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An extended compromise ratio method for fuzzy group multi-attribute decision making with SWOT analysis

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ABSTRACT

The technique for order preference by similarity to ideal solution (TOPSIS) is a well-known multi-attribute decision making (MADM) method that is used to identify the most attractive alternative solution among a finite set of alternatives based on the simultaneous minimization of the distance from an ideal solution (IS) and the maximization of the distance from the nadir solution (NS). We propose an alternative compromise ratio method (CRM) using an efficient and powerful distance measure for solving the group MADM problems. In the proposed CRM, similar to TOPSIS, the chosen alternative should be simultaneously as close as possible to the IS and as far away as possible from the NS. The conventional MADM problems require well-defined and precise data; however, the values associated with the parameters in the real-world are often imprecise, vague, uncertain or incomplete. Fuzzy sets provide a powerful tool for dealing with the ambiguous data. We capture the decision makers' (DMs') judgments with linguistic variables and represent their importance weights with fuzzy sets. The fuzzy group MADM (FGMADM) method proposed in this study improves the usability of the CRM. We integrate the FGMADM method into a strengths, weaknesses, opportunities and threats (SWOT) analysis framework to show the applicability of the proposed method in a solar panel manufacturing firm in Canada.

1. Introduction

Multi-criteria decision making (MCDM) methods are frequently used to solve real-world problems with multiple, conflicting, and incommensurate criteria. The aim is to help the decision maker (DM) take all important objective and subjective criteria of the problem into consideration using a more explicit, rational and efficient decision process [25,73]. MCDM problems are generally categorized as continuous or discrete, depending on the domain of alternatives. Hwang and Yoon [41] have classified the MCDM methods into two categories: multi-objective decision making (MODM) and multi-attribute decision making (MADM). MODM has been widely studied by means of mathematical programming methods with well-formulated theoretical frameworks. MODM methods have decision variable values that are determined in a continuous or integer domain with either an infinitive or a large number of alternative choices, the best of which should satisfy the DM constraints and preference priorities [26,42]. MADM methods, on the other hand, have been used to solve problems with discrete decision spaces and a predetermined or a limited number of alternative choices. The MADM solution process requires inter and intra-attribute comparisons and involves implicit or explicit tradeoffs [41].

MADM methods are used for circumstances that necessitate the consideration of different options that cannot be measured in a single dimension. Each method provides a different approach for selecting the best among several preselected alternatives [43]. The MADM methods help DMs learn about the issues they face, the value systems of their own and other parties, and the organizational values and objectives that will consequently guide them in identifying a preferred course of action. The primary goal in MADM is to provide a set of attribute-aggregation methodologies for considering the preferences and judgments of DMs [22]. Roy [62] argues that solving MADM problems is not searching for an optimal solution, but rather helping DMs master the complex judgments and data involved in their problems and advance toward an acceptable

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solution. Multi-attributes analysis is not an off-the-shelf recipe that can be applied to every problem and situation. The development of MADM models has often been dictated by real-life problems. Therefore, it is not surprising that methods have appeared in a rather diffuse way, without any clear general methodology or basic theory [71]. The selection of a MADM framework or method should be done carefully according to the nature of the problem, types of choices, measurement scales, dependency among the attributes, type of uncertainty, expectations of the DMs, and quantity and quality of the available data and judgments [71]. Finding the "best" MADM framework is an elusive goal that may never be reached [68].

A variety of MADM techniques such as Simple Additive Weighting (SAW), Analytic Hierarchy Process (AHP), ELimination and Choice Expressing Reality (ELECTRE), and the technique for order preference by similarity to ideal solution (TOPSIS) have been developed to help selection in the condition of multi-criteria [27,29]. Most MADM methods are usually used to solve MADM problems with single DM while more and more real-world MCDM problems are solved as group decision making (GDM) problems with several DMs. The GDM methods are used to find the most attractive alternative by considering different preferences of the DMs [66]. Recently, group multi-attribute decision making (GMADM) has received considerable attention in the MCDM literature [8,54,59,64]. The TOPSIS is a widely used MADM method initially developed by Hwang and Yoon [41]. It has been applied to a large number of application cases in advanced manufacturing [3,58], purchasing and outsourcing [44,65], and financial performance measurement [28].

The basic principle of TOPSIS is that the chosen alternatives should have the shortest distance from the ideal solution (IS) and the farthest distance from the nadir (negative-ideal) solution (NS) [48]. TOPSIS has been shown to be one of the best MADM methods in addressing the rank reversal issue, which is the change in the ranking of alternatives when a non-optimal alternative is introduced [88]. This consistency feature is largely appreciated in practical applications. Moreover, the rank reversal in TOPSIS is insensitive to the number of alternatives [88]. A relative advantage of TOPSIS is its ability to identify the best alternative quickly [60]. Tavana and Hatami-Marbini [66] developed a group MADM framework at the Johnson Space Center for the integrated human exploration mission simulation facility project to assess the priority of human spaceflight mission simulators. They investigated three different variations of TOPSIS including conventional, adjusted and modified TOPSIS methods in their proposed framework.

An important pitfall of some MADM methods is the need for precise measurement of the performance ratings and criteria weights [29]. However, in many real-world problems, ratings and weights cannot be measured precisely as some DMs may express their judgments using linguistic terms such as low, medium and high [15,69,87]. The fuzzy sets theory is ideally suited for handling this ambiguity encountered in solving MADM problems. Since Zadeh [86] introduced fuzzy set theory, and Bellman and Zadeh [7] described the decision making method in fuzzy environments, an increasing number of studies have dealt with uncertain fuzzy problems by applying fuzzy set theory [84,89]. According to Zadeh [87], it is very difficult for conventional quantification to reasonably express complex situations and it is necessary to use linguistic variables whose values are words or sentences in a natural or artificial language. In response, several researchers have studied and proposed various fuzzy MADM methods in the literature [9,13,17,20,89]. Chen [15] presented the TOPSIS method in fuzzy GDM using a crisp Euclidean distance between any two fuzzy numbers.

DMs sometimes use words in natural language or linguistic phrases instead of numerical values to express their judgments. There are also times when linguistic phrases are used because either precise quantitative information is not available or the cost for its computation is too high. The judgments provided by the DMs are often presented with different linguistic preference representation structures such as the traditional additive/multiplicative linguistic preference relations or uncertain additive/multiplicative linguistic preference relations. The following fuzzy linguistic modeling approaches are proposed to deal with linguistic group decision making problems: the approximate modeling based on the extension principle [19,33,59]; the ordered language modeling [8,35,75,79]; the 2-tuple fuzzy linguistic modeling [2,10–12,36,37]; the multi-granular fuzzy linguistic modeling [34,38] and the direct word modeling [76–78,80–82].

In this study, we focus on the compromise ratio method (CRM) for fuzzy group MADM (FGMADM) introduced by Li [50]. In TOPSIS, the basic principle is that the chosen alternative should have the shortest distance from the IS and the farthest distance from the NS. In a follow-up step, TOPSIS combines the IS and the NS to rank the alternative solutions. In contrast to TOPSIS, in the CRM, the chosen alternative should be as close as possible to the IS and as far away as possible from the NS simultaneously. Considering the fact that in real-world decision making problems, it is not possible to fulfill both conditions simultaneously; a relative importance is allocated to these two distances in CRM. Consequently, a distance measure is required to calculate these distances. Although there are several crisp distance measures proposed in the literature for fuzzy numbers [18,67,83], they are not suitable for fuzzy variables. Li [50] and Li [51] have proposed a precise distance measure for fuzzy variables. Guha and Chakraborty [30] further modified the crisp distance measure proposed by Li [50,51] to a fuzzy distance measure. However, their research encountered difficult computational issues since fuzzy numbers in the denominators of the compromise ratios may be neither positive nor negative. In addition, the fuzzy distance measure proposed by Guha and Chakraborty [30] could only be used for solving fuzzy MADM problems with a single DM. Recently, Li [63] extended the CRM by utilizing a fuzzy distance for solving FGMADM method problems in which the weights of the attributes and the ratings of the alternatives on the attributes are expressed with linguistic variables parameterized using triangular fuzzy numbers. They compared their extended method with other existing methods to represent its feasibility and effectiveness.

Since its inception in the early 1950s, SWOT analysis has been used with increasing success as a strategic planning tool by both researchers and practitioners [49,57]. The technique is used to segregate environmental factors and forces into internal strengths and weaknesses, and external opportunities and threats [23,70]. The SWOT matrix developed by Weihrich [74] for situational analysis is one of the most important references in the field. Even with its popularity, Novicevic et al. [56] observe that SWOT is a conceptual framework with limited prescriptive power. However, SWOT remains a useful tool for assisting DMs to structure complex and ill-structured problems [4,5,39].

In this study, we apply the fuzzy distance measure proposed by Guha and Chakraborty [31] to solve the FGMADM problems within the CRM framework. In addition, because of a lesser amount of vagueness and ambiguity, this fuzzy distance measure is more reasonable and efficient than other fuzzy distance measures proposed by Voxman [72] and Guha and Chakraborty [14]. We extend the CRM developed by Rui and Li [63] to solve the FGMADM problems with a number of DMs and a great deal of uncertainty in DMs' judgments. Furthermore, we enhance the fuzzy distance measure with a fuzzy ranking method. Finally, we integrate the FGMADM method into a strengths, weaknesses, opportunities and threats (SWOT) analysis framework to rank the strategic alternatives with respect to the internal strengths and weaknesses, and external opportunities and threats.

The remainder of the paper is organized as follows. We present a set of basic preliminaries and definitions in Section 2 followed by a step-by-step explanation of the proposed extended fuzzy CRM in Section 3. In Section 4, we discuss the novelty and contribution of our fuzzy CRM method and in Section 5 we present the applicability of the proposed method in a solar panel manufacturing firm in Canada. Our conclusions and remarks for future works are provided in Section 6.

2. Preliminaries and definitions

In this section, we first review the TOPSIS method and then introduce the preliminaries and definitions used throughout the paper.

2.1. TOPSIS method

Hwang and Yoon [41] developed the TOPSIS method based on the concept that the chosen alternative should have the shortest distance from the IS and the farthest distance from the NS. The method is briefly described as follows:

Considering m attributes, C_i (i = 1,2, ..., m), and n possible alternatives, A_j (j = 1,2 ..., n); a MADM problem can be expressed in a matrix form as $D = [x_{ij}]_{m\times n}$ where:

- x_{ij} is a score indicating the performance rating of the jth alternative with respect to the ith attribute, and
- w_i (i = 1,2 ..., m) is the importance weight of each attribute and $\sum_{i=1}^{m} w_i = 1$.

A normalized decision matrix is constructed to transform different scales of the attributes into comparable scales as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}(x_{ij})^2}}, \quad i = 1, 2, \ldots m; \; j = 1, 2, \ldots, n. \tag{1}$$

Considering the attribute weights, a weighted normalized decision matrix is obtained as follows:

$$v_{ij} = w_i \times r_{ij}, \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n. \tag{2}$$

The IS (A^*) and the NS (A^-) is defined as follows:

$$\begin{aligned} A^* &= (v_1^*, v_2^*, \ldots, v_m^*)^T = \{(\max_j v_{ij} \,|\, i \in B), (\min_j v_{ij} \,|\, i \in C)\}. \\ A^- &= (v_1^-, v_2^-, \ldots, v_m^-)^T = \{(\min_j v_{ij} \,|\, i \in B), (\max_j v_{ij} \,|\, i \in C)\}. \end{aligned} \tag{3}$$

where B and C are benefit and cost attribute sets. The Euclidean distance of each alternative from the ideal and the nadir solutions can be calculated as follows:

$$\begin{aligned} S_j^* &= \sqrt{\sum_{i=1}^{m}(v_{ij} - v_i^-)^2}, \quad j = 1, 2, \ldots, n. \\ S_j^- &= \sqrt{\sum_{i=1}^{m}(v_{ij} - v_i^*)^2}, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{4}$$

A closeness coefficient is calculated to determine the ranking preference order of the alternatives as follows:

$$CC_j = \frac{S_j^-}{S_j^- + S_j^*}, \quad 0 \le CC_j \le 1, \quad j = 1, 2, \ldots, n. \tag{5}$$

An alternative is closer to the IS and farther from the NS when CC_j approaches 1.

