A full ranking method in data envelopment analysis with multi-criteria decision analysis

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Abstract: This study presents a new hybrid Multi-Criteria Decision Analysis (MCDA) model for the full ranking of Decision-Making Units (DMUs) with multiple inputs and outputs. The Best-Worst Method (BWM) is used to rank the units, and the Charnes-Cooper-Rhodes (CCR) Data Envelopment Analysis (DEA) model is utilised to construct the pairwise comparison vector. The unit with the lowest efficiency is identified and compared with other units using DEA for each pair of units. Similarly, the unit with the highest efficiency is identified next and compared with the different units. A linear programming problem is formulated and solved to find the optimal weight of the units and rank them. The pairwise comparisons in the proposed BWM-DEA method are highly consistent because of the objective evaluation process. The proposed method has several advantages, including fewer and more consistent comparisons, leading to more reliable results than similar ranking methods in DEA.

Keywords: DEA; data envelopment analysis; MCDA; multi-criteria decision analysis; BWM; best worst method; analytics hierarchy process; pairwise comparison; ranking.

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1 Introduction

Data Envelopment Analysis (DEA) is a mathematical optimisation method for measuring the performance of a group of similar Decision-Making Units (DMUs) by solving linear programming problems. DEA models measure the relative efficiency of a group of DMUs that use multiple inputs to produce multiple outputs. Given the lack of necessity to find universal relationships among all the DMUs, DEA can be used as a decision analysis tool in a variety of fields, including economics (Afsharian and Ahn, 2017; Omrani et al., 2022), engineering (Huang et al., 2021a, 2021b), banking (Afsharian and Ahn, 2014; Ben Lahouel et al., 2022), insurance industry (Banker et al., 2022), fashion industry (Pourhejazy et al., 2022), manufacturing (Hahn et al., 2021), energy sector (Alizadeh et al., 2020; Costa et al., 2022), transportation (Saen et al., 2022), education (Le et al., 2021; Agasisti et al., 2022), environmental management (Bronner et al., 2022; Wu et al., 2024), network design (Afsharian, 2021; Boloori et al., 2016) and agriculture (Yang et al., 2022). Instead, DEA allows each unit to have its production function and compares the efficiency of each unit to the efficiency of all other units in the data set to measure its efficiency. Because it does not make functional assumptions on the considered factors or the underlying process, DEA can handle intricate relationships between inputs and outputs of varying types of units (Charnes et al., 1997).

Charnes et al. (1979) proposed the original DEA model. The production possibility set in this model, Charnes, Cooper and (CCR) is defined based on the assumption of constant returns to scale. Banker et al. (1984) proposed its generalisation. The BCC model Banker, Charnes and Cooper developed considers variable returns to scale. For the most useful results, decision-makers must consider whether applying the DEA model assumes constant or variable returns to scale. Units expressing the efficient frontier according to the CCR or BCC models receive an efficiency score of 1. The efficiency of a unit is understood such that there is no unit or a set of units that would achieve greater efficiency in the best possible scenario. A score of one is assigned to all efficient units, while it is below 1 for the inefficient ones. While inefficient DMUs can be ranked using the efficiency score, efficient DMUs cannot be ranked since they all have the same maximum efficiency score (Zahedi-Seresht et al., 2021). Specifically, DEA divides units into the efficient category with a score of 100% and the inefficient category with less than 100%.

Decision-Makers (DMs) often consider the dichotomic classification performed with DEA to be insufficient due to poor discriminating power since efficient units cannot be compared with each other due to the same efficiency score. DMs want to rank all units under evaluation rather than only label data as efficient or inefficient. A modified approach is needed to rank all DMUs to get around the discriminatory flaw of DEA. Several solutions have been suggested throughout the years to increase the DEA's discriminatory power (Wang, 2020). This problem has been widely studied by DEA researchers, as a result of which several techniques based on different approaches have been introduced. All DEA ranking techniques attempt to rank the DMUs from the best to worst, but they are based on various principles.

This study aimed to propose a new method for ranking decision-making units with multiple inputs and outputs, in which the concepts of BWM (Rezaei, 2015) are used for pairwise comparisons of units.

The main contribution of this paper is a new DEA ranking method that, different from existing MCDA-based methods like AHP-DEA, derives the ranking of DMUs based on pairwise comparisons in a novel way. This paper shows that the proposed approach uses less comparison data than other MCDA-based methods, leading to more reliable results compatible with DEA classification. The proposed method is simple enough, easy to use and especially useful for real-world problems with many DMUs. Pairwise comparisons in this method are highly consistent because the data is non-subjective, which means that the comparison vector is derived mathematically from the input/output data and is not based on the subjective evaluation of a decision maker. The proposed model eliminates the subjective evaluation of BWM and overcomes the inefficiency of DEA ranking.

The remainder of this paper is organised as follows. Section 2 summarises the research conducted on ranking DMUs and discusses some ranking methods that can be used in the context of DEA. Section 3 presents the mathematical details of the proposed model for ranking DMUs with multiple inputs and/or outputs. Section 4 includes some numerical examples of the proposed method and Section 5 presents result and the last Section 6 concludes and suggests some directions for future research.

2 Literature review

Since its introduction in 1978, the field of DEA has seen an explosion of publications and research, leading to significant developments in its methodology and models. In this development, a research subfield is focused on increasing the DEA's discriminatory power by improving its ability to fully rank units in each data set studied. As shown by the papers published by Aldamak and Zolfaghari (2017) and Labijak-Kowalska and Kadziński (2021), there are different approaches for ranking all units, which can be categorised into several main groups. Since the purpose of this study is not to provide a comprehensive overview of all of these approaches, apart from discussing the mathematical background, the most common approaches will be described in the following (for more details, refer to the cited references above).

The cross-efficiency evaluation, proposed by Sexton et al. (1986), is the most well-known ranking method used in the context of DEA (Zhu et al., 2021). It uses peer evaluation rather than pure self-evaluation to evaluate efficiency. Cross-evaluation eliminates unrealistic weight schemes without asking the DMs to provide a set of weight preferences (Tavana et al., 2021; Abolghasem et al., 2019). Despite its benefits, DEA cross-efficiency evaluation still has a major drawback related to the non-uniqueness of the optimal weights for each unit (Shi et al., 2019).

The second group of ranking methods uses the concept of super-efficiency, where the main idea is to evaluate the unit under evaluation over the linear combination of all other units (Andersen and Petersen, 1993). Although the super-efficiency approach is widely used in DEA due to its ability to identify outlying units and provide sensitivity analysis, its main drawback is the issue of infeasibility for specific units (Lin et al., 2019).

The third group of methods is based on post-statistical analysis, where a common weight is found and then used as a reference value for ranking all units. In this approach, decision-maker participation is high, and several works have been suggested in the literature to ensure that the criteria used by DMs remain unbiased (Hatami-Marbini et al., 2015).

The fourth group of methodologies is based on benchmarking units concerning their usefulness compared to other DMUs. It determines the relative importance of efficient DMUs concerning their role as references for inefficient units (Torgersen et al., 1996). In other words, it evaluates how frequently efficient DMUs are used to refer to inefficient DMUs. This approach is also broadly applied in DEA due to its simplicity (Krüger, 2018).

The fifth group is based on approaches that rank inefficient units. Since the standard DEA score for inefficient units is sufficient to rank them based on their scores, most existing methods do not seek to rank inefficient DMUs. In this regard, Bardhan et al. (1996) introduced a method to rank inefficient units. They ranked units according to a criterion known as efficiency dominance, which considers the input and output values of each DMU.

