An extension of the linear programming method with fuzzy parameters

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Abstract: In a recent paper, Jiménez et al. (2007) propose a 'general' and 'interactive' method for solving linear programming problems with fuzzy parameters. In this study, we propose a revision to the optimal crisp value of the objective function to eliminate the restrictive constraints imposed by Jiménez et al. (2007). The revised approach can be generalised to solve many real-world linear programming problems where the coefficients are fuzzy numbers. In contrary to the approach proposed by Jiménez et al. (2007), our method is rightfully general and interactive, as it provides an optimal solution that is not subject to specific restrictive conditions and supports the interactive participation of the Decision-Maker (DM) in all steps of the decision-making process. We also present a counterexample to illustrate the merits of the proposed method and the drawbacks of Jiménez et al.'s (2007) method.

Keywords: FLP; fuzzy linear programming; fuzzy sets; decision making.

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1 Introduction

In a recent paper, Jiménez et al. (2007) propose a method for solving linear programming problems where all the coefficients are fuzzy numbers. They use a fuzzy ranking method that allows them to work with the concept of feasibility degree and offer the Decision Maker (DM) with the optimal solution for several different degrees of feasibility. They establish a fuzzy goal and build a fuzzy subset whose membership function represents the balance between the satisfaction degree of constraints and the objective function value. They claim their 'general' method is "very useful to be applied in a lot of real-world problems where the information is uncertain or incomplete" (Jiménez et al., 2007, p.1608). They also claim their 'interactive' method "permits the interactive participation of DM in all steps of decision process" (Jiménez et al., 2007, p.1600). We present a counterexample and show their method cannot be generalised, as it provides an optimal solution under specific restrictive conditions and does not support the interactive participation in all steps of the decision process.

In addition, we propose a revision to the optimal crisp value of the objective function to eliminate the restrictive constraints imposed by Jiménez et al. (2007). The revised approach can be generalised to solve many real-world linear programming problems where the coefficients are fuzzy numbers (e.g., Chen and Ko, 2010; Jana and Roy, 2005; Peidro et al., 2010; Vasant, 2003). In contrary to the approach proposed by Jiménez et al. (2007), our method is rightfully general and interactive, as it provides an optimal solution that is not subject to specific restrictive conditions and supports the interactive participation of the DM in all steps of the decision-making process. This paper is organised into six sections. The next section presents the literature review. Section 3 presents the mathematical notations and definitions. In Section 4, we illustrate the details of the proposed mathematical model. In Section 5, we present a numerical example to illustrate the merits of the proposed method and the drawbacks of Jiménez et al.'s (2007) method. In Section 6, we conclude with our conclusions and future research directions.

2 Literature review

The Fuzzy Linear Programming (FLP) developed by Bellman and Zadeh (1970)'s concept of symmetrical decision model has received a great deal of attention in recent years. There are generally four FLP classifications in the literature:

- Zimmerman (1987) has classified FLP into two categories: symmetrical and non-symmetrical models. In a symmetrical fuzzy decision, there is no difference between the weight of the objectives and constraints whereas in the asymmetrical fuzzy decision, the objectives and constraints are not equally important and have different weights (Amid et al., 2006).
- Leung (1988) has classified FLP problems into four categories:
  - a precise objective and fuzzy constraint
  - a fuzzy objective and precise constraint
  - a fuzzy objective and fuzzy constraint
  - robust programming.
- Luhandjula (1989) has classified FLP into three categories:
  - flexible programming
  - mathematical programming with fuzzy parameters
  - fuzzy stochastic programming.
- Inuiguchi et al. (1990) have classified FLP into six categories:
  - flexible programming
  - possibilistic programming
  - possibilistic linear programming using fuzzy max
  - robust programming
  - possibilistic programming with fuzzy preference relations
  - possibilistic linear programming with fuzzy goals.