2.2. Fuzzy set theory

The conventional MADM problems require well-defined and precise data; however, the values associated with the parameters in the real-world are often imprecise, vague, uncertain or incomplete. Fuzzy sets introduced by Zadeh [86] provide a powerful tool for dealing with this kind of imprecise, vague, uncertain or incomplete data. Fuzzy set theory treats vague data as possibility distributions in terms of membership functions [61]. The non-numeric linguistic variables are often used in the fuzzy logic applications to facilitate the expression of rules and facts [87]. Fuzzy set theory is by no means devoid of numerical definitions; rather, it may be viewed as a higher level of complexity beyond conventional point-estimate numerical methods [55]. Hence, many experts have employed linguistic variables as fuzzy numbers to determine both importance of the attributes and performance of the alternatives in the presence of subjective or qualitative attributes. In this paper the importance weight of various criteria and the ratings of qualitative criteria are considered as linguistic variables. We also represent the importance weight of the DMs during the decision-making process with linguistic variables.

In this section, some basic definitions of fuzzy sets and numbers are reviewed from Buckley [9], Kaufmann and Gupta [45], Klir and Yuan [46], and Zadeh [87]:

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X, a real number in the interval [0,1]. The function value $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$.

Definition 2. A fuzzy set $\tilde{A}$ is normal if and only if the membership fuction of $\tilde{A}$ satisfies $\sup_x \mu_{\tilde{A}}(x) = 1$.

Definition 3. A fuzzy set $\tilde{A}$ in the universe of discourse X is convex if and only if for every pair of points x_1 and x_2 in the universe of discourse, the membership function of $\tilde{A}$ satisfies the inequality as follows:

$$\mu_{\tilde{A}}(\delta x_1 + (1-\delta)x_2) \ge \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \text{ where } \delta \in [0,1].$$

Definition 4. A generalized trapezoidal fuzzy number $\tilde{A}$ denoted by $\tilde{A} = (a^l, a^m, a^n, a^u; \rho)$ is described as any fuzzy subset of the real line $\mathbb{R}$ with membership function $\mu_{\tilde{A}}$ which satisfies the following properties:

- $\mu_{\tilde{A}}$ is a semi continuous mapping from $\mathbb{R}$ to the closed interval $[0, \rho]$, $0 \le \rho \le 1$,
- $\mu_{\tilde{A}}(x) = 0$, for all $x \in [-\infty, a^l]$,
- $\mu_{\tilde{A}}$ is increasing on $[a^l, a^m]$,
- $\mu_{\tilde{A}}(x) = \rho$ for all $x \in [a^m, a^n]$, where ρ is a constant and $0 < \rho \le 1$,
- $\mu_{\tilde{A}}$ is decreasing on $[a^n, a^u]$, $\mu_{\tilde{A}}(x) = 0$, for all $x \in [a^u, \infty]$,

where a^l, a^m, a^n and a^u are real numbers and ρ presents the degree of confidence of the expert about $\tilde{A}$.

Unless elsewhere specified, it is assumed that $\tilde{A}$ is convex and bounded; i.e., $-\infty < a^l, a^u < \infty$. If $\rho = 1$, $\tilde{A}$ is a normal fuzzy number, and if $0 < \rho < 1$, $\tilde{A}$ is a non-normal fuzzy number.

The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ can be expressed as

$$\mu_{\tilde{A}}(x) = \begin{cases} f^L(x), & a^l \le x \le a^m, \\ \rho, & a^m \le x \le a^n \\ f^R(x), & a^n \le x \le a^u, \\ 0, & O.W. \end{cases}$$

where $f^L : [a^l, a^m] \to [0, \rho]$ and $f^R : [a^n, a^u] \to [0, \rho]$.

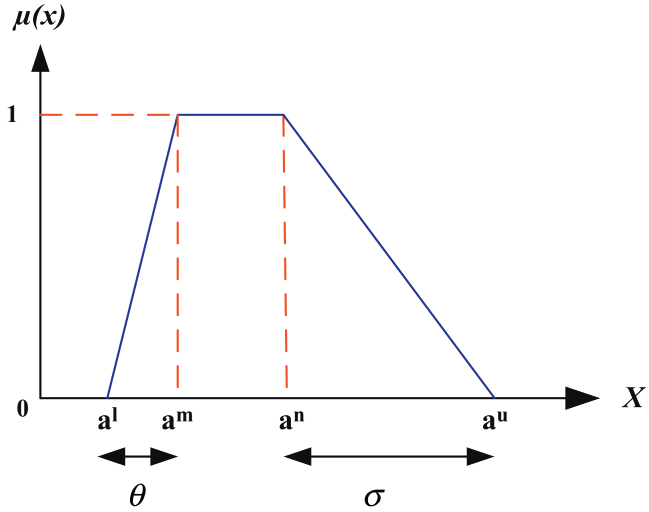


Fig. 1. A trapezoidal fuzzy number.

Definition 5. A fuzzy set $\tilde{A}=(a^l,a^m,a^n,a^u)$ on $\mathbb{R}$, $a^l\le a^m\le a^n\le a^u$, is called a (normal) trapezoidal fuzzy number where $[a^m,a^n]$ is a mode interval of $\tilde{A}$, and a^l and a^u are the left and the right spreads of $\tilde{A}$, respectively, as shown in Fig. 1. Note that $\rho=1$ and the membership function of a trapezoidal fuzzy number is represented as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a^l}{a^m-a^l}, & a^l\le x\le a^m,\\ 1, & a^m\le x\le a^n,\\ \dfrac{a^u-x}{a^u-a^n}, & a^n\le x\le a^u.\end{cases}$$

Note that $\tilde{A}=(a^m,a^n,\theta,\sigma)$ can be an alternative presentation of the trapezoidal fuzzy number in which a^m and a^n are defuzzifiers, and $\theta>0$ and $\sigma>0$ are the left and right fuzziness of the fuzzy number, respectively (see Fig. 1). The membership function of $\tilde{A}$ is represented as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{1}{\theta}(x-a^m+\theta), & a^m-\theta\le x\le a^m,\\ 1, & a^m\le x\le a^n,\\ \dfrac{1}{\sigma}(a^n-x+\sigma), & a^n\le x\le a^n+\sigma.\end{cases}$$

Definition 6. A fuzzy number $\tilde{A}$ is called a positive fuzzy number if $\mu_{\tilde{A}}(x)=0$ for all $x<0$.

Definition 7. Assuming that $\tilde{A}$ and $\tilde{B}$ are two positive trapezoidal fuzzy numbers parameterized by the quadruplet (a^l,a^m,a^n,a^u) and (b^l,b^m,b^n,b^u), respectively, and k is a positive scalar; the basic operations on trapezoidal fuzzy numbers can be shown as follows:

$$\tilde{A}+\tilde{B}=(a^l,a^m,a^n,a^u)+(b^l,b^m,b^n,b^u)=(a^l+b^l,a^m+b^m,a^n+b^n,a^u+b^u);$$
$$\tilde{A}-\tilde{B}=(a^l,a^m,a^n,a^u)-(b^l,b^m,b^n,b^u)=(a^l-b^u,a^m-b^n,a^n-b^m,a^u-b^l);$$
$$\tilde{A}\times\tilde{B}=(a^l,a^m,a^n,a^u)\times(b^l,b^m,b^n,b^u)=(a^l\times b^l,a^m\times b^m,a^n\times b^n,a^u\times b^u);$$
$$k\tilde{A}=(ka^l,ka^m,ka^n,ka^u).$$

Definition 8. The α-cut of the fuzzy set $\tilde{A}$, a crisp subset in the universe of discourse X, is denoted by $[\tilde{A}]_\alpha=\{x|\ \mu_{\tilde{A}}(x)\ge\alpha\}$ where $\alpha\in[0,1]$. For a trapezoidal fuzzy number $\tilde{A}=(a^l,a^m,a^n,a^u)$, the α-cut is represented as follows:

$$[\tilde{A}]_\alpha=[A^L(\alpha),A^R(\alpha)]=[(a^m-a^l)\alpha+a^l,-(a^u-a^n)\alpha+a^u].$$

where $A^L(\alpha)$ and $A^R(\alpha)$ are the lower and upper bounds of the closed interval, respectively.

Several crisp distance measures have been developed for fuzzy numbers in the literature [18,67,83]. However, in most decision making situations involving fuzziness in human judgments, the exact values are transformed into fuzzy numbers and the distance measures for precise values are no longer suited. Consequently, it is not reasonable to define an exact distance between two imprecise numbers and if the uncertainty in the form of fuzziness is within the fuzzy numbers, the distance value should be fuzzy [14]. Voxman [72] introduced the first fuzzy distance measure for two normal fuzzy numbers using the α-cut concept and Chakraborty and Chakraborty [14] improved Voxman's fuzzy distance method.

Recently, Guha and Chakraborty [31] presented a method to measure the fuzzy distance. They discussed the advantages of their method in comparison with the methods of Voxman [72] and Chakraborty and Chakraborty [14]. One of these advantages included the consideration of the confidence level for the DMs. For this reason, we use the fuzzy distance measure (see Definition 9) introduced by Guha and Chakraborty [31] to calculate the difference between the fuzzy numbers.

Definition 9. Let $\tilde{A}$ and $\tilde{B}$ be two generalized trapezoidal fuzzy numbers, where $\rho_1\in[0,1]$ and $\rho_2\in[0,1]$ are the degrees of confidence of the DM's opinion for two fuzzy numbers $\tilde{A}$ and $\tilde{B}$. Thus, the α-cut of $\tilde{A}$ and the α-cut of $\tilde{B}$ are represented by $[\tilde{A}]_\alpha=[A^L(\alpha),A^R(\alpha)]$ for $\alpha\in[0,\rho_1]$ and $[\tilde{B}]_\alpha=[B^L(\alpha),B^R(\alpha)]$ for $\alpha\in[0,\rho_2]$, respectively. Furthermore, the distance between $[\tilde{A}]_\alpha$ and $[\tilde{B}]_\alpha$ for every α can be defined as follows:

$$[\tilde{A}]_\alpha-[\tilde{B}]_\alpha \text{ if } \frac{A^L(\rho_1)+A^R(\rho_1)}{2}\ge\frac{B^L(\rho_2)+B^R(\rho_2)}{2}$$

$$[\tilde{B}]_\alpha-[\tilde{A}]_\alpha \text{ if } \frac{A^L(\rho_1)+A^R(\rho_1)}{2}<\frac{B^L(\rho_2)+B^R(\rho_2)}{2}$$

By defining a zero-unity variable η, we can combine both formulas as follows:

$$\eta([\tilde{A}]_\alpha-[\tilde{B}]_\alpha)+(1-\eta)([\tilde{B}]_\alpha-[\tilde{A}]_\alpha)=[L(\alpha),R(\alpha)] \tag{6}$$

where

$$\eta=\begin{cases}1, & \text{if } \dfrac{A^L(\rho_1)+A^R(\rho_1)}{2}\ge\dfrac{B^L(\rho_2)+B^R(\rho_2)}{2}\\ 0, & \text{if } \dfrac{A^L(\rho_1)+A^R(\rho_1)}{2}<\dfrac{B^L(\rho_2)+B^R(\rho_2)}{2}\end{cases} \tag{7}$$

and

$$L(\alpha)=\eta[A^L(\alpha)-B^L(\alpha)+A^R(\alpha)-B^R(\alpha)]+[B^L(\alpha)-A^R(\alpha)] \tag{8}$$

$$R(\alpha)=\eta[A^L(\alpha)-B^L(\alpha)+A^R(\alpha)-B^R(\alpha)]+[B^R(\alpha)-A^L(\alpha)] \tag{9}$$

The distance measure between two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ in terms of the α-cut approach is expressed as:

$$[d^L_\alpha,d^R_\alpha]=\begin{cases}[L(\alpha),R(\alpha)], & L(\alpha)\ge 0,\\ [0,[\,|L(\alpha)|\vee R(\alpha)]], & L(\alpha)\le 0\le R(\alpha).\end{cases} \tag{10}$$

where $\alpha\in[0,\rho]$ and $\rho=\min(\rho_1,\rho_2)$.

