013), and Fang et al. (2013), have built upon this foundation. Kurata et al. (2007), Choi and Fredj (2013), and Kotani and Sumita (2013) developed models to determine optimal pricing strategies for competing brands by considering how demand for each product depends not only on its price but also on the prices of competing products, reflecting the substitution effect where customers may switch brands in response to relative price differences.

Chen and Cui (2013) focused on improving the profitability of competitive supply chains by integrating customer utility and implementing price discrimination across brands. Similarly, Ma et al. (2013) analyzed pricing decisions that account for the impact of both manufacturer and retailer marketing efforts on product quality perceptions. In recent years, there has been renewed interest in the competitive dynamics between NB and SB, particularly amid rising retailer power and evolving consumer perceptions. Contemporary research highlights that SBs have progressed beyond their traditional role as low-price alternatives, increasingly entering premium and niche market segments (Gielens et al., 2021; Cheng & Hsu, 2023). Han et al. (2022) further show that online reviews and digital transparency have narrowed the perception gap between NB and SB products, intensifying price competition. Chakraborty (2022) examined quality competition between national and SBs, showing how quality positioning interacts with pricing to shape market shares.

Moreover, retailers are increasingly using data analytics to fine-tune their private label offerings (Kim et al., 2021). Game-theoretic approaches remain central to understanding SB–NB interactions. Recent works emphasize Stackelberg or Bertrand structures in this context. Luo et al. (2021) model the Stackelberg competition between NB manufacturers and SB retailers, considering the retailer's dual role and its impact on wholesale pricing. Their findings reinforce our insight that traditional contracts may fall short in coordinating asymmetric supply chain relationships. Liu et al. (2021) investigated a manufacturer's contract choice in the presence of competing online retail platforms, revealing how the contract form, wholesale versus agency, endogenously shifts pricing power and downstream competition. Wu and Wang (2023) used a Stackelberg game analysis of private-label supply chains under demand uncertainty to find that two-part tariffs outperform simple revenue-sharing contracts in aligning incentives. This outcome directly echoes our contribution. Huang and Zhang (2023) examined Stackelberg game dynamics in a competitive retail setting, highlighting the NB manufacturer's leadership role in setting prices but not addressing SB production by retailers.

2.2. Stochastic and correlated demand

The assumption of correlated and stochastic demand is increasingly prominent in modern supply chain modeling. Recent literature shows that demand correlation significantly affects pricing, inventory, and contract effectiveness. Karray & Martín-Herrán (2009), Fang et al. (2013), and Choi and Fredj (2013) developed deterministic models under uncertainty by applying stochastic demand rates in multi-brand competition models. Yang et al. (2010) focused on a model in which the demand rate is extended based on the customer's utility, which is dependent on the manufacturer's rebate and the retail price. Zhao et al. (2012) investigated a pricing problem in which the customer's utility is a linear function of the selling price and lead time. Neslin et al. (2014) extended a mathematical model for a supply chain based on customers' utility and studied the relationships between the brands and distribution channels. Fang et al. (2013) went one step further and defined a new random demand function based on customer utility, depending on the selling price and the product's quality level. Essentially, consumers prefer to buy a brand that meets their expectations and delivers greater value (Sethuraman & Gielens, 2014). This value can be gained from a lower price or arises from the brand's high utility, such as its higher quality. Noori-daryan et al. (2020) modeled advance-booking pricing in O2O commerce with demand leakage, illustrating how uncertain and diverted demand alters optimal price commitments. Fang et al. (2013) analyzed how the NB manufacturer can affect the market share by decreasing the selling price and improving the quality level.

Lin et al. (2020) examined correlated demand in competing product markets, concluding that neglecting such correlations leads to suboptimal decisions. Similarly, Zhao et al. (2022) analyzed stochastic demand correlation in dual-channel supply chains, showing it affects both pricing and the value of flexibility. Zhang et al. (2021) analyzed a dual-channel supply chain with NB and SB products, demonstrating that correlated demand significantly affects pricing decisions, with NB manufacturers adjusting prices to counter SB competition. Similarly, Li et al. (2022) developed a game-theoretic model to study pricing dynamics in a market with substitutable NB and SB products, finding that demand correlation amplifies price sensitivity and impacts retailer profitability. Karabağ et al. (2023) integrated pricing, manufacturing, and capacity decisions under correlated supply–demand processes, showing that pricing responsiveness declines as the degree of correlation increases. However, these studies assume simultaneous pricing decisions, overlooking the hierarchical decision-making structure often observed in NB-SB supply chains. Sharma et al. (2022) investigated private label quality investment under uncertain demand, finding that high substitutability between SB and NB intensifies channel conflict.

Additionally, Gong et al. (2023) showed that SB proliferation can reduce overall channel profits unless carefully coordinated. Chen and Wang (2023) extended a model by incorporating stochastic demand in an NB-SB competition model, showing that demand uncertainty exacerbates pricing conflicts but does not address coordination mechanisms to mitigate these issues. Koren et al. (2024) developed inventory models for substitutable products with stockout-based substitution, providing insights into demand uncertainty that map directly to price responses under correlated substitution.

2.3. Supply chain coordination

Supply chain coordination is essential to align the objectives of NB manufacturers and SB retailers, particularly when their products compete. Pal et al. (2015) studied the coordination of a two-level supply chain where the demand rate depends on the selling price and exhibits a non-linear relationship. They used three contracts: wholesale price discount, promotional cost-sharing, and RSC. They then compared the proposed contracts. Fang et al. (2013) found that simple revenue-sharing, quantity flexibility, sales rebates, and buy-back contracts cannot effectively coordinate the supply chain; therefore, they ultimately employed a minimum order quantity contract. Li and Liu (2015) considered a supply chain with a non-linear demand function dependent on sailing effort and selling price. They coordinated the model using a cost-sharing contract, a wholesale price contract, and a two-part tariff contract. El Ouardighi (2014) attempted to coordinate the supply chain through revenue-cost sharing and RSCs, while demand depends on the retail price and quality level. Related works can be found in Ma et al. (2013), Sarkar et al. (2013), Yin and Nishi (2014), and Sarkar et al. (2014).

Xu et al. (2021) analyzed RSCs in a supply chain with competing retailers, finding that simple RSCs improve coordination but fail when demand is highly correlated due to double marginalization. Wang et al. (2021) showed that a revenue-sharing contract may fail when retailers control competing SB products, suggesting the need for more sophisticated contracts. Bart et al. (2021) provided a comprehensive review of revenue-sharing contracts, identifying the conditions under which RSCs coordinate channel decisions and when hybrid or two-part tariffs are superior—findings that complement our contract design. Several researchers propose two-part tariffs or hybrid contracts for better coordination. Wang et al. (2021) investigated coordination in a dual-brand supply chain using a Stackelberg game in which the NB manufacturer acts as the leader. Their findings suggest that traditional contracts, such as wholesale price contracts, fail to achieve coordination due to conflicting incentives. Li et al. (2022) demonstrated that two-part contracts outperform simple RSCs in dual-brand scenarios by allowing more flexibility in profit sharing and risk allocation. Liu et al. (2022) explored coordination in an NB-SB supply chain with correlated demand and proposed a quantity-discount contract to align the retailers' and manufacturers' goals. However, their model assumes that the retailer sells only the NB product, neglecting scenarios in which the retailer also produces an SB product. Zhao et al. (2022) proposed a modified RSC to coordinate a supply chain with substitutable products, but their model focused on manufacturer-manufacturer competition rather than NB-SB dynamics. More recently, Yang and Zhou (2024) explored a two-part tariff contract in a multi-channel supply chain, demonstrating its effectiveness in coordinating pricing and inventory decisions under demand uncertainty. However, their study does not account for the retailer's dual role as both a seller of NB products and a producer of SB products, a critical aspect of our research.

According to the literature on coordination contracts, we observe that the impact of competitor policies, including product quality levels and selling prices, on customers' utility and preferences or behaviors under uncertainty, as well as on competitors' profits, is not well studied. Therefore, this study, to fill the aforementioned research gap, incorporates a random demand rate that depends on the selling prices and quality levels of both competitors' products. We employed a utility function to include the effects of consumers' behavior and preferences on the decision variables. Moreover, an RSC is applied to coordinate the supply chain effectively. According to the study performed by Fang et al. (2013), traditional revenue sharing cannot coordinate this supply chain. Therefore, in this research, a two-part tariff revenue-sharing contract is designed, and we find that this contract can have a considerable effect on profits.

The rest of the study is divided into six sections. The problem is described in Section 2. In Section 3, a mathematical model is presented for both NB and SB manufacturers, along with a solution method based on Stackelberg game theory. The coordination contract is given in Section 4. Numerical analysis is provided in Section 5. Finally, the conclusion and suggested directions for future research are summarized in Section 6.

