A Credibility and Strategic Behavior Approach in Hesitant Multiple Criteria Decision-Making With Application to Sustainable Transportation

Francisco J. Santos-Arteaga, Debora Di Caprio, Madjid Tavana, and Emilio Cerdá Tena

Abstract—**Multiple criteria decision-making (MCDM) methods do not account for the potentially strategic evaluations of experts. Once the ranking is delivered, decision makers (DMs) select the first alternative without questioning the credibility of the evaluations received from the experts. We formalize the selection problem of a DM who must choose from a set of alternatives according to both their characteristics and the credibility of the reports received. That is, we transform an MCDM setting into a game-theoretical scenario. We build our analysis on a recent extension of hesitant fuzzy numbers incorporated within the formal structure of technique for order of preference by similarity to ideal solution. We define the restrictions that must be imposed regarding the credibility of the evaluations and the capacity of experts to form coalitions and manipulate rankings based on their subjective preferences. This feature constitutes a considerable drawback in real-life scenarios, mainly when dealing with environmental and sustainable strategic problems. In this regard, sustainable transportation problems incorporate both technical variables and subjective assessments whose values can be strategically reported by experts. We extend a real-life study case accounting for the evaluations of several experts to demonstrate the importance of strategic incentives for the rankings obtained when implementing MCDM techniques. We numerically illustrate the interactions between the experts' reporting strategies and the formal tools available for the DMs to counteract potential manipulations of the final ranking.**

Index Terms—**Credibility, hesitant fuzzy numbers (HFNs), strategic behavior, sustainable transportation, technique for order of preference by similarity to ideal solution (TOPSIS).**

Manuscript received 20 April 2022; revised 14 June 2022; accepted 29 June 2022. Date of publication 6 July 2022; date of current version 3 February 2023. The work of Francisco J. Santos-Arteaga was supported by the María Zambrano contract from the Universidad Complutense de Madrid financed by the Ministerio de Universidades with funding from the European Union Next Generation Program. The work of Debora Di Caprio was supported by the University of Trento under Grant NEOASSUNTI 40202312. *(All authors contributed equally to this work.) (Corresponding author: Debora Di Caprio.)*

Francisco J. Santos-Arteaga and Emilio Cerdá Tena are with the Departamento de Análisis Económico y Economía Cuantitativa, Universidad Complutense de Madrid, 28223 Pozuelo de Alarcón, Spain (e-mail: fransant@ucm.es; ecerdate@ccee.ucm.es).

Debora Di Caprio is with the Department of Economics and Management, University of Trento, I-38122 Trento, Italy (e-mail: debora.dicaprio@unitn.it).

Madjid Tavana is with the Business Systems and Analytics Department, La Salle University, Philadelphia, PA 19141 USA, and also with the Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany (e-mail: tavana@lasalle.edu).

This article has supplementary material provided by the authors and color versions of one or more figures available at https://doi.org/10.1109/TFUZZ.2022.3188875.

Digital Object Identifier 10.1109/TFUZZ.2022.3188875

I. Introduction

One of the main problems faced by the literature on multiple criteria decision-making (MCDM) methods is that rankings are definitive and do not account for the uncertainty inherent to the potentially strategic evaluations of the experts. The lack of strategic considerations constitutes an intrinsic assumption that remains undiscussed in the literature. Once the ranking is delivered, Decision makers (DMs) should select the first alternative without questioning the validity of the evaluations and concluding the applicability and contribution of the corresponding model. These techniques do not consider the strategic interactions derived from the quality or credibility of the evaluations provided by the experts. This, however, constitutes a considerable problem in real-life environments.

MCDM models allow DMs to deal with imprecise information and uncertainty regarding the characteristics of the alternatives. However, a certainty constraint is implicitly imposed on this uncertainty; that is, all the evaluations received by the DM are assumed to be entirely credible. In other words, DMs account for the uncertainty inherent to the evaluations provided by the experts, while completely trusting them to report truthfully. We formalize the selection problem of a DM who must choose from a set of alternatives according to both their characteristics and the credibility of the reports received. That is, we transform an MCDM setting into a game-theoretical scenario.

Sustainable transportation problems incorporate both technical variables and subjective assessments whose values can be strategically reported by experts. This credibility problem, together with the capacity of experts to form coalitions and manipulate rankings based on their subjective preferences, motivates the development of the current research. Aggregation operators such as the maximize agreement heuristic could be used to generate some basic consensus among rankings. Even in this case, the credibility of the evaluations reported by each expert must be explicitly incorporated into the analysis, given the potential manipulability of the corresponding rankings.

