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A novel pairwise comparison method with linear programming for multi-attribute decision-making

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ABSTRACT

This study introduces a novel approach to effectively and efficiently solve Multi-Attribute Decision Making (MADM) problems with a considerable number of attributes. We demonstrate the need to categorize the attributes and facilitate a more systematic expert comparison. Our proposed method utilizes pairwise comparisons to assess attributes without requiring additional computations to evaluate the level of consistency. The proposed method offers greater flexibility and precision with reduced computational complexity. We present a comparative analysis with a widely used numerical example in the MADM literature to demonstrate the effectiveness and efficacy of the method proposed in this study.

1. Introduction and literature review

Multi-criteria decision-making (MCDM) methods are one of the most widely used essential methods for managers of organizations and project managers. The quality of management is a function of the quality of decision-making because the quality of plans, the effectiveness and efficiency of strategies, and the quality of the results all depend on the quality of the managers' decisions. In most cases, decisions are made when the Decision Maker (DM) is satisfied when the decision is based on several criteria (qualitative or quantitative). MCDM methods are divided into two main categories: Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM). MODM methods are generally used for design, and MADM methods are used to select the superior alternative. The main difference between MODM and MADM methods is that the former is defined in the continuous decision space and the latter in the discrete decision space. MODM methods are proposed to solve an optimization problem with multiple objective functions. Such methods are widely used to solve problems in engineering, management, economics, medical, and social sciences (Guo et al., 2022; Soltanifar, 2021; Han and Tong, 2020). MADM methods, on the other hand, are used to select the best alternative from a finite number of alternatives based on several criteria. Despite the significant advancements in both MADM and MODM methods in recent years, some researchers argue that MADM methods are more widely applicable to real-world multi-criteria problems due to their lower computational complexity, greater simplicity, and broader applicability (Alinezhad and Khalili, 2019; Topcu et al., 2021; Xu and Tao, 2012).

MADM methods can be divided from different perspectives. From one perspective, MADM methods fall into two categories: compensatory and non-compensatory. The DM in compensatory models is willing to exchange criteria and indicators. This means that a change in the values of one criterion can be compensated by the values of other criteria (Trade-off). The DM is unwilling to exchange criteria in non-compensatory models such as dominance, maximin, maximax, lexicographic, and conjunctive constraint methods. The advantage of another does not offset the weakness of one criterion. Apart from other criteria, each criterion is the basis for evaluating competing alternatives. In these models, the criteria are examined independently in the decision-making process. Most MADM methods fall into the category of compensatory methods. In these methods, based on the decision matrix and criteria weights, the optimal alternative is prioritized and selected and has high flexibility in uncertainty logic (Chen and Tsai, 2021a; 2021b; Fahmi et al., 2019; Ashraf et al., 2018; Zhang and Hu, 2024; Dai et al. 2024). Using compensatory methods to analyze the information obtained from

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the decision matrix is more useful and has mathematical accuracy. For this reason, here are some of the most common ones.

The Analytical Hierarchy Process (AHP) is an accurate, robust, and reliable method for quantifying subjective judgments in MADM, first proposed by Saaty (1980). In this method, the alternatives are weighted by drawing the problem in a hierarchical tree and using the pairwise comparison matrix. Researchers quickly appreciated this method, and its many applications in industry and society were presented (Ishizaka and Labib, 2011). Many researchers have tried to change the theory of this method to eliminate its shortcomings and provide an improved version of it. Some tried to reduce the amount of information obtained from the DM to increase their motivation for participation (Abastante et al., 2019; Liu and Hai, 2005; Soltanifar and Lotfi, 2011; Leal, 2020; Tavana et al., 2023; Faramondi et al., 2023; Soltanifar et al., 2023b); others studied other shortcomings such as inconsistencies in judgments (Aguarón et al., 2020; Carpitella et al., 2022; Sáenz-Royo et al., 2024); some noted phenomena such as rank reversal and suggested ways to prevent it (Triantaphyllou, 2001); and some, provide a network version of this method called Analytical Network Process (ANP) in cases where the criteria are not independent (Saaty, 2013; Chiang et al., 2016).

The Best-Worst method (BWM) (Rezaei, 2015) is a popular multi-criteria method that uses pairwise comparison matrices, similar to the AHP method. BWM is also used as a weighting method in combination with multi-criteria methods using decision matrices. However, despite its advantages, the BWM still suffers from certain limitations, particularly in addressing the issue of inconsistency in evaluations (Liang et al., 2020). Unlike traditional pairwise comparison methods, the BWM focuses solely on comparisons between the best option and other alternatives, as well as between the worst option and other alternatives, thereby ignoring many pairwise comparisons that could potentially enhance the completeness of the final result. This selective approach to pairwise comparisons may result in overlooking valuable information that could contribute to the precision of the outcome. Therefore, while the BWM offers benefits such as low computational load compared to other MADM methods (Lei et al., 2022; Wu et al., 2024), it is essential to acknowledge its limitations in terms of ignoring a significant portion of pairwise comparisons, which could impact the completeness and usability of the results.

In many MADM problems, attributes are used in specific groupings. For instance, attributes may be categorized into economic, environmental, social, and other similar perspectives. It would be highly beneficial to compare alternatives within each attribute category separately and then make a summary decision based on the priority set by the decision-maker for each attribute category. The KEmeny Median Indicator Ranks Accordance (KEMIRA) method was introduced to address such issues (Krylovas et al., 2014). In this method, the final ranking of the alternatives is determined by experts in the form of two different groups after deciding the priority and weight of the attributes. While the method was initially presented for categorizing attributes into two groups, it is extendable to accommodate multiple categories. However, this method has certain limitations in practice, such as a significant increase in computational complexity as the number of criterion categories increases. This method has been commonly used in its improved versions in many decision-making problems (Krylovas et al., 2016; Kosareva et al., 2016; Krylovas et al., 2017; Kaplinski et al., 2019; Kiş et al., 2020; Delice and Can, 2020; Soltanifar, 2022; Soltanifar et al., 2023a; Ouedraogo and Metchebon Takougang, 2023). Also, additional variations of this method, considering uncertainty, can be found in Toktaş and Can (2019) and Onar et al. (2021).

Other popular compensatory methods are the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method (Hwang and Yoon, 1981; Divya et al., 2020; Çelikbilek and Tüysüz, 2020; Lo and Liou, 2021), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method (Fei et al., 2018; Rafieyan et al., 2020; Soltanifar and Sharafi, 2021), The ELimination Et Choice Translating Reality (ELECTRE) method (Roy, 1968; Liu and Wan, 2019), the simple additive weighting (SAW) method (Fishburn, 1967) and like them. Each method, from one point of view, prioritizes or weighting the alternatives. The implementation process is simple and complex. Also, the interaction rate with DM is low in some and high in others. Therefore, DM selects one of these methods based on the problem's size and considers each method's point of view. Choosing the proper method of decision is an art. Methods should be chosen so that the interaction with DM, computational complexity, and simplicity in execution are such that they ultimately satisfy DM. For example, in small-scale problems, the AHP method will be good because of the low computational complexity and good interaction with the DM, leading to satisfactory results. However, the same approach can reduce DM motivation to provide information for larger issues due to the high probability of judgment inconsistencies.