3. Problem description

Consider a market in which one NB and one SB compete to sell two substitute products. Therefore, an increase in demand for one product leads to a decrease in demand for its substitute, and vice versa. Indeed, we consider a market with two similar products: one NB with a higher price and quality, and an SB with a lower price and quality.

Therefore, we consider a probability distribution function for the demand rate, which is a random variable and depends on the selling price and quality levels of both competitors' products. In this scenario, the retailer must decide on how to handle uncertainty.

So, under this uncertainty, the retailer must decide whether to use the appropriate policies, including: selling only the NB product, selling only his traditional product (SB-product), or offering both products simultaneously. The retailer needs to determine different decision variables based on selecting one of the described policies. If the retailer sells only NB products, then the order quantity of NB is Q_n. Suppose he chooses both NB- and SB-products. In that case, the retailer must determine the NB's order quantity, the production quantities of SB, Q_s, and the SB's retail prices, P_s. However, if the retailer decides to sell the SB product, meaning selling his substitute products, then only he has to determine the optimal values of P_s and Q_s. Surely, a retailer's decision about selecting an appropriate policy depends on the NB wholesale price set by the NB manufacturer. Fig. 1 shows the supply chain structure and

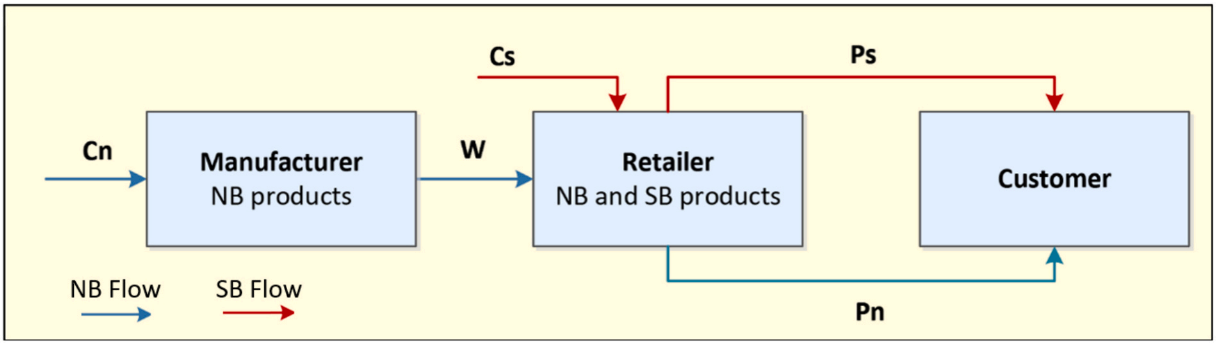

Fig. 1. A competitive chain including an NB manufacturer and an SB retailer.

the relationships among its members across different scenarios.

In Fig. 1, the blue flow represents the scenario where the retailer chooses to present only the NB-product. The red flow represents the scenario where the retailer decides to show only the SB product. The combination of red and blue flows represents the scenario in which the retailer adopts a coordinated policy of offering both products simultaneously. A detailed comparative analysis among these three policies, along with the identification of the optimal policy for the retailer, is subsequently provided and discussed in depth in Sections 4 and 5.

The Stackelberg game theory is applied to show the strategic relationship between the manufacturer and the retailer. In our model, the manufacturer serves as the leader, and the retailer acts as the follower. Therefore, the retailer must choose its strategy and optimal decision variables given the manufacturer's wholesale price. In this paper, we aim to adjust a contract that not only coordinates the supply chain and improves members' expected profits but also influences the retailer's decision-making process and increases the retailer's tendency to select NB products. Therefore, we adjust a two-part RSC tariff that allocates the benefit among the members of the supply chain. Moreover, the following assumptions are made.

- A two-level supply chain including one retailer and one manufacturer is mentioned.
- The problem is modeled in a single-period horizon planning.
- The retailer can produce the goods with a private label.
- The quality level of the NB product is higher than that of the SB product.
- The selling price of the NB product is more than that of the SB product.
- The quality value of each product from the customers' point of view is more than the selling price offered by the manufacturer.
- The price elasticity of demand for SB is greater than that of NB.

The following notations are used to model the problem.

Parameters

θ	The parameter to measure the intensity of a consumer's taste for quality.
N	The total market size.
q_n	The quality level of the NB product.
q_s	The quality level of the SB product
P_n	The selling price of the NB product.
C_n	The production cost for the NB product.
C_s	The production cost for the SB product.
γ_n	The price elasticity of the NB demand.
γ_s	The price elasticity of the SB demand.
$U(\theta)$	The utility function of customers.
x	The random variable is the number of potential customers.

Independent Decision Variables

Q_n	The production quantity of NB.
Q_s	The production quantity of SB.
P_s	The SB's retail price.
W	The NB's wholesale price.

Dependent Variables

π_T	The expected total profit of the system.
π_r	The expected profit of the retailer.
π_n	The expected profit of the supplier.

4. Modeling and solution method

Customers' purchasing decisions depend on the quality level and the selling price of the items. We assume product quality levels are exogenous parameters because quality determination is usually a long-term strategic decision that requires significant infrastructure investment, which may not be possible for an SB to change dynamically, and can be influenced by product category, which falls outside the operational time horizon of our model, focusing on short-term pricing decisions under uncertainty. Customers only purchase products that are profitable for them or that maximize their utility. Therefore, we assume that $U(\theta) = \theta q - p$ is a utility function applied by customers to buy a product, where θ is a parameter that shows the intensity of a consumer's taste concerning the quality level of the product. We assume that it has a uniform probability distribution function (PDF) within [0,1] (Moorthy, 1988). This function means that when a purchase is made, the $U(\theta)$ is positive. Therefore, $\theta > P/q$. In other words, $\theta = 1 - P/q$ is the tendency of each customer to buy the product, so $(1 - P/q)x$ is the part of the people who are willing to buy, where x has a uniform PDF within $[0, N]$ (see Tirole (1988)). Therefore, if the retailer decides to present only the SB-product, $(1 - \gamma_s p_s/q_s)x$ is the demand rate of the SB-product. Thus, the demand quantity of the SB product has a uniform PDF within $[0, (1 - \gamma_s P_s/q_s)N]$. If the retailer chooses only the NB-product, the demand quantity is equal to $(1 - \gamma_n P_n/q_n)x$, in result, the NB product has a uniform PDF within $[0, (1 - \gamma_n P_n/q_n)N]$.

Customers typically evaluate both the selling price and the quality of comparable products before placing an order. We define a new utility function based on the characteristics of both products as $U_s(\theta) = \theta(q_s - q_n) - \gamma_s(p_s - p_n)$ and $U_n(\theta) = \theta(q_n - q_s) - \gamma_n(p_n - p_s)$, when both substitutable products are presented simultaneously. The two utility functions are not independent, as each is influenced not only by its decision variables but also by the competitor's selling price and quality level. Therefore, the demand for the SB product when the retailer decides to offer both products is equal to $(1 - \gamma_s(P_s - P_n)/(q_s - q_n))x$, with uniform PDF within $[0, (1 - \gamma_s(P_s - P_n)/(q_s - q_n))N]$. It is noticeable that an increase in P_s or reduction in q_s leads to a decrease in the demand quantity of the SB-product. Moreover, an increase in P_n or reduction in q_n leads to an increase in the demand quantity of the SB product up to $(1 - \gamma_n(P_n - P_s)/(q_n - q_s))x$ with a uniform PDF from 0 to $(1 - \gamma_n(P_n - P_s)/(q_n - q_s))N$.

Table 1 shows that the retailer can influence the customer's utility function by adjusting the selling prices, thereby altering the customer's purchasing behavior. Depending on the price range (Ps), demand rates for the NB and the SB will vary. Depending on the three possible price

Table 1
The change in brands demands respect for the SB's retail price changes.

Scenario	Range for P_s	Demand rate of NB	Demand rate of SB
1	$P_s \le P_n - \frac{2(q_n - q_s)}{\gamma_s + \gamma_n}$	0	$D_s \sim u[0, N]$
2	$P_n - \frac{2(q_n - q_s)}{\gamma_s + \gamma_n} \le P_s < P_n$	$D_n \sim u[0, (1 - (\gamma_n + \gamma_s)\frac{p_n - p_s}{2(q_n - q_s)})N]$	$D_s \sim u[0, (\gamma_n + \gamma_s)\frac{p_n - p_s}{2(q_n - q_s)})N]$
3	$P_s = P_n$	$D_n \sim u[0, N]$	0

intervals for the selling price, different demand rates will be observed from customers.

Fig. 2 illustrates the customer's behavior under scenario 1. It shows if the retailer decreases P_s to less than $P_n - 2(q_n + q_s)/(\gamma_n + \gamma_s)$, the retailer can attract the attention of all customers to the SB product. As the utility of purchasing an NB product decreases for the customer, they tend to prefer the SB product instead.

