



Original articles

# An integrated data envelopment analysis and simulation method for group consensus ranking

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## Highlights

- Very few studies have integrated data envelopment analysis (DEA) and simulation.
- We propose an integrated DEA and simulation method for group consensus ranking.
- DEA is used to produce several rankings of the efficient candidates.
- Simulation is used to analyze these rankings and synthesize them into one group ranking.
- A case study demonstrates the applicability and efficacy of the proposed method.

## Abstract

Group consensus ranking is an important topic in performance evaluation and selection research. Data envelopment analysis (DEA) has been used for obtaining an efficiency score (preference score) for each candidate. We propose an integrated DEA and simulation method for group consensus ranking. The ranking method proposed in this study has several unique features. In contrast to most voting methods that assume equal voting power to voters, the proposed method classifies voters into different groups and allows for assigning a different voting power to each group. In spite of its effectiveness, though similarly to the competing methods in the literature, the proposed method may lead to more than one efficient candidate. Several ranking models are extended and used to discriminate among the efficient candidates. Despite the wealth of information provided to the decision maker(s), different extended ranking models may produce different rankings. Simulation is used to analyze these rankings and synthesize them into one overall group ranking. A case study is used to demonstrate the applicability and exhibit the efficacy of the proposed method. © 2015 International Association for Mathematics and Computers in Simulation (IMACS). Published by Elsevier B.V. All rights reserved.

**Keywords:** Data envelopment analysis; Simulation; Group ranking; Discrimination; Consensus

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## 1. Introduction

It is often necessary in decision making to rank a group of candidates using a voting system. In a typical ranked voting method, each voter selects a subset of candidates and ranks them from most to least preferred. Among these methods, a popular procedure for obtaining a group consensus ranking is the scoring method where fixed scores are assigned to different places. In this way, the score obtained by each candidate is the weighted sum of the scores he or she receives in different places. The plurality and Borda methods are the most widely used scoring methods. In the plurality method, the selected candidate is the one who obtains the most votes in the first place. In other words, in this method, the first place receives an importance weight of 100% while all other places receive a weight of zero. In Borda's method, the weight assigned to the first place equals to the number of candidates and each subsequent place receives one unit less than its preceding place. Although Borda's method has interesting properties, the utilization of a fixed scoring vector implies that the choice of the winner may depend upon which scoring vector is used [7].

Cook and Kress [8] have proposed a Data Envelopment Analysis (DEA) method for estimating the preference scores of the candidate without imposing a fixed weight to each place from the outset. Different sets of weights are used to calculate the best relative total scores, which are all restricted to be less than or equal to one. The candidate with the largest relative total score of unity is said to be efficient and may be considered as the selected candidate. However, this method often identifies more than one candidate as the efficient candidate. To avoid this shortcoming and to select a single candidate, Cook and Kress [8] proposed to maximize the gap between the consecutive weights of the scoring vector so that only one candidate is considered to be efficient. However, such an approach is equivalent to imposing a common set of weights on all the candidates. Therefore, Green et al. [16] applied the cross-efficiency evaluation technique in DEA to get the best efficient candidate. Noguchi et al. [26] also used the cross-efficiency evaluation to identify the best candidate and give a strong ordering constraint condition on the weights. Hashimoto [17] utilized the super-efficiency model in DEA to find the best candidate. Obata and Ishii [27] proved that these methods have a weak point since the order of efficient candidates may be changed by the existence of an inefficient candidate. Therefore, they proposed a new method to discriminate among the efficient candidates without any need for information about the inefficient candidates.

Foroughi and Tamiz [13] and Foroughi et al. [12] extended and simplified their method with fewer constraints and used it to rank the efficient and inefficient candidates. Wang and Chin [35] discriminated among the efficient candidates by considering their least relative total scores. However, their argument is not persuasive since the least relative total scores and the best relative total scores were not measured within the same range. They also proposed a model in which the total scores were measured within an interval. The upper bound of the interval was set to one, but they failed to determine the value of the lower bound for the interval. In response, Wang et al. [36] proposed a method for ranking multiple efficient candidates by comparing the least relative total score of the efficient candidate with the best and the worst relative total scores measured in the same range. They used a numerical example to identify the efficient candidates and showed that their model produces less efficient candidates than the model proposed by Wang and Chin [35]. Foroughi and Aouni [11] proposed new approaches to determine a common set of weights in a ranked voting system. In addition, other models have appeared in the literature to deal with these kinds of problems [2–4,10,18–20,31–33]. Llamazares and Pena [25] analyzed the principal ranking methods proposed in the literature to discriminate efficient candidates and explained that none of the proposed procedures is fully convincing.

Given the heterogeneity in the results obtained from the DEA ranking methods, decision makers will have to subjectively determine which methods they consider to be more reliable and assign them a higher relative weight when defining a final overall consensus ranking. In this regard, the social choice and economics literatures have dealt with the subjective selection of expert opinions when evaluating a set of candidates [9,34,37]. Even though we will not be pursuing this latter approach to its full extent, which would bring the paper to a purely strategic domain, we will assume that the decision maker can define an overall consensus ranking based on the subjective weights assigned to each of the DEA ranking methods considered.

The overall contribution of this study is fourfold: (1) In contrast to most existing voting methods that assume equal voting power to all voters, the proposed method classifies voters into different groups and assumes that each group is assigned a different voting power. (2) In spite of its effectiveness, though similarly to the competing methods in the literature, the proposed method may lead to more than one efficient candidate. Therefore, several ranking models are extended and used to discriminate among these efficient candidates. (3) Despite the wealth of information provided to the decision maker(s), different extended ranking models may produce different rankings. Although DEA

and simulation are methods that have been extensively studied and applied in the literature, there are very few studies which have integrated DEA and simulation. For example, recently, a few researchers have proposed hybrid methods for integrating DEA and simulation in farming [29], healthcare [1,23] and manufacturing [6,30]. However, integrated DEA and simulation frameworks are still in their early stages of development with simulation promising to be a very powerful tool for testing, improving and extending DEA models in hybrid uncertain environments [28]. In our study, we find simulation very useful to analyze the rankings and synthesize them into one overall group ranking of the candidates. (4) A case study is used to demonstrate the applicability and exhibit the efficacy of the proposed method.

The remainder of this paper is organized as follows. In Section 2, we review two DEA models commonly used for identifying efficient candidates in performance evaluation problems. In Section 3, we propose a new voting model for classification of voters with different voting powers. In Section 4, we extend several current ranking methods to discriminate among the efficient candidates in the proposed voting model. In Section 5, we use simulation to obtain a group consensus ranking for the efficient candidates. In Section 6, we present a case study to demonstrate the applicability and exhibit the efficacy of the proposed models and procedures. Finally, we present our conclusions and future research directions in Section 7.