In recent years, Lodwick and Jamison (2007) developed the theory underlying fuzzy, possibilistic, and mixed fuzzy/possibilistic optimisation and demonstrated the appropriate use of distinct solution methods associated with each type of optimisation dependent on the semantics of the problem. Mahdavi-Amiri and Nasseri (2007) developed some methods for solving FLP problems by introducing and solving certain auxiliary problems. They apply a linear ranking function to order trapezoidal fuzzy numbers and deduce some duality results by establishing the dual problem of the linear programming problem with trapezoidal fuzzy variables. Rommelfanger (2007) showed that both the probability distributions and the fuzzy sets should be used in parallel or in combination, to model imprecise data dependent on the real situation. Van Hop (2007) presented a model to measure attainment values of fuzzy numbers/fuzzy stochastic variables and used these new measures to convert the FLP problem or the fuzzy stochastic linear programming problem into the corresponding deterministic linear programming problem.

Ghodousian and Khorram (2008) studied the new linear objective function optimisation with respect to the fuzzy relational inequalities defined by max-min composition in which fuzzy inequality replaces ordinary inequality in the constraints. They showed that their method attains the optimal points that are better solutions than those resulting from the resolution of the similar problems with ordinary inequality constraints. Tan et al. (2008) developed an FLP extension of the general life cycle model using a concise and consistent linear model that makes identification of the optimal solution straightforward. Wu (2008) derived the optimality conditions for FLP problems by proposing two solution concepts based on similar solution concept, called the non-dominated solution, in the multiobjective programming problem.

Gupta and Mehlawat (2009) study a pair of fuzzy primal–dual linear programming problems and calculated duality results using an aspiration-level approach. Their approach is particularly important for FLP where the primal and dual objective values may not be bounded. Hosseinzadeh Lotfi et al. (2009) discussed full FLP problems of which all parameters and variables are triangular fuzzy numbers and used the concept of the symmetric triangular fuzzy number and introduce an approach to defuzzify a general fuzzy quantity. They propose a special ranking on fuzzy numbers to transform the full FLP model to a multiobjective linear programming model where all variables and parameters are crisp. Peidro et al. (2010) used fuzzy sets and developed an FLP to model the supply chain uncertainties. Baykasoğlu and Göçken (2008), Chen and Ko (2010), Inuiguchi and Ramík (2000) and Peidro et al. (2010) have developed a number of FLP models to solve problems ranging from supply chain management to product development.

3 Notations and definitions

In this section, we provide the basic fuzzy sets notions and definitions:

Definition 1: Let U be a universe set. A fuzzy set $\tilde{A}$ of U is determined by a membership function $\mu_{\tilde{A}}(x) \to [0,1]$, where $\mu_{\tilde{A}}(x)$, $\forall x \in U$, indicates the degree of membership of $\tilde{A}$ to U.

Definition 2: A fuzzy subset $\tilde{A}$ of universe set U is normal and convex if, respectively, $\sup_{x\in U} \mu_{\tilde{A}}(x) = 1$ and $\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y)), \forall x, y \in U$, $\forall \lambda \in [0,1]$, in which $\wedge$ indicates minimum operator.

Definition 3: A fuzzy set $\tilde{A}$ is a fuzzy number if $\tilde{A}$ is normal and convex.

Definition 4: A real fuzzy number $\tilde{A}$ denoted by $\tilde{A} = (a, b, c, d; w)$ is described as any fuzzy subset of the universe set *U* with membership function $\mu_{\tilde{A}}$, which satisfies the following properties:

- $\mu_{\tilde{A}}$ is a semicontinuous mapping from *R* to the closed interval $[0, w]$, $0 \leq w \leq 1$,
- $\mu_{\tilde{A}}(x) = 0,$ for all $x \in [-\infty, a]$,
- $\mu_{\tilde{A}}$ is increasing on $[a, b]$,
- $\mu_{\tilde{a}}(x) = w$ for all $x \in [b, c]$, where *w* is a constant and $0 < w \leq 1$, and
- $\mu_{\tilde{A}}$ is decreasing on $[c, d]$,
- $\mu_{\tilde{A}}(x) = 0,$ for all $x \in [d, \infty]$.

where *a*, *b*, *c* and *d* are real numbers. Furthermore, the membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ can be determined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} f^L(x), & a \leq x \leq b, \\ w, & b \leq x \leq c, \\ f^R(x), & c \leq x \leq d, \\ 0, & O.W, \end{cases} \quad (1)$$

where $f^L : [a, b] \rightarrow [0, w]$ and $f^R : [c, d] \rightarrow [0, w]$.

