A fully fuzzified data envelopment analysis model

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Abstract: In the conventional data envelopment analysis (DEA), all the data assumes the form of crisp numerical values. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Some researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA by constructing linear programming (LP) models with 'partial' fuzzy parameters. The main purpose of this study is to evaluate the performance of a set of decision making units (DMUs) in a fully fuzzified environment. We propose a novel fully fuzzified DEA (FFDEA) model by utilising a fully fuzzified LP (FFLP) model, where all decision parameters and variables are fuzzy numbers. The contribution of this paper is threefold: first, we consider ambiguous, uncertain and imprecise input and output data in DEA; second, we address the gap in the fuzzy DEA literature for solutions to fully fuzzified problems; and third, we present a numerical example to demonstrate the applicability and efficacy of the proposed model.

Keywords: data envelopment analysis; DEA; fuzzy decision parameters and variables; fuzzy efficiency; fuzzy linear programming; FLP.

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1 Introduction

Data envelopment analysis (DEA) is a widely used mathematical programming technique for comparing the inputs and outputs of a set of homogenous decision making units (DMUs) by evaluating their relative efficiency. The conventional DEA methods such as CCR (Charnes et al., 1978) and BBC (Banker et al., 1984) require accurate measurement of both the inputs and outputs. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. Imprecise evaluations may be the result of unquantifiable, incomplete and non-obtainable information. Some researchers have proposed various fuzzy methods for dealing with the impreciseness and ambiguity in DEA.

Sengupta (1992) proposed a fuzzy mathematical programming approach by incorporating fuzzy input and output data into a DEA model and defining tolerance levels for the objective function and constraint violations. Triantis and Girod (1998) proposed a mathematical programming approach by transforming fuzziness into a DEA model using membership function values. Guo and Tanaka (2001), León et al. (2003) and Lertworasirikul et al. (2003a) proposed three similar fuzzy DEA models by considering the uncertainties in fuzzy objectives and fuzzy constraints. Lertworasirikul et al. (2003b) proposed a fuzzy DEA model using the credibility approach where fuzzy variables were replaced by expected credits according to the credibility measures. Lertworasirikul et al. (2003c) further extended the fuzzy DEA through the possibility and credibility approaches.

Kao and Liu (2000b) transformed fuzzy input and output data into intervals by using α-level sets. The α-level set approach was extended by Saati et al. (2002), who defined the fuzzy DEA model as a possibilistic-programming problem and transformed it into an interval programming. Entani et al. (2002) extended the α-level set research by changing fuzzy input and output data into intervals. Dia (2004) proposed a fuzzy DEA model where a fuzzy aspiration level and a safety α-level were used to transform the fuzzy DEA model into a crisp DEA. Wang et al. (2005) proposed a pair of interval DEA models for dealing with imprecise data such as interval data, ordinal preference information, fuzzy data and their mixture. They introduced the minimax regret approach to the rank interval numbers based on the interval efficiency scores. Saati and Memariani (2005) further extended the α-level set approach so that all DMUs could be evaluated by using a common set of weights under a given α-level set. Soleimani-damaneh et al. (2006) addressed some of the limitations of the fuzzy DEA models proposed by Kao and Liu (2000a), León et al. (2003) and Lertworasirikul et al. (2003a) suggested a fuzzy DEA model to produce crisp efficiencies.

Liu (2008) and Liu and Chuang (2009) extended the α-level set approach by proposing the assurance region approach in the fuzzy DEA model. Hatami-Marbini et al. (2009) proposed a fuzzy DEA model to assess efficiency scores with fuzzy data based on the ranking fuzzy numbers method. Hatami-Marbini and Saati (2009) extended a fuzzy BCC model which considered fuzziness in the input and output data and the u_0 variable. Wang et al. (2009) developed two fuzzy DEA models using of fuzzy arithmetic to handle fuzziness in input and output data in DEA. Although these studies have made great strides in DEA research, none of them provide solutions to problems in a fully fuzzified environment.