## 2. Preliminaries

In this section, we consider a ranked voting problem where each voter selects a subset of  $k$  candidates from a set of  $m$  ( $m \geq k$ ) candidates  $\{A_1, A_1, \dots, A_m\}$  and ranks them from the top to the  $k$ th place. Each place is associated with a relative importance weight  $w_i$  ( $i = 1, 2, \dots, k$ ). Let  $v_i^r$  be the number of votes for candidate  $r$  being ranked in the place  $i$ . The total score of each candidate is defined as follows:

$$z_r = \sum_{i=1}^k w_i v_i^r, \quad r = 1, 2, \dots, m. \tag{1}$$

This is a linear function of the relative importance weights. Once the weights are determined, each candidate can be ranked in terms of its total score. To avoid subjectivity in obtaining the relative importance weights, Cook and Kress [8] have suggested the following DEA/AR which determines the most appropriate weights for candidate  $p$ :

$$\begin{aligned} z_p^* = \max \quad & z_p = \sum_{i=1}^k w_i v_i^p \\ \text{s.t.} \quad & \\ & \sum_{i=1}^k w_i v_i^r \leq 1, \quad r = 1, 2, \dots, m, \\ & w_i - w_{i+1} \geq d(i, \varepsilon), \quad i = 1, 2, \dots, k - 1, \\ & w_k \geq d(k, \varepsilon) \end{aligned} \tag{2}$$

where  $d(\cdot, \varepsilon)$  is called the discrimination intensify function. It is non-negative and monotonically increasing in a non-negative  $\varepsilon$  and satisfies  $d(\cdot, 0) = 0$ . Cook and Kress [8] showed that the choice of the discrimination intensify function form has a significant impact on the winner. Also, Noguchi et al. [26] showed that the choice of the winner depends upon the value of  $\varepsilon$ . To overcome these difficulties, Noguchi et al. [26] used the following strong ordering DEA model:

$$\begin{aligned} z_p^* = \max \quad & z_p = \sum_{i=1}^k w_i v_i^p \\ \text{s.t.} \quad & \\ & \sum_{i=1}^k w_i v_i^r \leq 1, \quad r = 1, 2, \dots, m, \\ & w_1 \geq 2w_2 \geq \dots \geq kw_k, \\ & w_k \geq \varepsilon = 2/Nk(k + 1) \end{aligned} \tag{3}$$

where  $N$  is the number of voters. This model will be used in the next section of the paper to illustrate our method.

Table 1  
Graphical illustration of the model.

| Group | Place | Candidate  |       |            |       |            |
|-------|-------|------------|-------|------------|-------|------------|
|       |       | 1          | ..... | r          | ..... | m          |
| 1     | 1     | $v_{11}^1$ | ..... | $v_{11}^r$ | ..... | $v_{11}^m$ |
|       | ..... | .....      | ..... | .....      | ..... | .....      |
|       | i     | $v_{i1}^1$ | ..... | $v_{i1}^r$ | ..... | $v_{i1}^m$ |
| j     | k     | $v_{k1}^1$ | ..... | $v_{k1}^r$ | ..... | $v_{k1}^m$ |
|       | 1     | $v_{1j}^1$ | ..... | $v_{1j}^r$ | ..... | $v_{1j}^m$ |
|       | ..... | .....      | ..... | .....      | ..... | .....      |
| t     | i     | $v_{ij}^1$ | ..... | $v_{ij}^r$ | ..... | $v_{ij}^m$ |
|       | ..... | .....      | ..... | .....      | ..... | .....      |
|       | k     | $v_{kj}^1$ | ..... | $v_{kj}^r$ | ..... | $v_{kj}^m$ |
| t     | 1     | $v_{1t}^1$ | ..... | $v_{1t}^r$ | ..... | $v_{1t}^m$ |
|       | ..... | .....      | ..... | .....      | ..... | .....      |
|       | i     | $v_{it}^1$ | ..... | $v_{it}^r$ | ..... | $v_{it}^m$ |
| t     | ..... | .....      | ..... | .....      | ..... | .....      |
|       | k     | $v_{kt}^1$ | ..... | $v_{kt}^r$ | ..... | $v_{kt}^m$ |

**Remark 1.** As argued by Hashimoto [20], in a ranked voting system each candidate is regarded as a decision making unit (DMU) in DEA. At the same time, each DMU is considered to have  $m$  outputs (ranked votes) and only one input with a unit value assigned. Note that this description corresponds to the pure output DEA model. This means that problem (3) is equivalent to the well-known DEA/AR model in which the first constraints are the usual ones in DEA (no candidate should have a preference score or an efficiency score greater than 1) and the remaining constraints represent the assurance region (AR).

### 3. Proposed model

It is assumed that in a ranked voting system, voters are classified into  $t$  distinct groups. The voters in each group select  $k$  candidates among  $m$  ( $m \geq k$ ) candidates  $\{A_1, A_1, \dots, A_m\}$  and rank them from the top to the  $k$ th place. Each place is associated with a relative importance weight  $w_i$  ( $i = 1, 2, \dots, k$ ) and each group is associated with a relative importance weight  $u_j$  ( $j = 1, 2, \dots, t$ ). As shown in Table 1, let  $v_{ij}^r$  be the number of votes for candidate  $r$  being ranked in the  $i$ th place by the  $j$ th group. The preference score of candidate  $r$  in the  $i$ th place is equal to  $\sum_{j=1}^t u_j v_{ij}^r$ . Thus, the total score for each candidate is determined as follows:

$$z_r = \sum_{i=1}^k \left( w_i \sum_{j=1}^t u_j v_{ij}^r \right), \quad r = 1, 2, \dots, m. \tag{4}$$

Note that if all groups have the same relative importance weight ( $u_j = 1, j = 1, 2, \dots, t$ ), then the preference score of candidate  $r$  in the  $i$ th place would be  $\sum_{j=1}^t v_{ij}^r$ , which is exactly the number of  $i$ th place votes that candidate  $r$  receives. In this case, our model is converted into that of Cook and Kress (2), in which all the voters are in one group. Thus, the value of  $z_r$  in (4) indicates the real score for each candidate.

