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ORIGINAL ARTICLE

A new fuzzy network data envelopment analysis model for measuring the performance of agility in supply chains

Kaveh Khalili-Damghani · Madjid Tavana

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Abstract Data envelopment analysis (DEA) is a linear programming method for assessing the efficiency and productivity of organizational units called decision-making units (DMUs). We propose a new network DEA (NDEA) model for measuring the performance of agility in supply chains. The uncertainty of the input and output data is modeled with linguistic terms parameterized with fuzzy sets. The proposed fuzzy NDEA model is linear and independent of the α-cut variables. The linear feature allows for a quick identification of the global optimum solution and the α-cut independency feature allows for a significant reduction in the computational efforts. We show that our model always generate solutions within a bounded feasible region. Our model also eliminates the potential for conflict by producing unique interval efficiency scores for each DMU. The proposed model is used to measure the performance of agility in a real-life case study in the dairy industry.

Keywords Fuzzy DEA · Network DEA · Mathematical programming · Supply chain performance

K. Khalili-Damghani (✉)
Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran
e-mail: kaveh.khalili@gmail.com
URL: kaveh-khalili.webs.com

M. Tavana
Business Systems and Analytics, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA
e-mail: tavana@lasalle.edu
URL: http://tavana.us/

1 Introduction

Agility directly impacts the sourcing, making, and delivery processes as well as the overall performance of the supply chains. Numerous agility frameworks [11, 32, 50] and performance measures [23–25] have been proposed in the supply chain management literature. However, agility frameworks and performance measures have not been considered comprehensively in an integrated supply chain management system. The integration of the agility frameworks with the performance measures is not a trivial task and may give rise to several important issues such as: How can the overall efficiency of a supply chain be decomposed into sub-efficiencies according to its internal processes? How can we measure the success or failure of the sub-processes in a complicated network? At what level is the agility of the sub-processes changed into the capabilities of agility in a supply chain? What proportion of a possible success or failure could be attributed to sourcing, making, and delivery processes in a supply chain? The lack of a comprehensive framework for measuring the efficiency of agility in supply chains and a desire to find answers to these unanswered questions motivated us to propose a new Data Envelopment Analysis (DEA) model in this study. The fuzzy sets are utilized in the new DEA model because the input and output data in most real-life performance measurement problems are often imprecise and uncertain. The proposed model is used to evaluate the relative efficiency of the sub-processes and the overall process of a supply chain in the context of agility by measuring the performance of the sourcing, making, and delivery processes.

The remainder of the paper is organized as follows. In Section 2, we present a review of the relevant literature.

In Section 3, we introduce the conceptual model of agility in supply chains. In Section 4, we propose a new fuzzy Network DEA (NDEA) model for measuring the performance of agility in supply chains. In Section 5, we demonstrate the applicability of the proposed model with a real-life case study in the dairy industry. In Section 6, we discuss our conclusions and future research directions.

2 Literature review

In this section, we review the relevant literature on supply chain agility, DEA, two-stage DEA, network DEA, and fuzzy DEA.

2.1 Agility literature

With an increasing amount of global competition, companies have witnessed significant changes in the market, such as high degree of market volatility, shortened lifecycles, uncertain demand and unreliable supply. Mass markets are continuing to fragment as customers' demands and expectations rise. These developments have caused a major shift in business priorities. The need to respond in a volatile environment is addressed through the concept of agility. Organizational agility is defined as "the ability to cope with unexpected challenges as opportunities" [57]. The research in this area has emphasized that the firm's ability to respond is a key measure of agility [18, 53]. Companies have recognized that agility is crucial for their survival and competitiveness. Agility is "the ability to detect opportunities for innovation and seize those competitive market opportunities by assembling requisite assets, knowledge and relationships with speed and surprise" [26, 54]. Agility is recognized as a winning competitive advantage [11, 12, 22, 44, 52, 59, 60]. Table 1 presents some influential definitions of agility in the literature.

Agility in supply chain management is also defined as the ability of a supply chain to rapidly respond to changes in the market and customer's demand [32, 58]. The powerful integration of supply chain management and agility, known as the Agile Supply Chain (ASC), is a significant source of competitiveness in organizations [11, 64, 65]. The systematic methods for establishing the agility paradigm in organizations and the effective management of supply chains have received considerable attention. However, the ability to build effective ASCs has developed more slowly than anticipated [58]. The lack of a systematic approach to manage agility does not allow organizations to develop the necessary proficiency needed to cope with change in turbulent environments [50]. The design and development of ASCs has become an essential step in acquiring various distinguishing capabilities to respond to the changing

Table 1 Definition of agility

Author(s)	Year	Definition
Kidd	1994	"Agility is synthesized use of developed and well-known technologies and methods of manufacturing. That is, it is mutuality compatible with lean manufacturing, CIM, TQM, MRP, BPR, employee empowerment, and OPT."
Goldman et al.	1995	"Agility means delivering value to customers, being ready for change, valuing human knowledge and skills and forming virtual partnership."
Gunasekaran	1998	"Agility is the capability of reaching unpredictable market changes in a cost-effective way, simultaneously prospering from the uncertainty."
Bullinger	1999	"Agility means mobility in an organization's behavior towards the environment and can, therefore, understand an extensive answer to continually changing markets. Agile companies are in a process of constant re-determination, or self-organization, self-configuration, and self-teaming."
Yusuf and Gunasekaran	1999	"Agility is successful exploration of competence bases (speed, flexibility, innovation, proactively, quality and profitability) through the integration of reconfigurable resources and best practices in a knowledge-rich environment to provide customer-driven products and services in a fast-changing market environment."
Naylor	1999	"Agility means using market knowledge and a virtual corporation to exploit profitable opportunities in a violate marketplace."
Christopher	2000	"Agility is defined as the ability of an organization to respond rapidly to changes in demand both in terms of volume and variety."
Stratton and Warburton	2003	"Innovative products and unstable demand typify agile supply drivers."
Harrison and Van Hoak	2005	"Agility is a supply chain wide capability that aligns organizational structures, information systems, logistics processes and, in particular, mindsets."

environments [11]. Sharp et al. [58] and Christopher [11] have identified these capabilities as follows:

- *Responsiveness*: the ability to identify changes and respond to them quickly, reactively or proactively, and also to recover from them.
- *Competency*: the ability to efficiently and effectively realize the strategic objectives in an enterprise.
- *Flexibility*: the ability to implement different process and apply different facilities to achieve the same goal.
- *Quickness*: the ability to complete an activity as quickly as possible.

Other characteristics for ASCs can found in the literature. An ASC requires some enablers like *Collaborative relationship* as the supply chain strategy, *Process integration* as the foundation of the supply chain, *Information integration* as the infrastructure of the supply chain, and finally *Customer/marketing sensitivity* as the mechanism of the supply chain [50]. Table 2 provides some conceptual models for ASC proposed in the literature.

2.2 DEA literature

DEA is a widely used mathematical programming technique that was originally developed by Charnes et al. [4] and was extended by Banker et al. [3] to include variable returns to scale. DEA generalizes the Farrell [21] single-input single-output technical efficiency measure to the multiple-input multiple-output case to evaluate the relative efficiency of peer units with respect to multiple performance measures [5, 15]. The units under evaluation in DEA are called decision-making units (DMUs). A DMU is considered efficient when no other DMUs can produce more outputs using an equal or lesser amount of inputs. The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs [15]. Unlike parametric methods which require a detailed knowledge of the process, DEA is non-parametric and does not require an explicit functional form relating inputs and outputs [see Cooper et al. [15] and Cook and Seiford [13] for an appraisal of the theoretical foundations and developments in DEA]. Numerous applications in recent years have been accompanied by new extensions and developments in the concept and methodology of DEA (see Seiford [56] and Emrouznejad et al. [19] for an extensive bibliography of DEA).

Table 2 Conceptual ASC models

Authors	Year	Primary emphasis
Christopher	2000	ASC's enablers
Yusuf et al.	2004	Agile supply chain capabilities
Lin et al.	2006	A conceptual model for assessing agility in supply chain
Sowford	2006	A process approach to ASC

2.3 Two-stage DEA literature

Single-stage DEA models are appropriate for measuring the relative efficiency of simple processes and cannot represent the internal operations of a DMU. These internal operations may consist of several sub-DMUs. The NDEA models are developed to effectively handle the internal operations of the DMUs. The NDEA essentially considers the internal operations of a DMU by modeling the complicated interactions among multiple sub-DMUs in the efficiency evaluation problems. By focusing on the efficiency score of a sub-DMU, managers can improve the performance of the sub-DMU which in turn improves the overall efficiency of the DMU [46].

A two-stage DEA model is a special variation of the NDEA where a DMU consists of two serial sub-DMUs. The sub-DMUs are related through a series relation and all the outputs of the first stage are used as inputs in the second stage. Moreover, no extra inputs are supplied for the second stage except for the outputs from the first stage. In other words, a two-stage DEA structure uses a set of inputs to produce a set of outputs in the first stage. The outputs of the first stage are named as intermediate measures and treated as inputs in the second stage to produce the outputs of the second stage [14]. [8] proposed a two-stage DEA model to measure the indirect impact of information technology on the firms' performance. [9] developed a non-linear programming DEA model to evaluate the impact of information technology on multiple stages of a business process to maximize the efficiency of the information technology-related resources. Kao and Hwang [34] divided the efficiency of a DMU into two sub-DMUs and used a conventional DEA model to identify the causes of inefficiency for each sub-DMU independently.

Liang et al. [49] studied non-cooperative, centralized and classic DEA approaches for solving two-stage DEA problems. Chen et al. [10] tried to improve the constraint of Kao and Hwang [34] using the return to scales concept. Kao and Hwang [34] considered the overall efficiency of a DMU as a product of the efficiencies of its sub-DMUs. Their method was only applicable to constant returns to scale situations. Chen et al. [10] proposed an approach for decomposing the overall efficiency of a DMU using the weighted sum of the efficiencies of the individual stages. Such decomposition can be applied under both constant returns to scale and variable returns to scale assumptions.

Wang and Chin [61] showed that the weighted harmonic mean of the efficiencies of two individual stages can also be applied in the two-stage models. Cook et al. [14] classified the solution procedures of the two-stage DEA models into four categories: the standard DEA approach, efficiency decomposition, network DEA, and game theoretic and established relations among various approaches.

2.4 Fuzzy DEA literature

In the conventional DEA, all the data assumes the form of specific numerical values. However, the observed values of the input and output data in real-life problems are often imprecise or vague. Fuzzy logic and fuzzy sets can represent ambiguous, uncertain or imprecise information in DEA by formalizing inaccuracy in decision-making. Fuzzy sets theory is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments [66, 69]. Numerous DEA models have been developed in the fuzzy environments. Hougaard [31] extended scores of technical efficiency used in DEA to fuzzy intervals and showed how the fuzzy scores allow the decision maker to use scores of technical efficiency in conjunction with other sources of performance information such as expert opinions. Kao and Liu [35] developed a procedure to measure the efficiencies of the DMUs with fuzzy observations. Their basic idea was based on transforming a fuzzy DEA model to a family of conventional crisp DEA models by applying the α-cut approach. Guo and Tanaka [28] proposed a fuzzy DEA model to deal with the efficiency evaluation problem with fuzzy input and output data. Entani et al. [20] proposed a DEA model with an interval efficiency consisting of the efficiencies obtained from the pessimistic and the optimistic viewpoints. They also developed this approach for fuzzy input and output data by using α-level sets. Kao and Liu [36] presented a method to rank the fuzzy efficiency scores without knowing the exact form of the membership functions. They provided a model which had no requirement of the membership functions consideration. Lertworasirikul et al. [45] developed DEA models using imprecise data represented by fuzzy sets. They also showed that fuzzy DEA models take the form of fuzzy linear programming which typically was solved with the aid of some methods to rank fuzzy sets. León et al. [47] developed some fuzzy versions of the BCC model by using some ranking methods based on the comparison of α-cuts. Zhu [68] developed a nice review of imprecise DEA.

Jahanshahloo et al.[33] proposed a methodology for assessing, ranking and imposing weight restrictions in the DEA problems with fuzzy input–output. They defined a fuzzy comparison of fuzzy numbers and a slack-based model in DEA was extended in a fuzzy situation. They showed that their model was convenient for using weight restrictions. They also compared the results of their proposed model with the results of Guo and Tanaka [28]. Karsak [38] introduced a DEA model that could take into account crisp, ordinal, and fuzzy data for evaluating flexible manufacturing systems. Zerafat Angiz et al. [67] proposed a model to evaluate DMUs under uncertainty using Fuzzy DEA and to include α-level to the model under a fuzzy environment. Hatami-Marbini et al. [30] have presented a comprehensive review of the FDEA methods in the literature. They proposed a classification scheme with four primary categories, namely, the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach.

Recently, Kao and Liu [37] proposed a new method for formulating two-stage fuzzy DEA problems. They used the DEA model proposed by Kao and Hwang [34] and extended it to a two-stage DEA model. They utilized two-level optimization modeling to overcome the problem of intermediate measures. Although their approach was useful in formulating DEA models with general left–right (L–R) fuzzy numbers, it had several drawbacks. First, their proposed method served non-linear mathematical models for the lower bound of the efficiency calculation which could imply the local optimum solutions in practice. Second, the proposed approach served α-cut dependent models which could be solved for different α-cuts. The latter drawback results in other problems in practice. The volume of a full fuzzy DEA analysis extremely increases in real-life problems. There are no known rules for determining the best step-size for the α-cut values. The step-size of an α-cut should be determined in a case-oriented fashion, depending on the problem. There was a probability to achieve different interval efficiency scores for a DMU using different α-cut values. The ranking of different interval efficiency scores was not a major issue. Third, they proposed a non-linear program for the lower bound of efficiency calculation and the procedure for finding the relative efficiencies of the sub-DMUs was not straightforward. Since the decision variables could not be achieved directly, they referred to the optimality theorems of linear programming in which the reduced costs of the slack variables of dual model were equal to the values of the decision variables in the primal model. Fourth, although they proposed a linear program for the upper bound of the efficiency calculation, they did not address the optimization problems for the sub-DMUs. They calculated the efficiency scores of the sub-DMUs using the optimum values of the decision variables in the main DMU model and the classic definition of relative efficiency. Since it is possible to have multiple optimum solutions for different values of the decision variables, different combinations of efficiency scores of the sub-DMUs may result in a single efficiency score for the main DMU. In summary, they did not suggest a procedure for finding the final ranking of the DMUs and their approach was not capable of testing the uniqueness of the optimal values of the decision variables. In this paper, we propose a new fuzzy NDEA model to address these drawbacks in the literature.

2.5 Supply chain applications of DEA

The DEA models used in supply chain studies can be grouped into deterministic and uncertain categories as follows:

2.5.1 Deterministic methods

Liang et al. [48] showed that a supply chain can be deemed as efficient while its members may be inefficient in DEA-terms. Liang et al. [48] developed several DEA-based approaches for characterizing and measuring supply chain efficiency when intermediate measures were incorporated into the performance evaluation. The models were illustrated in a seller-buyer supply chain context, when the relationship between the seller and buyer was treated first as one of leader-follower, and second as one that was cooperative. Dong and Zhi-Ping [17] used a DEA-based approach to survey the performance of a reverse logistic in a supply chain integration project. Wong and Wong [62] developed the technical efficiency and the cost efficiency model. Their models were further enhanced with scenario analysis to help supply chain managers in resource planning decisions. Chen [6] developed a DEA model to address some important issues concerning the evaluation and design of supply chain operations. He focused on two main topics: evaluation of operational performance of processes with an interrelated internal structure (e.g., operations in a supply chain) in a dynamic setting, and system design under risks and uncertainty.

Saranga and Moser [55] developed a performance measurement framework for purchasing and supply management using the classical and two-stage value chain DEA models. Their models made use of multiple measures at various stages and provided a single efficiency measure that estimated the all-round performance of a purchasing and supply management function and its contribution to the long term corporate performance. Halkos et al. [29] surveyed and classified supply chain DEA models which investigated the internal structures of a DMU. Halkos et al. [29] analyzed the two-stage and NDEA models. They also studied some variations of the models such as models with only intermediate measures between the first and the second stage and models with exogenous inputs in the second stage. Amirteimoori and Khoshandam [2] developed a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations. They proposed an additive efficiency decomposition for suppliers and manufacturers in supply chain operations.

Chen and Yan [7] proposed an alternative NDEA model to embody the internal structure of a supply chain performance evaluation. They took the perspective of organization mechanism to deal with the complex interactions in the supply chain. Three different NDEA models were introduced under the concept of centralized, decentralized and mixed organization mechanisms. Efficiency analysis including the relationship between supply chain and divisions, and the relationship among the three different organization mechanisms were discussed. They also investigated the internal resource waste in supply chains. Mishra [51] proposed a DEA-based approach to measure the performance of pharmacological supply chain in India. He used a single-stage constant return to scale model to assess the performance of pharmacological supply chains.