We build our framework of analysis on a recent extension of hesitant fuzzy numbers (HFNs), particularly well-suited to introduce credibility considerations in experts' evaluations. We incorporate these HFNs within a technique for order of preference by similarity to ideal solution (TOPSIS) setting, whose malleability allows for direct implementation of HFNs within its formal structure. Both these features allow for introducing strategic interactions into TOPSIS when selecting among several uncertain alternatives evaluated with different degrees of credibility.

We analyze both formally and numerically the restrictions that must be imposed regarding the credibility of the evaluation process and the number of expert opinions that may be considered. The implementation of the credibility scores assigned to the experts within an MCDM setting such as TOPSIS will be described and discussed. The capacity of experts to interact and form coalitions to alter the ranking obtained will also be analyzed. All in all, the incorporation of strategic and game-theoretical elements into an MCDM setting allows for a substantial number of extensions into a completely unstudied area of research.

The article proceeds as follows. Section II reviews the literature on fuzzy MCDM and sustainable transportation. Section III summarizes the main contribution of the current article. Section IV introduces basic definitions and notations regarding HFNs. Section V incorporates credibility via HFNs into TOPSIS. Section VI analyzes a real-life MCDM study case enhanced to incorporate credibility considerations via HFNs. Finally, Section VII concludes and suggests potential extensions.

II. Literature Review

There is a substantial amount of literature on the different MCDM techniques commonly applied to deal with real-life problems, such as the analytical hierarchy process (AHP), the linear programming technique for multidimensional analysis of preference, VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), and TOPSIS. These models have also been extended to deal with uncertain environments via fuzzy, interval, and probabilistic measures [2], [4], [11], [23], [24], [31], [46].

The formalization of uncertainty plays an increasingly prominent role in group decision-making scenarios dealing with multiple decision criteria. Pamučar et al. [32] exploited the interval-type knowledge inherent to data to introduce rough interval numbers within MCDM scenarios. Fan et al. [15] dealt with a similar group evaluation scenario, using intuitionistic fuzzy rough numbers to aggregate the information available. Pamučar et al. [33] focused on eliminating the subjectivity arising from the definition of fuzzy set (FS) borders via interval-valued fuzzy-rough numbers. A similar intuition applies to [30], who determined the weight assigned to the criteria in VIKOR through interval-valued intuitionistic hesitant fuzzy entropy.

TOPSIS remains widely applied as a reference MCDM technique when dealing with environmental and sustainable transportation problems. For instance, Awasthi et al. [7] applied this technique to evaluate three mobility projects in Luxemburg. Huang et al. [20] assessed the performance of Chengdu's subway system, while Lambas et al. [26] compared different public transportation systems in Spain and Italy. Shen et al. [40] applied TOPSIS to study the development of green traffic scenarios in Zhoushan and [3] to select locations for a consolidation facility in Melbourne.

Information uncertainty has also been incorporated into TOPSIS [34]. Samaie et al. [38] applied fuzzy TOPSIS to evaluate the penetration of electric vehicles in Teheran. Hajduk [18] implemented the entropy weight method to compute the weight of each criterion in TOPSIS and rank transport performance in several smart cities. Zhang et al. [51] applied the same formal enhancement to evaluate the different types of impacts of the transportation networks in several large cities of the Beijing–Tianjin–Hebei region.

A recent and complete review of the main MCDM methods applied in the literature on sustainable transportation is provided by [8]. The authors conclude that the techniques applied more frequently are AHP and TOPSIS, their fuzzy extensions, and those of the preference ranking organization method for enrichment evaluations (PROMETHEE). Moreover, the formal malleability of TOPSIS allows for complementarities with other techniques and the generation of hybrid MCDM models. For instance, Sobhani et al. [41] applied a hybrid AHP–TOPSIS model to evaluate the competitiveness and sustainability of unconventional transport modes in Dhaka. Liu et al. [29] integrated the failure modes and effects analysis with TOPSIS to assess and rank the risk of different failure modes.

