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A combinatorial data envelopment analysis with uncertain interval data with application to ICT evaluation

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ABSTRACT

Information and Communication Technologies (ICTs) have been extensively adopted by firms worldwide due to the significant positive effect on their performance. This fact contrasts with the uncertainty faced by decision makers when entering a country and selecting local firms with which to interact. Consider selecting Decision Making Units (DMUs) according to their relative efficiency, this efficiency being determined via Data Envelopment Analysis (DEA) based on the potential inputs consumed and outputs produced. The values of these variables are uncertain and defined through interval evaluations. Assume now that the interactions may be interrupted several times and new DMUs selected in place of previous ones. The new DMUs may require higher or lower amounts of inputs to produce variable amounts of outputs. The consequences derived from the potential realizations resolving the uncertainty should be incorporated into the DEA problem when deciding which DMUs to interact with and in which order. We study the combinatorial decision framework arising from the potential interactions with new DMUs. A numerical example is provided to complement the problem statement and outline the drawbacks of the existing approaches. It is shown that the selected DMUs and their order may differ substantially when accounting for the complementarities existing among all the DMUs. Moreover, the selection process and any subsequent decision vary with the number of modifications considered relative to the DMU initially selected. A case study analyzing the productive and environmental efficiency of a group of European countries displaying uncertain interval levels of ICT development is presented.

1. Introduction

We evaluate the efficiency of a set of Decision Making Units (DMUs) whose inputs and outputs are described using uncertain intervals defining different sets of potential realizations of these variables. That is, the actual amount of inputs required and outputs produced by a DMU may differ from those considered by a Decision Maker (DM). As a result, if a DM wants to select a DMU with which to interact, efficiency evaluations must consider the complementarities existing between the inputs and outputs of the DMU selected and any potential alternative. This type of evaluation structure relates to the design of different choice paths based on the potential information received and the order in which DMUs are selected. The actual choices are determined by the realizations of the input and output variables, which, at the same time, are conditioned by the uncertain intervals on which they are defined, and the density functions assigned to each interval.

We define a decision environment where a DM must select a DMU, namely, a firm or a country, with which to interact based on its efficiency performance while accounting for the potential consequences derived from this interaction. The percentage variables generally used to evaluate the relative efficiency of countries or firms provide DMs with uncertain evaluations that could be interpreted in terms of intervals of potential realizations. Nevertheless, these percentages are generally used to define the efficiency of DMUs through the direct application of Data Envelopment Analysis (DEA). Consider, for instance, the variable called “access and usage of ICT in business enterprises”. This variable was applied by Dzemydienė et al. (2022) to rank countries via Multiple Criteria Decision Making (MCDM) techniques. Clearly, higher values of

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the corresponding variable imply that larger percentages of firms and workers are familiar with the use of Information and Communication Technologies (ICTs).

Uncertainty is inherent to these percentages, which only provide some basic guidelines as to the potential inputs that may be required by a given DMU. That is, DMs must account for the possibility of observing realizations that differ from those expected when interacting with the DMUs. The uncertain quality of the interval evaluations may result in modifications of the DMU initially selected by the DM after different realizations are observed. This feature must constitute one of the main characteristics of the decision model. Therefore, the selection of a DMU must account for the different scenarios determined by the combinations of potential realizations across the DMUs when deciding to modify the initial choice. We analyze these combinations and the resulting efficiency evaluations, which are conditioned by the order in which DMUs are selected.

More precisely, we define an optimal set of sequential decisions within an interval uncertain DEA scenario where the DM can select different DMUs based on the combinatorial interactions arising from their potential realizations. The value functions defined to analyze these combinations are intuitive and easy to implement while satisfying two important requirements: the order in which DMUs are selected conditions the results and rewards are provided whenever a selection is optimal. The resulting evaluation and decision path describes the sequence of choices made by the DM when allowed to modify a given number of decisions and select different DMUs. In this regard, the path is conditioned by the number of DMUs that the DM is willing to consider.

This sequential path – determined by the potential realizations observed and the order in which DMUs are selected – must be incorporated into the DEA structure defined by the DM. We illustrate the consequences from introducing the combinatorial decision path within a DEA framework by analyzing the productive and environmental efficiency of a group of European countries displaying uncertain interval levels of ICT development. The empirical results highlight the fact that individually inefficient alternatives may be selected when potential complementarities can be exploited across DMUs. Moreover, efficiency varies with the number of modifications considered relative to the DMU initially selected.

Fig. 1 provides a graphical representation of the phases composing the proposed extended DEA model compared to the standard DEA approach.

The rest of the paper is divided as follows. The related literature is reviewed in Section 2. A standard input-oriented DEA framework is presented in Section 3. Section 4 defines the basics of the combinatorial scenarios. Section 5 focuses on the combinatorial framework involving two DMUs, formalizes the proposed extended DEA model and provides a numerical example. Sections 6 extends the combinatorial framework and formal analysis introduced in the previous section considering scenarios with three or more DMUs. Section 7 generalizes the Extended DEA model to the case of ordered combinations of k DMUs. Section 8 applies the model empirically and outlines some practical and theoretical implications. Finally, Section 9 concludes and suggests potential extensions.

2. Literature review

The economics and business literature has consistently highlighted the positive relationship existing between the development of the ICT structure of countries and economic growth (Fernández-Portillo et al., 2022; Warr and Ayres, 2012; Ho et al., 2011). The empirical evidence illustrates the causal relation existing between ICT development and economic growth (Fernández-Portillo et al., 2020; Vu et al., 2020), both at the country (Fernández-Portillo et al., 2019; Venturini, 2015) and firm levels (Eze et al., 2018; Albiman and Sulong, 2017; Gërguri-Rashiti et al., 2017).

There is however room for controversy, particularly when considering the economic impact of ICTs. The relationship between both concepts has been challenged both at the country (Pradhan et al., 2019; Thompson and Garbacz, 2011) and firm levels (Haller and Lyons, 2015; Bertschek et al., 2013). Despite this latter fact, ICTs have been adopted by firms worldwide, displaying a significant effect on the design of processes (Kumar et al., 2016; Vu, 2011). Indeed, digitalization has been widely adopted across different productive sectors due to its positive effect on the performance of firms (Albiman and Sulong, 2017; Venturini, 2015).

ICTs relate also to technological change through increments in productivity (Jorgenson and Vu, 2016) and knowledge propagation externalities (Fossen and Sorgner, 2021). As is usually the case, the actual

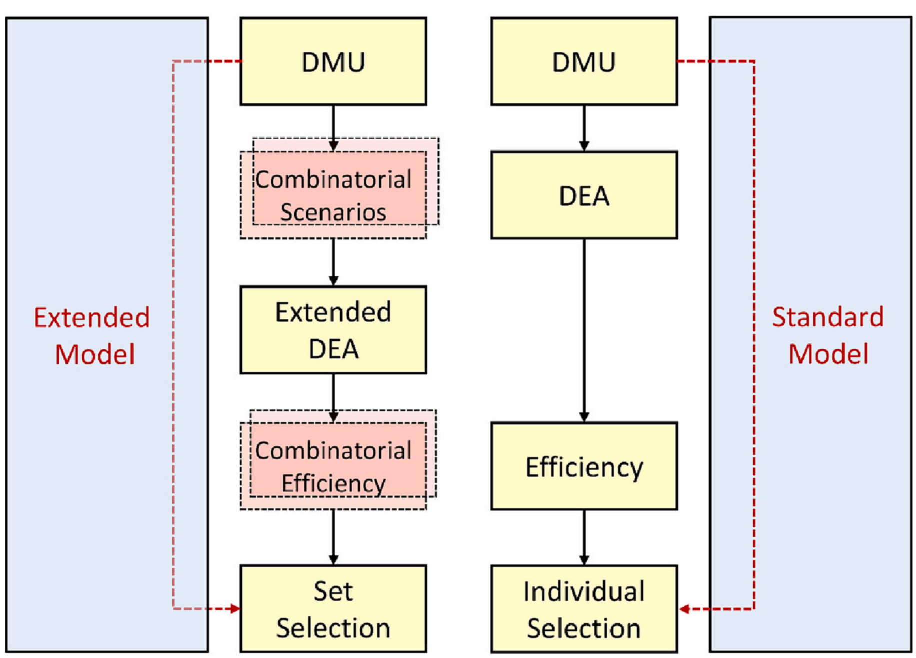


Fig. 1. Extended combinatorial DEA framework versus standard DEA.

effects derived from these interactions differ across firms. For instance, productivity increments require the complementary impact of human capital (Skorupinska and Torrent-Sellens, 2017). Competitive opportunities vary across firms conditioned by their innovation capacities (Bouwman et al., 2018). In this regard, the spread of innovations and knowledge constitutes an important decision factor among those firms that must select both a country to enter and local firms with which to interact (Sopha et al., 2021; Álvarez et al., 2015).

The uncertainty faced by firms when entering a given country has been consistently highlighted by the literature on international business (Kim et al., 2022; Klimas et al., 2022), focusing particularly on the importance of entry frictions (Nguyen et al., 2022; Sanna-Randaccio and Veugelers, 2007). Firms must account for a wide ranging set of potential frictions that follow from their entry choices, encompassing the risks derived from interacting and competing with local firms, inefficiencies arising due to human and capital incompatibilities, institutional barriers, and competitive losses to other firms making better decisions (Popli et al., 2022; Ragmoun, 2022; Strange et al., 2022; Baier-Fuentes et al., 2021; Guimarães et al., 2021; O'Connor et al., 2014).

This problem is generally analyzed from a strategic perspective, focusing on the interactions arising across firms and the resulting outcomes (Barnard, 2021; Findlay et al., 2021). However, the information available to evaluate the development of the ICT infrastructure of a country and infer the results from potential interactions with local firms is difficult to assess. Furthermore, the resulting effects should be incorporated into the analysis before selecting a firm, given the costs involved in retrieving information and the structural pecuniary ones resulting from a new selection (Arikan et al., 2022; Álvarez et al., 2016).