Note that if $w = 1$, $\tilde{A}$ is a normal fuzzy number, and $\tilde{A}$ is a non-normal fuzzy number if $0 < w < 1$.

4 The proposed mathematical model

Consider two fuzzy numbers, $\tilde{a}$ and $\tilde{b}$, and their membership functions, $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$, respectively. Jiménez (1996) suggests a fuzzy degree of satisfaction of $\tilde{a}$ over $\tilde{b}$ such that:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & E_2^a - E_1^b < 0, \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & 0 \in [E_1^a - E_2^b, E_2^a - E_1^b], \\ 1 & E_1^a - E_2^b > 0. \end{cases} \quad (2)$$

where $[E_1^a, E_2^a] = [\int_0^1 f(r)_a^{-1} dr, \int_0^1 g(r)_a^{-1} dr]$ and $[E_1^b, E_2^b] = [\int_0^1 f(r)_b^{-1} dr, \int_0^1 g(r)_b^{-1} dr]$ are *expected intervals* of $\tilde{a}$ and $\tilde{b}$, respectively. Note that $f^{-1}(r)_o$ and $g^{-1}(r)_o$, $r \in [0, 1]$, are the inverse functions of the left and right membership functions, respectively. If $\mu_M(\tilde{a}, \tilde{b}) \geq \alpha$, then $\tilde{a}$ is bigger than or equal to $\tilde{b}$ at least in a degree α denoted by $(\tilde{a} \geq \tilde{b})_\alpha$.

4.1 The Fuzzy Linear Programming Model

The aforementioned ranking of the fuzzy numbers are used to solve the Fuzzy Linear Programming Model (FLPM). Let us consider the following FLPM:

$$
\begin{aligned}
&\text{Minimise} \quad \tilde{Z} = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \cdots + \tilde{c}_n x_n \\
&\text{Subject to:} \\
&\quad \tilde{a}_{i1} x_1 + \tilde{a}_{i2} x_2 + \cdots + \tilde{a}_{in} x_n \le \tilde{b}_i \\
&\quad x_j \ge 0 \\
&\quad i = 1, 2, \ldots, m \\
&\quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{3}
$$

where $\tilde{c}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ represent the fuzzy parameters involved in the objective function and the constraints, and x_j represent the crisp decision variables.

4.2 The α-parametric linear programming model

Model (3) is then transformed into an α-parametric linear programming model based on the fuzzy degree of satisfaction equation (2):

$$
\begin{aligned}
&\text{Minimise} \quad Z_\alpha = EV(\tilde{c}_1)x_1 + EV(\tilde{c}_2)x_2 + \cdots + EV(\tilde{c}_n)x_n \\
&\text{Subject to:} \\
&\quad [(1-\alpha)E_2^{a_{i1}} + \alpha E_1^{a_{i1}}]\, x_1 + [(1-\alpha)E_2^{a_{i2}} + \alpha E_1^{a_{i2}}]\, x_2 + \cdots + \\
&\quad [(1-\alpha)E_2^{a_{in}} + \alpha E_1^{a_{in}}]\, x_n \le [(1-\alpha)E_1^{b_i} + \alpha E_2^{b_i}] \\
&\quad x_j \ge 0 \\
&\quad i = 1, 2, \ldots, m \\
&\quad j = 1, 2, \ldots, n
\end{aligned}
\tag{4}
$$

where $EV(\tilde{c}_j) = (E_1^{c_j} + E_2^{c_j})/2$ is the expected value of $\tilde{c}_j$. Then, $x_j^*(\alpha)$ is called the optimal solution of model equation (4) for each α-cut.