It should be noted that Ebrahimnejad [10] also assumed that in a ranked voting system voters are classified into  $t$  distinct categories in terms of their priority, and defined the total score of each candidate based on the following formulation:

$$z_r = \sum_{j=1}^t u_j \left( \sum_{i=1}^k v_{ij}^r \right) + \sum_{i=1}^k w_i \left( \sum_{j=1}^t v_{ij}^r \right), \quad r = 1, 2, \dots, m. \tag{5}$$

However, the scores defined by (5) cannot satisfactorily mimic the previous score condition. As a simple example, consider a ranking problem with two candidates using the votes from six voters in two categories. Let us further

assume that each category contains three voters and the votes of the first category are more important than those of the second one. Now, suppose that all three voters in the first category (the more important one) rank candidate 1 as their first choice and all three voters in the second category (the less important one) rank candidate 2 as their first choice. It is obvious that, for a properly defined score, the score of candidate 1 must be higher than the score of candidate 2. This should be the case since, although both candidates have three votes in their first place and three votes in their second place, the first place votes for candidate 1 are more important than those of candidate 2. However, using relation (5), we have that  $z_1 = z_2 = 3(u_1 + u_2 + w_1 + w_2)$ , which shows that candidates 1 and 2 have the same score for all possible selections of the weights. Note that the following values are defined for this example:

$$m = t = k = 2, \quad v_{11}^1 = 3, \quad v_{12}^1 = 0, \quad v_{21}^1 = 0, \quad v_{22}^1 = 3, \quad v_{11}^2 = 0, \\ v_{12}^2 = 3, \quad v_{21}^2 = 3, \quad v_{22}^2 = 0.$$

Therefore, the results reported by Ebrahimnejad [10] are invalid in the current context. In order to obtain a total ranking of candidates, we require that the weight vectors  $u = (u_1, \dots, u_t)$  and  $w = (w_1, \dots, w_k)$  given in formulation (4) satisfy the following strong ordering condition:

$$u_1 \geq 2u_2 \geq \dots \geq tu_t, \quad u_t \geq \varepsilon'_1 \tag{6}$$

$$w_1 \geq 2w_2 \geq \dots \geq kw_k, \quad w_k \geq \varepsilon'_2. \tag{7}$$

The following nonlinear model evaluates candidate  $p$  with the most favorable weight vectors:

$$z_p^* = \max \quad z_p = \sum_{i=1}^k \sum_{j=1}^t w_i u_j v_{ij}^p \\ \text{s.t.} \\ \sum_{i=1}^k \sum_{j=1}^t w_i u_j v_{ij}^r \leq 1, \quad r = 1, 2, \dots, m, \tag{8} \\ u_1 \geq 2u_2 \geq \dots \geq tu_t, \quad u_t \geq \varepsilon'_1, \\ w_1 \geq 2w_2 \geq \dots \geq kw_k, \quad w_k \geq \varepsilon'_2.$$

To transform the nonlinear model (8) into an equivalent linear model, let:

$$m_{ij} = w_i u_j, \quad i = 1, 2, \dots, k, \quad j = 1, 2, \dots, t. \tag{9}$$

Thus, the objective function defined in (8) is modified as follows:

$$z_r = \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r. \tag{10}$$

Similarly, we should redefine constraints (6) and (7) in terms of the above transformation in such a way that the priority among places and groups is preserved. To this end, we multiply the constraints defined in (6) and (7) by  $w_i$  ( $i = 1, 2, \dots, k$ ) and  $u_j$  ( $j = 1, 2, \dots, t$ ), respectively. Thus, we have:

$$m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon_{1i} = 2/Nt(t+1), \quad (i = 1, 2, \dots, k) \tag{11}$$

$$m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon_{2j} = 2/Nk(k+1), \quad (j = 1, 2, \dots, t). \tag{12}$$

Substituting (10)–(12) into model (8), the following linear model is obtained:

$$z_p^* = \max \quad z_p = \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^p \\ \text{s.t.} \\ \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r \leq 1, \quad r = 1, 2, \dots, m, \tag{13} \\ m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t.$$

In a similar way to Noguchi's model, we can choose  $\varepsilon$  in this model as follows,:

$$\varepsilon = \min \{2/Nt(t+1), 2/Nk(k+1)\}.$$

It is worth noting that in the proposed voting model (13),  $m_{ij}$  can be regarded as the relative importance weight of candidate  $r$  being ranked in the  $i$ th place by group  $j$ .

Similarly to model (3), several candidates may be recognized as efficient candidates in model (13). Therefore, we generalize six well-known ranking methods in the next section to discriminate the efficient candidate of model (13).

**Remark 2.** Similarly to model (3), the model (13) used in our proposed approach is equivalent to the traditional DEA/AR model. The first  $m$  constraints of model (13) are the standard ones in DEA and the remaining constraints represent the AR.

#### 4. Extension of ranking models

In this section, we apply the method proposed in this study to extend several current ranking methods in DEA and voting systems. Suppose that there are  $n$  efficient candidates in model (13) and denote by  $E$  the index set for these efficient candidates.

##### 4.1. Extension of Obata and Ishii's model

Obata and Ishii [27] considered using scoring vectors of the same size to compare the maximum score obtained by each candidate. They suggested normalizing the most favorable scoring vectors for each candidate. We apply Obata and Ishii's [27] method to our model (13) and propose model (14) as follows:

$$\begin{aligned} \frac{1}{\bar{z}_p^*} = \min \quad & z_p = \sum_{i=1}^k \sum_{j=1}^t m_{ij} \\ \text{s.t.} \quad & \\ \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^p = 1, & \\ \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r \leq 1, \quad & r \in E \setminus \{p\} \\ m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad & m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad & m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t. \end{aligned} \quad (14)$$

We use  $\bar{z}_p^*$  as a discrimination value to rank the efficient candidates.

##### 4.2. Extension of Hashimoto's model

Hashimoto [17] applied the DEA exclusion method introduced by Andersen and Petersen [5] to Cook and Kress's model. Here, we apply Hashimoto's [17] methodology to our model (13). This methodology enables an efficient candidate to achieve a score greater than one by removing the constraint relative to the aforementioned candidate in the formulation of model (13). The extension of this model is presented as model (15):

$$\begin{aligned} \bar{z}_p^* = \max \quad & z_p = \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^p \\ \text{s.t.} \quad & \\ \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r \leq 1, \quad & r = 1, 2, \dots, m, \quad r \neq p, \\ m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad & m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad & m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t. \end{aligned} \quad (15)$$

We use  $\bar{z}_p^*$  again as a discrimination value to rank the efficient candidates.

### 4.3. Common weights analysis extension

Liu and Peng [24] obtained and used a common set of weights to create the best efficiency score of a group composed of efficient units in DEA (see also Jahanshahloo et al. [21]). They then used that set of weights to evaluate the absolute efficiency of each efficient unit and rank them. Their methodology, which uses a ranking with a common set of weights, makes sense because a decision maker objectively chooses common weights for the purpose of maximizing the group efficiency. In order to extend this approach to our voting model, we define the ideal point as the multiplier bundle  $\tilde{m}$  for which every candidate is efficient. This means  $\sum_{i=1}^k \sum_{j=1}^t \tilde{m}_{ij} v_{ij}^r = 1$  or  $\alpha_r = 1 - \sum_{i=1}^k \sum_{j=1}^t \tilde{m}_{ij} v_{ij}^r = 0, (r \in E)$ . In fact,  $\alpha_r$  indicates the measure of deviation of candidate  $r$  from the efficiency score of one in terms of a multiplier bundle of an ideal point. In the absence of such an ideal point, a reasonable objective is to treat  $\alpha_r$  as the goal achievement variables. To this end, we must minimize  $\alpha_r$  for each efficient candidate. This leads to solving the following multiple objective linear programming (MOLP) problem:

$$\begin{aligned} \text{MOLP : } & \min (\alpha_1, \alpha_2, \dots, \alpha_n) \\ \text{s.t.} & \\ & \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha_r = 1, \quad r \in E \\ & \alpha_r \geq 0, \quad r \in E \\ & m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ & m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t. \end{aligned}$$

In order to solve this MOLP problem we apply some new approaches similar to the secondary goals in the DEA cross-efficiency evaluation proposed by Liang et al. [22]. In particular, we use the min-sum, max-ordering and minimizing mean absolute approaches to obtain an efficient solution for the above MOLP problem.