Moreover, if the price of the SB product is settled within $[P_n - 2(q_n + q_s)/(\gamma_n + \gamma_s)), P_n)$, there are demands for both types of brands, as shown in Fig. 3. This figure shows the SB product when $\theta \le \gamma_s(P_n - P_s)/(q_n - q_s)$ and the NB product when $\gamma_n(P_n - P_s)/(q_n - q_s) \le \theta \le 1$ have positive utilities for customers, and they will have their customers independently. But when $\gamma_n(P_n - P_s)/(q_n - q_s) \le \theta \le \gamma_s(P_n - P_s)/(q_n - q_s)$, both products have positive utilities simultaneously, and they can have their own share market. On the other hand, when $\gamma_n(P_n - P_s)/(q_n - q_s) \le \theta \le (\gamma_n + \gamma_s)(P_n - P_s)/2(q_n - q_s)$, the SB-product is more profitable, and when $(\gamma_n + \gamma_s)(P_n - P_s)/2(q_n - q_s) \le \theta \le \gamma_s(P_n - P_s)/(q_n - q_s)$, the NB product is more profitable than the SB product. It is important to note that for values of $\gamma_s(P_n - P_s)/(q_n - q_s) \le \theta \le 1$ and for the value of $\theta \le \gamma_n(P_n - P_s)/(q_n - q_s)$, none of the sellers can attract any customers.

In the following, we define the expected profit function for each member of the supply chain. We analyze the problem and its results across three scenarios. In the first case, the retailer presents only the NB product; in the second case, they offer only the SB product; finally, the retailer chooses to sell both the NB and SB products simultaneously. Initially, each scenario is examined in both decentralized and centralized cases, and then an appropriate contract is adjusted to coordinate the supply chain members.

4.1. Decentralized case

In the decentralized case, each member, in order to maximize their profit, makes decisions about their decision variables without considering the policies of other members.

4.1.1. NB only

In this case, we do not have a competitive supply chain, and our problem is simply a pricing issue. We have a supply chain with only one good, and in this case, the customer's utility is a function of P_n and q_n as follows:

$$U_n(\theta) = \theta q_n - \gamma_n P_n$$

And the retailer's profit is as follows:

$$\pi_r = \int_0^{Q_n} (P_n D - WQ_n) \frac{1}{(1 - (\gamma_n P_n / q_n))N} dD + \int_{Q_n}^{(1-(\gamma_n P_n/q_n))N} (P_n - W)Q_n \frac{1}{(1 - (\gamma_n P_n / q_n))N} dD$$

Because, when $Q_n < D$, the revenue that the retailer obtains from the sale of Q_n units minus the cost of providing these goods is $P_n D - WQ_n$. Moreover when $Q_n > D$ the retailer's profit is the difference between the income that he can obtain from the sale and the cost of providing Q_n units which is $P_n D - WQ_n$. After some algebra, we have:

$$E(\pi_r(Q_n)) = (P_n - W)Q_n - \frac{P_n Q_n^2}{2(1 - (\gamma_n P_n / q_n))N} \quad (1)$$

The first order derivative of Eq. (1) with respect to Q_n is as follows:

$$\frac{\partial \pi_r}{\partial Q_n} = (P_n - W) - \frac{P_n Q_n}{(1 - (\gamma_n P_n / q_n))N}$$

π_r is negative definite for all Q_n because $\partial^2 \pi_r / \partial Q_n^2 =$

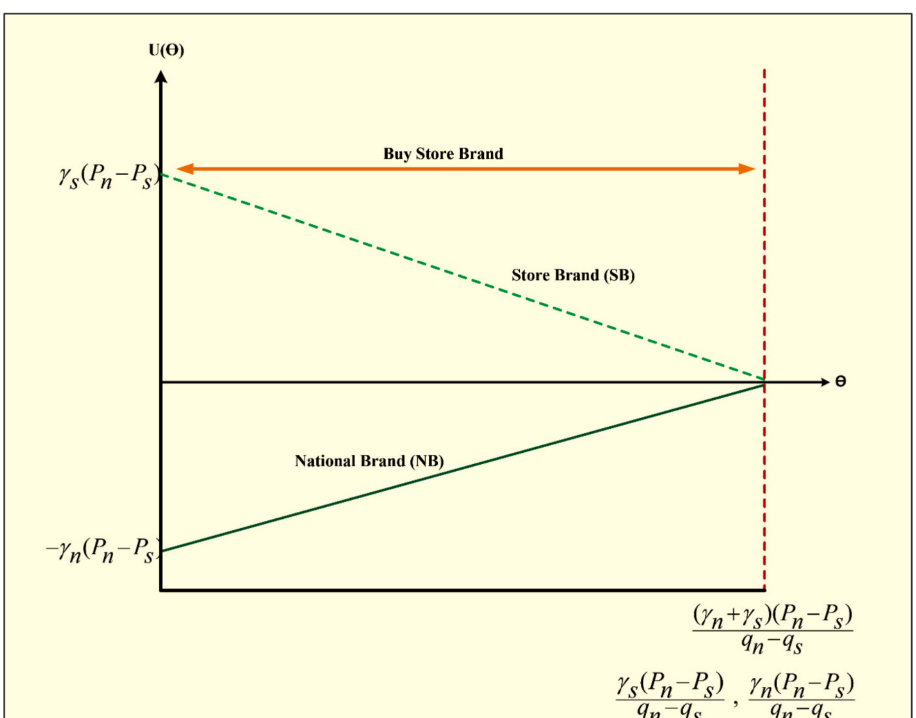

Fig. 2. Customer's behavior under Scenario 1.

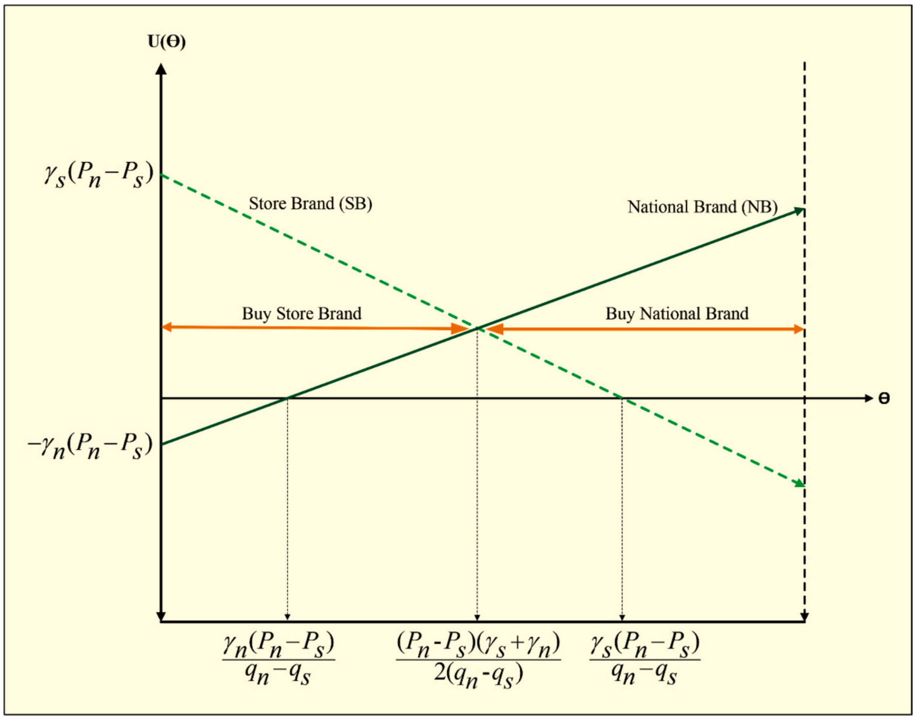

Fig. 3. Customer's behavior under scenario 2.

$-P_n/(1-\gamma_n P_n/q_n)N < 0$, therefore $\pi_r(Q_n)$ is concave, and by setting $\partial\pi_r/\partial Q_n$ equal to 0, the optimal order quantity for the NB product is obtained as follows:

$$Q_n^* = \frac{P_n - W}{P_n}(1 - (\gamma_n P_n/q_n))N \tag{2}$$

By substituting Eq. (2) in the profit function of the NB-manufacturer, we have:

$$E(\pi_m(W)) = (W - C_n)\frac{(P_n - W)}{P_n}(1 - \gamma_n\frac{P_n}{q_n})N \tag{3}$$

where $\frac{\partial\pi_m(w)}{\partial W} = \frac{(P_n-2W-C_n)}{P_n}(1-\gamma_n\frac{P_n}{q_n})N$ and $\frac{\partial^2\pi_m(w)}{\partial^2 w} = -\frac{2}{p_n}(1-\gamma_n\frac{p_n}{q_n})N < 0$.

