# Chapter 1:

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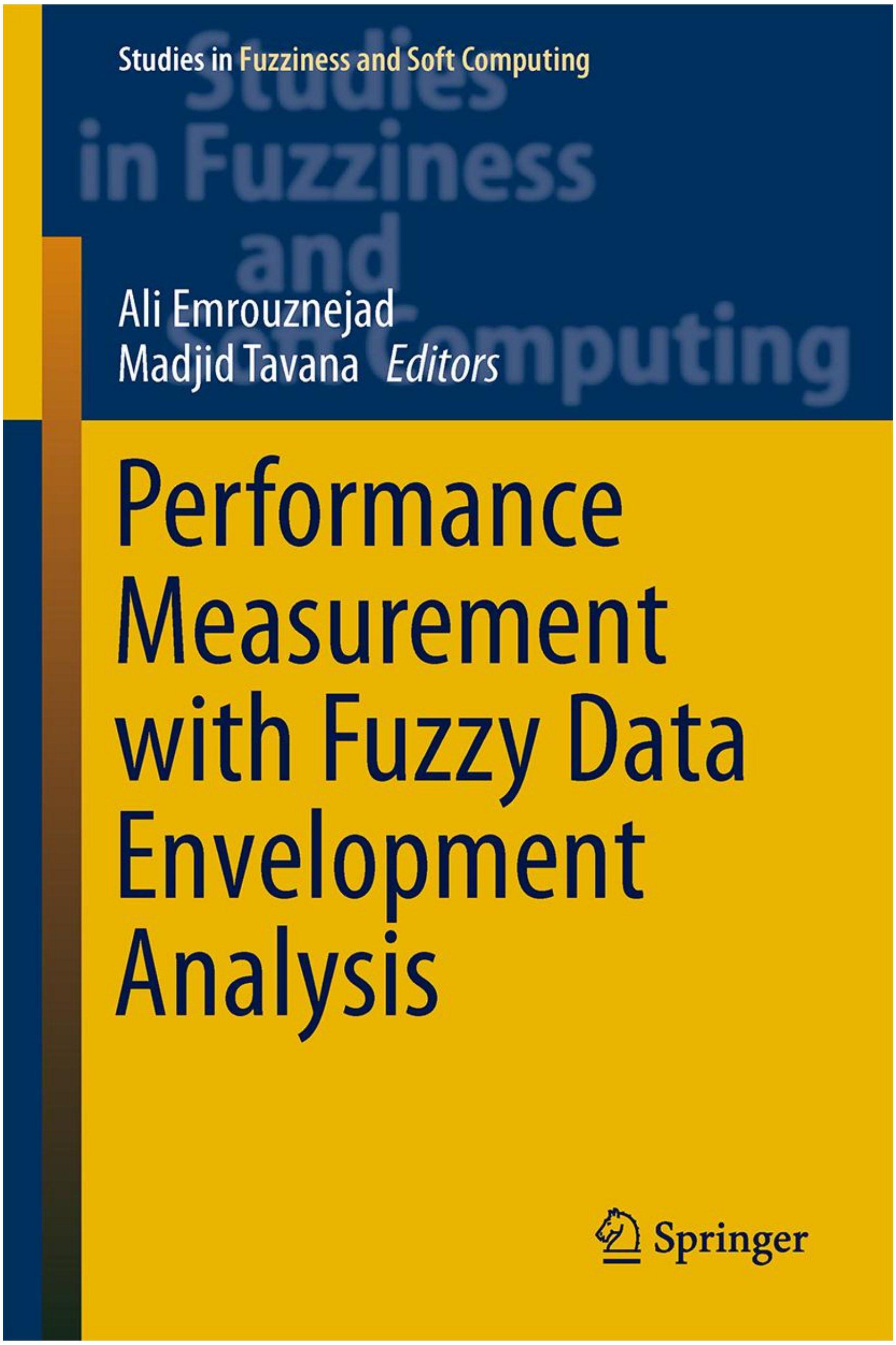


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# Chapter 1
# The State of the Art in Fuzzy Data Envelopment Analysis

**Ali Emrouznejad, Madjid Tavana and Adel Hatami-Marbini**

**Abstract** Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. Crisp input and output data are fundamentally indispensable in conventional DEA. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Many researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA. This chapter provides a taxonomy and review of the fuzzy DEA (FDEA) methods. We present a classification scheme with six categories, namely, the tolerance approach, the α-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set. We discuss each classification scheme and group the FDEA papers published in the literature over the past 30 years.

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An earlier version of this chapter was published as Hatami-Marbini et al. [1].

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**Keywords** Data envelopment analysis · Fuzzy sets · Tolerance approach · $\alpha$-level based approach · Fuzzy ranking approach · Possibility approach · Fuzzy arithmetic · Fuzzy random · Type-2 fuzzy set

## 1 Introduction

Data envelopment analysis (DEA) was first proposed by Charnes et al. [2], and is a non-parametric method of efficiency analysis for comparing units relative to their best peers (efficient frontier). Mathematically, DEA is a linear programming (LP)-based methodology for evaluating the relative efficiency of a set of decision making units (DMUs) with multi-inputs and multi-outputs. DEA evaluates the efficiency of each DMU relative to an estimated production possibility frontier determined by all the DMUs. The advantage of using DEA is that it does not require any assumption on the shape of the frontier surface and it makes no assumptions concerning the internal operations of a DMU. Since the original DEA study by Charnes et al. [2], there has been a continuous growth in the field. As a result, a considerable amount of published research papers and bibliographies have appeared in the DEA literature, including those of Seiford [3], Gattoufi et al. [4], Emrouznejad et al. [5], and Cook and Seiford [6].

The conventional DEA methods require accurate measurement of both the inputs and outputs. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Imprecise evaluations may be the result of unquantifiable, incomplete and non-obtainable information. Some researchers have proposed various fuzzy methods for dealing with this impreciseness and ambiguity in DEA. Since the original study by Sengupta [7, 8] there has been a continuous interest and increased development in fuzzy DEA (FDEA) literature. In this study, we review the FDEA methods and present a taxonomy by classifying the FDEA papers published over the past two decades into six primary categories, namely, the tolerance approach, the $\alpha$-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set as well as a secondary category to group the pioneering papers that do not fall into the six primary classifications. This study updates the previous review of Hatami-Marbini et al. [1] on FDEA and it provides the complete source of references on FDEA since its inception two decades ago. This chapter is organized into five sections. In Sect. 2, we present the fundamentals of DEA. In Sect. 3, we review the FDEA principles. In Sect. 4, we present a summary development of the FDEA followed by a detailed description of the FDEA methods in the literature. Conclusions and future research directions are presented in Sect. 5.

## 2 The Fundamentals of DEA

There are basically two main types of DEA models: a constant returns-to-scale (CRS) or CCR model that was initially introduced by Charnes et al. [2] and a variable returns-to-scale (VRS) or BCC model that was later developed by Banker et al. [9]. The BCC model is one of the extensions of the CCR model where the efficient frontiers set is represented by a convex curve passing through all efficient DMUs.

DEA can be either input- or output-orientated. In the first case, the DEA method defines the frontier by seeking the maximum possible proportional reduction in input usage, with output levels held constant, for each DMU. However, for the output-orientated case, the DEA method seeks the maximum proportional increase in output production, with input levels held fixed.

Figure 1 illustrates a simple VRS output-oriented DEA problem with two outputs, $Y$ and $Z$, and one input, $X$. The isoquant $L_1L_2$ represents the technical efficient frontier comprising $P_1$, $P_2$, and $P_3$ which are technically efficient DMUs and hence on the frontier. If a given DMU uses one unit of input and produces outputs defined by point $P$, the technical inefficiency of that DMU is represented as the distance PP′, which is the amount by which all outputs could be proportionally increased without increasing the input. In percentage terms, it is expressed by the ratio $OP$/OP′, which is the ratio by which all the outputs could be increased.

An input oriented DEA model with $m$ input variables $(x_1, \ldots, x_m)$, $s$ output variables $(y_1, \ldots, y_s)$ and $n$ decision making units $(j = 1, 2, \ldots, n)$ is presented in Model 1a (for CCR model) and Model 1b (for BCC model). The only difference between these two models is the inclusion of the convexity constraints of $\sum_{j=1}^{n} \lambda_j = 1$ in the BCC model.

| *Model 1a: A basic CCR model* | *Model 1b: A basic BCC model* |
|---|---|
| $\min \ \theta_p$ | $\min \ \theta_p$ |
| $s.t. \ \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip}, \ \forall i,$ | $s.t. \ \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_p x_{ip}, \quad \forall i,$ |
| $\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}, \quad \forall r,$ | $\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}, \quad \forall r,$ |
| $\lambda_j \ge 0, \ \forall j.$ | $\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0, \ \forall j.$ |

DEA applications are numerous in many applications such as financial services, agricultural, health care services, education, manufacturing, telecommunication, supply chain management, and many more. For a recent comprehensive bibliography of DEA see Emrouznejad et al. [5]. Recently fuzzy logic has been introduced to DEA for measuring efficiency of DMUs under uncertainty mainly when the precise data is not available. The rest of this chapter focuses on the use of fuzzy sets in DEA.

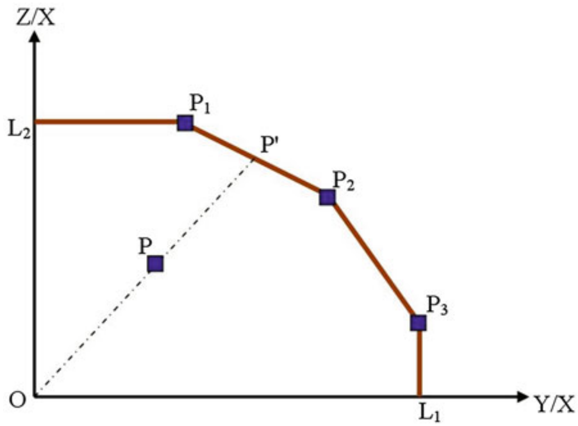


**Fig. 1** An output-oriented DEA with two outputs and one input

# 3 The FDEA Principles

The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non-obtainable information. Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers. In recent years, many researchers have formulated FDEA models to deal with situations where some of the input and output data are imprecise or vague.

## *3.1 Fuzzy Set Theory*

The theory of fuzzy set has been developed to deal with the concept of partial truth values ranging from absolutely true to absolutely false. Fuzzy set theory has become the prominent tool for handling imprecision or vagueness aiming at tractability, robustness and low-cost solutions for real-world problems. According to Zadeh [10], it is very difficult for conventional quantification to reasonably express complex situations and it is necessary to use linguistic variables whose values are words or sentences in a natural or artificial language. The potential of working with linguistic variables, low computational cost and easiness of understanding are characteristics that have contributed to the popularity of this approach. Fuzzy set algebra developed by Zadeh [11] is the formal body of theory that allows the treatment of imprecise and vague estimates in uncertain environments.

Zadeh [11], p. 339 states "*The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual frame-work which parallels in many respects the framework used in the case of ordinary sets, but is more general that the latter and, potentially, may prove to have a much wider scope of applicability*."

The application of fuzzy set theory in multi-attribute decision-making (MADM) became possible when Bellman and Zadeh [12] and Zimmermann [13] introduced fuzzy set into the field of MADM. They cleared the way for a new family of methods to deal with problems that had been unapproachable and unsolvable with standard techniques [see Chen and Hwang [14] for a numerical comparison of fuzzy and classical MADM models]. Bellman and Zadeh's [12] framework was based on the maximin principle and the simple additive weighing model of Yager and Basson [15] and Bass and Kwakernaak [16]. Bass and Kwakernaak's [16] method is widely known as the classic work of fuzzy MADM methods.