In many MADM problems, attributes are organized into diverse categories, such as economic, environmental, and social. It is advantageous to independently assess alternatives within each attribute category, facilitating a thorough and nuanced decision-making process. While methods like KEMIRA have been proposed to tackle such challenges, they possess certain limitations. These include inflexibility in managing an increasing number of attribute categories, reliance on attribute prioritization for weight determination, and complexity in calculating attribute weights. These limitations inevitably diminish the accuracy of final results due to the constrained information acquisition. Pairwise comparisons can mitigate these drawbacks instead of prioritization and exploiting linear programming problem properties to address these issues. This study introduces a method that not only overcomes the shortcomings of KEMIRA but also enhances the accuracy of final results and fosters a higher level of interaction with decision-makers. In essence, our approach aims to provide a more comprehensive interaction with decision-makers and improve the accuracy of final results. The subsequent sections of this paper are structured as follows. In Section 2, the motivation and algorithm of the proposed method are presented. In Section 3, the proposed method is used on a simple and familiar example in the MADM literature, and the results of the proposed method are compared with the results of one of the most widely used methods based on pairwise comparisons. In Section 4, to demonstrate the method's applicability, we apply it to a real-world problem. Section 5 contains managerial insight and suggestions for future research. Finally, the conclusion of the paper will be presented in Section 6.

2. Proposed Method: motivation and algorithm

In many MADM scenarios, especially when dealing with a considerable number of attributes that exhibit diverse characteristics, attributes are often categorized into two or more groups based on their characteristics. For instance, attributes may be grouped into economic, social, environmental, etc. This categorization facilitates a simpler and more logical comparison and expert prioritization of attributes within each group. Methods such as KEMIRA have been proposed to address the management of such complex issues. Similarly, this study introduces a method that offers greater flexibility, applicability, and accuracy compared to existing approaches. In the proposed method, instead of assigning priority to attributes within each group, pairwise comparisons of attributes within each group are solicited from experts. This acquisition of additional information from experts undoubtedly enhances the method's precision. While methods like AHP and its various versions also rely on pairwise comparisons, we propose an approach with reduced technical complexities compared to these methods. Next, we present the method for categorizing attributes into two groups, followed by its extension to a more significant number of groups.

Assume that n homogeneous alternatives $A_1, A_2, \ldots, A_n$ are to be evaluated considering the criteria. First, the decision-makers are asked to separate the criteria into two (or more) groups according to their type and characteristics and compile the decision matrix based on this classification. In many real-world problems, this type of segmentation is useful and sometimes necessary to better use the DM's opinions in prioritizing criteria. It is also assumed that in the first group, there are m criteria in the form of $C_1, C_2, \ldots, C_m$ and in the second group, s criteria in the form of $\overline{C}_1, \overline{C}_2, \ldots, \overline{C}_s$. Thus, the overall composition of the decision matrix will be in the form of Eq. (1).

$$D = \left[\begin{array}{ccc|ccc} x_{11} & \cdots & x_{1m} & y_{11} & \cdots & y_{1s} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} & y_{n1} & \cdots & y_{ns} \end{array}\right] \tag{1}$$

where the first m columns correspond to the first criteria category and the second s columns correspond to the second category criteria. Without losing the generality of the problem, assume that all criteria are of the profit type (otherwise, the elements related to the cost criteria are converted into profit by reversing them). Thus, the steps related to KEMIRA will be as follows.

Step 1: Normalize decision matrix (1) by utilizing Eqs. (2) and (3).

$$\widehat{x}_{ji} = \frac{x_{ji} - x_i^-}{x_i^+ - x_i^-} \qquad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{2}$$

$$\widehat{y}_{jr} = \frac{y_{jr} - y_r^-}{y_r^+ - y_r^-} \qquad r = 1, 2, \ldots,\ s;\ j = 1, 2, \ldots, n \tag{3}$$

where $x_i^- = \underset{1 \le j \le n}{min}\, x_{ji},\ i = 1, 2, .., m;\ x_i^+ = \underset{1 \le j \le n}{max} x_{ji},\ i = 1, 2, .., m\ \ y_r^- = \underset{1 \le j \le n}{min}\, y_{jr},\ r = 1, 2, .., s$ and $y_r^+ = \underset{1 \le j \le n}{max} y_{jr},\ r = 1, 2, .., s$. Other normalization methods can be found in Krylovas et al. (2018; 2019; 2020).

Step 2. In this step, with the help of experts, we determine the pairwise comparisons of criteria in each category based on Saaty's 9-point range (Saaty, 1980). Suppose $a_{iq}^k (i = 1, 2, \ldots, m;\ q = 1, 2, \ldots m)$ is the pairwise comparison of the i^{th} criterion to the q^{th} criterion in the first category and $\overline{a}_{lr}^k (l = 1, 2, \ldots, s;\ r = 1, 2, \ldots s)$ is the pairwise comparison of the l^{th} criterion over the r^{th} criterion in the second category according to the k^{th} expert ($k = 1, 2, \ldots, K$).

Step 3. We now find the distance between the pairwise comparisons of criteria by each expert with other experts in the two criteria categories identified through Eqs. (4) and (5).

$$\rho_k = \sum_{\overline{k}=1}^{K} \left(\sum_{i=1}^{m} \sum_{q=1}^{m} \left| a_{iq}^k - a_{iq}^{\overline{k}} \right| \right), \qquad k = 1, 2, \ldots K \tag{4}$$

$$\overline{\rho}_k = \sum_{\overline{k}=1}^{K} \left(\sum_{l=1}^{s} \sum_{r=1}^{s} \left| \overline{a}_{lr}^k - \overline{a}_{lr}^{\overline{k}} \right| \right), \quad k = 1, 2, \ldots K \tag{5}$$

Step 4. In this step, the expert's pairwise comparisons with a minimum distance of pairwise comparisons with other experts are determined as the reference pairwise comparisons in each criteria category. In other words, if we consider Eqs. (6) and (7), the expert k^* will be the reference expert for the first category criteria, the expert $k^\dagger$will be the reference expert for the second category criteria, and the pairwise comparisons of these experts will be used as a reference to determine the weights of the criteria called "median pairwise comparison components."

$$\rho_{k^*} = \underset{1 \le k \le K}{min}\, \rho_k \tag{6}$$

$$\overline{\rho}_{k^\dagger} = \underset{1 \le k \le K}{min}\, \overline{\rho}_k \tag{7}$$