Multi-objective models have played a crucial role in mathematical modelling and applications (Hernandes et al., 2007; Li and Hu, 2007; Perez et al., 2004). An important approach dealing with such problems is fuzzy linear programming (FLP) developed by Bellman and Zadeh (1970) which is a symmetrical decision model. Baykasoglu and Göçken (2008), Chen and Ko (2010), Ghodousian and Khorram (2008), Gupta and Mehlawat (2009), Inuiguchi and Ramík (2000), Lodwick and Jamison (2007), Mahdavi-Amiri and Nasseri (2007), Peidro et al. (2010), Rommelfanger (2007), Tan et al. (2008), Van Hop (2007) and Wu (2008) have studied FLP extensively and developed a number of FLP models to solve problems ranging from supply chain management to product development.

Recently, Buckley and Feuring (2000) introduced a general class of fuzzy linear programming (LP) models called fully fuzzified LP (FFLP) problems, where all decision parameters and variables are fuzzy numbers. Hosseinzadeh Lotfi and Mansouri (2008) also discussed FFLP problems of which all parameters and variables are triangular fuzzy numbers, used the concept of the symmetric triangular fuzzy number and introduced an approach to defuzzify a general fuzzy quantity. They proposed a special ranking on fuzzy numbers to transform the FFLP model to a multi objective LP model where all variables and parameters are crisp. Allahviranloo et al. (2008) also proposed a methodology for solving FFLP problems by applying the core of the nearest symmetric triangular fuzzy number for an approximation of the fuzzy numbers in the objective function and the coefficient matrix of the constraints.

We propose a novel DEA method by constructing a FFLP model. We apply the FFLP model developed by Allahviranloo et al. (2008) and transform the DEA model into a fully fuzzified DEA (FFDEA) model. This paper is organised into five sections. In Section 2, we present the preliminary definitions of fuzzy sets. In Section 3, we present the fuzzy DEA model. In Section 4, we illustrate the mathematical details of the proposed FFDEA framework. In Section 5, we present a numerical example to demonstrate the applicability and efficacy of the proposed framework. Finally, in Section 6, we sum up our conclusions and future research directions.

2 Preliminary definitions of fuzzy sets

The fuzzy set algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments. In this section, we review some of the basic definitions of fuzzy sets (Dubois and Prade, 1978; Zimmermann, 1996).

Definition 1. Let U be a universe set. A fuzzy set $\tilde{A}$ of U is defined by a membership function $\mu_{\tilde{A}}(x) \to [0,1]$, where $\mu_{\tilde{A}}(x), \forall x \in U$, indicates the degree of membership of $\tilde{A}$ to U.

Definition 2. A fuzzy subset $\tilde{A}$ of universe set U is convex if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq (\mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)), \forall\, x_1, x_2 \in U, \lambda \in [0, 1]$, where $\wedge$ denotes the minimum operator.

Definition 3. $\tilde{A}$ is a fuzzy number if $\tilde{A}$ is normal and a convex fuzzy set of U.

Definition 4. A trapezoidal fuzzy number $\tilde{A}$ is a kind of specific fuzzy number that can be defined as (M, N, α, β). The linear membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{\alpha}(x - M + \alpha), & M - \alpha \leq x \leq M, \\ 1, & M \leq x < N, \\ \frac{1}{\beta}(N - x + \beta), & N \leq x \leq N + \beta, \\ 0, & \textit{Otherwise.} \end{cases} \tag{1}$$

For a trapezoidal fuzzy number, if $M = N$, then $\tilde{A}$ is called a triangular fuzzy number.

Let $\tilde{A} = (M^A, \alpha^A, \beta^A)$ and $\tilde{B} = (M^B, \alpha^B, \beta^B)$ be two arbitrary triangular fuzzy numbers. Next, some basic fuzzy arithmetic operations on $\tilde{A}$ and $\tilde{B}$ are defined as follows:

- addition: $\tilde{A} \oplus \tilde{B} = (M^A + M^B,\ \alpha^A + \alpha^B,\ \beta^A + \beta^B)$
- subtraction: $\tilde{A} - \tilde{B} = (M^A - M^B,\ \alpha^A + \beta^B,\ \beta^A + \alpha^B)$
- multiplication: $\tilde{A} \otimes \tilde{B} \approx (M^A.M^B,\ \alpha^A.\alpha^B,\ \beta^A.\beta^B)$.