In 1992, Chen and Hwang [14] proposed an easy-to-use and easy-to-understand approach to reduce some of the cumbersome computations in the previous MADM methods. Their approach includes two steps: (1) converting fuzzy data into crisp scores; and (2) introducing some comprehensible and easy methods. In addition Chen and Hwang [14] made distinctions between fuzzy ranking methods and fuzzy MADM methods. Their first group contained a number of methods for finding a ranking: degree of optimality, Hamming distance, comparison function, fuzzy mean and spread, proportion to the ideal, left and right scores, area measurement, and linguistic ranking methods. Their second group was built around methods for assessing the relative importance of multiple attributes: fuzzy simple additive weighting methods, fuzzy analytic hierarchy process, fuzzy conjunctive/disjunctive methods, fuzzy outranking methods, and fuzzy maximin methods. The group with the most frequent contributions is fuzzy mathematical programming. Inuiguchi et al. [17] have provided a useful survey of fuzzy mathematical programming applications including: flexible programming, possibilistic programming, possibilistic programming with fuzzy preference relations, possibilistic LP using fuzzy max, possibilistic LP with fuzzy goals, and robust programming.

Recently, fuzzy set theory has been applied to a wide range of fields such as management science, decision theory, artificial intelligence, computer science, expert systems, logic, control theory and statistics, among others [18–30].

## 3.2 Fuzzy Set Theory and DEA

The data in the conventional CCR and BCC models assume the form of specific numerical values. However, the observed value of the input and output data are sometimes imprecise or vague. Sengupta [7, 8] was the first to introduce a fuzzy mathematical programming approach in which fuzziness was incorporated into the DEA model by defining tolerance levels on both the objective function and constraint violations.

Let us assume that $n$ DMUs consume varying amounts of $m$ different inputs to produce $s$ different outputs. Assume that $\tilde{x}_{ij}$ $(i = 1, 2, \ldots, m)$ and $\tilde{y}_{rj}$ $(r = 1, 2, \ldots, s)$ represent, respectively, the fuzzy input and fuzzy output of the $j$th

$\mathrm{DMU}_j$ $(j = 1, 2, \ldots, n)$. The primal and its dual *fuzzy CCR* models in input-oriented version can be formulated as:

*Primal CCR model* (*input-oriented*)

$$\begin{aligned} \min \quad & \theta_p \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rp}, \quad \forall r, \\ & \lambda_j \ge 0, \quad \forall j. \end{aligned} \tag{1}$$

*Dual CCR model* (*input-oriented*)

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0, \quad \forall j, \\ & u_r, v_i \ge 0, \quad \forall r, i. \end{aligned} \tag{2}$$

where $v_i$ and $u_r$ in model (2) are the input and output weights assigned to the $i$th input and $r$th output. If the constraint $\sum_{j=1}^{n} \lambda_j = 1$ is adjoined to (1), a *fuzzy BCC* model is obtained and this added constraint introduces an additional variable, $\tilde{u}_0$, into the dual model where these models are respectively shown as follows:

$$\begin{aligned} \min \quad & \theta_p \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \ge \tilde{y}_{rp}, \quad \forall r, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \ge 0, \quad \forall j. \end{aligned} \tag{3}$$

$$\begin{aligned} \max \quad & w_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} + u_0 \\ s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1, \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} + u_0 \le 0, \quad \forall j, \\ & u_r, v_i \ge 0, \quad \forall r, i. \end{aligned} \tag{4}$$

# 4 The FDEA Methods

The applications of fuzzy set theory in DEA are usually categorized into four groups Lertworasirikul et al. [31, 32], Lertworasirikul [33], Karsak [34]: (1) The tolerance approach, (2) The α-level based approach, (3) The fuzzy ranking approach, (4) The possibility approach. In this study, we expand this classification and add two new groups: the fuzzy arithmetic and the fuzzy random/type-2 fuzzy set.

In this section, we provide a mathematical description of each approach followed by a brief review of the most widely cited literature relevant to each of the six approaches. In addition to the six above mentioned approaches, we introduce a new category to group the pioneering papers that do not fall into any of the above classifications. A summary development of the FDEA is listed in Table 1.

## *4.1 The Tolerance Approach*

The tolerance approach was one of the first FDEA models that was developed by Sengupta [7] and further improved by Kahraman and Tolga [35]. In this approach the main idea is to incorporate uncertainty into the DEA models by defining tolerance levels on constraint violations. This approach fuzzifies the inequality or equality signs but it does not treat fuzzy coefficients directly. The intricate limitation of the tolerance approach proposed by Sengupta [7] is related to the design of a DEA model with a fuzzy objective function and fuzzy constraints which may or may not be satisfied (Triantis and Girod [36]. Although in most production processes fuzziness is present both in terms of not meeting specific objectives and in terms of the imprecision of the data, the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp.

## *4.2 The α-Level Based Approach*

The α-level approach is perhaps the most popular FDEA model. This is evident by the number of α-level based papers published in the FDEA literature. In this approach the main idea is to convert the FDEA model into a pair of parametric programs in order to find the lower and upper bounds of the α-level of the membership functions of the efficiency scores. Girod [37] used the approach proposed by Carlsson and Korhonen [38] to formulate the fuzzy BCC and free disposal hull (FDH) models which were radial measures of efficiency. In this model, the inputs could fluctuate between risk-free (upper) and impossible (lower) bounds and the outputs could fluctuate between risk-free (lower) and impossible

**Table 1** Fuzzy DEA reference classification from 1999 to 2013 (173 Papers)

| | | |
|---|---|---|
| *The tolerance approach (2 Papers)* | | |
| Sengupta [7] | Sengupta [8] | |
| *The α-level based approach (77 Papers)* | | |
| Liu [64] | Khalili-Damghani and Tavana [65] | Mugera [66] |
| Khalili-Damghani and Taghavifard [128] | Chen et al. [67] | Fathi and Izadikhah [79] |
| Srinivasa Raju and Nagesh Kumar [85] | Hatami-Marbini et al. [86] | Saati et al. [87] |
| Puri and Yadav [59] | Azadeh et al. [82] | Wang and Yan [127] |
| Kao and Lin [62] | Azadeh et al. [81] | Zhou et al. [91] |
| Zhou et al. [93] | Hatami-Marbini et al. [80] | Khalili-Damghani and Hosseinzadeh Lotfi [125] |
| Zerafat Angiz et al. [231] | Abtahi and Khalili-Damghani [121] | Khalili-Damghani and Taghavifard [124] |
| Ghapanchi et al. [83] | Rezaie et al. [84] | Khalili-Damghani et al. [126] |
| Mostafaee [118] | Khoshfetrat and Daneshvar [120] | Khalili-Damghani and Abtahi [235] |
| Kao and Liu [60] | Kao and Lin [54] | Noura et al. [115] |
| Zhou et al. [90] | Azadeh et al. [77] | Mansourirad et al. [117] |
| Azadeh et al. [77] | Azadeh and Alem [107] | Zerafat Angiz et al. [114] |
| Hatami-Marbini et al. [74] | Chiang and Che [58] | Saati and Memariani [75] |
| Liu and Chuang [92] | Wang et al. [110] | Noura and Saljooghi [109] |
| Tlig and Rebai [113] | Hatami-Marbini and Saati [73] | Hosseinzadeh Lotfi et al. [111] |
| Jahanshahloo et al. [89] | Li and Yang [56] | Liu [88] |
| Ghapanchi et al. [72] | Azadeh et al. [106] | Karsak [34] |
| Jahanshahloo et al. [100] | Kao and Liu [53] | Saneifard et al. [98] |
| Azadeh et al. [71] | Liu et al. [97] | Allahviranloo et al. [102] |
| Hosseinzadeh Lotfi et al. [103] | Saati and Memariani [69] | Hsu [96] |
| Wu et al. [70] | Kao and Liu [51] | Zhang et al. [52] |
| Kuo and Wang [55] | Entani et al. [95] | Triantis [40] |
| Kao and Liu [49] | Saati et al. [68] | Guh [48] |
| Kao [47] | Chen [18] | Kao and Liu [46] |

(continued)

**Table 1** (continued)

| | | |
|---|---|---|
| *The tolerance approach (2 Papers)* | | |
| Kao and Liu [42] | Girod and Triantis [39] | Triantis and Girod [36] |
| Maeda et al. [41] | Girod [37] | |
| *The fuzzy ranking approach (43 Papers)* | | |
| Beiranvand et al. [141] | Azadeh et al. [145] | Ahmady et al. [171] |
| Amindoust et al. [180] | Sefeedpari et al. [139] | Chang and Lee [134] |
| Moheb-Alizadeh et al. [179] | Hatami-Marbini et al. [1] | Emrouznejad et al. [170] |
| Azadeh et al. [143] | Azadeh et al. [169] | Hatami-Marbini et al. [168] |
| Azadeh et al. [144] | Hatami-Marbini et al. [140] | Sanei et al. [132] |
| Hosseinzadeh Lotfi et al. [178] | Jahanshahloo et al. [165] | Guo [131] |
| Soleimani-damaneh [162] | Hatami-Marbini et al. [163] | Juan [175] |
| Bagherzadeh valami [177] | Hosseinzadeh Lotfi et al. [167] | Hosseinzadeh Lotfi and Mansouri [155] |
| Zhou et al. [156] | Guo and Tanaka [130] | Jahanshahloo et al. [159] |
| Noora and Karami [157] | Soleimani-damaneh [161] | Hosseinzadeh Lotfi et al. [150] |
| Hosseinzadeh Lotfi et al. [149] | Pal et al. [153] | Jahanshahloo et al. [152] |
| Saati and Memariani [142] | Soleimani-damaneh et al. [147] | Lee et al. [174] |
| Molavi et al. [146] | Jahanshahloo et al. [76] | Lee [173] |
| Dia [172] | León et al. [136] | Lertworasirikul [33] |
| Guo and Tanaka [129] | | |
| *The possibility approach (21 Papers)* | | |
| Payan and Sharifi [202] | Nedeljković and Drenovac [190] | Zhao and Yue [189] |
| Wen et al. [198] | Wang and Chin [201] | Hossainzadeh et al. [200] |
| Lin [188] | Khodabakhshi et al. [194] | Wen et al. [196] |
| Wen and Li [195] | Jiang and Yang [193] | Wu et al. [187] |
| Ramezanzadeh et al. [191] | Garcia et al. [186] | Lertworasirikul et al. [31] |
| Lertworasirikul et al. [32] | Lertworasirikul et al. [185] | Lertworasirikul [33] |
| Lertworasirikul et al. [183] | Lertworasirikul et al. [184] | Guo et al. [182] |

(continued)

**Table 1** (continued)

| *The tolerance approach (2 Papers)* | | |
|---|---|---|
| *The fuzzy arithmetic* (*11 Papers*) | | |
| Azadi et al. [212] | Razavi Hajiagha et al. [211] | Alem et al. [210] |
| Mirhedayatian et al. [236] | Mirhedayatian et al. [237] | Mirhedayatian et al. [209] |
| Jafarian-Moghaddam and Ghoseiri [206] | Raei Nojehdehi et al. [208] | Abdoli et al. [205] |
| Wang et al. [204] | Wang et al. [203] | |
| *The fuzzy random/type-2 fuzzy set* (*7 Papers*) | | |
| Zerafat Angiz et al. [220] | Tavana et al. [222] | Tavana et al. [219] |
| Qin et al. [217] | Qin and Liu [215] | Qin and Liu [216] |
| Qin et al. [214] | | |
| *Other developments in fuzzy DEA* (*12 Papers*) | | |
| Zerafat Angiz and Mustafa [232] | Bagherzadeh Valami et al. [233] | Zerafat Angiz et al. [231] |
| Zerafat Angiz et al. (2010) | Zerafat Angiz et al. [230] | Zerafat Angiz et al. [228] |
| Qin and Liu [214] | Luban [227] | Uemura [226] |
| Hougaard [225] | Sheth and Triantis [224] | Hougaard [223] |

(upper) bounds. Triantis and Girod [36] followed up by introducing the fuzzy LP approach to measure technical efficiency based on Carlsson and Korhonen's [38] framework. Their approach involved three stages: First, the imprecise inputs and outputs were determined by the decision maker in terms of their risk-free and impossible bounds. Second, three fuzzy CCR, BCC and FDH models were formulated in terms of their risk-free and impossible bounds as well as their membership function for different values of $\alpha$. Third, they illustrated the implementation of their fuzzy BCC model in the context of a preprint and packaging line which inserts commercial pamphlets into newspapers. Furthermore, their paper was clarified in detail using the implementation road map by Girod and Triantis [39]. Triantis [40] extended his earlier work on FDEA [36] to fuzzy non-radial DEA measures of technical efficiency in support of an integrated performance measurement system. He also compared his method to the radial technical efficiency of the same manufacturing production line which was described in detail by Girod [37] and Girod and Triantis [39]. Meada et al. [41] used the $\alpha$-level based approach to obtain the fuzzy interval efficiency of DMUs.