Since the first-order derivative of Eq. (3) with respect to W is negative, this function is concave, and by solving $\partial\pi_m/\partial w = 0$ the optimal wholesale price of the NB product by the manufacturer can be derived as follows:

$$W^* = \frac{P_n + C_n}{2} \tag{4}$$

According to the optimal values of Q_n and W, the profit function of the retailer, NB manufacturer, and total profit are respectively as below:

$$\pi_r^* = \frac{(P_n - C_n)^2}{8P_n}(1 - \gamma_n\frac{P_n}{q_n})N \tag{5}$$

$$\pi_m^* = \frac{(P_n - C_n)^2}{4P_n}(1 - \gamma_n\frac{P_n}{q_n})N \tag{6}$$

$$\pi_T^* = \pi_r^* + \pi_m^* = \frac{3(P_n - C_n)^2}{8P_n}(1 - \gamma_n\frac{P_n}{q_n})N \tag{7}$$

4.1.2. SB only

In this case, the retailer chooses to sell only the SB product, so the retailer's profit function is used. Similar to the previous case, we define the following utility function for the customers when there is only the SB product: So, the retailer's profit is:

$$\pi_r(P_s, Q_s) = (P_s - C_s)Q_s - \frac{P_s Q_s^2}{2(1 - (\gamma_n P_n/q_n))N} \tag{8}$$

Lemma 1. The retailer needs to determine Q_s and P_s, and since the Hessian matrix of $\pi_r(P_s, Q_s)$ is a negative definite for all values of Q_s and P_s. So the following results from setting the first-order derivatives of Eq. (8) with respect to P_s and Q_s equal to zero can be derived.

$$P_s = \frac{q_s + \sqrt{q_s(8\gamma_s C_s + q_s)}}{4\gamma_s} \tag{9}$$

$$Q_s = \frac{P_s - C_s}{P_s}(1 - \gamma_s P_s/q_s)N \tag{10}$$

Proof. See Appendix A.

4.1.3. Both national and store brands

Retailer's Problem

When both types of products are sold together by the retailer, the customer's utility functions are changed to $U_s(\theta) = \theta(q_s - q_n) - \gamma_s(P_s - P_n)$ and $U_n(\theta) = \theta(q_n - q_s) - \gamma_n(P_n - P_s)$.

So the expected profit function of the retailer if he (she) decides to offer both NB- and SB-products, is the combination of profits he gains from selling both types of products as below:

$$\pi_r(P_s,Q_s,Q_n) = (P_s - C_s)Q_s - \frac{P_sQ_s^2}{((\gamma_n+\gamma_s)\frac{P_n-P_s}{q_n-q_s})N} + (P_n - W)Q_n - \frac{P_nQ_n^2}{2(1-(\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N} \tag{11}$$

where $P_s \in \left[\max(C_s, P_n - \frac{2(q_n-q_s)}{\gamma_s+\gamma_n}), P_n\right]$

Since the second-order derivatives of Eq. (11) with respect to Q_s and Q_n are negative:

$\frac{\partial^2\pi_r}{\partial Q_s^2} = -\frac{P_s(q_n-q_s)}{(\gamma_s+\gamma_n)(P_n-P_s)N} < 0$ and $\frac{\partial^2\pi_r}{\partial Q_n^2} = -\frac{P_n}{(1-(\gamma_s+\gamma_n)\frac{P_n-P_s}{q_n-q_s})N} < 0$, the optimal amounts of Q_s and Q_n to maximize $\pi_r(P_s, Q_s, Q_n)$ calculated based on P_s. We obtained them from the first-order derivatives of Eq. (11) with respect to Q_s and Q_n and setting them equal to zero, as follows:

$$\frac{\partial\pi_r}{\partial Q_s} = (P_s - C_s) - \frac{2P_sQ_s}{(\gamma_s+\gamma_n)\frac{P_n-P_s}{q_n-q_s}N} = 0 \tag{12}$$

$$\frac{\partial\pi_r}{\partial Q_n} = (P_n - W) - \frac{P_nQ_n}{(1-(\gamma_s+\gamma_n)\frac{P_n-P_s}{q_n-q_s})N} = 0 \tag{13}$$

From Eqs. (12) and (13), we have:

$$Q_s^* = \frac{P_s - C_s}{P_s}[(\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)}]N \tag{14}$$

$$Q_n^* = \frac{P_n - W}{P_n}[1-(\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)}]N \tag{15}$$

Then, by substituting Eqs. (14) and (15) into Eq. (11) we have:

$$\pi_r(P_s) = \frac{(P_s-C_s)^2}{2P_s}((\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N + \frac{(P_n-W)^2}{2P_n}(1 - (\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N$$

$$P_s \in \left[\max(C_s, P_n - \frac{2(q_n-q_s)}{\gamma_s+\gamma_n}), P_n\right] \tag{16}$$

According to the results presented in Appendix A, the retail price depends on the wholesale price, and the behavior of the profit function varies with changes in the wholesale price. Moreover, the decision variables can take different values. Therefore, the following Lemma can be used to derive the optimal retail price.

Lemma 2. If $W < C_s$ then $P_s^* = P_n$ and in this case, by substituting the amount of P_s^* in Eqs. (14) and (15), we have $Q_s^{III} = 0$, $Q_n^{III} = N(P_n - W)/P_n$. In fact, selling only NB is more profitable for the retailer. If $W > C_s$, the second-order condition of Eq. (16) respect to P_s is a negative definite, therefore, using the following equation, we have:

$$\frac{\partial\pi_r(p_s)}{\partial p_s} = \frac{(\gamma_n+\gamma_s)}{4(q_n-q_s)}N[\frac{(P_s^2-C_s^2)(P_n-P_s)}{P_s^2} - \frac{(P_s-C_s)^2}{P_s} + \frac{(P_n-W)^2}{P_n}] = 0$$

$$P_s^{III} = -2\sqrt{\frac{(P_n-W)^2+2P_nC_s+P_n^2}{3P_n^2C_s^2}}Cos(\frac{\alpha}{3}+\frac{\pi}{3}) \tag{17}$$

where cosα is $Cos(\alpha) = \frac{-1}{P_nC_s^2\sqrt{(\frac{(P_n-W)^2+2P_nC_s+P_n^2}{3P_n^2C_s^2})^3}}$. Meanwhile, only those values of P_s are acceptable, which belong to the satisfactory range mentioned in Eq. (16). Therefore $P_s^{III^*} = \max\{P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n), P_s^{III}\}$. Then we obtain $Q_n^{III^*}$ and $Q_s^{III^*}$ by substituting the amount of $P_s^{III^*}$ in Eqs. (14) and (15), respectively.

Since the retailer's desire to present the NB product decreases when W increases, there is a $W' \in (C_S, P_n]$ at which the retailer prefers to sell both brands. But when W' does not belong to the mentioned range, producing and selling only the SB product is the optimal strategy for the retailer. W' is a point that $P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$ and P_s^{III} are equal, and if $W' < W$, the optimal retail price of the SB product is P_s^{III} and otherwise $P_s^* = P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$.

Proof. See Appendix B.

Manufacturer's problem

According to the above description, the expected profit of the manufacturer changes to Eq. (18) when $W < C_s$:

$$\pi_m = \frac{(W-C_n)(P_n-W)}{P_n}N \tag{18}$$

And since the second order derivative of Eq. with respect to W, $\frac{\partial^2\pi_m}{\partial W^2} = -\frac{1}{P_n}$, is negative, we obtain the optimal wholesale price by setting the first derivative equal to 0, as follows:

$$W = \frac{P_n + C_n}{2}$$

In this case, since we assumed that $W < C_s$, the optimal wholesale price is $\min\{C_s, \frac{P_n+C_n}{2}\}$. Moreover, when $C_s < W < W'$, the wholesale price must be greater than the unit production cost; we have $\max\{C_s, C_n\} < W < W'$. So the manufacturer's function changes as follows:

$$\pi_m = \frac{(W-C_n)(P_n-W)}{P_n}(1-(\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N \tag{19}$$

By substituting $P_s^{III^*}$ in Eq. (19), we can gain the optimal wholesale price by setting the first order derivative of Eq. (19) with respect to W equal to zero as below:

Table 2
A summary of the order quantity against changes in wholesale price.

Range for W	Retailer Policies	The NB order quantity	The SB order quantity
$W \le \min\{C_s, \frac{P_n+C_n}{2}\}$	Only NB	$Q_n = \frac{P_n - w}{p_n}N$	0
$\max\{C_s, C_n\} < W < W'$	NB and SB	$Q_n^{III^*} = \frac{P_n-W}{P_n}(1-(\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N$	$Q_s^{III^*} = \frac{P_s-C_s}{P_s}((\gamma_n+\gamma_s)\frac{P_n-P_s}{2(q_n-q_s)})N$
$W' \le W < P_n$	Only SB	0	$Q_s = \frac{P_s - C_s}{P_s}N$

$$\frac{\partial \pi_m}{\partial W} = \frac{(P_n - 2W + C_n)}{P_n}(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N + \frac{(W - C_n)(P_n - W)^2}{2P_n}((\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N$$

$$[\frac{Cos(\frac{\alpha}{3} + \frac{\pi}{3})}{P_n C_s\sqrt{3((P_n - W)^2 + 2P_nC_s + P_n^2)}} - \frac{54P_n^3C_sSin(\frac{\alpha}{3} + \frac{\pi}{3})}{\sqrt{81P_n^4C_s^2 - 3((P_n - W)^2 + 2P_nC_s + P_n^2)^3}}] = 0$$

Finally, when $W' < W < P_n$, since in this condition the profit of the retailer from selling both types of SB- and NB-products is less than when he produces and sells only the SB-product, he has no desire to offer and present the NB-product. We summarize the brands' order quantities as a function of the wholesale price in Table 2.