Step 5. In this step, we obtain the final weights of the attributes by solving a multi-objective linear model. Recognizing that categorizing attributes based on their characteristics into two or more groups can simplify and rationalize their comparison, we can assign scores to alternative options based on the attributes of each group ($\sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji};\ j = 1, 2, \ldots, n$ scores for alternatives in the first group of attributes and $\sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr};\ j = 1, 2, \ldots, n$ scores for alternatives in the second group of attributes). If different categorizations are equally important to the decision-maker, then as a primary policy, we adjust the weights of the attributes to minimize the differences in scores for alternatives across different groups ($\min \sum_{j=1}^{n} \left(\left| \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr} \right| \right)$). However, there is flexibility for the decision-maker to weigh these scores differently, perhaps by making the score of one group twice that of another group, based on their preferences ($\min \sum_{j=1}^{n} \left(\left| \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - 2 \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr} \right| \right)$). Furthermore, since pairwise comparisons of attributes within each group are obtained, the second policy aims to align the attribute weights with these pairwise comparisons. Considering these two policies, the weights of the attributes are derived from the multi-objective programming model (8).

$$\begin{aligned} & \min \sum_{j=1}^{n} \left(\left| \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr} \right| \right) \\ & \min \xi_1 \\ & \min \xi_2 \\ & s.t. \\ & \left| v_i^{k^*} - a_{iq}^{k^*} v_q^{k^*} \right| \le \xi_1, \qquad i = 1, 2, \ldots, m;\ q = 1, 2, \cdots m \\ & \sum_{i=1}^{m} v_i^{k^*} = 1 \\ & \left| u_l^{k^\dagger} - \overline{a}_{lr}^{k^\dagger} u_r^{k^\dagger} \right| \le \xi_2, \qquad l = 1, 2, \cdots s;\ r = 1, 2, \ldots, s \\ & \sum_{r=1}^{s} u_r^{k^\dagger} = 1 \\ & v_i^{k^*} \ge 0, \qquad i = 1, 2, \ldots, m \\ & u_r^{k^\dagger} \ge 0, \qquad r = 1, 2, \ldots, s \end{aligned} \tag{8}$$

In this multi-objective programming model, the normalized weights of the attributes are calculated by considering two primary policies. The first policy, expressed in the form of the first objective function in the model, determines the weights of the attributes in two categories so that the difference in the final result of each alternative is minimized by considering the attributes of different categories. This will be done because the attributes are equally important in the two categories. We are looking for weights for the attributes to create the proper balance between the results of each attribute category. Of course, we can specify a weight for each set of attributes' final result, which can also be considered in the first objective function. The second policy is to find normalized weights to minimize inconsistencies in the judgments of reference experts in two sets of attributes. The second and third objective functions follow this policy and seek to determine the weight of the attributes to achieve the least inconsistency in judgments. It is clear that if a_{iq} is a pairwise comparison between attributes i and q with w_i and w_q as their weights then in the case of consistency $a_{iq} = \frac{w_i}{w_q}$, and therefore $|w_i - a_{iq} w_q|$ will show the rate of inconsistency of the judgment. The value of the second and third objective functions represents an indicator to determine the degree of inconsistency in judgments. The closer these values are to zero, the more consistent the judgments are and the greater their reliability. Indeed, one can examine acceptable inconsistency, similar to other methods based on pairwise comparisons, to ensure the rationality of evaluations (Liang et al., 2020).

Since the first objective function is a piecewise linear function, model (8) can be converted into a multi-objective linear model. For this purpose, by applying the variable change (9), the model (8) becomes the model (10).

$$\begin{cases} \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr} = P_j^+ - P_j^- , & j = 1,2,\ldots,n \\ P_j^+, P_j^- \geq 0 \end{cases} \tag{9}$$

In fact $P_j^+ = max\left\{ \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr}, 0 \right\}$ and $P_j^- = min\left\{ \sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr}, 0 \right\}$. Therefore $|\sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr}| = P_j^+ + P_j^-$ and $P_j^+ \times P_j^- = 0$.

$$\min \sum_{j=1}^{n} \left(P_j^+ + P_j^- \right) \tag{10}$$

$$\min \xi_1$$

$$\min \xi_2$$

$$s.t.$$

$$\sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji}^{k^*} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr}^{k^\dagger} = P_j^+ - P_j^- , \ j = 1,2,\ldots,n$$

$$\left| v_i^{k^*} - a_{iq}^{k^*} v_q^{k^*} \right| \leq \xi_1, \quad i = 1,2,\ldots,m;\ q = 1,2,\cdots m$$

$$\sum_{i=1}^{m} v_i^{k^*} = 1$$

$$\left| u_l^{k^\dagger} - \overline{a}_{lr}^{k^\dagger} u_r^{k^\dagger} \right| \leq \xi_2, \quad l = 1,2,\cdots s;\ r = 1,2,\ldots,s$$

$$\sum_{r=1}^{s} u_r^{k^\dagger} = 1$$

$$v_i^{k^*} \geq 0, \quad i = 1,2,\ldots,m$$

$$u_r^{k^\dagger} \geq 0, \quad r = 1,2,\ldots,s$$

$$P_j^+ \times P_j^- = 0,\ j = 1,2,\ldots,n$$

$$P_j^+, P_j^- \geq 0,\ j = 1,2,\ldots,n$$

Model (10) is a non-linear multi-objective model due to the existence of the constraint$P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$. By proving Theorem 1, these constraints can be removed from the model. Thus, model (10) (after removing constraints $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$) becomes a linear multi-objective model.

Theorem 1. *Constraints $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$ can be removed from the model (10).*

Proof. It should be shown that for every solution that exists in all constraints of model (10) except the $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$, there exists a feasible solution of model (10) with a better value of the objective function. Therefore, removing $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$ from the model (10) will not affect finding the optimal solution. For the solution $\left(v_1^{k^*}, \ldots, v_m^{k^*}, u_1^{k^\dagger}, \ldots, u_s^{k^\dagger}, \widehat{P}_1^+, \ldots, \widehat{P}_n^+, \widehat{P}_1^-, \ldots, \widehat{P}_n^-\right)$, assume that all the constraints of the model (10) except constraints $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$ are true. Also assume $P_j^{min} = min\left\{P_j^+, P_j^-\right\}$, $j = 1,2,\ldots,n$. It is clear that if $\widetilde{P}_j^+ = \widehat{P}_j^+ - P_j^{min}$, $\widetilde{P}_j^- = \widehat{P}_j^- - P_j^{min}; j = 1,2,\ldots,n$, then $\left(v_1^{k^*}, \ldots, v_m^{k^*}, u_1^{k^\dagger}, \ldots, u_s^{k^\dagger}, \widetilde{P}_1^+, \ldots, \widetilde{P}_n^+, \widetilde{P}_1^-, \ldots, \widetilde{P}_n^-\right)$ holds for all constraints of model (10) and also has a better objective function value than the previous solution.□