Kao and Liu [42] followed up on the basic idea of transforming a FDEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Their method found approximately the membership functions of the fuzzy efficiency measures by applying the $\alpha$-level approach and Zadeh's extension principle Zadeh [43], Zimmermann [44]. They transformed the FDEA model to a pair of parametric mathematical programs and used the ranking fuzzy numbers method proposed by Chen and Klein [45] to obtain the performance measure of the DMU. Solving this model at the given level of $\alpha$-level produced the interval efficiency for the DMU under consideration. A number of such intervals could be used to construct the corresponding fuzzy efficiency. Assume that there are $n$ DMUs under consideration. Each DMU consumes varying amounts of $m$ different fuzzy inputs to produce $s$ different fuzzy outputs. Specifically, $\text{DMU}_j$ consumes amounts $\tilde{x}_{ij}$ of inputs to produce amounts $\tilde{y}_{rj}$ of outputs. In the model formulation, $\tilde{x}_{ip}$ and $\tilde{y}_{rp}$ denote, respectively, the input and output values for the $\text{DMU}_p$. In order to solve the fuzzy BCC model (4), Kao and Liu [42] proposed a pair of two-level mathematical models to calculate the lower bound $(w_p)_\alpha^L$ and upper bound $(w_p)_\alpha^U$ of the fuzzy efficiency score for a specific $\alpha$-level as follows:

$$(w_p)_\alpha^L = \min_{\substack{(X_{ij})_\alpha^L \le x_{ij} \le (X_{ij})_\alpha^U \\ (Y_{rj})_\alpha^L \le y_{rj} \le (Y_{rj})_\alpha^U \\ \forall r,i,j}} \left\{ \begin{array}{l} \tilde{w}_p = \max \sum_{r=1}^{s} u_r y_{rp} + u_0 \\ \qquad s.t. \quad \sum_{i=1}^{m} v_i x_{ip} = 1, \\ \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad \forall j, \\ \qquad u_r, v_i \ge 0, \quad \forall r,i. \end{array} \right. \tag{5}$$

$$(w_p)^U_\alpha = \max_{\substack{(X_{ij})^L_\alpha \le x_{ij} \le (X_{ij})^U_\alpha \\ (Y_{rj})^L_\alpha \le y_{rj} \le (Y_{rj})^U_\alpha \\ \forall r,i,j}} \begin{cases} \tilde{w}_p = \max \quad \sum_{r=1}^{s} u_r y_{rp} + u_0 \\ \quad s.t. \quad \sum_{i=1}^{m} v_i x_{ip} = 1, \\ \qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad \forall j, \\ \qquad u_r, v_i \ge 0, \qquad \forall r, i. \end{cases} \tag{6}$$

where $\left[(X_{ij})^L_\alpha, (X_{ij})^U_\alpha\right]$ and $\left[(Y_{rj})^L_\alpha, (Y_{rj})^U_\alpha\right]$ are α-level form of the fuzzy inputs and the fuzzy outputs respectively. This two-level mathematical model can be simplified to the conventional one-level model as follows:

$$\begin{aligned} (w_p)^L_\alpha = \max \quad & \sum_{r=1}^{s} u_r (Y_{rp})^L_\alpha + u_0 \\ s.t. \quad & \sum_{r=1}^{s} u_r (Y_{rp})^L_\alpha - \sum_{i=1}^{m} v_i (X_{ip})^U_\alpha + u_0 \le 0, \\ & \sum_{r=1}^{s} u_r (Y_{rj})^U_\alpha - \sum_{i=1}^{m} v_i (X_{ij})^L_\alpha + u_0 \le 0, \quad \forall j, j \ne p, \\ & \sum_{i=1}^{m} v_i (X_{ip})^U_\alpha = 1, \quad u_r, v_i \ge 0, \quad \forall r, i. \end{aligned} \tag{7}$$

$$\begin{aligned} (w_p)^U_\alpha = \max & \sum_{r=1}^{s} u_r (Y_{rp})^U_\alpha + u_0 \\ s.t. \quad & \sum_{r=1}^{s} u_r (Y_{rp})^U_\alpha - \sum_{i=1}^{m} v_i (X_{ip})^L_\alpha + u_0 \le 0, \\ & \sum_{r=1}^{s} u_r (Y_{rj})^L_\alpha - \sum_{i=1}^{m} v_i (X_{ij})^U_\alpha + u_0 \le 0, \quad \forall j, j \ne p, \\ & \sum_{i=1}^{m} v_i (X_{ip})^L_\alpha = 1, \quad u_r, v_i \ge 0, \quad \forall r, i. \end{aligned} \tag{8}$$

Next, a membership function is built by solving the lower and upper bounds $\left[(w_p)^L_\alpha, (w_p)^U_\alpha\right]$ of the α-levels for each DMU using models (7) and (8). Kao and Liu [42] have used the ranking fuzzy numbers method of Chen and Klein [45] to rank the obtained fuzzy efficiencies. Kao and Liu [46] also used the method of Kao and Liu [42] to calculate the efficiency scores by considering the missing values in the FDEA based on the concept of the membership function in fuzzy set theory. In their approach, the smallest possible, most possible, and largest possible values of the missing data are derived from the observed data to construct a triangular membership function. They demonstrated the applicability of their approach by considered the efficiency scores of 24 university libraries in Taiwan with three missing values out of 144 observations. Kao [47] further introduced a method for ranking the fuzzy efficiency scores without knowing the exact form of their membership function. In this method, the efficiency rankings were determined by solving a pair of non-linear programs for each DMU. This approach was applied to the ranking of the twenty-four university libraries in Taiwan with fuzzy observations. Guh [48] used a FDEA model similar to Kao and Liu [42] to approximate the fuzzy efficiency measures. However, Kao and Liu [42] developed their model under the VRS assumption and Guh [48]'s model was developed under the CRS assumption.

Kao and Liu [49] integrated the maximum set–minimum set method of Chen [50] into the FDEA model proposed by Kao and Liu [42] and built pairs of non-linear programs and ranked the DMUs with fuzzy data. In their approach, there was no need for calculating the membership function of the fuzzy efficiency scores but the input and output membership functions must be known. Kao and Liu [51] applied their earlier method Kao and Liu [42] to determine the fuzzy efficiency scores of fifteen sampled machinery firms in Taiwan. Zhang et al. [52] proposed a macro model and a micro model for the efficiency evaluation of data warehouses by applying DEA and FDEA models. They used the FDEA solution proposed by Kao and Liu [42], which transformed FDEA models to bi-conventional crisp DEA models by a set of $\alpha$-level values. Kao and Liu [53] proposed a modification to the Kao and Liu's [46] method to handle missing values. In their method, they used a FDEA approach and obtained the efficiency scores of a set of DMUs by using the $\alpha$-level approach proposed by Kao and Liu [42].

Kao and Lin [54] first created the corresponding fuzzy numbers for ordinal data using the DEA multipliers and then adopted a pair of two-level mathematical programs of Kao and Liu [42] for measuring the fuzzy efficiencies for distinct $\alpha$-cuts. Kuo and Wang [55] applied a FDEA method to evaluate the performance of multinational corporations in the face of volatile exposure to exchange rate risk. They employed the FDEA model suggested by Kao and Liu [42] to the information technology industry in Taiwan. Li and Yang [56] proposed a FDEA-discriminant analysis methodology for classifying fuzzy observations into two groups based on the work of Sueyoshi [57]. They used the Kao and Liu's [42] method and replaced the fuzzy LP models by a pair of parametric models to determine the lower and upper bounds of the efficiency scores. By applying the Kao and Liu's [42] method and the fuzzy analytical hierarchy procedure, Chiang and Che [58] proposed a new weight-restricted FDEA methodology for ranking new product development projects at an electronic company in Taiwan. Puri and Yadav [59] proposed fuzzy CCR and fuzzy slack-based measurement (SBM) models and defined the fuzzy input mix-efficiency model (FIME) based on the $\alpha$-cut method developed by Kao and Liu [42]. To ensure the validity of their proposed FDEA model, they proposed a fuzzy correlation coefficient method by using the expected value approach for computing the expected interval and the expected value of the fuzzy correlation coefficient between the fuzzy inputs and fuzzy outputs. They then introduced a defuzzification method for ranking the DMUs using the FIME model. Kao and Liu [60] extended the fuzzy version of the two-stage DEA approach where its deterministic model was initially introduced by Kao and Hwang [61]. Their model was inspired by the work of Kao and Liu [42] in obtaining the lower and upper bounds of the efficiency of each DMU and its sub-DMUs for different α-cuts. Kao and Lin [62] explored a method for measuring the fuzzy efficiency of parallel production systems which involved a number of independent processes where the input/output data are fuzzy numbers. The incorporated parallel model with deterministic data can be found in Kao [63]. Based on the work of Kao and Liu [42], the two-level programming model was proposed to calculate the lower and upper bounds of efficiency for distinct α-cuts.

To expand the fuzzy two-stage DEA model proposed by Kao and Liu [60, 64] presented a ranking method for fuzzy overall efficiency scores using total utilities when the precise membership functions of the overall efficiencies obtained from fuzzy two-stage model are anonymous. He also took into account the possibility of imposing multiplier bounds to derive the efficiency rankings. Kao and Liu [60]'s study inspired Khalili-Damghani and Tavana [65] to propose a fuzzy network DEA model for measuring the performance of agility in supply chains. Mugera [66] exploited the FDEA method of Kao and Liu [42] to measure the technical efficiency of dairy farms. Chen et al. [67] incorporated Kao and Liu [42]'s technique into the SBM model to evaluate risk characteristics and estimate efficiencies in the banking sector.