The sixth group includes complex methods for ranking all efficient units via MCDM modelling. There are many MCDM papers in the field of DEA, but relatively few have focused on examining the discriminatory power of DEA models (Sinuany-Stern et al., 2000).

The seventh group involves introducing one or multiple virtual DMUs to the data set. This concept was first introduced by Wang and Luo (2006) when they suggested a pair of virtual DMUs, namely an ideal Decision-Making Unit (IDMU) and an Anti-ideal Decision-Making Unit (ADMU). IDMU consumes the lowest inputs and produces the maximum outputs, while ADMU consumes the maximum inputs to produce the minimum outputs (Kritikos, 2017). Table 1 presents some strengths and weaknesses of the main ranking methods.

Table 1 Main advantages and disadvantages of the main ranking methods

Method	*Advantages*	*Disadvantages*
Cross-efficiency	Peer and unbiased self-evaluation. Multiple weight vectors were considered.	Neglects actual score. A limited set of common weights.
Super-efficiency	Detecting outliers. Simplicity. Efficiency scores greater than 100%.	Occasional infeasible results. Ranks only efficient units.
Statistical-based methods	Ranks all units. Detect fitting errors. The common basis for the comparison of units.	Occasional infeasible results. Inefficient units can be ranked at the top. Complex application.
Benchmarking	Simple and direct application. Investigates the impact of efficient units on the inefficient ones.	Ranks only efficient units.
Inefficient DMUs	Detects worse units.	Ranks only inefficient units.
MCDA-based methods	Full ranking. Incorporates DMUs' cross-efficiency comparisons.	Inefficient units can be ranked at the top. Complex methodology.
Virtual DMU	Full ranking. Simplicity and flexibility.	Changes the original set of DMUs.

As mentioned, each method has its limitations, and none of these approaches provides a perfect model for the complete ranking of units in the context of DEA. In this paper, to make another attempt to fully rank units, a new hybrid model is developed that combines BWM and DEA as two popular and widely used methods to overcome their limitations while taking advantage of each. Many works in the literature have used BWM in the context of DEA, as presented in Table 2.

Table 2 Research works on the use of BWM in the DEA context

Reference	*Methodology*	*The reason for using BWM*
(Chen et al., 2022)	BWM, DEA Trapezoidal Interval Type-2 Fuzzy (TrIT2F)	To determine the weights of criteria and decision-makers.
(Eskandari et al., 2022)	BWM, DEA Strength-Weakness-Opportunity-Threat (SWOT) Analysis	To calculate the significance of each sub-indicator.
(Mobarezkhoo et al., 2022)	BWM, DEA Combinative Distance-Based Assessment (CODAS)	To determine the suitable location for constructing biorefinery based on economic, social, and environmental criteria.

Table 2 Research works on the use of BWM in the DEA context (continued)

Reference	*Methodology*	*The reason for using BWM*
(Jabbari et al., 2022)	BWM, DEA Decision support system (DSS)	To find the final outputs.
(Azizi et al., 2022)	BWM, DEA SWOT Analysis	To determine the relative importance of Resilience engineering indicators.
(Chetan et al., 2022)	BWM, DEA	To determine the weights of the criteria.
(Samieinasab et al., 2022)	BWM, DEA SWOT Analysis	To determine the relative importance of each indicator.
(Omrani et al., 2021b)	BWM, Robust DEA (RDEA)	To incorporate the DMs' preferences into the RDEA model.
(Huang et al., 2021a)	Fuzzy BWM, DEA	To calculate the weight values for each input and output indicator.
(Wang et al., 2021)	Fuzzy BWM, DEA	To determine the weights of the criteria.
(Omrani et al., 2021a)	BWM, Robust Credibility DEA (RCDEA)	To incorporate the DMs' judgment into the RCDEA model.
(Mei and Chen, 2021)	Rough-fuzzy BWM Rough-fuzzy DEA	To determine the relative weights of sustainability criteria.
(Qin et al., 2021)	BWM, DEA	To restrict weights of different criteria with decision-maker's preferences and judgments.
(Omrani et al., 2020b)	BWM, DEA -based Road Safety (DEA-RS)	To incorporate DMs' preferences into the decision-making process and overcome the weight flexibility shortcoming of the DEA-RS model.
(Chen and Ming, 2020)	Rough-fuzzy BWM Rough-fuzzy DEA	To determine the weights of evaluation criteria.
(Fan et al., 2020)	BWM, DEA, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)	To solve the shortcomings of the final result of the TOPSIS method.
(Kolagar et al., 2020)	Fuzzy BWM, DEA	To determine the weights of sustainability criteria.
(Omrani et al., 2020a)	BWM, Common Weight DEA (CWDEA)	To consider DMs' preferences in DEA.
(Starčević et al., 2019)	BWM, DEA, AHP, FUCOM	To determine the criteria weights and compare the results with AHP and DEA.
(Motevali and Torabi, 2018)	BWM, DEA SWOT Analysis	To obtain the weights of sustainable and resilience indicators.

As shown in Table 2, many researchers have used BWM in the DEA context in recent years, but with a main difference compared to the approach proposed in this research. In the literature, BWM has been applied to incorporate the decision maker's preferences into the DEA model to find the weights of inputs and outputs and, generally, to determine the criteria weights. At the same time, in our research, BWM will be used to rank the units.

3 The BWM-DEA ranking model

This model integrates the DEA and BWM models, which advances the DEA analysis beyond the simple classification of efficient or inefficient to a complete ranking by including some BWM components as a supplementary analysis. BWM uses pairwise comparisons between criteria/units that the decision maker subjectively evaluates to rank them. While the pairwise comparison matrix data in the original BWM are subjective (the decision maker's preferences), the data of the pairwise comparison matrix in our proposed model is non-subjective and based on running DEA, according to the input/output data of each pair of units. This objective approach is more convenient from the decision-makers' point of view because it eliminates the burden of evaluating alternatives subjectively. As a vector-based method, the proposed model requires fewer comparisons than matrix-based methods like AHP/DEA (Sinuany-Stern et al., 2000), which reduces the number of comparisons. Hence, there is no need to do many time-consuming calculations for pairwise comparison of all units.

The new BWM-DEA ranking model identifies the worst DMU (DMU with the lowest efficiency) first. Pairwise comparisons are conducted between this DMU (worst) and the other DMUs. The best DMU (DMU with the highest efficiency) is then identified concerning the obtained vector in the previous step. Pairwise comparisons are also conducted between this DMU (best) and the others. Finally, a maximin problem is formulated and solved to determine the ranks of DMUs.

3.1 Steps of BWM-DEA

In this section, we describe the steps of BWM-DEA that can be used to rank the units fully.

Step 1 (Identify the worst unit): In this step, the unit with the lowest efficiency score is considered as unit_W after solving the classical DEA model.

Step 2 (Execute the pairwise comparisons for $unit_W$): In this step, four DEA models are solved for the pairwise comparison of each unit with unit_W, as a result of which a_{jW} is obtained. The obtained Others-to-Worst vector is:

$$A_W = \left(a_{1W}, a_{2W}, \ldots, a_{nW}\right)^T$$

where a_{jW} indicates the evaluation of unit_j over unit_W. Suppose we have n organisational units. Each unit has m inputs and s outputs, where X_{ij} is input i of unit j and Y_{rj} is output r of unit j. we perform the following DEA runs, assuming that just unit_j and unit_W exist.