Thereby, we can obtain the fuzzy distance between $\tilde{A}$ and $\tilde{B}$ as

$$\tilde{d}(\tilde{A},\tilde{B})=(d^L_{\alpha=\rho},d^R_{\alpha=\rho},\theta,\sigma) \tag{11}$$

where

$$\theta = d^L_{\alpha=\rho} - \max\left[\int_0^\rho d^L_\alpha d\alpha, 0\right]$$
$$\sigma = \left|\left[\int_0^\rho d^R_\alpha d\alpha - d^R_{\alpha=\rho}\right]\right| \quad (12)$$

It should be note that in Eq. (12), $\rho = \min(\rho_1, \rho_2)$ and θ and σ are the left and right fuzziness of the fuzzy number (see Fig. 1).

Definition 10. Defuzzification is a process for mapping a fuzzy set to a crisp set. The Centroid method is a simple and popular method adapted to defuzzify fuzzy numbers [21]. For a trapezoidal fuzzy number $\tilde{A} = (a^l, a^m, a^n, a^u)$, the defuzzification centroid is computed as

$$\bar{A} = \frac{a^l\mu_{\tilde{A}}(a^l) + a^m\mu_{\tilde{A}}(a^m) + a^n\mu_{\tilde{A}}(a^n) + a^u\mu_{\tilde{A}}(a^u)}{\mu_{\tilde{A}}(a^l) + \mu_{\tilde{A}}(a^m) + \mu_{\tilde{A}}(a^n) + \mu_{\tilde{A}}(a^u)} = \frac{a^l + a^m + a^n + a^u}{4}$$

2.3. Ranking method for trapezoidal fuzzy numbers

The ranking of fuzzy numbers has an essential role in many real-world data analysis, artificial intelligence, and socioeconomic problems [6]. In response, several techniques have been proposed in the literature to rank fuzzy numbers [18,40,53]. In this paper, we obtain a ranking of the fuzzy numbers using the simple and efficient approach proposed by Abbasbandy and Hajjari [1].

Assuming that $\tilde{U} = (x_0, y_0, \theta, \sigma)$ is a trapezoidal fuzzy number and the parametric form of $\tilde{U}$ is a pair $(\underline{U}, \overline{U})$ of functions $\underline{U}(r), \overline{U}(r), 0 \le r \le 1$ where $\underline{U}(r) = x_0 - \theta + \theta r$ and $\overline{U}(r) = y_0 + \sigma - \sigma r$, the magnitude of the trapezoidal fuzzy number defined by Eq. (13) is used to rank the fuzzy numbers.

$$Mag(\tilde{U}) = \frac{1}{2}\left(\int_0^1 (\overline{U}(r) + \underline{U}(r) + x_0 + y_0)f(r)dr\right) \quad (13)$$

The function $f(r)$ is a non-negative and increasing function on [0,1] which can be considered as a weighting function. This function can be defined differently depending on the circumstances. In this paper, without loss of generality, we take into account this function as $f(r) = r$. The larger $Mag(\tilde{U})$ shows the larger fuzzy number. Thus, for any two trapezoidal fuzzy numbers like $\tilde{U}$ and $\tilde{V}$, the following policy is used to determine their ranking order:

(I) $Mag(\tilde{U}) > Mag(\tilde{V})$ if and only if $\tilde{U} > \tilde{V}$,
(II) $Mag(\tilde{U}) < Mag(\tilde{V})$ if and only if $\tilde{U} < \tilde{V}$, and
(III) $Mag(\tilde{U}) = Mag(\tilde{V})$ if and only if $\tilde{U} \sim \tilde{V}$.

See Abbasbandy and Hajjari [1], for further details of the above ranking fuzzy numbers method.

3. The extended fuzzy CRM

In this section, we present a step-by-step explanation of the proposed CRM using the fuzzy distance measure and the fuzzy ranking method depicted in Fig. 2. The distances between the fuzzy values in this measure are considered fuzzy rather than crisp and this measure enables us to consider the degree of confidence in expert opinions. Moreover, the fuzzy ranking method for trapezoidal fuzzy numbers facilitates the relative ranking of the fuzzy numbers.

Let us consider a FGMADM problem with n alternatives (A_j, $j = 1,2 \ldots, n$) and m attributes (C_i, $i = 1,2 \ldots, m$). Let us further assume that k DMs (E_k, $k = 1,2 \ldots, K$) are selected to determine the performance ratings and the importance weight of the attributes using linguistic variables. These linguistic variables are then transformed into trapezoidal fuzzy numbers. In addition, we consider the degree of confidence in DMs' opinions (ρ_k) and according to Guha and Chakraborty [31], $\rho_k = \min(\rho_{fk}, \rho_{gk})$ where ρ_{fk} and ρ_{gk} are the degrees of confidence of kth expert's opinion about two fuzzy numbers f and g. Thus, the performance ratings and the importance weights of the attributes can be constructed in matrix format for the DMs as follows:

$$\tilde{D}_k = [\tilde{x}_{ijk}]_{m\times n}, k = 1, 2, \ldots, K \quad (14)$$

$$\tilde{D}_k = \begin{bmatrix} \tilde{x}_{11k} & \tilde{x}_{12k} & \ldots & \tilde{x}_{1nk} \\ \tilde{x}_{21k} & \tilde{x}_{22k} & \ldots & \tilde{x}_{2nk} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1k} & \tilde{x}_{m2k} & \ldots & \tilde{x}_{mnk} \end{bmatrix}, \tilde{x}_{ijk} = (x^l_{ijk}, x^m_{ijk}, x^n_{ijk}, x^u_{ijk}, \rho_{ijk}) \quad (15)$$

where $\tilde{x}_{ijk}$ are the generalized trapezoidal fuzzy numbers indicating the performance rating of the jth alternative with regards to the ith attribute for the kth DM. We also presume the fuzzy relative importance of each DM as $\tilde{w}' = (\tilde{w}'_1, \tilde{w}'_2, \ldots, \tilde{w}'_K)^T$ where $\tilde{w}'_k = ({w_k}^l, {w_k}^m, {w_k}^n, {w_k}^u)$, $k = 1, 2, \ldots, K$ are the normal trapezoidal fuzzy numbers. In addition, the fuzzy importance of the attributes for the kth DM is expressed as

$$\tilde{w}_k = [\tilde{w}_{ik}]_{m\times 1},\ k = 1, 2, \ldots, K. \quad (16)$$

where $\tilde{w}_{ik} = (w^l_{ik}, w^m_{ik}, w^n_{ik}, w^u_{ik})$ is the normal trapezoidal fuzzy number i.e. $\rho_{ik} = 1$.

A linear normalization method is used to transform the different criteria scales into analogous scales. This normalization process is routinely used in multi-criteria decision making problems to preserve the homogeneity of the data in the decision matrix and to ensure that the ranges of the normalized trapezoidal fuzzy numbers belong to [0,1] [16,32]. The normalized fuzzy decision matrices for the DMs can be constructed as follows:

$$\tilde{R}_k = [\tilde{r}_{ijk}]_{m\times n}, k = 1, 2, \ldots, K. \quad (17)$$

where

$$\tilde{r}_{ijk} = (r^l_{ijk}, r^m_{ijk}, r^n_{ijk}, r^u_{ijk}; \rho_{ijk}) = \left(\frac{x^l_{ijk}}{I^*_{ik}}, \frac{x^m_{ijk}}{I^*_{ik}}, \frac{x^n_{ijk}}{I^*_{ik}}, \frac{x^u_{ijk}}{I^*_{ik}}; \rho_{ijk}\right), \quad j = 1, 2, \ldots, n,\ i \in B,$$
$$\tilde{r}_{ijk} = (r^l_{ijk}, r^m_{ijk}, r^n_{ijk}, r^u_{ijk}; \rho_{ijk}) = \left(\frac{I^-_{ik}}{x^u_{ijk}}, \frac{I^-_{ik}}{x^n_{ijk}}, \frac{I^-_{ik}}{x^m_{ijk}}, \frac{I^-_{ik}}{x^l_{ijk}}; \rho_{ijk}\right), \quad j = 1, 2, \ldots, n,\ i \in C. \quad (18)$$

$$I^*_{ik} = \max_j\{x^u_{ijk}\}, \quad i = 1, 2, \ldots, m,$$
$$I^-_{ik} = \min_j\{x^l_{ijk}\}, \quad i = 1, 2, \ldots, m.$$

and B and C are the benefit and cost attribute index sets, respectively.

Considering different fuzzy weights for the attributes, the weighted normalized fuzzy decision matrices can be computed for the DMs as follow:

$$\tilde{V}_k = [\tilde{v}_{ijk}]_{m\times n}, i = 1, 2, \ldots, m,\ \ j = 1, 2, \ldots, n,\ \ k = 1, 2, \ldots, K. \quad (19)$$

$$\tilde{v}_{ijk} = (v^l_{ijk}, v^m_{ijk}, v^n_{ijk}, v^u_{ijk}; \bar{\rho}_{ijk}) = \tilde{w}_{ik}(\times)\tilde{r}_{ijk} = (w^l_{ik}r^l_{ijk}, w^m_{ik}r^m_{ijk}, w^n_{ik}r^n_{ijk}, w^u_{ik}r^u_{ijk}; \min(\rho_{ijk}, 1)) \quad (20)$$