The strategic consequences derived from this type of uncertainty have also been studied by the operations research literature, particularly when dealing with risks in different technological settings (Rodríguez et al., 2016; Bahli and Rivard, 2005). For instance, Dzemydienė et al. (2022) applied two MCDM techniques to evaluate the access and usage of ICT in business enterprises among several European countries over the period 2013–2017. Similarly, Torkayesh and Torkayesh (2021) estimated the development of ICT structures using social and economic indicators within an integrated MCDM framework.

MCDM models have been extended to formalize the strategic consequences from uncertainty via fuzzy variables while disregarding the outcomes derived from the selection of a given alternative (Li et al., 2022a, 2022b; Karabašević et al., 2020). The same remark applies to MCDM models dealing with interval data, which focus mainly on their applicability (Dymova et al., 2013; Jahanshahloo et al., 2006). In this regard, we must note that the combinatorial possibilities defined throughout the different sets of potential interval realizations cannot be formalized via fuzzy variables or possibility theory (Alshahrani et al., 2022; Ruiz et al., 2022; Stawowy et al., 2021). This feature becomes relevant when implementing the corresponding models to real-life environments (Pashutan et al., 2022; Wachnik et al., 2022; Trzaska et al., 2021).

The literature on DEA and efficiency dealing with interval data has also ignored the sequential interactions among DMUs that arise after the initial decisions are made and potential realizations observed (Ebrahimi et al., 2018). This branch of the literature has mainly focused on eliminating the inherent uncertainty and evaluating the efficiency of the alternatives (Chen and Ming, 2020; Niroomand et al., 2018). This is the case despite the fact that the applicability of DEA extends beyond the operations research domain into applied economics and strategic management, especially when evaluating the efficiency of innovation and environmental systems (Bresciani et al., 2021; Kiani Mavi and Kiani Mavi, 2021; Wang and Ren, 2022).

Based on the studies reviewed above, the pros and cons of considering the development of the ICT structure of countries as a reference point to select local firms with which to interact are outlined as follows:

- Pros: ICTs are being adopted by firms worldwide given their significant effect on the design of production processes and, consequently, on the performance of firms. At the same time, ICTs relate to technological change through increments in productivity and knowledge propagation externalities. Both the performance of firms and the spread of innovations and knowledge constitute important decision factors for a firm that must select both a country to enter and local firms with which to interact.
- Cons: The information available to evaluate the development of the ICT infrastructure of a country and infer the results from potential interactions with local firms is difficult to assess. The resulting effects should be accounted for before selecting a firm. This is mainly due to the costs of retrieving information or establishing a new firm. However, the sequential interactions with different DMUs that arise after the initial decisions are made and potential realizations observed have been overlooked so far by the literature on MCDM as well as the one on DEA.

As mentioned above, the capacity of DMs to modify their initial decisions and select different DMUs when the uncertainty is resolved implies that the resulting consequences must be incorporated into the original DEA environment. We define and study different combinatorial scenarios that may arise depending on the width of the interval variables and the order of selection of the DMUs. These scenarios will be used to illustrate the complexities arising from increasing the number of modifications that the DM is willing to consider with respect to the DMUs already selected.

3. DEA framework

We illustrate the evaluation problems faced by a DM when using expectations to account for the potential realizations of the variables within a DEA framework. The assessment of efficiency provided by DEA is determined by the values of the different variables conditioned by their effect on and relative importance within the production process of firms. DEA computes the distance existing between the best performing DMUs composing the frontier and the remaining ones per input and output variable.

The framework of analysis corresponds to a standard input-oriented DEA. The model is composed by n DMUs ($j = 1,\dots,n$) endowed with m different inputs ($i = 1,\dots,m$). Inputs are used to produce a set of s outputs ($r = 1,\dots,s$). The inputs and outputs used and produced by DMU_j are denoted by x_{ij} ($i = 1,\dots,m$) and y_{rj} ($r = 1,\dots,s$), respectively.

Eq. (1) describes a standard input-oriented DEA problem. The structure of the model is designed to minimize the inputs used to produce a given amount of outputs. A non-Archimedean value of $\varepsilon = 0.001$ will be assumed throughout the whole set of basic simulations. The slacks variables s_r^+, $r = 1,2,\dots,s$, and s_i^-, $i = 1,2,\dots,m$, represent the amount of output and input lacked by a DMU to reach the frontier. Intuitively, if the input slacks are positive, the DMU is inefficient in the use of the corresponding input, whose value can be reduced while preserving the same level of output. The model assumes variable returns to scale by requiring all the reference values λ_j, $j = 1,2,\dots,n$, to add up to one.

$$\min \theta - \varepsilon\left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right) \tag{1}$$

subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io},\ i = 1,2,\dots,m;$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro},\ r = 1,2,\dots,s;$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n.$$

The next section presents a basic numerical example that illustrates the results derived from the incorporation of uncertainty in the efficiency evaluations of DMUs using expectations.

3.1. Basic efficiency setting

We build on the following basic textbook example (Zhu, 2014) to illustrate the main intuition on which the proposed extended DEA model is based. Consider the application of an input-oriented DEA model to the data described in Table 1(a). As stated above, the objective of the model is to minimize the inputs while keeping the outputs at their initial levels.

Five DMUs use two inputs, cost and response time, to produce a unique output, profit. Fig. 2(a) illustrates the corresponding evaluation framework representing these DMUs. The intuition provided by the figure is verified when implementing the DEA model and obtaining three efficient DMUs (the first three), one weakly efficient (the fifth one), and an inefficient DMU (the fourth one). The corresponding results are presented in Table 1(b). If a DM is asked to select two DMUs to cooperate in the development of a project, namely, an initial DMU as the preferred alternative and a second one in case the DMU chosen in first place does not perform as expected, he should be indifferent between the first three efficient alternatives.

This would also be the case if we introduce uncertainty and define the inputs and outputs in terms of the highest potential value that may be required or produced, respectively. That is, assume that the DM faces an uncertain environment characterized by interval variables describing each DMU. For expositional simplicity, we will assume that the lower limits defining the intervals equal zero, while the value retrieved defines the upper limit. That is, DMUs displaying higher input values are supposed to be more technologically developed and require a higher level of interaction from a firm, which should be compensated through a potentially higher output.

The same efficiency patterns would be obtained if a uniform probability function was used to formalize the interval uncertainty. In this case, the entries of the matrix would be given by the expected values of the variables conditioned by the width of the corresponding intervals. The efficient frontier would remain qualitatively unchanged and DMUs would obtain the same efficiency scores, as can be intuitively understood from Fig. 2(a). Thus, if uncertainty is resolved using the expectations operator, the DM would be indifferent between the first three DMUs.

Table 1
Basic DEA setting and DMU efficiencies

Table 1(a). Basic DEA setting

DMU	Cost ($ 100)	Response time (days)	Profit ($ 1000)	Input Oriented Efficiency θ
1	1	5	2	1
2	2	2	2	1
3	4	1	2	1
4	6	1	2	1 ($\lambda_3^* = 1, s_1^{-*} = 2$)
5	4	4	2	0.5 ($\lambda_2^* = 1$)

Table 1(b). DMU efficiencies

DMU	λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	s_1^{+*}	s_1^{-*}	s_2^{-*}	θ^*
1	1	0	0	0	0	0	0	0	1
2	0	1	0	0	0	0	0	0	1
3	0	0	1	0	0	0	0	0	1
4	0	0	1	0	0	0	2	0	1
5	0	1	0	0	0	0	0	0	0.5

We formalize the capacity of DMs to account for the set of potential interactions between inputs and outputs that could be observed after selecting the initial DMU. In addition, DMs should also consider the number of modifications that could be defined relative to a given set of choices. As can be intuitively inferred from the numerical example, the indifference derived from DEA vanishes once the DM assesses the whole set of potential realizations characterizing ordered combinations of DMUs.

4. Proposed combinatorial scenario: General mathematical assumptions

For expositional simplicity, we concentrate the analysis on a unique input per DMU. Let a denote one of the DMUs and x_{ia} represent the value that may be taken by the i-th input variable defining DMU a. The value x_{ia} is the initial value assigned to categorize DMU a and determines the position in the raking according to which the DM decides to interact with the DMUs.

We assume that DMU a displays a set of potential realizations $z_{ia} \in [0, x_{ia}^M]$ that may be observed after selecting DMU a. M indicates the upper limit value of the interval whose values can be taken by the variable x_{ia}.

Note that, while the upper limits of the sets of potential realizations of different variables (i.e., x_{ia} and x_{ib}) are in general different (i.e., $x_{ia}^M \neq x_{ib}^M$), the lower limits of the intervals have been all unified at the value of zero. That is, for every a and b, with $a \neq b$, we have $z_{ia} \in [0, x_{ia}^M]$ and $z_{ib} \in [0, x_{ib}^M]$. This assumption has been introduced to simplify both notations and computations, without leading to the generality of the results. Relaxing this assumption by assigning different positive lower limit values would complicate the presentation without modifying the qualitative results obtained.

The initial values observed by the DM are assumed to define the location of the intervals of potential realizations such that for every tuple of different DMUs, $(a_1, a_2, a_3, \ldots, a_k)$, with $k \leq n$, $x_{ia_1} > x_{ia_2} > x_{ia_3} > \ldots > x_{ia_k}$ implies $x_{ia_1}^M > x_{ia_2}^M > x_{ia_3}^M > \cdots > x_{ia_k}^M$.

Moreover, given the common lower limit of zero for all intervals, we assume that for every a, $x_{ia}^M = x_{ia}$. We have introduced this notation to differentiate between the potential realizations, z_{ia}, and the categorization implied by the initial values observed, x_{ia}^M.

