



Invited Review

A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making

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ABSTRACT

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. Crisp input and output data are fundamentally indispensable in conventional DEA. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Many researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA. In this study, we provide a taxonomy and review of the fuzzy DEA methods. We present a classification scheme with four primary categories, namely, the tolerance approach, the α -level based approach, the fuzzy ranking approach and the possibility approach. We discuss each classification scheme and group the fuzzy DEA papers published in the literature over the past 20 years. To the best of our knowledge, this paper appears to be the only review and complete source of references on fuzzy DEA.

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1. Introduction

Data envelopment analysis (DEA) was first proposed by Charnes et al. (1978), and is a non-parametric method of efficiency analysis for comparing units relative to their best peers (efficient frontier). Mathematically, DEA is a linear programming-based methodology for evaluating the relative efficiency of a set of decision making units (DMUs) with multi-inputs and multi-outputs. DEA evaluates the efficiency of each DMU relative to an estimated production possibility frontier determined by all DMUs. The advantage of using DEA is that it does not require any assumption on the shape of the frontier surface and it makes no assumptions concerning the internal operations of a DMU. Since the original DEA study by Charnes et al. (1978), there has been a continuous growth in the field. As a result, a considerable amount of published research and bibliographies have appeared in the DEA literature, including those of Seiford (1996), Gattoufi et al. (2004), Emrouznejad et al. (2008) and Cook and Seiford (2009).

The conventional DEA methods require accurate measurement of both the inputs and outputs. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Imprecise evaluations may be the result of unquantifiable, incomplete and non obtainable information. Some researchers have proposed various fuzzy methods for dealing with this impreciseness and ambiguity in DEA. Since the original study by Sengupta (1992a,b), there has been a continuous interest and increased development in fuzzy DEA literature. In this study, we review the fuzzy DEA methods and present a taxonomy by classifying the fuzzy DEA papers published over the past two decades into four primary categories, namely, the tolerance approach, the α -level based approach, the fuzzy ranking approach, and the possibility approach; and a secondary category to group the pioneering papers that do not fall into the four primary classifications. This study appears to be the only review and complete source of references on fuzzy DEA since its inception two decades ago. This paper is organized into five sections. In Section 2, we present the fundamentals of DEA. In Section 3, we review the fuzzy DEA principles. In Section 4, we present a summary development of the fuzzy DEA followed by a detailed description of the fuzzy DEA methods in the literature. Conclusions and future research directions are drawn in Section 5.

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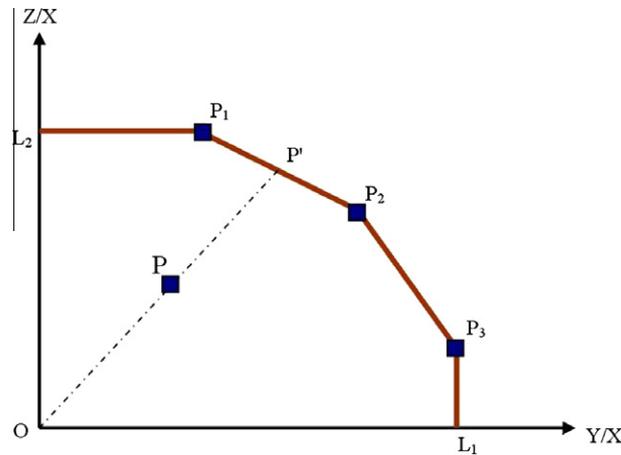


Fig. 1. An output-oriented DEA with two outputs and one input.

2. The fundamentals of DEA

There are basically two main types of DEA models: a constant returns-to-scale (CRS) or CCR model that initially introduced by Charnes et al. (1978) and a variable returns-to-scale (VRS) or BCC model that later developed by Banker et al. (1984). The BCC model is one of the extensions of the CCR model where the efficient frontiers set is represented by a convex curve passing through all efficient DMUs.

DEA can be either input- or output-orientated. In the first case, the DEA method defines the frontier by seeking the maximum possible proportional reduction in input usage, with output levels held constant, for each DMU. However, for the output-orientated case, the DEA method seeks the maximum proportional increase in output production, with input levels held fixed.

Fig. 1 illustrates a simple VRS output-oriented DEA problem with two outputs, Y and Z, and one input, X. The isoquant L1L2 represents the technical efficient frontier comprising P1, P2, and P3 which are technically efficient DMUs and hence on the frontier. If a given DMU uses one unit of input and produces outputs defined by point P, the technical inefficiency of that DMU are represented as the distance PP', this is the amount by which all outputs could be proportionally increased without increasing input. In percentage terms, it is expressed by the ratio OP/OP', which is the ratio by which all the outputs could be increased.

An input oriented DEA model with m input variables (x1, . . . , xm) and s output variables (y1, . . . , ys) with n decision making units (j = 1, 2, . . . , n) is presented in Model 1a (for CCR model) and Model 1b (for BCC model). The only difference between these two models is on inclusion of the convexity constraints of $\sum_{j=1}^n \lambda_j = 1$ in the BCC model.

Model 1a: A basic CCR model

$$\begin{aligned} \min \quad & \theta_p \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq \theta_p x_{ip}, \quad \forall i, \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rp}, \quad \forall r, \\ & \lambda_j \geq 0, \quad \forall j. \end{aligned}$$

Model 1b: A basic BCC model

$$\begin{aligned} \min \quad & \theta_p \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq \theta_p x_{ip}, \quad \forall i, \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{rp}, \quad \forall r, \\ & \sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0, \quad \forall j. \end{aligned}$$

DEA applications are numerous in financial services, agricultural, health care services, education, manufacturing, telecommunication, supply chain management, and many more. For a recent comprehensive bibliography of DEA see Emrouznejad et al. (2008). Recently fuzzy logic introduced to DEA for measuring efficiency of decision making units under uncertainty mainly when the precise data is not available. The rest of this paper focuses on the use of fuzzy sets in DEA.

3. The fuzzy DEA principles

The observed values in real-world problems are often imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and non obtainable information. Imprecise or vague data is often expressed with bounded intervals, ordinal (rank order) data or fuzzy numbers. In recent years, many researchers have formulated fuzzy DEA models to deal with situations where some of the input and output data are imprecise or vague.

3.1. Fuzzy set theory

The theory of fuzzy sets has been developed to deal with the concept of partial truth values ranging from absolutely true to absolutely false. Fuzzy set theory has become the prominent tool for handling imprecision or vagueness aiming at tractability, robustness and low-cost solutions for real-world problems. According to Zadeh (1975), it is very difficult for conventional quantification to reasonably express complex situations and it is necessary to use linguistic variables whose values are words or sentences in a natural or artificial language. The potential of working with linguistic variables, low computational cost and easiness of understanding are characteristics that have contributed to the popularity of this approach. Fuzzy set algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of imprecise and vague estimates in uncertain environments.

Zadeh (1965), p. 339 states “The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability.” The application of fuzzy set theory in multi-attribute decision-making (MADM) became possible when Bellman and Zadeh (1970) and Zimmermann (1978) introduced fuzzy sets into the field of MADM. They cleared the way for a new family of methods to deal with problems that had been unapproachable and unsolvable with standard techniques [see Chen and Hwang (1992) for numerical comparison of fuzzy and classical MADM models]. Bellman and Zadeh’s (1970) framework was based on the maximin and simple additive weighing model of Yager and Basson (1975) and Bass and Kwakernaak (1977). Bass and Kwakernaak’s (1977) method is widely known as the classic work of fuzzy MADM methods.

In 1992, Chen and Hwang (1992) proposed an easy-to-use and easy-to-understand approach to reduce some of the cumbersome computations in the previous MADM methods. Their approach includes two steps: (1) converting fuzzy data into crisp scores; and (2) introducing some comprehensible and easy methods. In addition Chen and Hwang (1992) made distinctions between fuzzy ranking methods and fuzzy MADM methods. Their first group contained a number of methods for finding a ranking: degree of optimality, Hamming distance, comparison function, fuzzy mean and spread, proportion to the ideal, left and right scores, area measurement, and linguistic ranking methods. Their second group was built around methods for assessing the relative importance of multiple attributes: fuzzy simple additive weighting methods, fuzzy analytic hierarchy process, fuzzy conjunctive/disjunctive methods, fuzzy outranking methods, and fuzzy maximin methods. The group with the most frequent contributions is fuzzy mathematical programming. Inuiguchi et al. (1990) have provided a useful survey of fuzzy mathematical programming applications including: flexible programming, possibilistic programming, possibilistic programming with fuzzy preference relations, possibilistic linear programming using fuzzy max, possibilistic linear programming with fuzzy goals, and robust programming.

Recently, fuzzy set theory has been applied to a wide range of fields such as management science, decision theory, artificial intelligence, computer science, expert systems, logic, control theory and statistics, among others (Chen, 2001; Chen and Tzeng, 2004; Chiou et al., 2005; Ding and Liang, 2005; Figueira et al., 2004; Geldermann et al., 2000; Ho et al., 2010; Triantaphyllou, 2000; Ölçer and Odabaşı, 2005; Wang and Lin, 2003; Wang et al., 2009c; Xu and Chen, 2007).