2.5.2 Uncertain methods

Xu et al. [63] studied the supply chain performance evaluation of the Chinese furniture manufacture industry. They identified the main uncertainty factors affecting the evaluation process, and then modeled and analyzed those using rough DEA models. They created a rough DEA model by integrating the classical DEA and rough set theory. Their proposed rough DEA method was discussed and employed to evaluate the supply chain network operation efficiency of the furniture manufacturing industry. Abtahi and Khalili-Damghani [1] proposed a mathematical formulation for measuring the performance of agility in supply chains using single-stage fuzzy DEA. Khalili-Damghani et al. [39] applied the proposed formulation of Abtahi and Khalili-Damghani [1] to measure the efficiency of agility in supply chains and used a simulation-based approach to rank the interval efficiency scores of Abtahi and Khalili-Damghani [1]. Khalili-Damghani and Taghavifard [40] proposed a fuzzy two-stage DEA approach for performance measurement in supply chain. They used linguistic terms parameterized by fuzzy sets in order to model qualitative and vague criteria in their proposed fuzzy two-stage approach. The proposed model resulted in computational savings for a full DEA analysis. Finally, they applied the proposed fuzzy two-stage DEA approach for agility performance measurement in supply chains.

Khalili-Damghani and Taghavifard [41] tried to elaborate the proposed approach by Khalili-Damghani and Taghavifard [40]. They proposed a three-stage fuzzy DEA approach to measure the performance of a serial process including just-in-time practices, agility indices, and the overall goals in a supply chain. They determined the most frequent practice for just-in-time and agility in supply chains through a conceptual framework and applied the proposed approach to measure the performance of just-in-time, agility, and the overall goals in a supply chain. Khalili-Damghani et al. [42] considered ordinal data in a two-stage DEA approach. They modeled the ordinal Likert-based data in a new two-stage DEA approach for agility performance in supply chain and illustrated the efficacy of their approach in a real-life supply chain.

Recently, Khalili-Damghani and Taghavifard [43] performed sensitivity and stability analysis in two-stage DEA models with fuzzy data. They proposed several models which could calculate the stability radius for a DMU. The stability radius represents the region in which an efficient DMU will not alter from efficient to inefficient status or vice versa. Their proposed analysis had an essential role for fuzzy two-stage DEA approaches in which inputs and outputs of the DMUs and the sub-DMUs presented considerable variations and uncertainties. They presented the applicability of their approach using illustrative instances.

3 Proposed network model of agility in supply chains

The Supply Chain Operations Reference (SCOR) model is a management tool designed to describe business activities associated with all phases of satisfying a customer's demand in a supply chain. The model is based on three major pillars: process modeling, performance measurements, and best practices. In this section, we present the conceptual framework of the SCOR model. The criteria applied in this conceptual framework have been discussed in several studies [11, 23–25, 32, 50]. The following criteria were considered most important in measuring the performance of agility in the fresh food and dairy supply chains. A brief description of each criterion is presented as follows for the sake of clarity for the readers.

3.1 Providers of agility (potential agility)

Collaborative Relationship: Strategic relationship with customers, lasting relationship with suppliers, and close relationship with suppliers.

Process Integration: Concurrent execution of activities and enterprise integration.

Information Integration: Information accessible to employees/suppliers/customers.

Customer/Market Sensitivity: New product introduction, customer-driven innovations, and response to market changes.

Figure 1 depicts a schematic view of the aggregation of the capabilities of agility and core processes in the SCOR model. The capabilities of agility are considered in sourcing, making, and delivery processes of the SCOR model.

3.2 Capabilities of agility (Emergent agility)

Flexibility: Product volume flexibility, product model/configuration flexibility, organization and organizational issues, flexibility, and people flexibility.

Quickness: Develop new products quickly for the market, products and services delivery quickness and timeliness, and fast operation time.

Responsiveness: Sensing, perceiving and anticipating changes, immediate reaction to changes by incorporating them into the system, and recovery from change.

Competency: Developing business practices difficult to copy. Samples of such practices are strategic actions, appropriate technology (hard and soft), sufficient technological ability, product/services quality, cost effectiveness, and a high rate of new product development.

Figure 2 depicts a schematic view of the aggregation of the providers of agility and core processes of the SCOR model. Providers of agility are considered in sourcing, making, and delivery processes of the SCOR model.

3.3 Supply chain goals

Cost: Providing products and services with a competitive price by utilizing efficient cost management strategies.

Time: Production and technology preparation time, period of manufacturing, speed of products design, and short development cycle time.

Quality: Quality over product life, first time right decision, and products and services with high information and value-added contents.

Service Level: Customer satisfaction, employee satisfaction, and customer enrichment.

The complicated interactions between the aforementioned concepts has been widely studied in the literature and presented in the conceptual model shown in Fig. 3.

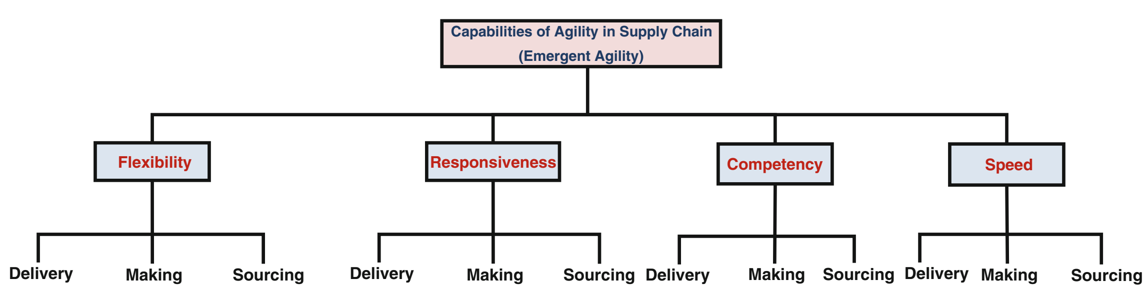

Fig. 1 Aggregation of the capabilities of agility and core processes of the SCOR model

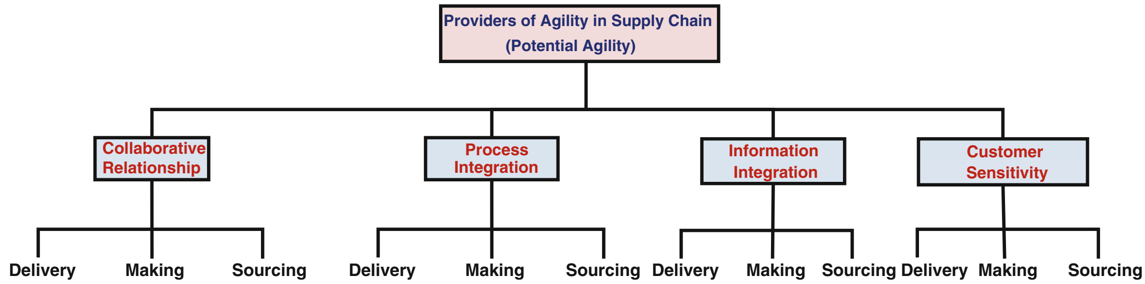

Fig. 2 Aggregation of the providers of agility and core processes of the SCOR model

Neither implementing the providers of the agility index nor the achievement of the agility capabilities are trivial jobs. These capabilities usually require a heavy investment. Therefore, the transformational processes contributing to the final goals in a supply chain deserve more consideration. The importance of the providers of agility, the capabilities of agility, and the goals of supply chain in sourcing, making, and delivery processes motivated us to propose the new fuzzy NDEA model described in the next section.

4 Proposed fuzzy NDEA model

We develop a fuzzy NDEA model based on the model proposed by Kao and Hwang [34] to measure the performance of agility in a supply chain. A schematic view of the DMU and its inputs, intermediates, and outputs are presented in Fig. 4. This complex network DMU is associated with the proposed conceptual model presented in Fig. 3. It is clear that the network DMU presented in Fig. 4 involves a series of complicated internal transactions between different sub-processes. It contains three serial sub-processes which are associated with sourcing, making, and delivery in the supply chain. Moreover, each sub-process has two sequential levels: providers of agility, and capabilities of agility. The outputs of the sub-processes are inputs to the final sub-process (sub-process 7) and the final outputs of the main DMU.

4.1 Crisp NDEA model

Let us consider the following assumptions and extend the usual DEA modeling notations to the proposed network format. In the sourcing process, each DMU$_j$ (j=1,2,…,n) consumes m_1 inputs $x^1_{ij}(i=1,2,\ldots,m_1)$ (i=1,2,…, m_1) in sub-process 1 to produce L_1 outputs $P^1_{lj}(l=1,2,\ldots,L_1)$. In

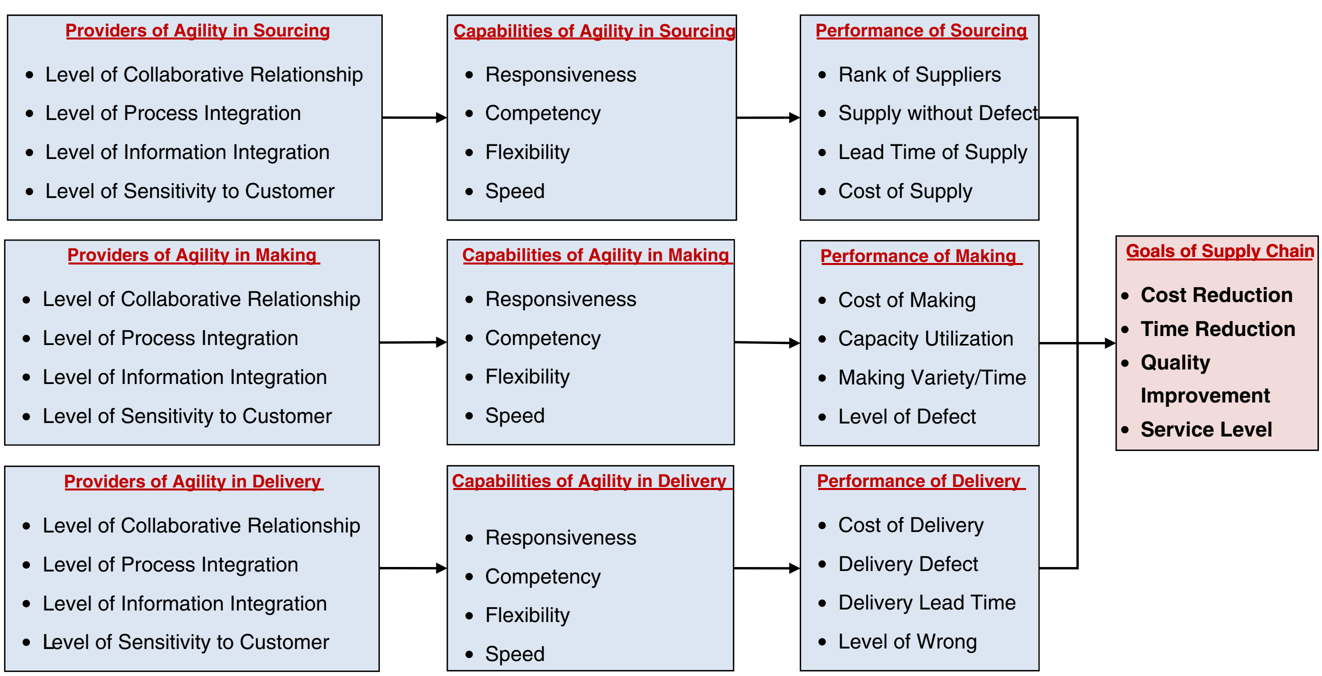

Fig. 3 Conceptual model of the providers of agility, capabilities of agility, performance goals, and the SCOR processes in the supply chain

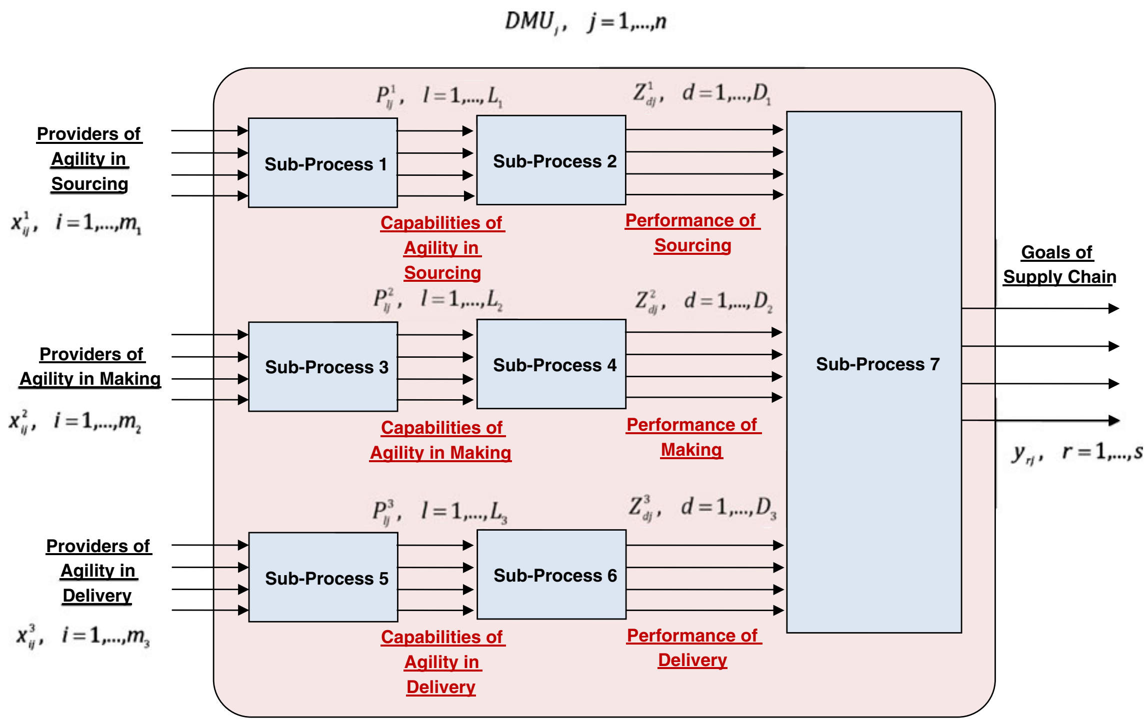

Fig. 4 Network DEA model of providers of agility, capabilities of agility, performance goals, and the SCOR processes in the supply chain

the making process, each DMU$_j$ (j=1,2,…,n) consumes m_2 inputs $x_{ij}^2(i=1,2,\ldots,m_2)$ in sub-process 3 to produce L_2 outputs $P_{lj}^2(l=1,2,\ldots,L_2)$. In the delivery process, each DMU$_j$ (j=1,2,…,n) consumes m_3 inputs $x_{ij}^3(i=1,2,\ldots,m_3)$ in sub-process 5 to produce L_3 outputs $P_{lj}^3(l=1,2,\ldots,L_3)$.

All the outputs of sub-processes 1, 3, and 5 (i.e., $P_{lj}^1\times(l=1,2,\ldots,L_1)$, $P_{lj}^2(l=1,2,\ldots,L_2)$, and $P_{lj}^3\times(l=1,2,\ldots,L_3)$) are treated as the first layer of the intermediate measures as well as the inputs of sub-processes 2, 4, and 6, respectively. These intermediate measures produce D_1, D_2, and D_3 outputs of $Z_{dj}^1(d=1,2,\ldots,D_1)$, $Z_{dj}^2\times(\mathrm{d}=1,2,\ldots,\mathrm{D}_2)$, and $Z_{dj}^3(d=1,2,\ldots,D_3)$ as outputs of sub-processes 2, 4, and 6. The outputs of sub-processes 2, 4, and 6 are the second intermediate measures used as the inputs in the next stage to produce s outputs y_{rj} (r=1,2,.., s) for the sub-process 7. For a given DMU$_\mathrm{j}$ with the aforementioned internal structure, the $e_j, e_j^1, e_j^2, e_j^3, e_j^4, e_j^5, e_j^6$, and e_j^7 are reserved for the overall efficiency score, and the efficiency score of its sub-processes, respectively. All the indices, the parameters, and the decision variables used in the proposed NDEA model are summarized in Table 3.

Using the constant return to scale input-oriented DEA model of Charnes et al. [4], considering equal multipliers for all intermediate measures of the associated model for each sub-process, and accomplishing proper variable changes, the following linear program is proposed to measure the overall efficiency of the DMU, and the sub-DMUs 1 through 7.

$$
\begin{aligned}
&\mathrm{e_o} = \mathrm{Max} \sum_{r=1}^{s} u_r y_{ro}^7\\
&s.t.\\
&\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k x^k{}_{io} = 1,\\
&\sum_{r=1}^{s} u_r y^7{}_{rj} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k x^k{}_{ij}\right) \le 0, \quad j=1,2,\ldots,n,\\
&\sum_{l=1}^{L_k} q_l^k p^k{}_{lj} - \sum_{i=1}^{m_k} v_i^k x^k{}_{ij} \le 0, \quad j=1,2,\ldots,n; \quad k=1,2,3,\\
&\sum_{d=1}^{D_k} w_d^k z^k{}_{dj} - \sum_{l=1}^{L_k} q_l^k p^k{}_{lj} \le 0, \quad j=1,2,\ldots,n; \quad k=1,2,3,\\
&\sum_{r=1}^{s} u_r y^7{}_{rj} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{dj}\right) \le 0, \quad j=1,2,\ldots,n\\
&q_l^k \ge \varepsilon, \quad l=1,2,\ldots,L_k; \quad k=1,2,3,\\
&w_d^k \ge \varepsilon, \quad d=1,2,\ldots,D_k; \quad k=1,2,3,\\
&v_i^k \ge \varepsilon, \quad i=1,2,\ldots,m_k; \quad k=1,2,3,\\
&u_r \ge \varepsilon, \quad r=1,2,\ldots,s
\end{aligned}
\tag{1}
$$

Model (1) presents the overall efficiency of the proposed network DMU depicted in Fig. 4 considering the abovementioned internal sub-processes.