Model (10), after removing constraints $P_j^+ \times P_j^- = 0$, $j = 1,2,\ldots,n$, will be a linear multi-objective model and, therefore, can be solved by multi-objective problem-solving methods such as the weighting method, absolute priority method, conversion of the objective function to constraints method, and goal programming method. The goal of all three objective functions is zero. Therefore, model (10) can be converted to linear programming model (11) using the goal programming method.

$$\begin{aligned}
&\min\ d + \xi_1 + \xi_2 \\
&s.t. \\
&\sum_{j=1}^{n} \left(P_j^+ + P_j^- \right) - d = 0 \\
&\sum_{i=1}^{m} v_i^{k^*} \widehat{x}_{ji}^{k^*} - \sum_{r=1}^{s} u_r^{k^\dagger} \widehat{y}_{jr}^{k^\dagger} = P_j^+ - P_j^- , \ j = 1,2,\ldots,n \\
&\left| v_i^{k^*} - a_{iq}^{k^*} v_q^{k^*} \right| \leq \xi_1, \quad i = 1,2,\ldots,m;\ q = 1,2,\cdots m \\
&\sum_{i=1}^{m} v_i^{k^*} = 1 \\
&\left| u_l^{k^\dagger} - \overline{a}_{lr}^{k^\dagger} u_r^{k^\dagger} \right| \leq \xi_2, \quad l = 1,2,\cdots s;\ r = 1,2,\ldots,s \\
&\sum_{r=1}^{s} u_r^{k^\dagger} = 1 \\
&v_i^{k^*} \geq 0, \quad i = 1,2,\ldots,m \\
&u_r^{k^\dagger} \geq 0, \quad r = 1,2,\ldots,s \\
&P_j^+, P_j^- \geq 0,\ j = 1,2,\ldots,n
\end{aligned} \tag{11}$$

Step 6. Finally, after solving the linear programming model (11) and finding the optimal weights for the criteria, the ranking basis for the j^{th} alternative will be obtained through Eq. (12). In other words, an alternative with a higher E_j, $j = 1,2,\ldots,n$ will have a better rating.

$$E_j = \sum_{i=1}^{m} v_i^* \widehat{x}_{ji} + \sum_{r=1}^{s} u_r^* \widehat{y}_{jr},\ j = 1,2,\ldots,n \tag{12}$$

A summary of the proposed method in the form of a flowchart can be seen in Fig. 1.

In the next section, we describe the method presented in this section using a simple and familiar numerical example in the MADM literature.

3. A simple example

In this section, we apply the proposed method to a familiar numerical example from the MADM literature (Bodin and Gass, 2004). Saaty (2013) introduced this numerical example for choosing the best car among three alternatives (Acura TL (A.TL.), Toyota Camry (T.C.), and Honda Civic (H.C.)) by considering the following criteria: Prestige, Comfort, Price, and Miles per gallon (MPG). This example is usually presented in the MADM literature as a hierarchical structure. For example, consider using the Group AHP (GAHP) method with three experts in this example. Once the group has agreed on the hierarchy, pairwise comparison matrices must be created at each level. There are two ways to create pairwise comparison matrices: "unanimous judgment" and "integration of personal judgments." Suppose the second

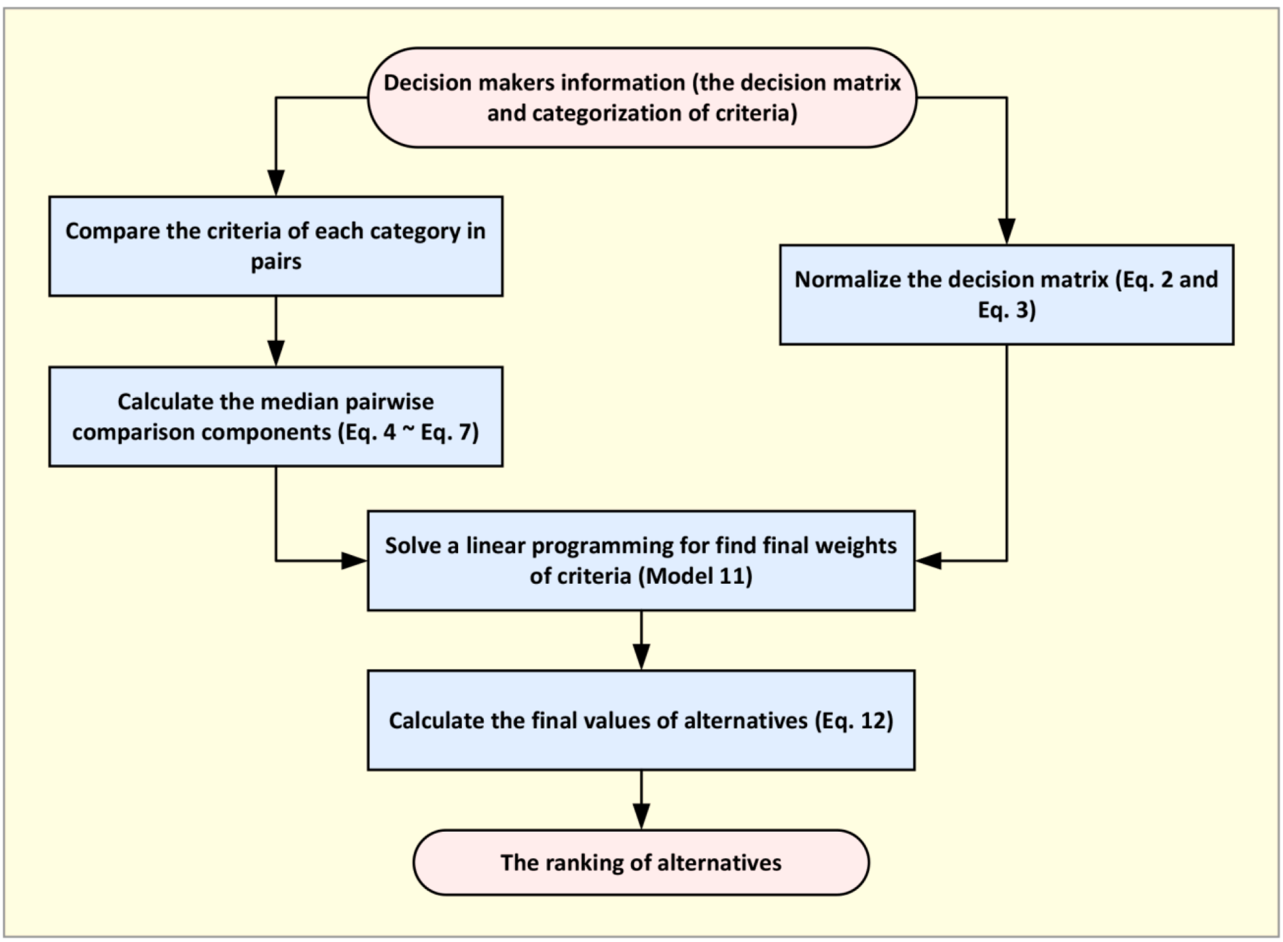

Fig. 1. Flowchart of the proposed method.