Note that a non-fuzzy number *r* can be expressed by (*r*, 0, 0).

Definition 5. In fuzzy linear programming, the min T-norm is usually applied to assess a linear combination of fuzzy quantities. Therefore, a given set of triangular fuzzy numbers $\tilde{a}_j = (a_j^M, a_j^\alpha, a_j^\beta),\ j = 1, 2, \ldots, n$ and $\lambda_j \geq 0, \sum_{j=1}^{n} \lambda_j \tilde{a}_j$ is defined as follows:

$$\sum_{j=1}^{n} \lambda_j \tilde{a}_j = \left(\sum_{j=1}^{n} \lambda_j a_j^M, \sum_{j=1}^{n} \lambda_j a_j^\alpha, \sum_{j=1}^{n} \lambda_j a_j^\beta \right) \tag{2}$$

where $\sum_{j=1}^{n} \lambda_j \tilde{a}_j$ denotes the combination $\lambda_1 \tilde{a}_1 \oplus \lambda_2 \tilde{a}_2 \oplus \ldots \oplus \lambda_n \tilde{a}_n$.

3 The fuzzy DEA model

Consider *n* DMUs under evaluation. The efficiency of DMU_j $(j = 1, 2, ..., n)$ with *m* inputs and *s* outputs is denoted by $x_{ij}\ (i = 1, 2, ..., m)$ and $y_{rj}\ (r = 1, 2, ..., s)$, respectively. The relative efficiency of the DMU_p can be obtained by using the following linear programming (LP) model (called CCR) proposed by Charnes et al. (1978):

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j \\ & u_r, v_i \geq 0, \quad \forall r, i. \end{aligned} \tag{3}$$

where $u_r (r = 1, \ldots, s)$ and $v_i (i = 1, \ldots, m)$ are the weights assigned to the *r*th output and *i*th input, respectively. Note that θ_p is the best relative efficiency and the DMUs with $\theta_p^* = 1,$ is called the efficient unit, and those units with $\theta_p^* \neq 1$ are called inefficient units.

In conventional DEA, all the data assume the form of specific numerical values. However, the observed value of the input and output data are sometimes imprecise or vague. In fuzzy DEA, uncertainty is represented in the LP model by using fuzzy coefficients. In most fuzzy DEA models, only the input and output data are represented by fuzzy numbers (e.g., Kao and Liu, 2000a; Kao and Liu, 2003; Lertworasirikul et al., 2003a; Saati and Memariani, 2005). However, in this study, we represent both the decision parameters and the input and output data with fuzzy numbers. According to this comprehensive representation, the relative efficiency of DMU_p can be obtained by using the following CCR model:

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{rp} \\ s.t. \quad & \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{ip} = 1, \\ & \sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{rj} - \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{ij} \leq 0, \quad \forall j \\ & \tilde{u}_r, \tilde{v}_i \geq 0, \quad \forall r, i. \end{aligned} \tag{4}$$

Triangular fuzzy numbers are used to quantify the vagueness in the decision parameters and the input and output data in the above model. The choice of triangular fuzzy numbers in this study is made due to their simplicity in modelling and ease of interpretation. The other types of fuzzy numbers may increase the computational complexity without substantially affecting the significance of the results (Wang and Elhag, 2006; Yang and Hung, 2007).