3.2. Fuzzy set theory and DEA

The data in the conventional CCR and BCC models assume the form of specific numerical values. However, the observed value of the input and output data are sometimes imprecise or vague. Sengupta (1992a,b) was the first to introduce a fuzzy mathematical programming approach in which fuzziness was incorporated into the DEA model by defining tolerance levels on both the objective function and constraint violations.

Let us assume that n DMUs consume varying amounts of m different inputs to produce s different outputs. Assume that \tilde{x}_{ij} ($i = 1, 2, \dots, m$) and \tilde{y}_{rj} ($r = 1, 2, \dots, s$) represent, respectively, the fuzzy input and fuzzy output of the j th DMU ($j = 1, 2, \dots, n$). The primal and its dual fuzzy CCR models in input-oriented version can be formulated as:

Primal CCR model (input-oriented)

$$\begin{aligned}
 &\min \quad \theta_p \\
 &\text{s.t.} \quad \sum_{j=1}^n \lambda_j \tilde{x}_{ij} \leq \theta_p \tilde{x}_{ip}, \quad \forall i, \\
 &\quad \quad \sum_{j=1}^n \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rp}, \quad \forall r, \\
 &\quad \quad \lambda_j \geq 0, \quad \forall j.
 \end{aligned} \tag{1}$$

Dual CCR model (input-oriented)

$$\begin{aligned}
 &\max \quad \theta_p = \sum_{r=1}^s u_r \tilde{y}_{rp} \\
 &\text{s.t.} \quad \sum_{i=1}^m v_i \tilde{x}_{ip} = 1, \\
 &\quad \quad \sum_{r=1}^s u_r \tilde{y}_{rj} - \sum_{i=1}^m v_i \tilde{x}_{ij} \leq 0, \quad \forall j, \\
 &\quad \quad u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{2}$$

where v_i and u_r in model (2) are the input and output weights assigned to the i th input and r th output. If the constraint $\sum_{j=1}^n \lambda_j = 1$ is adjoined to (1), a fuzzy BCC model is obtained and this added constraint introduces an additional variable, u_0 , into the dual model which these models are respectively, shown as follows:

$$\begin{aligned}
& \min \quad \theta_p \\
& \text{s.t.} \quad \sum_{j=1}^n \lambda_j \tilde{x}_{ij} \leq \theta_p \tilde{x}_{ip}, \quad \forall i, \\
& \quad \quad \sum_{j=1}^n \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{rp}, \quad \forall r, \\
& \quad \quad \sum_{j=1}^n \lambda_j = 1, \\
& \quad \quad \lambda_j \geq 0, \quad \forall j.
\end{aligned} \tag{3}$$

$$\begin{aligned}
& \max \quad w_p = \sum_{r=1}^s u_r \tilde{y}_{rp} + u_0 \\
& \text{s.t.} \quad \sum_{i=1}^m v_i \tilde{x}_{ip} = 1, \\
& \quad \quad \sum_{r=1}^s u_r \tilde{y}_{rj} - \sum_{i=1}^m v_i \tilde{x}_{ij} + u_0 \leq 0, \quad \forall j, \\
& \quad \quad u_r, v_i \geq 0, \quad \forall r, i.
\end{aligned} \tag{4}$$

4. The fuzzy DEA methods

The applications of fuzzy set theory in DEA are usually categorized into four groups (Lertworasirikul et al., 2003a,b; Lertworasirikul, 2002; Karsak, 2008): (1) The tolerance approach. (2) The α -level based approach. (3) The fuzzy ranking approach. (4) The possibility approach.

In this section, we provide a mathematical description of each approach followed by a brief review of the most widely cited literature relevant to each of the four approaches. In addition to the four abovementioned approaches, we introduce a new category to group the pioneering papers that do not fall into any of the above classifications. A summary development of the fuzzy DEA is listed in Table 1.

4.1. The tolerance approach

The tolerance approach was one of the first fuzzy DEA models that was developed by Sengupta (1992a) and further improved by Kahraman and Tolga (1998). In this approach the main idea is to incorporate uncertainty into the DEA models by defining tolerance levels on constraint violations. This approach fuzzifies the inequality or equality signs but it does not treat fuzzy coefficients directly. The intricate limitation of the tolerance approach proposed by Sengupta (1992a) is related to the design of a DEA model with a fuzzy objective function and fuzzy constraints which may or may not be satisfied (Triantis and Girod, 1998). Although in most production processes fuzziness is present both in terms of not meeting specific objectives and in terms of the imprecision of the data, the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp.

4.2. The α -level based approach

The α -level approach is perhaps the most popular fuzzy DEA model. This is evident by the number of α -level based papers published in the fuzzy DEA literature. In this approach the main idea is to convert the fuzzy DEA model into a pair of parametric programs in order to find the lower and upper bounds of the α -level of the membership functions of the efficiency scores. Girod (1996) used the approach proposed by Carlsson and Korhonen (1986) to formulate the fuzzy BCC and free disposal hull (FDH) models which were radial measures of efficiency. In this model, the inputs could fluctuate between risk-free (upper) and impossible (lower) bounds and the outputs could fluctuate between risk-free (lower) and impossible (upper) bounds. Triantis and Girod (1998) followed up by introducing the fuzzy LP approach to measure technical efficiency based on Carlsson and Korhonen's (1986) framework. Their approach involved three stages: First, the imprecise inputs and outputs were determined by the decision maker in terms of their risk-free and impossible bounds. Second, three fuzzy CCR, BCC and FDH models were formulated in terms of their risk-free and impossible bounds as well as their membership function for different values of α . Third, they illustrated the implementation of their fuzzy BCC model in the context of a preprint and packaging line which inserts commercial pamphlets into newspapers. Furthermore, their paper was clarified in detail the implementation road map by Girod and Triantis (1999). Triantis (2003) extended his earlier work on fuzzy DEA (Triantis and Girod, 1998) to fuzzy non-radial DEA measures of technical efficiency in support of an integrated performance measurement system. He also compared his method to the radial technical efficiency of the same manufacturing production line which was described in detail by Girod (1996) and Girod and Triantis (1999). Meada et al. (1998) used the α -level based approach to obtain the fuzzy interval efficiency of DMUs.

Kao and Liu (2000a) followed up on the basic idea of transforming a fuzzy DEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Their method found approximately the membership functions of the fuzzy efficiency measures by applying the α -level approach and Zadeh's extension principle (Zadeh, 1978; Zimmermann, 1996). They transformed the fuzzy DEA model to a pair of parametric mathematical programs and used the ranking fuzzy numbers method proposed by Chen and Klein (1997) to obtain the performance measure of the DMU. Solving this model at the given level of α -level produced the interval efficiency for the DMU under consideration. A number of such intervals could be used to construct the corresponding fuzzy efficiency. Assume that there are n DMUs under consideration. Each DMU consumes varying amounts of m different fuzzy inputs to produce s different fuzzy outputs. Specifically, DMU _{j} consumes amounts \tilde{x}_{ij} of inputs to produce amounts \tilde{y}_{rj} of outputs. In the model formulation, \tilde{x}_{ip} and \tilde{y}_{rp} denote, respectively, the input and output values for the DMU _{p} . In order to solve the fuzzy BCC model (4), Kao and Liu (2000a) proposed a pair of two-level mathematical models to calculate the lower bound $(w_p)_\alpha^L$ and upper bound $(w_p)_\alpha^U$ of the fuzzy efficiency score for a specific α -level as follows:

$$(w_p)_\alpha^L = \min \left\{ \begin{array}{l} (X_{ij})_\alpha^L \leq x_{ij} \leq (X_{ij})_\alpha^U \\ (Y_{rj})_\alpha^L \leq y_{rj} \leq (Y_{rj})_\alpha^U \\ \forall r, i, j \end{array} \right. \left\{ \begin{array}{l} \tilde{w}_p = \max \sum_{r=1}^s u_r y_{rp} + u_0 \\ \text{s.t.} \sum_{i=1}^m v_i x_{ip} = 1, \\ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 \leq 0, \quad \forall j, \\ u_r, v_i \geq 0, \quad \forall r, i. \end{array} \right. \quad (5)$$

$$(w_p)_\alpha^U = \max \left\{ \begin{array}{l} (X_{ij})_\alpha^L \leq x_{ij} \leq (X_{ij})_\alpha^U \\ (Y_{rj})_\alpha^L \leq y_{rj} \leq (Y_{rj})_\alpha^U \\ \forall r, i, j \end{array} \right. \left\{ \begin{array}{l} \tilde{w}_p = \max \sum_{r=1}^s u_r y_{rp} + u_0 \\ \text{s.t.} \sum_{i=1}^m v_i x_{ip} = 1, \\ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + u_0 \leq 0, \quad \forall j, \\ u_r, v_i \geq 0, \quad \forall r, i. \end{array} \right. \quad (6)$$

Table 1
Fuzzy DEA reference classification.