Table 1
Pairwise comparison matrices for choosing the best car (Bodin and Gass, 2004).

a. Goal-level pairwise comparisons				
Goal	Prestige	Price	MPG	Comfort
Prestige	1	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
Price	4	1	3	$\frac{3}{2}$
MPG	3	$\frac{1}{3}$	1	$\frac{1}{3}$
Comfort	2	$\frac{2}{3}$	3	1
b. Prestige pairwise comparisons				
Prestige	A.TL.	T.C.	H.C.	
A.TL.	1	8	4	
T.C.	$\frac{1}{8}$	1	$\frac{1}{4}$	
H.C.	$\frac{1}{4}$	4	1	
c. Price pairwise comparisons				
Price	A.TL.	T.C.	H.C.	
A.TL.	1	$\frac{1}{4}$	$\frac{1}{9}$	
T.C.	4	1	$\frac{1}{5}$	
H.C.	9	5	1	
d. MPG pairwise comparisons				
MPG	A.TL.	T.C.	H.C.	
A.TL.	1	$\frac{2}{3}$	$\frac{1}{3}$	
T.C.	$\frac{3}{2}$	1	$\frac{1}{2}$	
H.C.	3	2	1	
e. Comfort pairwise comparisons				
Comfort	A.TL.	T.C.	H.C.	
A.TL.	1	4	7	
T.C.	$\frac{1}{4}$	1	3	
H.C.	$\frac{1}{7}$	$\frac{1}{3}$	1	

method is chosen for this purpose. We ask each expert to provide their desired pairwise comparison matrices at each level. Then, we integrate the pairwise comparison matrices obtained from each expert at each level. Aczel and Saaty (1983) showed that the "Geometric Mean (GM) method" is the best way to integrate judgments in the GAHP. Assume that the pairwise comparison matrices in Table 1 result from integrating three expert judgments by the GM method. Applying the GAHP method to these matrices leads to the results presented in Fig. 2.

In this section, we will form a decision matrix and implement the proposed method by dividing the criteria into objective (Price and Miles per gallon) and subjective (Prestige and Comfort). Table 2 shows the decision matrix and normalized decision matrix (using Eqs. (2) and (3)) for choosing the best car.

We now ask the experts to provide pairwise comparisons of the criteria in each category. Suppose the results of these comparisons are presented in Table 3. Note that the GM of these pairwise comparisons is used in the GAHP method.

We now calculate the distance between the pairwise comparisons of criteria by each expert with other experts in two categories of criteria

Table 2
Decision matrix and normalized decision matrix for choosing the best car.

Decision Matrix	Subjective Criteria		Objective Criteria	
	Prestige	Comfort	MPG	Price
Acura TL	$x_{11}=0.707$	$x_{12}=0.705$	$y_{11}=0.182$	$y_{12}=0.063$
Toyota Camry	$x_{21}=0.070$	$x_{22}=0.211$	$y_{21}=0.273$	$y_{22}=0.194$
Honda Civic	$x_{31}=0.223$	$x_{32}=0.084$	$y_{31}=0.545$	$y_{32}=0.743$
Normalized Decision Matrix	**Subjective Criteria**		**Objective Criteria**	
	Prestige	**Comfort**	**MPG**	**Price**
Acura TL	$\hat{x}_{11}=1$	$\hat{x}_{12}=1$	$\hat{y}_{11}=0$	$\hat{y}_{12}=0$
Toyota Camry	$\hat{x}_{21}=0$	$\hat{x}_{22}=0.20$	$\hat{y}_{21}=0.25$	$\hat{y}_{22}=0.19$
Honda Civic	$\hat{x}_{31}=0.24$	$\hat{x}_{32}=0$	$\hat{y}_{31}=1$	$\hat{y}_{32}=1$

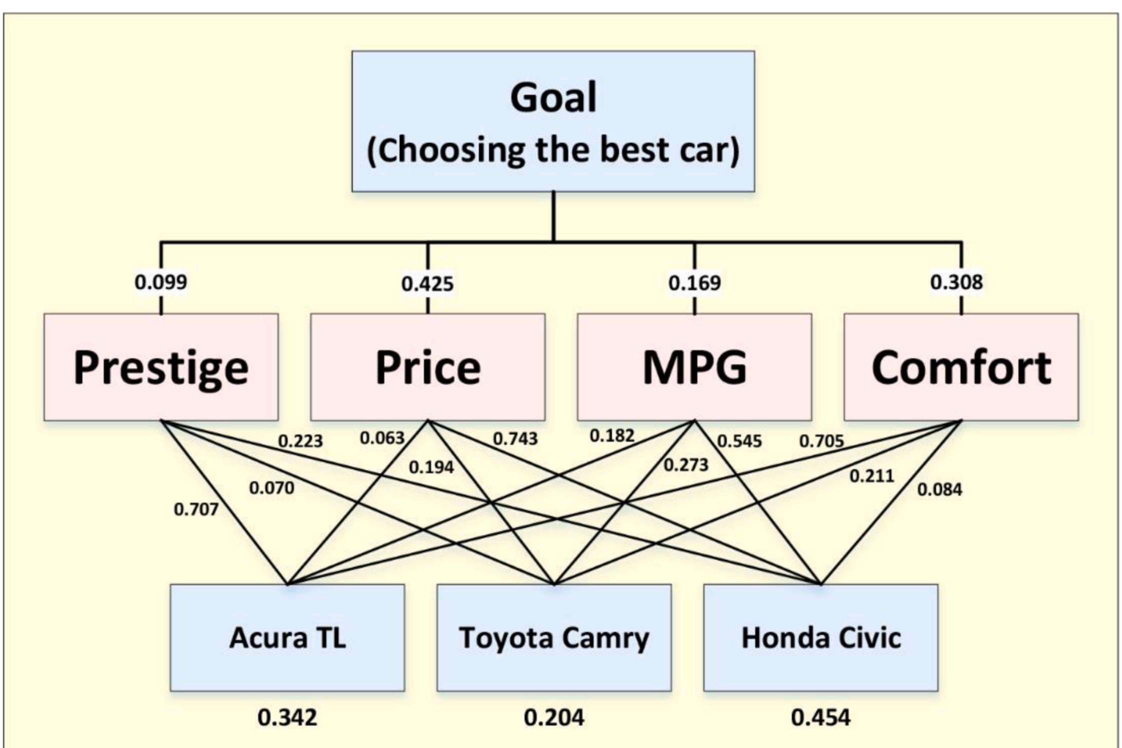

Fig. 2. Results of choosing the best car using the GAHP method (Bodin and Gass, 2004).

Table 3
The pairwise comparison matrices for the criteria in each category.