The min-sum approach leads to the following model that could be used to obtain the common set of weights:

$$\begin{aligned} \min & \sum_{r \in E} \alpha_r, \\ \text{s.t.} & \\ & \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha_r = 1, \quad r \in E \\ & \alpha_r \geq 0, \quad r \in E \\ & m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ & m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t. \end{aligned} \tag{16}$$

Solving the MOLP problem based on the max-ordering approach leads to the following model:

$$\begin{aligned} \min & \max(\alpha_1, \alpha_2, \dots, \alpha_n) \\ \text{s.t.} & \\ & \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha_r = 1, \quad r \in E \\ & \alpha_r \geq 0, \quad r \in E \\ & m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\ & m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t. \end{aligned}$$

By introducing an auxiliary variable  $\alpha = \max(\alpha_1, \alpha_2, \dots, \alpha_n)$ , the above model can be written as follows:

$$\begin{aligned}
& \min \alpha \\
& \text{s.t.} \\
& \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha \geq 1, \quad r \in E \\
& \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r \leq 1, \quad r \in E \\
& m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\
& m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t.
\end{aligned} \tag{17}$$

Also, the minimizing mean absolute approach leads to the following model that computes the mean absolute deviation of a set of data, namely, the average of the absolute deviations of the data points from their mean:

$$\begin{aligned}
& \min \frac{1}{n} \sum_{r \in E} |\alpha_r - \bar{\alpha}| \\
& \text{s.t.} \\
& \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha_r = 1, \quad r \in E \\
& m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\
& m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t
\end{aligned}$$

where  $\bar{\alpha} = \frac{1}{n} \sum_{r \in E} \alpha_r$  and  $n$  is the number of efficient candidates ( $|E| = n$ ).

To show that this nonlinear model can be transformed into an equivalent linear model, assume that  $a_r = \frac{1}{2} (|\alpha_r - \bar{\alpha}| + (\alpha_r - \bar{\alpha}))$  and  $b_r = \frac{1}{2} (|\alpha_r - \bar{\alpha}| - (\alpha_r - \bar{\alpha}))$ . Thus, the above model can be written as follow:

$$\begin{aligned}
& \min \frac{1}{n} \sum_{r \in E} (a_r + b_r) \\
& \text{s.t.} \\
& \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r + \alpha_r = 1, \quad r \in E \\
& a_r - b_r = \alpha_r - \frac{1}{n} \sum_{r \in E} \alpha_r, \quad r \in E \\
& \alpha_r, a_r, b_r \geq 0, \quad r \in E \\
& m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\
& m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t.
\end{aligned} \tag{18}$$

Finally, we generalize a model such that a manager can choose the most favorable weights for the group that comprises all candidates under his/her governance. In other words, a set of weights that maximizes the group comprehensive score is used as the common set of weights for all the units to obtain each individual's comprehensive score. To this end, define  $\sum_{r=1}^m v_{ij}^r$  as the number of votes received by the aggregated candidate in group  $j$  for the  $i$ th place. Then, the common set of weights is obtained by solving the following model:

$$\begin{aligned}
& \max \sum_{i=1}^k \sum_{j=1}^t m_{ij} \left( \sum_{r=1}^m v_{ij}^r \right) \\
& \text{s.t.} \\
& \sum_{i=1}^k \sum_{j=1}^t m_{ij} v_{ij}^r \leq 1, \quad r \in E \\
& m_{i1} \geq 2m_{i2} \geq \dots \geq tm_{it}, \quad m_{it} \geq \varepsilon, \quad i = 1, 2, \dots, k, \\
& m_{1j} \geq 2m_{2j} \geq \dots \geq km_{kj}, \quad m_{kj} \geq \varepsilon, \quad j = 1, 2, \dots, t.
\end{aligned} \tag{19}$$

We are now in a position to give a ranking of efficient candidates based on the optimal common weights obtained from models (16)–(19). To this end, it is sufficient to substitute the optimal weights given by models (16)–(19) into  $\bar{z}_p^* = \sum_{i=1}^k \sum_{j=1}^t m_{ij}^* v_{ij}^p$  as a ranking score for each efficient candidate.

### 5. Consensus ranking with simulation

Consider the six ranking models introduced to discriminate among the multiple efficient candidates proposed by model (13). As intuition suggests, the rankings obtained for the set of efficient candidates will differ among these models. We apply four different methods to generate overall consensus rankings based on the efficiency scores obtained for each candidate using models (14)–(19). Throughout this section, we maintain the notation of  $E$  as the index set of the efficient candidates obtained in model (13). We will denote the numerical values assigned to each one of the  $n$  efficient candidates within the set  $E$  using  $(r_h)_{h \in H}$ , where  $r \in E$  refers to the candidate being considered and  $h$  to the ranking method that is applied.

- The simplest way to obtain a consensus ranking consists of weighting equally the efficiency scores obtained by each model and calculating the average efficiency score for each candidate. Formally, the *simple average criterion* is based on the set of average ranking scores obtained for each candidate and denoted by  $\{\overline{r_h}\} = \left\{ \frac{\sum_{h \in H} r_h}{|H|} : r \in E \right\}$ , with  $|H|$  referring to the number of ranking criteria comprising the set  $H$ . The resulting set is ordered from the highest to the lowest value and the candidate chosen is given by the  $\text{argmax}\{\overline{r_h}\}$ .
- We also adapt two formal criteria defined in the game theoretical literature to study the selection of strategies by decision makers based on the potential payoffs obtained [14]. These criteria will be denoted as *max–min* and *max–max*. The first criterion aims at guaranteeing a minimum score to the decision maker and implies choosing the alternative (candidate) with the highest among the lowest efficiency scores obtained for each and every alternative. In this case, the decision maker maximizes the efficiency score obtained in the “worst evaluation scenario”. The max–max criterion follows a similar approach, with the decision maker selecting the alternative (candidate) with the highest among the highest efficiency scores obtained for each and every alternative.