Classification	Reference	Reference
The tolerance approach	Sengupta (1992a)	Sengupta (1992b)
The α -level based approach	Azadeh and Alem (2010) Zerafat Angiz et al. (2010a) Hatami-Marbini et al. (in pressd) Saati and Memariani (2009) Noura and Saljooghi (2009) Wang et al. (2009b) Liu and Chuang (2009) Karsak (2008) Ghapanchi et al. (2008) Hossein-zadeh Lotfi et al. (2007c) Allahviranloo et al. (2007) Kuo and Wang (2007) Liu et al. (2007) Zhang et al. (2005) Wu et al. (2005) Kao and Liu (2005) Kao and Liu (2003) Entani et al. (2002) Chen (2001) Kao and Liu (2000b) Girod and Triantis (1999) Triantis and Girod (1998)	Chiang and Che (2010) Hatami-Marbini et al. (2010a) Hatami-Marbini and Saati (2009) Tlig and Rebai (2009) Jahanshahloo et al. (2009a) Hossein-zadeh Lotfi et al. (2009a) Li and Yang (2008) Azadeh et al. (2008) Liu (2008) Saneifard et al. (2007) Azadeh et al. (2007) Kao and Liu (2007) Jahanshahloo et al. (2007b) Saati and Memariani (2005) Hsu (2005) Triantis (2003) Saati et al. (2002) Guh (2001) Kao (2001) Kao and Liu (2000a) Meada et al. (1998) Girod (1996)
The fuzzy ranking approach	Hatami-Marbini et al. (2010b) Hatami-Marbini et al. (in presse) Jahanshahloo et al. (2009b) Hossein-zadeh Lotfi et al. (2009b) Guo (2009) Juan (2009) Zhou et al. (2008) Noora and Karami (2008) Jahanshahloo et al. (2008) Hossein-zadeh Lotfi et al. (2007b) Pal et al. (2007) Soleimani-damaneh et al. (2006) Lee et al. (2005) Jahanshahloo et al. (2004a) Lee (2004) Lertworasirikul (2002)	Hatami-Marbini et al. (in pressc) Hatami-Marbini et al. (2009) Soleimani-damaneh (2009) Bagherzadeh valami (2009) Hossein-zadeh Lotfi et al. (2009c) Sanei et al. (2009) Guo and Tanaka (2008) Soleimani-Damaneh (2008) Hossein-zadeh Lotfi and Mansouri (2008) Jahanshahloo et al. (2007a) Hossein-zadeh Lotfi et al. (2007a) Saati and Memariani (2006) Molavi et al. (2005) Dia (2004) Leon et al. (2003) Guo and Tanaka (2001)
The possibility approach	Wen et al. (2010) Wen and Li (2009) Wu et al. (2006) Ramezanzadeh et al. (2005) Lertworasirikul et al. (2003b) Lertworasirikul et al. (2002a) Lertworasirikul (2002)	Khodabakhshi et al. (2010) Jiang and Yang (2007) Garcia et al. (2005) Lertworasirikul et al. (2003a) Lertworasirikul et al. (2003c) Lertworasirikul et al. (2002b) Guo et al. (2000)
Other developments in fuzzy DEA	Zerafat Angiz et al. (2010b) Wang et al. (2009a) Qin et al. (2009) Uemura (2006) Hougaard (2005) Hougaard (1999)	Qin and Liu (2010) Luban (2009) Qin and Liu (2009) Wang et al. (2005) Sheth and Triantis (2003)

where $[(X_{ij})_{\alpha}^L, (X_{ij})_{\alpha}^U]$ and $[(Y_{rj})_{\alpha}^L, (Y_{rj})_{\alpha}^U]$ are α -level form of the fuzzy inputs and the fuzzy outputs, respectively. This two-level mathematical model can be simplified to the conventional one-level model as follows:

$$\begin{aligned}
 (w_p)_{\alpha}^L &= \max \sum_{r=1}^s u_r (Y_{rp})_{\alpha}^L + u_0 \\
 \text{s.t. } &\sum_{r=1}^s u_r (Y_{rp})_{\alpha}^L - \sum_{i=1}^m v_i (X_{ip})_{\alpha}^U + u_0 \leq 0, \\
 &\sum_{r=1}^s u_r (Y_{rj})_{\alpha}^U - \sum_{i=1}^m v_i (X_{ij})_{\alpha}^L + u_0 \leq 0, \quad \forall j, j \neq p, \\
 &\sum_{i=1}^m v_i (X_{ip})_{\alpha}^U = 1, \quad u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 (w_p)_{\alpha}^U &= \max \sum_{r=1}^s u_r (Y_{rp})_{\alpha}^U + u_0 \\
 \text{s.t. } &\sum_{r=1}^s u_r (Y_{rp})_{\alpha}^U - \sum_{i=1}^m v_i (X_{ip})_{\alpha}^L + u_0 \leq 0, \\
 &\sum_{r=1}^s u_r (Y_{rj})_{\alpha}^L - \sum_{i=1}^m v_i (X_{ij})_{\alpha}^U + u_0 \leq 0, \quad \forall j, j \neq p, \\
 &\sum_{i=1}^m v_i (X_{ip})_{\alpha}^L = 1, \quad u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{8}$$

Next, a membership function is built by solving the lower and upper bounds $[(w_p)_{\alpha}^L, (w_p)_{\alpha}^U]$ of the α -levels for each DMU using models (7) and (8). Kao and Liu (2000a) have used the ranking fuzzy numbers method of Chen and Klein (1997) to rank the obtained fuzzy efficiencies. Kao and Liu (2000b) also used the method of Kao and Liu (2000a) to calculate the efficiency scores by considering the missing values in the fuzzy DEA based on the concept of the membership function in the fuzzy set theory. In their approach, the smallest possible, most possible, and largest possible values of the missing data are derived from the observed data to construct a triangular membership function. They demonstrated the applicability of their approach by considered the efficiency scores of 24 university libraries in Taiwan with three missing values out of 144 observations. Kao (2001) further introduced a method for ranking the fuzzy efficiency scores without knowing the exact form of their membership function. In this method, the efficiency rankings were determined by solving a pair of non-linear programs for each DMU. This approach was applied to the ranking of the 24 university libraries in Taiwan with fuzzy observations. Guh (2001) used a fuzzy DEA model similar to Kao and Liu (2000a) to approximate the fuzzy efficiency measures. However, Kao and Liu (2000a) developed their model under the VRS assumption and Guh (2001)'s model was developed under the CRS assumption.

Kao and Liu (2003) used the maximum set–minimum set method of Chen (1985) into the fuzzy DEA model proposed by Kao and Liu (2000a) and built pairs of nonlinear programs and ranked the DMUs with fuzzy data. In their approach, there was no need for calculating the membership function of the fuzzy efficiency scores but the input and output membership functions must be known. Kao and Liu (2005) applied their earlier method (Kao and Liu, 2000a) to determine the fuzzy efficiency scores of fifteen sampled machinery firms in Taiwan. Zhang et al. (2005) proposed a macro model and a micro model for the efficiency evaluation of data warehouses by applying DEA and fuzzy DEA models. They used the fuzzy DEA solution proposed by Kao and Liu (2000a), which transforming fuzzy DEA models to bi-conventional crisp DEA models by a set of α -level values. Kao and Liu (2007) proposed a modification to the Kao and Liu's (2000b) method to handle missing values. In their method, they used a fuzzy DEA approach and obtained the efficiency scores of a set of DMUs by using the α -level approach proposed by Kao and Liu (2000a). Kuo and Wang (2007) applied a fuzzy DEA method to evaluate the performance of multinational corporations in face of volatile exposure to exchange rate risk. They employed the fuzzy DEA model suggested by Kao and Liu (2000a) to information technology industry in Taiwan. Li and Yang (2008) proposed a fuzzy DEA–discriminant analysis methodology for classifying fuzzy observations into two groups based on the work of Sueyoshi (2001). They used the Kao and Liu's (2000a) method and replaced the fuzzy linear programming models by a pair of parametric models to determine the lower and upper bounds of the efficiency scores. By applying the Kao and Liu's (2000a) method and the fuzzy analytical hierarchy procedure, Chiang and Che (2010) proposed a new weight-restricted fuzzy DEA methodology for ranking new product development projects at an electronic company in Taiwan.