Theorem #1 *Model (1) is feasible and bounded. Its optimal objective function is also equal to unit.*

Table 3 The notations used in the proposed network DEA

Indices		
j	The number of DMUs	j=1,2,…,n
m_1	The number of inputs of sub-process 1	i=1,2,…,m_1
m_2	The number of inputs of sub-process 3	i=1,2,…,m_2
m_3	The number of inputs of sub-process 5	i=1,2,…,m_3
L_1	The number of outputs of sub-process 1; the number of inputs of sub-process 2	l=1,2…,L_1
L_2	The number of outputs of sub-process 3; the number of inputs of sub-process 4	l=1,2,…,L_2
L_3	The number of outputs of sub-process 5; the number of inputs of sub-process 6	l=1,2,…,L_3
D_1	The number of outputs of sub-process 2; some parts of inputs of sub-process 7	d=1,2,…,D_1
D_2	The number of outputs of sub-process 4; some parts of inputs of sub-process 7	d=1,2,…,D_2
D_3	The number of outputs of sub-process 6; some parts of inputs of sub-process 7	d=1,2,…,D_3
r	The number of outputs of sub-process 7	r=1,2,…,s
Parameters		
X_{ij}^1	The ith input of sub-process 1 of DMU$_j$	
X_{ij}^2	The ith input of sub-process 3 of DMU$_j$	
X_{ij}^3	The ith input of sub-process 5 of DMU$_j$	
P_{lj}^1	The lth output of sub-process 1 of DMU$_j$; the lth input of sub-process 2 of DMU$_j$	
P_{lj}^2	The lth output of sub-process 3 of DMU$_j$; the lth input of sub-process 4 of DMU$_j$	
P_{lj}^3	The lth output of sub-process 5 of DMU$_j$; the lth input of sub-process 6 of DMU$_j$	
Z_{dj}^1	The dth output of sub-process 2 of DMU$_j$; the dth input of sub-process 7 of DMU$_j$	
Z_{dj}^2	The dth output of sub-process 4 of DMU$_j$; the dth input of sub-process 7 of DMU$_j$	
Z_{dj}^3	The dth output of sub-process 6 of DMU$_j$; the dth input of sub-process 7 of DMU$_j$	
Y_{rj}	The rth output of sub-process 7 of DMU$_j$	
Decision variables		
v_i^1	The multiplier of the ith input of sub-process 1	i=1,2,…,m_1
v_i^2	The multiplier of the ith input of sub-process 3	i=1,2,…,m_2
v_i^3	The multiplier of the ith input of sub-process 5	i=1,2,…,m_3
q_l^1	The multiplier of the lth output of sub-process 1; the multiplier of the lth input of sub-process 2	l=1,2,…,L_1
q_l^2	The multiplier of the lth output of sub-process 3; the multiplier of the lth input of sub-process 4	l=1,2,…,L_2
q_l^3	The multiplier of the lth output of sub-process 5; the multiplier of the lth input of sub-process 6	l=1,2,…,L_3
w_d^1	The multiplier of the dth output of sub-process 2; the multiplier of the dth input of sub-process 7	d=1,2,…,D_1
w_d^2	The multiplier of the dth output of sub-process 4; the multiplier of the dth input of sub-process 7	d=1,2,…,D_2
w_d^3	The multiplier of the dth output of sub-process 6; the multiplier of the dth input of sub-process 7	d=1,2,…,D_3
u_r	The multiplier of the rth output of sub-process 7	r=1,2,…,s
e_j	The overall efficiency score of DMU$_j$	j=1,2,…,n
e_j^k	The efficiency score of sub-process k of DMU$_j$	j=1,2,…,n; k=1,2,…,7.

Proof The dual form of model (1) can be written as follows:

$$
\begin{aligned}
&\mathrm{Min}\,\theta\\
&S.t.\\
&\sum_{j=1}^{n}\lambda_j^1 x^k{}_{ij}+\sum_{j=1}^{n}\lambda_j^{(k+1)}x^k{}_{ij}\le\theta x^k{}_{io},\quad i=1,2,\ldots,m_k;k=1,2,3\\
&\sum_{j=1}^{n}\lambda_j^{(k+1)}p^k{}_{lj}-\sum_{j=1}^{n}\lambda_j^{(k+4)}p^k{}_{lj}\ge 0,\quad l=1,2,\ldots,L_k;k=1,2,3\\
&\sum_{j=1}^{n}\lambda_j^{(k+4)}z^k{}_{dj}-\sum_{j=1}^{n}\lambda_j^{8}z^k{}_{dj}\ge 0,\quad d=1,2,\ldots,D_k;k=1,2,3\\
&\sum_{j=1}^{n}\lambda_j^1 y^7{}_{rj}+\sum_{j=1}^{n}\lambda_j^8 y^7{}_{rj}\ge y^7{}_{ro},\quad r=1,2,\ldots,s,\\
&\lambda_j^k\ge 0,\quad k=1,2,\ldots,8;\ j=1,2,\ldots,n,\\
&\theta,\quad free.
\end{aligned}
$$

Let θ=1, and $\lambda_j^k=\begin{cases}\frac{1}{2} & for\ j=o\\ 0 & for\ j\ne o\end{cases}, k=1,2,\ldots,8;\quad j=1,2,\ldots,n.$ A solution to the dual model always exists. Since the dual model is feasible, the primal is feasible as well. Since the objective function of the dual model is, Min θ the optimal value of objective function, called θ*, is less than or equal to θ. This solution implies $\theta^*\le\theta\le 1$. The optimal solution, θ*, yields an efficiency score for a particular DMU. By the virtue of the dual theorem of linear programming, we have $\theta^*=e_o^*$. Hence, the optimal solution of the primal model (i.e., model (1)) is also bounded and $\theta^*=e_o^*\le 1$. This completes the proof.

Theorem #2 *The optimal solution of the dual model satisfies the following conditions:*

$$\theta^* = 1, \lambda_j^{*k} = \begin{cases} 0, & for\ j \neq o \\ 1 & for\ j = o \end{cases}, k = 1,2,\ldots,8; j = 1,2,\ldots,n, \text{and},$$
$$S_{i1}^{*-} = 0, \quad i = 1,2,\ldots,m_1; S_{i2}^{*-} = 0, \quad i = 1,2,\ldots,m_2; S_{i3}^{*-} = 0, \quad i = 1,2,\ldots,m_3; S_{l1}^{*+} = 0, \quad l = 1,2,\ldots,L_1; S_{l2}^{*+} = 0, \quad l = 1,2,\ldots,L_2;$$
$$S_{l3}^{*+} = 0, \quad l = 1,2,\ldots,L_3; S_{d1}^{*+} = 0, \quad d = 1,2,\ldots,D_1; S_{d2}^{*+} = 0, \quad d = 1,2,\ldots,D_2; S_{d3}^{*+} = 0, \quad d = 1,2,\ldots,D_3; S_r^{*+} = 0, \quad r = 1,2,\ldots,S$$

where $S_{i1}^{*-}, S_{i2}^{*-}, S_{i3}^{*-}, S_{l1}^{*+}, S_{l2}^{*+}, S_{l3}^{*+}, S_{d1}^{*+}, S_{d2}^{*+}, S_{d3}^{*+}, S_r^{*+}$ are reserved as the slack and the surplus variables of the dual model. In other words, in a strong optimal solution, the objective function is equal to unity and all the slack variables are equal to zero.

Proof Suppose that there is an optimal solution in which the objective value of the dual model is not equal to unity. According to Theorem #1, the optimal value of the objective function should be less than unity (i.e., $\theta^* < 1$). The following constraints can be derived according to the first set of constraints in the dual model:

$$\sum_{j=1}^{n} \lambda_j^1 x^1{}_{ij} + \sum_{j=1}^{n} \lambda_j^2 x^1{}_{ij} \leq \theta^* x^1{}_{io}, \quad i = 1,2,\ldots,m_1,$$
$$x^1{}_{io} + x^1{}_{io} + S_{i1}^{*-} = \theta^* x^1{}_{io}, \quad i = 1,2,\ldots,m_1$$
$$S_{i1}^{*-} = \theta^* x^1{}_{io} - x^1{}_{io} - x^1{}_{io}, \quad i = 1,2,\ldots,m_1$$
$$S_{i1}^{*-} = x^1{}_{io}(\theta^* - 1) - x^1{}_{io}, \quad i = 1,2,\ldots,m_1$$

Since the optimal objective function of the dual model is assumed to be less than unity ($\theta^* < 1$), and considering the fact that the values of $x^1{}_{io}, \quad i = 1,2,\ldots,m_1$ are positive, the last equation implies that all the slack variables of the first set of constraints must be less than zero ($S_{i1}^{*-} < 0, \quad i = 1,2,\ldots, m_1$) which is a contradiction. This completes the proof.

Corollary 1 *If there are no multiple optimal solutions in the dual model, in the optimal solution of the dual model the slack variables are equal to zero. This solution (known as "strong efficient solution") satisfies the following conditions:*

$\theta^* = 1$, and,

$$S_{i1}^{*-} = 0, i = 1,2,\ldots,m_1; \ S_{i2}^{*-} = 0, i = 1,2,\ldots,m_2; \ S_{i3}^{*-} = 0,$$
$$i = 1,2,\ldots,m_3; S_{l1}^{*+} = 0, \ l = 1,2,\ldots,L_1; S_{l2}^{*+} = 0,$$
$$l = 1,2,\ldots,L_2; S_{l3}^{*+} = 0, \quad l = 1,2,\ldots,L_3; S_{d1}^{*+} = 0,$$
$$d = 1,2,\ldots,D_1; S_{d2}^{*+} = 0, \ d = 1,2,\ldots,D_2;$$
$$S_{d3}^{*+} = 0, \quad d = 1,2,\ldots,D_3; \quad S_r^{*+} = 0, \ r = 1,2,\ldots,S$$

Corollary 2 *Complementary conditions hold between the optimal solutions of model (1) and its dual model. Assume that the optimal solutions of these models are as follows:*

Optimal solution of model (1) : $v_i^{1*}, v_i^{2*}, v_i^{3*}, q_l^{1*}, q_l^{1*}, q_l^{1*}, w_d^{1*}, w_d^{1*}, w_d^{1*}, u_r^*$

Optimal Solution of the dual model : $\lambda_j^{*k}, S_{i1}^{*-}, S_{i2}^{*-}, S_{i3}^{*-}, S_{l1}^{*+}, S_{l2}^{*+}, S_{l3}^{*+}, S_{d1}^{*+}, S_{d2}^{*+}, S_{d3}^{*+}, S_r^{*+}$

Therefore, the following equations are satisfied:

$$v_i^{t*} \times S_{it}^{*-} = 0, \ t = 1,2,3, \ q_l^{t*} \times S_{lt}^{*+} = 0,$$
$$t = 1,2,3, w_d^{t*} \times S_{dt}^{*+} = 0, \quad t = 1,2,3, \text{ and } \mathrm{u_r^*} \times \mathrm{S_r^{*+}} = 0.$$

This means that if any main decision variable in model (1) is positive, then, the corresponding slack or surplus variable in the dual model must be zero, and conversely. It is also possible for both of them to be zero simultaneously.

If the optimal solution of model (1) can be uniquely achieved, the efficiency of all the sub-processes can be calculated easily using the basic definition of relative efficiency (i.e., division of the weighted output by the weighted inputs). It is notable that the sequence of the sub-processes in Fig. 4 is assumed to be based on the SCOR model as sourcing, making, and delivery. The priorities of the agility levels are based on their cause and effect relations (i.e., the providers of agility and also the capabilities of agility).

4.2 Fuzzy NDEA model

In most real-life cases, the decision makers often use qualitative indices with ambiguities to measure the inputs and/or the outputs in each stage of the NDEA model. In those cases, linguistic terms parameterized through the fuzzy numbers can be supplied to model the vagueness of the qualitative factors.

The trapezoidal fuzzy numbers (TrFNs), which are commonly used in the real-life problems, are used in the development of the new fuzzy NDEA model.

Let us consider the following definitions and notations suggested by Zimmermann [69] in a fuzzy environment. Let X be the universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy set $\widetilde{A}$ of X is a set of order pairs $\left\{\left(x_1, \mu_{\widetilde{A}}(x_1)\right), \left(x_2, \mu_{\widetilde{A}}(x_2)\right), \ldots \left(x_n, \mu_{\widetilde{A}}(x_n)\right)\right\}$ where $\mu_{\widetilde{A}} : X \to [0,1]$ is the membership function of $\widetilde{A}$, and $\mu_{\widetilde{A}}(x_i)$ stands for the membership degree of x_i in $\widetilde{A}$.

Definition 2.1 The α-cut. $\widetilde{A}_a$, and the strong α-cut $\widetilde{A}_{\alpha+}$ of the fuzzy set $\widetilde{A}$ in the universe of discourse X is defined by $\widetilde{A}_\alpha = \left\{x_i : \mu_{\widetilde{A}}(x_i) \geq \alpha, x_i \in X\right\}$ where $\alpha \in [0,1]$ and, $\widetilde{A}_{\alpha+} = \{x_i : \mu_{\widetilde{A}}(x_i) > \alpha, x_i \in X\}$ where $\alpha \in [0,1]$, respectively.

Definition 2.2 A TrFN can be defined as $\widetilde{x} = (x^1, x^2, x^3, x^4)$, where the membership function $\mu_{\widetilde{m}}$ of $\widetilde{m}$ is given by the following equation:

$$\mu(x) = \begin{cases} \frac{x - x^1}{x^2 - x^1}\,(x^1 \leq x \leq x^2) \\ 1 \quad (x^2 \leq x \leq x^3) \\ \frac{x^4 - x}{x^4 - x^3}\,(x^3 \leq x \leq x^4) \end{cases} \tag{2}$$

where $[x^2, x^3]$ is called a mode interval of $\widetilde{x}$, and x^1 and x^2 are the lower and the upper limits of $\widetilde{x}$, respectively. As stated earlier in Section 4.2, the TrFNs are used in the development of the new fuzzy NDEA model.

4.2.1 Fuzzy NDEA model formulation

In this sub-section, we propose a new fuzzy NDEA model for measuring the efficiency and the sub-efficiencies of the proposed network structure of Fig. 4 based on the existing models of Despotis and Smirlis [16], Kao and Hwang [34], and Kao and Liu [37]. Consider TrFNs in the L–R spread format as the inputs, the intermediate measures, and the outputs of n DMUs with network structure.

In the sourcing process, each DMU$_j$ (j=1,2,…,n) consumes m_1 fuzzy inputs $\widetilde{x}^1_{ij}, i = 1, 2, \ldots, m_1$ in the sub-process 1 to produce L_1 fuzzy outputs $\widetilde{P}^1_{lj}, l = 1, 2, \ldots, L_1$. In the making process, each DMU$_j$ (j=1,2,…,n) consumes m_2 fuzzy inputs $\widetilde{x}^2_{ij}, i = 1, 2, \ldots, m_2$ in sub-process 3 to produce L_2 fuzzy outputs $\widetilde{P}^2_{lj}, l = 1, 2, \ldots, L_2$. In the delivery process, each DMU$_j$ (j=1,2,…,n) consumes m_3 fuzzy inputs $\widetilde{x}^3_{ij}, i = 1, 2, \ldots, m_3$ in sub-process 5 to produce L_3 fuzzy outputs $\widetilde{P}^3_{lj}$, $l = 1, 2, \ldots, L_3$. All of the outputs of the sub-processes 1, 3, and 5 are treated as *the* first intermediate measures as well as the fuzzy inputs of sub-processes 2, 4, and 6, respectively. These intermediate measures produce $D_1\left(\widetilde{z}^1_{dj}, d = 1, 2, \ldots, d_1\right)$, $D_2\left(\widetilde{z}^2_{dj}, d = 1, 2, \ldots d_2\right)$, and $D_3\left(\widetilde{z}^3_{\mathrm{dj}}, d = 1, 2, \ldots d_3\right)$, as the outputs of the sub-processes 2, 4, and 6, respectively. All the fuzzy outputs of the sub-processes 2, 4, and 6 are assumed as the second intermediate measures to produce s fuzzy outputs $\widetilde{y}_{rj}, r = 1, 2, \ldots, s$ of sub-process 7. For a given DMU$_j$ with the aforementioned internal structure, the e^U_j, $e^{U_i}_j, i = 1, 2, \ldots, 7$ are reserved for the upper bound of the overall efficiency score, and the efficiency score of the sub-processes, respectively. The e^L_j, $e^{L_i}_j, i = 1, 2, \ldots, 7$ are reserved for the lower bound of the overall efficiency score, and the efficiency score of the sub-processes, respectively.