In order to provide a formal definition of these criteria we must define first the set of minimum efficiency scores attained by each candidate, which is given by:

$$\{\underline{r_h}\} = \{\min\{r_h : h \in H\} : r \in E\} \tag{20}$$

and the set of maximum efficiency scores attained by each candidate, which is given by:

$$\{\overline{r_h}\} = \{\max\{r_h : h \in H\} : r \in E\}. \tag{21}$$

Thus, the resulting rankings will be based on the sets of minimum and maximum score values attained by each candidate. If the decision maker applies the max–min criterion, the choice of candidate will be given by the  $\text{argmax}\{\underline{r_h}\}$ , while the max–max criterion implies choosing according to the  $\text{argmax}\{\overline{r_h}\}$ .

Given the current max–min and max–max settings, we must also analyze the case where the lowest or the highest scores attained by two or more candidates coincide. In this case, the decision maker will have to account for the lowest or the highest efficiency scores attained by these candidates using the remaining ranking methods. That is, consider the max–min criterion (the same argument applies to the max–max criterion).

- If  $r_h \neq s_g, \forall r_h, s_g \in \{\underline{r_h}\}$ , with  $r, s \in E, r \neq s$ , and  $h, g \in H$ , then the set  $\{\underline{r_h}\}$  does not contain two identical scores and the max–min criterion completely ranks all its elements.
- If  $r_h = s_g$ , for some  $r_h, s_g \in \{\underline{r_h}\}$ , with  $r, s \in E, r \neq s$ , and  $h, g \in H$ , then the set  $\{\underline{r_h}\}$  contains two or more identical scores for different candidates. Assume, for expositional simplicity, that two candidates have identical scores within  $\{\underline{r_h}\}$ . Then, the ranking must be completed using  $r_l$  and  $s_m$ , with  $r, s \in E, r \neq s$ , and  $h \neq l, g \neq m \in H$ . These new values are the second lowest efficiency scores attained by both candidates. If both these values are also identical, the decision maker must proceed with the third lowest ones for both candidates, and so on. If all values are identical, then both candidates are equivalent.
- The last consensus ranking method implemented consists of distributing the six (extension) ranking models in two different categories. The models using a common set of weights, i.e. models (16)–(19), are grouped within a given category while models (14) and (15) form a second category. We then allow the decision maker to weight

each category subjectively, depending on the preferred ranking model. The resulting weighted efficiency score (WES) determines the ranking of candidates based on the subjective weights,  $\alpha$  and  $\beta$ , assigned by the decision maker to each ranking model

$$\text{WES}(r) = \frac{1}{\alpha} \left( \frac{1}{4}r_1 + \frac{1}{4}r_2 + \frac{1}{4}r_3 + \frac{1}{4}r_4 \right) + \left( \frac{\alpha - 1}{\alpha} \right) \left[ \frac{1}{\beta}r_5 + \left( \frac{\beta - 1}{\beta} \right) r_6 \right]. \quad (22)$$

Eq. (22) defines the weighted efficiency score for candidate  $r \in E$ . The common weight models presented in Eqs. (16) to (19) are defined as ranking methods 1 to 4, respectively, within the initial category to which the decision maker assigns a weight of  $\frac{1}{\alpha}$  using the variable  $\alpha \geq 1$ . Note that each model within this category has been given an identical default weight of  $1/4$ . Similarly, the remaining ranking models, (14) and (15), which are defined as ranking methods 5 and 6, respectively, have each been given a  $\beta$ -based subjective weight within their  $\frac{\alpha-1}{\alpha}$ -weighted category. In the next section, we will illustrate how modifications of the relative weights affect the corresponding consensus rankings obtained.

## 6. Case study

In this section, we present a numerical example to demonstrate the applicability and exhibit the efficacy of the proposed models and procedures. The Department of Industrial Engineering at the Pennsylvania Institute of Technology<sup>1</sup> with 34 full-time faculty members is considering 12 candidates screened by their search committee for faculty appointment. The full-time faculty members are asked to participate in the hiring process and select one candidate for the open-rank position. The voting faculty includes 8 Professors, 10 Associate Professors, 9 Assistant Professors and 7 Instructors.

Each faculty member was given four votes to allocate to the available candidates. Thus, candidates could be potentially ranked by each voter in a position from the first (most preferred) to the fourth. The numbers of votes received by each of the 12 candidates are presented in Table 2. The entrances defining Table 2 consist of the total number of votes received by each candidate for each of the four groups of voters. That is, section (a) within Table 2 accounts for a total of 32 votes given by the 8 faculty professors, each of whom issues four votes. The main aim is to obtain the preference score of each candidate based on data given in Table 2.

As emphasized at the end of Section 3, in order to obtain the (preference) efficiency score of each candidate, we must determine the relative importance weight ( $m_{ij}$ ) assigned to each candidate for being ranked in the  $i$ th position by the  $j$ th group. We then use model (13) to calculate the efficiency scores of all the candidates, which are presented in Table 3:

Although the efficiency scores in model (13) can be used to rank the inefficient candidates, they are useless for ranking the efficient candidates. Model (13) signals candidates 3, 4, 6, 9 and 10 as the efficient candidates. Then, we use models (14)–(19) to rank these five efficient candidates. The efficiency scores and the respective ranking positions of the five efficient candidates obtained using these six ranking models are presented in Table 4.

As it can be observed in Table 4, the ranking of the efficient candidates differs depending on the model considered. For example, the efficient candidate 9 is ranked 2, 3, 4, 3, 4, and 4 according to models (14), (15), (16), (17), (18), and (19), respectively. Therefore, we need to find an aggregated score for each efficient candidate to identify the most preferred one.

We have applied four different methods to analyze these rankings and synthesize them into one overall group ranking. Table 5 illustrates the rankings obtained for the set of efficient candidates after implementing the four consensus ranking methods. As can be observed, the rankings provided by each consensus method differ among each other and the choice of candidate depends on the extension ranking model that the decision maker wants to emphasize.

The results derived from using the simple average approach, which assigns equal weights to all the extension models in order to find the average efficiency score for each candidate, are presented in the second column of Tables 5 and 6.

Now consider the max–min and max–max criteria. Table 6 illustrates how implementing the max–min criterion requires a second set of evaluations to provide the final ranking between candidates 6 and 10. Note how the rankings

<sup>1</sup> The name is changed to protect the anonymity of the university.