Saati et al. (2002) suggested a fuzzy CCR model as a possibilistic programming problem and transformed it into an interval programming problem using α -level based approach. The resulting interval programming problem could be solved as a crisp LP model for a given α with some variable substitutions. Model (9) proposed by Saati et al. (2002) is derived for a particular case where the inputs and outputs are triangular fuzzy numbers:

$$\begin{aligned}
 \max \quad &w_p = \sum_{r=1}^s y'_{rp} \\
 \text{s.t. } &\sum_{r=1}^s y'_{rj} - \sum_{i=1}^m x'_ij \leq 0, \quad \forall j, \\
 &v_i (\alpha x_{ij}^m + (1 - \alpha)x_{ij}^l) \leq x'_ij \leq v_i (\alpha x_{ij}^m + (1 - \alpha)x_{ij}^u), \quad \forall i, j, \\
 &u_r (\alpha y_{rj}^m + (1 - \alpha)y_{rj}^l) \leq y'_{rj} \leq u_r (\alpha y_{rj}^m + (1 - \alpha)y_{rj}^u), \quad \forall r, j, \\
 &\sum_{i=1}^m x'_{ip} = 1, \quad u_r, v_i \geq 0, \quad \forall r, j
 \end{aligned} \tag{9}$$

where $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$ are the triangular fuzzy inputs and the triangular fuzzy outputs, and x'_{ij} and y'_{rj} are the decision variables obtained from variable substitutions used to transform the original fuzzy model proposed into a parametric LP model with

$\alpha \in [0, 1]$. Saati and Memariani (2005) suggested a procedure for determining a common set of weights in fuzzy DEA based on the α -level method proposed by Saati et al. (2002) with triangular fuzzy data. In this method, the upper bounds of the input and output weights were determined by solving some fuzzy LP models and then a common set of weights were obtained by solving another fuzzy LP model. Wu et al. (2005) developed a buyer–seller game model for selecting purchasing bids in consideration of fuzzy values. They adopted the fuzzy DEA model proposed by Saati and Memariani (2005) to obtain a common set of weights in fuzzy DEA. Azadeh et al. (2007) proposed an integrated model of fuzzy DEA and simulation to select the optimal solution between some scenarios which obtained from a simulation model and determined optimum operators' allocation in cellular manufacturing systems. They used a fuzzy DEA model to rank a set of DMUs based on the Saati et al. (2002)'s method. In addition, they clustered fuzzy DEA ranking of DMUs by fuzzy C-Means method to show a degree of desirability for operator allocation. Ghapanchi et al. (2008) employed fuzzy DEA to evaluate the enterprise resource planning (ERP) packages performance. In their approach, inputs and outputs indices were first determined by experts' opinions which were evaluated using linguistic variables characterized by triangular fuzzy numbers and then a set of potential ERP systems was considered as DMUs. They applied a possibilistic-programming approach proposed by Saati et al. (2002) and obtained the efficiency scores of the ERP systems at different α values.

Hatami-Marbini and Saati (2009) developed a fuzzy BCC model which considered fuzziness in the input and output data as well as the u_0 variable. Consequently, they obtained the stability of the fuzzy u_0 as an interval by means of the method proposed by Saati et al. (2002). Hatami-Marbini et al. (2010a) used the method of Saati et al. (2002) and proposed a four-phase fuzzy DEA framework based on the theory of displaced ideal. Two hypothetical DMUs called the ideal and nadir DMUs are constructed and used as reference points to evaluate a set of information technology investment strategies based on their Euclidean distance from these reference points. Chen (2001) modified the α -level approach and proposed an alternative fuzzy DEA to handle both the crisp and fuzzy data. Saati and Memariani (2009) developed a fuzzy slack-based measure (SBM) based on the α -level approach. They transformed their fuzzy SBM model into a LP problem by using the approach proposed by Saati et al. (2002). Moreover, they compared their model to the method of Jahanshahloo et al. (2004a) and demonstrated the advantages of their proposed model. Hatami-Marbini et al. (in pressd) proposed a fuzzy additive DEA model for evaluating the efficiency of peer DMUs with fuzzy data by utilizing the Saati et al. (2002)'s α -level approach.

Liu (2008) developed a fuzzy DEA method to find the efficiency measures embedded with assurance region (AR) concept when some observations were fuzzy numbers. He applied an α -level approach and Zadeh's extension principle (Zadeh, 1978; Zimmermann, 1996) to transform the fuzzy DEA/AR model into a pair of parametric mathematical programs and worked out the lower and upper bounds of the efficiency scores of the DMUs. The membership function of the efficiency was approximated by using different possibility levels. Thereby, he used the Chen and Klein's (1997) method for ranking the fuzzy numbers and calculating the crisp values. Let us consider the relative importance of the inputs and outputs as $\frac{L_{i\delta}^L \leq v_i \leq U_{i\delta}^L}{U_{iq}^L \leq v_q \leq L_{iq}^L}$, $\delta < q = 2, \dots, m$; and $\frac{L_{o\delta}^U \leq u_o \leq U_{o\delta}^U}{U_{oq}^U \leq u_q \leq L_{oq}^U}$, $\delta < q = 2, \dots, s$; respectively.

The two parametric mathematical programs proposed by Liu (2008) are as follows:

$$\begin{aligned}
 (W_p)_\alpha^L &= \max \sum_{r=1}^s u_r (y_{rp})_\alpha^L \\
 \text{s.t. } &\sum_{r=1}^s u_r (y_{rj})_\alpha^U - \sum_{i=1}^m v_i (x_{ij})_\alpha^L \leq 0, \quad \forall j, j \neq p, \\
 &-v_\delta + I_{\delta q}^L v_q \leq 0, v_\delta - I_{\delta q}^U v_q \leq 0, \quad \forall \delta < q, \\
 &-u_\delta + O_{\delta q}^L u_q \leq 0, u_\delta - O_{\delta q}^U u_q \leq 0, \quad \forall \delta < q, \\
 &\sum_{i=1}^m v_i (x_{ip})_\alpha^U = 1, \quad u_r, v_i \geq 0, \quad \forall r, j.
 \end{aligned}
 \tag{10}$$

$$\begin{aligned}
 (W_p)_\alpha^U &= \max \sum_{r=1}^s u_r (y_{rp})_\alpha^U \\
 \text{s.t. } &\sum_{r=1}^s u_r (y_{rj})_\alpha^L - \sum_{i=1}^m v_i (x_{ij})_\alpha^U \leq 0, \quad \forall j, j \neq p, \\
 &-v_\delta + I_{\delta q}^L v_q \leq 0, v_\delta - I_{\delta q}^U v_q \leq 0, \quad \forall \delta < q, \\
 &-u_\delta + O_{\delta q}^L u_q \leq 0, u_\delta - O_{\delta q}^U u_q \leq 0, \quad \forall \delta < q, \\
 &\sum_{i=1}^m v_i (x_{ip})_\alpha^L = 1, \quad u_r, v_i \geq 0, \quad \forall r, j.
 \end{aligned}
 \tag{11}$$

where $I_{\delta q}^L = \frac{L_{i\delta}^L}{U_{iq}^L}$, $I_{\delta q}^U = \frac{U_{i\delta}^L}{L_{iq}^L}$, $O_{\delta q}^L = \frac{L_{o\delta}^U}{U_{oq}^U}$ and $O_{\delta q}^U = \frac{U_{o\delta}^U}{L_{oq}^U}$. Jahanshahloo et al. (2009a) proposed some corrections to the Liu's (2008) model. Liu and Chuang (2009) applied the fuzzy DEA/AR model suggested by Liu (2008) and evaluated the performance of 24 university libraries in Taiwan based on the method proposed by Kao and Liu (2000b).

Entani et al. (2002) proposed a DEA model with an interval efficiency consisting of the efficiencies obtained from the pessimistic and the optimistic viewpoints. They also developed this approach for fuzzy input and output data by using α -level sets. Hsu (2005) applied a simple fuzzy DEA model to balanced scorecard with an application to multi-national research and development projects. The fuzzy DEA method included both crisp and linguistic variables processed by a four-step framework. Liu et al. (2007) developed a modified fuzzy DEA model to handle fuzzy and incomplete information on weight indices in product design evaluation. They transformed fuzzy information into trapezoidal fuzzy numbers and considered incomplete information on indices weights as constraints. They used an α -level approach to convert their fuzzy DEA model into a family of conventional crisp DEA models. Saneifard et al. (2007) developed a model to evaluate the relative performance of DMUs with crisp data based on l_2 - norm. They used the ranking fuzzy numbers method of Jiménez (1996) to determine a crisp α -parametric model and solve the fuzzy l_2 - norm model.

Jahanshahloo et al. (2007b) developed a fuzzy l_1 - norm model with trapezoidal fuzzy inputs/outputs that was initially suggested by Jahanshahloo et al. (2004c) for solving the crisp data in DEA. They applied the ranking fuzzy numbers method of Jiménez (1996) to the fuzzy l_1 - norm model and obtained a crisp α -parametric model. Allahviranloo et al. (2007) introduced the notion of fuzziness to deal with

imprecise data in DEA. They proposed fuzzy production possibility set with constant returns to scale to calculate the upper and lower relative efficiency scores of the DMUs by using the α -level approach. Hosseinzadeh Lotfi et al. (2007c) applied the method of DEA-discriminant analysis proposed by Sueyoshi (1999) to the imprecise environment. They first modified Sueyoshi's model with crisp data and then developed it to fuzzy inputs and outputs based on the concept of α -level approach. Karsak (2008) proposed an extension of Cook et al. (1996)'s model to evaluate crisp, ordinal and fuzzy inputs and outputs in flexible manufacturing systems by determining the optimistic (the upper bound) and pessimistic (the lower bound) of the α -level of the membership function of the efficiency scores. Azadeh et al. (2008) used a triangular form of fuzzy inputs and outputs instead of the crisp data and proposed a fuzzy DEA model for calculating the efficiency scores of the DMUs under uncertainty with application to the power generation sector. They transformed the fuzzy CCR model into a pair of parametric programs using the α -level approach and found the lower and upper bounds of the efficiency for different α -values. Their contribution to the fuzzy DEA literature is in the development of the membership functions and not the crisp measure of the efficiencies. They used the α -level to transform the fuzzy DEA model into a series of conventional crisp DEA models. Azadeh and Alem (2010) also used this fuzzy DEA method (Azadeh et al., 2008) for vendor selection problem which was taken from Wu and Olson (2008).