For an arbitrary α-cut, the lower and the upper bound of each input, the intermediate measure, and the output membership function are calculated by Eqs. (3)–(10).

$$\left(x^{kL}_{ij}\right)_{\alpha_i} = x^{k1}_{ij} + \alpha_i\left(x^{k2}_{ij} - x^{k1}_{ij}\right), \quad \alpha_i \in [0,1], i = 1, \ldots, m; j = 1, \ldots, n; \quad k = 1, 2, 3. \tag{3}$$

$$\left(x^{kU}_{ij}\right)_{\alpha_i} = x^{k4}_{ij} - \alpha_i\left(x^{k4}_{ij} - x^{k3}_{ij}\right), \quad \alpha_i \in [0,1], i = 1, \ldots, m; j = 1, \ldots, n; \quad k = 1, 2, 3. \tag{4}$$

$$\left(p^{kL}_{lj}\right)_{\alpha_l} = p^{k1}_{lj} + \alpha_l\left(p^{k2}_{lj} - p^{k1}_{lj}\right), \quad \alpha_l \in [0,1], l = 1, \ldots, L; j = 1, \ldots, n; \quad k = 1, 2, 3. \tag{5}$$

$$\left(p_{lj}^{kU}\right)_{\alpha_l} = p_{lj}^{k4} - \alpha_l\left(p_{lj}^{k4} - p_{lj}^{k3}\right), \quad \alpha_l \in [0,1], l = 1,\ldots,L; j = 1,\ldots,n; \quad k = 1,2,3. \tag{6}$$

$$\left(z_{dj}^{kL}\right)_{\alpha_d} = z_{dj}^{k1} + \alpha_d\left(z_{dj}^{k2} - z_{dj}^{k1}\right), \quad \alpha_d \in [0,1], d = 1,\ldots,D; j = 1,\ldots,n; \quad k = 1,2,3. \tag{7}$$

$$\left(z_{rj}^{kU}\right)_{\alpha_d} = z_{dj}^{k4} - \alpha_d\left(z_{dj}^{k4} - z_{dj}^{k3}\right), \quad \alpha_d \in [0,1], d = 1,\ldots,D; j = 1,\ldots,n; \quad k = 1,2,3. \tag{8}$$

$$\left(y_{rj}^{L}\right)_{\alpha_r} = y_{rj}^{1} + \alpha_r\left(y_{rj}^{2} - y_{rj}^{1}\right), \quad \alpha_r \in [0,1], r = 1,\ldots,s; j = 1,\ldots,n. \tag{9}$$

$$\left(y_{rj}^{U}\right)_{\alpha_r} = y_{rj}^{4} - \alpha_r\left(y_{rj}^{4} - y_{rj}^{3}\right), \quad \alpha_r \in [0,1], r = 1,\ldots,s; j = 1,\ldots,n. \tag{10}$$

The upper $\left(e_o^U\right)$ and the lower $\left(e_o^L\right)$ bound of the overall efficiency values can be calculated using model (11), model (12), respectively. It should be noted that the optimum values of the intermediate measures of models (11) and (12) should be determined using the upstream optimization models. This will be achieved through multi-level optimization models.

$$\begin{aligned}
&\mathrm{e}_{\mathrm{o}}^{\mathrm{U}} = \mathrm{Max} \sum_{\mathrm{r}=1}^{\mathrm{s}} \mathrm{u_r}\left(\mathrm{y}_{\mathrm{ro}}^{\mathrm{U}}\right)_{\alpha_\mathrm{r}}\\
&S.t.\\
&\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kL}\right)_{\alpha_i} = 1\\
&\sum_{r=1}^{s} u_r\left(y_{rj}^{L}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{ij}^{kU}\right)_{\alpha_i}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o\\
&\sum_{r=1}^{s} u_r\left(y_{ro}^{U}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kL}\right)_{\alpha_i}\right) \le 0,\\
&\sum_{l=1}^{L_k} q_l^k p^k{}_{lj} - \sum_{i=1}^{m_k} v_i^k\left(x_{ij}^{kU}\right)_{\alpha_i} \le 0, \quad j = 1,2,\ldots,n, \quad j \ne o, \quad k = 1,2,3\\
&\sum_{l=1}^{L_k} q_l^k p^k{}_{lo} - \sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kL}\right)_{\alpha_i} \le 0, \quad k = 1,2,3\\
&\sum_{d=1}^{D_k} w_d^k z^k{}_{dj} - \sum_{l=1}^{L_k} q_l^k p^k{}_{lj} \le 0, \quad j = 1,2,\ldots,n, \quad k = 1,2,3\\
&\sum_{r=1}^{s} u_r\left(y_{rj}^{L}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{dj}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o\\
&\sum_{r=1}^{s} u_r\left(y_{ro}^{U}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{do}\right) \le 0\\
&q_l^k \ge \varepsilon, \quad l = 1,2,\ldots,L_k; \quad k = 1,2,3,\\
&w_d^k \ge \varepsilon, \quad d = 1,2,\ldots,D_k; \quad k = 1,2,3,\\
&v_i^k \ge \varepsilon, \quad i = 1,2,\ldots,m_k; \quad k = 1,2,3,\\
&u_r \ge \varepsilon, \quad r = 1,2,\ldots,s
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\mathrm{e}_{\mathrm{o}}^{\mathrm{L}} = \mathrm{Max} \sum_{\mathrm{r}=1}^{\mathrm{s}} \mathrm{u_r}\left(\mathrm{y}_{\mathrm{ro}}^{\mathrm{L}}\right)_{\alpha_\mathrm{r}}\\
&S.t.\\
&\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kU}\right)_{\alpha_i} = 1\\
&\sum_{r=1}^{s} u_r\left(y_{rj}^{U}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{ij}^{kL}\right)_{\alpha i}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o\\
&\sum_{r=1}^{s} u_r\left(y_{ro}^{L}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kU}\right)_{\alpha_i}\right) \le 0,\\
&\sum_{l=1}^{L_k} q_l^k p^k{}_{lj} - \sum_{i=1}^{m_k} v_i^k\left(x_{ij}^{kL}\right)_{\alpha_i} \le 0, \quad j = 1,2,\ldots,n, \quad j \ne o, \quad k = 1,2,3\\
&\sum_{l=1}^{L_k} q_l^k p^k{}_{lo} - \sum_{i=1}^{m_k} v_i^k\left(x_{io}^{kU}\right)_{\alpha_i} \le 0, \quad k = 1,2,3\\
&\sum_{d=1}^{D_k} w_d^k z^k{}_{dj} - \sum_{l=1}^{L_k} q_l^k p^k{}_{lj} \le 0, \quad j = 1,2,\ldots,n, \quad k = 1,2,3\\
&\sum_{r=1}^{s} u_r\left(y_{rj}^{U}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{dj}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o\\
&\sum_{r=1}^{s} u_r\left(y_{ro}^{L}\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{do}\right) \le 0\\
&q_l^k \ge \varepsilon, \quad l = 1,2,\ldots,L_k; \quad k = 1,2,3,\\
&w_d^k \ge \varepsilon, \quad d = 1,2,\ldots,D_k; \quad k = 1,2,3,\\
&v_i^k \ge \varepsilon, \quad i = 1,2,\ldots,m_k; \quad k = 1,2,3,\\
&u_r \ge \varepsilon, \quad r = 1,2,\ldots,s
\end{aligned} \tag{12}$$

The determination of the intermediate measures for model (11) and model (12) will result in multi-level optimization problems such as model (13) and model (14), respectively.

$$\underset{\left(z_{dj}^{kL}\right)_{\alpha_d}\le z^k{}_{dj}\le\left(z_{dj}^{kU}\right)_{d_l},k=1,2,3.}{Max}\left\{\underset{\left(p_{lj}^{kL}\right)_{\alpha_l}\le p^k{}_{lj}\le\left(p_{lj}^{kU}\right)_{\alpha_l},k=1,2,3.}{Max}\left\{\begin{array}{l}
\mathrm{e_o^U}=\mathrm{Max}\sum_{\mathrm{r}=1}^{\mathrm{s}}\mathrm{u_r}\left(\mathrm{y_{ro}^U}\right)_{\alpha_\mathrm{r}}\\
S.t.\\
\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kL}\right)_{\alpha_i}=1\\
\sum_{r=1}^{s}u_r\left(y_{rj}^L\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{ij}^{kU}\right)_{\alpha_i}\right)\le 0,\quad j=1,2,\ldots,n,j\ne o\\
\sum_{r=1}^{s}u_r\left(y_{ro}^U\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kL}\right)_{\alpha_i}\right)\le 0,\\
\sum_{l=1}^{L_k}q_l^kp^k{}_{lj}-\sum_{i=1}^{m_k}v_i^k\left(x_{ij}^{kU}\right)_{\alpha_i}\le 0,\quad j=1,2,\ldots,n,\quad j\ne o,\quad k=1,2,3\\
\sum_{l=1}^{L_k}q_l^kp^k{}_{lo}-\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kL}\right)_{\alpha_i}\le 0,\quad k=1,2,3\\
\sum_{d=1}^{D_k}w_d^kz^k{}_{dj}-\sum_{l=1}^{L_k}q_l^kp^k{}_{lj}\le 0,\quad j=1,2,\ldots,n,\quad k=1,2,3\\
\sum_{r=1}^{s}u_r\left(y_{rj}^L\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{d=1}^{D_k}w_d^kz^k{}_{dj}\right)\le 0,\quad j=1,2,\ldots,n,j\ne o\\
\sum_{r=1}^{s}u_r\left(y_{ro}^U\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{d=1}^{D_k}w_d^kz^k{}_{do}\right)\le 0\\
q_l^k\ge\varepsilon,\quad l=1,2,\ldots,L_k;\quad k=1,2,3,\\
w_d^k\ge\varepsilon,\quad d=1,2,\ldots,D_k;\quad k=1,2,3,\\
v_i^k\ge\varepsilon,\quad i=1,2,\ldots,m_k;\quad k=1,2,3,\\
u_r\ge\varepsilon,\quad r=1,2,\ldots,s
\end{array}\right.\right. \tag{13}$$

$$\underset{\left(z_{dj}^{kL}\right)_{\alpha_d}\le z^k{}_{dj}\le\left(z_{dj}^{kU}\right)_{d_l},k=1,2,3.}{Min}\left\{\underset{\left(p_{lj}^{kL}\right)_{\alpha_l}\le p^k{}_{lj}\le\left(p_{lj}^{kU}\right)_{\alpha_l},k=1,2,3.}{Min}\left\{\begin{array}{l}
\mathrm{e_o^L}=\mathrm{Max}\sum_{\mathrm{r}=1}^{\mathrm{s}}\mathrm{u_r}\left(\mathrm{y_{ro}^L}\right)_{\alpha_\mathrm{r}}\\
S.t.\\
\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kU}\right)_{\alpha_i}=1\\
\sum_{r=1}^{s}u_r\left(y_{rj}^U\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{ij}^{kL}\right)_{\alpha_i}\right)\le 0,\quad j=1,2,\ldots,n,j\ne o\\
\sum_{r=1}^{s}u_r\left(y_{ro}^L\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kU}\right)_{\alpha_i}\right)\le 0,\\
\sum_{l=1}^{L_k}q_l^kp^k{}_{lj}-\sum_{i=1}^{m_k}v_i^k\left(x_{ij}^{kL}\right)_{\alpha_i}\le 0,\quad j=1,2,\ldots,n,\quad j\ne o,\quad k=1,2,3\\
\sum_{l=1}^{L_k}q_l^kp^k{}_{lo}-\sum_{i=1}^{m_k}v_i^k\left(x_{io}^{kU}\right)_{\alpha_i}\le 0,\quad k=1,2,3\\
\sum_{d=1}^{D_k}w_d^kz^k{}_{dj}-\sum_{l=1}^{L_k}q_l^kp^k{}_{lj}\le 0,\quad j=1,2,\ldots,n,\quad k=1,2,3\\
\sum_{r=1}^{s}u_r\left(y_{rj}^U\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{d=1}^{D_k}w_d^kz^k{}_{dj}\right)\le 0,\quad j=1,2,\ldots,n,j\ne o\\
\sum_{r=1}^{s}u_r\left(y_{ro}^L\right)_{\alpha_r}-\left(\sum_{k=1}^{3}\sum_{d=1}^{D_k}w_d^kz^k{}_{do}\right)\le 0\\
q_l^k\ge\varepsilon,\quad l=1,2,\ldots,L_k;\quad k=1,2,3,\\
w_d^k\ge\varepsilon,\quad d=1,2,\ldots,D_k;\quad k=1,2,3,\\
v_i^k\ge\varepsilon,\quad i=1,2,\ldots,m_k;\quad k=1,2,3,\\
u_r\ge\varepsilon,\quad r=1,2,\ldots,s
\end{array}\right.\right. \tag{14}$$

Model (13) and model (14) contain seven optimization problems. These models cannot be solved in their current form. They should be transformed into single-level optimization problems. Since model (13) contains seven optimization models with a maximum objective function, it can be easily reduced to a single-level optimization problem such as model (15).

$$
\begin{aligned}
& e_o^U = \text{Max} \sum_{r=1}^{s} u_r \left(y_{ro}^U\right)_{\alpha_r} \\
& S.t. \\
& \sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k \left(x_{io}^{kL}\right)_{\alpha_i} = 1 \\
& \sum_{r=1}^{s} u_r \left(y_{rj}^L\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k \left(x_{ij}^{kU}\right)_{\alpha_i}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o \\
& \sum_{r=1}^{s} u_r \left(y_{ro}^U\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k \left(x_{io}^{kL}\right)_{\alpha_i}\right) \le 0, \\
& \sum_{l=1}^{L_k} q_l^k p^k{}_{lj} - \sum_{i=1}^{m_k} v_i^k \left(x_{ij}^{kU}\right)_{\alpha_i} \le 0, \quad j = 1,2,\ldots,n, j \ne o, \quad k = 1,2,3 \\
& \sum_{l=1}^{L_k} q_l^k p^k{}_{lo} - \sum_{i=1}^{m_k} v_i^k \left(x_{io}^{kL}\right)_{\alpha_i} \le 0, \quad k = 1,2,3 \\
& \sum_{d=1}^{D_k} w_d^k z^k{}_{dj} - \sum_{l=1}^{L_k} q_l^k p^k{}_{lj} \le 0, \quad j = 1,2,\ldots,n, \quad k = 1,2,3 \\
& \sum_{r=1}^{s} u_r \left(y_{rj}^L\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{dj}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o \\
& \sum_{r=1}^{s} u_r \left(y_{ro}^U\right)_{\alpha_r} - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} w_d^k z^k{}_{do}\right) \le 0 \\
& \left(p_{lj}^{kL}\right)_{\alpha_l} \le p^k{}_{lj} \le \left(p_{lj}^{kU}\right)_{\alpha_l}, k = 1,2,3, j = 1,2,\ldots,n; l = 1,2,\ldots,L_k \\
& \left(z_{dj}^{kL}\right)_{\alpha_d} \le z^k{}_{dj} \le \left(z_{dj}^{kU}\right)_{\alpha_d}, k = 1,2,3, j = 1,2,\ldots,n; d = 1,2,\ldots,D_k \\
& q_l^k \ge \varepsilon, \quad l = 1,2,\ldots,L_k; k = 1,2,3 \\
& w_d^k \ge \varepsilon, \quad d = 1,2,\ldots,D_k; k = 1,2,3 \\
& v_i^k \ge \varepsilon, \quad i = 1,2,\ldots,m_k; k = 1,2,3 \\
& u_r \ge \varepsilon, \quad r = 1,2,\ldots,s
\end{aligned} \tag{15}
$$

Model (15) is a non-linear optimization model that is dependent on the α-cut concept. By replacing relations (3)–(10) in model (15) and using the transformation set (16), model (17) is constructed as follows:.