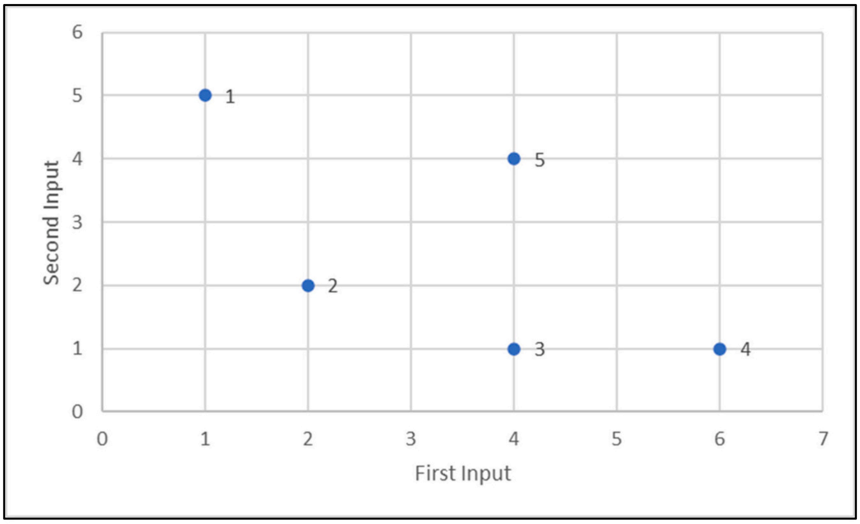
Finally, for every DMU a, the beliefs of the DM about the distribution of potential realizations of each input variable, x_{ia}, are formalized through a probability density function $f_{ia} : [0, x_{ia}^M] \rightarrow [0, \ 1]$. These probability density functions will be used to describe the expected values that arise when choosing any of the DMUs.

4.1. Interval uncertainty

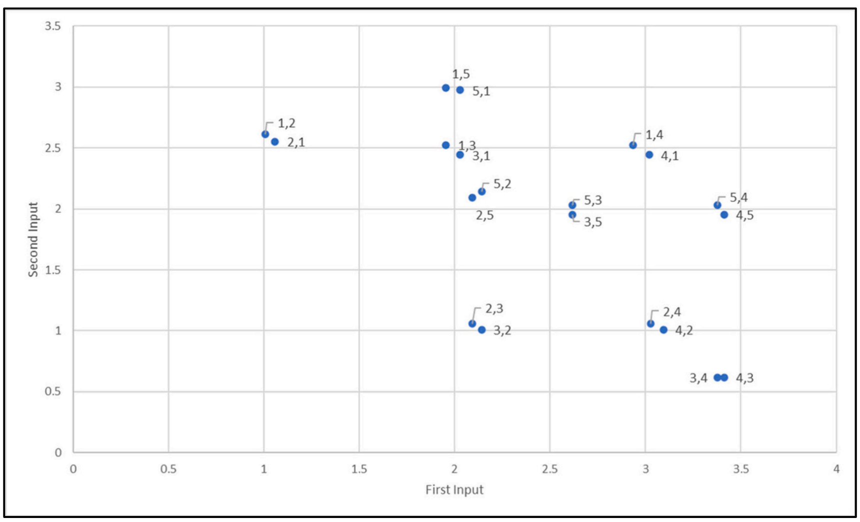
For every i-th input variable, DMU a is categorized via the width of the interval of potential realizations, this width being equal to x_{ia}. We have simplified this feature by assuming a lower limit value of zero and $x_{ia}^M = x_{ia}$. That is, the interval of potential realizations of $x_{ia}$is given by $[0, x_{ia}^M] = [0, x_{ia}]$. The DM must consider the whole set of potential realizations for each variable and its combinations with those of other DMUs when making the initial decision. To maximize information entropy, we assume that a uniform density is defined on $[0, x_{ia}^M]$, that is:

$$f_{ia}(z_{ia}) = \begin{cases} \dfrac{1}{x_{ia}} & \text{if } z_{ia} \in [0, x_{ia}^M] \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The qualitative results obtained do not depend on the type of density function selected, though modifications in the expected values will follow from assuming different probability density functions.



(a). Basic input environment



(b). Combinatorial input environment

Fig. 2. Basic and combinatorial input environments.

5. Combinatorial scenario with two DMUs

Consider the case of a DM who must select a DMU with which to develop a project while accounting for the possibility of having to modify the choice and select different DMUs if the performance of the ones previously selected is not optimal. The DM must compute the whole set of input and output combinations that may be realized as determined by their corresponding interval domains and the order of selection on which the interactions are based.

In this section, we focus our attention on the combinatorial scenario derived from considering interactions with two DMUs. By default, the order of selection of the DMUs will be given by the sequence a, b. This is the order according to which the DM is assumed to interact with the DMUs from the corresponding countries.

5.1. $x_{ia}^M \leq x_{ib}^M$ framework

We first study the scenario with two DMUs, allowing for one change in the initial selection. This scenario results in two potential frameworks determined by the relative values of the variables categorizing the inputs. The same intuition and formalization apply to the outputs.

The first framework considers the case where the DMU initially selected, DMU a, displays an upper limit defining the set of potential input realizations lower than the second, DMU b. Hence, $x_{ia}^M \leq x_{ib}^M$..

In order to assign a value to the pair of inputs (x_{ia}, x_{ib}) and, hence, to the sequential path represented by the ordered pair $(a, b)$of DMUs with respect to the i-th input variable, the whole set of potential realizations – that is, all the pairs of the form (z_{ia}, z_{ib}) – must be considered.

The following expression defines the value to assign to (x_{ia}, x_{ib}) considering all the combinations of inputs that may be required when selecting two DMUs, also the case when the performance of the one selected first is suboptimal.

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_0^{x_{ia}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ib}} (z_{ia}) dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \right] dz_{ia} \tag{3}$$

The limits of the domains of potential realizations, $x_{ia}^M \leq x_{ib}^M$, categorize alternatives a and b. Note how the intervals of potential realizations where z_{ia} and z_{ib} vary, have each an associated density function given by $\frac{1}{x_{ia}}$ and $\frac{1}{x_{ib}}$, respectively.

After the DM selects DMU a and observes z_{ia}, he must also account for the potential realizations of the variable defining DMU b. The realizations, z_{ib}, are distributed within the intervals $[0, z_{ia}]$ and $[z_{ia}, x_{ib}^M]$.

The integrals $\int_0^{z_{ia}} \frac{1}{x_{ib}}(z_{ia})dz_{ib}$ and $\int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}}\left(z_{ib} - c^b\right)dz_{ib}$ defining the expression above account for the cases where $z_{ib} \in [0, z_{ia}]$ and $z_{ib} \in \left[z_{ia}, x_{ib}^M\right]$, respectively.

As already stated, the upper limit of the domain defining the first DMU is lower than that of the second. Thus, the input requirements imposed by the first DMU are expected to be lower than those of the second. Whenever higher, the inputs of the first DMU imply a suboptimal choice relative to those of the second, as described by the first term of the equation. On the other hand, if the inputs are lower, the DMU selected in first place constitutes the right choice. A reward c^b has been introduced to account for the optimality of the initial decision and condition the expression in Eq. (3) on the order of choice.

Fig. 3(a) describes different potential realizations and combinations of z_{ia} and z_{ib} throughout their domains within the $x_{ia}^M \leq x_{ib}^M$ framework.

5.2. $x_{ia}^M > x_{ib}^M$ framework

Consider now the case where the DMU initially selected, DMU a, displays a higher limit for the interval of input realizations, that is, $x_{ia}^M > x_{ib}^M$. We must modify the value function and adapt it to the domains used to categorize the DMUs as follows:

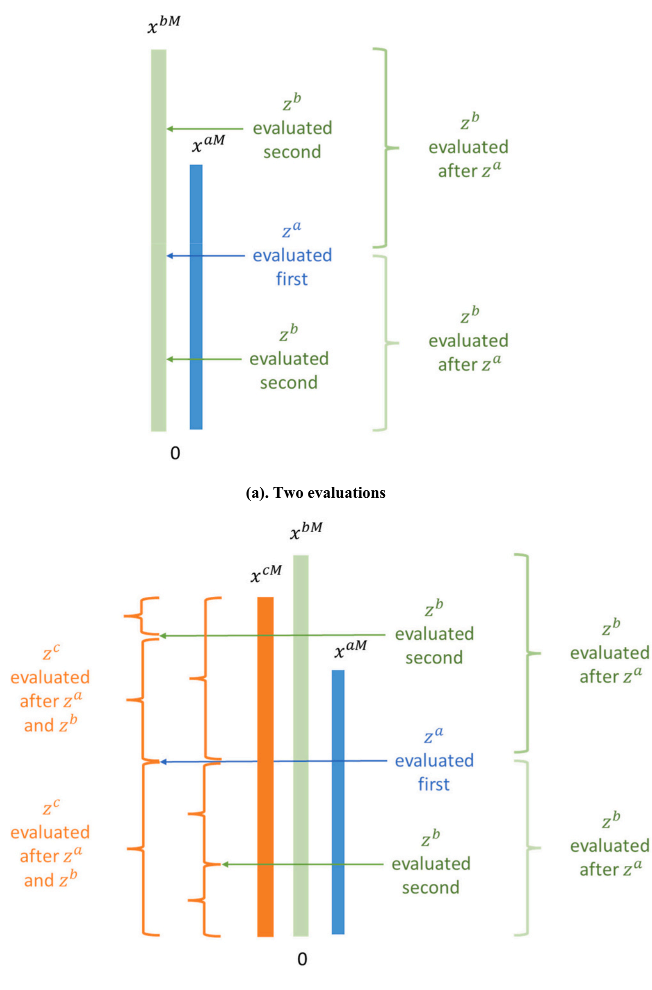


Fig. 3. Sets of potential realizations with two and three evaluations.

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_{x_{ib}^M}^{x_{ia}^M} \frac{1}{x_{ia}}[z_{ia}]dz_{ia} + \int_0^{x_{ib}^M} \frac{1}{x_{ia}}\left[\int_0^{z_{ia}} \frac{1}{x_{ib}}(z_{ia})dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}}\left(z_{ib} - c^b\right)dz_{ib}\right]dz_{ia} \quad (4)$$

The first term describes the case where the input realizations z_{ia} are higher than z_{ib}, with x_{ib}^M denoting the upper limit of the potential input realizations z_{ib}. The second DMU may then either require a lower input and improve upon the first one or require a higher input leading to a reward of c^b. In this regard, the second term encompasses two expressions describing the input realizations $z_{ib} \in [0, z_{ia}]$ and $z_{ib} \in \left[z_{ia}, x_{ib}^M\right]$, respectively. Let us emphasize that the reward parameter has been introduced to allow for the order of choice to determine the value of the function and reflect the consequences from making an initially optimal or suboptimal choice. We will assume that $c^b = 0.1$ in all frameworks.