A. *Strategic Incentives and Subjective Evaluations*

One of the main qualities of the MCDM models applied to study sustainable transportation problems is the significant number of criteria considered by the experts and the absence of strategic reporting. This feature remains unaccounted for when defining the characteristics of the experts. The lack of strategic incentives determining the subsequent reports constitutes a significant drawback of these models, particularly in uncertain environments. Fuzzy variables do not describe the strategic quality of the reports received or the experts' characteristics. In addition, several features may define the credibility of an expert, ranging from his area of expertise to the existence of personal strategic interests.

The use of fake experts represents one of the main communication strategies among climate skeptics [39]. As a result, the substantial amount of misinformation regarding environmental and climate change questions constitutes an important problem nowadays [21], [45]. Other than a few exceptions [13], the strategic interactions that may be defined among information sources are not incorporated into the analyses, which generally focus on the uncertainty inherent to the perception and evaluations of experts within fuzzy environments.

It is important to differentiate between the strategic reports provided by the experts and the uncertainty inherent in their evaluations. The latter has been widely analyzed in the literature using fuzzy techniques [6], [19], [47]. The former remains unstudied, even though warnings regarding the manipulability of MCDM models when considering strategic reporting and asymmetric information between experts and DMs have already been issued [14]. The game-theoretical literature has consistently focused on real-life environments dealing with sustainable production structures [1], [10]. However, these strategic interactions have never been formalized within MCDM settings.

The resulting complexities and lack of formal instruments may be responsible for this analytical void. However, recent and well documented study cases, such as that of Madrid Central

[37], [42], highlight both the difference of opinions among experts as well as their strategic quality. These features require extending the framework of analysis of MCDM models beyond their current limits. Sustainable transportation problems, dealing with multiple environmental indicators and policy criteria, are highly strategic and require formal analyses that account for these features and analyze the potential consequences.

III. Contribution

MCDM techniques are designed to select among alternatives composed of several and often conflicting criteria. A standard constraint of these models is the assumption that the value of the characteristics composing the alternatives is known with complete certainty. A considerable variety of fuzzy tools has been defined to deal with the different types of uncertainty faced when applying these techniques. These tools range from type-1 and type-2 FSs [50] to intuitionistic fuzzy sets (IFSs) [5], and hesitant FSs (HFSs) [44], all of which have been widely analyzed in the literature.

One of the most recent developments is given by HFSs where a finite set of hesitant fuzzy elements (HFEs) defined within [0, 1] is used to describe the membership degree of the hesitation expressed by DMs. Hybrid models composed of FSs, IFSs, and HFSs and different variants of the latter HFSs have been recently developed and implemented in the literature [17], [25], [27].

HFNs [35], [36] constitute a novel extension of HFSs where type-1 fuzzy numbers give the HFEs. The membership degrees defined within the reference interval provided by the type-1 fuzzy numbers allow for infinitely many potential interpretations of uncertainty. Keikha [22] weakens this constraint by defining subjective qualitative assessments contained in [0, 1].

The basic structure of the HFNs introduced by [22] is given by $\langle a, \{\gamma_1, \gamma_2, \ldots, \gamma_n\}\rangle$. The number is composed by two main elements, a positive real value, $a \in \mathbb{R}^+$, and an HFE defined by a set of hesitation degrees, $\gamma_i \in [0, 1]$. This type of HFN combines subjective qualitative beliefs together with standard quantitative evaluations [16]. Therefore, it allows for the formalization of the interactions between the evaluations received and the credibility assigned to the experts performing the evaluations.

The capacity of these HFNs to operate with and combine credibility scores among experts makes them particularly suitable to be implemented within MCDM settings. Consider a DM who must rank several projects based on the evaluations received from a given expert, whose main subjective motivations and preferences are partially known, leading the DM to assign a credibility score to his evaluations. The credibility assigned to the expert must interact with the crisp evaluation provided when defining the corresponding MCDM problem.

For instance, experts may evaluate the implementation quality of two projects conditioned by private interests, subjective perceptions, and different degrees of familiarity with the characteristics and objectives of the projects. The credibility assigned by the DM to these subjective features of the expert is represented through HFEs, such as $\{0.3, 0.75, 0.8\}$. Note that the uncertainty of the system is given by the credibility assigned to the expert, not the evaluations provided by the latter. We will present several numerical comparisons to highlight the importance of credibility and its strategic implications when compared to crisp and standard fuzzy evaluation scenarios.