$$
\begin{aligned}
& \overline{p}_{lj}{}^k = q_l^k p^k{}_{lj}, j = 1,2,\ldots,n; l = 1,2,\ldots,L_k, k = 1,2,3. \\
& \overline{z}_{dj}{}^k = w_d^k z^k{}_{dj}, j = 1,2,\ldots,n; d = 1,2,\ldots,D_1, k = 1,2,3. \\
& \lambda_i^k = v_i^k \alpha_i^k, \quad i = 1,2,\ldots,m_k, \quad k = 1,2,3. \\
& \eta_r = u_r \alpha_r, r = 1,2,\ldots,s \\
& \theta_d^k = w_d^k \alpha_d^k, \quad d = 1,2,\ldots,D_k, \quad k = 1,2,3 \\
& \gamma_l^k = q_l^k \alpha_l^k, \quad l = 1,2,\ldots,L_k, \quad k = 1,2,3
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
& e_o^U = \text{Max} \sum_{r=1}^{s} u_r y_{ro}^4 - \eta_r \left(y_{ro}^4 - y_{ro}^3\right) \\
& S.t. \\
& \sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k x_{io}^{k1} + \lambda_i^k \left(x_{io}^{k2} - x_{io}^{k1}\right) = 1 \\
& \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r \left(y_{rj}^2 - y_{rj}^1\right) - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k x_{ij}^{k4} - \lambda_i^k \left(x_{ij}^{k4} - x_{ij}^{k3}\right)\right) \le 0, \\
& \qquad j = 1,2,\ldots,n, j \ne o \\
& \sum_{r=1}^{s} u_r y_{ro}^4 - \eta_r \left(y_{ro}^4 - y_{ro}^3\right) - \left(\sum_{k=1}^{3}\sum_{i=1}^{m_k} v_i^k x_{io}^{k1} + \lambda_i^k \left(x_{io}^{k2} - x_{io}^{k1}\right)\right) \le 0, \\
& \sum_{l=1}^{L_k} \overline{p}^k{}_{lj} - \sum_{i=1}^{m_k} v_i^k x_{ij}^{k4} - \lambda_i^k \left(x_{ij}^{k4} - x_{ij}^{k3}\right) \le 0, \quad j = 1,2,\ldots,n, j \ne o, \\
& \qquad k = 1,2,3. \\
& \sum_{l=1}^{L_k} \overline{p}^k{}_{lo} - \sum_{i=1}^{m_k} v_i^k \left(x_{io}^{k1} + \lambda_i^k \left(x_{io}^{k2} - x_{io}^{k1}\right) \le 0, \quad k = 1,2,3. \\
& \sum_{d=1}^{D_k} \overline{z}^k{}_{dj} - \sum_{l=1}^{L_k} \overline{p}^k{}_{lj} \le 0, \quad j = 1,2,\ldots,n, \quad k = 1,2,3. \\
& \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r \left(y_{rj}^2 - y_{rj}^1\right) - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} \overline{z}^k{}_{dj}\right) \le 0, \ j = 1,2,\ldots,n, j \ne o \\
& \sum_{r=1}^{s} u_r y_{ro}^4 - \eta_r \left(y_{ro}^4 - y_{ro}^3\right) - \left(\sum_{k=1}^{3}\sum_{d=1}^{D_k} \overline{z}^k{}_{do}\right) \le 0 \\
& q_l^k p_{lj}^{k1} + \gamma_l^k \left(p_{lj}^{k2} - p_{lj}^{k1}\right)\Big) \le \overline{p}^k{}_{lj} \le q_l^k p_{lj}^{k4} - \gamma_l^k \left(p_{lj}^{k4} - p_{lj}^{k3}\right), \\
& \qquad k = 1,2,3, j = 1,2,\ldots,n; l = 1,2,\ldots,L_k \\
& w_d^k z_{dj}^{k1} + \theta_d^k \left(z_{dj}^{k2} - z_{dj}^{k1}\right) \le \overline{z}^k{}_{dj} \le w_d^k z_{dj}^{k4} - \theta_d^k \left(z_{dj}^{k4} - z_{dj}^{k3}\right), \\
& \qquad k = 1,2,3, j = 1,2,\ldots,n; d = 1,2,\ldots,D_k \\
& \overline{z}^k{}_{dj} \ge 0, \quad k = 1,2,3, j = 1,2,\ldots,n; d = 1,2,\ldots,D_k \\
& \overline{p}^k{}_{lj} \ge 0, \quad k = 1,2,3, j = 1,2,\ldots,n; l = 1,2,\ldots,L_k \\
& q_l^k \ge \varepsilon, \quad l = 1,2,\ldots,L_k, k = 1,2,3 \\
& w_d^k \ge \varepsilon, \quad d = 1,2,\ldots,D_1, k = 1,2,3 \\
& v_i^k \ge \varepsilon, \quad i = 1,2,\ldots,m_1, k = 1,2,3 \\
& u_r \ge \varepsilon, \quad r = 1,2,\ldots,s \\
& q_l^k \ge \gamma_l^k \ge 0, \quad l = 1,2,\ldots,L_k, k = 1,2,3 \\
& w_d^k \ge \theta_d^k \ge 0, \quad d = 1,2,\ldots,D_1, k = 1,2,3 \\
& v_i^k \ge \lambda_i^k \ge 0, \quad i = 1,2,\ldots,m_1, k = 1,2,3 \\
& u_r \ge \eta_r \ge 0, \quad r = 1,2,\ldots,s
\end{aligned} \tag{17}
$$

Model (17) is a successful approach for the calculation of the upper bound of the proposed network of sourcing, making, and delivery processes in the agile supply chain. First, model (17) is independent of the α-cut variables and therefore it should be solved just once for each DMU in order to complete a full fuzzy NDEA analysis. This reduces the computational efforts substantially. Second, there is no need to determine the step-size of the α-cut variables using a trial and error method. Third, this results in a unique upper bound in the efficiency score for each DMU and there are never any conflicts in the ranking of the DMUs using model (17). Fourth, it is possible to show that model (17) is always feasible and its objective function is always bounded.

Theorem #3 *Model (17) is feasible and bounded. Its optimal objective function is also equal to unity.*

Proof The dual form of model (17) can be written as follows:

$$\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t.} \\
&\theta x_{io}^{k1} - \sum_{\substack{j=1\\ j\neq o}}^{n} x_{ij}^{k4}\alpha_j - x_{io}^{k1}\alpha_o - \sum_{\substack{j=1\\ j\neq o}}^{n} x_{ij}^{k4}\tau_j^k - x_{io}^{k1}\tau_o^k - Q^k \geq 0, i=1,2,\ldots,m_k, k=1,2,3, \\
&\sum_{j=1}^{n}\tau_j^k - \sum_{j=1}^{n}\beta_j^k - m_{jl}^k + m_{jl}'^k \geq 0, l=1,2,\ldots,L_k, k=1,2,3, \\
&\sum_{j=1}^{n}\beta_j^k - \sum_{j=1}^{n}\varphi_j - n_{jd}^k + n_{jd}'^k \geq 0, d=1,2,\ldots,D_k, k=1,2,3, \\
&\sum_{\substack{j=1\\ j\neq o}}^{n} y_{rj}^1\alpha_j + y_{ro}^4\alpha_o + \sum_{\substack{j=1\\ j\neq o}}^{n} y_{rj}^1\varphi_j + y_{ro}^4\varphi_o - R^k \geq y_{ro}^4, r=1,2,\ldots,s, \\
&\theta\left(x_{io}^{k2} - x_{io}^{k1}\right) + \sum_{\substack{j=1\\ j\neq o}}^{n}\left(x_{ij}^{k4} - x_{ij}^{k3}\right)\alpha_j - \left(x_{io}^{k2} - x_{io}^{k1}\right)\alpha_o + \sum_{\substack{j=1\\ j\neq o}}^{n}\left(x_{ij}^{k4} - x_{ij}^{k3}\right)\tau_j^k - \left(x_{io}^{k2} - x_{io}^{k1}\right)\tau_o^k + Q^k \geq 0, i=1,2,\ldots,m_k, k=1,2,3, \\
&\sum_{\substack{j=1\\ j\neq o}}^{n}\left(y_{rj}^2 - y_{rj}^1\right)\alpha_j - \left(y_{ro}^4 - y_{ro}^3\right)\alpha_o + \sum_{\substack{j=1\\ j\neq o}}^{n}\left(y_{rj}^2 - y_{rj}^1\right)\varphi_j - \left(y_{ro}^4 - y_{ro}^3\right)\varphi_o + R^k \geq -\left(y_{ro}^4 - y_{ro}^3\right), r=1,2,\ldots,s, \\
&p_{lj}^{k1}m_{jl}^k - p_{lj}^{k4}m_{jl}'^k - O^k \geq 0, l=1,2,\ldots,L_k, k=1,2,3, j=1,2,\ldots,n, \\
&z_{dj}^{k1}n_{jd}^k - z_{dj}^{k4}n_{jd}'^k - P^k \geq 0, d=1,2,\ldots,D_k, k=1,2,3, j=1,2,\ldots,n, \\
&\left(p_{lj}^{k2} - p_{lj}^{k1}\right)m_{jl}^k + \left(p_{lj}^{k4} - p_{lj}^{k3}\right)m_{jl}'^k + O^k \geq 0, l=1,2,\ldots,L_k, k=1,2,3, j=1,2,\ldots,n, \\
&\left(z_{dj}^{k2} - z_{dj}^{k1}\right)n_{jd}^k - \left(z_{dj}^{k4} - z_{dj}^{k3}\right)n_{jd}'^k + P^k \geq 0, d=1,2,\ldots,D_k, k=1,2,3, j=1,2,\ldots,n, \\
&\alpha_j, \varphi_j \geq 0, j=1,2,\ldots,n, \\
&\tau_j^k, \beta_j^k \geq 0, j=1,2,\ldots,n, k=1,2,3, \\
&m_{jl}^k, m_{jl}'^k \geq 0, j=1,2,\ldots,n, l=1,2,\ldots,L_k, k=1,2,3, \\
&n_{jd}^k, n_{jd}'^k \geq 0, j=1,2,\ldots,n, d=1,2,\ldots,D_k, k=1,2,3, \\
&O^k, P^k, Q^k, R^k \geq 0, k=1,2,3, \\
&\theta \quad \textit{free}
\end{aligned}$$

Consider an arbitrary solution for dual model as follows:

$$\begin{aligned}
&\alpha_j, \varphi_j = 0, j=1,2,\ldots,n, \\
&\tau_j^k, \beta_j^k = 0, j=1,2,\ldots,n, k=1,2,3, \\
&m_{jl}^k, m_{jl}'^k = 0, j=1,2,\ldots,n, l=1,2,\ldots,L_k, k=1,2,3, \\
&n_{jd}^k, n_{jd}'^k = 0, j=1,2,\ldots,n, d=1,2,\ldots,D_k, k=1,2,3, \\
&O^k, P^k, Q^k, R^k = 0, k=1,2,3, \\
&\theta = 1
\end{aligned}$$

It is clear that this solution is always feasible and independent of the inputs, the intermediate measures, and the outputs. Hence, the dual model is always feasible and has at least one feasible solution. Therefore, the primal model (i.e., model (17)) is also feasible.

According to the objective function and the considered necessary optimality condition of the dual form of model (17), we have $\theta^* \leq 1$. Hence, the dual model is bounded. Due to the optimality theorems in linear programming, the optimum values of the objective functions of the dual and the primal models are equal $\left(e_o^{U*} = \theta^*\right)$. Therefore, $e_o^{U*} \leq 1$ and model (17) is also bounded. This completes the proof.

Since the objective functions in the multi-level optimization model (14) are not compatible, it cannot easily be reduced to a single-level optimization problem. The following procedure is proposed to for the purpose of simplification.

It is notable that the inner optimization problem is assumed to be a non-linear program as in the upstream optimization problems of multi-level optimization problems (14).

Let us consider the inner optimization problem. It is clear that this inner problem is assumed to be a linear program in an arbitrary α-cut level. Therefore, the theorems of linear programming are applicable to the inner optimization problem. Due to the dual theorems of linear programming, the optimal values of the primal and the dual are equal. Therefore, the dual form of the inner optimization problem is developed as model (18).

$$
\begin{array}{l}
Min \quad \theta \\
s.t. \\
\left(x_{io}^{kU}\right)_{\alpha_i} \times \theta - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^1 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^1 - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^2 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^2 \geq 0, \quad i = 1,2,\ldots,m_k, k = 1,2,3 \\
\sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+1)} - \sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+4)} \geq 0, \quad l = 1,2,\ldots,L_k, k = 1,2,3 \\
\sum_{j=1}^{n} z_{dj}^k \times \lambda_j^{(k+4)} - \sum_{j=1}^{n} z_{dj}^k \times \lambda_j^8 \geq 0, \quad d = 1,2,\ldots,D_k, k = 1,2,3 \\
\sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^1 + \left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^1 + \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^8 + \left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^8 \geq \left(y_{ro}^L\right)_{\alpha_r}, r = 1,2,\ldots,s. \\
\lambda_j^k \geq 0, j = 1,2,\ldots,n; k = 2,3,\ldots,8, \\
\theta \quad free
\end{array}
\tag{18}
$$

We will prove that model (18) will always generate solutions within a bounded feasible region. By the virtue of the dual theorem in linear programming, the optimum value of the objective function of the inner optimization problem of model (14) is equal to the objective function of model (18). Thus, we can replace model (18) with model (14).

Theorem #4 *Model (18) is feasible and bounded.*

Proof Consider an arbitrary solution for model (18) as follows:

$$
\begin{array}{l}
\lambda_j^k = 0, j = 1,2,\ldots,n; k = 2,3,\ldots,8, \\
\lambda_j^1 = 0, j = 1,2,\ldots,n, j \neq o \\
\theta = 1
\end{array}
$$

It is clear that this solution is always feasible and independent of the inputs, the intermediate measures, and the outputs of the model (18). Hence, model (18) has always at least one feasible solution and is always feasible. According to the objective function of model (18), it is clear that the relation $\theta^* \leq 1$ is satisfied in the optimal condition. Therefore, model (18) is bounded. This completes the proof.

Now, we construct model (19) by replacing model (18) with model (14).

$$
\underset{\left(z_{dj}^{kL}\right)_{\alpha_d} \leq z^k{}_{dj} \leq \left(z_{dj}^{kU}\right)_{d_l}, k=1,2,3.}{Min}
\left\{
\underset{\left(p_{lj}^{kL}\right)_{\alpha_l} \leq p^k{}_{lj} \leq \left(p_{lj}^{kU}\right)_{\alpha_l}, k=1,2,3.}{Min}
\left\{
\begin{array}{l}
Min \quad \theta \\
s.t. \\
\left(x_{io}^{kU}\right)_{\alpha_i} \times \theta - \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^1 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^1 \\
- \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^2 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^2 \geq 0, \quad i = 1,2,\ldots,m_k, k = 1,2,3 \\
\sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+1)} - \sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+4)} \geq 0, \quad l = 1,2,\ldots,L_k, k = 1,2,3 \\
\sum_{j=1}^{n} z_{dj}^k \times \lambda_j^{(k+4)} - \sum_{j=1}^{n} z_{dj}^k \times \lambda_j^8 \geq 0, \quad d = 1,2,\ldots,D_k, k = 1,2,3 \\
\sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^1 + \left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^1 + \sum_{\substack{j=1 \\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^8 + \\
\left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^8 \geq \left(y_{ro}^L\right)_{\alpha_r}, r = 1,2,\ldots,s. \\
\lambda_j^k \geq 0, j = 1,2,\ldots,n; k = 2,3,\ldots,8, \\
\theta \quad free
\end{array}
\right.
\right.
\tag{19}
$$

The multi-level optimization model (19) can now be reduced to a single-level optimization model (20) since the direction of all the optimization models in model (19) are similar.

$$
\begin{aligned}
&Min \quad \theta \\
&s.t. \\
&\left(x_{io}^{kU}\right)_{\alpha_i} \times \theta - \sum_{\substack{j=1\\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^1 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^1 - \sum_{\substack{j=1\\ j\neq 0}}^{n} \left(x_{ij}^{kL}\right)_{\alpha_i} \times \lambda_j^2 - \left(x_{io}^{kU}\right)_{\alpha_i} \times \lambda_o^2 \geq 0, \quad i = 1,2,\ldots,m_k, k = 1,2,3. \\
&\sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+1)} - \sum_{j=1}^{n} p_{lj}^k \times \lambda_j^{(k+4)} \geq 0, \quad l = 1,2,\ldots,L_k, k = 1,2,3. \\
&\sum_{j=1}^{n} z_{dj}^k \times \lambda_j^{(k+4)} - \sum_{j=1}^{n} z_{dj}^k \times \lambda_j^8 \geq 0, \quad d = 1,2,\ldots,D_k, k = 1,2,3. \\
&\sum_{\substack{j=1\\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^1 + \left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^1 + \sum_{\substack{j=1\\ j\neq 0}}^{n} \left(y_{rj}^U\right)_{\alpha_r} \times \lambda_j^8 + \left(y_{ro}^L\right)_{\alpha_r} \times \lambda_o^8 \geq \left(y_{ro}^L\right)_{\alpha_r}, r = 1,2,\ldots,s. \\
&\left(p_{lj}^{kL}\right)_{\alpha_l} \leq p^k{}_{lj} \leq \left(p_{lj}^{kU}\right)_{\alpha_l}, k = 1,2,3, j = 1,2,\ldots,n \\
&\left(z_{dj}^{kL}\right)_{\alpha_d} \leq z^k{}_{dj} \leq \left(z_{dj}^{kU}\right)_{\alpha_d}, k = 1,2,3, j = 1,2,\ldots,n \\
&\lambda_j^k \geq 0, j = 1,2,\ldots,n; k = 2,3,\ldots,8, \\
&\theta \quad free
\end{aligned}
\tag{20}
$$

The single-level optimization model (20) is dependent on the α-cut variables and is assumed to be a non-linear programming problem. Replacing equations (3)–(10) with equations (21) will result in the linear model (22) which is independent of the α-cut variables.