Therefore, we express the inputs and outputs of each DMU and their weights by the following triangular fuzzy numbers: $\tilde{x}_{ij} = (x_{ij}^M, x_{ij}^\alpha, x_{ij}^\beta)$, $\tilde{y}_{rj} = (y_{rj}^M, y_{rj}^\alpha, y_{rj}^\beta)$, $v_i = (v_i^M, v_i^\alpha, v_i^\beta)$ and $\tilde{u}_r = (u_r^M, u_r^\alpha, u_r^\beta)$, respectively. Next, we rewrite model (3) as:

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} \left(u_r^M, u_r^\alpha, u_r^\beta\right) \otimes \left(y_{rp}^M, y_{rp}^\alpha, y_{rp}^\beta\right) \\ s.t. \quad & \sum_{i=1}^{m} \left(v_i^M, v_i^\alpha, v_i^\beta\right) \otimes \left(x_{ip}^M, x_{ip}^\alpha, x_{ip}^\beta\right) = (1,0,0), \\ & \sum_{r=1}^{s} \left(u_r^M, u_r^\alpha, u_r^\beta\right) \otimes \left(y_{rj}^M, y_{rj}^\alpha, y_{rj}^\beta\right) - \\ & \sum_{i=1}^{m} \left(v_i^M, v_i^\alpha, v_i^\beta\right) \otimes \left(x_{ij}^M, x_{ij}^\alpha, x_{ij}^\beta\right) \leq (0,0,0), \quad \forall j, \\ & \left(u_r^M, u_r^\alpha, u_r^\beta\right), \ \left(v_i^M, v_i^\alpha, v_i^\beta\right) \geq 0, \quad \forall r, i. \end{aligned} \tag{5}$$

Note that the triangular fuzzy numbers $(\bullet^M, \bullet^\alpha, \bullet^\beta)$ in the above model are the centre, the left spread and the right spread, respectively. We incorporate the FFLP model developed by Allahviranloo et al. (2008) in the FFDEA model proposed in this study to represent both the decision parameters and the input and output data with fuzzy numbers. In the next section, we present the mathematical details of the proposed FFDEA framework.

4 The proposed FFDEA framework

The main purpose of this study is to evaluate the performance of a set of DMUs in a full fuzzy environment. We use triangular fuzzy input-output data and their fuzzy weights in the input oriented CCR model and introduce a new method which considers all fuzzy data and fuzzy parameters in DEA. We apply the FFLP model developed by Allahviranloo et al. (2008) and transform the fuzzy DEA model (5) to the following LP model:

$$\begin{aligned}
\max\ \ & \theta_p = \sum_{r=1}^{s}\left[u_r^M\left(y_{rp}^M + \left(\frac{1}{4}\right)y_{rp}^{\beta} - \left(\frac{1}{4}\right)y_{rp}^{\alpha}\right) + u_r^{\beta}\left(\left(\frac{1}{4}\right)y_{rp}^M\right) - u_r^{\alpha}\left(\left(\frac{1}{4}\right)y_{rp}^M\right)\right] \\
s.t.\ \ & \sum_{i=1}^{m}\left[v_i^M\left(x_{ip}^M + \left(\frac{1}{4}\right)x_{ip}^{\beta} - \left(\frac{1}{4}\right)x_{ip}^{\alpha}\right) + v_i^{\beta}\left(\left(\frac{1}{4}\right)x_{ip}^M\right) - v_i^{\alpha}\left(\left(\frac{1}{4}\right)x_{ip}^M\right)\right] = 1, \\
& \sum_{r=1}^{s}\left[u_r^M\left(y_{rj}^M + \left(\frac{1}{4}\right)y_{rj}^{\beta} - \left(\frac{1}{4}\right)y_{rj}^{\alpha}\right)\right] + u_r^{\beta}\left(\left(\frac{1}{4}\right)y_{rj}^M\right) - u_r^{\alpha}\left(\left(\frac{1}{4}\right)y_{rj}^M\right) \le \\
& \qquad \sum_{i=1}^{m}\left[v_i^M\left(x_{ij}^M + \left(\frac{1}{4}\right)x_{ij}^{\beta} - \left(\frac{1}{4}\right)x_{ij}^{\alpha}\right) + v_i^{\beta}\left(\left(\frac{1}{4}\right)x_{ij}^M\right) - v_i^{\alpha}\left(\left(\frac{1}{4}\right)x_{ij}^M\right)\right], \qquad \forall j, \\
& u_r^M - u_r^{\alpha} \ge 0, \qquad \forall r, \\
& u_r^M - \left(\frac{1}{4}\right)u_r^{\alpha} + \left(\frac{1}{4}\right)u_r^{\beta} \ge 0, \qquad \forall r, \\
& v_i^M - v_i^{\alpha} \ge 0, \qquad \forall i, \\
& v_i^M - \left(\frac{1}{4}\right)v_i^{\alpha} + \left(\frac{1}{4}\right)v_i^{\beta} \ge 0, \qquad \forall i.
\end{aligned} \tag{6}$$