Saati et al. [68] suggested a fuzzy CCR model as a possibilistic programming problem and transformed it into an interval programming problem using the α-level based approach. The resulting interval programming problem could be solved as a crisp LP model for a given α with some variable substitutions. Model (9) proposed by Saati et al. [68] is derived for a particular case where the inputs and outputs are triangular fuzzy numbers:

$$\begin{aligned}
\max \quad & w_p = \sum_{r=1}^{s} y'_{rp} \\
s.t. \quad & \sum_{r=1}^{s} y'_{rj} - \sum_{i=1}^{m} x'_{ij} \le 0, \quad \forall j, \\
& v_i(\alpha x^m_{ij} + (1-\alpha)x^l_{ij}) \le x'_{ij} \le v_i(\alpha x^m_{ij} + (1-\alpha)x^u_{ij}), \quad \forall i, j, \\
& u_r(\alpha y^m_{rj} + (1-\alpha)y^l_{rj}) \le y'_{rj} \le u_r(\alpha y^m_{rj} + (1-\alpha)y^u_{rj}), \quad \forall r, j, \\
& \sum_{i=1}^{m} x'_{ip} = 1, \quad u_r, v_i \ge 0, \quad \forall r, j.
\end{aligned} \tag{9}$$

where $\tilde{x}_{ij} = (x^l_{ij}, x^m_{ij}, x^u_{ij})$ and $\tilde{y}_{rj} = (y^l_{rj}, y^m_{rj}, y^u_{rj})$ are the triangular fuzzy inputs and the triangular fuzzy outputs, and $x'_{ij}$ and $y'_{rj}$ are the decision variables obtained from variable substitutions used to transform the original fuzzy model into a parametric LP model with $\alpha \in [0, 1]$. Saati and Memariani [69] suggested a procedure for determining a common set of weights in FDEA based on the α-level method proposed by Saati et al. [68] with triangular fuzzy data. In this method, the upper bounds of the input and output weights were determined by solving some fuzzy LP models and then a common set of weights were obtained by solving another fuzzy LP model. Wu et al. [70] developed a buyer-seller game model for selecting purchasing bids using fuzzy values. They adopted the FDEA model proposed by Saati and Memariani [69] to obtain a common set of weights in FDEA. Azadeh et al. [71] proposed an integrated model of FDEA and simulation to select the optimal solution between some scenarios which were obtained from a simulation model and determined the optimum operators' allocation in cellular manufacturing systems. They used a FDEA model to rank a set of DMUs based on Saati et al. [68]'s method. In addition, they clustered the FDEA ranking of the DMUs by the fuzzy C-Means method to show a degree of desirability for operator allocation. Ghapanchi et al. [72] employed FDEA to evaluate the performance of enterprise resource planning (ERP) packages. In their approach, inputs and outputs indices were first determined by experts' opinions which were evaluated using linguistic variables characterized by triangular fuzzy numbers and then a set of potential ERP systems was considered as DMUs. They applied a possibilistic-programming approach proposed by Saati et al. [68] and obtained the efficiency scores of the ERP systems at different $\alpha$ values.

Hatami-Marbini and Saati [73] developed a fuzzy BCC model which considered fuzziness in the input and output data as well as the $u_0$ variable. Consequently, they obtained the stability of the fuzzy $u_0$ as an interval by means of the method proposed by Saati et al. [68]. Hatami-Marbini et al. [74] used the method of Saati et al. [68] and proposed a four-phase FDEA framework based on the theory of the displaced ideal. Two hypothetical DMUs called the ideal and nadir DMUs are constructed and used as reference points to evaluate a set of information technology investment strategies based on their Euclidean distance from these reference points. Chen [18] modified the $\alpha$-level approach and proposed an alternative FDEA to handle both the crisp and fuzzy data. Saati and Memariani [75] developed a fuzzy SBM based on the $\alpha$-level approach. They transformed their fuzzy SBM model into a LP problem by using the approach proposed by Saati et al. [68].

Azadeh et al. [77] applied FDEA, fuzzy C-means and computer simulation to determine the optimal scenario selection in cellular manufacturing. They developed a computer simulation model to determine distinct operator layouts. The output of their simulation was converted into fuzzy numbers to preserve information and subsequently the FDEA proposed by Saati et al. [68] was used to evaluate the simulation alternatives at different levels of uncertainty. A degree of desirability for the operator allocation was ultimately identified by the clusters obtained from the fuzzy C-means method. Saati et al. [78] used the idea of Saati et al. [68] to present a FDEA model with discretionary and non-discretionary factors in both the input and output-oriented CCR models. Fathi and Izadikhah [79] duplicated Saati et al. [78]'s method in an alternative method. Hatami-Marbini et al. [80] proposed a fuzzy additive DEA model for evaluating the efficiency of peer DMUs with fuzzy data by utilizing Saati et al. [68]'s $\alpha$-level approach.

Azadeh et al. [81] explored an integrated approach for performance evaluation of health safety environment divisions, involving DEA and FDEA, to lessen the human error and the data imprecision. They applied the method proposed by Saati et al. [68] to deal with the FDEA model. Azadeh et al. [82] presented an integrated decision support system, called AutoAssess, to measure performance and analyze the continuous improvement of the DMUs. AutoAssess utilizes FDEA, principle component analysis, DEA, numerical taxonomy, and the Spearman correlation experiment. They used the method proposed Saati et al. [68] when inputs and outputs are characterized by triangular fuzzy numbers. Ghapanchi et al. [83] proposed a four-step method for project portfolio selection involving (i) modeling the problem; (ii) assessing the projects and selecting potential candidate projects using FDEA based on Saati et al. [68]'s method; (iii) generating portfolios and determining the maximal portfolios; and (iv) assessing the maximal portfolios using FDEA based on Saati at al. [68]'s method. Rezaie et al. [84] employed the FDEA method of Saati et al. [68] to evaluate and rank 50 companies in the Tehran Stock Exchange. Srinivasa Raju and Nagesh Kumar [85] explored the performance evaluation of an irrigation system in the fuzzy environments. The FDEA method proposed by Saati et al. [68] was adopted to deal with the impreciseness in the irrigation systems. Hatami-Marbini et al. [86] presented a FDEA model to provide the positive-normative use of fuzzy logic in a NATO enlargement application by using the $\alpha$-level technique developed by Saati et al. [68]. Saati et al. [87] used Saati et al. [68]'s method to present a FDEA method for clustering operating units in a fuzzy environment by considering the priority between the clusters and the priority between the operating units in each cluster simultaneously.

Liu [88] developed a FDEA method to find the efficiency measures embedded with the assurance region (AR) concept when some observations were fuzzy numbers. He applied an $\alpha$-level approach and Zadeh's extension principle [43, 44] to transform the FDEA/AR model into a pair of parametric mathematical programs and worked out the lower and upper bounds of the efficiency scores of the DMUs. The membership function of the efficiency was approximated by using different possibility levels. Thereby, he used the Chen and Klein's [45] method for ranking the fuzzy numbers and calculating the crisp values. Let us consider the relative importance of the inputs and outputs as $\frac{L_{I_\delta}}{U_{I_q}} \le \frac{v_\delta}{v_q} \le \frac{U_{I_\delta}}{L_{I_q}}$, $\delta < q = 2, \ldots, m$; and $\frac{L_{O_\delta}}{U_{O_q}} \le \frac{u_\delta}{u_q} \le \frac{U_{O_\delta}}{L_{O_q}}, \delta < q = 2, \ldots, s$; respectively.

The two parametric mathematical programs proposed by Liu [88] are as follows:

$$
\begin{aligned}
(W_p)^L_\alpha = \max \quad & \sum_{r=1}^{s} u_r (y_{rp})^L_\alpha \\
s.t. \quad \sum_{r=1}^{s} u_r (y_{rj})^U_\alpha - \sum_{i=1}^{m} v_i (x_{ij})^L_\alpha \le 0, \quad & \forall j, j \ne p, \\
-v_\delta + I^L_{\delta q} v_q \le 0, \quad v_\delta - I^U_{\delta q} v_q \le 0, \quad & \forall \delta < q, \\
-u_\delta + O^L_{\delta q} u_q \le 0, \quad u_\delta - O^U_{\delta q} u_q \le 0, \quad & \forall \delta < q, \\
\sum_{i=1}^{m} v_i (x_{ip})^U_\alpha = 1, \quad u_r, v_i \ge 0, \quad & \forall r, j.
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
(W_p)^U_\alpha = \max \quad & \sum_{r=1}^{s} u_r (y_{rp})^U_\alpha \\
s.t. \quad & \sum_{r=1}^{s} u_r (y_{rj})^L_\alpha - \sum_{i=1}^{m} v_i (x_{ij})^U_\alpha \le 0, \quad \forall j, j \ne p, \\
& -v_\delta + I^L_{\delta q} v_q \le 0, \quad v_\delta - I^U_{\delta q} v_q \le 0, \quad \forall \delta < q, \\
& -u_\delta + O^L_{\delta q} u_q \le 0, \quad u_\delta - O^U u_q \le 0, \quad \forall \delta < q, \\
& \sum_{i=1}^{m} v_i (x_{ip})^L_\alpha = 1, \quad u_r, v_i \ge 0, \qquad \forall r, j.
\end{aligned}
\tag{11}
$$

where $I^L_{\delta q} = \frac{L_{I_\delta}}{U_{I_q}}$, $I^U_{\delta q} = \frac{U_{I_\delta}}{L_{I_q}}$, $O^L_{\delta q} = \frac{L_{O_\delta}}{U_{O_q}}$ and $O^U_{\delta q} = \frac{U_{O_\delta}}{L_{O_q}}$. Jahanshahloo et al. [89], Zhou et al. [90] and Zhou et al. [91] proposed some corrections to Liu's [88] model. Liu and Chuang [92] applied the FDEA/AR model suggested by Liu [88] and evaluated the performance of 24 university libraries in Taiwan based on the method proposed by Kao and Liu [46]. Zhou et al. [93] developed a generalized FDEA model with assurance regions based on the generalized precise DEA model of Yu et al. [94]. They used the α-cut based approach to calculate the upper and lower bounds of the efficiency score for a given α.

Entani et al. [95] proposed a DEA model with an interval efficiency consisting of the efficiencies obtained from the pessimistic and the optimistic viewpoints. They also developed this approach for fuzzy input and output data by using α-level sets. Hsu [96] applied a simple FDEA model to a balanced scorecard with an application to multi-national research and development projects. The FDEA method included both crisp and linguistic variables processed by a four-step framework. Liu et al. [97] developed a modified FDEA model to handle fuzzy and incomplete information on weight indices in product design evaluation. They transformed fuzzy information into trapezoidal fuzzy numbers and considered incomplete information on indices weights as constraints. They used an α-level approach to convert their FDEA model into a family of conventional crisp DEA models. Saneifard et al. [98] developed a model to evaluate the relative performance of DMUs with crisp data based on the $l_2 - norm$. They used the ranking fuzzy numbers method of Jiménez [99] to determine a crisp α-parametric model and solve the fuzzy $l_2 - norm$ model.

Jahanshahloo et al. [100] developed a fuzzy $l_1 - norm$ model with trapezoidal fuzzy inputs/outputs that was initially suggested by Jahanshahloo et al. [238] for solving the crisp data in DEA. They applied the ranking fuzzy numbers method of Jiménez [99] to the fuzzy $l_1 - norm$model and obtained a crisp α-parametric model. Allahviranloo et al. [102] introduced the notion of fuzziness to deal with imprecise data in DEA. They proposed a fuzzy production possibility set with constant returns to scale to calculate the upper and lower relative efficiency scores of the DMUs by using the α-level approach. Hosseinzadeh Lotfi et al. [103] applied the method of DEA-discriminant analysis proposed by Sueyoshi [104] to the imprecise environment. They first modified Sueyoshi's model with crisp data and then developed it using fuzzy inputs and outputs based on the concept of the α-level approach. Karsak [34] proposed an extension of Cook et al. [105]'s model to evaluate crisp, ordinal and fuzzy inputs and outputs in flexible manufacturing systems by determining the optimistic (the upper bound) and pessimistic (the lower bound) of the α-level of the membership function of the efficiency scores. Azadeh et al. [106] used a triangular form of fuzzy inputs and outputs instead of the crisp data and proposed a FDEA model for calculating the efficiency scores of the DMUs under uncertainty with application to the power generation sector. They transformed the fuzzy CCR model into a pair of parametric programs using the α-level approach and found the lower and upper bounds of the efficiency for different α-values. Their contribution to the FDEA literature is in the development of the membership functions and not the crisp measure of the efficiencies. They used the α-level to transform the FDEA model into a series of conventional crisp DEA models. Azadeh and Alem [107] also used this FDEA method [106] for the vendor selection problem which was taken from Wu and Olson [108].