$$
\begin{aligned}
&\beta_i = \theta\alpha_i, \quad i = 1,2,\ldots,m_1, \\
&\gamma_{ij}^l = \lambda_j^l \alpha_i, j = 1,2,\ldots,n, \quad i = 1,2,\ldots,m_1, \quad l = 1,2. \\
&\gamma_{ij}'^l = \lambda_j^l \alpha_i, j = 1,2,\ldots,n, \quad i = 1,2,\ldots,m_2, \quad l = 1,2. \\
&\gamma_{ij}''^l = \lambda_j^l \alpha_i, j = 1,2,\ldots,n, \quad i = 1,2,\ldots,m_3, \quad l = 1,2. \\
&\eta_{ij}^1 = \lambda_j^1 \alpha_r, j = 1,2,\ldots,n, \quad r = 1,2,\ldots,s; \eta_{ij}^8 = \lambda_j^8 \alpha_r, j = 1,2,\ldots,n, \quad r = 1,2,\ldots,s \\
&q_l' k = q_l^k \alpha_l, l = 1,2,\ldots,L_k, k = 1,2,3; w_d'^k = w_d^k \alpha_d, d = 1,2,\ldots,D_k, k = 1,2,3 \\
&\overline{p}^k{}_{lj} = q_l^k p^k{}_{lj}, l = 1,2,\ldots,L_k, k = 1,2,3, j = 1,2,\ldots,n \\
&\overline{z}^k{}_{dj} = w_d^k z^k{}_{dj}, d = 1,2,\ldots,D_k, k = 1,2,3, j = 1,2,\ldots,n
\end{aligned}
\tag{21}
$$

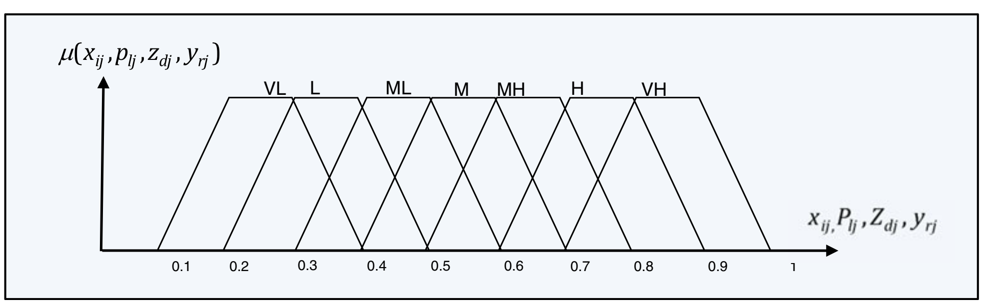

Fig. 5 Membership functions of the linguistic terms

$$
\begin{aligned}
&Min \quad \theta \\
&s.t. \\
&(\theta-\lambda_o^1-\lambda_o^2)x_{io}^{14}-(\beta_i-\gamma_{io}^1-\gamma_{io}^2)(x_{io}^{14}-x_{io}^{13})-\sum_{\substack{j=1\\ j\neq 0}}^{n}\left(x_{ij}^{11}\left(\lambda_j^1+\lambda_j^2\right)+\left(\gamma_{ij}^1+\gamma_{ij}^2\right)\left(x_{ij}^{12}-x_{ij}^{11}\right)\right)\geq 0,\quad i=1,2,\ldots,m_1,\\
&(\theta-\lambda_o^1-\lambda_o^2)x_{io}^{24}-(\beta_i-\gamma_{io}'^1-\gamma_{io}'^2)(x_{io}^{24}-x_{io}^{23})-\sum_{\substack{j=1\\ j\neq 0}}^{n}\left(x_{ij}^{21}\left(\lambda_j^1+\lambda_j^2\right)+\left(\gamma_{ij}'^1+\gamma_{ij}'^2\right)\left(x_{ij}^{22}-x_{ij}^{21}\right)\right)\geq 0,\quad i=1,2,\ldots,m_2,\\
&(\theta-\lambda_o^1-\lambda_o^2)x_{io}^{34}-\left(\beta_i-\gamma''^1_{io}-\gamma''^2_{io}\right)(x_{io}^{34}-x_{io}^{33})-\sum_{\substack{j=1\\ j\neq 0}}^{n}\left(x_{ij}^{31}\left(\lambda_j^1+\lambda_j^2\right)+\left(\gamma''^1_{ij}+\gamma''^2_{ij}\right)\left(x_{ij}^{32}-x_{ij}^{31}\right)\right)\geq 0,\quad i=1,2,\ldots,m_3,\\
&\sum_{j=1}^{n}p_{lj}^k\times\lambda_j^{(k+1)}-\sum_{j=1}^{n}p_{lj}^k\times\lambda_j^{(k+4)}\geq 0,\quad l=1,2,\ldots,L_k,k=1,2,3.\\
&\sum_{j=1}^{n}z_{dj}^k\times\lambda_j^{(k+4)}-\sum_{j=1}^{n}z_{dj}^k\times\lambda_j^{8}\geq 0,\quad d=1,2,\ldots,D_k,k=1,2,3.\\
&\sum_{\substack{j=1\\ j\neq 0}}^{n}\left(y_{rj}^4\left(\lambda_j^1+\lambda_j^8\right)-\left(\eta_{ij}^1+\eta_{ij}^8\right)\left(y_{rj}^4-y_{rj}^3\right)\right)+\left(y_{ro}^1(\lambda_o^1+\lambda_o^8)+(\eta_{io}^1+\eta_{io}^8)(y_{ro}^2-y_{ro}^1)\right)\geq\left(y_{ro}^1+(y_{ro}^2-y_{ro}^1)\right),\quad r=1,2,\ldots,s.\\
&q_l^kp_{lj}^{k1}+q_l'^k\left(p_{lj}^{k2}-p_{lj}^{k1}\right)\leq\overline{p}^k{}_{lj}\leq q_l^kp_{lj}^{k4}-q_l'^k\left(p_{lj}^{k4}-p_{lj}^{k3}\right),k=1,2,3,j=1,2,\ldots,n\\
&w_d^kz_{dj}^{k1}+w_d'^k\left(zd_{lj}^{k2}-z_{dj}^{k1}\right)\leq\overline{z}^k{}_{dj}\leq w_d^kz_{dj}^{k4}-w_d'k\left(zd_{lj}^{k4}-z_{dj}^{k3}\right),k=1,2,3,j=1,2,\ldots,n\\
&\overline{p}^k{}_{lj}\geq 0,l=1,2,\ldots,L_k,k=1,2,3,j=1,2,\ldots,n\\
&\overline{z}^k{}_{dj}\geq 0,d=1,2,\ldots,D_k,k=1,2,3,j=1,2,\ldots,n\\
&w_d^k,w_d'k\geq 0,d=1,2,\ldots,D_k,k=1,2,3\\
&q_l^k,q_l'k\geq 0,l=1,2,\ldots,L_k,k=1,2,3\\
&0\leq\beta_i\leq\alpha_i,\quad i=1,2,\ldots,m_1,\\
&0\leq\gamma_{ij}^l\leq\alpha_i,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_1,\quad l=1,2.\\
&0\leq\gamma_{ij}'^l\leq\alpha_i,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_2,\quad l=1,2.\\
&0\leq\gamma''^l_{ij}\leq\alpha_i,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_3,\quad l=1,2.\\
&0\leq\eta_{ij}^1\leq\alpha_r,j=1,2,\ldots,n,\quad r=1,2,\ldots,s\\
&0\leq\eta_{ij}^8\leq\alpha_r,j=1,2,\ldots,n,\quad r=1,2,\ldots,s\\
&\lambda_j^k\geq 0,j=1,2,\ldots,n;k=2,3,\ldots,8,\\
&\theta\quad free
\end{aligned}
\tag{22}
$$

Theorem #5 *Model (22) is feasible and bounded. Therefore, it always has a bounded optimum solution.*

Proof Consider an arbitrary solution for model (22) as follows:

$$
\begin{aligned}
&\lambda_j^k=0,j=1,2,\ldots,n;k=2,3,\ldots,8,\\
&\lambda_j^1=0,j=1,2,\ldots,n,j\neq o\\
&\overline{p}^k{}_{lj}=0,l=1,2,\ldots,L_k,k=1,2,3,j=1,2,\ldots,n\\
&\overline{z}^k{}_{dj}=0,d=1,2,\ldots,D_k,k=1,2,3,j=1,2,\ldots,n\\
&w_d^k,w_d'^k=0,d=1,2,\ldots,D_k,k=1,2,3\\
&q_l^k,q_l'^k=0,l=1,2,\ldots,L_k,k=1,2,3\\
&\beta_i=\alpha_i=0,\quad i=1,2,\ldots,m_1,\\
&\gamma_{ij}^l=\alpha_i=0,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_1,l=1,2.\\
&\gamma_{ij}'^l=\alpha_i=0,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_2,l=1,2.\\
&\gamma''^l_{ij}=\alpha_i=0,j=1,2,\ldots,n,\quad i=1,2,\ldots,m_3,l=1,2.\\
&\eta_{ij}^1=\alpha_r=0,j=1,2,\ldots,n,\quad r=1,2,\ldots,s\\
&\eta_{ij}^8=\alpha_r=0,j=1,2,\ldots,n,\quad r=1,2,\ldots,s\\
&\theta=1
\end{aligned}
\tag{23}
$$

It is clear that this solution is always feasible and independent of the inputs, the intermediate measures, and the outputs of the model (22). Hence, model (22) is always feasible and has at least one feasible solution. According to the objective function of model (22), it is clear that the relation $\theta^*\leq 1$ is satisfied in the optimal condition. Therefore, model (22) is bounded. This completes the proof.

4.3 Upper and lower bound of the efficiency of sub-DMUs

The optimum values of the decision variables, achieved from model (17), are used to calculate the upper bound of the efficiency scores of the sub-DMUs based on the classical definition of the relative efficiency. As a result of this variable interchange, model (22) becomes linear and independent of the α-cut. The new variables cannot help us to calculate the efficiency of sub-DMUs. Therefore, the same procedure proposed for model (17) and model (22), including the multi-level modeling, the reduction to single-level optimization modeling, and the required variable interchanges, should be used for each sub-process in the proposed network presented in Fig. 4. Since the modeling procedure for the sub-DMUs is similar to what was accomplished in models (17)–(22), additional details are omitted in favor of brevity.

4.3.1 Classification of the efficiency scores of the DMUs and the sub-DMUs

The uncertainty of the input and the output measures was modeled through fuzzy numbers for each DMU. In addition to their flexibility in choosing the weights, the DMUs are also free to adjust their levels of inputs and outputs in a favorable manner within the fuzzy membership functions. Therefore, many DMUs and sub-DMUs are likely to be proved efficient. Moreover, the accomplished variable change increased the likelihood of more freedom for each DMU to select the most preferred α-cut value as well as the lower bound and the upper bound of the inputs and/or the outputs. Hence, further discrimination of the efficient units becomes more essential for fuzzy NDEA. Considering the resulting interval efficiency scores of each DUM, and sub-DMU, they can be categorized using (24)–(25).

$$\begin{aligned}&\text{DMU}\\&E^{++}=\left\{j\in J\middle|e_j^L=1\right\},\\&E^{+}=\left\{j\in J\middle|e_j^L<1\text{ and }\mathrm{e}_\mathrm{j}^*=1\right\},\\&E^{-}=\left\{j\in J\middle|e_j^*<1\right\}.\end{aligned}\tag{24}$$

$$\begin{aligned}&\text{Sub}-\text{DMU}_\text{k}\\&E^{++}=\left\{j\in J\middle|\left[e_o^{k-}\right]^L=1\right\},\quad k=1,2,\ldots,7\\&E^{+}=\left\{j\in J\middle|\left[e_o^{k-}\right]^L<1\text{ and }\mathrm{e}_\mathrm{o}^{\mathrm{k}-}=1\right\},\quad k=1,2,\ldots,7\\&E^{-}=\left\{j\in J\middle|e_o^{k-}<1\right\},\quad k=1,2,\ldots,7.\end{aligned}$$

where, J is the set of all DMUs with cardinality of n (i.e., $|J|=n$), $e_j^*=Max\left(e_j^L,e_j^U\right)$, $e_o^{k+}=Max\left(\left[e_o^{k+}\right]^L,\left[e_o^{k+}\right]^U\right)$, $k=1,2,\ldots,7$, $e_o^{k-}=Max\left(\left[e_o^{k-}\right]^L,\left[e_o^{k-}\right]^U\right)$, $k=1,2,\ldots,7$.

5 Real-life case study

As in most food industries, the fresh food industry is characterized by "repetitive production operations carrying out specific physical (e.g., blending or milling) or chemical reactions" [27]. Process industries usually show a higher complexity than discrete manufacturing, which is caused by factors such as the perishability of the products, the high number of end products, a great variety of possible production paths, special storage equipment, co- and by-products, and variable recipes. Most production systems in the fresh food industries—as well as the food industry in general—contain "processing" and "packaging". The number of products involved increases with each production step. Out of a limited number of raw materials (e.g., raw milk), a moderate number of intermediate products are produced within the processing step. High product complexity typically occurs at the packaging level due to different tastes and packaging forms. In general, the supply, production, distribution, and delivery of fresh foods are accomplished through cooperation among different companies in the supply chain. According to these characteristics, the fresh food industry (which operates in a supply chain) requires varying levels of agility for the sourcing, making, and delivery processes. We apply the proposed fuzzy NDEA model developed in this study to measure the performance of agility in the top forty dairy companies in Iran. Each dairy company is as an independent supply chain with a network structure similar to the one presented in Fig. 4.

5.1 Measurement scales

The measurement indices for the providers of agility, the capabilities of agility, and the goals of supply chain in sourcing, making, and delivery processes are often vague and subjective. Therefore, without loss of generality, the linguistic terms parameterized through the TrFNs were used to measure them. The linguistic terms and their associated trapezoidal fuzzy numbers used in the proposed fuzzy NDEA are presented in Fig. 5.

5.2 Data gathering

The judgments from 40 experts in the Iranian dairy industry were used to determine the agility indices used in the proposed fuzzy NDEA model. A questionnaire was used to collect the expert judgments. The experts who responded to the questionnaire were experienced managers with 10.35 years of tenure in the dairy industry. A set of 40 comparable diary supply chains producing cheese products was selected and a manager at each supply chain was invited to complete the questionnaire. The respondents were allowed to use linguistic terms provided in Table 4 to record their opinions. The aggregation of the collective opinions from the supply chain managers presented in Tables 5, 6, and 7 formed the values of the indices for the

Table 4 Linguistic terms and their associated TrFNs

Linguistic terms	TrFN
Very low (VL)	(0.1, 0.2, 0.3, 0.4)
Low (L)	(0.2, 0.3, 0.4, 0.5)
Medium low (ML)	(0.3, 0.4, 0.5, 0.6)
Medium (M)	(0.4, 0.5, 0.6, 0.7)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.6, 0.7, 0.8, 0.9)
Very high (VH)	(0.7, 0.8, 0.9, 1)