Table 2  
Vote breakdown for different faculty groups.

| Place                                       | Candidate |   |   |   |   |   |   |   |   |    |    |    |
|---|-----------|---|---|---|---|---|---|---|---|----|----|----|
|   | 1         | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| (a) Professors group ( $n = 8$ )            |           |   |   |   |   |   |   |   |   |    |    |    |
| 1   | 1         | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 2  | 0  | 1  |
| 2   | 0         | 0 | 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0  | 0  | 2  |
| 3   | 0         | 0 | 0 | 1 | 1 | 2 | 0 | 1 | 0 | 2  | 1  | 0  |
| 4   | 1         | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 0 | 0  | 2  | 0  |
| (b) Associate professors group ( $n = 10$ ) |           |   |   |   |   |   |   |   |   |    |    |    |
| 1   | 0         | 0 | 1 | 2 | 0 | 0 | 2 | 2 | 2 | 1  | 0  | 0  |
| 2   | 0         | 0 | 1 | 1 | 1 | 3 | 0 | 0 | 2 | 0  | 1  | 1  |
| 3   | 0         | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1  | 1  | 2  |
| 4   | 0         | 0 | 1 | 2 | 1 | 1 | 3 | 0 | 0 | 0  | 2  | 0  |
| (c) Assistant professors group ( $n = 9$ )  |           |   |   |   |   |   |   |   |   |    |    |    |
| 1   | 1         | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 2 | 0  | 2  | 1  |
| 2   | 0         | 2 | 3 | 2 | 0 | 0 | 0 | 1 | 0 | 0  | 1  | 0  |
| 3   | 0         | 0 | 0 | 1 | 0 | 1 | 1 | 5 | 1 | 0  | 0  | 0  |
| 4   | 0         | 4 | 0 | 0 | 0 | 2 | 2 | 1 | 0 | 0  | 0  | 0  |
| (d) Instructors group ( $n = 7$ )           |           |   |   |   |   |   |   |   |   |    |    |    |
| 1   | 0         | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0  | 3  | 0  |
| 2   | 1         | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0  | 0  | 0  |
| 3   | 0         | 0 | 1 | 1 | 0 | 0 | 1 | 2 | 0 | 2  | 0  | 0  |
| 4   | 0         | 0 | 1 | 0 | 0 | 1 | 2 | 0 | 2 | 0  | 0  | 1  |

Table 3  
Efficiency scores of the candidates.

| Candidate | Efficiency score |
|-----------|------------------|
| 1         | 0.54             |
| 2         | 0.44             |
| 3         | 1                |
| 4         | 1                |
| 5         | 0.6              |
| 6         | 1                |
| 7         | 0.84             |
| 8         | 0.72             |
| 9         | 1                |
| 10        | 1                |
| 11        | 0.78             |
| 12        | 0.99             |

Table 4  
Efficiency scores and their respective rankings in the six ranking models.

| Candidate | Efficiency score (ranking) |            |            |            |            |            |
|-----------|----------------------------|------------|------------|------------|------------|------------|
|           | Model (14)                 | Model (15) | Model (16) | Model (17) | Model (18) | Model (19) |
| 3         | 1.270 (3)                  | 1.120 (4)  | 0.900 (5)  | 0.940 (3)  | 0.086 (5)  | 0.870 (5)  |
| 4         | 1.260 (4)                  | 1.250 (2)  | 1.000 (1)  | 1.000 (1)  | 0.104 (1)  | 1.000 (1)  |
| 6         | 1.110 (5)                  | 1.040 (5)  | 0.970 (3)  | 0.940 (3)  | 0.094 (2)  | 0.990 (3)  |
| 9         | 1.370 (2)                  | 1.180 (3)  | 0.950 (4)  | 0.940 (3)  | 0.089 (4)  | 0.950 (4)  |
| 10        | 1.800 (1)                  | 1.890 (1)  | 1.000 (1)  | 1.000 (1)  | 0.094 (2)  | 1.000 (1)  |

produced by these criteria, which are presented in the third and fourth columns of Table 5, differ considerably from that of the simple average method, particularly in the max–min case.

There is no a priori way for a decision maker to know which extension ranking model is objectively more reliable. This uncertainty reintroduces the subjectivity that was eliminated from the analysis by Cook and Kress [8] when

Table 5  
Summary of the candidate ranking positions for all consensus methods implemented.

| Ranking position | Simple average | Max–max | Max–min | Weighted efficiency score |              |              |
|------------------|----------------|---------|---------|---------------------------|--------------|--------------|
|                  |                |         |         | $\alpha = 2$              | $\alpha = 2$ | $\alpha = 2$ |
|                  |                |         |         | $\beta = 1$               | $\beta = 2$  | $\beta = 3$  |
| 1                | 10             | 10      | 4       | 10                        | 10           | 10           |
| 2                | 4              | 9       | 10      | 9                         | 4            | 4            |
| 3                | 9              | 3       | 6       | 4                         | 9            | 9            |
| 4                | 3              | 4       | 9       | 3                         | 3            | 3            |
| 5                | 6              | 6       | 3       | 6                         | 6            | 6            |

Table 6  
Efficiency scores and rankings when applying the simple average, max–min, and max–max consensus criteria.

| Candidate | Efficiency score (ranking) |            |            |            |            |            |            |
|-----------|----------------------------|------------|------------|------------|------------|------------|------------|
|           | Average                    | Model (14) | Model (15) | Model (16) | Model (17) | Model (18) | Model (19) |
| 3         | 0.864 (4)                  | 1.270 (3)  | 1.120 (4)  | 0.900 (5)  | 0.940 (3)  | 0.086 (5)  | 0.870 (5)  |
| 4         | 0.936 (2)                  | 1.260 (4)  | 1.250 (2)  | 1.000 (1)  | 1.000 (1)  | 0.104 (1)  | 1.000 (1)  |
| 6         | 0.857 (5)                  | 1.110 (5)  | 1.040 (5)  | 0.970 (3)  | 0.940 (3)  | 0.094 (2)  | 0.990 (3)  |
| 9         | 0.913 (3)                  | 1.370 (2)  | 1.180 (3)  | 0.950 (4)  | 0.940 (3)  | 0.089 (4)  | 0.950 (4)  |
| 10        | 1.131 (1)                  | 1.800 (1)  | 1.890 (1)  | 1.000 (1)  | 1.000 (2)  | 0.094 (2)  | 1.000 (1)  |
| Criterion | Simple Average             | Max–max    | Max–max    |            | Max–min    | Max–min    |            |

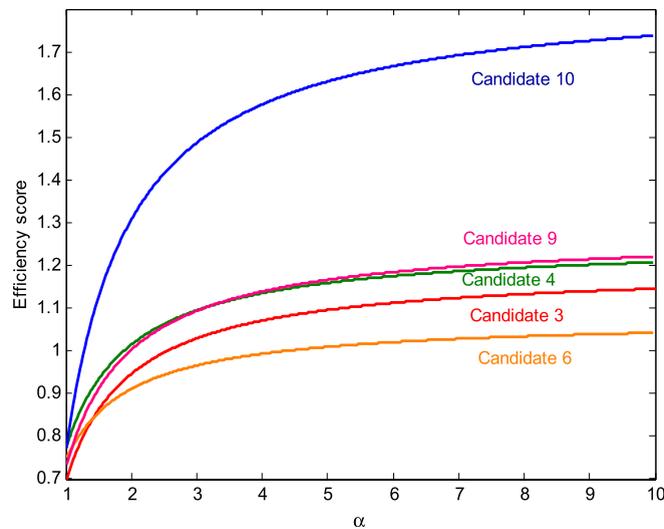


Fig. 1. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 10]$  and  $\beta = 2$ .

using DEA to obtain the relative importance weights for each candidate. That is, the choice of ranking model (and, therefore, candidate) made by decision makers depends on their subjective preferences. Despite this fact, the choice among the average, max–min and max–max criteria can be related to the analysis of the attitude of decision makers towards uncertainty common to the economic literature [15].