Problems *WW*:

$$
\begin{aligned}
& E_{WW} = \max_{u_r, v_i} \sum_{r=1}^{s} u_r y_{rW} \\
& st: \sum_{i=1}^{m} v_i x_{iW} = 1 \\
& \sum_{r=1}^{s} u_r y_{rW} \le 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{,} v_i x_{ij} \le o \\
& u_r, v_i \ge o
\end{aligned}
\tag{1}
$$

Problems *jW*:

$$
\begin{aligned}
& E_{jW} = \max_{u_r, v_i} \sum_{r=1}^{s} u_r y_{rj} \\
& st: \sum_{i=1}^{m} v_i x_{ij} = 1 \\
& \sum_{r=l}^{s} u_r y_{rj} \le 1 \\
& \sum_{r=l}^{s} u_r y_{rW} - E_{WW} \sum_{i=l}^{m} v_i x_{iW} = 0 \\
& u_r, v_i \ge o
\end{aligned}
\tag{2}
$$

where E_{jW} is the optimal cross-evaluation of unit_j and E_{WW} is the optimal efficiency value of unit_W. Similarly, E_{Wj} and E_{jj} are determined after solving Problems Wj and jj. Using the results of the paired DEA mentioned above, we then construct the first pairwise comparison vector required for BWM-DEA, so that for every pair of unit_W and unit_j, we have:

$$
a_{jW} = \frac{E_{jj} + E_{jW}}{E_{WW} + E_{Wj}} \tag{3}
$$

Step 3 (Identify the best unit): One of the a_{jW} obtained in step 2 belongs to the evaluation of the best unit over unit_W, and the largest a_{jW} represents this issue. This a_{jW} is called a_{BW} and its j indicates the unit with the highest efficiency. The corresponding unit is considered as unit_B.

Step 4 (Execute the pairwise comparisons for $unit_B$): In this step, for the pairwise comparison of unit_B with each unit, similar to step 2, four DEA models are solved, as a result of which a_{Bj} is obtained. The obtained Best-to-Others vector is:

$$A_B = (a_{B1}, a_{B2}, \dots, a_{Bn})$$

where a_{Bj} indicates the evaluation of unit_B over unit_j. The following DEA runs are performed for any pair of units, unit_B and unit_j.

Problems *BB*:

$$\begin{aligned}
&E_{BB} = \max_{u_r, v_i} \sum_{r=1}^{s} u_r y_{rB} \\
&st: \sum_{i=1}^{m} v_i x_{iB} = 1 \\
&\sum_{r=1}^{s} u_r y_{rB} \leq 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{,} v_i x_{ij} \leq o \\
&u_r, v_i \geq o
\end{aligned} \tag{4}$$

Problems *jB*:

$$\begin{aligned}
&E_{jB} = \max_{u_r, v_i} \sum_{r=1}^{s} u_r y_{rj} \\
&st: \sum_{i=1}^{m} v_i x_{ij} = 1 \\
&\sum_{r=l}^{s} u_r y_{rj} \leq 1 \\
&\sum_{r=l}^{s} u_r y_{rB} - E_{BB} \sum_{i=l}^{m} v_i x_{iB} = 0 \\
&u_r, v_i \geq o
\end{aligned} \tag{5}$$

where E_{jB} is the optimal cross-evaluation of unit_j and E_{BB} is the optimal efficiency value of unit_B. Similarly, E_{Bj} and E_{jj} are determined after solving Problems Bj and jj. Using the results of the paired DEA mentioned above, we then construct the second pairwise comparison vector required for BWM-DEA so that for every pair of unit_B and unit_j, we have:

$$a_{Bj} = \frac{E_{BB} + E_{Bj}}{E_{jj} + E_{jB}} \tag{6}$$

Step 5 (Find the optimal weights): The optimal weight for the units is the one where for each pair of w_B / w_j and w_j / w_W, we should have $w_B / w_j = a_{Bj}$ and $w_j / w_W = a_{jW}$. To satisfy these conditions for all j, we should determine a solution where the maximum absolute gaps $\left|\frac{w_B}{w_j} - a_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - a_{jW}\right|$ for all j are minimised. Therefore, to determine the optimal weights, the constrained optimisation problem can be obtained as follows:

$$\begin{aligned}
&\min \max_j \left\{ \left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right| \right\} \\
&st: \sum_{j=1}^{n} w_j = 1 \\
&w_j \geq o, \text{for all } j
\end{aligned} \tag{7}$$

Equation (7) can be transformed into the following non-linear problem:

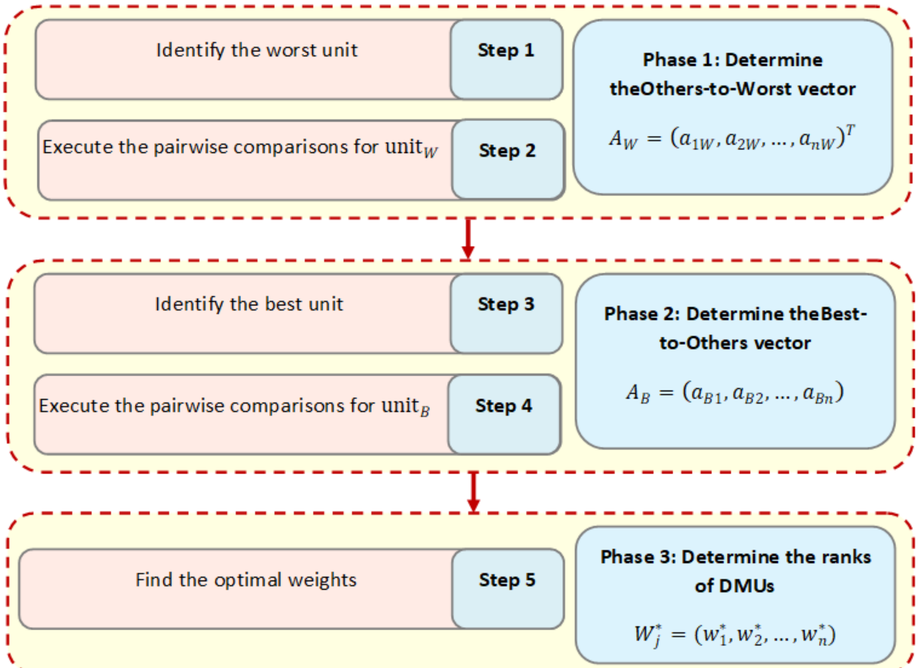
$$\begin{aligned}
&\min \xi \\
&st \\
&\left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi, \text{ for all } j \\
&\left|\frac{w_j}{w_W} - a_{jW}\right| \leq \xi, \text{for all } j \\
&\sum_{j=1}^{n} w_j = 1 \\
&w_j \geq o, \text{for all } j
\end{aligned} \tag{8}$$

The linear model of the problem (8) is presented as follows (Rezaei, 2016).

$$\begin{aligned}
&\min \xi \\
&st: \\
&\left|w_B - a_{Bj} w_j\right| \leq \xi, \text{ for all } j \\
&\left|w_j - a_{jW} w_W\right| \leq \xi, \text{for all } j \\
&\sum_{j=1}^{n} w_j = 1 \\
&w_j \geq o, \text{for all } j
\end{aligned} \tag{9}$$

The optimal weights $\left(w_1^*, w_2^*, ..., w_n^*\right)$ and ξ^* are obtained by solving the problem (9). We give the rank 1 to the DMU with the maximum value of w_j, etc., in descending order of w_j. The consistency ratio can be presented using ξ^*. There is no inconsistency because the data in the pairwise comparison vector is non-subjective. A flowchart that summarises the method is shown in Figure 1.