In a nutshell, this scenario considers the intervals where the input limit of the first DMU is located above that of the second, implying a higher probability of being inefficient, as reflected by the first and second terms of Eq. (4), while the efficient subset of realizations and the corresponding reward are described by the third expression. These combinations define the value of the inputs obtained when all potential realizations and the corresponding pairs are computed by the DM. Clearly, DMUs may belong to categories displaying higher limit inputs but require lower amounts than those located in categories with lower limit inputs.

The results derived from this combinatorial process must be considered by the DM before evaluating the DMUs through DEA. That is, the results from the potential combinations must define the inputs and outputs of the corresponding DEA problem.

5.3. Modifying the reference values and relative performances

Two important remarks regarding the evaluation functions follow. Note that we are not considering a reference certainty equivalent or expected value to compare the outcome observed. Thus, the benchmark of reference is given by the realizations observed after selecting a given DMU. In the cases above, the reference benchmark is given by the realizations of the first alternative or DMU chosen. That is, whenever the second alternative performs worse than the first one, the DM knows that the initial choice was the correct one, implying that a positive payment must be incorporated into the equation. Since we are dealing with inputs, a positive payment implies a decrease in the value of the inputs required.

As a first remark, we must note that the reference framework could be shifted to the second DMU. This possibility implies that the realizations of the first DMU located above those of the second are suboptimal and a penalty should be incorporated into the corresponding equations as follows:

$x_{ia}^M \leq x_{ib}^M$ framework

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_0^{x_{ia}^M} \frac{1}{x_{ia}}\left[\int_0^{z_{ia}} \frac{1}{x_{ib}}\left(z_{ia} + c^b\right)dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}}(z_{ib})dz_{ib}\right]dz_{ia} \quad (3^*)$$

$x_{ia}^M > x_{ib}^M$ framework

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_{x_{ib}^M}^{x_{ia}^M} \frac{1}{x_{ia}}\left[z_{ia} + c^b\right]dz_{ia} + \int_0^{x_{ib}^M} \frac{1}{x_{ia}}\left[\int_0^{z_{ia}} \frac{1}{x_{ib}}\left(z_{ia} + c^b\right)dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}}(z_{ib})dz_{ib}\right]dz_{ia} \quad (4^*)$$

Clearly, the change in benchmark DMUs modifies the expressions of the value functions. The results obtained would be quantitatively different, though qualitatively, the same intuition remains. The equations defining the value functions must be adapted depending on the benchmark of reference chosen, whether it is

- the first DMU selected, implying that the second choice performing relatively worse validates the first alternative as the correct one, leading to a positive compensation effect in the form of lower inputs,
- or the second DMU, in which case, any performance of the first alternative above the second implies a suboptimal initial choice, leading to a penalty, namely, a higher input requirement in a DEA environment.

However, adding inputs artificially to a DMU is counterintuitive when considering the basic premises on which a DEA model is built.

The second remark is based on the relative performance of the alternatives. That is, the model could be defined for every relative value of each realization as follows:

$x_{ia}^M \leq x_{ib}^M$ framework

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_0^{x_{ia}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ib}} \left(z_{ia} + c^b[z_{ia} - z_{ib}]\right) dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b[z_{ib} - z_{ia}]\right) dz_{ib} \right] dz_{ia} \tag{3**}$$

$x_{ia}^M > x_{ib}^M$ framework

$$V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) = \int_{x_{ib}^M}^{x_{ia}^M} \frac{1}{x_{ia}} \left[z_{ia} + c^b[z_{ia} - z_{ib}]\right] dz_{ia} + \int_0^{x_{ib}^M} \frac{1}{x_{ia}} \begin{bmatrix} \int_0^{z_{ia}} \frac{1}{x_{ib}} \left(z_{ia} + c^b[z_{ia} - z_{ib}]\right) dz_{ib} + \\ \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b[z_{ib} - z_{ia}]\right) dz_{ib} \end{bmatrix} dz_{ia} \tag{4**}$$

Once again, the quantitative results would differ, since frictions would be determined by the relative width of the domains, while the qualitative ones would remain unchanged. In this case, we are considering opportunity costs and a relative penalty is added in case the second DMU is expected to perform relatively better, while opportunity rewards are applied whenever the opposite occurs.

As already stated, the artificial addition of inputs to incorporate opportunity costs does not align well with the basics of DEA. The selection of a DMU requiring a higher input is penalized by not considering the lower values realized from the second unit, while the decrease in inputs is incorporated to allow for the equation to differentiate outcomes by the order of selection. Further, selecting a different DMU implies having to deal with additional costs characterizing the transfer process. The version of the equations used to define the value functions in the paper has been selected due to its simpler formulation that allows to formalize all the required intuition while being easily implementable.

5.4. Extended DEA with two DMUs

The extension of the DEA problem that incorporates Eqs. (3) and (4) within the corresponding constraints is given by

$$min\ \theta - \varepsilon\left(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-\right) \tag{5}$$

subject to

$$\sum_{(a,b)\in W_2(n)} \lambda_{(a,b)} V\left(x_{ia}, x_{ib}, x_{ia}^M, x_{ib}^M\right) + s_i^- = \theta V\left(x_{ia_0}, x_{ib_0}, x_{ia_0}^M, x_{ib_0}^M\right),\ i = 1, 2, \dots, m;$$

$$\sum_{(a,b)\in W_2(n)} \lambda_{(a,b)} V\left(y_{ra}, y_{rb}, y_{ra}^M, y_{rb}^M\right) - s_r^+ = V\left(y_{ra_0}, y_{rb_0}, y_{ra_0}^M, y_{rb_0}^M\right),\ r = 1, 2, \dots, s;$$

$$\sum_{(a,b)\in W_2(n)} \lambda_{(a,b)} = 1$$

$$\lambda_{(a,b)} \geq 0, \qquad (a,b) \in W_2(n).$$

where $W_2(n)$ is the set of all ordered combinations (or permutations) of the total of n DMUs taken, 2 and $\lambda_{(a,b)}$, with $(a,b) \in W_2(n)$, are the reference values of the model.

Note that $W_2(n)$ coincides with the set of all ordered pairs that can be obtained considering all the available DMUs, $W_2(n) = \{(a,b) : a \text{ and } b \text{ are DMUs}\}$. Moreover, the cardinality of $W_2(n)$ is given by $|W_2(n)| = \frac{n!}{(n-2)!}$. For instance, the five alternatives described in Table 1(a) would lead to a total of 20 pairs.

5.5. Numerical example

The sequential interactions among DMUs and their effect on efficiency are illustrated using the basic DEA scenario described in Section 3.1. Table 2 presents all possible ordered combinations of two DMUs that can be generated based on the set of the DMUs defining the initial efficiency problem. The value assigned to each combined pair of DMUs is determined by the set of potential realizations and the reward assigned based on the order of selection, i.e., $c^b = 0.1$. The results obtained when solving the minimization problem are described in Table 3.

To simplify the presentation, in Tables 2 and 3, set notations have been introduced to denote the different ordered combinations of two DMUs. These sets are listed in the 1st column of both tables and

Table 2
Extended DEA: Combinatorial structure derived from the basic DEA setting.

SET	DMU 1st	DMU 2nd	Cost Input		Combined Value	Response Time Input		Combined Value	Profit Output	Combined Value
			DMU 1st	DMU 2nd		DMU 1st	DMU 2nd			
S1	1	2	1	2	1.0083	5	2	2.6133	2	1.2833
S2	1	3	1	4	1.9542	5	1	2.5233	2	1.2833
S3	1	4	1	6	2.9361	5	1	2.5233	2	1.2833
S4	1	5	1	4	1.9542	5	4	2.9933	2	1.2833
S5	2	1	2	1	1.0583	2	5	2.5533	2	1.2833
S6	2	3	2	4	2.0917	2	1	1.0583	2	1.2833
S7	2	4	2	6	3.0278	2	1	1.0583	2	1.2833
S8	2	5	2	4	2.0917	2	4	2.0917	2	1.2833
S9	3	1	4	1	2.0292	1	5	2.4433	2	1.2833
S10	3	2	4	2	2.1417	1	2	1.0083	2	1.2833
S11	3	4	4	6	3.3778	1	1	0.6167	2	1.2833
S12	3	5	4	4	2.6167	1	4	1.9542	2	1.2833
S13	4	1	6	1	3.0194	1	5	2.4433	2	1.2833
S14	4	2	6	2	3.0944	1	2	1.0083	2	1.2833
S15	4	3	6	4	3.4111	1	1	0.6167	2	1.2833
S16	4	5	6	4	3.4111	1	4	1.9542	2	1.2833
S17	5	1	4	1	2.0292	4	5	2.9733	2	1.2833
S18	5	2	4	2	2.1417	4	2	2.1417	2	1.2833
S19	5	3	4	4	2.6167	4	1	2.0292	2	1.2833
S10	5	4	4	6	3.3778	4	1	2.0292	2	1.2833

numbered from 1 to 20, namely, S1, S2, S3…, S20. Each of these sets corresponds to one of the possible ordered pairs that can be formed using the 5 DMUs. The generic ordered pair is denoted by $\left(\text{DMU }1^{\text{st}}, \text{DMU }2^{\text{nd}}\right)$ and all the possible ordered pairs are specified in the 2nd and 3rd columns of both tables. Furthermore, the 4th and 5th columns of Table 2 report the cost inputs of each ordered combination $\left(\text{DMU }1^{\text{st}}, \text{DMU }2^{\text{nd}}\right)$, while the response time inputs are given in the 7th and 8th columns. Finally, in Table 3, the reference values have been indexed using the same indexes as the corresponding sets. That is, the values $\lambda_1^*, \lambda_2^*, \lambda_3^*, \ldots, \lambda_{20}^*$ listed in the 4th to the 23rd columns of Table 3 represent the optimal values of the variables $\lambda_{\left(\text{DMU }1^{\text{st}}, \text{DMU }2^{\text{nd}}\right)} \geq 0$ with $\left(\text{DMU }1^{\text{st}}, \text{DMU }2^{\text{nd}}\right) \in W_2(5)$.