4.2. Coordinating contracts

To coordinate the supply chain, we evaluate the model's behavior in the centralized case and then introduce an appropriate contract for each scenario.

4.2.1. Centralized case

In this section, we integrate the supply chain members and assume a single decision maker who seeks to determine the optimal levels of the decision variables to maximize the supply chain's expected profit.

4.2.1.1. NB only. We can obtain the profit of the centralized case from the sum of the profits of the retailer and the manufacturer as follows:

$$\pi_{SC}^{CI} = \pi_r^{CI} + \pi_m^{CI} = (P_n - C_n)Q_n - \frac{P_nQ_n^2}{2(1 - \gamma_n\frac{P_n}{q_n})N} \quad (20)$$

Since the second-order derivative of Eq. (20) with respect to Q_n is negative, $(-P_nN/(1 - \gamma_nP_n/q_n))$, the optimal order quantity can be derived by setting the first-order derivative equal to 0, as shown in Eq. (21).

$$Q_n^{CI} = \frac{P_n - C_n}{P_n}(1 - \gamma_n\frac{P_n}{q_n})N \quad (21)$$

4.2.1.2. Both SBs and NBs. When the retailer decides to present both SB and NB products simultaneously, the profit function for the centralized supply chain can be expressed as follows:

$$\pi_{SC}^{CIII} = (P_s - C_s)Q_s - \frac{P_sQ_s^2}{((\gamma_n + \gamma_s)\frac{P_n - P_s}{q_n - q_s})N} + (P_n - C_n)Q_n - \frac{P_nQ_n^2}{2(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N} \quad (22)$$

$$P_s \in \left[\max(C_s, P_n - \frac{2(q_n - q_s)}{\gamma_s + \gamma_n}), P_n\right]$$

Since the second-order derivatives of the equation for Q_s and Q_n are negative:

$$\frac{\partial^2 \pi_{SC}^{CIII}}{\partial Q_s^2} = -\frac{P_s(q_n - q_s)}{(\gamma_s + \gamma_n)(P_n - P_s)N} < 0$$

$$\frac{\partial^2 \pi_{SC}^{CIII}}{\partial Q_n^2} = -\frac{P_n}{(1 - (\gamma_s + \gamma_n)\frac{P_n - P_s}{q_n - q_s})N} < 0$$

One can obtain the optimal amount of Q_n and Q_s by setting the first-order derivatives of Eq. for Q_n and Q_s equal to zero as below:

$$Q_n^{CIII} = \frac{P_n - C_n}{P_n}[1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)}]N \quad (23)$$

$$Q_S^{CIII} = \frac{P_s - C_s}{P_s}[(\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)}]N \quad (24)$$

By substituting Eqs. (23) and (24) into Eq. (22), we obtain:

$$\pi_{SC}^{CIII}(P_s) = \frac{(P_s - C_s)^2}{2P_s}((\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N + \frac{(P_n - C_n)^2}{2P_n}(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N$$

$$P_s \in \left[\max(C_s, P_n - \frac{2(q_n - q_s)}{\gamma_s + \gamma_n}), P_n\right] \quad (25)$$

Lemma 3. If $C_s > C_n$, then $P_s^* = P_n$, so by substituting the amount of P_s^* in Eqs. (23) and (24), we have $Q_s^{III} = 0$, $Q_n^{III} = N(P_n - P_s)/P_n$. Selling only NB is more profitable for the retailer. If $C_s < C_n$, then the second order derivative of Eq. (25) respect to P_s is definitely negative, therefore we can obtain the optimal SB's retail price by solving the following equation:

$$\frac{\partial \pi_{SC}^{CIII}}{\partial P_s^{SCIII}} = \frac{(\gamma_n + \gamma_s)}{4(q_n - q_s)}N\left[\frac{(P_n - P_s)(P_s^2 - C_s^2)}{P_s^2} - \frac{(P_s - C_s)^2}{P_s} + \frac{(P_n - C_n)^2}{P_n}\right] \quad (26)$$

$$P_s^{CIII} = -2\sqrt{\frac{(P_n - C_n)^2 + 2P_nC_s + P_n^2}{3P_n^2C_s^2}}Cos(\frac{\alpha}{3} + \frac{\pi}{3}) \quad (27)$$

where $Cos\alpha = \frac{-1}{P_nC_s^2\sqrt{\left(\frac{(P_n - C_n)^2 + 2P_nC_s + P_n^2}{3P_n^2C_s^2}\right)^3}}$.

Proof. See Appendix C.

4.3. Revenue sharing-NB only

In this sub-section, an RSC to coordinate the supply chain is applied when the retailer decides to present only the NB product. Under RSC, the retailer divides the revenue with the manufacturer, whereas the manufacturer reduces the wholesale price. Therefore, α_n is a fraction of the retailer's earnings from selling each unit of the NB product that he pays to the manufacturer. The profit functions of the brands under coordination using RSC change as follows:

$$\pi_r^{RSI}(Q_n|\alpha_n, W_n) = (\alpha_nP_n - W_n)Q_n - \frac{\alpha_nP_nQ_n^2}{2(1 - (\gamma_nP_n/q_n))N} \quad (28)$$

$$\pi_m^{RSI} = (W_n - C_n)Q_n + (1 - \alpha_n)P_nQ_n - \frac{(1 - \alpha_n)Q_n^2P_n}{2(1 - (\gamma_nP_n/q_n))N} \quad (29)$$

Table 3
Input data.

Retailer			Manufacturer			
C_s	q_s	γ_s	C_n	P_n	q_n	γ_n
7 ($W > C_s$)	20	0.75	8	22	25	0.25
16 ($W < C_s$)	20	0.75	8	22	25	0.25

And by setting the first order derivative of Eq. (28) equal to zero, we have:

$$Q_n^{RSI} = \frac{(\alpha_n P_n - W_n)}{\alpha_n P_n}(1 - (\gamma_n P_n / q_n))N \tag{30}$$

$$\pi_r^{RSIII}(Q_n, P_s|\alpha_s, \alpha_n, W_n) = (\alpha_n P_n - W_n)Q_n - \frac{\alpha_n P_n Q_n^2}{2[1 - (\gamma_n + \gamma_s)((P_n - P_s)/2(q_n - q_s))]N} + (\alpha_s P_s - C_s)Q_s - \frac{\alpha_s P_s Q_s^2}{[(\gamma_n + \gamma_s)((P_n - P_s)/(q_n - q_s))]N} \tag{32}$$

$$\pi_m^{RSIII} = (W_n - C_n)Q_n + (1 - \alpha_n)P_n Q_n - \frac{(1 - \alpha_n)Q_n^2 P_n}{2[1 - (\gamma_n + \gamma_s)((P_n - P_s)/2(q_n - q_s))]N} + (1 - \alpha_s)P_s Q_s - \frac{(1 - \alpha_s)P_s Q_s^2}{[(\gamma_n + \gamma_s)((P_n - P_s)/(q_n - q_s))]N} \tag{33}$$

4.3.1. Conditions for participating members

In this section, we will examine the conditions under which supply chain members participate in a contract. Coordination in a decentralized supply chain is said to have optimally occurred if the Q_ns of revenue sharing and centralized cases become equal to each other. So by setting Eqs. (30) and (23) are equal simultaneously, we obtain an equation for W_n as a function of α_n, as follows:

$$Q_n^{RSI} = Q_n^{CI}; \; W_n = \alpha_n C_n \tag{31}$$

The retailer and the manufacturer accept the contract if and only if the profit from an RSC is higher than the profit that they can obtain under the decentralized case. Therefore, we obtain $\alpha_n > 1/4$ as the lower limit of α_n under the above-mentioned relationship $\pi_r^{RS\,I} > \pi_r^{I}$ between Eq. (28) and Eq. (1) for the retailer. Moreover, by solving this relationship $\pi_m^{RS\,I} > \pi_m^{I}$ between Eqs. (29) and (3), we gain $\alpha_n \le 1/2$ as the upper limit of α_n. Therefore, if the limitations of $\alpha_n \in [1/4, 1/2]$ are met, not only are all of the SC members willing to accept the contract, but also the total profit of SC is improved.

$$\pi_m^{RSIII}(P_s|\alpha_s, \alpha_n, W_n) = \left(\frac{(W_n - C_n)(\alpha_n P_n - W_n)}{\alpha_n P_n} + \frac{(1 - \alpha_n)(\alpha_n^2 P_n^2 - W_n^2)}{2\alpha_n^2 P_n}\right)\left(1 - (\gamma_n + \gamma_s)\frac{(P_n - P_s)}{2(q_n - q_s)}\right)N + \frac{(1 - \alpha_s)(P_s^2 - C_s^2)}{2P_s}\left((\gamma_n + \gamma_s)\frac{(P_n - P_s)}{2(q_n - q_s)}\right)N \tag{37}$$

4.3.2. Two-part tariff RSC-national and store brands

The traditional revenue-sharing contract with (W_n, α_n) (α_n is the retailer's share of income from the sale of NB and W_n is the wholesale price after revenue-sharing) cannot coordinate the newsvendor problem while the demand is price-dependent (Cachon, 2003).