We run model (6) and obtain the following optimal solutions: u_r^{M*}, $u_r^{\alpha*}$, $u_r^{\beta*}$, v_i^{M*}, $v_i^{\alpha*}$ and $v_i^{\beta*}$. Next, we use these optimal solutions to build the optimal weight of the inputs and outputs as triangular fuzzy numbers, denoted by $\tilde{u}_r^* = (u_r^{M*}, u_r^{\alpha*}, u_r^{\beta*})$ and $\tilde{v}_i^* = (v_i^{M*}, v_i^{\alpha*}, v_i^{\beta*})$. Finally, the objective function of model (4) is used to work out the optimal fuzzy efficiency of each DMU $(\theta_p^{M*}, \theta_p^{\alpha*}, \theta_p^{\beta*})$ as follows:

$$\tilde{\theta}_p^* = \sum_{r=1}^{s} \tilde{u}_r^* \otimes \tilde{y}_{rp} \tag{7}$$

5 A numerical example

In this section, we use the numerical example of Guo and Tanaka (2001) to demonstrate the applicability and efficacy of the proposed framework. Consider Guo and Tanaka's (2001) example presented in Table 1 with five DMUs that consume two fuzzy inputs to obtain two fuzzy outputs.

Table 1 The numerical example of Gou and Tanaka (2001)

DMU	*Input-1*	*Input-2*	*Output-1*	*Output-2*
A	(4, 0.5, 0.5)	(2.1, 0.2, 0.2)	(2.6, 0.2, 0.2)	(4.1, 0.3, 0.3)
B	(2.9, 0, 0)	(1.5, 0.1, 0.1)	(2.2, 0, 0)	(3.5, 0.2, 0.2)
C	(4.9, 0.5, 0.5)	(2.6, 0.4, 0.4)	(3.2, 0.5, 0.5)	(5.1, 0.8, 0.8)
D	(4.1, 0.7, 0.7)	(2.3, 0.1, 0.1)	(2.9, 0.4, 0.4)	(5.7, 0.2, 0.2)
E	(6.5, 0.6, 0.6)	(4.1, 0.5, 0.5)	(5.1, 0.7, 0.7)	(7.4, 0.9, 0.9)

Guo and Tanaka (2001) applied the comparison of fuzzy numbers to determine the efficiency scores of the DMUs for different α-cuts presented in Table 2. According to their method, $S_0 = \{B, C, D, E\}$, $S_{0.5} = \{B, D\}$, $S_{0.75} = \{B, D\}$ and $S_1 = \{B, D, E\}$ are classified into the α-possibilistic efficient DMUs since the maximum value of the fuzzy efficiency at that α-cut is greater than or equal to 1.

Table 2 The fuzzy efficiencies of Gou and Tanaka (2001)

α	*A*	*B*	*C*	*D*	*E*
0.00	(0.81,0.15,0.18)	(0.98,0.10,0.11)	(0.82,0.22,0.3)	(0.93,0.22,0.32)	(0.79,0.18,0.23)
0.50	(0.83,0.08,0.09)	(0.97,0.03,0.03)	(0.83,0.12,0.14)	(0.97,0.12,0.15)	(0.82,0.10,0.11)
0.75	(0.84,0.04,0.04)	(0.99,0.03,0.03)	(0.83,0.06,0.07)	(0.98,0.06,0.07)	(0.83,0.05,0.06)
1.00	(0.85,0.00,0.00)	(1.00, 0.00,0.00)	(0.86,0.00,0.00)	(1.00,0.00,0.00)	(1.00,0.00,0.00)