In this regard, the max–min criterion is associated with decision makers who are uncertainty averse, due to the guarantee provided by the choice of the highest score among the lowest evaluations. Similarly, the simple average approach generally relates to individuals with a neutral attitude towards uncertainty, while decision makers less concerned with the low efficiency scores of the alternatives will tend to follow a max–max approach. A similar intuition will be provided when describing the fourth consensus ranking method below.

Finally consider the rankings that follow from the common weights extension models. Table 4 illustrates how the common weights-based models produce highly similar rankings while models (14) and (15) constitute the main source of ranking heterogeneity. Thus, the main differences among the consensus rankings are caused by the (subjective) im-

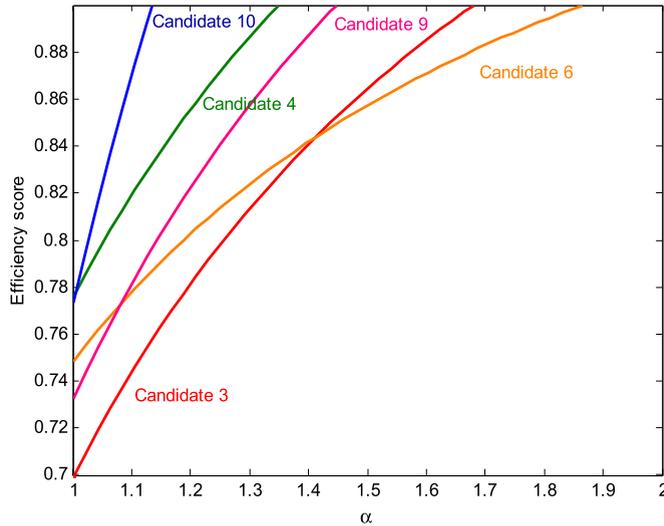
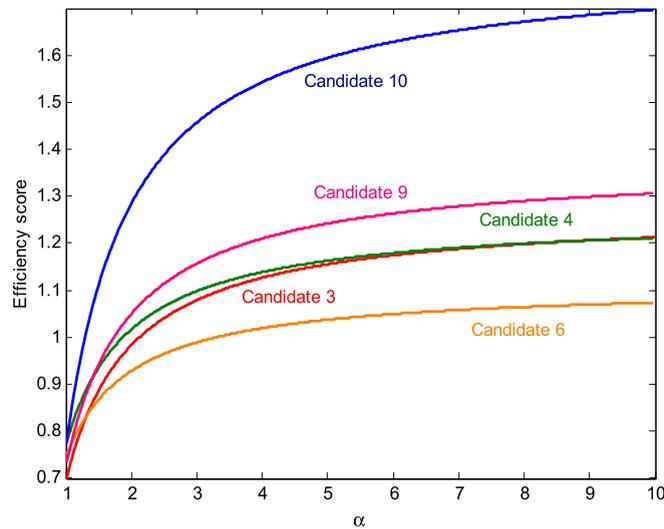


Fig. 2. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 2]$  and  $\beta = 2$ .



\*Note: At  $\alpha = 10$ ,  $WES(3) = 1.2129$  while  $WES(4) = 1.2116$

Fig. 3. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 10]$  and  $\beta = 1^*$ .

portance given to models (14) and (15) relative to the common weights ones. The weighted efficiency score method builds on this fact. Several potential rankings are presented in the last three columns of Table 5 for different values of the  $\alpha$  and  $\beta$  parameters. We have assumed a value of  $\alpha = 2$  in order to guarantee that the same importance is given to the common weights extension models and to those in Eqs. (14) and (15).

It should be noted that the weighted efficiency score method with  $\alpha = 2$  and  $\beta = 2$  is not equivalent to the simple average one, despite the coincidence in ranking results. However, this choice of weights relates intuitively to that of a decision maker exhibiting a neutral attitude towards uncertainty, given the equal weight assigned to the common weight and the alternative extension ranking models. Similarly, a preference for model (14) would lead to a final ranking coinciding with that of the max–max criterion. In this regard, the choice of weights can be related to the attitude towards uncertainty of the decision maker, providing a strategic perspective to the ranking elaboration process of decision makers.

Figs. 1, 3 and 5 illustrate the behavior of the WES ranking method for different values of  $\beta$  when  $\alpha \in [1, 10]$ , while Figs. 2, 4 and 6 present the respective rankings when  $\alpha \in [1, 2]$ . It follows from these figures and the last three columns of Table 5 that increasing the weight given to model (14) leads to candidate 9 being preferred to candidate 4

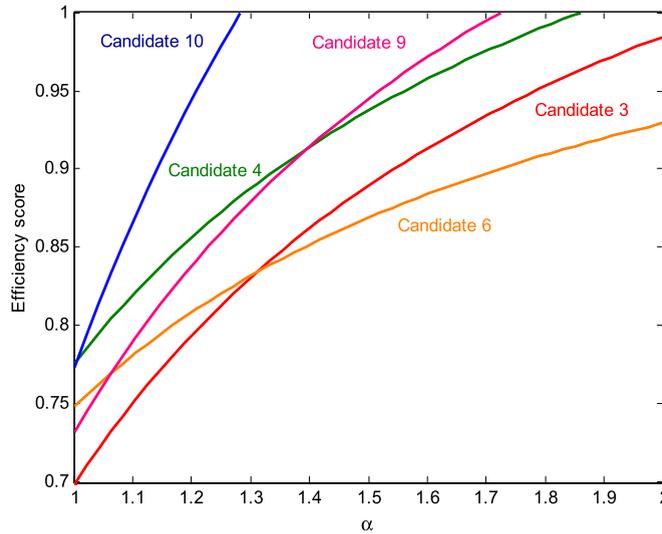


Fig. 4. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 2]$  and  $\beta = 1$ .