Noura and Saljooghi (2009) proposed an extension of a definite class of weight function in fuzzy DEA based on the principle of maximum entropy in order to provide circumstances for the compatibility and stability in ranking of interval efficiency scores of DMUs at various α values. Wang et al. (2009b) proposed a fuzzy DEA–Neural approach with a self-organizing map for classification in their neural network. They used the upper and lower bounds of efficiency score at different possibilistic levels in their model. Hosseinzadeh Lotfi et al. (2009a) developed two methods for solving fuzzy CCR model with respect to fuzzy, ordinal and exact data. They used an analogue function to transform the fuzzy data into exact values in the first method. In the second approach, they applied an α -level approach based on the Kao (2006)'s method to obtain the interval efficiency scores for DMUs. Tlig and Rebai (2009) proposed an approach based on the ordering relations between LR-fuzzy numbers to solve the primal and the dual of FCCR. They suggested a procedure based on the resolution of a goal programming problem to transform the fuzzy normalisation equality in the primal of FCCR. Zerafat Angiz et al. (2010a) show the advantages and shortcomings of the fuzzy ranking approach, the defuzzification approach, the tolerance approach and the α -level based approach. They proposed an α -level approach to retain fuzziness of the model by maximizing the membership functions of inputs and outputs. They also compared their results with the results from Saati et al. (2002).

4.3. The fuzzy ranking approach

The fuzzy ranking approach is also another popular technique that has attracted a great deal of attention in the fuzzy DEA literature. In this approach the main idea is to find the fuzzy efficiency scores of the DMUs using fuzzy linear programs which require ranking fuzzy sets. The fuzzy ranking approach of efficiency measurement was initially developed by Guo and Tanaka (2001). They proposed a fuzzy CCR model in which fuzzy constraints (including fuzzy equalities and fuzzy inequalities) were converted into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers. Assuming there are n DMUs under evaluation, the efficiency of the DMU _{j} with m symmetrical triangular fuzzy inputs and s symmetrical triangular fuzzy outputs is denoted by $\tilde{x}_{ij} = (x_{ij}, c_{ij})$ and $\tilde{y}_{rj} = (y_{rj}, d_{rj})$, respectively, where x_{ij} and y_{rj} are the center, and c_{ij} and d_{rj} are the spread of fuzzy numbers. Guo and Tanaka (2001) proposed the following linear programming (LP) model with two objective functions:

$$\begin{aligned} \max_{u,v} \theta_p &= \sum_{r=1}^s (u_r y_{rp} - (1 - \alpha) u_r d_{rp}) \\ \text{s.t.} \quad \max_v \quad &\sum_{i=1}^m v_i c_{ip} \\ \text{s.t.} \quad &\sum_{i=1}^m (v_i x_{ip} - (1 - \alpha) v_i c_{ip}) = 1 - (1 - \alpha) e, \\ &\sum_{i=1}^m (v_i) x_{ip} + (1 - \alpha) v_i c_{ip} \leq 1 + (1 - \alpha) e, \\ &v_i \geq 0, \forall i. \end{aligned} \left. \vphantom{\begin{aligned} \max_{u,v} \theta_p &= \sum_{r=1}^s (u_r y_{rp} - (1 - \alpha) u_r d_{rp}) \\ \text{s.t.} \quad \max_v \quad &\sum_{i=1}^m v_i c_{ip} \\ \text{s.t.} \quad &\sum_{i=1}^m (v_i x_{ip} - (1 - \alpha) v_i c_{ip}) = 1 - (1 - \alpha) e, \\ &\sum_{i=1}^m (v_i) x_{ip} + (1 - \alpha) v_i c_{ip} \leq 1 + (1 - \alpha) e, \\ &v_i \geq 0, \forall i. \end{aligned}} \right\} \text{model (12-1)} \tag{12}$$

$$\begin{aligned} \sum_{r=1}^s (u_r y_{rj} + (1 - \alpha) u_r d_{rj}) &\leq \sum_{i=1}^m (v_i x_{ij} + (1 - \alpha) v_i c_{ij}), \quad \forall j, \\ \sum_{r=1}^s (u_r y_{rj} - (1 - \alpha) u_r d_{rj}) &\leq \sum_{i=1}^m (v_i x_{ij} - (1 - \alpha) v_i c_{ij}), \quad \forall j, \\ u_r &\geq 0, \forall r. \end{aligned}$$

where $\alpha \in [0, 1]$ is a predetermined possibility level by decision-makers and unity number in the right hand side of the first constraint of the model (4) is supposedly a symmetrical triangular fuzzy number $1 = (1, e)$. Note that if $c_{ij} = d_{rj} = 0$, then, the traditional CCR is obtained and if $\max [c_{p1}/x_{p1}, \dots, c_{p1}/x_{ps}] \leq e$ in (12-1), there exists an optimal solution in (12-1).

The fuzzy efficiency of each DMU under evaluation with the symmetrical triangular fuzzy inputs \tilde{x}_{ip} and outputs \tilde{y}_{rp} is obtained for each α possibility level as a non-symmetrical triangular fuzzy number $\tilde{\theta}_p = (e_p^l, e_p^m, e_p^u)$ as follows:

$$e_p^m = \frac{u_r^* y_{rp}}{v_i^* x_{ip}}, \quad e_p^l = e_p^m - \frac{u_r^* (y_{rp} - d_{rp}(1 - \alpha))}{v_i^* (x_{ip} + c_{ip}(1 - \alpha))}, \quad e_p^u = \frac{u_r^* (y_{rp} + d_{rp}(1 - \alpha))}{v_i^* (x_{ip} - c_{ip}(1 - \alpha))} - e_p^m$$

where u_r^* and v_i^* are obtained from (12-1), and e_p^l , e_p^m and e_p^u are the left, right spreads and the center of the fuzzy efficiency $\tilde{\theta}_p$, respectively. Because of using predefined $\alpha \in [0, 1]$ Guo and Tanaka (2001)'s method can also be classified within α -level approaches.

Guo and Tanaka (2008) extended their earlier work (Guo and Tanaka, 2001) and introduced a fuzzy aggregation model to objectively rank a set of DMUs by integrating multiple attribute fuzzy values. Guo (2009) further applied a fuzzy DEA model in a case study for a restaurant location problem in China by integrating the fuzzy DEA model proposed by Guo and Tanaka (2001) with the fuzzy aggregation model for proposed by Guo and Tanaka (2008). Sanei et al. (2009) used the sensitivity analysis model of Cooper et al. (2001) with fuzzy data, and they applied the approach of Guo and Tanaka (2001) to build their fuzzy model for determining the stability radius for different α values. Similar to the approach proposed by Guo and Tanaka (2001), Leon et al. (2003) developed a fuzzy BCC model (3). However, in Guo

and Tanaka (2001)'s method, a fuzzy efficiency score is obtained for each possibility level α while in Leon et al. (2003)'s method, a crisp efficiency score is obtained for either all or each of the possibility levels. Leon et al. (2003) proposed two different fuzzy DEA models depending on the ranking method used to interpret the fuzzy inequalities. The first model uses the ranking method of Ramík and Řimánek (1985) to obtain a crisp efficiency score of DMU_p in which all the possible values of the various variables for all the DMUs at all the possibility levels are considered. This model can be expressed as follows:

$$\begin{aligned}
 \min \quad & \theta_p \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^L \leq \theta_p x_{ip}^L, \quad \forall i, & \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{rp}^L, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j x_{ij}^R \leq \theta_p x_{ip}^R, \quad \forall i, & \sum_{j=1}^n \lambda_j y_{rj}^R \geq y_{rp}^R, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j x_{ij}^L - \sum_{j=1}^n \lambda_j c_{ij}^L \leq \theta_p x_{ip}^L - \theta_p c_{ip}^L, \quad \forall i, & \sum_{j=1}^n \lambda_j y_{rj}^L - \sum_{j=1}^n \lambda_j d_{rj}^L \geq y_{rp}^L - d_{rp}^L, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j x_{ij}^R + \sum_{j=1}^n \lambda_j c_{ij}^R \leq \theta_p x_{ip}^R + \theta_p c_{ip}^R, \quad \forall i, & \sum_{j=1}^n \lambda_j y_{rj}^R + \sum_{j=1}^n \lambda_j d_{rj}^R \geq y_{rp}^R + d_{rp}^R, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j = 1, & \lambda_j \geq 0, \quad \forall j.
 \end{aligned} \tag{13}$$