Table 3 highlights the complementarities existing between DMU3 and DM4 in Table 1(b). Indeed, the fact that $\lambda_3^* = 1$ for DMU4 may justify the weak optimality of their combination. More importantly, note how not all combinations of the three efficient DMUs, namely, DMU1, DMU2 and DMU3, are efficient. Half of their potential combinations are not considered efficient by the extended model. Fig. 2(b) complements the results presented in Table 3 and provides additional intuition regarding the efficiencies obtained.

6. Combinatorial scenario with *k* DMUs

Value functions become increasingly complex as new DMUs are incorporated to the evaluation process. When adding a new DMU, two different sets of combinations must be defined, those resulting from the set of DMUs selected and the order in which they are chosen. That is, the combinatorial value obtained from the different sets of potential realizations changes according to the limits of the domains defining the different potential inputs and outputs and the order of selection of the DMUs.

6.1. Introducing a third DMU

Consider the case where DMs may observe three different realizations, denoted by $z_{ia}, z_{ib},$ andz_{ic}, from DMUs a, b, and c, with $a \neq b \neq c$.

Considering potentially suboptimal realizations from a second and third DMU implies introducing increasing rewards, which will be assumed linear and equal to c^b and $2c^b$, respectively.

Six permutations must be now computed to completely define the corresponding value function $V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right)$.

In the following section, we consider the $x_{ia} \leq x_{ib} \leq x_{ic}$ framework. The same intuition applies to the remaining combinatorial scenarios described in the appendix section and determined by the relative values of the upper interval limits and the order of choice.

Fig. 3(b) illustrates different sets of potential realizations z_{ib} and z_{ic} that may be observed relative to the initial realizations z_{ia}.

6.2. $x_{ia} \leq x_{ib} \leq x_{ic}$ framework

We consider the $x_{ia} \leq x_{ib} \leq x_{ic}$ scenario. Note that the capacity of x_{ib} to improve upon x_{ia} is limited to the realizations composing the section of the domain of z_{ib} located below x_{ia}^M. The potential realizations of x_{ic} relative to x_{ia}^M and x_{ib}^M follow the same intuition. Eq. (6) incorporates these extensions into the analysis:

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_0^{x_{ia}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \displaystyle\int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \displaystyle\int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ib}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} \end{array} \right] dz_{ia} \tag{6}$$

Clearly, the limits of the densities are determined by the support of the intervals of potential realizations per DMU. The value function is therefore composed by four different terms.

The first two fall within the domain composed by the input requirements of the second DMU located below those of the first and the third. The first term corresponds to the scenario where the initial DMU selected requires the highest amount of input. The second term describes a situation where the last DMU selected is the one requiring the highest amount of input. The correct selection of the first DMU implies a reward of $2c^b$.

The third and fourth terms, composing the lower expression, account for the input realizations of the second DMU located above those of the first. These terms describe the scenarios where the input realizations of the third DMU are respectively lower and higher than those of the second. This intuition is reinforced by the rewards defined within each term.

Table 3
Extended DEA: Combinatorial results following from the simple DEA setting.

SET	DMU 1st	DMU 2nd	λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	λ_6^*	λ_7^*	λ_8^*	λ_9^*	λ_{10}^*	λ_{11}^*	λ_{12}^*	λ_{13}^*	λ_{14}^*	λ_{15}^*	λ_{16}^*	λ_{17}^*	λ_{18}^*	λ_{19}^*	λ_{20}^*	s_1^{+*}	s_1^{-*}	s_2^{-*}	θ^*
S1	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S2	1	3	0.5561	0	0	0	0	0.4439	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7621
S3	1	4	0.2974	0	0	0	0	0.7026	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.6027
S4	1	5	0.6675	0	0	0	0	0.3325	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7003
S5	2	1	0.9567	0	0	0	0	0.0433	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9971
S6	2	3	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S7	2	4	0	0	0	0	0	0	0	0	0	0.6847	0.3153	0	0	0	0	0	0	0	0	0	0	0	0	0.8361
S8	2	5	0.3917	0	0	0	0	0.6083	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7971
S9	3	1	0.5107	0	0	0	0	0.4893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7582
S10	3	2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
S11	3	4	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
S12	3	5	0.2131	0	0	0	0	0.7869	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.711
S13	4	1	0.2608	0	0	0	0	0.7392	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.599
S14	4	2	0	0	0	0	0	0	0	0	0	0.6092	0.3908	0	0	0	0	0	0	0	0	0	0	0	0	0.848
S15	4	3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0.0333	0	1
S16	4	5	0.0644	0	0	0	0	0.9356	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5928
S17	5	1	0.6385	0	0	0	0	0.3615	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.6899
S18	5	2	0.3917	0	0	0	0	0.6083	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7785
S19	5	3	0.2354	0	0	0	0	0.7646	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.7019
S20	5	4	0.0899	0	0	0	0	0.9101	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5904

6.3. *Further extensions: $x_{ia} \le x_{ib} \le x_{ic} \le x_{id} \le x_{ie}$ framework*

We add a fifth set of potential realizations per DMU to the analysis so as to illustrate how the complexity of the subsequent combinatorial problem increases. As an example, we describe the $x_{ia} \le x_{ib} \le x_{ic} \le x_{id} \le x_{ie}$ framework out of a total of 120 potential scenarios. The intuition provided reflects a decision environment determined by the order in which DMUs are selected and the resulting modifications in the interval structure that conditions the results.

We focus on this framework due to its relative simplicity among those generated, an intuition that follows from those analyzed when considering three DMUs. The way alternatives are selected, namely, following an increasing ordered pattern, allows for an intuitive description of the interactions arising among the different domains that determine the integration limits.

Consider the initial combinatorial decision made by the DM.

- The realization from the first DMU, z_{ia}, endowed with the potentially smallest input requirement, i.e., displaying the lowest upper value of the domain,
  - may not be exceeded by the next DMU, $z_{ib} \in [0, z_{ia}]$, defining the upper section of Eq. (7);
  - may be exceeded by the next DMU, that is, $z_{ib} \in [z_{ia}, z_{ib}^M]$, which leads the lower section of Eq. (7).

We must then continue with the next DMU, defining the realization z_{ic}, which

- may (when $z_{ic} \in [z_{ib}, z_{ic}^M]$) or may not (when $z_{ic} \in [0, z_{ib}]$) exceed input requirements relative to z_{ib} whenever the latter does not exceed z_{ia} (that is, $z_{ib} \in [0, z_{ia}]$), defining the next set of realizations within the upper level expression;
- may (when $z_{ic} \in [z_{ia}, z_{ic}^M]$) or may not (when $z_{ic} \in [0, z_{ia}]$) exceed input requirements relatively to z_{ia} whenever z_{ib} exceeds z_{ia} (that is, $z_{ib} \in [z_{ia}, z_{ib}^M]$), defining the next set of realizations within the lower level expression.

- We continue the analysis of Eq. (7) considering the fourth DMU, whose realizations, z_{id}, may be located
- below or above z_{ia} when all the previous realizations are below z_{ia};
- below or above z_{ic} whenever this variable is above z_{ia} and z_{ib} but this latter variable is located below z_{ia},

which constitutes the upper part of the value function.

At the same time, the realizations of z_{id} may be located

- below or above z_{ib} when the previous realizations are above z_{ia} but below z_{ib};
- below or above z_{ic} whenever this last variable is above z_{ia} and z_{ib},

defining the lower section of the value function.

- Finally, we have the realizations of z_{ie} defined within the corresponding brackets
- relative to z_{ia} and z_{id} depending on whether the latter exceeds or not z_{ia};
- relative to z_{ic} and z_{id} depending on whether the latter exceeds or not z_{ic}.

Both these sets are contained within the upper side of the equation.

- The lower side of the equation accounts for the realizations z_{ie} defined within the corresponding brackets,
- relative to z_{ib} and z_{id} depending on whether the latter exceeds or not z_{ib};

- ∘ relative to z_{ic} and z_{id} depending on whether the latter exceeds or not z_{ic} given the fact that this last realization exceeds z_{ib} and z_{ia}.

These sequential realization patterns provide a consistent description of the potential requirements that may be observed depending on those of the previous variables.

$$V\left(x_{ia},x_{ib},x_{ic},x_{id},x_{ie},x_{ia}^M,x_{ib}^M,x_{ic}^M,x_{id}^M,x_{ie}^M\right)=$$

$$\int_0^{x_{ia}^M}\frac{1}{x_{ia}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ia}}\frac{1}{x_{ib}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ia}}\frac{1}{x_{ic}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ia}}\frac{1}{x_{id}}\left[\int_0^{z_{ia}}\frac{1}{x_{ie}}(z_{ia})dz_{ie}+\int_{z_{ia}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}+\\
\displaystyle\int_{z_{ia}}^{x_{id}^M}\frac{1}{x_{id}}\left[\int_0^{z_{id}}\frac{1}{x_{ie}}\left(z_{id}-3c^b\right)dz_{ie}+\int_{z_{id}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}
\end{array}\right]dz_{ij}^c+\\
\displaystyle\int_{z_{ia}}^{x_{ic}^M}\frac{1}{x_{ic}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ic}}\frac{1}{x_{id}}\left[\int_0^{z_{ic}}\frac{1}{x_{ie}}\left(z_{ic}-2c^b\right)dz_{ie}+\int_{z_{ic}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}+\\
\displaystyle\int_{z_{ic}}^{x_{id}^M}\frac{1}{x_{id}}\left[\int_0^{z_{id}}\frac{1}{x_{ie}}\left(z_{id}-3c^b\right)dz_{ie}+\int_{z_{id}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}
\end{array}\right]dz_{ic}
\end{array}\right]dz_{ib}+\\
\displaystyle\int_{z_{ia}}^{x_{ib}^M}\frac{1}{x_{ib}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ib}}\frac{1}{x_{ic}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ib}}\frac{1}{x_{id}}\left[\int_0^{z_{ib}}\frac{1}{x_{ie}}\left(z_{ib}-c^b\right)dz_{ie}+\int_{z_{ib}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}+\\
\displaystyle\int_{z_{ib}}^{x_{id}^M}\frac{1}{x_{id}}\left[\int_0^{z_{id}}\frac{1}{x_{ie}}\left(z_{id}-3c^b\right)dz_{ie}+\int_{z_{id}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}
\end{array}\right]dz_{ic}+\\
\displaystyle\int_{z_{ib}}^{x_{ic}^M}\frac{1}{x_{ic}}\left[\begin{array}{l}
\displaystyle\int_0^{z_{ic}}\frac{1}{x_{id}}\left[\int_0^{z_{ic}}\frac{1}{x_{ie}}\left(z_{ic}-2c^b\right)dz_{ie}+\int_{z_{ic}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}+\\
\displaystyle\int_{z_{ic}}^{x_{id}^M}\frac{1}{x_{id}}\left[\int_0^{z_{id}}\frac{1}{x_{ie}}\left(z_{id}-3c^b\right)dz_{ie}+\int_{z_{id}}^{x_{ie}^M}\frac{1}{x_{ie}}\left(z_{ie}-4c^b\right)dz_{ie}\right]dz_{id}
\end{array}\right]dz_{ic}
\end{array}\right]dz_{ib}
\end{array}\right]dz_{ia} \tag{7}$$

We conclude by noting that Eq. (7) incorporates the rewards that must be accounted for based on the order of choice selected by the DM and the actual realizations observed.