Expert 1	Prestige	Comfort	**Expert 2**	Prestige	Comfort	**Expert 3**	Prestige	Comfort
Prestige	1	1	Prestige	1	$\frac{1}{4}$	Prestige	1	$\frac{1}{2}$
Comfort	1	1	Comfort	4	1	Comfort	2	1
Expert 1	MPG	Price	**Expert 2**	MPG	Price	**Expert 3**	MPG	Price
MPG	1	$\frac{1}{3}$	MPG	1	$\frac{1}{5}$	MPG	1	$\frac{5}{9}$
Price	3	1	Price	5	1	Price	$\frac{9}{5}$	1

using Eqs. (4) and (5) and present the results as follows:

$\rho_{Expert\,1} = 5.25$	$\overline{\rho}_{Expert\,1} = 3.56$
$\rho_{Expert\,2} = 6$	$\overline{\rho}_{Expert\,2} = 5.69$
$\rho_{Expert3} = 3.75$	$\overline{\rho}_{Expert3} = 4.98$

Based on Eqs. (6) and (7), Expert 3 and Expert 1 are the reference experts for the first and second criteria categories, respectively. Therefore, using pairwise comparisons identified by these experts, we calculate the weight of the criteria by solving model (11) as follows:

Criteria	**Subjective Criteria**		**Objective Criteria**	
	Prestige	**Comfort**	**MPG**	**Price**
Weights	0.33	0.67	0.25	0.75

Now, using Eq. (12), the score and rank of the alternatives can be obtained as follows:

Alternatives	**Acura TL**	**Toyota Camry**	**Honda Civic**
Score	1	0.34	1.08
Rank	2	3	1

As shown here, the ranking results are consistent with those obtained in GAHP. It should be noted, however, that in all pairwise comparison-based methods, including GAHP, the possibility of inconsistency in pairwise comparison results exists. Complex methods are often proposed to investigate acceptable inconsistency to ensure the rationality of evaluations. In the proposed method, additional computations to examine this issue are unnecessary. The optimal values of ξ_1 and ξ_2 in the model (11) can serve as a suitable basis for expressing acceptable inconsistency. A value of zero for these parameters indicates the compatibility of evaluations, and an approximation close to zero can be considered to ensure the rationality of evaluations.

In many MADM methods, attributes are categorized into different groups based on their characteristics, and their priorities within each group are determined according to expert opinions. This categorization allows for comparability among attributes within each group and makes their prioritization more logical for experts. Subsequently, the final weights of the attributes are calculated based on their priorities within each group. One such familiar method in this domain is the KEMIRA method. In KEMIRA, alongside considering the priority of attributes within each group in weight calculation, weights are calculated to maximize the similarity among alternative evaluations within different groups.

Similarly, our study introduces a method that, in certain aspects, outperforms similar methods. You can follow the solution to a simple example using the KEMIRA method in the appendix. Unlike KEMIRA, which precisely selects weights from a predefined list of authentic weights in priority, our method selects weights from the continuous feasible space of the linear programming problem due to employing a linear programming formulation. While the computational complexity of KEMIRA significantly increases with the precision of weights or the number of attribute categorizations, our proposed method only adds a limited number of constraints to the linear programming problem, hence providing greater flexibility in practical applications. Moreover, in KEMIRA, experts only consider the priority of attributes within each group. In contrast, in our proposed method, pairwise comparisons among attributes within each group are determined in interaction with experts, resulting in a significantly higher accuracy of the proposed method for obtaining more detailed insights from experts. In the next section, we present a problem where its attributes are categorized into three groups.

4. Case Study

One of the important factors for survival in today's highly competitive environment is the reduction of production costs. Choosing the right suppliers can significantly reduce purchasing costs and increase the organization's and manufacturing companies' competitiveness because, in most industries, the cost of raw materials and components of the product is a large part of the cost price. A variety of methods have been proposed by researchers as decision-support tools to help decision-makers cope with the complexities of choosing the right supplier (De Boer et al., 2001; Deshmukh and Chaudhari, 2011; Aouadni et al., 2019; Naqvi and Amin, 2021; Soltanifar and Sharafi, 2021). Organizations have now realized that the bid price is not the only criterion for selection and cooperation with the supplier. Supplier selection is a complex process that involves various quantitative and qualitative criteria. In this section, to show the applicability of the proposed method, the issue of selecting and rating potential suppliers in the automotive industry is discussed.

The criteria for selecting a supplier and identifying potential suppliers have been examined in an automotive group to supply a specific part. For this purpose, four experts in this field have been used. They examined the selection criteria in 3 main categories, and in each category, after discussion and review, they identified four criteria using the Delphi method (Helmer, 1977; Mauksch et al., 2020). Then, four potential suppliers were identified, and potential suppliers were evaluated using 12 criteria. Then, the final decision matrix is presented in Table 4. In this matrix, the scores of each supplier in each criterion are entered after converting the cost criteria into profit criteria.

Experts selected product quality, price, flexibility, supply time, green production, green transport, pollution control, environmental response, financial ability, technology, reliability, and standard certificate as the criteria for supplier selection after studying the criteria discussed in the literature and using the Delphi method. They also classified these criteria into three categories: operational, environmental, and credit (Suraraksa and Shin, 2019; Bhatia and Ganagwani, 2021; Dweiri et al., 2016). This matrix is normalized by Eqs. (2) and (3) in Table 5.

Experts compared the criteria in pairs to determine their importance in each category. Four experts in Table 6 present the pairwise comparison matrices of the criteria in each category.

At this stage, the reference expert in each category must be specified. A reference expert is an expert who has made judgments in each category closer to those of other experts. Therefore, first, the distance between the judgments of each expert and the judgments of other experts should be calculated through Eqs. (4) and (5), and then an expert with the shortest distance of judgments should be selected as the reference expert in each category. Judgments of this expert in each category will be the basis for decision-making. It should be noted that the method presented in Section (2) for dividing the criteria into two categories was presented, while in this case, we are faced with dividing the criteria into three categories, and we must update the steps based on the fact; that it

Table 4
Supplier selection decision matrix.

Suppliers	Operational				Environmental				Credit			
	Product quality	Price	Flexibility	Supply time	Green production	Green transport	Pollution control	Environmental response	Financial ability	Technology	Reliability	Standard certificate
Supplier A	8.23	8.59	8.52	8.28	9.24	8.43	8.14	8.12	8.55	9.43	9.27	8.76
Supplier B	7.52	8.14	7.51	7.56	8.09	7.23	7.58	7.57	7.87	7.21	8.54	7.01
Supplier C	9.54	7.15	9.03	9.19	9.51	9.22	9.52	9.87	9.45	9.58	9.51	9.45
Supplier D	6.13	7.56	7.81	5.43	5.65	4.51	6.09	5.57	6.52	6.65	7.76	6.23

Table 5
Supplier selection normalized decision matrix.