Figure 1 Flowchart of the proposed method (see online version for colours)



4 Numerical illustrations

In this section, we have provided four numerical examples to illustrate the reliability and applicability of the proposed method.

Example 1: *In this example, we consider the numerical example that Sinuany-Stern et al. (2000) used to introduce the AHP/DEA method. Table 3 shows data, including five units with two inputs and two outputs. In the last two columns of the table, efficiency scores are computed by the CCR model, and AHP/DEA ranking is presented. According to the efficiency scores of the units, the lowest efficiency score is related to unit 3, so unit 3 is considered as* unit_w.

Table 3 Inputs, outputs and efficiency scores

DMUs	*Inputs*		*Outputs*		*CCR score*	*AHP/DEA ranks*
	i_1	i_2	o_1	o_2		
DMU_1	50	55	10	56	1	2–3
DMU_2	130	60	12	78	1	2–3
DMU_3	68	96	45	9	0.85	4
DMU_4	45	30	35	18	1	1

After the unit with the lowest efficiency score is determined, four DEA models according to equations (1) and (2) are solved for the pairwise comparison of each unit with unit_W, as a result of which a_{jW} is obtained according to equation (3) (see Table 4). Each a_{jW} indicates the evaluation of unit_j over unit_W.

Table 4 Pairwise comparison vector for the worst DMU

DMUs	*Worst DMU: DMU_3*
DMU_1	1
DMU_2	1
DMU_4	1.1754

One of the a_{jW} obtained in the previous step is related to the evaluation of the unit with the highest efficiency over unit_W. Certainly, the largest number calculated in a_{jW} indicates this evaluation, which is the same as a_{BW} and its corresponding unit is the unit with the highest efficiency. After unit_B is determined, four DEA models according to equations (4) and (5) are solved for the pairwise comparison of unit_B with each unit, as a result of which a_{Bj} is obtained according to equation (6) (see Table 5).

Table 5 Pairwise comparison vector for the best DMU

DMUs	DMU_1	DMU_2
Best DMU: DMU_4	1	1

After the pairwise comparison vector for unit_B and unit_W is determined, using the obtained A_W and A_B, the problem is formulated according to equation (9) as follows:

$$\min \xi$$

$$|W_4 - W_1| \leq \xi \ , \ |W_4 - W_2| \leq \xi$$

$$|W_4 - 1.1754 * W_3| \leq \xi \ , \ |W_1 - W_3| \leq \xi \ \#$$

$$|W_2 - W_3| \leq \xi \ , \ W_1 + W_2 + W_3 + W_4 = 1 \ \#$$

$$W_1, W_2, W_3, W_4 \geq 0$$

The optimal weights of DMUs are obtained by solving the above problem (see Table 6). Unit 3 in the DEA classification was inefficient and had the lowest efficiency score, ranking fourth in the proposed method's ranking. In the comparison vector A_W, the largest a_{jW} was related to the evaluation of unit 4, which represented the unit with the highest efficiency and was considered as unit_B. After solving the problem, we see that unit 4 is ranked first. The ranking of other units is also compatible with the DEA classification.

Table 6 Values *Wj* and final ranks of DMUs

DMUs	DMU_1	DMU_2	DMU_3	DMU_4
Wj	0.2500	0.2500	0.2362	0.2638
Rank	2-3	2-3	4	1

This example is solved by the AHP/DEA method in Sinuany-Stern et al. (2000), the ranking of which is given in Table 3. As can be seen in this simple example, the ranking is the same in both methods, with the difference that in the proposed method, the ranking is obtained through a simple process and with only two comparison vectors (without the need for an $n \times n$ pairwise comparison matrix). Obviously, the ranking of the two methods is not always the same, so in order to better evaluate the proposed method and compare it with the AHP/DEA, three real-life numerical examples are examined in the following.

Example 2: *In this example, we consider the data set related to five retail store branches with two inputs (material costs and labour costs) and two outputs (Sales of shoes and Sales of bags), presented in Table 7. In the last column of this table, the efficiency score of each DMU is calculated, which shows that units 1, 4 and 5 are efficient and units 2 and 3 are inefficient. The lowest efficiency score is related to unit 2, which is considered as* $unit_W$.

Table 7 Inputs, outputs, and efficiency scores

DMUs	*Inputs*		*Outputs*		*CCR score*
	i_1	i_2	o_1	o_2	
DMU_1	408	1545	693	1284	1
DMU_2	2949	1068	270	36	0.21
DMU_3	1662	1905	597	54	0.34
DMU_4	2880	2310	2757	855	1
DMU_5	2541	2679	69	2259	1

Now that unit_W is identified, the other units are compared with unit 2 to obtain the pairwise comparison vector A_W (see Table 8).

Table 8 Pairwise comparison vector for the worst DMU

DMUs	*Worst DMU:* DMU_2
DMU_1	1.7744
DMU_3	1
DMU_4	4.721
DMU_5	1

In the comparison vector A_W , the largest a_{jW} corresponds to unit 4, so unit_B is unit 4 and is compared with other units to obtain the pairwise comparison vector A_B (see Table 9).

Table 9 Pairwise comparison vector for the best DMU

DMUs	DMU_1	DMU_3	DMU_5
Best DMU: DMU_4	1	2.665	1

Now that the comparison vectors A_W and A_B are obtained, the problem is formulated and solved as follows to obtain the optimal weights of DMUs (see Table 10).

$$\min \xi$$

$$|W_4 - 1 * W_1| \le \xi \ , \ |W_4 - 4.721 * W_2| \le \xi$$

$$|W_4 - 2.665 * W_3| \le \xi \ , \ |W_4 - 1 * W_5| \le \xi \ , \ |W_1 - 1.774 * W_2| \le \xi$$

$$|W_3 - 1 * W_2| \le \xi \ , \ |W5 - 1 * W_2| \le \xi \ , \ W_1 + W_2 + W_3 + W_4 + W_5 = 1 \ \#$$

$$W_1, W_2, W_3, W_4, W_5 \ge 0$$

Unit 2 had the lowest efficiency score in the DEA ranking, ranked 5th here. In the calculations of the proposed method, it was found that unit 4 is the most efficient, which is also assigned the 1st rank. The ranking of other units is also compatible with the ranks of the DEA efficiency score.

Table 10 Values *Wj* and final ranks of DMUs

DMUs	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
Wj	0. 2619	0. 0869	0. 1539	0. 3024	0. 1946
Rank	2	5	4	1	3

Example 3: *In this example, the data of six private banks in Shiraz are considered. These banks consume three inputs (number of employees, assets, and equity) to produce three outputs (interest income, deposits and loans), which are presented in Table 11. Each unit's efficiency scores were calculated, showing that units 1, 2 and 6 are efficient and units 3, 4 and 5 are inefficient. Among the inefficient units, the lowest efficiency score is related to unit 3, which is considered as* $unit_W$.

Table 11 Inputs, outputs and efficiency scores

DMUs	*Inputs*			*Outputs*			*CCR score*
	i_1	i_2	i_3	o_1	o_2	o_3	
DMU_1	10	3000	3000	84	160	160	1
DMU_2	14	9000	3200	94	192	177	1
DMU_3	24	8000	6000	64	170	150	0.492
DMU_4	22	7000	5000	88	150	169	0.618
DMU_5	30	12000	3600	50	170	151	0.719
DMU_6	18	6000	2400	92	190	140	1

After identifying $unit_W$, we compare other units with it to obtain the pairwise comparison vector A_W (see Table 12).