Now, we use an innovative two-part tariff RSC to coordinate the brands. Under the two-part tariff RSC, the manufacturer receives a share of revenue that the retailer obtains from selling both NB- and SB-products. We denote this contract as (W_n, α_s, α_n) where W_n is the wholesale price after the contract and $0 < \alpha_s$ and $\alpha_n < 1$ respectively show the fraction of revenue that the retailer obtains from selling SB- and NB-products. Moreover, the manufacturer's share from selling these goods is $(1 - \alpha_s)$ and $(1 - \alpha_n)$. Finally, the profits of the retailer and manufacturer change as follows:

By taking the first-order derivatives of Eq. (32) with respect to Q_n and Q_s, and setting them equal to 0, we obtain:

$$Q_n^{RSIII} = \frac{\alpha_n P_n - W_n}{\alpha_n P_n}\left(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)}\right)N \tag{34}$$

$$Q_s^{RSIII} = \frac{P_s - C_s}{2P_s}\left((\gamma_n + \gamma_s)\frac{P_n - P_s}{q_n - q_s}\right)N \tag{35}$$

Substituting Eqs. (34) and (35) into Eq. (32) and (33), we have:

$$\pi_r^{RSIII}(Q_n, P_s|\alpha_s, \alpha_n, W_n) = \frac{(\alpha_n P_n - W_n)^2}{2\alpha_n P_n}\left(1 - (\gamma_n + \gamma_s)\frac{(P_n - P_s)}{2(q_n - q_s)}\right)N + \frac{(P_s - C_s)(\alpha_s(P_s + C_s) - 2C_s)}{2P_s}\left((\gamma_n + \gamma_s)\frac{(P_n - P_s)}{2(q_n - q_s)}\right)N \tag{36}$$

We can obtain the optimal decision of P_s with the first order derivative of Eq. (36) with respect to P_s, as follows:

Table 4
Decision variables.

		P_s	W_n	Q_s	Q_n	α_n	α_s	π_r	π_m	π_{SC}
$W > C_s$	Decentralized	18.22	13	23.30	25.43	–	–	245.09	127.16	372.25
	Revenue sharing	21.26	5.05	4.95	58.94	0.63	0.82	282.91	164.97	447.88
$W < C_s$	Decentralized	22.00	15.00	0.00	31.82	–	–	111.36	222.73	334.09
	Revenue sharing	22.00	3	0.00	63.64	0.38	0.46	167.05	278.41	445.45

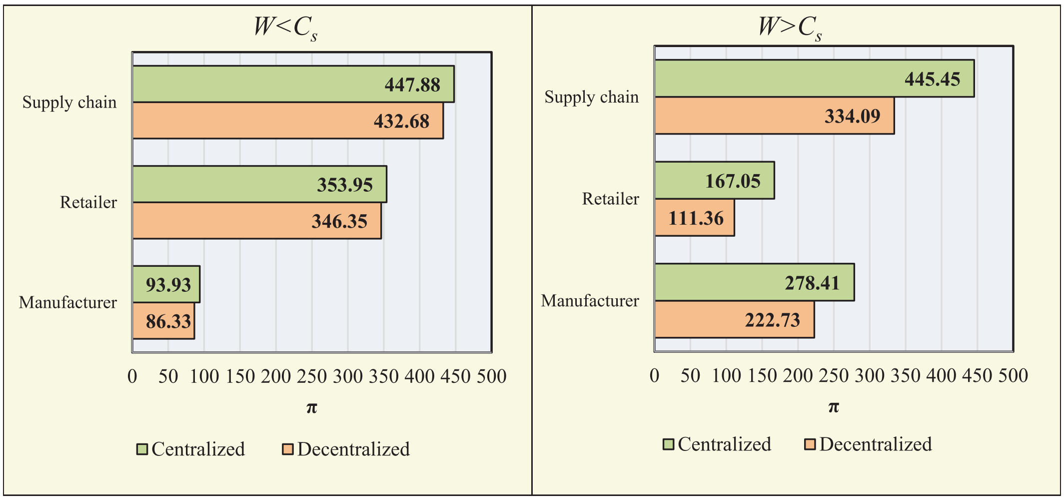

Fig. 4. The profit of members when $W < C_s$ and $W > C_s$.

Table 5
The effect of variation on wholesale price on the decision variables.

W	Decentralized			Two-part tariff revenue-sharing			
	P_s	Q_s	Q_n	P_s	W_n	Q_s	Q_n
10	19.91	13.53	43.16	21.26	6.32	4.95	58.94
11	19.30	17.19	36.52	21.26	5.75	4.95	58.94
12	18.74	20.44	30.63	21.26	5.34	4.95	58.94
13	18.22	23.30	25.43	21.26	5.05	4.95	58.94
14	17.74	25.78	20.88	21.26	4.86	4.95	58.94
15	17.32	27.90	16.91	21.26	4.75	4.95	58.94
16	16.94	29.69	13.47	21.26	4.71	4.95	58.94
17	16.62	31.16	10.49	21.26	4.72	4.95	58.94
18	16.35	32.33	7.90	21.26	4.78	4.95	58.94
19	16.13	33.21	5.64	21.26	4.87	4.95	58.94

$$\frac{\partial \pi_r^{RSIII}}{\partial P_s^{RSIII}} = \frac{(\gamma_n + \gamma_s)}{4(q_n - q_s)} N[\frac{C_s(P_n - P_s)(\alpha_s(P_s + C_s) - 2C_s)}{P_s^2} + \frac{(P_s - C_s)(2C_s + \alpha_s(P_n - 2P_s - C_s))}{P_s} + \frac{\alpha_n(P_n - C_n)^2}{P_n}] \quad (38)$$

Under the RSC, we seek to determine the optimal values of the decision variables in the centralized case. Therefore, Eqs. (23) and (34) are equal to each other, then we obtain an equation for W_n as a function of α_n, as follows:

$$Q_n^{RSIII} = Q_n^{CIII},\ W_n = \alpha_n C_n$$

Also, it is necessary to examine the equality between the retail prices of the centralized and RSC cases, but to avoid the complexities of the trigonometric problem, Eqs. (26) and (38) are set equal to obtain a function for α_n as a function of α_s as below:

$$\frac{\partial \pi_r^{RSIII}}{\partial P_s^{RSIII}} = \frac{\partial \pi_{sc}^{C\ III}}{\partial P_s^{C\ III}}$$

$$\alpha_n = 1 + \frac{(1 - \alpha_s)P_n}{(P_n - C_n)^2}\left(\frac{P_n C_s^2}{P_s^2} + P_n - 2P_s\right)$$

As mentioned in the last contract, the members accept the contract if and only if the profits generated under the coordination contract exceed the profits they obtained in the decentralized case. Therefore, with examination of this relationship between Eqs. (36) and (16), we obtain the following inequality for α_s:

Table 6
The effect of variation on wholesale price on the profit functions and revenue sharing fractions.