Suppliers	Operational				Environmental				Credit			
	Product quality	Price	Flexibility	Supply time	Green production	Green transport	Pollution control	Environmental response	Financial ability	Technology	Reliability	Standard certificate
Supplier A	0.62	1.00	0.66	0.76	0.93	0.83	0.60	0.59	0.69	0.95	0.86	0.79
Supplier B	0.41	0.69	0.00	0.57	0.63	0.58	0.43	0.47	0.46	0.19	0.45	0.24
Supplier C	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Supplier D	0.00	0.28	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6
The pairwise comparison matrices for the criteria in each category.

Operational				
Expert 1	**Product quality**	**Price**	**Flexibility**	**Supply time**
Product quality	1.00	7.00	4.00	5.00
Price	0.14	1.00	0.20	0.17
Flexibility	0.25	5.00	1.00	0.50
Supply time	0.20	2.00	2.00	1.00
Expert 2	**Product quality**	**Price**	**Flexibility**	**Supply time**
Product quality	1.00	8.00	7.00	6.00
Price	0.13	1.00	0.33	0.33
Flexibility	0.14	3.00	1.00	0.50
Supply time	0.17	3.00	2.00	1.00
Expert 3	**Product quality**	**Price**	**Flexibility**	**Supply time**
Product quality	1.00	6.00	5.00	4.00
Price	0.17	1.00	0.33	0.50
Flexibility	0.20	3.00	1.00	3.00
Supply time	0.25	2.00	0.33	1.00
Expert 4	**Product quality**	**Price**	**Flexibility**	**Supply time**
Product quality	1.00	3.00	2.00	4.00
Price	0.33	1.00	0.50	6.00
Flexibility	0.50	2.00	1.00	6.00
Supply time	0.25	0.17	0.17	1.00
Environmental				
Expert 1	**Green production**	**Green transport**	**Pollution control**	**Environmental response**
Green production	1.00	4.00	2.00	3.00
Green transport	0.25	1.00	0.33	0.50
Pollution control	0.50	3.00	1.00	3.00
Environmental response	0.33	2.00	0.33	1.00
Expert 2	**Green production**	**Green transport**	**Pollution control**	**Environmental response**
Green production	1.00	3.00	0.25	0.33
Green transport	0.33	1.00	0.20	0.33
Pollution control	4.00	5.00	1.00	4.00
Environmental response	3.00	3.00	0.25	1.00
Expert 3	**Green production**	**Green transport**	**Pollution control**	**Environmental response**
Green production	1.00	5.00	2.00	3.00
Green transport	0.20	1.00	0.33	0.50
Pollution control	0.50	3.00	1.00	3.00
Environmental response	0.33	2.00	0.33	1.00
Expert 4	**Green production**	**Green transport**	**Pollution control**	**Environmental response**
Green production	1.00	4.00	2.00	3.00
Green transport	0.25	1.00	0.50	0.33
Pollution control	0.50	2.00	1.00	0.50
Environmental response	0.33	3.00	2.00	1.00
Credit				
Expert 1	**Financial ability**	**Technology**	**Reliability**	**Standard certificate**
Financial ability	1.00	0.25	0.33	0.50
Technology	4.00	1.00	4.00	5.00
Reliability	3.00	0.25	1.00	3.00
Standard certificate	2.00	0.20	0.33	1.00
Expert 2	**Financial ability**	**Technology**	**Reliability**	**Standard certificate**
Financial ability	1.00	0.33	0.50	0.25
Technology	3.00	1.00	4.00	0.50
Reliability	2.00	0.25	1.00	0.25
Standard certificate	4.00	2.00	4.00	1.00
Expert 3	**Financial ability**	**Technology**	**Reliability**	**Standard certificate**
Financial ability	1.00	0.20	0.50	0.33
Technology	5.00	1.00	4.00	0.50
Reliability	2.00	0.25	1.00	0.50
Standard certificate	3.00	2.00	2.00	1.00
Expert 4	**Financial ability**	**Technology**	**Reliability**	**Standard certificate**
Financial ability	1.00	0.20	0.50	0.33
Technology	5.00	1.00	3.00	0.50
Reliability	2.00	0.33	1.00	0.50
Standard certificate	3.00	2.00	2.00	1.00

is a simple task. The distance between the judgments of each expert and other experts in each category of criteria is as follows:

Distance	Operational	Environmental	Credit
Expert 1	44.01	**23.60**	46.00
Expert 2	49.62	53.37	29.23
Expert 3	**39.41**	25.70	**20.40**
Expert 4	73.57	34.27	22.57

Therefore, based on the above results, in the first category (Operational), the third expert, in the second category (Environmental), the first expert, and in the third category (Credit), the third expert will be the reference experts. The results of their judgments will be the basis for the decision. In fact, in each category, we base our conclusions on expert judgments most similar to those of other experts in that category. We are looking for a supplier who considers the criteria in the three categories in a balanced way, and therefore, we must solve model (11) to achieve the criteria weights. Of course, this model is designed for dual categorization of criteria, and before implementation, we must update the model for triple categorization, which is a simple task. The following weights are the result of these calculations:

Criteria	Operational				Environmental				Credit			
	Product quality	Price	Flexibility	Supply time	Green production	Green transport	Pollution control	Environmental response	Financial ability	Technology	Reliability	Standard certificate
Weights	0.64	0.08	0.16	0.12	0.48	0.11	0.29	0.13	0.10	0.32	0.13	0.45

The proposed model for this problem provides values to assess the consistency of judgments. These values are obtained for pairwise comparison matrices provided by reference experts as $\xi_1 = 0.18132$, $\xi_2 = 0.09524$ and $\xi_3 = 0.19231$. The closer these values are to zero, the greater the consistency of the judgments. The values obtained in this case provide an acceptable consistency for the results provided by the experts. If we calculate the Inconsistency Ratio (I.R.) of the pairwise comparison matrices presented in Table 6 from the method presented in AHP, the following values are obtained:

I.R.	Operational	Environmental	Credit
Expert 1	0.02	0.03	0.08
Expert 2	0.06	0.10	0.05
Expert 3	0.08	0.02	0.09
Expert 4	0.10	0.05	0.07

All the Inconsistency Ratios of all the pairwise comparison matrices are acceptable. Thus, according to the weights obtained for the criteria, the score of each supplier can be calculated through Eq. (12), and based on the score obtained, the final ranking of suppliers can be presented. The scoring and ranking of each supplier is as follows:

Suppliers	Scores	Ranks
Supplier A	2.29	2
Supplier B	1.20	3
Supplier C	2.92	1
Supplier D	0.05	4

In this way, not only was the priority of suppliers determined, but according to the scores obtained by each supplier, its distance from other suppliers could be determined, and a powerful tool for decision support could be provided to the DM. The next section will provide some managerial insights and suggestions for future research.