7. Extended DEA with k DMUs

The extension of the DEA problem for the case where ordered combinations of k DMUs can be considered by the DM is formulated as follows.

$$min\ \theta-\varepsilon\left(\sum_{r=1}^{s}s_r^{+}+\sum_{i=1}^{m}s_i^{-}\right) \tag{8}$$

subject to

$$\sum_{\vec{a}\in W_k(n)}\lambda_{\vec{a}}V\left(x_{ia_1},x_{ia_2},\dots,x_{ia_k},x_{ia_1}^M,x_{ia_2}^M,\dots,x_{ia_k}^M\right)+s_i^{-}=\theta V\left(x_{ia_{1,0}},x_{ia_{2,0}},\dots,x_{ia_{k,0}},x_{ia_{1,0}}^M,x_{ia_{2,0}}^M,\dots,x_{ia_{k,0}}^M\right)\quad i=1,2,\dots,m;$$

$$\sum_{\vec{a}\in W_k(n)}\lambda_{\vec{a}}V\left(y_{ra_1},y_{ra_2},\dots,y_{ra_k},y_{ra_1}^M,y_{ra_2}^M,\dots,y_{ra_k}^M\right)-s_r^{+}=V\left(y_{ra_{1,0}},y_{ra_{2,0}},\dots,y_{ra_{k,0}},y_{ra_{1,0}}^M,y_{ra_{2,0}}^M,\dots,y_{ra_{k,0}}^M\right)\quad r=1,2,\dots,s;$$

Table 4
Input and output variables defining the ICT and Environmental framework of analysis.

		Input Variables									Output Variables		
		Eurostat Variables						OECD Variables		ITU Variable	Eurostat Variables		
		[TIN00115]	[TIN00110]	[TIN00125]	[TIN00116]	[TIN00090]	[TIN00111]	A1	C5B	IDI Relative	[SDG_08_10]	[TEC00116]	[TEN00135]
1	Belgium	26	31	72	27	98	24	64.32	87.68	86.97	35,050	129.6	3.2
2	Bulgaria	17	5	51	13	89	7	30.16	63.41	76.39	6,120	46.2	1.5
3	Czechia	12	31	79	16	98	24	48.83	84.64	79.73	17,490	82.5	2.7
4	Estonia	15	16	74	15	95	16	48.47	88.10	90.65	13,530	74.5	2.3
5	Ireland	12	33	76	27	96	30	55.70	81.24	89.31	53,400	183.5	0.7
6	Greece	10	4	52	15	85	11	43.71	69.89	80.51	17,110	74.2	1.2
7	Spain	17	16	80	28	98	20	58.89	84.60	86.75	24,440	101.9	1.6
8	France	13	19	75	18	99	17	59.78	86.56	91.76	32,360	114.7	1.9
9	Croatia	19	11	83	12	95	18	50.24	67.10	80.62	11,750	74.9	2.4
10	Italy	11	10	71	18	96	8	50.04	70.97	78.40	26,730	107.3	1.8
11	Lithuania	28	13	88	24	100	22	45.95	77.62	80.07	12,760	75.2	1.7
12	Hungary	9	20	70	9	91	13	47.17	76.75	77.17	12,020	67.9	1.9
13	Netherlands	19	15	77	28	100	16	72.48	95.33	94.54	40,730	110.5	2.4
14	Poland	21	15	70	16	95	10	45.61	75.99	76.73	11,800	74.8	1.9
15	Portugal	17	16	71	18	98	18	42.32	73.79	79.40	17,650	76.0	1.6
16	Romania	7	8	50	13	82	8	35.56	63.75	72.16	8,360	65.6	0.6
17	Slovenia	15	16	81	13	99	18	56.63	78.89	82.18	19,440	81.8	1.9
18	Slovakia	15	22	82	17	95	15	50.42	81.63	78.62	15,000	74.1	1.9
19	Finland	22	21	92	23	100	21	76.37	93.68	87.75	36,380	109.6	1.6
20	Sweden	13	19	77	20	97	29	78.02	96.19	93.65	43,430	112.9	2
21	Norway	16	21	79	20	94	29	75.49	97.55	94.32	69,130	131.9	1.5

$$\sum_{\vec{a} \in W_k(n)} \lambda_{\vec{a}} = 1$$

$$\lambda_{\vec{a}} \geq 0 \qquad \vec{a} \in W_k(n).$$

where $W_k(n)$ is the set of all ordered combinations (or permutations), $\vec{a} = (a_1, a_2, \ldots, a_k)$, of the n DMUs taken, k and $\lambda_{\vec{a}}$, with $\vec{a} \in W_k(n)$, are the reference values of the model. The cardinality of $W_k(n)$ is given by $|W_k(n)| = \frac{n!}{(n-k)!}$.

8. Numerical evaluations and frequency distributions

The empirical analysis aims to illustrate the consequences of introducing the combinatorial structures described throughout the previous sections into a DEA framework, allowing DMs to modify the DMUs selected when facing uncertain interval realizations of the input and output variables.

Table 4 presents the values of the input and output variables considered, which condition the sets of potential realizations that may be observed from the different countries analyzed. The year selected to study the efficiency of these countries, as determined by their ICT development levels and environmental expenses, is 2017. The choice is conditioned by the fact that the ICT Development Index provided by the International Telecommunication Union (ITU) of the United Nations is not available after this year. The model could be extended to account for dynamic interactions, but this would reduce the list of countries analyzed due to the substantial amount of missing data.

We retrieve data from three different institutions to study ICT assimilation and implementation differences across countries inferred through the characteristics of their production processes and environmental expenses. We have selected variables according to their availability while avoiding overlaps in the concepts measured across institutions. It has been assumed that all variables are equally important. We have retrieved data from Eurostat, ITU, and the Organisation for Economic Co-operation and Development (OECD). The data is public and available from the corresponding websites of these institutions.

Most input variables have been retrieved from Eurostat (https://ec.europa.eu/eurostat/web/main/data/database). These variables have been selected to provide a general perspective regarding the ICT capabilities of countries and the firms within them as well as the relative importance assigned to the ICT sector by the employed population.

1. Enterprises whose business processes are automatically linked to those of their suppliers and/or customers [TIN00115]
2. Share of enterprises' turnover on e-commerce - % [TIN00110]
3. Enterprises giving portable devices for a mobile connection to the Internet to their employees [TIN00125]
4. Enterprises using software solutions, like CRM, to analyze information about clients for marketing purposes [TIN00116]
5. Enterprises with broadband access [TIN00090]
6. Enterprises having received orders online (at least 1 %) - % of enterprises [TIN00111]

We have also retrieved input data from the OECD database (https://stats.oecd.org/Index.aspx?DataSetCode=ICT_BUS) focusing on the use of ICTs among the employed and general population so as to prevent any potential overleap between these variables and those from Eurostat:

7. A1: Persons employed regularly using a computer in their work (%)
8. C5B: Individuals using the Internet - last 3 m (%) (All individuals aged 16–74)

The final input composing the model is the value of the last ICT Development Index available from the ITU database (https://www.itu.int/en/ITU-D/Statistics/Pages/IDI/default.aspx). We have normalized the value of the 2017 index relative to the highest one, which corresponds to Iceland.

9. IDI Value Relative to that of Iceland

An alternative measure to the ICT development index is provided by Dobrota et al. (2012), who defined an I-distance method to quantify information development by combining 11 indicators across the access, use and skills categories.

Output variables have also been retrieved from Eurostat. We have focused on the main indicators derived from production processes together with the environmental commitment of countries.

1. Real GDP per capita [SDG_08_10]
2. Nominal labor productivity per person employed (ESA 2010) [TEC00116]
3. National expenditure on environmental protection [TEN00135]

We must elaborate on the interpretation of the input variables selected. It could be argued that these variables should be reinterpreted as undesirable inputs, since DMUs should try to maximize their corresponding values. However, like any regular input such as capital or labor, a more developed ICT infrastructure confers a competitive advantage to the corresponding country. This advantage has been accumulated throughout the years, in the same way as technology and human capital. Indeed, DMs should expect to observe the resulting consequences in the GPD and productivity values. Thus, we treat these variables as regular inputs.

We have considered two sets of outputs to measure the efficiency of countries. The first one disregards environmental factors, while the second incorporates the national expenditure on environmental protection as a third output variable. In each case, we compare the efficiency results derived from a direct and an extended combinatorial application of DEA to the data. The intuition regarding the inclusion of environmental expenditures follows the work of Li et al. (2022a, 2022b), who designed a digital and green economy efficiency index using a panel of 277 cities in China from 2011 to 2018. They found that the digital economy significantly improved the efficiency of the green economy in the region through technological innovation.

All in all, the input variables have been selected to provide an approximate description of the ICT capabilities of countries, the firms within them, and the local employed population. The variables presented in Table 4 are proxies for the performance of the firms in the respective countries, describing the potential inputs that may be required and output obtained when interacting with them and the local institutions.

Note that a firm displaying lower input requirements than another may use a higher amount. This outcome should be less probable than the opposite one but remains a potential result that must be considered by the DM when choosing a firm with which to interact. The fact that the potential inputs required and outputs obtained may differ from those described and expected could force the DM to interact with firms from a different country. Dealing with this type of uncertainty motivates the model introduced in this paper.