W	Decentralized			Two-part tariff revenue-sharing				
	π_r	π_m	π_{SC}	π_r	π_m	π_{SC}	α_n	α_s
10	346.35	86.33	432.68	353.95	93.93	447.88	0.79	0.9
11	306.57	109.55	416.13	322.45	125.43	447.88	0.72	0.86
12	273.06	122.5	395.56	299.22	148.66	447.88	0.67	0.84
13	245.09	127.16	372.25	282.91	164.97	447.88	0.63	0.82
14	221.99	125.29	347.27	272.29	175.59	447.88	0.61	0.81
15	203.13	118.40	321.54	266.31	181.57	447.88	0.59	0.80
16	187.98	107.78	295.76	264.04	183.84	447.88	0.59	0.80
17	176.04	94.42	270.45	264.75	183.13	447.88	0.59	0.80
18	166.87	79.03	245.90	267.86	180.02	447.88	0.60	0.80
19	160.12	62.01	222.13	273.00	174.88	447.88	0.61	0.81

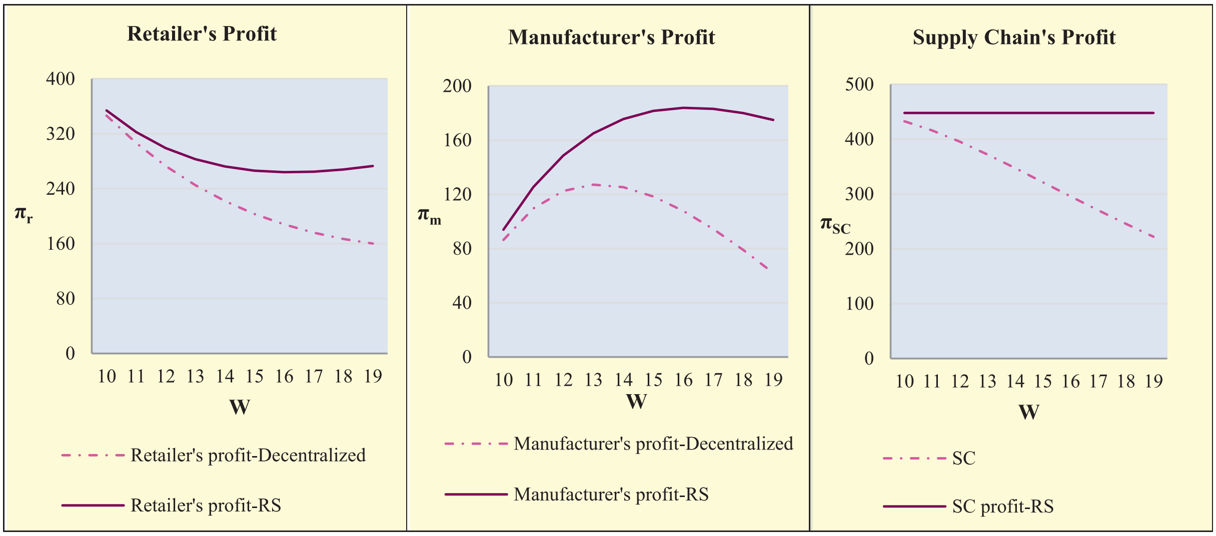

Fig. 5. The profit functions under the variation of the wholesale price.

$$\pi_r^{RSIII} \geq \pi_r^{III}$$

$$\alpha_s \geq \frac{2\pi_r^{III} P_s^2 P_n - [((P_n - C_n)^2 P_s^2 + P_n^2 C_s^2 + P_n^2 P_s^2 - 2P_s^3 P_n)(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N] + P_s P_n C_s (P_s - C_s)(P_n - P_s)((\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N}{P_s C_s (P_s^2 - C_s^2)(P_n - P_s)((\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N - (P_n C_s^2 + P_n P_s^2 - 2P_s^3)(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N}$$

Additionally, from the manufacturer's point of view, the profit generated under the RSC exceeds the profit obtained in the decentralized case. Therefore, similarly performing the same comparison between Eqs. (37) and (18), we have:

$$\pi_m^{RSIII} \geq \pi_m^{III}$$

$$\alpha_s \geq 1 - \frac{2\pi_m^{III}}{\frac{(P_s^2 - C_s^2)}{P_s}((\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N - (\frac{P_n C_s^2}{P_s^2} + P_n - 2P_s)(1 - (\gamma_n + \gamma_s)\frac{P_n - P_s}{2(q_n - q_s)})N}$$

5. Numerical analysis

Numerical analyses are provided to indicate the effectiveness of the models. Moreover, we show the significant impacts of the two-part tariff RSC on the expected profit of the supply chain members. Table 3 shows the input parameters meeting the conditions $W > C_s,\ W < C_s$.

The results of numerical analysis are presented in Table 4.

The result shows that the two-part tariff RSC can affect the retailer's policies and enable him to increase his selling price. Moreover, not only his profit but also the manufacturer's profit is improved under the two-part tariff RSC (See Fig. 4). The wholesale price is one of the contract parameters that plays an essential role in the policies of supply chain members. Therefore, we analyzed the effects of wholesale price on the decision variables when it changes within [10, 19] as presented in Table 5. We found that with increasing W, the SB product retail price reduces, and then the amount of SB production rises, as a result Q_n reduces. Under RSC, the amount of $P_s, Q_s,$ and Q_n are independent of wholesale price, but their variation can lead to small changes in the W_n.

Table 6 and Fig. 5 show that, in decentralized cases, the profit function of all members decreases with increasing *W*, whereas in centralized cases, expected profits are improved. But according to Eq. (22), the SC profit function is independent of W and the amount of π_{SC} is constant when *W* changes. Also, the last two columns of Table 6 illustrate the shares of retailers that decrease with the increase in wholesale price, and then W_n reduces too. Therefore, with the increasing wholesale price, using the RSC is more economical for members.

The results indicate that the RSC can have a significant positive effect on members' expected profit and the entire SC, as well as on members' performance. Moreover, this contract can influence the retailer's decision-making, potentially leading them to increase orders for the NB product.

6. Managerial insights

Our numerical analysis demonstrates that while standard mechanisms such as revenue-sharing may fail to coordinate the supply chain under correlated demand, the implementation of a two-part tariff RSC offers a robust solution. This specific contract structure yields critical prescriptive insights for managers of both NBs and SBs.

6.1. *Overcoming inefficiency with the two-part tariff RSC*

Managers should recognize that the two-part tariff RSC is essential for maximizing system performance where simple contracts fail. The two-part tariff RSC significantly affects the expected profits of supply chain members, consistently outperforming the decentralized model. For instance, our results show that under this specific contract, the total supply chain profit (π_{SC}) increases substantially (e.g., from 372.25 to 447.88) compared to the decentralized case. Moreover, crucially, this contract structure ensures that both the retailer's and the manufacturer's profits are improved simultaneously. This makes the two-part tariff RSC a viable "win–win" strategy for negotiation.

6.2. *Strategic shift in retailer pricing and inventory*

For NB manufacturers, the two-part tariff RSC is a strategic tool to influence the retailer's behavior favorably. In a decentralized setting, as the wholesale price (W) increases, the retailer typically adopts a defensive strategy by reducing the SB retail price (P_s) and increasing SB production (Q_n) while reducing NB orders (Q_s). Additionally, the two-part tariff RSC effectively alters this policy. It incentivizes the retailer to increase the selling price of the SB product and, significantly, to increase the order quantity for the NB product. This suggests that the two-part tariff RSC helps mitigate the cannibalization threat from SBs.

6.3. Decoupling operations from wholesale price volatility

A key advantage of the two-part tariff RSC is its ability to stabilize operational decisions against fluctuations in the wholesale price. Unlike the decentralized case, where profits and quantities fluctuate with W, under the two-part tariff RSC, the optimal decision variables (P_s, Q_s, Q_n) and the total system profit (π_{SC}) become independent of the wholesale price. Moreover, while operations remain stable, the contract handles economic distribution through its parameters. As the wholesale price (W) rises, the retailer's direct margin shrinks; to maintain the coordinated state, the revenue-sharing fractions (α_n, α_s) paid to the manufacturer decreases. This highlights that as the wholesale price increases, adopting the two-part tariff RSC becomes even more economical and necessary for the members to sustain profitability. Ultimately, for decision-makers managing the competitive landscape between NBs and SBs, the two-part tariff RSC is an effective mechanism for supply chain coordination, as it stabilizes operations against wholesale price volatility while strategically incentivizing retailers to optimize NB and SB sales for maximal system-wide profit.

7. Conclusion and future research

This paper developed a stochastic pricing model for two substitutable products from different brands in a two-stage supply chain. Based on the studied framework, the study derives equilibrium pricing strategies and reveals key insights into brand competition and supply chain coordination. A brand-specific utility-based demand model is developed to capture customer substitution behavior and account for the correlation between the two demands. The demand structure is characterized by a competitive relationship, whereby an increase in demand for one brand is assumed to cannibalize demand for the competing brand. The strategic interactions between the manufacturer and the retailer are formally analyzed using a Stackelberg game framework, with the manufacturer designated as the market leader.

Initially, a basic revenue-sharing contract is investigated in the restricted case where the retailer exclusively offered the national-brand product. Recognizing the inherent limitations of this conventional contract in a dual-brand environment, we subsequently developed and proposed a novel two-part tariff revenue-sharing contract. This contract is designed to facilitate a more balanced, efficient, and Pareto-improving profit distribution among the supply chain members.

The findings, substantiated by comprehensive numerical examples and sensitivity analyses, confirm that the proposed two-part tariff revenue-sharing contract significantly enhances the economic performance of both parties and strengthens the NB's competitive market position. This model is particularly valuable for competitive supply chains characterized by asymmetric power dynamics, such as those involving a dominant national-brand manufacturer and a smaller store-brand retailer. Furthermore, the framework offers broad applicability across industries where such dynamics are prevalent, including quick-service restaurants, apparel, and electronics. For illustrative purposes, this structure mirrors scenarios in which a global leader (e.g., McDonald's in fast food) competes against a local, smaller-scale retailer offering a private-label alternative.