Noura and Saljooghi [109] proposed an extension of a definite class of weight function in FDEA based on the principle of maximum entropy in order to provide circumstances for the compatibility and stability in ranking of interval efficiency scores of DMUs at various α values. Wang et al. [110] proposed a FDEA–Neural approach with a self-organizing map for classification in their neural network. They used the upper and lower bounds of efficiency scores at different possibilistic levels in their model. Hosseinzadeh Lotfi et al. [111] developed two methods for solving the fuzzy CCR model with respect to fuzzy, ordinal and exact data. They used an analogue function to transform the fuzzy data into exact values in the first method. In the second approach, they applied an α-level approach based on the Kao [112]'s method to obtain the interval efficiency scores for DMUs. Tlig and Rebai [113] proposed an approach based on the ordering relations between LR-fuzzy numbers to solve the primal and the dual of FCCR. They suggested a procedure based on the resolution of a goal programming problem to transform the fuzzy normalisation equality in the primal of FCCR. Zerafat Angiz et al. [114] show the advantages and shortcomings of the fuzzy ranking approach, the defuzzification approach, the tolerance approach and the α-level based approach. They proposed an α-level approach to retain fuzziness of the model by maximizing the membership functions of inputs and outputs. They also compared their results with the results from Saati et al. [68].

Noura et al. [115] developed a fuzzy version of the DEA and the fuzzy preference relation introduced by Wu [116] using the α-cut based approach. Mansourirad et al. [117] proposed a DEA model in favor of efficiency measurement where the output weights are characterized by fuzzy numbers. The concept of the α-cut based approach was taken into consideration to calculate the components of the fuzzy output weights. Mostafaee [118] extended the economic efficiency models in non-convex technologies to both the interval approach and fuzzy set. Mostafaee [118] applied the idea of Mostafaee and Saljooghi [119] to deal with uncertainty. In the fuzzy case, he transformed the model to an interval model using the α-cut approach in order to calculate the interval revenue efficiency for a given number of α levels. Khoshfetrat and Daneshvar [120] demonstrated that a unique non-Archimedean infinitesimal, called Epsilon, as a lower bound of all multipliers of fuzzy inputs and fuzzy outputs may not precisely measure the efficiency scores of weak efficient DMUs. In response, they proposed a method for identifying an adequate lower-bound for each weight in FDEA based on the α-cut approach.

Abtahi and Khalili-Damghani [121] proposed a FDEA approach to measure the efficiency of just-in-time implementation and supply chains, respectively, based on the idea of Despotis and Smirlis [122] and the α-cut method. Zerafat Angiz et al. [123] extended the α-cut based approach for solving FDEA models by defining the concept of the "local α-level" and considering uncertainty. Their contribution led to a multi-objective LP method which was transformed to a LP model using Archimedean goal programming. Khalili-Damghani and Taghavifard [124] developed a fuzzy three-stage DEA method for measuring the efficiency of serial processes and sub-processes for just-in-time practices. The relational two-stage DEA model of Kao and Hwang [61] was extended to a three-stage DEA model with fuzzy observations. They first obtained the interval data using the α-cut based method and then used Despotis and Smirlis [122]'s method to solve the FDEA model.

Khalili-Damghani and Hosseinzadeh Lotfi [125] developed a method for measuring the productivity of Iranian traffic centers where the input and the output data were characterized by fuzzy numbers. The basic idea was proposed by Despotis and Smirlis [122] for interval data. They first used the concept of the α-cuts method to obtain the interval inputs and outputs for a given α level. They then mathematically re-formed the proposed model by removing the α variable from the proposed model. Khalili-Damghani et al. [126] extended the two-stage DEA model of Kao and Hwang [61] to a fuzzy programming model and calculated the efficiency of the process and its sub-processes. They used the method of Despotis and Smirlis [122] and the α-cuts method to obtain the LP models with optimistic and pessimistic viewpoints. Wang and Yan [127] presented FDEA/AR evaluation method to select the most appropriate manufacturing mode using the α-level approach. Khalili-Damghani and Taghavifard [128] considered a special case of network DEA consisting of two sub-processes for each DMU with fuzzy parameters. They converted the precise form of the model proposed by Kao and Hwang [61] to the fuzzy form in order to calculate the interval efficiency score of a DMU and its two sub-DMUs using the α-cut approach. To discriminate between the DMUs (and sub-DMUs), Khalili-Damghani and Taghavifard [128] proposed three distinct categories based on the interval efficiency bounds. They also applied the sensitivity analysis of Jahanshahloo et al. [76] to specify the radius of stability of the optimistic and pessimistic situations for the DMUs and sub-DMUs.

## *4.3 The Fuzzy Ranking Approach*

The fuzzy ranking approach is also another popular technique that has attracted a great deal of attention in the FDEA literature. In this approach the main idea is to find the fuzzy efficiency scores of the DMUs using fuzzy linear programs which require ranking the fuzzy set. The fuzzy ranking approach of efficiency measurement was initially developed by Guo and Tanaka [129]. They proposed a fuzzy CCR model in which fuzzy constraints (including fuzzy equalities and fuzzy inequalities) were converted into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers. Assuming there are $n$ DMUs under evaluation, the efficiency of the DMU$_j$ with $m$ symmetrical triangular fuzzy inputs and $s$ symmetrical triangular fuzzy outputs is denoted by $\tilde{x}_{ij} = (x_{ij}, c_{ij})$ and $\tilde{y}_{rj} = (y_{rj}, d_{rj})$, respectively, where $x_{ij}$ and $y_{rj}$ are the center, and $c_{ij}$ and $d_{rj}$ are the spread of fuzzy numbers. Guo and Tanaka [129] proposed the following LP model with two objective functions:

$$
\begin{aligned}
\max_{u,v}\quad & \theta_p = \sum_{r=1}^{s} (u_r y_{rp} - (1-\alpha) u_r d_{rp}) \\
& \left.\begin{aligned}
s.t.\ \max_{v}\ & \sum_{i=1}^{m} v_i c_{ip} \\
s.t.\quad & \sum_{i=1}^{m} (v_i x_{ip} - (1-\alpha) v_i c_{ip}) = 1 - (1-\alpha) e, \\
& \sum_{i=1}^{m} (v_i x_{ip} + (1-\alpha) v_i c_{ip}) \le 1 + (1-\alpha) e, \\
& v_i \ge 0, \quad \forall i.
\end{aligned}\right\} \rightarrow \text{Model}(12-1) \\
& \sum_{r=1}^{s} (u_r y_{rj} + (1-\alpha) u_r d_{rj}) \le \sum_{i=1}^{m} (v_i x_{ij} + (1-\alpha) v_i c_{ij}), \quad \forall j, \\
& \sum_{r=1}^{s} (u_r y_{rj} - (1-\alpha) u_r d_{rj}) \le \sum_{i=1}^{m} (v_i x_{ij} - (1-\alpha) v_i c_{ij}), \quad \forall j, \\
& u_r \ge 0, \quad \forall r.
\end{aligned}
\tag{12}
$$

where $\alpha \in [0, 1]$ is a predetermined possibility level by decision-makers and the unity number in the right hand side of the first constraint of model (2) is supposedly a symmetrical triangular fuzzy number $1 = (1, e)$. Note that if $c_{ij} = d_{rj} = 0$, then, the traditional CCR is obtained and if $\max[c_{p1}/x_{p1}, \ldots, c_{p1}/x_{ps}] \le e$ in (12-1), there exists an optimal solution in (12).

The fuzzy efficiency of each DMU under evaluation with the symmetrical triangular fuzzy inputs $\widetilde{x_{ip}}$ and outputs $\widetilde{y_{rp}}$ is obtained for each $\alpha$ possibility level as a non-symmetrical triangular fuzzy number $\widetilde{\theta_p} = \left(e_p^l, e_p^m, e_p^u\right)$ as follows:

$$
e_p^m = \frac{u_r^* y_{rp}}{v_i^* x_{ip}}, e_p^l = e_p^m - \frac{u_r^*\left(y_{rp} - d_{rp}(1-\alpha)\right)}{v_i^*\left(x_{ip} + c_{ip}(1-\alpha)\right)}, e_p^u = \frac{u_r^*\left(y_{rp} + d_{rp}(1-\alpha)\right)}{v_i^*\left(x_{ip} - c_{ip}(1-\alpha)\right)} - e_p^m
$$

where $u_r^*$ and $v_i^*$ are obtained from (12), and, $e_p^l, e_p^u$ and $e_p^m$ are the left, right spreads and the center of the fuzzy efficiency $\widetilde{\theta_p}$, respectively. Because of using a predefined $\alpha \in [0, 1]$ Guo and Tanaka [129]'s method can also be classified within $\alpha$-level approaches.

Guo and Tanaka [130] extended their earlier work [129] and introduced a fuzzy aggregation model to objectively rank a set of DMUs by integrating multiple attribute fuzzy values. Guo [131] further applied a novel FDEA model in a case study for a restaurant location problem in China by integrating the FDEA model proposed by Guo and Tanaka [129] with the fuzzy aggregation model proposed by Guo and Tanaka [130]. Sanei et al. [132] used the sensitivity analysis model of Cooper et al. [133] with fuzzy data, and they applied the approach of Guo and Tanaka [129] to build their fuzzy model for determining the stability radius for different $\alpha$ values. Chang and Lee [134] extended the integrated DEA and knapsack models proposed by Cook and Green [135] to select an optimal group of projects in the fuzzy environment. The proposed fuzzy programming model was converted into a non-linear programming model based on the method proposed by Guo and Tanaka [129]. Similar to the approach proposed by Guo and Tanaka [129], León et al. [136] developed a fuzzy BCC model (3). However, in Guo and Tanaka [129]'s method, a fuzzy efficiency score is obtained for each possibility level $\alpha$ while in León et al. [136]'s method, a crisp efficiency score is obtained for either all or each of the possibility levels. León et al. [136] proposed two different FDEA models depending on the ranking method used to interpret the fuzzy inequalities. The first model uses the ranking method of Ramík and Rímánek [137] to obtain a crisp efficiency score of $\text{DMU}_p$ in which all the possible values of the various variables for all the DMUs at all the possibility levels are considered. This model can be expressed as follows:

$$\begin{aligned}
&\min \quad \theta_p \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_p x_{ip}^L, \quad \forall i, && \sum_{j=1}^{n} \lambda_j y_{rj}^L \ge y_{rp}^L, \quad \forall r, \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^R \le \theta_p x_{ip}^R, \quad \forall i, && \sum_{j=1}^{n} \lambda_j y_{rj}^R \ge y_{rp}^R, \quad \forall r, \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^L - \sum_{j=1}^{n} \lambda_j c_{ij}^L \le \theta_p x_{ip}^L - \theta_p c_{ip}^L, \quad \forall i, && \sum_{j=1}^{n} \lambda_j y_{rj}^L - \sum_{j=1}^{n} \lambda_j d_{rj}^L \ge y_{rp}^L - d_{rp}^L, \quad \forall r, \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^R + \sum_{j=1}^{n} \lambda_j c_{ij}^R \le \theta_p x_{ip}^R + \theta_p c_{ip}^R, \quad \forall i, && \sum_{j=1}^{n} \lambda_j y_{rj}^R + \sum_{j=1}^{n} \lambda_j d_{rj}^R \ge y_{rp}^R + d_{rp}^R, \quad \forall r, \\
&\sum_{j=1}^{n} \lambda_j = 1, && \lambda_j \ge 0, \quad \forall j.
\end{aligned} \tag{13}$$