8.1. Efficiency analysis and combinatorial selection

Assume that a DM wants to develop a project with a local firm belonging to one of the countries described. As already stated, the indicators measuring the ICT development level of countries are used to approximate the potential behavior of firms and institutions within them. When introducing the possibility of selecting different DMUs, the resulting interactions among the inputs and outputs defining each country and the firms within them must be incorporated into the analysis. The same intuition applies when considering several firms from a given country whenever the corresponding data are available.

Fig. 4 describes the efficiency of countries and their paired combinations absent environmental expenditures. Fig. 4(a) presents the efficiencies obtained from directly applying DEA to the data described in Table 4. There are a total of eight efficient countries. The group is quite heterogeneous, encompassing less developed countries such as Bulgaria, Croatia, Greece, Hungary, and Romania and more developed ones such as Ireland, Italy, and Norway. Intuitively, the relatively underdeveloped ICT structures of countries such as Bulgaria and Romania are compensated through reasonable GDP and labor productivity values.

Among the less efficient ones, we find countries such as Estonia, Lithuania, Spain, and Sweden, while others like Finland, France, and the Netherlands occupy intermediate positions. The same intuition applies in these cases, where technologically developed countries are matched with higher GDP and productivity values but insufficient to be considered fully efficient.

More developed countries exhibit higher GDPs and labor productivity, though these are not necessarily unique consequences of exploiting their ICT resources. Many other factors, such as the quality of infrastructures and human capital, determine GDP and productivity outcomes. The current analysis focuses on how ICT development, considered a sign of economic growth, reflects this quality to a certain extent. Allowing for interactions across countries incorporates a compensating mechanism in using resources and outputs obtained. As a result, entry and exit strategies across countries exhibiting different degrees of technological development can be defined by a DM according to the potential input requirements and outputs observed.

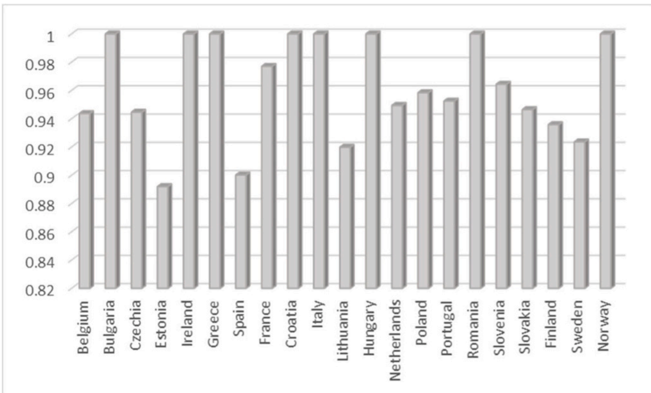
For instance, when considering one potential change in the DMU selected, we obtain a total of 420 combined pairs. The efficiencies derived from these pairs are presented in Fig. 4(b), while Fig. 4(c) focuses on the subset of 52 efficient pairs and their frequency distribution across countries. As can be observed, the set of efficient paired combinations includes DMUs that are not individually efficient. Fig. 4(c) illustrates the relative dominance of Ireland and Romania within the efficient pairs, with Norway and Italy losing some ground while Bulgaria, Hungary, Greece, and Croatia become less represented.

A similar intuition follows from the analysis of the triples of DMUs described in Fig. 5. Fig. 5(a) presents the efficient triples of DMUs, while Fig. 5(b) describes the frequency distribution of the countries composing these triples. The higher number of combinations derived from the triples (7980) increments substantially the set of potentially efficient countries (229). However, it must be noted that Finland, Lithuania, Slovakia, and Spain are not included in the set of efficient triples.

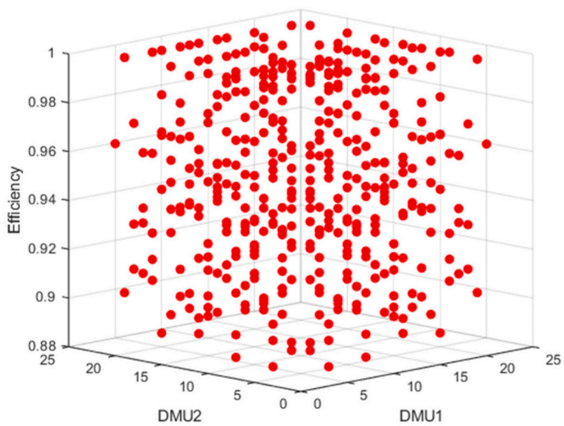
Figs. 6 and 7 incorporate environmental expenses into the analysis as an output. The results and interpretation are similar to those of Figs. 4 and 5. However, the number of efficient pairs and triples is now higher since developed countries can compensate for inefficiencies in GDP and productivity via environmental expenses. Fig. 6(a) illustrates how countries such as France, the Netherlands, and Sweden, become efficient. On the other hand, Finland remains inefficient, given its relatively smaller environmental expenditures.

A DM selecting a pair or triple of countries to interact with should be indifferent between the efficient ones in the interaction order. When considering efficient paired combinations, Romania displays the highest frequency, followed by Ireland, Norway, and Italy. On the other hand, Bulgaria, Croatia, Greece, and Hungary lose some influence within the potentially efficient pairs. Note how a group of countries remains out of the efficient pairs delivered by the model. This situation changes when considering combinations of triples, where countries can further compensate for their relative inefficiencies, and none of them is excluded from the set of efficient triples.

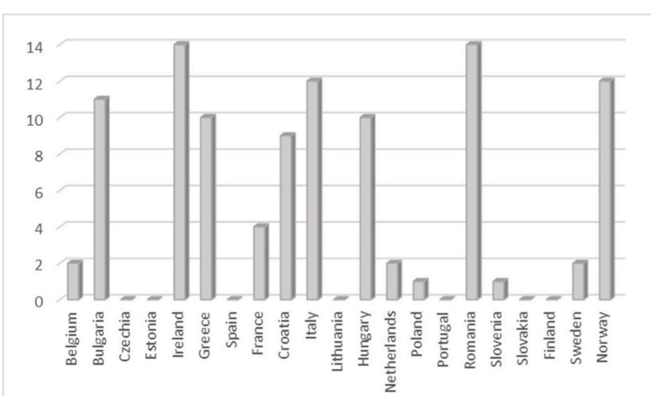
We highlight the substantial information acquisition and evaluation costs that must be incurred to compute the whole set of potential combinations. These costs must be added to the pecuniary and strategic ones derived from changes in the DMUs selected, which limit the number of combinatorial frameworks that may be considered by the DM.



(a). DEA country efficiency without environmental expenses

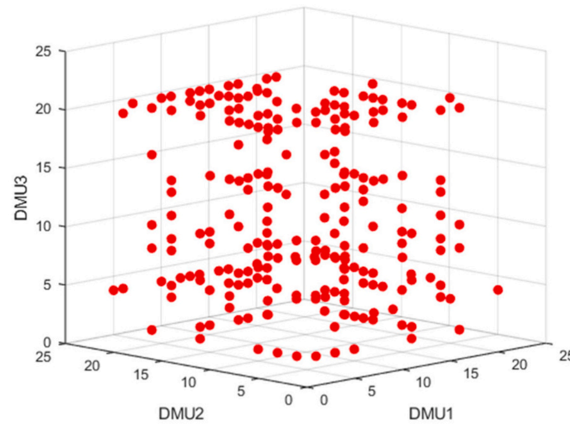


(b). The efficiency of combined pairs absent environmental expenses: 52 efficient out of a total of 420

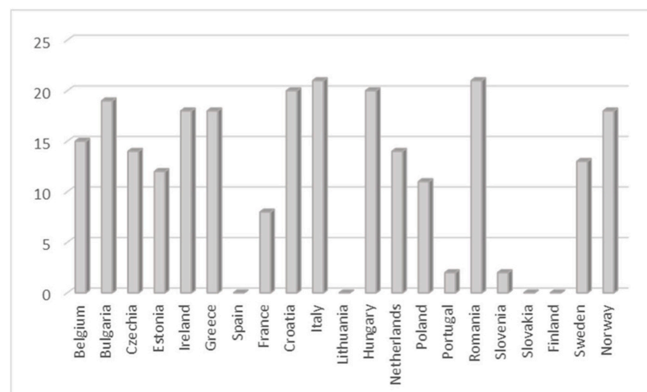


(c). Frequency distribution within 52 pairs absent environmental expenses

Fig. 4. Paired evaluation without environmental expenses.



(a). Efficient triples absent environmental expenses: total 229 out of 7980



(b). Frequency distribution within 229 pairs absent environmental expenses

Fig. 5. Triples evaluation without environmental expenses.

8.2. Practical and theoretical implications

The practical and theoretical implications of the results obtained in the previous sections can be outlined as follows.

From a practical viewpoint, the relevance of solving the problem faced by DMs who need to select countries and/or local firms within a country with which to interact is unquestionable. The uncertainty related to the input and output information available to DMs when assessing countries or local firms constitutes a serious obstacle to performing an objective evaluation of the available alternatives and defining a ranking that properly reflects the expectations of the DM.

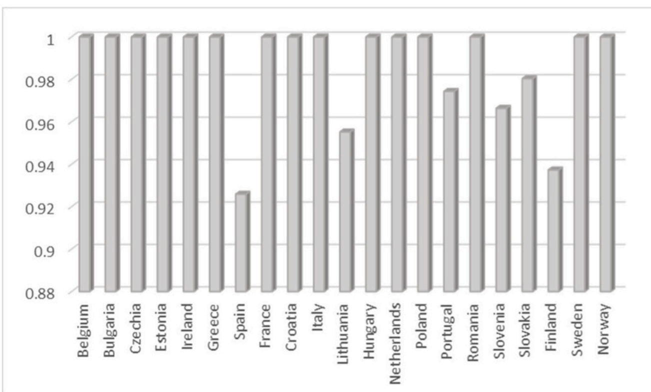
There is a concrete possibility that the DM observes realizations from an initially selected country or local firm that differ from those expected. As a consequence, the DM may want to modify the initial choice. Clearly, the same reasoning applies to any of the countries or local firms classified as second, third, and so on, based on a well-defined performance measure.