Table 5 Aggregation of DMs' opinions on sourcing

Brand	DMU	Providers of agility in sourcing				Capabilities of agility in sourcing (Intermediate measures 1)				Performance of sourcing (Intermediate measures 2)				Final goals of supply chain (Outputs)			
		X_1^1	X_2^1	X_3^1	X_4^1	P_1^1	P_2^1	P_3^1	P_4^1	Z_1^1	Z_2^1	Z_3^1	Z_4^1	Y_1	Y_2	Y_3	Y_4
Aban-Shir Ardabil	1	MH	M	M	L	M	MH	H	H	L	MH	VL	MH	H	MH	VL	ML
Abshar-Sepid Shiraz	2	H	H	ML	VH	L	ML	L	VL	H	VL	VH	ML	M	MH	VL	M
Alborz Laban	3	ML	L	M	MH	L	M	M	VL	L	L	H	H	M	VH	H	L
Arak Dairy	4	MH	VH	L	M	M	H	VL	VL	VL	VH	MH	VL	M	VL	VL	H
Aryan-ShirAlborz	5	L	VL	L	ML	ML	MH	M	M	L	VH	VH	ML	L	VL	VL	MH
Ashayer	6	ML	ML	MH	L	M	H	H	ML	M	VL	VL	M	VL	ML	H	L
Azar nosh Sharq	7	MH	ML	L	H	L	ML	L	M	M	M	ML	VL	M	MH	H	H
AzarShiraneh	8	VL	M	H	ML	VL	VL	ML	H	MH	M	M	ML	L	ML	L	VL
Barekat	9	L	VH	VL	ML	M	M	MH	H	ML	ML	VL	ML	L	M	M	VL
Bistoon	10	VH	MH	M	ML	L	L	ML	L	VL	MH	ML	MH	M	H	VL	VL
Choopan	11	M	M	VL	MH	L	L	M	M	H	H	MH	VL	ML	MH	M	M
Damdaran	12	H	ML	VH	ML	M	M	H	VL	M	M	MH	VL	M	H	H	ML
Ishmali	13	L	M	H	H	M	VL	VL	M	M	M	VH	H	L	ML	L	M
Kalber	14	VH	L	MH	VL	VH	L	MH	VL	M	M	VL	VL	H	MH	VH	L
Kaleh	15	VL	L	M	L	VL	L	M	L	L	L	VL	VL	M	L	M	L
Kamel-Novin	16	ML	MH	VL	M	ML	MH	VL	M	VL	VL	ML	H	L	ML	ML	MH
Laban Razavi	17	ML	L	ML	VL	ML	L	ML	VL	M	M	MH	H	H	MH	ML	L
Maadi-Mimas	18	M	H	M	ML	M	H	M	ML	L	L	ML	L	VL	VL	M	H
Mihan	19	H	VL	L	ML	H	VL	L	ML	L	L	M	M	VL	L	VH	VL
Namli	20	MH	M	ML	MH	ML	L	M	MH	M	M	H	VL	VL	VH	MH	M
Pak Ara	21	ML	ML	MH	L	MH	VH	L	M	M	VL	VL	M	VL	ML	H	L
Pak Pey	22	MH	ML	L	H	L	VL	L	ML	M	M	ML	VL	M	MH	H	H
Pakban	23	VL	M	H	ML	ML	ML	MH	L	MH	M	M	ML	L	ML	L	VL
Paksar Sari	24	VH	L	MH	VL	MH	ML	L	H	M	M	VL	VL	H	MH	VH	L
Pak-Tehran	25	VL	L	M	L	VL	M	H	ML	L	L	VL	VL	M	L	M	L
Pars-Pooyan Zagros	26	ML	MH	VL	M	L	VH	VL	ML	VL	VL	ML	H	L	ML	ML	MH
Pazhan	27	ML	L	ML	VL	L	MH	VL	MH	M	M	MH	H	H	MH	ML	L
Pegah Fars	28	M	H	M	ML	H	VL	VH	ML	L	L	ML	L	VL	VL	M	H
Pegah-Gilan	29	H	VL	L	ML	L	L	H	H	L	L	M	M	VL	L	VH	VL
Pegah-Golpaygan	30	ML	L	M	MH	VL	VH	MH	VL	L	L	H	H	M	VH	H	L
Pegah-Hamedan	31	MH	VH	L	M	L	VH	VH	ML	VL	VH	MH	VL	M	VL	VL	H
Pegah-Kerman	32	L	VL	L	ML	M	VL	VL	M	L	VH	VH	ML	L	VL	VL	MH
Pegah-Khuzestan	33	ML	ML	MH	L	M	M	ML	VL	M	VL	VL	M	VL	ML	H	L
Pegah-Lorestan	34	MH	ML	L	H	MH	M	M	ML	M	M	ML	VL	M	MH	H	H
Pegah-Tehran	35	VL	M	H	ML	ML	L	VL	VL	MH	M	M	ML	L	ML	L	VL
Pegah-Zanjan	36	L	VH	VL	ML	M	M	VL	L	ML	ML	VL	ML	L	M	M	VL
Ramak	37	VH	MH	M	ML	H	VL	VL	VH	VL	MH	ML	MH	M	H	VL	VL
Roozaneh	38	M	M	VL	MH	VL	M	VL	ML	H	H	MH	VL	ML	MH	M	M
Sahar	39	H	ML	VH	ML	ML	VL	M	MH	M	M	MH	VL	M	H	H	ML
Ta'rif	40	L	M	H	H	M	ML	L	ML	M	M	VH	H	L	ML	L	M

Table 6 Aggregation of the DMs' opinions on making

Brand	DMU	Providers of agility in making				Capabilities of agility in making (Intermediate measures 1)				Performance of making (Intermediate measures 2)				Final goals of supply chain (Outputs)			
		X_1^2	X_2^2	X_3^2	X_4^2	P_1^2	P_2^2	P_3^2	P_4^2	Z_1^2	Z_2^2	Z_3^2	Z_4^2	Y_1	Y_2	Y_3	Y_4
Aban-Shir Ardabil	1	M	L	M	MH	H	H	L	MH	MH	M	M	MH	H	MH	VL	ML
Abshar-Sepid Shiraz	2	ML	VH	L	ML	L	VL	H	VL	H	H	L	ML	M	MH	VL	M
Alborz Laban	3	M	MH	L	M	M	VL	L	L	ML	L	L	M	M	VH	H	L
Arak Dairy	4	L	M	M	H	VL	VL	VL	VH	MH	VH	M	H	M	VL	VL	H
Aryan-ShirAlborz	5	L	ML	ML	MH	M	M	L	VH	L	VL	ML	MH	L	VL	VL	MH
Ashayer	6	MH	L	M	H	H	ML	M	VL	ML	ML	M	H	VL	ML	H	L
Azar nosh Sharq	7	L	H	L	ML	L	M	M	M	MH	ML	L	ML	M	MH	H	H
AzarShiraneh	8	H	ML	VL	VL	ML	H	MH	M	VL	M	VL	VL	L	ML	L	VL
Barekat	9	VL	ML	M	M	MH	H	ML	ML	L	VH	M	M	L	M	M	VL
Bistoon	10	M	ML	L	L	ML	L	VL	MH	VH	MH	L	L	M	H	VL	VL
Choopan	11	VL	MH	L	L	M	M	H	H	M	M	L	L	ML	MH	M	M
Damdaran	12	VH	ML	M	M	H	VL	M	M	H	ML	M	M	M	H	H	ML
Ishmali	13	H	H	M	VL	VL	M	M	M	L	M	M	VL	L	ML	L	M
Kalber	14	MH	VL	VH	L	MH	VL	M	M	VH	L	VH	L	H	MH	VH	L
Kaleh	15	M	L	VL	L	M	L	L	L	VL	L	VL	L	M	L	M	L
Kamel-Novin	16	VL	M	ML	MH	VL	M	VL	VL	ML	MH	ML	MH	L	ML	ML	MH
Laban Razavi	17	ML	VL	ML	L	ML	VL	M	M	ML	L	ML	L	H	MH	ML	L
Maadi-Mimas	18	M	ML	M	H	M	ML	L	L	M	H	M	H	VL	VL	M	H
Mihan	19	L	ML	H	VL	L	ML	L	L	H	VL	H	VL	VL	L	VH	VL
Namli	20	ML	MH	ML	L	M	MH	M	M	MH	M	ML	L	VL	VH	MH	M
Pak Ara	21	MH	L	MH	VH	L	M	M	VL	ML	ML	MH	VH	VL	ML	H	L
Pak Pey	22	L	H	L	VL	L	ML	M	M	MH	ML	L	VL	M	MH	H	H
Pakban	23	H	ML	ML	ML	MH	L	MH	M	VL	M	ML	ML	L	ML	L	VL
Paksar Sari	24	MH	VL	MH	ML	L	H	M	M	VH	L	MH	ML	H	MH	VH	L
Pak-Tehran	25	M	L	VL	M	H	ML	L	L	VL	L	VL	M	M	L	M	L
Pars-Pooyan Zagros	26	VL	M	L	VH	VL	ML	VL	VL	ML	MH	L	VH	L	ML	ML	MH
Pazhan	27	ML	VL	L	MH	VL	MH	M	M	ML	L	L	MH	H	MH	ML	L
Pegah Fars	28	M	ML	H	VL	VH	ML	L	L	M	H	H	VL	VL	VL	M	H
Pegah-Gilan	29	L	ML	L	L	H	H	L	L	H	VL	L	L	VL	L	VH	VL
Pegah-Golpaygan	30	M	MH	VL	VH	MH	VL	L	L	ML	L	VL	VH	M	VH	H	L
Pegah-Hamedan	31	L	M	L	VH	VH	ML	VL	VH	MH	VH	L	VH	M	VL	VL	H
Pegah-Kerman	32	L	ML	M	VL	VL	M	L	VH	L	VL	M	VL	L	VL	VL	MH
Pegah-Khuzestan	33	MH	L	M	M	ML	VL	M	VL	ML	ML	M	M	VL	ML	H	L
Pegah-Lorestan	34	L	H	MH	M	M	ML	M	M	MH	ML	MH	M	M	MH	H	H
Pegah-Tehran	35	H	ML	ML	L	VL	VL	MH	M	VL	M	ML	L	L	ML	L	VL
Pegah-Zanjan	36	VL	ML	M	M	VL	L	ML	ML	L	VH	M	M	L	M	M	VL
Ramak	37	M	ML	H	VL	VL	VH	VL	MH	VH	MH	H	VL	M	H	VL	VL
Roozaneh	38	VL	MH	VL	M	VL	ML	H	H	M	M	VL	M	ML	MH	M	M
Sahar	39	VH	ML	ML	VL	M	MH	M	M	H	ML	ML	VL	M	H	H	ML
Ta'rif	40	H	H	M	ML	L	ML	M	M	L	M	M	ML	L	ML	L	M

Table 7 Aggregation of the DMs' opinions on delivery

Brand	DMU	Providers of agility in delivery				Capabilities of agility indelivery (Intermediate measures 1)				Performance of delivery (Intermediate measures 2)				Final goals of supply chain (Outputs)			
		X_1^3	X_2^3	X_3^3	X_4^3	P_1^3	P_2^3	P_3^3	P_4^3	Z_1^3	Z_2^3	Z_3^3	Z_4^3	Y_1	Y_2	Y_3	Y_4
Aban-Shir Ardabil	1	M	MH	H	H	M	MH	M	MH	H	H	H	MH	H	MH	VL	ML
Abshar-Sepid Shiraz	2	L	ML	L	VL	L	ML	L	ML	L	VL	VL	H	M	MH	VL	M
Alborz Laban	3	L	M	M	VL	L	M	L	M	M	VL	VL	ML	M	VH	H	L
Arak Dairy	4	M	H	VL	VL	M	H	M	H	VL	VL	VL	MH	M	VL	VL	H
Aryan-ShirAlborz	5	ML	MH	M	M	ML	MH	ML	MH	M	M	M	L	L	VL	VL	MH
Ashayer	6	M	H	H	ML	M	H	M	H	H	ML	ML	ML	VL	ML	H	L
Azar nosh Sharq	7	L	ML	L	M	L	ML	L	ML	L	M	M	MH	M	MH	H	H
AzarShiraneh	8	VL	VL	ML	H	VL	VL	VL	VL	ML	H	H	VL	L	ML	L	VL
Barekat	9	M	M	MH	H	M	M	M	M	MH	H	H	L	L	M	M	VL
Bistoon	10	L	L	ML	L	L	L	L	L	ML	L	L	VH	M	H	VL	VL
Choopan	11	L	L	M	M	L	L	L	L	M	M	M	M	ML	MH	M	M
Damdaran	12	M	M	H	VL	M	M	M	M	H	VL	VL	H	M	H	H	ML
Ishmali	13	M	VL	VL	M	M	VL	M	VL	VL	M	M	L	L	ML	L	M
Kalber	14	VH	L	MH	VL	VH	L	VH	L	MH	VL	VL	VH	H	MH	VH	L
Kaleh	15	VL	L	M	L	VL	L	VL	L	M	L	L	VL	M	L	M	L
Kamel-Novin	16	ML	MH	VL	M	ML	MH	ML	MH	VL	M	M	ML	L	ML	ML	MH
Laban Razavi	17	ML	L	ML	VL	ML	L	ML	L	ML	VL	VL	ML	H	MH	ML	L
Maadi-Mimas	18	M	H	M	ML	M	H	M	H	M	ML	ML	M	VL	VL	M	H
Mihan	19	H	VL	L	ML	H	VL	H	VL	L	ML	ML	H	VL	L	VH	VL
Namli	20	ML	L	M	MH	ML	L	ML	L	M	MH	MH	MH	VL	VH	MH	M
Pak Ara	21	MH	VH	L	M	MH	VH	MH	VH	L	M	M	ML	VL	ML	H	L
Pak Pey	22	L	VL	L	ML	L	VL	L	VL	L	ML	ML	MH	M	MH	H	H
Pakban	23	ML	ML	MH	L	ML	ML	ML	ML	MH	L	L	VL	L	ML	L	VL
Paksar Sari	24	MH	ML	L	H	MH	ML	MH	ML	L	H	H	VH	H	MH	VH	L
Pak-Tehran	25	VL	M	H	ML	VL	M	VL	M	H	ML	ML	VL	M	L	M	L
Pars-Pooyan Zagros	26	L	VH	VL	ML	L	VH	L	VH	VL	ML	ML	ML	L	ML	ML	MH
Pazhan	27	L	MH	VL	MH	L	MH	L	MH	VL	MH	MH	ML	H	MH	ML	L
Pegah Fars	28	H	VL	VH	ML	H	VL	H	VL	VH	ML	ML	M	VL	VL	M	H
Pegah-Gilan	29	L	L	H	H	L	L	L	L	H	H	H	H	VL	L	VH	VL
Pegah-Golpaygan	30	VL	VH	MH	VL	VL	VH	VL	VH	MH	VL	VL	ML	M	VH	H	L
Pegah-Hamedan	31	L	VH	VH	ML	L	VH	L	VH	VH	ML	ML	MH	M	VL	VL	H
Pegah-Kerman	32	M	VL	VL	M	M	VL	M	VL	VL	M	M	L	L	VL	VL	MH
Pegah-Khuzestan	33	M	M	ML	VL	M	M	M	M	ML	VL	VL	ML	VL	ML	H	L
Pegah-Lorestan	34	MH	M	M	ML	MH	M	MH	M	M	ML	ML	MH	M	MH	H	H
Pegah-Tehran	35	ML	L	VL	VL	ML	L	ML	L	VL	VL	VL	VL	L	ML	L	VL
Pegah-Zanjan	36	M	M	VL	L	M	M	M	M	VL	L	L	L	L	M	M	VL
Ramak	37	H	VL	VL	VH	H	VL	H	VL	VL	VH	VH	VH	M	H	VL	VL
Roozaneh	38	VL	M	VL	ML	VL	M	VL	M	VL	ML	ML	M	ML	MH	M	M
Sahar	39	ML	VL	M	MH	ML	VL	ML	VL	M	MH	MH	H	M	H	H	ML
Ta'rif	40	M	ML	L	ML	M	ML	M	ML	L	ML	ML	L	L	ML	L	M

sourcing, making, and delivery processes in the supply chain (i.e., the main DMU, the sub-DMUs, etc.). The experts were also instructed to rate the indices according to these similar products in the supply chains.

5.3 Experimental results

The proposed fuzzy three-stage DEA models were coded using LINGO 11.0 software. The codes of the proposed