In model (13), the fuzzy inputs and the fuzzy outputs, respectively, are $\tilde{x}_{ij} = \left(x_{ij}^L, x_{ij}^R, c_{ij}^L, c_{ij}^R\right)$ and $\tilde{y}_{rj} = \left(y_{rj}^L, y_{rj}^R, d_{rj}^L, d_{rj}^R\right)$ in which $x_{ij}^L$ and $y_{rj}^L$ are the left centers, $y_{rj}^R$ and $x_{ij}^R$ are the right centers of the inputs and outputs, respectively, while $c_{ij}^L$ and $d_{rj}^L$ are the left spreads, and $c_{ij}^R$ and $d_{rj}^R$ are the right spreads of the inputs and outputs, respectively. The second model of León et al. [136] uses the ranking method of Tanaka et al. [138] to calculate the efficiency score of DMU$_p$ for each possibility level $\alpha \in [0, 1]$. This model can be formulated as follows:

$$
\begin{aligned}
\min \quad & \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_p x_{ip}^L, \quad \forall i, \quad \sum_{j=1}^{n} \lambda_j x_{ij}^R \le \theta_p x_{ip}^R, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^L - L_i^*(\alpha) \sum_{j=1}^{n} \lambda_j c_{ij}^L \le \theta_p x_{ip}^L - L_i^*(\alpha)\theta_p c_{ip}^L, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^R + R_i^*(\alpha) \sum_{j=1}^{n} \lambda_j c_{ij}^R \le \theta_p x_{ip}^R + R_i^*(\alpha)\theta_p c_{ip}^R, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^L \ge y_{rp}^L, \quad \forall r, \quad \sum_{j=1}^{n} \lambda_j y_{rj}^R \ge y_{rp}^R, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^L - L_i'^*(\alpha) \sum_{j=1}^{n} \lambda_j d_{rj}^L \ge y_{rp}^L - L_i'^*(\alpha) d_{rp}^L, \quad \forall r \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^R + R_i'^*(\alpha) \sum_{j=1}^{n} \lambda_j d_{rj}^R \ge y_{rp}^R + R_i'^*(\alpha) d_{rp}^R, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0, \quad \forall j.
\end{aligned}
\tag{14}
$$

A fuzzy set of efficient DMUs can be defined based on the optimal solution for model (14) so that the decision maker is able to identify sensitive DMUs and to select the appropriate possibility level. When the data are assumed to be symmetrical triangular fuzzy numbers that are denoted by $\tilde{x}_{ij} = (x_{ij}, c_{ij})$ and $\tilde{y}_{rj} = (y_{rj}, d_{rj})$, respectively, where $x_{ij}$ and $y_{rj}$ are the center, and $c_{ij}$ and $d_{rj}$ are the spread of fuzzy numbers, model (14) can be written as:

$$
\begin{aligned}
\min \quad & \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - (1-\alpha) \sum_{j=1}^{n} \lambda_j c_{ij} \le \theta_p x_{ip} - (1-\alpha)\theta_p c_{ip}, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} + (1-\alpha) \sum_{j=1}^{n} \lambda_j c_{ij}^R \le \theta_p x_{ip} + (1-\alpha)\theta_p c_{ip}, \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - (1-\alpha) \sum_{j=1}^{n} \lambda_j d_{rj} \ge y_{rp} - (1-\alpha) d_{rp}, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + (1-\alpha) \sum_{j=1}^{n} \lambda_j d_{rj} \ge y_{rp} + (1-\alpha) d_{rp}, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \ge 0, \quad \forall j.
\end{aligned}
\tag{15}
$$

Note that $L_i^*(\alpha) = R_i^*(\alpha) = L_i'^*(\alpha) = R_i'^*(\alpha) = 1 - \alpha$, $\alpha \in [0, 1]$. We can also categorize León et al. [136]'s method as an $\alpha$-level approach since they used $\alpha \in [0, 1]$ in their model. Sefeedpari et al. [139] evaluated the technical efficiency of poultry eggs producers using the FDEA model proposed by León et al. [136].

Hatami-Marbini et al. [140] extended a fuzzy CCR model for evaluating the DMUs from the perspective of the best and the worst possible relative efficiency by utilizing León et al. [136]'s approach. Beiranvand et al. [141] applied a genetic algorithm to identify the optimal $\alpha$ possibility level of León et al [136]'s FDEA method. Then, in order to rank all of the DMUs, a closeness coefficient index was obtained by combining the two various efficiencies. Jahanshahloo et al. [76] proposed a fuzzy ranking method for solving the SBM model in DEA when the input-output data are triangular fuzzy numbers. Saati and Memariani [142] addressed some shortcomings of the FDEA proposed by Jahanshahloo et al. [76] and suggested several corrections to their method. Azadeh et al. [143] developed an integrated algorithm containing DEA and FDEA for measuring the efficiency of wireless communication sectors for 42 countries with uncertain data. They used principal component analysis and the Spearman correlation technique to verify and validate the DEA models. The FDEA method proposed by Jahanshahloo et al. [76] was implemented to study the effects of government subsidies and interventions (if any) through regions. Azadeh et al. [144] proposed a hybrid method for the location optimization of solar plants using an artificial neural network and FDEA. Their study took account of the FDEA model explored by Jahanshahloo et al. [76]. For validation of the results of the FDEA, they used DEA when $\alpha$=1. They then determined the best $\alpha$-cut based on the normality test. Azadeh et al. [145] proposed an adaptive-network-based fuzzy inference system-FDEA algorithm for improving the long-term natural gas consumption forecasting and analysis. They used the FDEA method proposed by Jahanshahloo et al. [76] to study the behavior of gas consumption.

Molavi et al. [146] introduced two FDEA models in which the objective function and fuzzy constraints of the fuzzy CCR model were transformed into crisp conditions by using *LR*-fuzzy numbers and the ranking method of Ramík and Římánek [137]. Soleimani-damaneh et al. [147] addressed some computational and theoretical shortcomings of several FDEA models including Kao and Liu [42], León et al. [136], Lertworasirikul et al. [31], Guo and Tanaka [129] and Jahanshahloo et al. [76]. Furthermore, they proposed a fuzzy BCC model using the fuzzy number ranking method proposed by Yao and Wu [148] for trapezoidal fuzzy data in DEA.

Hosseinzadeh Lotfi et al. [149] applied trapezoidal fuzzy data to Jahanshahloo et al.'s [101] DEA method, in which a fuzzy fixed cost was equitably assigned to all DMUs in such a way that the efficiency scores were not changed. They used a fuzzy ranking method to solve the fuzzy model in which each fuzzy constraint transformed to three crisp constraints. Hosseinzadeh Lotfi et al. [150] adopted the linear ranking function proposed by Maleki [151] to present fuzzy CCR models with triangular fuzzy data. Jahanshahloo et al. [152] suggested a method to deal with the DEA-based Malmquist productivity index for all DMUs with triangular fuzzy inputs and outputs. They applied a linear ranking function proposed by Maleki [151] to transform their fuzzy LP model into a group of the conventional crisp DEA models. Pal et al. [153] used a FDEA approach and $\alpha$-parametric inequalities in quality function deployment. They used a fuzzy CCR model based on the method proposed by Lai and Hwang [154].

Hosseinzadeh Lotfi and Mansouri [155] considered the extended DEA-discriminant analysis method proposed by Sueyoshi [57] as fuzzy data and changed their fuzzy model into a crisp model using the linear ranking function proposed by Maleki [151]. Zhou et al. [156] developed a FDEA method to evaluate the efficiency performance of real estate investment programs. They applied the ranking fuzzy numbers to solve their model and designed a "relatively effective controller" which considered controlling the diversity of the method. Noora and Karami [157] adopted triangular fuzzy data to establish a fuzzy non-radial DEA model and applied a ranking function proposed by Maleki et al. [158] to transform the fuzzy LP into the crisp DEA models. Jahanshahloo et al. [159] applied the linear ranking function proposed by Mahdavi-Amiri and Nasseri [160] to change the fuzzy cost efficiency model into a conventional LP problem. Soleimani-damaneh [161] used the fuzzy signed distance and the fuzzy upper bound concepts to formulate a fuzzy additive model in DEA with fuzzy input-output data. Soleimani-damaneh [162] put forward a theorem on the FDEA model which was proposed by Soleimani-damaneh [161] in order to show the existence of distance-based upper bound for the objective function of the model.

Hatami-Marbini et al. [163] proposed a FDEA model to assess the efficiency scores in the fuzzy environment. They used the proposed ranking method in Asady and Zendehnam [164] and obtained precise efficiency scores for the overall rankings of the DMUs. They compared their method with the FDEA methods proposed by Soleimani et al. [147] and León et al. [136]. They also applied their model to sixteen bank's branches. Jahanshahloo et al. [165] further introduced an alternative approach for solving the fuzzy $l_1 - norm$ method in DEA with fuzzy data based on the comparison of fuzzy numbers proposed by Tran and Duckstein [166] to change fuzzy LP to a crisp model form. Hosseinzadeh Lotfi et al. [167] generalized a multi-activity network DEA to fuzzy inputs and outputs which were formed by triangular membership functions. They used a ranking function to convert the multi-activity network FDEA into a multi-activity network crisp DEA model. Hatami-Marbini et al. [168] proposed an interactive evaluation process for measuring the efficiency of peer DMUs in FDEA with consideration of the decision makers' preferences. By applying the fuzzy LP of Jiménez et al. (2007) based on the ranking method introduced by Jiménez [99], they constructed a DEA model with fuzzy parameters to calculate the fuzzy efficiency of the DMUs for different $\alpha$ levels. Then, the decision maker identifies his/her most preferred fuzzy goal for each DMU under evaluation and a ranking order of the DMUs can be obtained by a modified Yager index.

Azadeh et al. [169] presented a three-step algorithm for tackling a special case of single-row facility layout problems. In the first step, discrete-event-simulation with deterministic and fuzzy data was used to model the process. In the second step, the verification and validation of the previous results were studied using crisp simulation. In the final step, the range-adjusted measure as a measure in the non-radial DEA model was utilized with fuzzy parameters for ranking the simulation results and finding the optimal layout design. Similar to Hatami-Marbini et al. [168], the authors used a possibilistic programming method proposed by Jiménez et al. (2007) to convert the FDEA model to an equivalent DEA model for different $\alpha$- cuts. Emrouznejad et al. [170] developed two methods for measuring the overall profit Malmquist productivity index (MPI) when the inputs, outputs, and price vectors are fuzzy or vary in intervals. They first extended a fuzzy version of the conventional MPI model using a ranking method and then defined an interval for the overall profit MPI of each DMU. They classified the DMUs into six groups according to the intervals obtained for their overall profit efficiency and MPIs. Ahmady et al. [171] generalized DEA with double frontiers (Wang and Chin 2009) from precise mathematical DEA modelling to fuzzy formulations in order to cope with ambiguity and fuzziness in supplier selection problems. They employed the ranking method defined by Zimmermann [44] to provide the optimal solutions for the FDEA models.

In this section, we also review a related method, called "defuzzification approach", proposed by Lertworasirikul [33]. In this approach, which is essentially a fuzzy ranking method, fuzzy inputs and fuzzy outputs are first defuzzified into crisp values. These crisp values are then used in a conventional crisp DEA model which can be solved by an LP solver. Dia [172] developed an alternative FDEA model based on fuzzy arithmetic operations and ranking of fuzzy numbers. A fuzzy aspiration level was used to change the model into a crisp DEA model and the fuzzy results outlined the practical and robustness aspects of the fuzzy methodology. Lee [173] and Lee et al. [174] have also proposed FDEA models for CCR and BCC by defuzzifying fuzzy inputs and outputs into crisp values and using them in conventional DEA models. Juan [175] proposed a two-stage decision support model by using a hybrid DEA and case-based reasoning model. In this approach, the center of gravity method (CGM) suggested in [176] was used to transform the fuzzy data into crisp data and build a conventional CCR model. Bagherzadeh valami [177] introduced a cost efficiency model with triangular fuzzy input prices and proposed a method for comparing the production cost of the target DMU with the minimum cost fuzzy set. Hosseinzadeh Lotfi et al. [178] proposed a FDEA model to evaluate a set of DMUs where all parameters and decision variables were fuzzy numbers. They changed their fuzzy model into a multiple objective LP model and solved the LP model by using a lexicography method. The defuzzification approach is simple but the uncertainty in input and output variables (i.e., possible range of values at different $\alpha$-levels) is not effectively considered [114]. Hatami-Marbini et al. [1] developed a fully fuzzified CCR model to obtain the fuzzy efficiency of the DMUs by utilizing a fully fuzzified LP model, in which all of the input-output data and variables (including their weights) were fuzzy numbers. In spite of its simplicity, the defuzzification approach has not been used by DEA researchers and practitioners. The lack of interest in the defuzzification approach might be due to the fact that with the defuzzification approach the fuzziness in the inputs and outputs is effectively ignored [113].