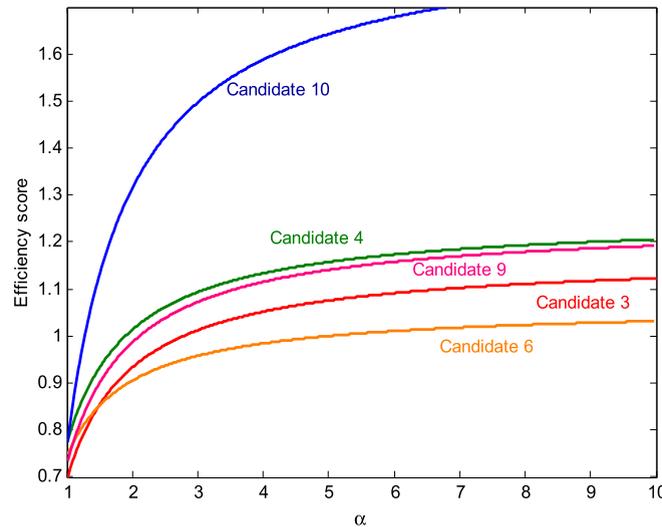


Fig. 5. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 10]$  and  $\beta = 3$ .

as a second choice. We illustrate this case in Fig. 7 for a continuous set of  $\alpha$  and  $\beta$  values. In this case, the  $\beta$  parameter determines the ranking results, with candidate 4 being preferred to candidate 9 as the value of  $\beta$  increases.

A similar result follows when comparing candidates 4 and 10. In this case, it is the  $\alpha$  parameter the one that determines the ranking between both candidates. Candidate 4 is ranked above candidate 10 for small values of  $\alpha$ , in which case the common weights extension models are given a higher relative weight than models (14) and (15). This is also illustrated in Fig. 8, where the effect of the  $\alpha$  parameter on the ranking of both candidates is observable. Note that, when  $\alpha = 1$ , the weighted efficiency score defined in Eq. (22) becomes  $WES(r) = \frac{1}{4}(r_1 + r_2 + r_3 + r_4)$ , with  $WES(4) = 0.7760$  and  $WES(10) = 0.7735$  for all  $\beta$  values.

### 7. Conclusions and future research directions

Different methods have been proposed to allow voters to express their preferences on a set of candidates. In ranked voting methods, each voter selects a subset of candidates and ranks them from the most to the least preferred one. The score obtained by each candidate is the weighted sum of the scores received from the different voters. The principal drawback of such scoring rules is that they assume the votes of all the voters to have equal importance, without any preference being defined among them. In this paper, we generalized the existing scoring methods to overcome the

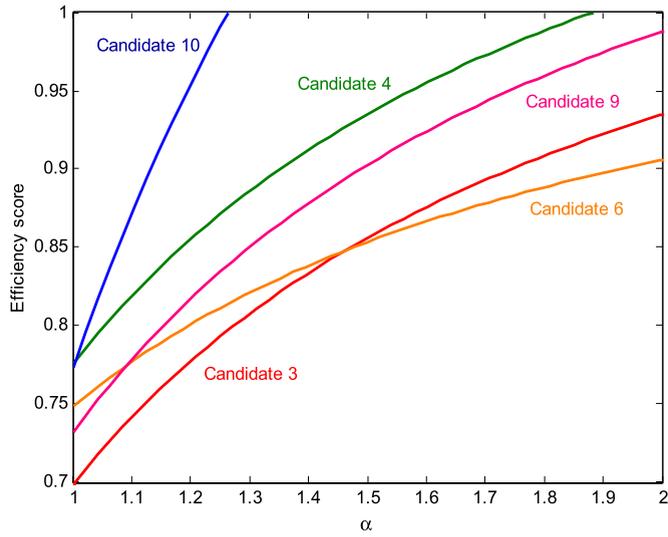


Fig. 6. Efficiency scores and candidate rankings with the WES method for  $\alpha \in [1, 2]$  and  $\beta = 3$ .

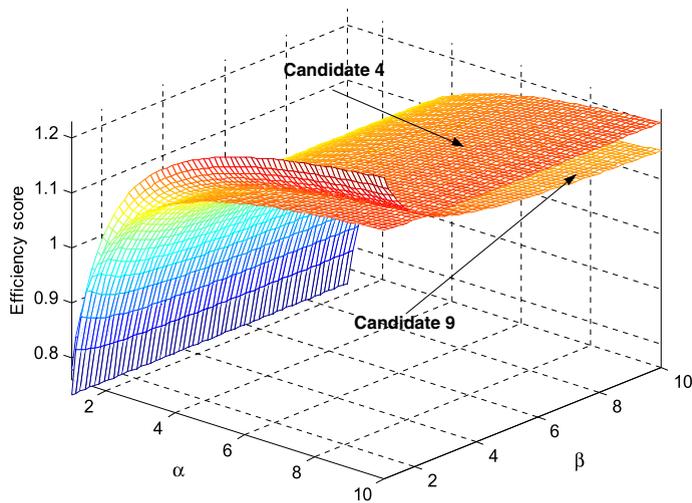


Fig. 7. Shift in ranking positions between candidates 4 and 9 with the WES method for  $\alpha \in [1, 10]$  and  $\beta \in [1, 10]$ .

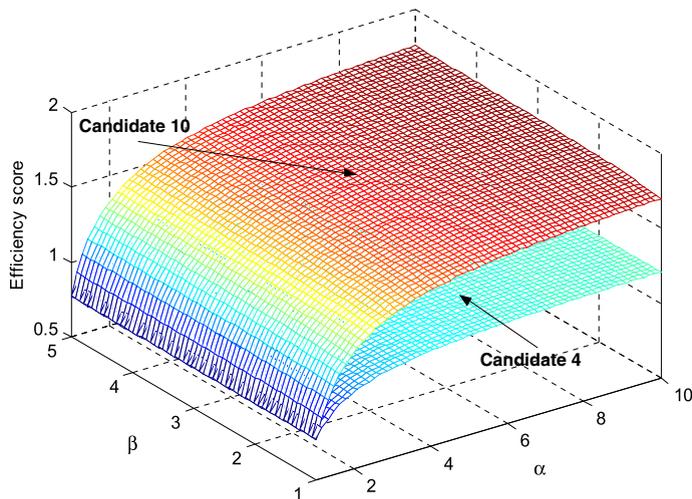


Fig. 8. Shift in ranking positions between candidates 4 and 10 with the WES method for  $\alpha \in [1, 10]$  and  $\beta \in [1, 5]$ .

forementioned drawback. We incorporated several ranking methods in our proposed model since the ranking of the efficient candidates changed with the particular method being implemented. We used the scores obtained from the ranking models and proposed an aggregated score for each efficient candidate. Finally, we illustrated the performance of our method with a real world application.

The aggregation of the different methods proposed into an overall consensus ranking has been done assuming that the decision maker assigns a subjective importance weight to each one of these ranking methods. In this regard, the choice of ranking methods and their effect in determining the final consensus ranking could be treated from a strategic perspective. This is the case since it is the subjective weight assigned by the decision maker to the different ranking methods employed what determines the final consensus ranking of the candidates. Thus, the current paper could be extended to analyze the choice of ranking methods using potential evaluations from a strategic viewpoint, providing a bridge between the operational research and economic approaches when ranking sets of alternatives.

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