Table 8 Interval efficiency scores and classification of DMUs and sub-DMUs

DMU	Class	e^U	e^L	Sub-DMU_1 class	$[e^{+1}]^U$	$[e^{+1}]^L$	Sub-DMU_2 class	$[e^{+2}]^U$	$[e^{+2}]^L$	Sub-DMU_3 class	$[e^{+3}]^L$	$[e^{+3}]^L$
DMU_1	E^-	0.878289	0.318845	E^+	1	0.70311	E^+	1	0.70311	E^+	1	0.70311
DMU_2	E^-	0.671102	0.220233	E^+	1	0.72332	E^+	1	0.72332	E^+	1	0.72332
DMU_3	E^-	0.488546	0.119396	E^+	1	0.88858	E^+	1	0.88858	E^+	1	0.88858
DMU_4	E^-	0.719061	0.201509	E^+	1	0.81652	E^+	1	0.81652	E^+	1	0.81652
DMU_5	E^-	0.71122	0.397249	E^+	1	0.88816	E^+	1	0.88816	E^+	1	0.88816
DMU_6	E^-	0.830302	0.29426	E^+	1	0.71999	E^+	1	0.71999	E^+	1	0.71999
DMU_7	E^-	0.64614	0.264905	E^+	1	0.7258	E^+	1	0.7258	E^+	1	0.7258
DMU_8	E^-	0.527599	0.427	E^+	1	0.88503	E^+	1	0.88503	E^+	1	0.88503
DMU_9	E^-	0.683883	0.227877	E^+	1	0.77176	E^+	1	0.77176	E^+	1	0.77176
DMU_{10}	E^-	0.776133	0.258398	E^+	1	0.64859	E^+	1	0.64859	E^+	1	0.64859
DMU_{11}	E^-	0.590287	0.208036	E^+	1	0.92626	E^+	1	0.92626	E^+	1	0.92626
DMU_{12}	E^-	0.883816	0.300212	E^+	1	0.64284	E^+	1	0.64284	E^+	1	0.64284
DMU_{13}	E^-	0.591059	0.318305	E^+	1	0.90673	E^+	1	0.90673	E^+	1	0.90673
DMU_{14}	E^-	0.854122	0.323316	E^+	1	0.80355	E^+	1	0.80355	E^+	1	0.80355
DMU_{15}	E^-	0.775755	0.34798	E^+	1	0.59499	E^+	1	0.59499	E^+	1	0.59499
DMU_{16}	E^-	0.812351	0.333385	E^+	1	0.89488	E^+	1	0.89488	E^+	1	0.89488
DMU_{17}	E^-	0.634715	0.240422	E^+	1	0.89489	E^+	1	0.89489	E^+	1	0.89489
DMU_{18}	E^-	0.620228	0.331787	E^-	0.85628	0.41681	E^-	0.85628	0.41681	E^-	0.85628	0.41681
DMU_{19}	E^-	0.824502	0.419615	E^+	1	0.78696	E^+	1	0.78696	E^+	1	0.78696
DMU_{20}	E^-	0.614776	0.215864	E^+	1	0.66055	E^+	1	0.66055	E^+	1	0.66055
DMU_{21}	E^-	0.867305	0.310307	E^+	1	0.70311	E^+	1	0.70311	E^+	1	0.70311
DMU_{22}	E^-	0.74228	0.252409	E^+	1	0.72332	E^+	1	0.72332	E^+	1	0.72332
DMU_{23}	E^-	0.797385	0.264114	E^+	1	0.88858	E^+	1	0.88858	E^+	1	0.88858
DMU_{24}	E^-	0.843132	0.282129	E^+	1	0.81652	E^+	1	0.81652	E^+	1	0.81652
DMU_{25}	E^-	0.7219	0.505602	E^+	1	0.88816	E^+	1	0.88816	E^+	1	0.88816
DMU_{26}	E^-	0.795597	0.281506	E^+	1	0.71999	E^+	1	0.71999	E^+	1	0.71999
DMU_{27}	E^+	0.91633	0.415216	E^+	1	0.7258	E^+	1	0.7258	E^+	1	0.7258
DMU_{28}	E^-	0.527599	0.414948	E^+	1	0.88503	E^+	1	0.88503	E^+	1	0.88503
DMU_{29}	E^-	0.814029	0.416796	E^+	1	0.77176	E^+	1	0.77176	E^+	1	0.77176
DMU_{30}	E^-	0.657961	0.176078	E^+	1	0.64859	E^+	1	0.64859	E^+	1	0.64859
DMU_{31}	E^-	0.742315	0.299342	E^+	1	0.92626	E^+	1	0.92626	E^+	1	0.92626
DMU_{32}	E^-	0.88779	0.318159	E^+	1	0.64284	E^+	1	0.64284	E^+	1	0.64284
DMU_{33}	E^-	0.604067	0.37433	E^+	1	0.90673	E^+	1	0.90673	E^+	1	0.90673
DMU_{34}	E^-	0.817298	0.307213	E^+	1	0.80355	E^+	1	0.80355	E^+	1	0.80355
DMU_{35}	E^-	0.792916	0.447938	E^+	1	0.59499	E^+	1	0.59499	E^+	1	0.59499
DMU_{36}	E^-	0.837919	0.365225	E^+	1	0.89488	E^+	1	0.89488	E^+	1	0.89488
DMU_{37}	E^-	0.61903	0.269156	E^+	1	0.89489	E^+	1	0.89489	E^+	1	0.89489
DMU_{38}	E^-	0.562977	0.195426	E^-	0.85628	0.41681	E^-	0.85628	0.41681	E^-	0.85628	0.41681
DMU_{39}	E^-	0.788523	0.29287	E^+	1	0.78696	E^+	1	0.78696	E^+	1	0.78696
DMU_{40}	E^-	0.736665	0.22403	E^+	1	0.66055	E^+	1	0.66055	E^+	1	0.66055
DMU	Sub-DMU_4 class	$[e^{+4}]^U$	$[e^{+4}]^L$	Sub-DMU_5 $[e^{+4}]^L$	$[e^{+5}]^U$	$[e^{+5}]^L$	Sub-DMU_6 class	$[e^{+6}]^U$	$[e^{+6}]^L$	Sub-DMU_7 Class	$[e^{+7}]^U$	$[e^{+7}]^L$
DMU_1	E^+	1	0.90756	E^+	1	0.90756	E^+	1	0.90756	E^+	1	0.90438
DMU_2	E^+	1	0.86894	E^+	1	0.86894	E^+	1	0.86894	E^+	1	0.98434
DMU_3	E^+	1	0.93726	E^+	1	0.93726	E^+	1	0.93726	E^-	0.90111	0.2303
DMU_4	E^+	1	0.89029	E^+	1	0.89029	E^+	1	0.89029	E^+	1	0.73051
DMU_5	E^+	1	0.58986	E^+	1	0.58986	E^+	1	0.58986	E^+	1	0.93601
DMU_6	E^+	1	0.86294	E^+	1	0.86294	E^+	1	0.86294	E^+	1	1
DMU_7	E^+	1	0.91086	E^+	1	0.91086	E^+	1	0.91086	E^-	0.74645	0.43997

Table 8 (continued)

DMU_8	E^-	0.84896	0.46367	E^-	0.84896	0.46367	E^-	0.84896	0.46367	E^+	1	1
DMU_9	E^+	1	0.80485	E^+	1	0.80485	E^+	1	0.80485	E^+	1	0.45019
DMU_{10}	E^+	1	0.90221	E^+	1	0.90221	E^+	1	0.90221	E^+	1	1
DMU_{11}	E^+	1	0.665	E^+	1	0.665	E^+	1	0.665	E^-	0.90192	0.40059
DMU_{12}	E^+	1	0.90155	E^+	1	0.90155	E^+	1	0.90155	E^+	1	0.7169
DMU_{13}	E^-	0.90864	0.49599	E^-	0.90864	0.49599	E^-	0.90864	0.49599	E^+	1	0.88523
DMU_{14}	E^+	1	0.91498	E^+	1	0.91498	E^+	1	0.91498	E^+	1	0.6205
DMU_{15}	E^+	1	0.81203	E^+	1	0.81203	E^+	1	0.81203	E^+	1	0.72703
DMU_{16}	E^+	1	0.84846	E^+	1	0.84846	E^+	1	0.84846	E^+	1	0.64104
DMU_{17}	E^+	1	0.75789	E^+	1	0.75789	E^+	1	0.75789	E^+	1	0.4753
DMU_{18}	E^+	1	0.89585	E^+	1	0.89585	E^+	1	0.89585	E^+	1	1
DMU_{19}	E^+	1	0.90476	E^+	1	0.90476	E^+	1	0.90476	E^+	1	1
DMU_{20}	E^+	1	0.93611	E^+	1	0.93611	E^+	1	0.93611	E^+	1	0.69918
DMU_{21}	E^+	1	0.90756	E^+	1	0.90756	E^+	1	0.90756	E^+	1	0.9344
DMU_{22}	E^+	1	0.86894	E^+	1	0.86894	E^+	1	0.86894	E^+	1	0.89108
DMU_{23}	E^+	1	0.93726	E^+	1	0.93726	E^+	1	0.93726	E^+	1	0.57197
DMU_{24}	E^+	1	0.89029	E^+	1	0.89029	E^+	1	0.89029	E^+	1	0.75959
DMU_{25}	E^+	1	0.58986	E^+	1	0.58986	E^+	1	0.58986	E^+	1	1
DMU_{26}	E^+	1	0.86294	E^+	1	0.86294	E^+	1	0.86294	E^+	1	0.87746
DMU_{27}	E^+	1	0.91086	E^+	1	0.91086	E^+	1	0.91086	E^+	1	0.71686
DMU_{28}	E^-	0.84896	0.46367	E^-	0.84896	0.46367	E^-	0.84896	0.46367	E^+	1	1
DMU_{29}	E^+	1	0.80485	E^+	1	0.80485	E^+	1	0.80485	E^+	1	0.77054
DMU_{30}	E^+	1	0.90221	E^+	1	0.90221	E^+	1	0.90221	E^+	1	0.61489
DMU_{31}	E^+	1	0.665	E^+	1	0.665	E^+	1	0.665	E^+	1	0.60653
DMU_{32}	E^+	1	0.90155	E^+	1	0.90155	E^+	1	0.90155	E^+	1	0.85507
DMU_{33}	E^-	0.90864	0.49599	E^-	0.90864	0.49599	E^-	0.90864	0.49599	E^+	1	1
DMU_{34}	E^+	1	0.91498	E^+	1	0.91498	E^+	1	0.91498	E^+	1	0.63306
DMU_{35}	E^+	1	0.81203	E^+	1	0.81203	E^+	1	0.81203	E^+	1	0.99322
DMU_{36}	E^+	1	0.84846	E^+	1	0.84846	E^+	1	0.84846	E^+	1	0.76169
DMU_{37}	E^+	1	0.75789	E^+	1	0.75789	E^+	1	0.75789	E^+	1	0.53105
DMU_{38}	E^+	1	0.89585	E^+	1	0.89585	E^+	1	0.89585	E^+	1	0.84903
DMU_{39}	E^+	1	0.90476	E^+	1	0.90476	E^+	1	0.90476	E^+	1	0.6993
DMU_{40}	E^+	1	0.93611	E^+	1	0.93611	E^+	1	0.93611	E^+	1	0.7168

mathematical models were executed on a Pentium IV portable PC with Core 2 duo CPU, 2 GHz, and Windows XP using 1 GB of RAM.

5.3.1 Calculating the interval efficiency scores and interpreting the results

Running the proposed models yielded unique interval efficiency scores for all the DMUs. The relative efficiency and the classification of the DMUs and the sub-DMUs are presented in Table 8. This table shows the source of inefficiency in each sub-DMU. It is notable that using the proposed network structure for a DMU, it is possible to find the inefficiency of sourcing, making, or delivery processes of a chain in each level of the providers of agility and the capabilities of agility.

Afterward, the possible efficiency of making, sourcing, and delivery processes can be easily calculated. These results are shown Table 9. The possible efficiency of the providers of agility and the capabilities of agility was also achieved and the results are presented in Table 10.

This information is useful for identifying the sub-processes with weak efficiencies and improving the overall performance of the supply chains in the context of the providers of agility and the capabilities of agility in the sourcing, the making, and the delivery processes. More formally, the proposed

Table 9 Maximum and minimum efficiency scores of sourcing, making, and delivery processes

DMU	Sourcing process		Making process		Delivery process	
	Maximum efficiency	Minimum efficiency	Maximum efficiency	Minimum efficiency	Maximum efficiency	Minimum efficiency
DMU_1	1	0.494369	1	0.638118	1	0.823665
DMU_2	1	0.523188	1	0.62852	1	0.755059
DMU_3	1	0.78957	1	0.832824	1	0.878447
DMU_4	1	0.666699	1	0.726936	1	0.792614
DMU_5	1	0.78882	1	0.523886	1	0.347933
DMU_6	1	0.518386	1	0.62131	1	0.74467
DMU_7	1	0.526783	1	0.661102	1	0.829669
DMU_8	1	0.783274	0.848964	0.410359	0.720741	0.214988
DMU_9	1	0.595606	1	0.621151	1	0.647791
DMU_{10}	1	0.420665	1	0.585164	1	0.81399
DMU_{11}	1	0.857966	1	0.615964	1	0.442222
DMU_{12}	1	0.413249	1	0.579559	1	0.812798
DMU_{13}	1	0.822157	0.908635	0.449728	0.825618	0.246006
DMU_{14}	1	0.645699	1	0.735233	1	0.837181
DMU_{15}	1	0.354013	1	0.483149	1	0.65939
DMU_{16}	1	0.800814	1	0.759269	1	0.719879
DMU_{17}	1	0.80082	1	0.678221	1	0.57439
DMU_{18}	0.733214	0.173728	0.856279	0.373396	1	0.802547
DMU_{19}	1	0.619309	1	0.71201	1	0.818588
DMU_{20}	1	0.436327	1	0.618345	1	0.876294
DMU_{21}	1	0.494369	1	0.638118	1	0.823665
DMU_{22}	1	0.523188	1	0.62852	1	0.755059
DMU_{23}	1	0.78957	1	0.832824	1	0.878447
DMU_{24}	1	0.666699	1	0.726936	1	0.792614
DMU_{25}	1	0.78882	1	0.523886	1	0.347933
DMU_{26}	1	0.518386	1	0.62131	1	0.74467
DMU_{27}	1	0.526783	1	0.661102	1	0.829669
DMU_{28}	1	0.783274	0.848964	0.410359	0.720741	0.214988
DMU_{29}	1	0.595606	1	0.621151	1	0.647791
DMU_{30}	1	0.420665	1	0.585164	1	0.81399
DMU_{31}	1	0.857966	1	0.615964	1	0.442222
DMU_{32}	1	0.413249	1	0.579559	1	0.812798
DMU_{33}	1	0.822157	0.908635	0.449728	0.825618	0.246006
DMU_{34}	1	0.645699	1	0.735233	1	0.837181
DMU_{35}	1	0.354013	1	0.483149	1	0.65939
DMU_{36}	1	0.800814	1	0.759269	1	0.719879
DMU_{37}	1	0.80082	1	0.678221	1	0.57439
DMU_{38}	0.733214	0.173728	0.856279	0.373396	1	0.802547
DMU_{39}	1	0.619309	1	0.71201	1	0.818588
DMU_{40}	1	0.436327	1	0.618345	1	0.876294

approach can be utilized to distinguish the relative efficiency scores of the sub-processes in a complicated network. Managers can identify the inefficient sub-DMUs and develop strategies to improve their performance and hence improve the overall efficiency of the sourcing, the making, and the delivery processes.

Table 10 Maximum and minimum efficiency scores of providers of agility and capabilities of agility

DMU	Providers of agility		Capabilities of agility	
	Maximum efficiency	Minimum efficiency	Maximum efficiency	Minimum efficiency
DMU_1	1	0.44867	1	0.579131
DMU_2	1	0.45462	1	0.546147
DMU_3	1	0.740028	1	0.780568
DMU_4	1	0.593555	1	0.647183
DMU_5	1	0.465292	1	0.309018
DMU_6	1	0.447337	1	0.536155
DMU_7	1	0.479827	1	0.602172
DMU_8	0.8489644	0.363179	0.720741	0.19027
DMU_9	1	0.479376	1	0.499936
DMU_{10}	1	0.37953	1	0.527943
DMU_{11}	1	0.570545	1	0.409614
DMU_{12}	1	0.372566	1	0.522503
DMU_{13}	0.908635	0.407781	0.825618	0.22306
DMU_{14}	1	0.590799	1	0.67272
DMU_{15}	1	0.287468	1	0.39233
DMU_{16}	1	0.679457	1	0.644207
DMU_{17}	1	0.60693	1	0.514013
DMU_{18}	0.7332142	0.155634	0.856279	0.334507
DMU_{19}	1	0.560325	1	0.644197
DMU_{20}	1	0.408448	1	0.578836
DMU_{21}	1	0.44867	1	0.579131
DMU_{22}	1	0.45462	1	0.546147
DMU_{23}	1	0.740028	1	0.780568
DMU_{24}	1	0.593555	1	0.647183
DMU_{25}	1	0.465292	1	0.309018
DMU_{26}	1	0.447337	1	0.536155
DMU_{27}	1	0.479827	1	0.602172
DMU_{28}	0.8489644	0.363179	0.720741	0.19027
DMU_{29}	1	0.479376	1	0.499936
DMU_{30}	1	0.37953	1	0.527943
DMU_{31}	1	0.570545	1	0.409614
DMU_{32}	1	0.372566	1	0.522503
DMU_{33}	0.908635	0.407781	0.825618	0.22306
DMU_{34}	1	0.590799	1	0.67272
DMU_{35}	1	0.287468	1	0.39233
DMU_{36}	1	0.679457	1	0.644207
DMU_{37}	1	0.60693	1	0.514013
DMU_{38}	0.7332142	0.155634	0.856279	0.334507
DMU_{39}	1	0.560325	1	0.644197
DMU_{40}	1	0.408448	1	0.578836

6 Conclusions and future research directions

Agility is a competitive advantage in supply chains. Measuring the performance of agility in supply chains is a complex task which involves a multitude of conflicting criteria. In this paper, we proposed a new NDEA model for measuring the performance of agility in supply chains. First, a network structure was constructed for a DMU and several sub-DMUs with uncertain inputs, intermediate measures, and outputs. The sub-DMUs formed the main processes of agility in the supply chain including the sourcing, the making, and the delivery processes. A sequence, in which the agility levels were transformed into a set of supply chain goals were considered for the sourcing, the making, and the delivery processes. The efficiency score of the DMU was decomposed into a series of efficiency scores for its sub-DMUs. We studied the bounded feasible region generated by the proposed model. The uncertainty of the inputs, the intermediate measures, and the outputs of the DMUs and the sub-DMUs were modeled with linguistic terms parameterized with fuzzy sets. A fuzzy NDEA model was constructed and the optimistic and pessimistic scenarios, including the multi-level optimization problems, were considered to calculate an efficiency score for each DMU and sub-DMU. The multi-level optimization problems were reduced into single-level optimization problems. Variable exchange was used to make the models linear and independent of the α-cut variables.

The proposed fuzzy NDEA approach served several distinctive features. First, the computational efforts were substantially reduced because the models were independent of the α-cut variables. Second, the global optimum solution was identified quickly since the proposed models were linear. Third, the models were proven to be always feasible and bounded. Fourth, there were no conflicting interpretation since the achieved ranks and interval efficiency scores were unique for each DMU. The Iranian dairy supply chains were selected to illustrate the applicability of the proposed model. The results were promising and the computational efforts were significantly reduced. The DMUs and the sub-DMUs were categorized based on their efficiency scores.

The proposed approach can be investigated in the envelopment form of the DEA models. The uncertainty of the inputs, the intermediate measures, and the outputs can be modeled in a more generic form for the L–R fuzzy numbers. We have assumed that the overall efficiency score of a DMU is the product of the efficiency scores of the sub-DMUs. This assumption can alternatively be changed using the weighted mean or geometric mean of the efficiency scores of sub-DMUs. The sensitivity and stability analysis of the efficiencies of the proposed fuzzy NDEA model can be another interesting research stream. Additional assumptions of the return to scale and an analysis of the proposed fuzzy NDEA in a multi-period planning horizon may also be considered in future research directions.

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