The model is subject to certain assumptions. It assumes a single-period horizon, as in many studies in this domain, and does not explicitly determine optimal product quality levels. Future research could address these limitations by extending the model to multiple periods and incorporating quality optimization. The model can be extended to investigate scenarios in which SBs are assumed to have sufficient infrastructure and flexibility to adjust their quality levels (e.g., in response to market changes or competition). This approach could yield profound insights into how firms simultaneously adjust these two strategic factors and how contract mechanisms might need to be redesigned to ensure supply chain coordination when quality becomes a strategic variable. Additionally, for the analysis of inventory accumulation, price adjustments, learning effects, or temporal variations in demand correlation, multiple periods can be considered. Exploring alternative contract designs, employing different game-theoretic approaches, adopting different demand distributions, and conducting comparative analyses may offer further insights. Additionally, examining the role of advertising and the impact of various distribution channels could be valuable extensions.

8. Declarations

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CRediT authorship contribution statement

Ata Allah Taleizadeh: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Validation. **Madjid Tavana:** Methodology, Formal analysis, Writing – original draft, Writing – review & editing, Visualization, Validation. **Razieh Sadeghi:** Validation, Writing – original draft, Writing – review & editing, Visualization. **Hamidreza Abedsoltan:** Validation, Writing – original draft, Writing – review & editing, Visualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Not applicable.

Appendix A. Proof of Lemma 1

From Eq. (8) we have;

$$\pi_r(Q_s) = (P_s - C_s)Q_s - \frac{P_s Q_s^2}{2(1-\gamma_{s\frac{P_s}{q_s}})N}$$

So the Hessian matrix should be determined as below: (A1)

$$H(Q_s, P_s) = \begin{bmatrix} \frac{\partial^2 \pi_r}{\partial Q_s^2} & \frac{\partial^2 \pi_r}{\partial Q_s \partial P_s} \\ \frac{\partial^2 \pi_r}{\partial Q_s \partial P_s} & \frac{\partial^2 \pi_r}{\partial P_s^2} \end{bmatrix} \tag{A2}$$

Where

$$\frac{\partial^2 \pi_r}{\partial Q_s^2} = -\frac{P_s}{(1 - \gamma_s \frac{P_s}{q_s})N} < 0 \tag{A3}$$

$$\frac{\partial^2 \pi_r}{\partial Q_s \partial P_s} = 1 - \frac{Q_s}{(1 - \gamma_s \frac{P_s}{q_s})^2 N} \tag{A4}$$

$$\frac{\partial^2 \pi_r}{\partial P_s^2} = -\frac{\gamma_s Q_s^2}{N q_s (1 - \gamma_s \frac{P_s}{q_s})^3} < 0$$

So the determinant of the Hessian matrix is; (A5)

$$\left(\frac{\partial^2 \pi_r}{\partial Q_s^2} \times \frac{\partial^2 \pi_r}{\partial P_s^2}\right) - \left(\frac{\partial^2 \pi_r}{\partial Q_s \partial P_s}\right)^2 = \frac{\gamma_s P_s Q_s^2}{N^2 q_s (1 - \gamma_s \frac{P_s}{q_s})^4} - (1 - \frac{Q_s}{(1 - \gamma_s \frac{P_s}{q_s})^2 N})^2 > 0 \tag{A6}$$

So it shows that the retailer's profit function is a negative definite for all P_s and Q_s and thus the profit function of the retailer is concave with respect to P_s and Q_s. After some algebra, we obtain the optimum values for P_s and Q_s and among the two obtained values for P_s, since $P_s = \frac{q_s - \sqrt{q_s(8\gamma_s C_s + q_s)}}{4\gamma_s}$ is negative, $P_s = \frac{q_s + \sqrt{q_s(8\gamma_s C_s + q_s)}}{4\gamma_s}$ should be used.

Appendix B. Proof of Lemma 2

Taking the first and the second order derivatives of Eq. (16) with respect to P_s, yield to:

$$\frac{\partial \pi_r(p_s)}{\partial p_s} = \frac{(\gamma_n + \gamma_s)}{4(q_n - q_s)} N[\frac{(P_s^2 - C_s^2)(p_n - p_s)}{P_s^2} - \frac{(P_s - C_s)^2}{P_s} + \frac{(P_n - W)^2}{P_n}] \tag{B1}$$

$$\frac{\partial^2 \pi_r(P_s)}{\partial P_s^2} = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s) p_s^3} N[C_s^2 P_n - P_s^3] \tag{B2}$$

Eq. (B1) at the beginning and end of the range is as follows:

$$\frac{\partial \pi_r(p_s)}{\partial p_s}|(P_s = C_s) = \frac{(\gamma_n + \gamma_s)N}{4(q_n - q_s)p_n}(P_n - W)^2 > 0 \tag{B3}$$

$$\frac{\partial \pi_r(p_s)}{\partial p_s}|(P_s = P_n) = \frac{(\gamma_n + \gamma_s)N}{4(q_n - q_s)P_n}[(W - C_s)(C_s + W - 2P_n)] \tag{B4}$$

And then, we check the concavity of the function at these points.

$$\frac{\partial^2 \pi_r(P_s)}{\partial P_s^2}|(P_s = C_s) = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s)C_s} N(p_n - C_s) > 0 \tag{B5}$$

$$\frac{\partial^2 \pi_r(P_s)}{\partial P_s^2}|(P_s = P_n) = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s)P_n^2} N(C_s^2 - P_n^2) < 0 \tag{B6}$$

With this assumption that $P_n > P_s > C_s$, when $W > C_s$, since the first order derivative at the beginning and the end of the range are positive and negative, respectively, the functional behaviors up to P_s^{III} is ascending and then descending.

At this stage, we need to examine that P_s^{III} is greater than $P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$ or not. If P_s^{III} is less than $P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$, according to the above calculations, the functional behaviors within $[P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n), P_n]$ is decreasing, and since we're looking to maximize the profit function, $P_s^* = P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$ is the optimal point. When P_s^{III} is bigger than $P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$, P_s^{III} is within the acceptable range, so the maximum profit occurs in the optimal value of P_s which is P_s^{III}.

Also, when $W < C_s$ the first order derivative at the beginning and end of the acceptable range of P_s is positive, so we have an increasing function, and the maximum value occurs when P_s^* is equal to P_n.

Appendix C. Proof of Lemma 3

The first and the second-order derivatives of Eq. (25) with respect to P_s are:

$$\frac{\partial \pi_{SC}^{CIII}(p_s)}{\partial p_s} = \frac{(\gamma_n + \gamma_s)}{4(q_n - q_s)} N[\frac{(P_s^2 - C_s^2)(p_n - p_s)}{P_s^2} - \frac{(P_s - C_s)^2}{P_s} + \frac{(P_n - C_n)^2}{P_n}] \tag{C1}$$

$$\frac{\partial^2 \pi_{SC}^{CIII}(P_s)}{\partial P_s^2} = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s)p_s^3} N[C_s^2 P_n - P_s^3] \tag{C2}$$

Similar to Lemma 2, the treats of it in the beginning and end of the range should be examined as below:

$$\frac{\partial \pi_{SC}^{CIII}(p_s)}{\partial p_s} |(P_s = C_s) = \frac{(\gamma_n + \gamma_s)N}{4(q_n - q_s)p_n} (P_n - C_n)^2 > 0 \tag{C3}$$

$$\frac{\partial \pi_{SC}^{CIII}(p_s)}{\partial p_s} |(P_s = P_n) = \frac{(\gamma_n + \gamma_s)N}{4(q_n - q_s)P_n} [(C_n - C_s)(C_s + C_n - 2P_n)] \tag{C4}$$

And then, we check the concavity of the function at these points.

$$\frac{\partial^2 \pi_{SC}^{CIII}(P_s)}{\partial P_s^2} |(P_s = C_s) = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s)C_s} N(p_n - C_s) > 0 \tag{C5}$$

$$\frac{\partial^2 \pi_{SC}^{CIII}(P_s)}{\partial P_s^2} |(P_s = P_n) = \frac{(\gamma_n + \gamma_s)}{2(q_n - q_s)P_n^2} N(C_s^2 - P_n^2) < 0 \tag{C6}$$

If $C_n > C_s$, then the first order derivative at the beginning and the end of the range, respectively, are positive and negative, so the functional behaviors up to P_s^{III} is ascending and then is descending. If $P_s^{III} < P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$, therefore, the functional behaviors within $[P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n), P_n]$ is decreasing, so P_s^* is equal to $P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$. Otherwise when $P_s^{III} > P_n - 2(q_n - q_s)/(\gamma_s + \gamma_n)$, P_s^{III} is within the acceptable range, so $P_s^* = P_s^{III}$.

Also, when $C_n < C_s$ the first order derivative at the beginning and end of the acceptable range of P_s is positive, so we have an increasing function, and the maximum value occurs when P_s^* is equal to P_n.

Data availability

Data will be made available on request.

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