Table 12 Pairwise comparison vector for the worst DMU

DMUs	*Worst DMU: DMU_3*
DMU_1	1.8822
DMU_2	1.0039
DMU_4	1
DMU_5	1
DMU_6	1.2444

The largest member in the comparison vector A_W corresponds to the evaluation of Unit 1 over Unit 3, indicating that Unit 1 is the most efficient unit. Unit 1 is then compared with other units to obtain the pairwise comparison vector A_B (see Table 13).

Table 13 Pairwise comparison vector for the best DMU

DMUs	*DMU_2*	*DMU_4*	*DMU_5*	*DMU_6*
Best DMU: DMU_1	1	1.5778	1.1294	1

After obtaining the comparison vectors A_W and A_B, to determine the ranks of DMUs, the problem is formulated and solved as follows (Table 14).

$$\min \xi$$

$$|W_1 - W_2| \leq \xi \ , \ |W_1 - 1.8822 * W_3| \leq \xi$$

$$|W_1 - 1.5778 * W_4| \leq \xi \ , \ |W_1 - 1.1294 * W_5| \leq \xi \ \#$$

$$|W_1 - W_6| \leq \xi \ , \ |W_2 - 1.0039 * W_3| \leq \xi \ , \ |W_4 - W_3| \leq \xi \ \#$$

$$|W_5 - W_3| \leq \xi \ , \ |W_6 - 1.2444 * W_3| \leq \xi \ , \ W_1 + W_2 + W_3 + W_4 + W_5 + W_6 = 1 \ \#$$

$$W_1, W_2, W_3, W_4, W_5, W_6 \geq 0$$

Table 14 Values *Wj* and final ranks of DMUs

DMUs	*DMU_1*	*DMU_2*	*DMU_3*	*DMU_4*	*DMU_5*	*DMU_6*
Wj	0.2009	0.1639	0.1264	0.1508	0.1634	0.1943
Rank	1	3	6	5	4	2

The lowest efficiency score in the DEA ranking was related to unit 3, ranked 6th in the proposed method. The highest efficiency was related to unit 1, also ranked first here. The ranking of other units is also compatible with the DEA classification.

Example 4: *In this example, we consider 12 insurance agencies with four inputs, including fixed asset value, operating expenses, number of employees, and liquid investment, and three outputs, including profits, total investment income and premium issued. The data for these insurance agencies can be seen in Table 15.*

Table 15 Inputs, outputs, and efficiency scores

DMUs	*Inputs*				*Outputs*			*CCR score*
	i_1	i_2	i_3	i_4	o_1	o_2	o_3	
DMU_1	50	40	4	12	266272	2350694	1769432	0.841
DMU_2	90	110	6	12	1090210	7833828	5498064	1
DMU_3	70	150	6	10	306634	2612282	1713094	0.323
DMU_4	80	32	4	12	185210	1791798	538784	0.377
DMU_5	20	26	4	12	165682	1054108	227732	0.374
DMU_6	24	24	4	12	293106	2641014	1203338	0.996
DMU_7	60	140	4	12	2384294	15203492	6355946	1
DMU_8	70	50	4	12	342544	2046518	536072	0.402
DMU_9	70	36	6	12	539142	5972550	2045520	1
DMU_{10}	40	42	6	12	285110	2047866	773902	0.408
DMU_{11}	30	38	4	12	178596	2272850	1596948	0.847
DMU_{12}	40	32	4	10	100462	1344182	84370	0.301

The CCR model computes efficiency scores in the last column of Table 15. It can be seen that the lowest efficiency score is related to unit 12, so this unit is considered as unit_W. In the next step, other units are compared with this unit, as a result of which the pairwise comparison vector A_W is obtained (see Table 16).

Table 16 Pairwise comparison vector for the worst DMU

DMUs	*Worst DMU: DMU_{12}*
DMU_1	1.4
DMU_2	2.59
DMU_3	1.1
DMU_4	1
DMU_5	1
DMU_6	1.639
DMU_7	7.575
DMU_8	1
DMU_9	2.544
DMU_{10}	1.016
DMU_{11}	1.41

The largest member of the vector A_W indicates the evaluation of the unit with the highest efficiency over unit_W. As can be seen, $a_{7W} = 7.575$ is the largest member of this vector, which shows that unit 7 is the most efficient unit. Therefore, unit 7 is considered as unit_B and is compared with other units to obtain the pairwise comparison vector A_B (Table 17).

Table 17 Pairwise comparison vector for the best DMU

DMUs	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_8	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}
Best DMU: DMU_7	1.449	1	1	1.941	2.673	1	1.426	1	2.232	1.038	7.575

Now that the comparison vectors A_W and A_B are obtained, the problem is formulated and solved as follows to obtain W_j^* (see Table 18):

$$\min \xi$$

$$|W_7 - 1.449 * W_1| \le \xi \ , \ |W_7 - W_2| \le \xi \ , \ |W_7 - W_3| \le \xi$$

$$|W_7 - 1.941 * W_4| \le \xi \ , \ |W_7 - 2.673 * W_5| \le \xi \ , \ |W_7 - W_6| \le \xi \ \#$$

$$|W_7 - 1.426 * W_8| \le \xi \ , \ |W_7 - W_9| \le \xi \ , \ |W_7 - 2.232 * W_{10}| \le \xi$$

$$|W_7 - 1.038 * W_{11}| \le \xi \ , \ |W_7 - 7.575 * W_{12}| \le \xi \ , \ |W_1 - 1.4 * W_{12}| \le \xi \ \#$$

$$|W_2 - 2.59 * W_{12}| \le \xi \ , \ |W_3 - 1.1 * W_{12}| \le \xi \ , \ |W_4 - W_{12}| \le \xi \ , \ |W_5 - W_{12}| \le \xi$$

$$|W_6 - 1.639 * W_{12}| \le \xi \ , \ |W_8 - W_{12}| \le \xi \ , \ |W_9 - 2.544 * W_{12}| \le \xi \ , \ |W_{10} - 1.016 * W_{12}| \le \xi$$

$$|W_{11} - 1.41 * W_{12}| \le \xi \ ,$$

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} + W_{11} + W_{12} = 1$$

$$W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11}, W_{12} \ge 0$$

In the DEA efficiency score ranks, unit 12 had the lowest efficiency, which is also ranked last in the proposed method. In addition, the evaluation of unit 7 over unit 12 was larger than others and was considered as unit_B which is also ranked first here. The ranking of other units also has acceptable match with the DEA ranking.

Table 18 Values Wj and final ranks of DMUs

DMUs	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}
Wj	0.084	0.1121	0.0769	0.0745	0.0669	0.0896	0.1278	0.0745	0.1110	0.0749	0.0842	0.0236
Rank	6	2	7	9-10	11	4	1	9-10	3	8	5	12

5 Results and comparisons

Four illustrative examples are employed to verify the applicability of the proposed method and for all four examples, ranking was also done by AHP/DEA to compare the results (see Table 19). In this table, the DEA classification (CCR version) is also given so that the results can be compared with it. In examples 1 and 2, the ranking of the proposed method is the same as the AHP/DEA, and both methods are perfectly compatible with the DEA results. In example 3, the ranking of the proposed method was relatively superior to the AHP/DEA. This means that the ranking of both methods is compatible with the DEA classification, and inefficient units did not get a higher rank than efficient units. Still, the proposed method is more compatible with the DEA classification than AHP/DEA because the ranking order of inefficient units has not changed. In example 4, the ranking of the proposed method is compatible with the DEA classification, which means that the inefficient units are not ranked better than the efficient units. However, this compatibility is not seen in AHP/DEA, and unit 6, which was inefficient, was placed higher than unit 9, which was efficient in the DEA ranking.