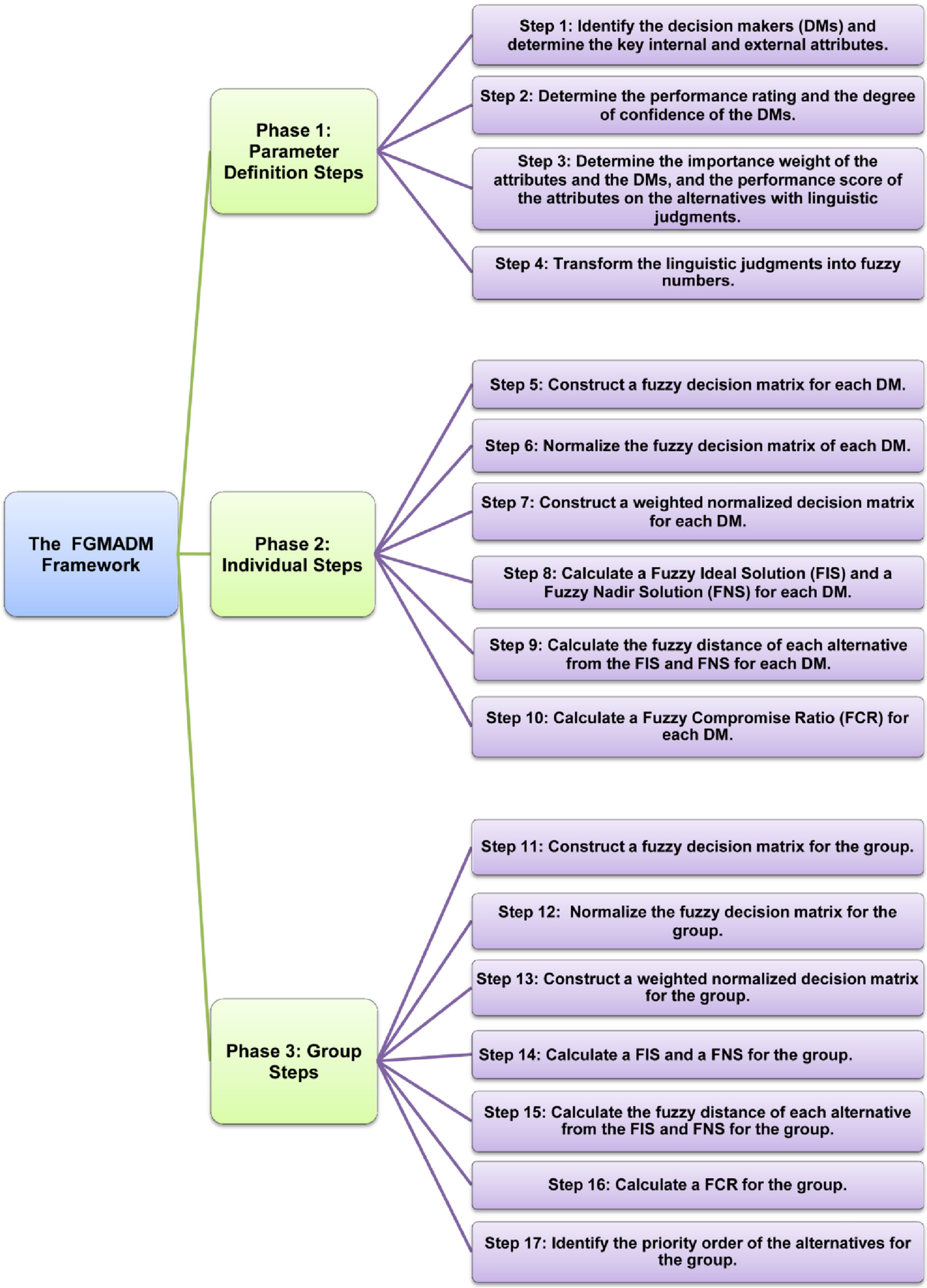
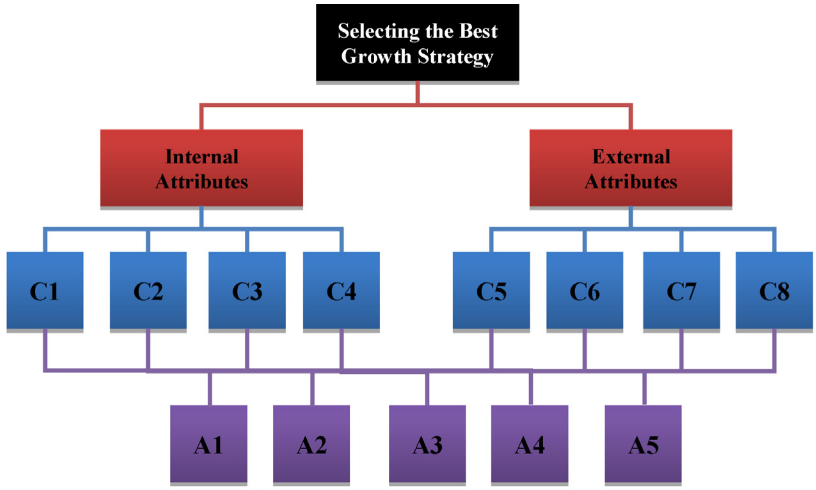


Fig. 2. The proposed framework.



Internal Attributes	**Strengths**	**(C1) Strong R&D capabilities**
		(C2) Innovative corporate culture
	Weaknesses	**(C3) Single production facility**
		(C4) High debt liability
External Attributes	**Opportunities**	**(C5) Government subsidies and tax credit**
		(C6) Growing market trends
	Threats	**(C7) Highly competitive market**
		(C8) U.S. tariffs on imported solar panels
Growth Alternative Strategies: • **(A1) Internal Expansion** • **(A2) Backward Vertical Integration** • **(A3) Forward Vertical Integration** • **(A4) Horizontal integration** • **(A5) Diversified integration**		

Fig. 3. The SWOT matrix.

The fuzzy IS (FIS) $(\tilde{A}_k^*)$ and the fuzzy NS (FNS) $(\tilde{A}_k^-)$ for the DMs can be determined [52] as follows:

$$\tilde{A}_k^* = (\tilde{v}_{1k}^*, \tilde{v}_{2k}^*, \ldots, \tilde{v}_{mk}^*)^T, \quad k = 1, 2, \ldots, K.$$
$$\tilde{A}_k^- = (\tilde{v}_{1k}^-, \tilde{v}_{2k}^-, \ldots, \tilde{v}_{mk}^-)^T, \quad k = 1, 2, \ldots, K. \tag{21}$$

where

$$\tilde{v}_{ik}^* = (\max_j\{v_{ijk}^l\}, \max_j\{v_{ijk}^m\}, \max_j\{v_{ijk}^n\}, \max_j\{v_{ijk}^u\}; \tilde{\rho}_{ijk}), \quad i = 1, 2, \ldots, m.$$
$$\tilde{v}_{ik}^- = (\min_j\{v_{ijk}^l\}, \min_j\{v_{ijk}^m\}, \min_j\{v_{ijk}^n\}, \min_j\{v_{ijk}^u\}; \tilde{\rho}_{ijk}), \quad i = 1, 2, \ldots, m \tag{22}$$

Next, the fuzzy distance of each alternative from the FIS $(\tilde{A}_k^*)$ and the FNS $(\tilde{A}_k^-)$ for the DMs can be calculated using the fuzzy distance measure in Eq. (11) as follows:

$$\tilde{d}_{kj}(A_{kj}, \tilde{A}_k^*) = \sum_{i=1}^{m} \tilde{d}(\tilde{v}_{ijk}, \tilde{A}_k^*), j = 1, 2, \ldots, n \tag{23}$$

$$\tilde{d}_{kj}(A_{kj}, \tilde{A}_k^-) = \sum_{i=1}^{m} \tilde{d}(\tilde{v}_{ijk}, \tilde{A}_k^-), j = 1, 2, \ldots, n \tag{24}$$

The FIS and FNS are used to determine the ranking preference orders among the alternatives. The alternatives with smaller distances from the FIS are preferred to the alternatives with larger distances from the FIS. On the other hand, the alternatives with larger distances from the FNS are preferred to the alternatives with smaller distances from the FNS. Therefore, the fuzzy compromise ratios $(\tilde{\zeta}_{kj})$ of the alternatives A_{kj}, $j = 1,2 \ldots, n$ for the kth DM can then be determined as follows:

$$\tilde{\zeta}_{kj} = \varepsilon_k \tilde{d}(\tilde{d}_k^-(\tilde{A}_k^*), \tilde{d}_{kj}(A_j, \tilde{A}_k^*)) + (1 - \varepsilon_k)\tilde{d}(\tilde{d}_{kj}(A_j, \tilde{A}_k^-), \tilde{d}_k^-(\tilde{A}_k^-)) \tag{25}$$

where

$$\tilde{d}_k^-(\tilde{A}_k^*) = \max_j\{\tilde{d}_{kj}(A_j, \tilde{A}_k^*)\},$$
$$\tilde{d}_k^-(\tilde{A}_k^-) = \min_j\{\tilde{d}_{kj}(A_j, \tilde{A}_k^-)\}.$$

Notice that $\tilde{d}_k^-(\tilde{A}_k^*)$ and $\tilde{d}_k^-(\tilde{A}_k^-)$ are calculated based on the defuzzification method introduced in Definition 10. Furthermore, parameters $\varepsilon_k \in [0,1]$ are the indicators of the attitudinal factors for the DMs. When $\varepsilon_k = 0$, the DM gives more weight to the distance from the FIS. Likewise, when $\varepsilon_k = 0.5$, equal weight is given to both distances.

Obviously, $\tilde{\zeta}_{kj} = (\zeta_{kj}^l, \zeta_{kj}^m, \zeta_{kj}^n, \zeta_{kj}^u)$ are still trapezoidal fuzzy numbers and we can construct the fuzzy decision matrix for the group as follows:

$$\tilde{D}' = [\tilde{\zeta}_{kj}]_{k \times n}$$
$$\tilde{D}' = \begin{bmatrix} \tilde{\zeta}_{11} & \tilde{\zeta}_{12} & \cdots & \tilde{\zeta}_{1n} \\ \tilde{\zeta}_{21} & \tilde{\zeta}_{22} & \cdots & \tilde{\zeta}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\zeta}_{K1} & \tilde{\zeta}_{K2} & \cdots & \tilde{\zeta}_{Kn} \end{bmatrix} \tag{26}$$

The weight vector of the DMs is available in the form of trapezoidal fuzzy numbers as follows:

$$\tilde{w}' = (\tilde{w}'_1, \tilde{w}'_2, \ldots, \tilde{w}'_K)^T$$
$$\tilde{w}'_k = (\tilde{w}'^l_k, \tilde{w}'^m_k, \tilde{w}'^n_k, \tilde{w}'^u_k), k = 1, 2, \ldots, K \quad (27)$$

The normalized fuzzy decision matrix for the group of DMs can then be constructed as follows:

$$\tilde{R}' = [\tilde{r}'_{kj}]_{k\times n}$$
$$\tilde{r}'_{kj} = (r'^l_{kj}, r'^m_{kj}, r'^n_{kj}, r'^u_{kj}) = \left(\frac{\zeta^l_{kj}}{T^*_k}, \frac{\zeta^m_{kj}}{T^*_k}, \frac{\zeta^n_{kj}}{T^*_k}, \frac{\zeta^u_{kj}}{T^*_k}\right), j = 1, 2, \ldots, n. \quad (28)$$

where

$$T^*_k = \max_j\{\zeta^u_{kj}\}, k = 1, 2, \ldots, K. \quad (29)$$

Therefore, the weighted normalized fuzzy decision matrix can be computed for the group of DMs as follows:

$$\tilde{V}' = [\tilde{v}'_{kj}]_{k\times n}, j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, K \quad (30)$$

$$\tilde{v}'_{kj} = (v'^l_{kj}, v'^m_{kj}, v'^n_{kj}, v'^u_{kj}) = \tilde{w}'_k \times \tilde{r}'_{kj} = (w'^i_k r'^l_{kj}, w'^m_k r'^m_{kj}, w'^n_k r'^n_{kj}, w'^u_k r'^u_{kj}) \quad (31)$$

Next, the fuzzy FIS ($\tilde{A}'^*$) and the FNS ($\tilde{A}'^-$) for the group can be defined as follows:

$$\tilde{A}'^* = (\tilde{v}'^*_1, \tilde{v}'^*_2, \ldots, \tilde{v}'^*_k)^T$$
$$\tilde{A}'^- = (\tilde{v}'^-_1, \tilde{v}'^-_2, \ldots, \tilde{v}'^-_k)^T \quad (32)$$

where

$$\tilde{v}'^*_k = (\max_j\{v'^l_{kj}\}, \max_j\{v'^m_{kj}\}, \max_j\{v'^n_{kj}\}, \max_j\{v'^u_{kj}\}), \quad k = 1, 2, \ldots, K$$
$$\tilde{v}'^-_k = (\min_j\{v'^l_{kj}\}, \min_j\{v'^m_{kj}\}, \min_j\{v'^n_{kj}\}, \min_j\{v'^u_{kj}\}), \quad k = 1, 2, \ldots, K \quad (33)$$