In model (13), the fuzzy inputs and the fuzzy outputs, respectively, are $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^R, c_{ij}^L, c_{ij}^R)$ and $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^R, d_{rj}^L, d_{rj}^R)$ in which x_{ij}^L and y_{rj}^L are the left centers, x_{ij}^R and y_{rj}^R are the right centers of the inputs and outputs, respectively, while c_{ij}^L and d_{rj}^L are the left spreads, and c_{ij}^R and d_{rj}^R are the right spreads of the inputs and outputs, respectively. The second model of Leon et al. (2003) uses the ranking method in Tanaka et al. (1984) to calculate the efficiency score of DMU_p for each possibility level $\alpha \in [0, 1]$. This model can be formulated as follows:

$$\begin{aligned}
 \min \quad & \theta_p \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^L \leq \theta_p x_{ip}^L, \quad \forall i, & \sum_{j=1}^n \lambda_j x_{ij}^R \leq \theta_p x_{ip}^R, \quad \forall i, \\
 & \sum_{j=1}^n \lambda_j x_{ij}^L - L_i^*(\alpha) \sum_{j=1}^n \lambda_j c_{ij}^L \leq \theta_p x_{ip}^L - L_i^*(\alpha) \theta_p c_{ip}^L, \quad \forall i, \\
 & \sum_{j=1}^n \lambda_j x_{ij}^R + R_i^*(\alpha) \sum_{j=1}^n \lambda_j c_{ij}^R \leq \theta_p x_{ip}^R + R_i^*(\alpha) \theta_p c_{ip}^R, \quad \forall i, \\
 & \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{rp}^L, \quad \forall r, & \sum_{j=1}^n \lambda_j y_{rj}^R \geq y_{rp}^R, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j y_{rj}^L - L_i^r(\alpha) \sum_{j=1}^n \lambda_j d_{rj}^L \geq y_{rp}^L - L_i^r(\alpha) d_{rp}^L, \quad \forall r \\
 & \sum_{j=1}^n \lambda_j y_{rj}^R + R_i^r(\alpha) \sum_{j=1}^n \lambda_j d_{rj}^R \geq y_{rp}^R + R_i^r(\alpha) d_{rp}^R, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0, \quad \forall j.
 \end{aligned} \tag{14}$$

A fuzzy set of efficient DMUs can be defined based on the optimal solution for model (14) so that the decision maker is able to identify sensitive DMUs and to select the appropriate possibility level. When the data are assumed to be symmetrical triangular fuzzy numbers that are denoted by $\tilde{x}_{ij} = (x_{ij}, c_{ij})$ and $\tilde{y}_{rj} = (y_{rj}, d_{rj})$, respectively, where x_{ij} and y_{rj} are the center, and c_{ij} and d_{rj} are the spread of fuzzy numbers, model (14) can be written as:

$$\begin{aligned}
 \min \quad & \theta_p \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} - (1 - \alpha) \sum_{j=1}^n \lambda_j c_{ij} \leq \theta_p x_{ip} - (1 - \alpha) \theta_p c_{ip}, \quad \forall i, \\
 & \sum_{j=1}^n \lambda_j x_{ij} + (1 - \alpha) \sum_{j=1}^n \lambda_j c_{ij} \leq \theta_p x_{ip} + (1 - \alpha) \theta_p c_{ip}, \quad \forall i, \\
 & \sum_{j=1}^n \lambda_j y_{rj} - (1 - \alpha) \sum_{j=1}^n \lambda_j d_{rj} \geq y_{rp} - (1 - \alpha) d_{rp}, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j y_{rj} + (1 - \alpha) \sum_{j=1}^n \lambda_j d_{rj} \geq y_{rp} + (1 - \alpha) d_{rp}, \quad \forall r, \\
 & \sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0, \quad \forall j.
 \end{aligned} \tag{15}$$

Note that $L_i^*(\alpha) = R_i^*(\alpha) = L_i^r(\alpha) = R_i^r(\alpha) = 1 - \alpha, \alpha \in [0, 1]$. We can also categorize Leon et al. (2003)'s method as an α -level approach since they used $\alpha \in [0, 1]$ in their model.

Hatami-Marbini et al. (2010b) extended a fuzzy CCR model for evaluating the DMUs from the perspective of the best and the worst possible relative efficiency by utilizing Leon et al. (2003)'s approach. Then, in order to rank all of the DMUs, a closeness coefficient index was obtained by combining the two various efficiencies. Jahanshahloo et al. (2004a) proposed a fuzzy ranking method for solving slack-based measure (SBM) model in DEA when input–output data are triangular fuzzy numbers. Saati and Memariani (2006) addressed some shortcomings of the fuzzy DEA proposed by Jahanshahloo et al. (2004a) and suggested several corrections to their method. Molavi et al. (2005) introduced two fuzzy DEA models in which the objective function and fuzzy constraints of the fuzzy CCR model were transformed into

crisp conditions by using LR-fuzzy numbers ranking method of Ramík and Římanek (1985). Soleimani-damaneh et al. (2006) addressed some computational and theoretical shortcomings of several fuzzy DEA models including Kao and Liu (2000a), Leon et al. (2003), Lertworasirikul et al. (2003a), Guo and Tanaka (2001) and Jahanshahloo et al. (2004a). Furthermore, they proposed a fuzzy BCC model using the fuzzy number ranking method proposed by Yao and Wu (2000) for trapezoidal fuzzy data in DEA.

Hosseinzadeh Lotfi et al. (2007a) applied trapezoidal fuzzy data to Jahanshahloo et al.'s (2004b) DEA method, in which a fuzzy fixed cost was equitably assigned to all DMUs in such a way that the efficiency scores were not changed. They used a fuzzy ranking method to solve the fuzzy model in which each fuzzy constraint transformed to three crisp constraints. Hosseinzadeh Lotfi et al. (2007b) adopted the linear ranking function proposed by Maleki (2002) to present fuzzy CCR models with triangular fuzzy data. Jahanshahloo et al. (2007a) suggested a method to deal with the DEA-based Malmquist productivity index for all DMUs with triangular fuzzy inputs and outputs. They applied a linear ranking function proposed by Maleki (2002) to transform their fuzzy linear programming model into a group of the conventional crisp DEA models. Pal et al. (2007) used a fuzzy DEA approach and α -parametric inequalities in quality function deployment. They used a fuzzy CCR model based on the method proposed by Lai and Hwang (1992).

Hosseinzadeh Lotfi and Mansouri (2008) considered the extended DEA-discriminant analysis method proposed by Sueyoshi (2001) as fuzzy data and changed their fuzzy model into a crisp model using the linear ranking function proposed by Maleki (2002). Zhou et al. (2008) developed a fuzzy DEA method to evaluate the efficiency performance of real estate investment programs. They applied the ranking fuzzy numbers to solve their model and designed a “relatively effective controller” which considered controlling the diversity of the method. Noora and Karami (2008) adopted triangular fuzzy data to establish a fuzzy non-radial DEA model and applied a ranking function proposed by Maleki et al. (2000) to transform the fuzzy linear programming into the crisp DEA models. Jahanshahloo et al. (2008) applied the linear ranking function proposed by Mahdavi-Amiri and Nasseri (2006) to change the fuzzy cost efficiency model into a conventional linear programming problem. Soleimani-Damaneh (2008) used the fuzzy signed distance and the fuzzy upper bound concepts to formulate a fuzzy additive model in DEA with fuzzy input–output data. Soleimani-damaneh (2009) put forward a theorem on the fuzzy DEA model which was proposed by Soleimani-Damaneh (2008) in order to show the existence of distance-based upper bound for the objective function of the model.

Hatami-Marbini et al. (2009) proposed a fuzzy DEA model to assess the efficiency scores in the fuzzy environment. They used the proposed ranking method in Asady and Zendehtnam (2007) and obtained precise efficiency scores for the overall rankings of the DMUs. They compared their method with the fuzzy DEA methods proposed by Soleimani-damaneh et al. (2006) and Leon et al. (2003). They also applied their model to sixteen bank's branches with fuzzy data. Jahanshahloo et al. (2009b) further introduced an alternative approach for solving fuzzy l_1 – norm method in DEA with fuzzy data based on the comparison of fuzzy numbers proposed by Tran and Duckstein (2002) to change fuzzy linear programming to crisp model form. Hosseinzadeh Lotfi et al. (2009c) generalized a multi-activity network DEA to fuzzy inputs and outputs which were formed by triangular membership functions. They used a ranking function to convert the multi-activity network fuzzy DEA into a multi-activity network crisp DEA model. Hatami-Marbini et al. (in presse) proposed an interactive evaluation process for measuring the efficiency of peer DMUs in fuzzy DEA with consideration of the decision makers' preferences. By applying the fuzzy ranking method of Jiménez (1996), they constructed a linear programming model with fuzzy parameters and calculated the fuzzy efficiency of the DMUs for different α levels. Then, the decision maker identifies his/her most preferred fuzzy goal for each DMU under evaluation and a ranking order of the DMUs can be obtained by a modified Yager index.