Of course, it should be said that the proposed method cannot provide this maximum compatibility for all problems, but applying this method to different problems showed that it provides a reasonable and acceptable ranking. In addition, the number of comparisons is greatly reduced, and a simple process greatly shortens the problem-solving time. Also, in the proposed method, pairwise comparisons of BWM are used, which perform better than other methods based on pairwise comparisons, such as AHP, because they make more consistent comparisons and produce more reliable results. In AHP/DEA, a large number of pairwise comparison matrix values is 1, which affects the ranking. Still, since only two comparison vectors are obtained in the proposed method, it reduces the number of generated 1 s and provides a more accurate ranking. Also, in the proposed method, the data in the pairwise comparison vector is non-subjective and derived mathematically, so there is no need to calculate consistency. However, the consistency ratio for all four problems was tested and was below 0.1.

Although it cannot be said that there is a complete compatibility between BWM-DEA and DEA, in practice, we found a perfect match for many examples. However, the goodness of fit between the two models can be tested statistically. Therefore, the performance of the Spearman rank-order correlation coefficient was examined for these four examples to measure the compatibility between BWM-DEA and DEA. In addition, the rank order correlation between AHP/DEA and DEA was also calculated for comparison, the results of which are shown in Table 20. Spearman's rank correlation measures the strength and direction of association between two ranked variables.

Table 19 Results of ranking for three methods: DEA, AHP-DEA and BWM-DEA

Example	*DMUs*	*DEA score*	*DEA ranks*	*AH-DEA score*	*AH-DEA ranks*	*BWM-DEA score*	*BWM-DEA ranks*
Example 1	DMU_1	1	1	0.4994	2-3	0.2500	2–3
	DMU_2	1	1	0.4994	2-3	0.2500	2–3
	DMU_3	0.85	4	0.4800	4	0.2362	4
	DMU_4	1	1	0.5204	1	0.2638	1
Example 2	DMU_1	1	1	0.2312	2	0.2619	2
	DMU_2	0.211	5	0.1199	5	0.0869	5
	DMU_3	0.344	4	0.1412	4	0.1539	4
	DMU_4	1	1	0.2956	1	0.3024	1
	DMU_5	1	1	0.2121	3	0.1946	3
Example 3	DMU_1	1	1	0.2018	1	0.2009	1
	DMU_2	1	1	0.1737	3	0.1639	3
	DMU_3	0.492	6	0.1423	6	0.1264	6
	DMU_4	0.618	5	0.153	4	0.1508	5
	DMU_5	0.719	4	0.1476	5	0.1634	4
	DMU_6	1	1	0.1812	2	0.1943	2

Table 19 Results of ranking for three methods: DEA, AHP-DEA and BWM-DEA (continued)

Example	*DMUs*	*DEA score*	*DEA ranks*	*AH-DEA score*	*AH-DEA ranks*	*BWM-DEA score*	*BWM-DEA ranks*
Example 4	DMU_1	0.841	6	0.0802	6	0.0840	6
	DMU_2	1	1	0.1028	2	0.1121	2
	DMU_3	0.323	11	0.0800	7	0.0769	7
	DMU_4	0.377	9	0.0721	10	0.0745	9
	DMU_5	0.374	10	0.0720	11	0.0669	11
	DMU_6	0.996	4	0.0897	3	0.0896	4
	DMU_7	1	1	0.1262	1	0.1278	1
	DMU_8	0.402	8	0.0790	8	0.0745	10
	DMU_9	1	1	0.0887	4	0.1110	3
	DMU_{10}	0.408	7	0.0726	9	0.0749	8
	DMU_{11}	0.846	5	0.0837	5	0.0842	5
	DMU_{12}	0.301	12	0.0528	12	0.0236	12

Table 20 Spearman's correlation coefficients

	AHP/DEA & DEA	*BWM-DEA & DEA*
Example 1	0.775	0.775
Example 2	0.894	0.894
Example 3	0.880	0.941
Example 4	0.901	0.916

As can be seen from Table 20, The ranking of BWM-DEA is compatible with the DEA classification, and in some examples, this compatibility is more than AHP/DEA.

6 Conclusion and future research

This paper proposed a new method called BWM-DEA for ranking decision-making units with multiple inputs and outputs. BWM is used for ranking, and the CCR model is used to construct the pairwise comparison vector. It obtains the ranking based on comparing the worst and the best units with the other units. A five-step procedure was used to rank the units fully. Pairwise comparisons are highly consistent in this method because the data of the comparison vector is non-subjective. It is derived mathematically from the input/output data and is not based on a decision-maker's subjective evaluation.

The results of the four illustrative examples show our proposed model's applicability. In addition, the results of BWM-DEA were compared with AHP/DEA. They showed that the proposed method performs better than AHP/DEA and can obtain a more reliable ranking, which is also compatible with DEA classification. This method is vector-based and requires less comparison data compared to matrix-based methods such as AHP/DEA. Only $2n - 3$ comparisons are necessary for BWM-DEA, while $n(n-1)/2$ comparisons are required in AHP/DEA.

It was shown that the proposed method is simple, easy to use and especially useful for real-world problems with many DMUs. It should be noted that BWM-DEA is not a substitute for DEA and provides supplementary analysis of DEA to achieve a complete ranking of units. To test the applicability of the proposed method and improve its validity, we suggest using it in other real-world applications and comparing its results with other ranking methods in data envelopment analysis. We also recommend extending this method to contain imprecise data, such as fuzzy data, for future research.

References

Abolghasem, S., Toloo, M. and Amézquita, S. (2019) 'Cross-efficiency evaluation in the presence of flexible measures with an application to healthcare systems', *Health Care Management Science*, Vol. 22, pp.512–533.

Afsharian, M. (2021) 'A frontier-based facility location problem with a centralised view of measuring the performance of the network', *Journal of the Operational Research Society*, Vol. 72, No. 5, pp.1058–1074.

Afsharian, M. and Ahn, H. (2014) 'The Luenberger indicator and directions of measurement: a bottoms-up approach with an empirical illustration to German savings banks', *International Journal of Production Research*, Vol. 52, No. 20, pp.6216–6233.

Afsharian, M. and Ahn, H. (2017) 'Multi-period productivity measurement under centralized management with an empirical illustration to German saving banks', *Or Spectrum*, Vol. 39, pp.881–911.

Agasisti, T., De Oliveira Ribeiro, C. and Montemor, D.S. (2022) 'The efficiency of Brazilian elementary public schools', *International Journal of Educational Development*, Vol. 93. Doi: 10.1016/j.ijedudev.2022.102627.

Aldamak, A. and Zolfaghari, S. (2017) 'Review of efficiency ranking methods in data envelopment analysis', *Measurement: Journal of the International Measurement Confederation*, Vol. 106, pp.161–172. Doi: 10.1016/j.measurement.2017.04.028.

Alizadeh, R., Gharizadeh Beiragh, R., Soltanisehat, L., Soltanzadeh, E. and Lund, P.D. (2020) 'Performance evaluation of complex electricity generation systems: a dynamic network-based data envelopment analysis approach', *Energy Economics*, Vol. 91. Doi: 10.1016/j.eneco.2020.104894.