4.3 The fuzzy goal model

Model (4) is used to obtain the fuzzy objective function value, $\tilde{Z}_\alpha = \sum_j \tilde{c}_j x_j^*(\alpha)$, for specific values of $\alpha \in [0, 1]$. The DM then identifies a fuzzy goal $(\tilde{G})$ with a membership function $(\mu_{\tilde{G}}(Z))$ for each α-cut. Dubois et al. (1978) review several methods for computing the degree of satisfaction of the fuzzy goal for each α-cut. Next, the Yager index (Yager, 1978) is used for each optimal solution of the objective function value $(\tilde{Z}_\alpha)$ and the satisfaction of the fuzzy goal $(\tilde{G})$:

$$K_{\tilde{G}}(\tilde{Z}_\alpha) = \int_{-\infty}^{+\infty} \mu_{\tilde{G}}(z).\mu_{\tilde{Z}_\alpha}(z)\mathrm{d}z \Big/ \int_{-\infty}^{+\infty} \mu_{\tilde{Z}_\alpha}(z)\mathrm{d}z.$$

Next, the properties and advantages of the suggested index are highlighted and the balance solution for each α is defined as:

$$\mu_{\tilde{D}}(x_\alpha *) = \alpha * K_{\tilde{G}}(\tilde{Z}_\alpha) \tag{5}$$

where * is the algebraic product.

Finally, we look for a balanced solution between the feasibility degree and the degree of satisfaction. An acceptable optimal solution is obtained when two conditions, the highest degree (α) and the highest $\mu_{\tilde{D}}(x_\alpha^*)$, are satisfied simultaneously.

In contrary to the approach proposed by Jiménez et al. (2007), our method provides an optimal solution under various conditions and supports the interactive participation of the DM in all steps of the decision-making process. In addition, the proposed method supports real-world decision-making situations where the interactive participation of the DM is an integral part of the decision process.

5 A numerical example

In this section, we present a counterexample and show that the method proposed by Jiménez et al. (2007) cannot be generalised as it provides an optimal solution under specific restrictive conditions and does not support the interactive participation in all steps of the decision process.

The Data Envelopment Analysis (DEA) model provided next is used to measure the relative efficiency of a Decision-Making Unit (DMU) in producing two outputs from two inputs. Inputs and outputs are assumed to be symmetrical triangular fuzzy numbers, denoted by the triplet consisting of the corresponding left, centre and right points along the horizontal axis, $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u), i = 1, \dots, m; \ j = 1, \dots, n.$ This results in the following linear programming formulation:

$$\text{Maximise} \quad \tilde{Z} = (2.2, 2.2, 2.2)x_1 + (3.3, 3.5, 3.7)x_2$$

Subject to:

$$\begin{aligned}
&(2.9, 2.9, 2.9)x_3 + (1.4, 1.5, 1.6)x_4 = (1, 1, 1) \\
&(2.4, 2.6, 2.8)x_1 + (3.8, 4.1, 4.4)x_2 - (3.5, 4.0, 4.5)x_3 - (1.9, 2.1, 2.3)x_4 \le (0, 0, 0) \\
&(2.2, 2.2, 2.2)x_1 + (3.3, 3.5, 3.7)x_2 - (2.9, 2.9, 2.9)x_3 - (1.4, 1.5, 1.6)x_4 \le (0, 0, 0) \\
&(2.7, 3.2, 3.7)x_1 + (4.3, 5.1, 5.9)x_2 - (4.4, 4.9, 5.4)x_3 - (2.2, 2.6, 3.0)x_4 \le (0, 0, 0) \\
&(2.5, 2.9, 3.3)x_1 + (5.5, 5.7, 5.9)x_2 - (3.4, 4.1, 4.8)x_3 - (2.2, 2.3, 2.4)x_4 \le (0, 0, 0) \\
&(4.4, 5.1, 5.8)x_1 + (6.5, 7.4, 8.3)x_2 - (5.9, 6.5, 7.1)x_3 - (3.6, 4.1, 4.6)x_4 \le (0, 0, 0) \\
&x_j \ge 0 \\
&j = 1, 2, 3, 4.
\end{aligned} \tag{6}$$

Note that some coefficient values in model (5) are crisp parameters. Crisp parameters are often used to solve practical problems in a wide variety of situations. On the basis of model (3), the above-mentioned model can be written as:

Maximise $\tilde{Z}_\alpha = 2.2\, x_1 + 3.5 x_2$

Subject to:

$$((1-\alpha)2.90 + 2.90\alpha)x_3 + ((1-\alpha)1.55 + 1.45\alpha)x_4 = 1$$

$$((1-\alpha)2.70 + 2.50\alpha)x_1 + ((1-\alpha)4.25 + 3.95\alpha)x_1 - ((1-\alpha)4.25 + 3.75\alpha)$$

$$x_3 - ((1-\alpha)2.20 + 2.00\alpha)x_4 \le 0$$

$$((1-\alpha)2.20 + 2.20\alpha)x_1 + ((1-\alpha)3.60 + 3.40\alpha)x_1 - ((1-\alpha)2.90 + 2.90\alpha)$$

$$x_3 - ((1-\alpha)1.55 + 1.45\alpha)x_4 \le 0$$

$$((1-\alpha)3.45 + 2.95\alpha)x_1 + ((1-\alpha)5.50 + 4.70\alpha)x_1 - ((1-\alpha)5.15 + 4.65\alpha)$$

$$x_3 - ((1-\alpha)2.80 + 2.40\alpha)x_4 \le 0$$

$$((1-\alpha)3.10 + 2.70\alpha)x_1 + ((1-\alpha)5.80 + 5.60\alpha)x_1 - ((1-\alpha)4.45 + 3.75\alpha)$$

$$x_3 - ((1-\alpha)2.35 + 2.25\alpha)x_4 \le 0$$

$$((1-\alpha)5.45 + 4.75\alpha)x_1 + ((1-\alpha)7.85 + 6.95\alpha)x_1 - ((1-\alpha)6.80 + 6.20\alpha)$$

$$x_3 - ((1-\alpha)4.35 + 3.85\alpha)x_4 \le 0$$

$$x_j \ge 0 \tag{7}$$

$$j = 1, 2, 3, 4.$$

Running the linear programming model (6) with different α-cuts resulted in the fuzzy optimal objective function values presented in Table 1.

Table 1 Counterexample results

α-cuts	*Decision variables* x_1	*Decision variables* x_2	*Fuzzy optimal objective function values* $\tilde{Z}_\alpha = (Z_\alpha^l, Z_\alpha^m, Z_\alpha^u)$	*Yager indices*	*Balance degree indices*
0	0.4545	0	(0.9999, 0.9999, 0.9999)	0	0
0.1	0.4545	0	(0.9999, 0.9999, 0.9999)	0	0
0.2	0.4545	0	(0.9999, 0.9999, 0.9999)	0	0
0.3	0.4545	0	(0.9999, 0.9999, 0.9999)	0	0
0.4	0.4545	0	(0.9999, 0.9999, 0.9999)	0	0
0.5	0.3102	0.0907	(0.9818, 0.9999, 1.0180)	0.580	0.290
0.6	0.1304	0.2049	(0.9631, 1.0040, 1.0450)	0.588	0.353
0.7	0.1224	0.2112	(0.9662, 1.0085, 1.0507)	0.596	0.417
0.8	0.1153	0.2170	(0.9698, 1.0132, 1.0566)	0.606	0.485
0.9	0.1089	0.2223	(0.9732, 1.0176, 1.0621)	0.614	0.553
1	0.1032	0.2273	(0.9771, 1.0226, 1.0681)	0.623	0.623

The last two columns of Table 1 present the Yager indices and the balance degree indices (the final balance between the feasibility degree of constraints and the satisfaction degree of goal). In this example, it is also noted that the aspiration level (goal) of the DM is obtained as follows:

$$\mu_{\tilde{G}_j}(z) = \begin{cases} 0, & z \le \min\{Z_\alpha^l\}, \\ \dfrac{z - \min\{Z_\alpha^l\}}{\max\{Z_\alpha^u\} - \min\{Z_\alpha^l\}}, & \min\{Z_\alpha^l\} \le z \le \max\{Z_\alpha^u\}, \\ 1, & z \ge \max\{Z_\alpha^u\}. \end{cases}$$

As shown in Table 1, when $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4\}$, the optimal solutions of this model are $(x_1 = 0.4545, x_2 = 0)$, and its fuzzy optimal value of objective function are (0.9999, 0.9999, 0.9999). In other words, the optimal value of the objective function is a crisp value and hence the Yager and balance degree indices are virtually zero. Therefore, the method presented by Jiménez et al. (2007) has an optimal solution under a specific restrictive condition and is not 'general' and cannot be utilised in a lot of real-world problems where the information is uncertain or incomplete.