Similarly, the fuzzy distance of each alternative from the FIS ($\tilde{A}'^*$) and FNS ($\tilde{A}'^-$) can be derived by utilizing Eq. (11) as follows:

$$\tilde{d}(A'_j, \tilde{A}'^*) = \sum_{k=1}^{K} \tilde{d}(\tilde{v}'_{kj}, \tilde{A}'^*), j = 1, 2, \ldots, n \quad (34)$$

$$\tilde{d}(A'_j, \tilde{A}'^-) = \sum_{k=1}^{K} \tilde{d}(\tilde{v}'_{kj}, \tilde{A}'^-), j = 1, 2, \ldots, n \quad (35)$$

The fuzzy compromise ratios of the alternatives for the group of DMs can be calculated as follows:

$$\tilde{\zeta}'_j = \varepsilon'\tilde{d}[\tilde{d}^-(\tilde{A}'^*), \tilde{d}(A'_j, \tilde{A}'^*)] + (1-\varepsilon')\tilde{d}[\tilde{d}(A'_j, \tilde{A}'^-), \tilde{d}^-(\tilde{A}'^-)],$$
$$j = 1, 2, \ldots, n \quad (36)$$

where

$$\tilde{d}^-(\tilde{A}'^*) = \max_j\{\tilde{d}(A'_j, \tilde{A}'^*)\},$$
$$\tilde{d}^-(\tilde{A}'^-) = \min_j\{\tilde{d}(A'_j, \tilde{A}'^-)\}$$

The parameter $\varepsilon' \in [0,1]$ represents the attitudinal factor of the group of DMs and we apply the formula of Definition 10 to calculate $\tilde{d}^-(\tilde{A}'^*)$ and $\tilde{d}^-(\tilde{A}'^-)$. The priority ranking of the alternative strategies can be generated according to the fuzzy compromise ratios. $\tilde{\zeta}'_j, j = 1, 2, \ldots, n$ are clearly trapezoidal fuzzy numbers and the larger the value $\tilde{\zeta}'_j$, the better the performance of the alternative A_j.

4. Novelty and contribution

The basic premise of the CRM is that the chosen alternative should be the shortest distance to the ideal solution and the longest distance from the negative-ideal solution simultaneously. Many real-world decision making problems inherently involve uncertainty, vagueness and impreciseness, particularly when they consider human judgments which are fuzzy in nature. Fuzzy set theory has been widely used to provide a consistent and reliable mechanism for evaluating the alternatives in MCDM problems with uncertain or vague variables. The CRM with fuzzy variables was introduced by Li [50] and Li [51] using a precise distance measure. Guha and Chakraborthy [30] questioned the rationality of defining the distance between two fuzzy numbers with a precise measure and proposed their own fuzzy distance measure. However, their proposed method encountered difficult computational complexities and could only be used for a single DM. Rui and Li [63] applied the method presented by Li [50,51] for solving FGMADM using the fuzzy distance measure proposed by Chakraborthy and Chakraborthy [14].

Recently, Guha and Chakraborthy [31] presented a method for measuring the fuzzy distance. They discussed the advantages of their method in comparison with the methods of Voxman [72] and Chakraborthy and Chakraborthy [14]. They also showed the distance measure proposed by Chakraborthy and Chakraborthy [14] is not always effective. The methods of Voxman [72] and Chakraborthy and Chakraborthy [14] calculated the distance between two normal fuzzy numbers while the method proposed by Guha and Chakraborthy [31] computed the fuzzy distance measure between two generalized fuzzy numbers. In addition, they used "fuzzy similarity measure" to show the superiority of their method in comparison with the methods of Chen [90], Lee [91] and Chen and Chen [92]. We use the fuzzy distance measure introduced by Guha and Chakraborthy [31] to calculate the distance between fuzzy numbers and accordingly extend an alternative CRM method that considers more generality in the fuzzy environment. In other words, the existing methods only compute the distance between two normal fuzzy numbers (see Definition 5) whereas the approach proposed in this study is more general, less restrictive (since a normal fuzzy number is a special case of a generalized fuzzy number) and can calculate the distance between two generalized fuzzy numbers (see Definition 4).

In addition, we conducted a concise review of the literature and could not find any methods that considered the confidence level of the DM in group decision making. In some real-world decision making problems it is useful to know the confidence level of the DMs in their judgments. For example, one DM might have "full confidence" in his judgment while another DM might be "somewhat confident" in his judgment. We should note that while both DMs agree on a subject matter, one DM has full-confidence in his judgment and another DM is somewhat confident in his judgment. Therefore, we suggest considering the confidence level of the DMs when addressing human judgments in uncertain environments.

In this study, we use the fuzzy distance measure proposed by Guha and Chakraborthy [31] to solve the FGMADM problems within the CRM framework. The proposed fuzzy distance measure is more applicable and less restrictive to the real-world problems in comparison with the competing fuzzy distance measures proposed by Voxman [72] and Chakraborthy and Chakraborthy [14] because of the generality of the model and the lack of restrictions. We further extend the CRM developed by Rui and Li [63] to solve the FGMADM problems with a number of DMs and a great deal of uncertainties surrounding the DMs' judgments. Furthermore, we enhance the fuzzy distance measure with a fuzzy ranking method. Finally, we integrate the FGMADM method into a SWOT analysis framework to rank the strategic alternatives with respect to the internal strengths and weaknesses, and external opportunities and threats.

5. Case study

Sunlite[1] is one of the largest producers of solar panels in Canada. The company has been slow to expand compared to the fast growing companies in the solar panel industry. A group of five DMs $E_k (k=1,2 \ldots, 5)$ were chosen to participate in this study and select a suitable growth strategy for Sunlite. The five DMs were well-educated. Three of them held graduate degrees in engineering and two of them held masters of business administration. All five DMs were experienced managers with 18–26 years of experience in the solar panel industry. They all had a wide range of expertise in manufacturing, strategic management, and capital budgeting.

The first task for this group of five DMs was the articulation of the relevant growth strategy attributes at Sunlite. All five DMs were asked to provide a list of attributes that could be used to evaluate different growth strategies. The individual responses were compiled into a comprehensive list with 13 attributes. Eight attributes, $C_i (i=1,2 \ldots, 8)$, that were common to all five DMs were chosen for assessing organizational growth using external and internal environmental analysis. Attributes C_1, C_2, C_5 and C_6 were considered as benefit attributes and the remaining attributes were considered as cost ones. The questionnaire shown in Appendix A was filled out individually by each DM. Each DM was asked to check the box that best describes the relative importance of each attribute in his or her opinion using the scale from "Very Low" to "Very High" provided in this questionnaire. These attributes were used in a SWOT matrix with a hierarchical structure depicted in Fig. 3.

Sunlite is considering the following growth strategies to increase their sales and market share:

(a) Internal expansion: In order to expand internally, Sunlite will need to retain sufficient profits to be able to purchase new assets, including new technology. Over time, the total value of a firm's assets could rise and provide collateral to enable it to borrow to fund further expansion.
(b) External expansion: The second alternative for Sunlite to achieve growth is to integrate with other solar panel companies in Canada. Sunlite is considering several external expansion strategies including vertical integration, horizontal integration, and diversified integration. With Vertical integration the company can merge with other solar manufacturers at different stages of production. Sunlite is considering two types of vertical integration, backwards and forwards. With backward vertical integration, Sunlite can merge with another Canadian solar panel manufacturer which is nearer to the source of the product. With forward vertical integration, Sunlite can merge with another Canadian solar panel manufacturer to move nearer to the consumer. With horizontal integration, Sunlite can merge with another Canadian solar panel manufacturer at the same stage of production. With diversified integration Sunlite can operate in a completely different market by retaining their name but owned by a 'holding' company.

In summary, Sunlite is considering the following five strategic alternatives A_j $(j=1,2, \ldots, 5)$ for expansion and growth: Internal Expansion (A1), Backward Vertical Integration (A2), Forward Vertical Integration (A3), Horizontal integration (A4), and Diversified integration (A5).We consider the linguistic variables used by Chen et al. [16] and Hatami-Marbini and Tavana [32] to determine the

[1] The name is changed to protect the anonymity of the company.

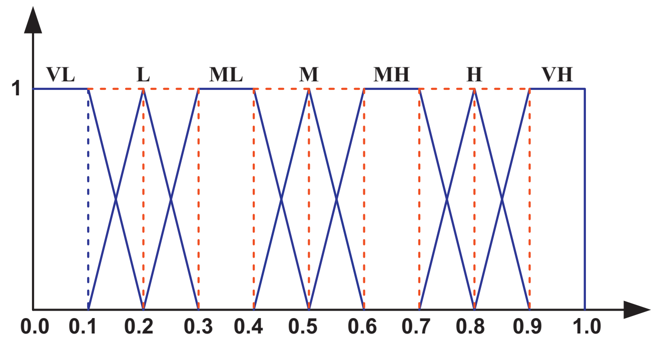


Fig. 4. The membership function of the importance weights.

Table 1
The linguistic variables for the importance weights and their associated fuzzy numbers.

Linguistic variable	Fuzzy number
Very low (VL)	(0, 0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.2, 0.3)
Moderately low (ML)	(0.2, 0.3, 0.4, 0.5)
Moderate (M)	(0.4, 0.5, 0.5, 0.6)
Moderately high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1, 1)

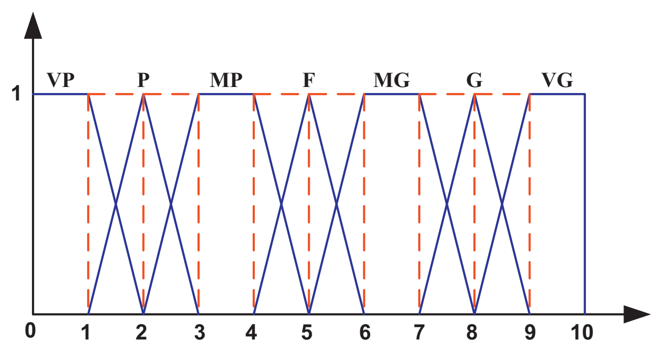


Fig. 5. The membership function of the performance scores.

importance weight of the attributes (shown in Fig. 4 and Table 1) and the performance rating of the alternative strategies (shown in Fig. 5 and Table 2). We should note that Fig. 5 and Table 2 only show the normal fuzzy numbers ($\rho_{ijk}=1$) while ρ_{ijk} can be changed in (0,1].

The importance weight of the attributes and the performance scores of the alternative strategies with respect to the eight attributes provided by each individual DM are presented in Tables 3 and 4, respectively. Note that the degree of the confidence in DMs' opinions for each performance score is presented in the parenthesis for each cell of Table 4. The linguistic assessments

Table 2
The linguistic variables for the performance scores and their associated fuzzy numbers.

Linguistic variable	Fuzzy number[a]
Very poor (VP)	(0, 0, 1, 2)
Poor (P)	(1, 2, 2, 3)
Moderately poor (MP)	(2, 3, 4, 5)
Fair (F)	(4, 5, 5, 6)
Moderately good (MG)	(5, 6, 7, 8)
Good (G)	(7, 8, 8, 9)
Very good (VG)	(8, 9, 10, 10)

[a] It is a normal trapezoidal fuzzy number (i.e. $\rho=1$).

Table 3
The importance weight of the attributes provided by the five DMs.