In this section, we also review a related method, called “defuzzification approach”, proposed by Lertworasirikul (2002). In this approach, which is essentially a fuzzy ranking method, fuzzy inputs and fuzzy outputs are first defuzzified into crisp values. These crisp values are then used in a conventional crisp DEA model which can be solved by an LP solver. Dia (2004) developed an alternative fuzzy DEA model based on fuzzy arithmetic operations and ranking of fuzzy numbers. A fuzzy aspiration level was used to change the model into a crisp DEA model and the fuzzy results outlined the practical and robustness aspects of the fuzzy methodology. Lee (2004) and Lee et al. (2005) have also proposed fuzzy DEA models for CCR and BCC by defuzzifying fuzzy inputs and outputs into crisp value and using them in conventional DEA models. Juan (2009) proposed a two-stage decision support model by using a hybrid DEA and case-based reasoning model. In this approach, the center of gravity method (CGM) suggested in Bojadziew and Bojadziew (1997) was used to transform the fuzzy data into crisp data and build a conventional CCR model. Bagherzadeh Valami (2009) introduced a cost efficiency model with triangular fuzzy input prices and proposed a method for comparing the production cost of the target DMU with the minimum cost fuzzy set. Hosseinzadeh Lotfi et al. (2009b) proposed a fuzzy DEA model to evaluate a set of DMUs where all parameters and decision variables were fuzzy numbers. They changed their fuzzy model into a multiple objective LP model and solved the LP model by using a lexicography method. The defuzzification approach is simple but the uncertainty in input and output variables (i.e., possible range of values at different α -levels) is not effectively considered (Zerafat Angiz et al., 2010a). Hatami-Marbini et al. (in pressc) developed a fully fuzzified CCR model to obtain the fuzzy efficiency of the DMUs by utilizing a fully fuzzified LP model, in which all of the input–output data and variables (including their weights) were fuzzy numbers. In spite of its simplicity, the defuzzification approach has not been used by DEA researchers and practitioners. The lack of interest in the defuzzification approach might be due to the fact that with the defuzzification approach the fuzziness in the inputs and outputs is effectively ignored (Tlig and Rebai, 2009).

4.4. The possibility approach

The fundamental principles of the possibility theory are entrenched in Zadeh's (1978) fuzzy set theory. Zadeh (1978) suggests that a fuzzy variable is associated with a possibility distribution in the same manner that a random variable is associated with a probability distribution. In fuzzy LP models, fuzzy coefficients can be viewed as fuzzy variables and constraint can be considered to be fuzzy events. Hence, the possibilities of fuzzy events (i.e., fuzzy constraints) can be determined using possibility theory. Dubois and Prade (1988) provide a comprehensive overview of the possibility theory.

Guo et al. (2000) initially built fuzzy DEA models based on possibility and necessity measures and then Lertworasirikul (2002) and Lertworasirikul et al. (2002a,b) have proposed two approaches for solving the ranking problem in fuzzy DEA models called the “possibility approach” and the “credibility approach.” They introduced the possibility approach from both optimistic and pessimistic view points by considering the uncertainty in fuzzy objectives and fuzzy constraints with possibility measures. In their credibility approach, fuzzy DEA model was transformed into a credibility programming-DEA model and fuzzy variables were replaced by “expected credits,” which were obtained by using credibility measures. The mathematical details of the credibility model can be found in Lertworasirikul et al. (2003b).

Lertworasirikul et al. (2003a,c) proposed a possibility approach for solving a fuzzy CCR model in which fuzzy constraints were treated as fuzzy events. They transformed the fuzzy DEA model into a possibility LP problem by using the possibility measures of the fuzzy events. In the special case, if the fuzzy data were assumed to be trapezoidal fuzzy numbers, the possibility DEA model becomes a LP model. The proposed possibility CCR model of Lertworasirikul et al. (2003a) where they applied the concept of chance-constrained programming (CCP) and possibility of fuzzy events are represented by the following model:

$$\begin{aligned}
 & \max \quad \theta_p = \bar{f} \\
 & \text{s.t.} \quad \left(\sum_{r=1}^s u_r \tilde{y}_{rp} \right)_{\beta}^U \geq \bar{f}, \\
 & \quad \left(\sum_{i=1}^m v_i \tilde{x}_{ip} \right)_{\alpha_0}^U \geq 1, \\
 & \quad \left(\sum_{i=1}^m v_i \tilde{x}_{ip} \right)_{\alpha_0}^L \leq 1, \\
 & \quad \left(\sum_{r=1}^s u_r \tilde{y}_{rj} - \sum_{i=1}^m v_i \tilde{x}_{ij} \right)_{\alpha}^L \leq 0, \quad \forall j, \\
 & \quad u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{16}$$

where $\beta \in [0, 1]$, $\alpha_0 \in [0, 1]$ and $\alpha \in [0, 1]$ are predetermined admissible levels of possibility. The purpose of (16) is to maximize \bar{f} so that $\sum_{r=1}^s u_r \tilde{y}_{rp}$ of the first constraint can achieve with “possibility” level β or higher, subject to the possibility levels being at least α_0 and α in other constraints. In other words, at the optimal solution, the value of $\sum_{r=1}^s u_r \tilde{y}_{rp}$ is obtained at least equal to \bar{f} with possibility level β ; while at the same time all constraints are satisfied at the predetermined possibility levels.

Lertworasirikul et al. (2003b) further developed possibility and credibility approaches for solving fuzzy BCC models. They applied the concept of chance-constrained programming (CCP) and possibility of fuzzy events (fuzzy constraints) to the primal and dual of the fuzzy BCC models in order to obtain possibility BCC models. This approach also disclosed the relationship between the primal and dual models of the fuzzy BCC. The efficiency obtained through their possibility approach to the primal and dual models provided the upper bound and the lower bound for each DMU for a given possibility level. Next, in the credibility approach, they replaced the “expected credits” with fuzzy variables to cope with the uncertainty in fuzzy objectives and fuzzy constraints. Hence, their fuzzy BCC model was transformed into a credibility programming-DEA model. An efficiency score for each DMU was obtained from the credibility approach as a representative of its possible range. Unlike the possibility approach, the decision makers did not have to determine any parameters or rank fuzzy efficiency values in the credibility approach. According to the possibility BCC approach proposed in Lertworasirikul et al. (2003b), the primal proposed model can be represented by the following model:

$$\begin{aligned}
 & \max \quad \theta_p = \left(\sum_{r=1}^s u_r \tilde{y}_{rp} \right)_{\beta}^U - u_0 \\
 & \text{s.t.} \quad \left(\sum_{i=1}^m v_i \tilde{x}_{ip} \right)_{\alpha_0}^U \geq 1, \\
 & \quad \left(\sum_{i=1}^m v_i \tilde{x}_{ip} \right)_{\alpha_0}^L \leq 1, \\
 & \quad \left(\sum_{r=1}^s u_r \tilde{y}_{rj} - \sum_{i=1}^m v_i \tilde{x}_{ij} \right)_{\alpha}^L - u_0 \leq 0, \quad \forall j, \\
 & \quad u_r, v_i \geq 0, \quad \forall r, i.
 \end{aligned} \tag{17}$$

where $\beta \in [0, 1]$, $\alpha_0 \in [0, 1]$ and $\alpha \in [0, 1]$ are the predetermined admissible levels of possibility.

Similarly, according to the possibility approach (Lertworasirikul et al., 2003b), the dual proposed model can be represented by the following model:

$$\begin{aligned}
 & \min \quad \theta_B \\
 & \text{s.t.} \quad \left(\theta_B \tilde{x}_{ip} - \sum_{j=1}^n \lambda_j \tilde{x}_{ij} \right)_{\bar{\alpha}_1}^U \geq 0, \quad \forall i, \\
 & \quad \left(\sum_{j=1}^n \lambda_j \tilde{y}_{rj} - \tilde{y}_{rp} \right)_{\bar{\alpha}_2}^U \geq 0, \quad \forall r, \\
 & \quad \sum_{j=1}^n \lambda_j = 1, \\
 & \quad \lambda_j \geq 0, \quad \forall j.
 \end{aligned} \tag{18}$$

where $\bar{\alpha}_1 \in [0, 1]$ and $\bar{\alpha}_2 \in [0, 1]$ are predetermined admissible levels of possibility. Garcia et al. (2005) introduced a fuzzy DEA approach to rank failure modes identified by means of the occurrence, severity and detection indices. Their method allowed the experts to use linguistic variables in assigning more important values to the considered indices. They utilized the possibility approach proposed by Lertworasirikul et al. (2003a) in their model to solve their fuzzy DEA problem. Similarly, Wu et al. (2006) used the formulation of Lertworasirikul et al. (2003a) in their fuzzy DEA model to cope with the quantitative and linguistic variables in the efficiency analysis of cross-region bank branches in Canada.