Jiménez et al. (2007) also claim their method is 'interactive' and the DM can intervene in all the steps of the decision process. It is imperative to validate the results in real-world problems where the DM must be confident that his or her decision is both accurate and appropriate. Using the model proposed by Jiménez et al. (2007), a DM is required to establish a fuzzy goal and build a fuzzy subset whose membership function represents the balance between the satisfaction degree of constraints and the objective function value. However, their model cannot be generalised to reliably provide accurate information for establishing a fuzzy goal. We could not verify their claim that "We offer the DM the optimal solution for several different degrees of feasibility. With this information the DM is able to establish a fuzzy goal" (Jiménez et al., 2007, p.1599).

Using their model, a fuzzy optimal solution for $\tilde{Z}_\alpha$ is obtained as a crisp value. It is easy to modify this solution with $(Z_\alpha^l - \varepsilon, Z_\alpha^m, Z_\alpha^u + \varepsilon)$. Note that ε is infinitesimal and it allows for calculating the Yager index by keeping it away from zero. We recalculate the Yager indices using the proposed approach for crisp values of the objective function. The results for different ε and the value of the objective function (0.9999, 0.9999, 0.9999) are shown in Table 2.

Table 2 Yager indices for different ε

ε	*Yager indices*
5×10^{-2}	0.5801367266
3×10^{-3}	0.5801367267
2×10^{-3}	0.5801367265
1×10^{-3}	0.5801367265
8×10^{-4}	0.5801367265
1×10^{-4}	0.5801367265
1×10^{-5}	0.5801367265
1×10^{-7}	0.5801367265
1×10^{-10}	0.5801367265

As shown in Table 2, the value of Yager index is 0.580136726. This revision of the crisp value of the objective function eliminates the restrictive constraints imposed by Jiménez et al. (2007). The revised approach is 'truly' useful in solving real-world problems where the information is uncertain or incomplete.

6 Conclusions and future research directions

In this paper, we examined the FLP method proposed by Jiménez et al. (2007) and presented a counterexample that showed that their method is not 'general' and 'interactive' and should be used cautiously. We showed their method cannot be generalised, as it provides an optimal solution under specific restrictive conditions. Furthermore, we could not verify their claim that their method offers DMs with reliable information to establish fuzzy goals in real-world problems. We proposed a revision to the optimal crisp value of the objective function. This fine-tuning eliminates the restrictive constraints imposed by Jiménez et al. (2007) and the revised approach can be generalised to solve many real-world linear programming problems where all the coefficients are fuzzy numbers.

Future research will concentrate on the comparison of results obtained with those that might be obtained with other methods. In addition, we plan to extend the FLP approach proposed here to deal with fuzzy non-linear optimisation problems with multiple objectives where the vagueness or impreciseness appears in all the components of the optimisation problem such as the objectives, constraints and coefficients. Such an extension also implies the study of new practical experiments. Finally, we plan to focus on the use of co-evolutionary algorithms to solve fuzzy optimisation problems. This approach would permit the search for solutions covering optimality, diversity and interpretability.

Over the past few decades, researchers have proposed many FLP models with different levels of sophistication. However, many of these models have limited real-world applications because of their methodological complexities, inflexible assumptions and impractical solutions. It is noteworthy that the FLP model proposed here supports the interactive participation of the DM and allows the uncertain data and soft constraints to be formalised through fuzzy approach (Mula et al., 2008). The managerial implication of the proposed approach is its applicability to a wide range of real-world problems such as supply chain management, performance evaluation by means of DEA, marketing management, failure mode and effect analysis and product development (Baykasoğlu and Göçken, 2008; Chen and Ko, 2010; Inuiguchi and Ramík, 2000; Peidro et al., 2010).

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