Attributes	Decision maker				
	E1	E2	E3	E4	E5
C1	MH	H	H	VH	MH
C2	M	M	MH	MH	MH
C3	H	H	H	VH	VH
C4	M	MH	MH	M	H
C5	H	H	MH	VH	VH
C6	L	ML	M	M	L
C7	M	MH	H	M	M
C8	L	ML	ML	M	M

produced by the DMs are then transformed into normal or generalized trapezoidal fuzzy numbers, and consequently the performance ratings and the importance weights of the attributes are constructed in matrix form for each DM. Following this step, a linear normalization method described earlier is used to eliminate any anomalies with various measurement units according to Eq. (18).

Eq. (20) is then used to construct a weighted normalized fuzzy decision matrix for each DM. Next, the FIS ($\tilde{A}^*$) and FNS ($\tilde{A}^-$) are determined for each DM using Eq. (21). We then calculate the distance values of each alternative from the FIS and FNS for each DM using the fuzzy distance method described earlier (see formulas (23) and (24)). In the next step, the fuzzy compromise ratios of the alternatives for the DMs are determined using Eq. (25), where $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$, $\varepsilon_3 = 0.2$, $\varepsilon_4 = 0.5$ and $\varepsilon_5 = 0.2$. For the sake of brevity, these steps are presented in Tables 5–8 for the first DM (E1).

Next we constructed the fuzzy decision matrix presented in Table 9 for our group of DMs according to Eq. (25). Using linear normalization Eq. (28), the normalized fuzzy decision matrix for the group is constructed and presented in Table 10. Using the weight vector of the DMs determined as $\tilde{w}'_1 = (0.7, 0.8, 0.8, 0.9)$, $\tilde{w}'_2 = (0.8, 0.9, 1, 1)$, $\tilde{w}'_3 = (0.8, 0.9, 1, 1)$, $\tilde{w}'_4 = (0.7, 0.8, 0.8, 0.9)$ and $\tilde{w}'_5 = (0.7, 0.8, 0.8, 0.9)$, the group's weighted normalized fuzzy decision matrix presented in Table 11 is constructed.

Next, the FIS and FNS are determined using Eqs. (32) and (33), respectively. The distances of the alternative from the FIS and FNS presented in Table 12 are then calculated for the group. The fuzzy compromise ratios of alternatives for the group are identified using Eq. (36) where $\varepsilon' = 0.3$. In the final step, the priority order of the alternative strategies for the group is determined according to $Mag(\tilde{U})$ defined by formula (13).

The last column of Table 12 presents an overall ranking of the five alternative strategies for growth. For example (0,0,0,0) for an alternative strategy indicates that this alternative has the shortest distance from the NS and the farthest distance from the IS. Therefore, $Mag(\tilde{U})$ for alternative A2 is equal to 0.00. The overall ranking of the five alternative growth strategies for Sunlite is $A3 > A4 > A1 > A5 > A2$>. The team identified Forward Vertical Integration (A3) as the most effective growth strategy and Backward

Table 4
The performance scores of the alternative strategies with respect to the eight attributes provided by the five DMs.

Attributes	Alternative strategies	Decision makers				
		E1	E2	E3	E4	E5
	A1	G (1)	MG (0.5)	G (0.1)	MG (1)	VG (1)
	A2	MG (0.8)	VG (1)	F (0.8)	VG (0.5)	MP (1)
C1	A3	G (1)	G (0.1)	VG (1)	G (0.9)	MG (0.8)
	A4	F (0.2)	MP (0.3)	MG (0.5)	VG (0.5)	G (0.7)
	A5	VG (1)	G (0.1)	MG (0.5)	G (0.9)	F (0.2)
	A1	G (0.8)	F (0.15)	VG (0.8)	G (0.7)	MG (0.8)
	A2	MG (0.6)	VG (1)	G (1)	P (1)	VG (1)
C2	A3	G (0.8)	VG (1)	G (1)	MG (0.8)	G (0.7)
	A4	VG (1)	G (0.8)	G (1)	G (0.7)	VG (1)
	A5	P (1)	MG (0.2)	MG (1)	VG (1)	F (0.2)
	A1	VG (1)	G (0.7)	MG (0.5)	MG (0.7)	G (0.1)
	A2	G (0.7)	MG (0.6)	MP (0.3)	VG (1)	F (1)
C3	A3	F (0.1)	VG (1)	MP (0.3)	G (1)	VG (1)
	A4	MG (0.6)	MG (0.6)	G (0.7)	G (1)	MG (0.5)
	A5	VG (1)	F (0.1)	VG (1)	P (1)	G (0.1)
	A1	F (0.1)	P (1)	G (1)	MP (1)	F (1)
	A2	MG (0.6)	VG (1)	F (0.1)	F (0.1)	G (0.1)
C4	A3	G (0.8)	G (1)	VG (1)	VG (1)	VG (1)
	A4	VG (1)	MG (0.7)	P (1)	G (0.8)	MG (0.5)
	A5	MP (0.4)	MG (0.7)	MG (0.7)	G (0.8)	F (1)
	A1	G (0.1)	P (0.3)	VG (0.8)	VG (0.5)	G (1)
	A2	F (1)	MG (0.5)	MG (1)	F (1)	P (1)
C5	A3	MG (0.5)	VG (1)	MG (1)	G (0.1)	MG (0.5)
	A4	VG (1)	G (0.1)	G (1)	MG (0.5)	VG (1)
	A5	VG (1)	F (1)	G (1)	VG (0.5)	G (1)
	A1	VG (1)	MG (0.9)	F (0.6)	G (0.8)	G (0.9)
	A2	G (0.9)	G (0.8)	VG (0.8)	G (0.8)	VG (1)
C6	A3	G (0.9)	VG (1)	MG (1)	VG (1)	MP (0.4)
	A4	VG (1)	VG (1)	MP (1)	G (0.8)	F (0.5)
	A5	MP (0.4)	F (0.6)	G (0.1)	MG (0.6)	MG (0.7)
	A1	G (1)	F (1)	VP (0.1)	VG (1)	G (0.8)
	A2	MG (0.6)	VG (0.2)	G (1)	MG (0.6)	MG (0.6)
C7	A3	VG (0.7)	G (0.4)	VG (0.7)	F (0.1)	F (0.1)
	A4	P (1)	G (0.4)	VG (0.7)	G (0.8)	VG (1)
	A5	MG (0.6)	MG (0.9)	F (0.6)	VG (1)	MG (0.6)
	A1	G (0.6)	VG (1)	G (1)	P (1)	VG (1)
	A2	G (0.6)	G (0.1)	MG (0.1)	VG (0.7)	MG (0.6)
C8	A3	G (0.6)	G (0.1)	VG (0.1)	G (1)	F (0.1)
	A4	VG (0.5)	P (1)	F (1)	MG (0.6)	MG (0.6)
	A5	F (1)	MG (0.3)	F (0.3)	VG (0.7)	G (0.8)

Table 5
The fuzzy decision matrix for the first DM (E1).

Attributes	A1	A2	A3	A4	A5
C1	(7, 8, 8, 9; 1)	(5, 6, 7, 8; 0.8)	(7, 8, 8, 9; 1)	(4, 5, 5, 6; 0.2)	(8, 9, 10, 10; 1)
C2	(7, 8, 8, 9; 0.8)	(5, 6, 7, 8; 0.6)	(7, 8, 8, 9; 0.8)	(8, 9, 10, 10; 1)	(1, 2, 2, 3; 1)
C3	(8, 9, 10, 10; 1)	(7, 8, 8, 9; 0.7)	(4, 5, 5, 6; 0.1)	(5, 6, 7, 8; 0.6)	(8, 9, 10, 10; 1)
C4	(4, 5, 5, 6; 0.1)	(5, 6, 7, 8; 0.8; 0.6)	(7, 8, 8, 9; 0.8)	(8, 9, 10, 10; 1)	(2, 3, 4, 5; 0.4)
C5	(7, 8, 8, 9; 0.1)	(4, 5, 5, 6; 1)	(5, 6, 7, 8; 0.5)	(8, 9, 10, 10; 1)	(8, 9, 10, 10; 1)
C6	(8, 9, 10, 10; 1)	(7, 8, 8, 9; 0.9)	(7, 8, 8, 9; 0.9)	(8, 9, 10, 10; 1)	(2, 3, 4, 5; 0.4)
C7	(7, 8, 8, 9; 1)	(5, 6, 7, 8; 0.8; 0.6)	(8, 9, 10, 10; 0.7)	(1, 2, 2, 3; 1)	(5, 6, 7, 8; 0.6)
C8	(7, 8, 8, 9; 0.6)	(7, 8, 8, 9; 0.6)	(7, 8, 8, 9; 0.6)	(8, 9, 10, 10; 0.5)	(4, 5, 5, 6; 1)

Table 6
The weighted normalized fuzzy decision matrix for the first DM (E1).

Attributes	A1	A2	A3	A4	A5
C1	(0.35, 0.48, 0.56, 0.72)	(0.25, 0.36, 0.49, 0.64)	(0.35, 0.48, 0.56, 0.72)	(0.20, 0.30, 0.35, 0.48)	(0.40, 0.54, 0.70, 0.80)
C2	(0.28, 0.40, 0.40, 0.54)	(0.20, 0.30, 0.35, 0.48)	(0.28, 0.40, 0.40, 0.54)	(0.32, 0.45, 0.50, 0.60)	(0.04, 0.10, 0.10, 0.18)
C3	(0.28, 0.32, 0.35, 0.45)	(0.30, 0.40, 0.40, 0.51)	(0.46, 0.64, 0.64, 0.90)	(0.35, 0.45, 0.52, 0.72)	(0.28, 0.32, 0.35, 0.45)
C4	(0.13, 0.20, 0.20, 0.30)	(0. 10, 0.14, 0.16, 0.24)	(0.08, 0.12, 0.12, 0.16)	(0.08, 0.10, 0.11, 0.15)	(0.16, 0.25, 0.33, 0.60)
C5	(0.49, 0.64, 0.64, 0.81)	(0.28, 0.40, 0.40, 0.54)	(0.35, 0.48, 0.56, 0.72)	(0.56, 0.72, 0.80, 0.90)	(0.56, 0.72, 0.80, 0.90)
C6	(0.08, 0.18, 0.20, 0.30)	(0.07, 0.16, 0.16, 0.27)	(0.07, 0.16, 0.16, 0.27)	(0.08, 0.18, 0.20, 0.30)	(0.02, 0.06, 0.08, 0.15)
C7	(0.04, 0.06, 0.06, 0.08)	(0.04, 0.07, 0.08, 0.12)	(0.04, 0.05, 0.05, 0.07)	(0.13, 0.25, 0.25, 0.60)	(0.04, 0.07, 0.08, 0.12)
C8	(0.04, 0.10, 0.10, 0.17)	(0.04, 0.10, 0.10, 0.17)	(0.04, 0.10, 0.10, 0.17)	(0.04, 0.08, 0.08, 0.15)	(0.06, 0.16, 0.16, 0.30)

Vertical Integration (A2) as the least effective growth strategy for Sunlite.

6. Conclusions and future research directions

Most real-world strategic decision problems take place in a complex environment and involve conflicting systems of criteria, uncertainty and imprecise information. A wide range of methods have been proposed to solve multi-criteria problems when available information is precise. However, uncertainty and fuzziness inherent in the structure of information make rigorous mathematical models unsuitable for solving multi-criteria problems with imprecise information [7,73,87,89]. MCDM forms an important

Table 7
The FIS and FNS for the first DM (E1).