Andersen, P. and Petersen, N.C. (1993) 'A procedure for ranking efficient units in data envelopment analysis', *Management Science*, Vol. 39, No. 10, pp.1261–1264. Doi: 10.1287/mnsc.39.10.1261.

Azizi, F., Tavakkoli-Moghaddam, R., Hamid, M., Siadat, A. and Samieinasab, M. (2022) 'An integrated approach for evaluating and improving the performance of surgical theaters with resilience engineering', *Computers in Biology and Medicine*, Vol. 141. Doi: 10.1016/j.compbiomed.2021.105148.

Banker, R.D., Amirteimoori, A. and Sinha, R.P. (2022) 'An integrated data envelopment analysis and generalized additive model for assessing managerial ability with application to the insurance industry', *Decision Analytics Journal*, Vol. 4. Doi: 10.1016/j.dajour.2022.100115.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984) 'Some models for estimating technical and scale inefficiencies in data envelopment analysis', *Management Science*, Vol. 30, No. 9, pp.1078–1092. Doi: 10.1287/mnsc.30.9.1078.

Bardhan, I., Bowlin, W.F., Cooper, W.W. and Sueyoshi, T. (1996) 'Models and measures for efficiency dominance in DEA – Part II: free disposal hull (FDH) and Russell measure (RM) approaches', *Journal of the Operations Research Society of Japan*, Vol. 39, No. 3, pp.333–344. https://doi.org/10.15807/jorsj.39.333.

Ben Lahouel, B., Taleb, L. and Kossai, M. (2022) 'Nonlinearities between bank stability and income diversification: a dynamic network data envelopment analysis approach', *Expert Systems with Applications*, Vol. 207. Doi: 10.1016/j.eswa.2022.117776.

Boloori, F., Afsharian, M. and Pourmahmoud, J. (2016) 'Equivalent multiplier and envelopment DEA models for measuring efficiency under general network structures', *Measurement*, Vol. 80, pp.259–269.

Bronner, M., See, K.F. and Yu, M.M. (2022) 'Circular water economy performance evaluation based on dynamic network data envelopment analysis', *Journal of Cleaner Production*, Vol. 367. Doi: 10.1016/j.jclepro.2022.132474.

Charnes, A., Cooper, W., Lewin, A.Y. and Seiford, L.M. (1997) 'Data envelopment analysis theory, methodology and applications', *Journal of the Operational Research Society*, Vol. 48, No. 3, pp.332–333.

Charnes, A., Cooper, W.W. and Rhodes, E. (1979) 'Measuring the efficiency of decision-making units', *European Journal of Operational Research*, Vol. 3, No. 4. Doi: 10.1016/0377-2217(79)90229-7.

Chen, Z. and Ming, X. (2020) 'A rough–fuzzy approach integrating best–worst method and data envelopment analysis to multi-criteria selection of smart product service module', *Applied Soft Computing*, Vol. 94. Doi: 10.1016/j.asoc.2020.106479.

Chen, Z.H., Wan, S.P. and Dong, J.Y. (2022) 'An efficiency-based interval type-2 fuzzy multi-criteria group decision making for makeshift hospital selection', *Applied Soft Computing*, Vol. 115. Doi: 10.1016/j.asoc.2021.108243.

Chetan, T.G., Jenamani, M. and Sarmah, S.P. (2022) 'Iterative multi-attribute procurement auction with decision support for bid formulation', *Asia-Pacific Journal of Operational Research*, Vol. 39, No. 3. Doi: 10.1142/S0217595921500366.

Costa, M.A., Salvador, C.V.M. and Da Silva, A.V. (2022) 'Stochastic data envelopment analysis applied to the 2015 Brazilian energy distribution benchmarking model', *Decision Analytics Journal*, Vol. 3. Doi: 10.1016/j.dajour.2022.100061.

Eskandari, M., Hamid, M., Masoudian, M. and Rabbani, M. (2022) 'An integrated lean production-sustainability framework for evaluation and improvement of the performance of pharmaceutical factory', *Journal of Cleaner Production*, Vol. 376. Doi: 10.1016/j.jclepro.2022.134132.

Fan, J.P., Guo, Z.W. and Wu, M.Q. (2020) 'An improvement of DEA cross-efficiency aggregation based on BWM-TOPSIS', *IEEE Access*, Vol. 8, pp.37334–37342. Doi: 10.1109/ACCESS.2020.2975240.

Haghighi, S.M. and Torabi, S.A. (2018) 'A novel mixed sustainability-resilience framework for evaluating hospital information systems', *International Journal of Medical Informatics*, Vol. 118, pp.16–28. Doi: 10.1016/j.ijmedinf.2018.07.003.

Hahn, G.J., Brandenburg, M. and Becker, J. (2021) 'Valuing supply chain performance within and across manufacturing industries: a DEA-based approach', *International Journal of Production Economics*, Vol. 240. Doi: 10.1016/j.ijpe.2021.108203.

Hatami-Marbini, A., Tavana, M., Agrell, P.J., Hosseinzadeh Lotfi, F. and Ghelej Beigi, Z. (2015) 'A common-weights DEA model for centralized resource reduction and target setting', *Computers and Industrial Engineering*, Vol. 79, pp.195–203. Doi: 10.1016/j.cie.2014.10.024.

Huang, B., Zhang, L., Ma, L., Bai, W. and Ren, J. (2021a) 'Multi-criteria decision analysis of China's energy security from 2008 to 2017 based on Fuzzy BWM-DEA-AR model and Malmquist productivity index', *Energy*, Vol. 228. Doi: 10.1016/j.energy.2021.120481.

Huang, M., Xia, T., Chen, Z., Pan, E. and Xi, L. (2021b) 'A DEA integrated grey factor analysis approach for efficiency evaluation and ranking in uncertain systems', *Computers and Industrial Engineering*, Vol. 162. Doi: 10.1016/j.cie.2021.107681.

Jabbari, M., Sheikh, S., Rabiee, M. and Oztekin, A. (2022) 'A collaborative decision support system for multi-criteria automatic clustering', *Decision Support Systems*, Vol. 153. Doi: 10.1016/j.dss.2021.113671.

Kolagar, M., Hosseini, S.M.H., Felegari, R. and Fattahi, P. (2020) 'Policy-making for renewable energy sources in search of sustainable development: a hybrid DEA-FBWM approach', *Environment Systems and Decisions*, Vol. 40, No. 4, pp.485–509. Doi: 10.1007/s10669-019-09747-x.

Kritikos, M.N. (2017) 'A full ranking methodology in data envelopment analysis based on a set of dummy decision making units', *Expert Systems with Applications*, Vol. 77, pp.211–225. Doi: 10.1016/j.eswa.2017.01.042.

Krüger, J.J. (2018) 'Direct targeting of efficient DMUs for benchmarking', *International Journal of Production Economics*, Vol. 199, pp.1–6. Doi: 10.1016/j.ijpe.2018.02.009.

Labijak-Kowalska, A. and Kadziński, M. (2021) 'Experimental comparison of results provided by ranking methods in data envelopment analysis', *Expert Systems with Applications*, Vol. 173. Doi: 10.1016/j.eswa.2021.114739.

Le, M.H., Afsharian, M. and Ahn, H. (2021) 'Inverse frontier-based benchmarking for investigating the efficiency and achieving the targets in the Vietnamese education system', *Omega*, Vol. 103. Doi: 10.1016/j.omega.2021.102427.