Next, we use model (6) to solve this performance evaluation problem. For example, model (6) with respect to DMU A can be written as follows:

$$
\begin{aligned}
\max \quad & \theta_A = \left(2.6u_1^M + 4.1u_2^M + 0.65u_1^\beta + 1.025u_2^\beta - 0.65u_1^\alpha - 1.025u_2^\alpha\right) \\
s.t. \quad & \left(4v_1^M + 2.1v_2^M + v_1^\beta + 0.525v_2^\beta - v_1^\alpha - 0.525v_2^\alpha\right) = 1, \\
& \left(2.6u_1^M + 4.1u_2^M + 0.65u_1^\beta + 1.025u_2^\beta - 0.65u_1^\alpha - 1.025u_2^\alpha\right) \leq \\
& \left(4v_1^M + 2.1v_2^M + v_1^\beta + 0.525v_2^\beta - v_1^\alpha - 0.525v_2^\alpha\right), \\
& \left(2.2u_1^M + 3.5u_2^M + 0.55u_1^\beta + 0.875u_2^\beta - 0.55u_1^\alpha - 0.875u_2^\alpha\right) \leq \\
& \left(2.9v_1^M + 1.5v_2^M + 0.725v_1^\beta + 0.375v_2^\beta - 0.725v_1^\alpha - 0.375v_2^\alpha\right), \\
& \left(3.2u_1^M + 5.1u_2^M + 0.8u_1^\beta + 1.275u_2^\beta - 0.8u_1^\alpha - 1.275u_2^\alpha\right) \leq \\
& \left(4.9v_1^M + 2.6v_2^M + 1.225v_1^\beta + 0.65v_2^\beta - 1.225v_1^\alpha - 0.65v_2^\alpha\right),
\end{aligned}
$$

$$\left(2.9u_1^M + 5.7u_2^M + 0.725u_1^\beta + 1.425u_2^\beta - 0.725u_1^\alpha - 1.425u_2^\alpha\right) \le$$
$$\left(4.1v_1^M + 2.3v_2^M + 1.025v_1^\beta + 0.575v_2^\beta - 1.025v_1^\alpha - 0.575v_2^\alpha\right),$$
$$\left(5.1u_1^M + 7.4u_2^M + 1.275u_1^\beta + 1.85u_2^\beta - 1.275u_1^\alpha - 1.85u_2^\alpha\right) \le$$
$$\left(6.5v_1^M + 4.1v_2^M + 1.625v_1^\beta + 1.025v_2^\beta - 1.625v_1^\alpha - 1.025v_2^\alpha\right),$$
$$u_1^M - u_1^\alpha \ge 0, \qquad u_1^M - 0.25u_1^\alpha + 0.25u_1^\beta \ge 0$$
$$u_2^M - u_2^\alpha \ge 0, \qquad u_2^M - 0.25u_2^\alpha + 0.25u_2^\beta \ge 0$$
$$v_1^M - v_1^\alpha \ge 0, \qquad v_1^M - 0.25v_1^\alpha + 0.25v_1^\beta \ge 0$$
$$v_2^M - v_2^\alpha \ge 0, \qquad v_1^M - 0.25v_1^\alpha + 0.25v_1^\beta \ge 0$$

The fuzzy efficiencies scores of DMU A, B, C, D and E are (0.855, 0.066, 0.066), (0.999, 0.018, 0.018), (0.861, 0.135, 0.135), (0.999, 0.035, 0.035) and (0.999, 0.132, 0.132), respectively. By applying the concept of the method proposed by Guo and Tanaka (2001), DMU B, D and E are efficient. Furthermore, according to Guo and Tanaka's (2001) method, units B and D are also efficient for all α-cuts and unit E is efficient when α is equal to 0 and 1. As shown here, the fuzzy results obtained from our method are very similar to Guo and Tanaka's (2001) results. However, our method is significantly less complicated than Guo and Tanaka's (2001) approach. Finally, we applied three fuzzy ranking methods proposed by Wang et al. (2006), Asady and Zendehnam (2007) and Abbasbandy and Hajjari (2009) to Guo and Tanaka's (2001) example. The results are presented in Table 3.