Attributes	FIS	FNS
C1	(0.40, 0.54, 0.70, 0.80; 1)	(0.20, 0.30, 0.35, 048; 0.2)
C2	(0.32, 0.45, 0.50, 0.60; 1)	(0.04, 0.10, 0.10, 0.18; 1)
C3	(0.46, 0.64, 0.64, 0.90; 0.1)	(0.28, 0.32, 0.35, 0.45; 1)
C4	(0.16, 0.25, 0.33, 0.60; 0.4)	(0.08, 0.10, 0.11, 0.15; 1)
C5	(0.56, 0.72, 0.80, 0.90; 1)	(0.28, 0.40, 0.40, 0.54; 1)
C6	(0.08, 0.18, 0.20, 0.30; 1)	(0.02, 0.06, 0.08, 015; 0.4)
C7	(0.13, 0.25, 0.25, 0.60; 1)	(0.04, 0.05, 0.05, 0.07; 0.7)
C8	(0.06, 0.16, 0.16, 0.30; 1)	(0.04, 0.08, 0.08, 0.15; 0.5)

Table 8
The fuzzy compromise ratio for the first DM (E1).

Alternatives	Fuzzy distance from FIS	Fuzzy distance from FNS	$\tilde{\zeta}_j$
A1	(0.12, 0.22, 2.09, 3.69)	(0.15, 0.26, 1.80, 2.91)	(0, 0, 2.00, 2.62)
A2	(0.26, 0.49, 2.36, 3.52)	(0.04, 0.12, 1.16, 1.51)	(0, 0, 0, 0)
A3	(0.13, 0.24, 1.57, 2.38)	(0.15, 0.29, 1.98, 3.10)	(0, 0, 2.04, 2.66)
A4	(0.01, 0.06, 1.64, 2.81)	(0.48, 0.82, 1.59, 1.95)	(0, 0, 2.05, 2.53)
A5	(0.33, 0.55, 1.39, 2.29)	(0.18, 0.38, 1.58, 2.42)	(0, 0, 1.70, 2.32)

Table 9
The fuzzy decision matrix for the group.

DMs	A1	A2	A3	A4	A5
E1	(0, 0, 2.00, 2.62)	(0, 0, 0, 0)	(0, 0, 2.04, 2.66)	(0, 0, 2.05, 2.53)	(0, 0, 1.70, 2.32)
E2	(0, 0, 0, 0)	(0, 0, 2.36, 3.33)	(0, 0, 2.36, 3.33)	(0, 0, 2.06, 2.94)	(0, 0, 2.29, 3.35)
E3	(0, 0, 2.47, 3.18)	(0, 0, 2.06, 2.48)	(0, 0, 2.14, 2.40)	(0, 0, 1.75, 2.42)	(0, 0, 0, 0)
E4	(0, 0, 2.27, 2.77)	(0, 0, 0, 0)	(0, 0, 1.94, 2.58)	(0, 0, 2.03, 2.57)	(0, 0, 2.24, 2.79)
E5	(0, 0, 2.29, 3.05)	(0, 0, 0, 0)	(0, 0, 2.13, 3.07)	(0, 0, 2.70, 3.45)	(0, 0, 1.87, 2.43)

Table 10
The normalized fuzzy decision matrix for the group.

DMs	A1	A2	A3	A4	A5
E1	(0, 0, 0.75, 0.98)	(0, 0, 0, 0)	(0, 0, 0.76, 1.00)	(0, 0, 0.77, 0.95)	(0, 0, 0.64, 0.87)
E2	(0, 0, 0, 0)	(0, 0, 0.70, 0.99)	(0, 0, 0.70, 0.99)	(0, 0, 0.61, 0.87)	(0, 0, 0.68, 1.00)
E3	(0, 0, 0.77, 1.00)	(0, 0, 0.64, 0.77)	(0, 0, 0.67, 0.75)	(0, 0, 0.55, 0.76)	(0, 0, 0, 0)
E4	(0, 0, 0.81, 0.99)	(0, 0, 0, 0)	(0, 0, 0.69, 0.92)	(0, 0, 0.72, 0.92)	(0, 0, 0.80, 1.00)
E5	(0, 0, 0.66, 0.88)	(0, 0, 0, 0)	(0, 0, 0.61, 0.88)	(0, 0, 0.78, 1.00)	(0, 0, 0.54, 0.70)

Table 11
The weighted normalized fuzzy decision matrix for the group.

DMs	A1	A2	A3	A4	A5
E1	(0, 0, 0.60, 0.88)	(0, 0, 0, 0)	(0, 0, 0.60, 0.90)	(0, 0, 0.61, 0.85)	(0, 0, 0.51, 0.78)
E2	(0, 0, 0, 0)	(0, 0, 0.70, 0.99)	(0, 0, 0.70, 0.99)	(0, 0, 0.61, 0.87)	(0, 0, 0.68, 1.00)
E3	(0, 0, 0.77, 1.00)	(0, 0, 0.64, 0.77)	(0, 0, 0.67, 0.75)	(0, 0, 0.55, 0.76)	(0, 0, 0, 0)
E4	(0, 0, 0.64, 0.89)	(0, 0, 0, 0)	(0, 0, 0.55, 0.82)	(0, 0, 0.57, 0.82)	(0, 0, 0.64, 0.90)
E5	(0, 0, 0.52, 0.79)	(0, 0, 0, 0)	(0, 0, 0.48, 0.79)	(0, 0, 0.62, 0.90)	(0, 0, 0.43, 0.63)

Table 12
The final ranking of the alternatives strategies based on the $Mag(\tilde{U})$ values.

Alternatives	Fuzzy distance from FIS	Fuzzy distance from FNS	$\tilde{\zeta}'_j$	$Mag(\tilde{U})$	Rank
A1	(0, 0, 2.56, 3.11)	(0, 0, 2.53, 3.03)	(0, 0, 2.77, 3.04)	1.40	3
A2	(0, 0, 3.34, 4.00)	(0, 0, 1.34, 1.54)	(0, 0, 0, 0)	0.00	5
A3	(0, 0, 3.33, 4.00)	(0, 0, 3.00, 3.61)	(0, 0, 3.10, 3.41)	1.57	1
A4	(0, 0, 2.72, 3.25)	(0, 0, 2.96, 3.57)	(0, 0, 3.07, 3.38)	1.56	2
A5	(0, 0, 2.68, 3.23)	(0, 0, 2.26, 2.78)	(0, 0, 2.58, 2.86)	1.31	4

part of the decision process for complex problems and the theory of fuzzy set is well-suited to handle the ambiguity and impreciseness inherent in multi-criteria decision problems. TOPSIS is a well-established MADM method that has a history of successful real-world applications [15,48,52,64,66].

In this paper, we proposed a CRM using an effective distance measure for solving the FGMADM problems. The contribution of this paper is sixfold: (1) we addressed the gap in the MADM literature for problems involving conflicting systems of criteria, uncertainty and imprecise information; (2) we proposed a fuzzy distance measure which is more applicable and less restrictive to the real-world problems in comparison with the competing fuzzy distance measures proposed in the literature; (3) we considered the confidence level of the DMs when addressing human judgments in uncertain environments; (4) we solved the FGMADM problem within the CRM framework with a measure that is less vague than the existing measures in the literature; (5) we integrated the FGMADM method into a SWOT analysis framework to rank the strategic alternatives with respect to the internal strengths and weaknesses and external opportunities and threats; and (6) we presented a real-world case study to elucidate the details of the proposed method.

In spite of these contributions, we cannot claim that our method produces a better solution because different MADM methods involve various types of underlying assumptions, information requirements from a DM, and evaluation principles ([41], p. 213). There are compatibilities and incompatibilities with various MADM methods. As to which MADM method(s) we should use, there are no specific rules. Different MADM methods are introduced for different decision situations ([41], p. 210). There are many MADM methods and models, but none can be considered the “best” and/or appropriate for all situations [47]. Solving MADM problems is not searching for some kind of optimal solution, but rather helping DMs master the (often complex) data involved in their problem and advance toward a solution [62]. The method proposed in this study was developed after attempting to address a real-life strategic decision making problem. As often happens in applied mathematics, the development of multi-criteria models is dictated by real-life problems. It is therefore not surprising that methods have appeared in a rather diffuse way, without any clear general methodology or basic theory [71]. A stream of future research can extend our method by developing other hybrid approaches for the integrated use of our distance measure, not only for hybrids of different MADM methods but also for hybrids of MAVT and numerical optimization.

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Appendix A. Individual questionnaire

Direction: Please check the box that best describes the relative importance of each attributes.

Attributes	Very low (VL)	Low (L)	Moderately low (ML)	Moderate (M)	Moderately high (MH)	High (H)	Very high (VH)
Strong R&D capabilities	☐	☐	☐	☐	☐	☐	☐
Innovative corporate culture	☐	☐	☐	☐	☐	☐	☐
Single production facility	☐	☐	☐	☐	☐	☐	☐
High-debt liability	☐	☐	☐	☐	☐	☐	☐
Government subsidies and tax credit	☐	☐	☐	☐	☐	☐	☐
Growing market trends	☐	☐	☐	☐	☐	☐	☐
Highly competitive market	☐	☐	☐	☐	☐	☐	☐
U.S. tariffs on imported solar panels	☐	☐	☐	☐	☐	☐	☐

References

[1] S. Abbasbandy, T. Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, Computers and Mathematics with Applications 57 (2009) 413–419.
[2] S. Alonso, F.J. Cabrerizo, F. Chiclana, F. Herrera, E. Herrera-Viedma, Group decision making with incomplete fuzzy linguistic preference relations, International Journal of Intelligent Systems 24 (2) (2009) 201–222.
[3] V.P. Agrawal, V. Kohli, S. Gupta, Computer aided robot selection: the multiple attribute decision making approach, International Journal of Production Research 29 (8) (1991) 1629–1644.
[4] S.H. Amin, J. Razmib, G. Zhang, Supplier selection and order allocation based on fuzzy SWOT analysis and fuzzy linear programming, Expert Systems with Applications 38 (2011) 334–342.
[5] C. Anderson, J. Vince, Strategic Marketing Management, Houghton Mifflin, Boston, 2002.
[6] B. Asady, A. Zendehnam, Ranking fuzzy numbers by distance minimization, Applied Mathematical Modelling 31 (2007) 2589–2598.
[7] R. Bellman, L.A. Zadeh, Decision making in a fuzzy environment, Management Science 17B (4) (1970) 141–164.
[8] G. Bordogna, M. Fedrizzi, G. Passi, A linguistic modelling of consensus in group decision making based on OWA operators, IEEE Transactions on Systems, Man and Cybernetics 27 (1) (1997) 126–132.
[9] J.J. Buckley, Fuzzy hierarchical analysis, Fuzzy Sets and Systems 17 (1985) 233–247.