Moheb-Alizadeh et al. [179] adopted a multi-criteria DEA for location–allocation problems with fuzzy input and output quantities. They determined the location of facilities as well as the volume of assigned demands for each located facility such that not only the total cost involving transportation and fixed location costs is minimized but also the efficiency scores of the facilities are maximized. They presented a two-step approach to solve their fuzzy multi-objective non-linear programming problem. In the first step, the problem was transformed into a crisp multi-objective programming using the defuzzification method of Carlsson and Korhonen [38] and then an efficient solution of such a multi-objective programming problem was computed using the minimum deviation method. To quantify the impreciseness in the environmental factors, Amindoust et al. [180] incorporated fuzzy logic into DEA to assess the green suppliers. They defuzzified the FDEA model with the center of area method.

## *4.4 The Possibility Approach*

The fundamental principles of the possibility theory are entrenched in Zadeh's [43] fuzzy set theory. Zadeh [43] suggests that a fuzzy variable is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution. In fuzzy LP models, fuzzy coefficients can be viewed as fuzzy variables and the constraints can be considered to be fuzzy events. Hence, the possibilities of fuzzy events (i.e., fuzzy constraints) can be determined using possibility theory. Dubois and Prade [181] provide a comprehensive overview of possibility theory.

Guo et al. [182] initially built FDEA models based on possibility and necessity measures and then Lertworasirikul [33] and Lertworasirikul et al. [183], [184] have proposed two approaches for solving the ranking problem in FDEA models called the "possibility approach" and the "credibility approach". They introduced the possibility approach from both optimistic and pessimistic view points by considering the uncertainty in fuzzy objectives and fuzzy constraints with possibility measures. In their credibility approach, the FDEA model was transformed into a credibility programming-DEA model and fuzzy variables were replaced by 'expected credits', which were obtained by using credibility measures. The mathematical details of the credibility model can be found in Lertworasirikul et al. [32].

Lertworasirikul et al. [31], [185] proposed a possibility approach for solving a fuzzy CCR model in which fuzzy constraints were treated as fuzzy events. They transformed the FDEA model into a possibility LP problem by using the possibility measures of the fuzzy events. In the special case, if the fuzzy data were assumed to be trapezoidal fuzzy numbers, the possibility DEA model becomes a LP model. The proposed possibility CCR model of Lertworasirikul et al. [31] where they applied the concept of chance-constrained programming (CCP) and possibility of fuzzy events are represented by the following:

$$\begin{aligned}
&\max \quad \theta_p = \bar{f} \\
&s.t. \quad \Big(\sum_{r=1}^{s} u_r \tilde{y}_{rp}\Big)_{\beta}^{U} \geq \bar{f}, \\
&\Big(\sum_{i=1}^{m} v_i \tilde{x}_{ip}\Big)_{\alpha_0}^{U} \geq 1, \\
&\Big(\sum_{i=1}^{m} v_i \tilde{x}_{ip}\Big)_{\alpha_0}^{L} \leq 1, \\
&\Big(\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij}\Big)_{\alpha}^{L} \leq 0, \quad \forall j, \\
&u_r, v_i \geq 0, \quad \forall r, i.
\end{aligned} \tag{16}$$

where $\beta \in [0,1]$, $\alpha_0 \in [0,1]$ and $\alpha \in [0,1]$ are predetermined admissible levels of possibility. The purpose of (16) is to maximize $\bar{f}$ so that $\sum_{r=1}^{s} u_r \tilde{y}_{rp}$of the first constraint can achieve with a "possibility" level $\beta$ or higher, subject to the possibility levels being at least $\alpha_0$ and $\alpha$ in other constraints. In other words, at the optimal solution, the value of $\sum_{r=1}^{s} u_r \tilde{y}_{rp}$ is obtained at least equal to $\bar{f}$ with the possibility level $\beta$; while at the same time all constraints are satisfied at the predetermined possibility levels.

Lertworasirikul et al. [32] further developed possibility and credibility approaches for solving fuzzy BCC models. They applied the concept of chance-constrained programming (CCP) and possibility of fuzzy events (fuzzy constraints) to the primal and dual of the fuzzy BCC models in order to obtain possibility BCC models. This approach also studied the relationship between the primal and dual models of the fuzzy BCC. The efficiency obtained through their possibility approach to the primal and dual models provided the upper bound and the lower bound for each DMU for a given possibility level. Next, in the credibility approach, they replaced the ''expected credits'' with fuzzy variables to cope with the uncertainty in fuzzy objectives and fuzzy constraints. Hence, their fuzzy BCC model was transformed into a credibility programming-DEA model. An efficiency score for each DMU was obtained from the credibility approach as a representative of its possible range. Unlike the possibility approach, the decision makers did not have to determine any parameters or rank fuzzy efficiency values in the credibility approach. According to the possibility BCC approach proposed in [32], the primal proposed model can be represented by the following model:

$$\begin{aligned}
\max \quad & \theta_p = \Big(\sum_{r=1}^{s} u_r \tilde{y}_{rp}\Big)_{\beta}^{U} - u_0 \\
s.t. \quad & \Big(\sum_{i=1}^{m} v_i \tilde{x}_{ip}\Big)_{\alpha_0}^{U} \geq 1, \\
& \Big(\sum_{i=1}^{m} v_i \tilde{x}_{ip}\Big)_{\alpha_0}^{L} \leq 1, \\
& \Big(\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij}\Big)_{\alpha}^{L} - u_0 \leq 0, \quad \forall j, \\
& u_r, v_i \geq 0, \quad \forall r, i.
\end{aligned} \tag{17}$$

where $\beta \in [0, 1]$, $\alpha_0 \in [0, 1]$ and $\alpha \in [0, 1]$ are the predetermined admissible levels of possibility.

Similarly, according to the possibility approach [32], the dual proposed model can be represented by the following model:

$$\begin{aligned}
\min \quad & \theta_B \\
s.t. \quad & \Big(\theta_B \tilde{x}_{ip} - \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}\Big)_{\bar{\alpha}_1}^{U} \geq 0, \quad \forall i, \\
& \Big(\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} - \tilde{y}_{rp}\Big)_{\bar{\alpha}_2}^{U} \geq 0, \quad \forall r, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \quad \forall j.
\end{aligned} \tag{18}$$

where $\bar{\alpha}_1 \in [0, 1]$ and $\bar{\alpha}_2 \in [0, 1]$ are predetermined admissible levels of possibility. Garcia et al. [186] introduced a FDEA approach to rank failure modes identified by means of the occurrence, severity and detection indices. Their method allowed the experts to use linguistic variables in assigning more important values to the considered indices. They utilized the possibility approach proposed by Lertworasirikul et al. [31] in their model to solve their FDEA problem. Similarly, Wu et al. [187] used the formulation of Lertworasirikul et al. [31] in their FDEA model to cope with the quantitative and linguistic variables in the efficiency analysis of cross-region bank branches in Canada.

Lin [188] proposed a three-phase method as a decision support tool using an integrated analytic network process and FDEA approach to tackle the personnel selection problem. In phase 1, a fuzzy scheme was employed to appraise the applicants using linguistic variables. In phase 2, the analytic network process technique was employed to obtain the global criteria weights with regards to the decision–makers' preferences. In phase 3, a possibility DEA-CCR model based on Lertworasirikul et al. [31] was used and the global criteria weights from the previous phase were considered as weight restrictions to measure the relative effectiveness of the applicants at different possibility levels. Zhao and Yue [189] evaluated the mutual funds management companies in China, in which their internal structure involved investment research competence and marketing service competence. They extended the two-subsystem FDEA model based on the FDEA model proposed by Lertworasirikul et al. [31]. Nedeljković and Drenovac [190] used the possibility approach of Lertworasirikul et al. [31] to measure the efficiency of delivery post offices.

Ramezanzadeh et al. [191] proposed a CCR model with a chance-constrained programming approach and used the α-level method and a fuzzy probability measure to rectify the randomness by the classical mean-variance method of Cooper et al. [192]. Jiang and Yang [193] proposed a fuzzy chance-constrained creditability programming DEA model. Khodabakhshi et al. [194] formulated two alternative fuzzy and stochastic additive models to determine returns to scale in DEA. They developed the fuzzy and stochastic DEA models based on the possibility approach and the concept of chance constraint programming, respectively. Wen and Li [195] proposed a hybrid algorithm integrating fuzzy simulation and a genetic algorithm to solve a FDEA model based on the credibility measure. Recently, Wen et al. [196] extended the CCR model to a FDEA model based on the credibility measure presented by Liu [197]. They designed a hybrid algorithm combining a fuzzy simulation and a genetic algorithm to rank all the DMUs with fuzzy inputs and outputs.

Wen et al. [198] investigated the sensitivity and stability analysis of the FDEA model developed by Wen and You [199] with respect to the concept of the credibility measure. Hossainzadeh et al. [200] presented a FDEA method using CCP. The FDEA model was first converted into a multi-objective programming model by considering optimistic, pessimistic and expected values. A goal programming technique was applied to obtain the LP model. Wang and Chin [201] introduced a FDEA method to measure the optimistic and the pessimistic efficiencies of DMUs using a fuzzy expected value approach with either fuzzy or crisp multipliers. They used the geometric average to aggregate the two extreme efficiencies and rank the DMUs. Payan and Sharifi [202] extended a method for measuring the fuzzy MPI using credibility theory.

## 4.5 The Fuzzy Arithmetic

The papers published in this group focus on the fact that decision makers are not allowed to convert a fuzzy fractional programming to a LP model using conventional methods. That is to say, $\sum_{r=1}^{s} u_r\tilde{y}_{rj} \big/ \sum_{i=1}^{m} v_i\tilde{x}_{ij}$ cannot be transformed into $\sum_{r=1}^{s} u_r\tilde{y}_{rj}$ by setting $\sum_{i=1}^{m} v_i\tilde{x}_{ij} = \tilde{1}$. Though $\tilde{1}$ is presumed to be crisp (i.e., Dual CCR model (2)) there are a variety of methods for measuring efficiency in a fuzzy environment that mostly involve extensive computational efforts. As a result, methods in this category focus heavily on fuzzy arithmetic to cope with the fuzziness of the input and output data in the DEA models.