Lin, R., Yang, W. and Huang, H. (2019) 'A modified slacks-based super-efficiency measure in the presence of negative data. *Computers and Industrial Engineering*, Vol. 135, pp.39–52. Doi: 10.1016/j.cie.2019.05.030.

Mei, M. and Chen, Z. (2021) 'Evaluation and selection of sustainable hydrogen production technology with hybrid uncertain sustainability indicators based on rough-fuzzy BWM-DEA', *Renewable Energy*, Vol. 165, pp.716–730. Doi: 10.1016/j.renene.2020.11.051.

Mobarezkhoo, H., Saidi-Mehrabad, M. and Sahebi, H. (2022) 'Life cycle evaluation of microalgae based biorefinery supply network: a sustainable multi-criteria approach', *Algal Research*, Vol. 66. Doi: 10.1016/j.algal.2022.102816.

Omrani, H., Alizadeh, A. and Naghizadeh, F. (2020a) 'Incorporating decision makers' preferences into DEA and common weight DEA models based on the best–worst method (BWM)', *Soft Computing*, Vol. 24, No. 6, pp.3989–4002. Doi: 10.1007/s00500-019-04168-z.

Omrani, H., Alizadeh, A., Emrouznejad, A. and Teplova, T. (2021a) 'Data envelopment analysis model with decision makers' preferences: a robust credibility approach', *Annals of Operations Research*, pp.1–38. Doi: 10.1007/s10479-021-04262-2.

Omrani, H., Amini, M. and Alizadeh, A. (2020b) 'An integrated group best-worst method – data envelopment analysis approach for evaluating road safety: a case of Iran', *Measurement: Journal of the International Measurement Confederation*, Vol. 152. Doi: 10.1016/j.measurement.2019.107330.

Omrani, H., Emrouznejad, A., Shamsi, M. and Fahimi, P. (2022) 'Evaluation of insurance companies considering uncertainty: a multi-objective network data envelopment analysis model with negative data and undesirable outputs', *Socio-Economic Planning Sciences*, Vol. 82. Doi: 10.1016/j.seps.2022.101306.

Omrani, H., Valipour, M. and Emrouznejad, A. (2021b) 'A novel best worst method robust data envelopment analysis: Incorporating decision makers' preferences in an uncertain environment', *Operations Research Perspectives*, Vol. 8. Doi: 10.1016/j.orp.2021.100184.

Pourhejazy, P., Thamchutha, P. and Namthip, T.I. (2022) 'A DEA-based decision analytics framework for product deletion in the luxury goods and fashion industry', *Decision Analytics Journal*, Vol. 2. Doi: 10.1016/j.dajour.2021.100019.

Qin, J., Zeng, Y. and Zhou, Y. (2021) 'Context-dependent DEASort: a multiple criteria sorting method for ecological risk assessment problems', *Information Sciences*, Vol. 572, pp.88–108. Doi: 10.1016/j.ins.2021.04.085.

Rezaei, J. (2015) 'Best-worst multi-criteria decision-making method', *Omega*, Vol. 53, pp.49–57.

Rezaei, J. (2016) 'Best-worst multi-criteria decision-making method: Some properties and a linear model', *Omega (United Kingdom)*, Vol. 64, pp.126–130. Doi: 10.1016/j.omega.2015.12.001.

Saen, R.F., Karimi, B. and Fathi, A. (2022) 'Assessing the sustainability of transport supply chains by double frontier network data envelopment analysis', *Journal of Cleaner Production*, Vol. 354. Doi: 10.1016/j.jclepro.2022.131771.

Samieinasab, M., Hamid, M. and Rabbani, M. (2022) 'An integrated resilience engineering-lean management approach to performance assessment and improvement of clinical departments', *Socio-Economic Planning Sciences*. Doi: 10.1016/j.seps.2022.101425.

Sexton, T.R., Silkman, R.H. and Hogan, A.J. (1986) 'Data envelopment analysis: critique and extensions', *New Directions for Program Evaluation*, pp.73–105. Doi: 10.1002/ev.1441.

Shi, H., Wang, Y. and Chen, L. (2019) 'Neutral cross-efficiency evaluation regarding an ideal frontier and anti-ideal frontier as evaluation criteria', *Computers and Industrial Engineering*, Vol. 132, pp.385–394. Doi: 10.1016/j.cie.2019.04.035.

Sinuany-Stern, Z., Mehrez, A. and Hadad, Y. (2000) 'An AHP/DEA methodology for ranking decision making units', *International Transactions in Operational Research*, Vol. 7, No. 2, pp.109–124. Doi: 10.1111/j.1475-3995.2000.tb00189.x.

Starčević, S., Bojović, N., Junevičius, R. and Skrickij, V. (2019) 'Analytical hierarchy process method and data envelopment analysis application in terrain vehicle selection', *Transport*, Vol. 34, No. 5, pp.600–616. Doi: 10.3846/transport.2019.11710.

Tavana, M., Toloo, M., Aghayi, N. and Arabmaldar, A. (2021) 'A robust cross-efficiency data envelopment analysis model with undesirable outputs', *Expert Systems with Applications*, Vol. 167. Doi: 10.1016/j.eswa.2020.114117.

Torgersen, A.M., Førsund, F.R. and Kittelsen, S.A.C. (1996) 'Slack-adjusted efficiency measures and ranking of efficient units', *Journal of Productivity Analysis*, Vol. 7, No. 4, pp.379–398.

Wang, C.N., Nhieu, N.L., Nguyen, H.P. and Wang, J.W. (2021) 'Simulation-based optimization integrated multiple criteria decision-making framework for wave energy site selection: a case study of Australia', *IEEE Access*, Vol. 9, pp.167458–167476. Doi: 10.1109/ACCESS.2021.3134656.

Wang, D.D. (2020) 'Ranking multiple-input and multiple-output units: a comparative study of data envelopment analysis and rank aggregation', *Expert Systems with Applications*, Vol. 160. Doi: 10.1016/j.eswa.2020.113687.

Wang, Y.M. and Luo, Y. (2006) 'DEA efficiency assessment using ideal and anti-ideal decision making units', *Applied Mathematics and Computation*, Vol. 173, No. 2, pp.902–915.

Wu, Y., Li, K. and Fu, X. (2024) 'An integrated zero-sum game and data envelopment analysis model for efficiency analysis and regional carbon emission allocation', *Decision Analytics Journal*, Vol. 10. Doi: 10.1016/j.dajour.2023.100387.

Yang, L., Zhou, Y., Meng, B., Li, H., Zhan, J., Xiong, H., Zhao, H., Cong, W., Wang, X., Zhang, W., Lakshmanan, P., Deng, Y., Shi, X., Chen, X. and Zhang, F. (2022) 'Reconciling productivity, profitability and sustainability of small-holder sugarcane farms: a combined life cycle and data envelopment analysis', *Agricultural Systems*, Vol. 199. Doi: 10.1016/j.agsy.2022.103392.

Zahedi-Seresht, M., Khosravi, S., Jablonsky, J. and Zykova, P. (2021) 'A data envelopment analysis model for performance evaluation and ranking of DMUs with alternative scenarios', *Computers and Industrial Engineering*, Vol. 152. Doi: 10.1016/j.cie.2020.107002.

Zhu, Q., Li, F., Wu, J. and Sun, J. (2021) 'Cross-efficiency evaluation in data envelopment analysis based on the perspective of fairness utility', *Computers and Industrial Engineering*, Vol. 151. Doi: 10.1016/j.cie.2020.106926.