Ramezanzadeh et al. (2005) proposed a CCR model with chance-constrained programming approach and used α -level method and fuzzy probability measure to rectify the randomness by classical mean–variance method of Cooper et al. (1996). Jiang and Yang (2007) proposed a fuzzy chance-constrained creditability programming DEA model and introduced a procedure for converting the fuzzy programming to confirm programming. Khodabakhshi et al. (2010) formulated two alternative fuzzy and stochastic additive models to determine returns to scale in DEA. They developed the fuzzy and stochastic DEA models based on the possibility approach and the concept of chance constraint programming, respectively. Wen and Li (2009) proposed a hybrid algorithm integrating fuzzy simulation and genetic algorithm to solve a fuzzy DEA model established based on the credibility measure. Recently, Wen et al. (2010) extended the CCR model to a fuzzy DEA model based on the credibility measure presented by Liu (2004). They designed a hybrid algorithm combined with fuzzy simulation and genetic algorithm to rank all the DMUs with fuzzy inputs and outputs.

4.5. Other developments in fuzzy DEA

In this section, we review several fuzzy DEA models that do not fall into the fuzzy ranking approach, the tolerance approach, the α -level based approach, or the possibility approach categories. Hougaard (1999) extended scores of technical efficiency used in DEA to fuzzy intervals and showed how the fuzzy scores allow the decision maker to use scores of technical efficiency in combination with other sources of available performance information such as expert opinions, key figures, etc. Sheth and Triantis (2003) introduced a fuzzy goal DEA framework to measure and evaluate the goals of efficiency and effectiveness in a fuzzy environment. They defined a membership function for each fuzzy constraint associated with the efficiency and effectiveness goals and represented the degree of achievement of that constraint.

Hougaard (2005) introduced a simple approximation for the assessment of efficiency scores with regards to fuzzy production plans. This approach did not require the use of fuzzy LP techniques and had a clear economic interpretation where all the necessary calculations could be performed in a spreadsheet making it highly operational. Wang et al. (2005) proposed a pair of interval DEA models for dealing with imprecise data such as interval data, ordinal preference information, fuzzy data and their mixture. In their method, the efficiency scores were obtained as interval numbers and a minimax regret approach was used to rank the interval numbers. Uemura (2006) introduced a fuzzy goal based on the evaluation ratings of individual outputs obtained from the fuzzy loglinear analysis and then proposed a fuzzy goal into the DEA. Luban (2009) proposed a method inspired by Sheth and Triantis's (2003) work and used the fuzzy dimension of the DEA models to select the membership function, the bound on the inputs and outputs, the global targets, and the bound of the global targets. Wang et al. (2009a) proposed two fuzzy DEA models with fuzzy inputs and outputs by means of fuzzy arithmetic. They converted each proposed fuzzy CCR model into three LP models in order to calculate the efficiencies of DMUs as fuzzy numbers. In addition they developed a fuzzy ranking approach to rank the fuzzy efficiencies of the DMUs.

Qin et al. (2009) developed a DEA model with type-2 fuzzy inputs and outputs to deal with linguistic uncertainties as well as numerical uncertainties with respect to fuzzy membership functions. Based on the expected value of fuzzy variable, they used a reduction method for type-2 fuzzy variables and built a fuzzy DEA model by means of the generalized credibility measure. Qin and Liu (2009) proposed a class of fuzzy random DEA (FRDEA) models with fuzzy random inputs and outputs when randomness and fuzziness coexisted in an evaluation system and the fuzzy random data were characterized with known possibility and probability distributions. They also proposed a hybrid genetic algorithm and stochastic simulation approach to assess the objective function of the proposed DEA. Qin and Liu (2010) also proposed another approach similar to the method proposed in Qin and Liu (2009). They included the chance functions in the objective and constraint functions which were subsequently converted to the equivalent stochastic programming forms and solved with a hybrid genetic algorithm and Monte Carlo simulation method.

Zerafat Angiz et al. (2010b) proposed an alternative ranking approach based on DEA in the fuzzy environment to aggregate preference rankings of a group of decision makers. They applied their method to a preferential voting system with four stages. Although they considered data as ordinal relations, stage 1 defined a fuzzy membership function for ranking a set of alternatives to find the ideal alternative. In the Second stage they used the fuzzy DEA model proposed in Zerafat Angiz et al. (2006) to obtain the ideal solution. In the last two stages, they proposed a method to aggregate the results to a single score using subjective weights obtained from comparative judgments for ranking the alternatives.

5. Conclusions, limitations and directions for future research

There are relatively a large number of papers in the fuzzy DEA literature. Fuzzy sets theory has been used widely to model uncertainty in DEA. Although other models such as probabilistic/stochastic DEA and statistical preference (e.g., bootstrapping) are also used to model uncertainty in DEA, in this paper we focus on the fuzzy sets DEA papers published in the English-language academic journals. The applications of fuzzy sets theory in DEA are usually categorized into four groups: the tolerance approach, the α -level based approach, the fuzzy ranking approach and the possibility approach. While most of these approaches are powerful, they usually have some theoretical and/or computational limitations and sometimes applicable to a very specific situation (e.g., Soleimani-damaneh et al. (2006)). For example, the tolerance approach uses fuzzy inequalities and equalities instead of fuzzy inputs and fuzzy outputs. The most popular fuzzy DEA group, α -level based approach, often provides a fuzzy efficiency score whose membership function is constructed from α -level even though models related to this approach are not computationally efficient because this group mostly requires a large number of linear programming models according to various α -levels (e.g., Soleimani-damaneh et al. (2006)). In this study, the importance of fuzzy ranking approach in the literature is ranked second (see Table 1) while considerable limitation of this group is that different fuzzy ranking methods may result in different efficiency scores. In the possibility approach, the proposed models may not be adapted to other DEA models (e.g., Soleimani-damaneh et al. (2006)), and we believe that this approach requires complicated numerical computations compared to other approaches.

In summary, fuzzy DEA is best known for its distinct treatment of the imprecise or vague input and output data in the real-world problems. As shown in Fig. 2, fuzzy DEA is a growing field with many practical and theoretical developments. Nevertheless, we believe that fuzzy DEA is still in its early stages of development.

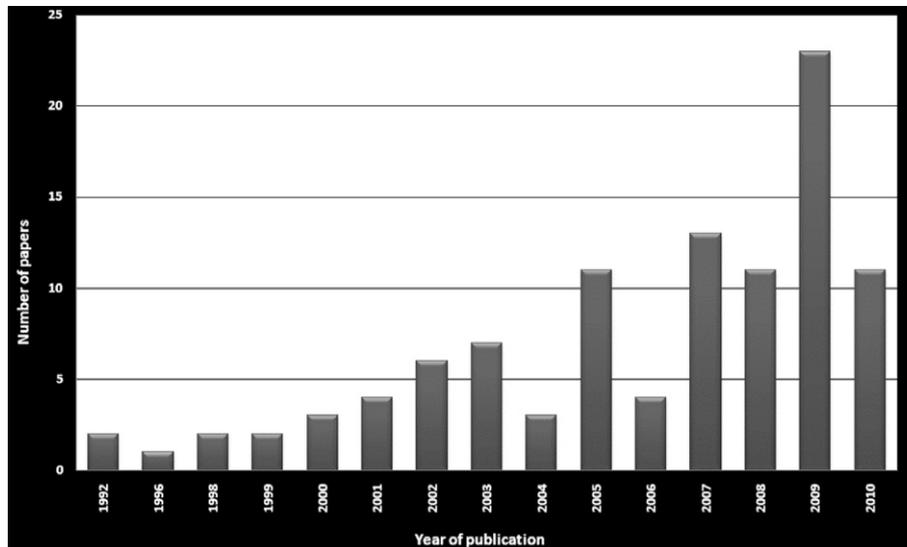


Fig. 2. Two decades of fuzzy DEA development (1990–2010).

A wide variety of applications and proliferation of models have demonstrated that fuzzy DEA is an effective approach for performance measurement in problems with imprecise and vague data. Nevertheless, there are a number of challenges involved in the fuzzy DEA research that provide a great deal of fruitful scope for future research:

- A unified process: It is imperative to provide a unified fuzzy DEA approach for practicing managers and novice users. This need is clearly illustrated by the large number of models and the proliferation of frameworks, at times confusing or even contradictory. A unified process similar to COOPER-framework (Emrouznejad and De Witte, 2010) can provide the novice users with a clear-cut procedure for solving fuzzy DEA problems. Experienced users can use the unified process for modeling depth and breadth.
- User-friendly software: Although there are several DEA software packages in the market, none of them are capable of handling fuzzy data and fuzzy DEA modeling.
- Real-life applications: Most of the papers published in the literature have used simple examples or small sets of hypothetical data to illustrate the applicability of the models. We encourage researchers to use real-world case studies in demonstrating the applicability of their models and exhibit the efficacy of their procedures and algorithms.
- Sensitivity analysis: There is a need for comprehensive studies focusing on sensitivity analysis strategies in fuzzy DEA. Fuzzy data by definition are not fixed. As a result, the results from fuzzy models are less robust and more likely to change over a period of time or even during the model-building phase. Consequently, there is a need for elaborate and comprehensive sensitivity analysis methods and procedures to deal with the changing nature of fuzzy DEA models.

We hope that our research will benefit a wide range of users who desire to solve real-life DEA problems with vague or imprecise data. The taxonomy and the comprehensive review of the literature provided here should lead to a better understanding of fuzzy DEA and its applications.

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