[10] F.J. Cabrerizo, R. Heradio, I.J. Pérez, E. Herrera-Viedma, A selection process based on additive consistency to deal with incomplete fuzzy linguistic information, Journal of Universal Computer Science 16 (1) (2010) 62–81.
[11] F.J. Cabrerizo, I.J. Pérez, E. Herrera-Viedma, Managing the consensus in group decision making in an unbalanced fuzzy linguistic context with incomplete information, Knowledge-Based Systems 23 (2) (2010) 169–181.
[12] F.J. Cabrerizo, S. Alonso, E. Herrera-Viedma, A consensus model for group decision making problems with unbalanced fuzzy linguistic information, International Journal of Information Technology and Decision Making 8 (1) (2009) 109–131.
[13] C. Carlsson, Tackling an MCDM-problem with the help of some results from fuzzy set theory, European Journal of Operational Research 10 (3) (1982) 270–281.
[14] C. Chakraborty, D. Chakraborty, A theoretical development on a fuzzy distance measure for fuzzy numbers, Mathematical and Computer Modelling 43 (2006) 254–261.
[15] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (1) (2000) 1–9.
[16] C.T. Chen, C.T. Lin, S.F. Huang, A fuzzy approach for supplier evaluation and selection in supply chain management, International Journal of Production Economics 102 (2006) 289–301.
[17] S.J. Chen, C.L. Hwang, Fuzzy Multiple Attribute Decision Making: Methods and Applications, Springer-Verlag, New York, 1992.
[18] C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95 (3) (1998) 307–317.
[19] F. Chiclana, F. Herrera, E. Herrera-Viedma, Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations, Fuzzy Sets and Systems 97 (1) (1998) 33–48.
[20] H.K. Chiou, G.H. Tzeng, D.C. Cheng, Evaluating sustainable fishing development strategies using fuzzy MCDM approach, Omega 33 (2005) 223–234.
[21] S.Y. Chou, Y.H. Chang, A decision support system for supplier selection based on a strategy-aligned fuzzy SMART approach, Expert Systems with Application 34 (2008) 2241–2253.
[22] M. Doumpos, C. Zopounidis, Multicriteria Decision Aid Classification Methods, Kluwer Academic Publishers, Boston, 2002.
[23] C. Duarte, L.P. Ettkin, M.M. Helms, M.S. Anderson, The challenge of Venezuela: a SWOT analysis, Competitiveness Review 16 (3/4) (2006) 233–247.
[25] I.N. Durbach, T.J. Stewart, Using expected values to simplify decision making under uncertainty, Omega 37 (2) (2009) 312–330.
[26] M. Ehrgott, M.M. Wiecek, Multiobjective programming, in: J. Figueira, S. Greco, M. Ehrgott (Eds.), Multiple Criteria Decision Analysis: State of the Art Surveys, Springer Science + Business Media, Inc., New York, USA, 2005, pp. 667–722.
[27] M. Ehrgott, X. Gandibleux, Multiple Criteria Optimization: State of the Art Annotated Bibliographic Surveys, Springer-Verlag, Berlin, 2003.
[28] C.-M. Feng, R.-T. Wang, Considering the financial ratios on the performance evaluation of highway bus industry, Transport Reviews 21 (4) (2001) 449–467.
[29] J. Figueira, S. Greco, M. Ehrgott, Multiple Criteria Decision Analysis: State of the Art Surveys, Springer, New York, 2005.
[30] D. Guha, D. Chakraborty, Compromise ratio method for decision making under fuzzy environment using fuzzy distance measure, International Journal of Mathematical, Physical and Engineering Science 1 (1) (2008) 1–7.
[31] D. Guha, D. Chakraborty, A new approach to fuzzy distance measure and similarity measure between two generalized fuzzy numbers, Applied Soft Computing 10 (2010) 90–99.
[32] A. Hatami-Marbini, M. Tavana, An extension of the Electre I method for group decision-making under a fuzzy environment, Omega 39 (2011) 373–386.
[33] F. Herrera, E. Herrera-Viedma, Linguistic decision analysis: steps for solving decision problems under Linguistic information, Fuzzy Sets and Systems 115 (1) (2000) 67–82.
[34] F. Herrera, E. Herrera-Viedma, L. Martínez, A fusion approach for managing multi-granularity linguistic term sets in decision making, Fuzzy Sets and Systems 114 (2000) 43–58.
[35] F. Herrera, E. Herrera-Viedma, J.L. Verdegay, Direct approach processes in group decision making using linguistic OWA operators, Fuzzy Sets and Systems 79 (2) (1996) 175–190.
[36] F. Herrera, L. Martínez, A 2-tuple fuzzy linguistic representation model for computing with words, IEEE Transactions on Fuzzy Systems 8 (6) (2000) 746–752.
[37] F. Herrera, L. Martínez, The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 9 (2001) 33–48.
[38] F. Herrera, L. Martínez, A model based on linguistic 2-tuples for dealing with multi-granular hierarchical linguistic contexts in multi-expert decision-making, IEEE Transaction on Systems, Man and Cybernetics 31 (2) (2001) 227–233.
[39] M.A. Hitt, R.D. Ireland, R.E. Hoskisson, Strategic Management: Competitiveness and Globalization, 4th ed., South-Western College Publishing, Cincinnati, 2000.
[40] W. Huijun, A. Jianjun, A new approach for ranking fuzzy numbers based on fuzzy simulation analysis method, Applied Mathematics and Computation 174 (1) (2006) 755–767.
[41] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making: Methods and Applications, Springer-Verlag, New York, 1981.
[42] C.L. Hwang, A.S. Masud, Multi Objective Decision Making, Methods and Applications, Springer, Berlin, 1979.
[43] M. Janic, A. Reggiani, An application of the multiple criteria decision making (MCDM) analysis to the selection of a new hub airport, European Journal of Transport and Infrastructure Research 2 (2) (2002) 113–141.
[44] C. Kahraman, O. Engin, O. Kabak, I. Kaya, Information systems outsourcing decisions using a group decision-making approach, Engineering Applications of Artificial Intelligence 22 (2009) 832–841.
[45] A. Kaufmann, M.M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Application, Van Nostrand Reinhold, New York, 1991.
[46] G.J. Klir, B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice-Hall, New York, 1995.
[47] E. Kujawski, Multi-criteria Decision Analysis: Limitations, Pitfalls and Practical Difficulties, Lawrence Berkeley National Laboratory, 2003 http://escholarship.org/uc/item/0cp6j7sj, 2003
[48] Y.-J. Lai, T.-Y. Liu, C.L. Hwang, TOPSIS for MODM, European Journal of Operational Research 76 (3) (1994) 486–500.
[49] E.P. Learned, C.R. Christensen, K.R. Andrews, W.D. Guth, Business Policy: Text and Cases, Irwin, Homewood, IL, 1965.
[50] D.F. Li, Compromise ratio method for fuzzy multi-attribute group decision making, Applied Soft Computing 7 (3) (2007) 807–817.
[51] D.F. Li, A fuzzy closeness approach to fuzzy multi-attribute decision making, Fuzzy Optimization and Decision Making 6 (3) (2007) 237–254.
[52] I. Mahdavi, N. Mahdavi-Amiri, A. Heidarzade, R. Nourifar, Designing a model of fuzzy TOPSIS in multiple criteria decision making, Applied Mathematics and Computation 206 (2008) 607–617.
[53] M. Modarres, S. Sadi-Nezhad, Ranking fuzzy numbers by preference ratio, Fuzzy Sets and Systems 118 (2001) 429–436.
[54] D.C. Morais, A.T.D. Almeida, Group decision making on water resources based on analysis of individual rankings, Omega 40 (2012) 42–45.
[55] H.T. Nguyen, Some mathematical tools for linguistic probabilities, Fuzzy Sets and Systems 2 (1) (1979) 53–65.
[56] E.P. Novicevic, M.M. Harvey, M. Autry, C.W. Bond III, Dual-perspective SWOT: a synthesis of marketing intelligence and planning, Marketing Intelligence and Planning 22 (1) (2004) 84–94.
[57] G. Panagiotou, Bringing SWOT into focus, Business Strategy Review 14 (2) (2003) 8–10.
[58] C. Parkan, M.-L. Wu, Decision-making and performance measurement models with applications to robot selection, Computers and Industrial Engineering 36 (1999) 503–523.
[59] R.O. Parreiras, P.Y. Ekel, J.S.C. Martini, R.M. Palhares, A flexible consensus scheme for multicriteria group decision making under linguistic assessments, Information Sciences 180 (2010) 1075–1089.
[60] C. Paxkan, M.L. Wu, On the equivalence of operational performance measurement and multiple attribute decision making, International Journal of Production Research 35 (11) (1997) 2963–2988.
[61] B. Pérez-Galdish, I. Gonzalez, A. Bilbao-Terol, M. Arenas-Parra, Planning a TV advertising campaign: a crisp multiobjective programming model from fuzzy basic data, Omega 38 (1–2) (2010) 84–94.
[62] B. Roy, Decision-aid and decision making, European Journal of Operational Research 45 (1990) 324–331.
[63] Z. Rui, D.F. Li, Fuzzy distance based FMAGDM compromise ratio method and application, Journal of Systems Engineering and Electronics 21 (2010) 455–460.
[64] H.S. Shih, H.J. Shyur, E.S. Lee, An extension of TOPSIS for group decision making, Mathematical and Computer Modelling 45 (7–8) (2007) 801–813.
[65] H.-J. Shyura, H.-S. Shih, A hybrid MCDM model for strategic vendor selection, Mathematical and Computer Modelling 44 (2006) 749–761.
[66] M. Tavana, A. Hatami-Marbini, A group AHP-TOPSIS framework for human spaceflight mission planning at NASA, Expert Systems with Applications 38 (2011) 13588–13603.
[67] L. Tran, L. Duckstein, Comparison of fuzzy numbers using a fuzzy distance measures, Fuzzy Sets and Systems 130 (3) (2002) 331–341.
[68] E. Triantaphyllou, Multi-criteria Decision Making Methods: a Comparative Study, Kluwer Academic Publishers, Boston, 2000.
[69] S.H. Tsaur, T.Y. Chang, C.H. Yen, The evaluation of airline service quality by fuzzy MCDM, Tourism Management 23 (2002) 107–115.
[70] E.K. Valentin, SWOT analysis from a resource-based view, Journal of Marketing Theory and Practice 9 (2) (2001) 54–68.
[71] P. Vincke, Multicriteria Decision Aid, Wiley, New York, 1992.
[72] W. Voxman, Some remarks on distances between fuzzy numbers, Fuzzy Sets and Systems 100 (1998) 353–365.
[73] X. Wang, E. Triantaphyllou, Ranking irregularities when evaluating alternatives by using some ELECTRE methods, Omega 36 (2008) 45–63.
[74] H. Weihrich, The TOWS matrix-a tool for situational analysis, Long Range Planning 15 (2) (1982) 54–66.
[75] Z.S. Xu, EOWA and EOWG operators for aggregating linguistic labels based on linguistic preference relations, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 12 (6) (2004) 791–810.
[76] Z.S. Xu, Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment, Information Sciences 168 (3) (2004) 171–184.
[77] Z.S. Xu, An approach to group decision making based on incomplete linguistic preference relations, International Journal of Information Technology and Decision Making 4 (1) (2005) 153–160.
[78] Z.S. Xu, Deviation measures of linguistic preference relations in group decision making, Omega 33 (3) (2005) 249–254.
[79] Z.S. Xu, Induced uncertain linguistic OWA operators applied to group decision making, Information Fusion 7 (2006) 231–238.
[80] Z.S. Xu, An approach based on the uncertain LOWG and the induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations, Decision Support Systems 41 (2006) 488–499.

[81] Z.S. Xu, A practical procedure for group decision making under incomplete multiplicative linguistic preference relations, Group Decision and Negotiation 15 (2006) 581–591.
[82] Z.S. Xu, Group decision making based on multiple types of linguistic preference relations, Information Sciences 178 (2008) 452–467.
[83] R. Xu, C. Li, Multidimensional least-squares fitting with a fuzzy model, Fuzzy Sets and Systems 119 (2001) 215–223.
[84] R.R. Yager, Multiple objective decision-making using fuzzy sets, International Journal of Man–Machine Studies 9 (1977) 375–382.
[86] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[87] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Information Sciences 8 (1975), 199–249(I), 301–357(II).
[88] S.H. Zanakis, A. Solomon, N. Wishart, S. Dublish, Multi-attribute decision making: a simulation comparison of select methods, European Journal of Operational Research 107 (1998) 507–529.
[89] H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer Academic Publishers, Boston, 1996.
[90] S.M. Chen, New methods for subjective mental workload assessment and fuzzy risk analysis, Cybernetics and Systems 27 (1996) 449–472.
[91] H.S. Lee, An optimal aggregation method for fuzzy opinions of group decision, Proceedings of IEEE International Conference on Systems, Man, and Cybernetics 3 (1999) 314–319.
[92] S.J. Chen, S.M. Chen, Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers, IEEE Transactions on Fuzzy Systems 11 (2003) 45–56.