5. Managerial insights and suggestions for future research

Decision-making is an integral part of management and is evident in every management task; decision-making is essential in determining the organization's policies, designing the organization, selecting and evaluating, and all management actions. According to many decision scientists, decision-making is the essence of management, and management can even be considered synonymous with it. Therefore, there is always a need to provide methods to support the decision. Managers tend to use methods that interact with experts while using scientific approaches to increase their motivation and participation in the decision-making process. Methods without solid mathematical backing or confuse and demotivate experts in the decision-making process do not provide applicable results. One of the new and widely used methods to support decision-making is the AHP method, which is presented in a group version called GAHP. This method is based on pairwise comparisons of elements at the criteria level, sub-criteria, and alternatives relative to the criteria. But in cases where the number of criteria increases, experts are forced to complete pairwise comparison matrices with more dimensions, increasing the possibility of inconsistency of judgments, and the DM is forced to achieve an acceptable inconsistency ratio by a trade-off. This process sometimes causes experts to be unmotivated.

In this paper, a MADM method based on pairwise comparisons is presented. The method in which criteria are first divided into several categories based on their nature and pairwise comparisons are made between the criteria of each category. In this way, the experts compare the comparable criteria in pairs and provide the final matrices. Then, the results of pairwise comparisons of experts in each category are compared, and the results presented by an expert most similar to those of other experts are selected. As a suggestion for future research, we could combine the results of the pairwise comparison matrices provided by the experts using the GM method. It is also possible to design the proposed method considering the uncertainty logic. The criteria weights in this method are obtained by solving a linear programming problem. The proposed linear programming problem not only suggests the weights of the criteria but also provides an indicator for determining the inconsistency of the judgments in each category. Additionally, as a subject for future research, a study of acceptable inconsistency, similar to other pairwise comparison-based methods, can be proposed to ensure the rationality of evaluations (Liang et al., 2020). Given that the proposed method balances mathematical concepts and the need for interaction with experts, it can be a powerful tool to support decision-making, especially in subjects with multiple criteria and flexibility for designing other extensions, such as fuzzy, interval, and stochastic, among others.

6. Conclusion

Decision-making is the process and selection of operations to solve a particular problem. Suppose we define management as a decision-making authority in planning and using factors and resources to achieve the set goals. In that case, we will see that good decisions fundamentally impact industry and society. This highlights the need to design and provide scientific tools for decision-making. In this paper, a tool for decision support was presented by designing a MADM method. In the proposed method, the criteria were first divided into several comparable categories, and expert pairwise comparisons were made in each category of criteria. This facilitates pairwise comparisons and explores alternatives from multiple perspectives. Also, the degree of inconsistency in judgments is significantly reduced. Then, the experts' judgments in each category of criteria are evaluated, and expert judgments in each category are the basis for decisions that have the shortest distance to the judgments of other experts. The local weight of the criteria is then extracted from solving a multi-objective linear programming problem. The goal programming method determines the relative weights of the criteria and an index to measure the degree of inconsistency of the judgments. We used the proposed method on a familiar example in the MADM literature and compared it with the well-known GAHP method. The same results were obtained with less calculation and less interaction with experts. Then, we implemented the method for selecting a supplier in an automotive group with 12 criteria in three categories. The simplicity of application, fewer calculations compared to similar methods, and high accuracy of results are some features of the proposed method. Finally, we provide some managerial insights and suggestions for future research.

CRediT authorship contribution statement

Mehdi Soltanifar: Conceptualization, Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Madjid Tavana:** Formal analysis, Methodology, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The above authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

The application of the KEMIRA method to a basic illustration

In this segment, we showcase the outcomes obtained through KEMIRA for a straightforward scenario involving car selection. Our implementation of this approach involves consulting three specialists to establish attribute priorities. These experts have classified the three attributes into distinct clusters based on the provided classification attributes.

Table A1
Expert prioritization of attributes.

Prioritization of attributes	Subjective attributes	Objective attributes
Expert 1	Prestige $\prec$ Comfort	MPG $\succ$ Price
Expert 2	Prestige $\succ$ Comfort	MPG $\prec$ Price
Expert 3	Prestige $\prec$ Comfort	MPG $\prec$ Price

Table A2 displays the matrices of priorities within each attribute cluster.

Table A2
The matrices of priorities within each attribute cluster.

Expert 1	Prestige	Comfort	Expert 2	Prestige	Comfort	Expert 3	Prestige	Comfort
Prestige	0	0	**Prestige**	0	1	**Prestige**	0	0
Comfort	1	0	**Comfort**	0	0	**Comfort**	1	0
Expert 1	**MPG**	**Price**	**Expert 2**	**MPG**	**Price**	**Expert 3**	**MPG**	**Price**
MPG	0	1	**MPG**	0	0	**MPG**	0	0
Price	0	0	**Price**	1	0	**Price**	1	0

In the KEMIRA method, experts' judgments are compared, and the judgment with the least deviation from other experts' judgments is chosen for each feature category. Table A2 displays the results of comparing the experts' judgments (Krylovas et al., 2014).

$\rho_{Expert\ 1=2}$	$\rho'_{Expert\ 1=4}$
$\rho_{Expert\ 2=4}$	$\rho'_{Expert\ 2=2}$
$\rho_{Expert3=2}$	$\rho'_{Expert3=2}$

Utilizing the input from Experts 1 and 3 within the subjective cluster, alongside the input from Experts 2 and 3 within the objective cluster, where the lowest ρ-value was observed, we establish the attribute priorities within each cluster as follows: (Prestige $\prec$ Comfort) and (MPG $\prec$ Price). With these attribute priorities in mind, the potential weight combinations within each cluster are detailed in Table A3 (weights rounded to one decimal point).

Table A3
The possible sets of attribute weights.

Subjective attributes			Objective attributes		
Set of attribute weights	Prestige	Comfort	Set of attribute weights	MPG	Price
$\mathbf{v_1}$	0.1	0.9	$\mathbf{u_1}$	0.1	0.9
$\mathbf{v_2}$	0.2	0.8	$\mathbf{u_2}$	0.2	0.8
$\mathbf{v_3}$	0.3	0.7	$\mathbf{u_3}$	0.3	0.7
$\mathbf{v_4}$	0.4	0.6	$\mathbf{u_4}$	0.4	0.6

Based on the attribute weights provided in Table A3, we generate Table A4 (Krylovas et al., 2014).

Table A4
$F(X,Y)$ for combined attribute weights in two clusters.

$F(X,Y)$	$\mathbf{u_1}$	$\mathbf{u_2}$	$\mathbf{u_3}$	$\mathbf{u_4}$
$\mathbf{v_1}$	1.990	1.996	2.002	2.008
$\mathbf{v_2}$	1.987	1.993	1.998	2.004
$\mathbf{v_3}$	1.983	1.989	1.995	2.001
$\mathbf{v_4}$	1.980	1.985	1.991	1.997

Vector $(\mathbf{v_4}, \mathbf{u_1})$ is selected according to the steps of the KEMIRA method, and then the score and rank of each alternative are determined based on Table A5 (Krylovas et al., 2014).

Table A5
Final alternative scores and rankings.

Alternatives	Score	Rank
Acura TL	1	2
ToyotaCamry	0.321	3
Honda Civic	1.096	1

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