Wang et al. [203] briefly argued how to treat fuzzy data using fuzzy arithmetic. Along this line, Wang et al. [204] proposed two FDEA models with fuzzy inputs and outputs by means of fuzzy arithmetic. They converted each proposed fuzzy CCR model into three LP models in order to calculate the efficiencies of the DMUs as fuzzy numbers. In addition they developed a fuzzy ranking approach to rank the fuzzy efficiencies of the DMUs. Abdoli et al. [205] studied the productivity of a set of knowledge workers using a FDEA-based approach in which the fuzzy efficiencies were computed using Wang et al. [204]'s method. Jafarian-Moghaddam and Ghoseiri [206] extended a multi-objective static DEA model of Chiang and Tzeng [207] to the fuzzy dynamic multi-objective DEA model. They transformed the fuzzy multiple objectives programming problem to one objective programming problem using the method introduced by Zimmermann [44]. Furthermore, Jafarian-Moghaddam and Ghoseiri [206] incorporated missing values, as triangular fuzzy numbers, into their model. After obtaining the LP model using the previous method, they solved the model based on the FDEA method developed by Wang et al. [204]. Raei Nojehdehi et al. [208] studied the production frontier and production possibility set in FDEA. They defined the production possibility set as a fuzzy set where all production plans are treated as its member with different degrees of membership. Mirhedayatian et al. [236] used FDEA to obtain weights in the fuzzy analytic hierarchy process (AHP). They considered the fuzzy arithmetic approach to solve the FDEA model. Their proposed approach was utilized for determining the superior tunnel ventilation system. Mirhedayatian et al. [237] proposed a network DEA model for evaluating the firms in green supply chain management. Their method encompasses dual-role factors, undesirable outputs and fuzzy data and they used the principal fuzzy calculation to solve the FDEA model. Similar to Hatami-Marbini et al. [74], Mirhedayatian et al. [209] used FDEA in the technique for order performance by similarity to the ideal solution (TOPSIS) by means of the fuzzy arithmetic method proposed in Wang et al. [204] to examine the relative welding process selection factors and evaluate different welding processes. Alem et al. [210] proposed an efficiency analysis method using FDEA and fuzzy AHP. They first calculated the fuzzy efficiency via the FDEA method of Wang et al. [204] and the applied the fuzzy AHP to rank the fuzzy efficiency scores. Razavi Hajiagha et al. [211] extended a FDEA model by using intuitionistic fuzzy data consisting of a membership and non-membership functions. They applied an arithmetical operator to obtain the LP problem. Azadi et al. [212] applied fuzzy input-output data to present a goal directed benchmarking model for benchmarking and selecting the best suppliers. To resolve their fuzzy mathematical DEA model, Azadi et al. [212] used fuzzy arithmetic to obtain the performance and benchmarks of each supplier. Khalili-Damghani and Taghavifard [124] and Khalili-Damghani and Taghavifard [128] used fuzzy arithmetic to remove the $\alpha$ from the $\alpha$-cut model and achieve a LP model.

### 4.6 The Fuzzy Random/Type-2 Fuzzy Set

Zadeh [10] initially introduced the type-2 fuzzy set as an extension of the prevalent fuzzy set in which uncertainty is incorporated into the membership function of a fuzzy set. In addition, many complex systems often involve randomness and fuzziness simultaneously. In response, Kwakernaak [213] proposed fuzzy random variables to tackle performance measurement in such systems. In this section, we review a few recent studies on the application of type-2 fuzzy set and fuzzy random variables in DEA.

Qin et al. [214] developed a DEA model with type-2 fuzzy inputs and outputs to deal with linguistic uncertainties as well as numerical uncertainties with respect to fuzzy membership functions. Based on the expected value of a fuzzy variable, they used a reduction method for type-2 fuzzy variables and built a FDEA model using the generalized credibility measure. Qin and Liu [215] proposed a class of fuzzy random DEA (FRDEA) models with fuzzy random inputs and outputs where randomness and fuzziness coexisted in an evaluation system and the fuzzy random data were characterized with known possibility and probability distributions. They also proposed a hybrid genetic algorithm and stochastic simulation approach to assess the objective function of the proposed DEA. Qin and Liu [216] also proposed another approach similar to the method proposed in [215]. They included the chance functions in the objective and constraint functions which were subsequently converted into the equivalent stochastic programming forms and solved with a hybrid genetic algorithm and Monte Carlo simulation method. Qin et al. [217] incorporated the fuzzy possibility theory of Liu and Liu [218] into DEA models in which the inputs and outputs were type-2 fuzzy with known possibility distributions. Based on mean reduction methods, the fuzzy generalized expectation DEA models were developed where the inputs and outputs were independent type-2 triangular fuzzy variables.

Qin et al. [217] presented the equivalent parametric forms of the constraints and the generalized expectation objective. Tavana et al. [219] developed three FDEA models with respect to probability-possibility, probability-necessity and probability-credibility constraints in which fuzziness and randomness coexisted in the evaluation problems. In addition, they illustrated the proposed model by using a case study for the base realignment and closure (BRAC) decision process at the U.S. department of defense (DoD). Zerafat Angiz et al. [220] used a fuzzy non-radial DEA method to determine the ideal solution and distance function of type-2 attributes using a TOPSIS methodology. The principal idea of FDEA was to maximize all the membership functions of the fuzzy parameters. The proposed FDEA model was thus transformed into the multi-objective program that was solved using a method proposed by Zerafat Angiz et al. [221].

Tavana et al. [222] first developed three DEA models for estimating the radial efficiency of DMUs in the presence of random-fuzzy (Ra-Fu) variables with Poisson, uniform and normal distributions. They then advanced the formulation of the possibility-probability and the necessity-probability DEA models with Ra-Fu parameters for where the Ra-Fu data contained normal distributions with fuzzy means and variances. Tavana et al. [222] lastly presented the general possibility-probability and necessity-probability DEA models with fuzzy thresholds.

## 4.7 Other Developments in Fuzzy DEA

In this section, we review several FDEA models that do not fall into the fuzzy ranking approach, the tolerance approach, the α-level based approach, the possibility approach the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set categories. Hougaard [223] extended scores of technical efficiency used in DEA to fuzzy intervals and showed how the fuzzy scores allow the decision maker to use scores of technical efficiency in combination with other sources of available performance information such as expert opinions, key figures, etc. Sheth and Triantis [224] introduced a fuzzy goal DEA framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. They defined a membership function for each fuzzy constraint associated with the efficiency and effectiveness goals and represented the degree of achievement of that constraint. Hougaard [225] introduced a simple approximation for the assessment of efficiency scores with regards to fuzzy production plans. This approach did not require the use of fuzzy LP techniques and had a clear economic interpretation where all the necessary calculations could be performed in a spreadsheet making it highly operational. Uemura [226] introduced a fuzzy goal based on the evaluation ratings of individual outputs obtained from the fuzzy loglinear analysis and then proposed a fuzzy goal into the DEA. Luban [227] proposed a method inspired by Sheth and Triantis's [224] work and used the fuzzy dimension of the DEA models to select the membership function, the bound on the inputs and outputs, the global targets, and the bound of the global targets.

Zerafat Angiz et al. [228] proposed an alternative ranking approach based on DEA in the fuzzy environment to aggregate preference rankings of a group of decision makers. They applied their method to a preferential voting system with four stages. Although they considered data as ordinal relations, stage 1 defined a fuzzy membership function for ranking a set of alternatives to find the ideal alternative. In the second stage they used the FDEA model proposed in Zerafat Angiz et al. [229] to obtain the ideal solution. In the last two stages, they proposed a method to aggregate the results to a single score using subjective weights obtained from comparative judgments for ranking the alternatives. Zerafat Angiz et al. [230] proposed a multi-objective mathematical model using the fuzzy concept on the multipliers for ranking the efficient units.

Zerafat Angiz et al. [231] also presented a ranking method in the preferential voting system using DEA and the concept of a fuzzy set. Their method first constructed fuzzy numbers based on the number of votes for the first ranked DMU. Second, the nearest point to a fuzzy number concept was used to define a dummy ideal alternative. Finally, the efficiency of the alternatives in a pairwise comparison with the dummy ideal alternative was calculated using the DEA method. Zerafat Angiz and Mustafa [232] used fuzzy concepts to deal with non-discretionary data embedded in DEA models. Bagherzadeh Valami et al. [233] considered the production possibility set as a fuzzy set where the input and output data vary in the interval.

## 5 Conclusions, Limitations and Directions for Future Research

Fuzzy set theory has been used widely to model uncertainty in DEA. Although other models such as probabilistic/stochastic DEA and statistical preference (e.g. bootstrapping) are also used to model uncertainty in DEA, in this chapter we focus on the fuzzy set DEA papers published in the English-language academic journals. We present a classification scheme with six primary categories, namely, the tolerance approach, the α-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 fuzzy set. While most of these approaches are powerful, they usually have some theoretical and/or computational limitations and sometimes applicable to a very specific situation (e.g., Soleimani-damaneh et al. [147]). For example, the tolerance approach uses fuzzy inequalities and equalities instead of fuzzy inputs and fuzzy outputs. The most popular FDEA group, α-level based approach, often provides a fuzzy efficiency score whose membership function is constructed from α-level even though models related to this approach are not computationally efficient because this group mostly requires a large number of LP models according to various α-levels (e.g., Soleimani-damaneh et al. [147]). In this study, the importance of fuzzy ranking approach in the literature is ranked second (see Table 1) while a considerable limitation of this group is that different fuzzy ranking methods may result in different efficiency scores. In the possibility approach, the proposed models may not be adapted to other DEA models (e.g., Soleimani-damaneh et al. [147]), and we believe that this approach requires complicated numerical computations compared to other approaches.

In summary, FDEA is best known for its distinct treatment of the imprecise or vague input and output data in the real-world problems. As shown in Fig. 2, FDEA is a growing field with many practical and theoretical developments. Nevertheless, we believe that FDEA is still in its early stages of development.

A wide variety of applications and proliferation of models have demonstrated that FDEA is an effective approach for performance measurement in problems with imprecise and vague data. Nevertheless, there are a number of challenges involved in the FDEA research that provide a great deal of fruitful scope for future research:

- A unified process: It is imperative to provide a unified FDEA approach for practicing managers and novice users. This need is clearly illustrated by the

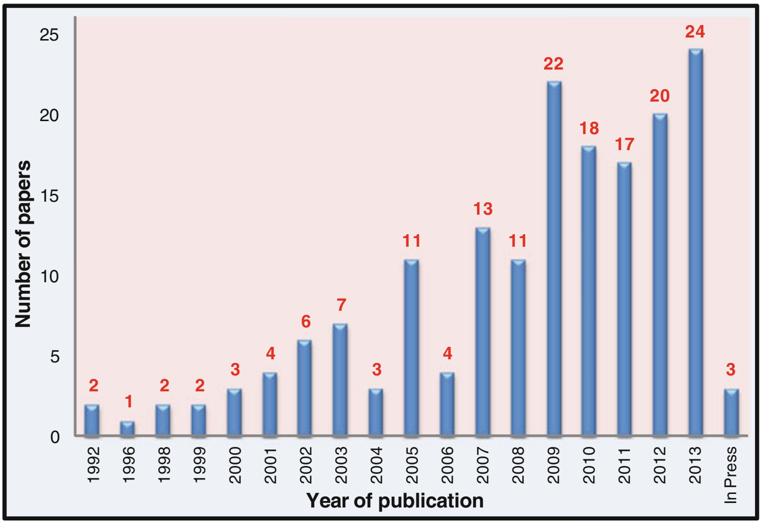


**Fig. 2** Three decades of fuzzy DEA development (1992–2013)

large number of models and the proliferation of frameworks, at times confusing or even contradictory. A unified process similar to the COOPER-framework [234] can provide the novice users with a clear-cut procedure for solving FDEA problems. Experienced users can use the unified process for modeling depth and breadth.

- User-friendly software: Although there are several DEA software packages in the market, none of them are capable of handling fuzzy data and FDEA modeling.
- Real-life applications: Most of the papers published in the literature have used simple examples or small sets of hypothetical data to illustrate the applicability of the models. We encourage researchers to use real-world case studies in demonstrating the applicability of their models and exhibit the efficacy of their procedures and algorithms.
- Sensitivity analysis: There is a need for comprehensive studies focusing on sensitivity analysis strategies in FDEA. Fuzzy data by definition are not fixed. As a result, the results from fuzzy models are less robust and more likely to change over a period of time or even during the model-building phase. Consequently, there is a need for elaborate and comprehensive sensitivity analysis methods and procedures to deal with the changing nature of FDEA models.

We hope that our research will benefit a wide range of users who desire to solve real-life DEA problems with vague or imprecise data. The taxonomy and the comprehensive review of the literature provided here should lead to a better understanding of FDEA and its applications.

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