Table 3 The comparative results

DMU	*Fuzzy efficiencies*	*Wang et al. (2006)*		*Asady and Zendehnam (2007)*		*Abbasbandy and Hajjari (2009)*	
		Euclidean distances	*Rank*	*Nearest points*	*Rank*	*Magnitude values*	*Rank*
A	(0.855, 0.066, 0.066)	1.090	3	0.855	3	0.855	3
B	(0.999, 0.018, 0.018)	1.154	1	0.999	1	0.999	1
C	(0.861, 0.135, 0.135)	1.092	2	0.861	2	0.861	2
D	(0.999, 0.035, 0.035)	1.154	1	0.999	1	0.999	1
E	(0.999, 0.132, 0.132)	1.154	1	0.999	1	0.999	1

Wang et al. (2006) introduced a modified centroid-based distance method which used the *Euclidean distances* from the origin to the centroid point of each fuzzy number. The third and fourth columns in Table 3 show the Euclidean distances of the fuzzy numbers and their respective rankings based on Wang et al.'s (2006) method. Asady and Zendehnam

(2007) proposed a defuzzification method based on the *nearest points* of the fuzzy numbers. The fifth and sixth columns in Table 3 show the nearest points of the fuzzy numbers and their respective rankings based on Asady and Zendehnam's (2007) method. Recently, Abbasbandy and Hajjari (2009) proposed a modified fuzzy ranking method of Asady and Zendehnam (2007) by defining a *magnitude* index. The seventh and eight columns in Table 3 show the magnitude values of the fuzzy numbers and their respective rankings based on Abbasbandy and Hajjari's (2009) method. In summary, as shown in Table 3, the rankings obtained from our method are identical to the rankings of Wang et al. (2006), Asady and Zendehnam (2007) and Abbasbandy and Hajjari (2009).

6 Conclusions and future research directions

The field of DEA has grown exponentially since the pioneering papers of Farrell (1957) and Charnes et al. (1978). DEA is generally used to measure the relative efficiencies of a set of DMUs producing multiple outputs from multiple inputs. The conventional DEA methods require accurate measurement of both the inputs and outputs. However, the observed values of the input and output data in real-world problems are sometimes imprecise or vague. To deal with imprecise data, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of FLP problems which are typically solved using some methods for the ranking of fuzzy sets.

In this study, we proposed a novel DEA method by constructing a FFLP model. We applied the FFLP model developed by Allahviranloo et al. (2008) and transformed the DEA model into a FFDEA model. The contribution of this paper was threefold:

1. we considered ambiguous, uncertain and imprecise input and output data in DEA
2. we addressed the gap in the fuzzy DEA literature for solutions to fully fuzzified problems
3. we presented a numerical example to demonstrate the applicability and efficacy of the proposed model by comparing its results with those of Guo and Tanaka (2001), Wang et al. (2006), Asady and Zendehnam (2007) and Abbasbandy and Hajjari (2009).

We showed that in spite of its simplicity, our approach produces similar results.

The proposed FFDEA model enables analysts to assimilate the imprecise and vague data in a formal systematic approach. However, there are a number of challenges involved in the proposed research that provide a great deal of fruitful scope for future research. The application of the FFDEA framework to hierarchical structures is an important area for future research. Many organisational problems tend to exhibit such a profile. In addition, the principles are also somewhat related to and could be applied to the concepts and structures studied in the network DEA model of Färe and Grosskopf (2000). The framework developed in this study can potentially lend itself to many practical applications. We plan to implement the proposed framework in the real-world and write a follow-up paper demonstrating the practical implications of our model in real-life problems.

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