Our study shows that modeling countries/local firms as DMUs within a DEA efficiency evaluation environment offers a reasonable solution, but this model must be integrated with a combinatorial framework that systematically allows the DM to modify the initial choices made and select the best possible alternate DMU without having to evaluate again the whole set of DMUs.

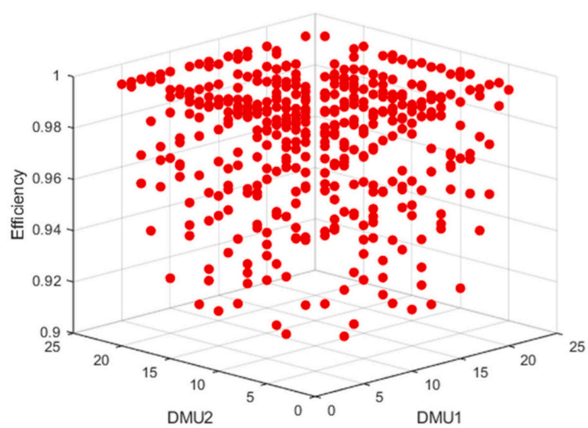
The results of the case study show that the implementation of the proposed combinatorial analysis constitutes a coherent procedure for the DM, who will be able to not only select a set of alternate DMUs to interact with in place of the one initially selected, but also determine the optimal order to follow when switching to new DMUs. The possibility of knowing from the very beginning, even before choosing the initial DMU, which DMUs will be the best ones to interact with and the order of interaction represents a clear, practical advantage given the costs involved in retrieving information and the structural pecuniary consequences that the selection of a new DMU would imply.

From a more theoretical viewpoint, it must be underlined that the DMUs selected and the order in which they are selected may differ substantially when accounting for the complementarities existing among DMUs. Moreover, the selection process and any subsequent DM's decision regarding the DMU to choose next vary with the number of modifications that the DM is willing to consider relative to the DMU initially selected. These are key features of the proposed formal environment and relate to the non-recursive character of the structure of the evaluation functions V defined for the single scenarios.

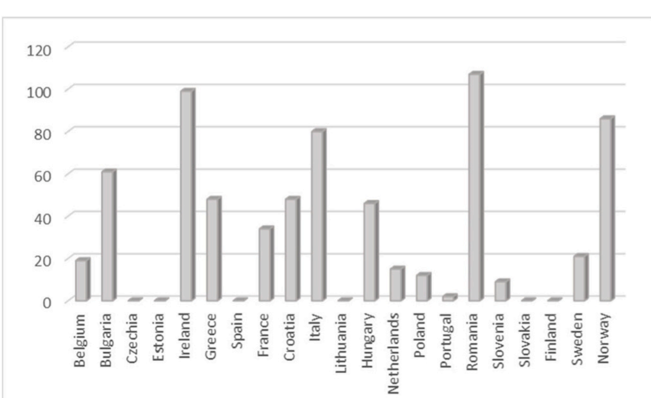
As a consequence, the formal and computational complexity of the analysis performed within the single combinatorial scenarios increases considerably as the number of changes that the DM is willing to consider increases. This limitation of the proposed approach does not necessarily impact the practical applications. Indeed, as mentioned above, the information acquisition and evaluation costs associated with the computation of V when considering ordered combinations of DMUs of large dimensions are substantial. These costs must be added to the pecuniary and strategic ones derived from the decision to select new DMUs. This fact inevitably limits the number of combinatorial frameworks that may be considered by a DM, providing a sort of self-regulating mechanism.



(a). DEA country efficiency with environmental expenses

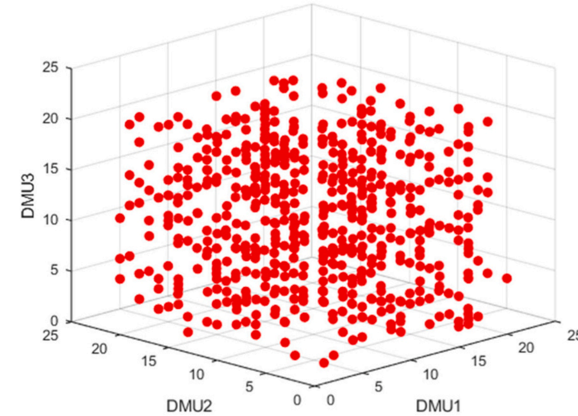


(b). The efficiency of combined pairs with environmental expenses: 123 efficient out of a total of 420

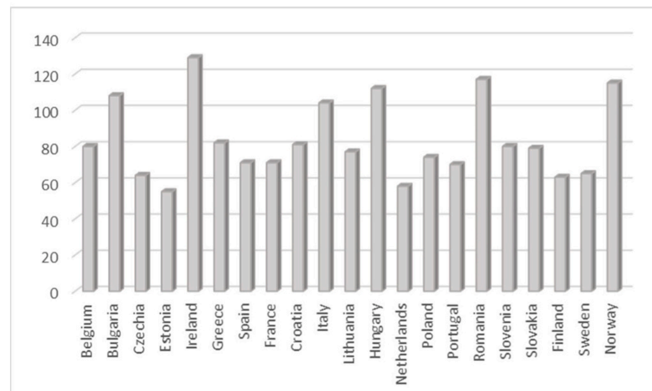


(c). Frequency distribution within 123 pairs and environmental expenses

Fig. 6. Paired evaluation with environmental expenses.



(a). Efficient triples with environmental expenses: total 585 out of 7980



(b). Frequency distribution within 585 triples and environmental expenses

Fig. 7. Triples evaluation with environmental expenses.

9. Conclusion

The input and output information available to DMs when assessing DMUs is generally imprecise. We have analyzed the consequences derived from this type of uncertainty, represented through interval variables when evaluating the efficiency of a series of countries based on their ICT development levels. Environmental expenditures have been introduced as outputs, allowing countries to compensate for the potential inefficiencies arising from their production processes. The combinatorial evaluations obtained are determined by the domains of the input and output variables defining the DMUs and the order in which the latter are selected. The increasing complexity of the combinatorial setting as additional DMUs are incorporated into the analysis has been highlighted.

Given the dependence of the results on the number of modifications in the DMU chosen, a potential line of research should focus on defining heuristic methods to incorporate additional combinatorial frameworks into the analysis. The DMs should limit the number of changes they will consider, given the different costs involved when modifying an already selected DMU.

An additional extension could be defined by introducing strategic information transmission to evaluate DMUs. Modifying the inputs that may be required or the outputs produced by different DMUs would allow for introducing beliefs and signals into the analysis with the corresponding reporting strategies and equilibria. The assignment of credibility weights to the reporters would constitute a relevant extension that could be incorporated into various DEA-related environments.

CRediT authorship contribution statement

Francisco J. Santos-Arteaga: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Debora Di Caprio:** Methodology, Validation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing. **Madjid Tavana:** Formal analysis, Methodology, Investigation, Writing – original draft, Writing – review & editing, Visualization.

Data availability

Data will be made available on request.

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Appendix A. Combinations of triples

The current appendix completes the combinatorial frameworks defining the potential triples of DMUs based on the relative widths of the domains of the variables and the selection order applied by the DM.

A.1. $x_{ia} \leq x_{ic} \leq x_{ib}$ framework

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_0^{x_{ia}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \displaystyle\int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \displaystyle\int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ib}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \int_{x_{ic}^M}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \end{array} \right] dz_{ia} \tag{A.1}$$

A.2. $x_{ic} \leq x_{ia} \leq x_{ib}$ framework

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_{x_{ic}^M}^{x_{ia}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ib}} (z_{ia}) dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \right] dz_{ia} + \int_0^{x_{ic}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \displaystyle\int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \displaystyle\int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ib}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \int_{x_{ic}^M}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \end{array} \right] dz_{ia} \tag{A.2}$$

A.3. $x_{ic} \leq x_{ib} \leq x_{ia}$ framework

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_{x_{ib}^M}^{x_{ia}^M} \frac{1}{x_{ia}} [z_{ia}] dz_{ia} + \int_{x_{ic}^M}^{x_{ib}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ib}} (z_{ia}) dz_{ib} + \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \right] dz_{ia} + \int_0^{x_{ic}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \displaystyle\int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \displaystyle\int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ib}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \int_{x_{ic}^M}^{x_{ib}^M} \frac{1}{x_{ib}} \left(z_{ib} - c^b\right) dz_{ib} \end{array} \right] dz_{ia} \tag{A.3}$$

A.4. $x_{ib} \leq x_{ia} \leq x_{ic}$ framework

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_{x_{ib}^M}^{x_{ia}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ia} + \int_0^{x_{ib}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ij}^b}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} \end{array} \right] dz_{ia} \tag{A.4}$$

A.5. $x_{ib} \leq x_{ic} \leq x_{ia}$ framework

$$V\left(x_{ia}, x_{ib}, x_{ic}, x_{ia}^M, x_{ib}^M, x_{ic}^M\right) = \int_{x_{ic}^M}^{x_{ia}^M} \frac{1}{x_{ia}} [z_{ia}] dz_{ia} + \int_{x_{ib}^M}^{x_{ic}^M} \frac{1}{x_{ia}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ia}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ia} + \int_0^{x_{ib}^M} \frac{1}{x_{ia}} \left[\begin{array}{l} \int_0^{z_{ia}} \frac{1}{x_{ib}} \left[\int_0^{z_{ia}} \frac{1}{x_{ic}} (z_{ia}) dz_{ic} + \int_{z_{ij}^a}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} + \\ \int_{z_{ia}}^{x_{ib}^M} \frac{1}{x_{ib}} \left[\int_0^{z_{ib}} \frac{1}{x_{ic}} \left(z_{ib} - c^b\right) dz_{ic} + \int_{z_{ib}}^{x_{ic}^M} \frac{1}{x_{ic}} \left(z_{ic} - 2c^b\right) dz_{ic} \right] dz_{ib} \end{array} \right] dz_{ia} \tag{A.5}$$

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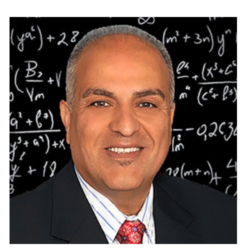

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