We build on the study case analyzed by [6], who were among the first scholars to incorporate the evaluations of several experts within an uncertain setting. The authors formalized these evaluations as triangular fuzzy numbers (TFNs) and added them up within a fuzzy TOPSIS environment to rank different sustainable transportation alternatives. The main characteristics of these experts and their potential strategic motivation were not considered in the analysis. We extend their analysis to illustrate the importance of strategic incentives for the rankings obtained when applying MCDM techniques.

IV. Hesitant Fuzzy Numbers

The initial evaluations provided by the experts regarding the characteristics of each alternative consist of crisp values conditioned by their credibility. Consider a DM who receives an evaluation a_{ij}^h from an expert $h = 1, 2, \ldots, k$ regarding the jth characteristics of the ith alternative. We will assume that the DM describes the credibility assigned to the hth expert, $h = 1, 2, \ldots, k$, through different values in $[0, 1]$ summarized via $h(a_{ij}^h) = \{\gamma_{h1}, \gamma_{h2}, \ldots, \gamma_{hl}\}$. The values contained in $h(a_{ij}^h)$ refer to the set of different features determining the credibility of the hth expert regarding the evaluation of the jth characteristics of the ith alternative. The evaluation of the alternatives provided by each expert are described through HFNs $\langle a_{ij}^h; h(a_{ij}^h)\rangle$, endowed with an associated credibility given by $h(a_{ij}) = h(a_{ij}^1) \cup h(a_{ij}^2) \cup \ldots \cup h(a_{ij}^k)$.

A. Definitions and Notations

HFNs provide a flexible and consistent technique to measure and aggregate credibility and deal with uncertainty, endowing DMs with the capacity to weight the potentially strategic evaluations assigned by each expert to the characteristic of a given alternative. In this regard, the score of an HFN constitutes one of its main characteristics, allowing DMs to compare and order HFNs based on both their real and hesitant parts.

Keikha [22] applies the power average (PA) operator defined by [48] and [49] to compute the score assigned to a given HFN defining the characteristics of the corresponding alternatives. The PA operator is designed to foster the support of its arguments for each other through the aggregation process, promoting consistency in terms of lower distances and avoiding dispersion. In particular, this tool reflects the consistency of a given evaluation through a support degree based on the proximity of the credibility scores assigned to the expert.

Let $\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_n$ be n positive real numbers. The support degree $\mathrm{Supp}(\mathfrak{a}_i, \mathfrak{a}_j)$ of $\mathfrak{a}_i$ from $\mathfrak{a}_j$ satisfies the following conditions:

1) $Supp(\mathfrak{a}_i, \mathfrak{a}_j) = Supp(\mathfrak{a}_j, \mathfrak{a}_i)$.
2) $Supp(\mathfrak{a}_i, \mathfrak{a}_j) \in [0, 1]$.
3) If $|x - y| < |\mathfrak{a}_i - \mathfrak{a}_j|$, then, $Supp(\mathfrak{a}_i, \mathfrak{a}_j) < Supp(x, y)$.

Definition 1: The PA of $\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_n$, denoted by $PA(\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_n)$, is a mapping: $\mathbb{R}^n \to \mathbb{R}$, defined as

$$PA(\mathfrak{a}_1, \mathfrak{a}_2, \ldots, \mathfrak{a}_n) = \frac{\sum_{i=1}^{n}(1+T(\mathfrak{a}_i))\mathfrak{a}_i}{\sum_{i=1}^{n}(1+T(\mathfrak{a}_i))}$$

where $T(\mathfrak{a}_i) = \sum_{\substack{j=1 \\ j\neq i}}^{n} Supp(\mathfrak{a}_i, \mathfrak{a}_j)$.

Definition 2 [5]: Let X be a universal set. The set $\tilde{A} = \{(x, m_{\tilde{A}}(x)) | x \in X\}$ is called an FS of X, with $m_{\tilde{A}} : X \to [0, 1]$ representing the membership function, such that $\forall x \in X$, $m_{\tilde{A}}(x)$ describes the degree of membership of x in $\tilde{A}$.

B. Hesitant Fuzzy Sets and HFNs

HFSs generalize FSs by allowing each member to be endowed with several membership degrees [43]. Let *X* be a fixed set. For every $x \in X$, the HFE $h(x)$ describes the membership degrees of x to *E* as a finite set of values in $[0, 1]$. The ensuing HFS E is defined as $E = \{< x, h(x) > | x \in X\}$, with $h(x) = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ [17].

Definition 3: Let λ be a positive real number. The main arithmetic operations that can be defined on a collection of HFEs, $h_j (j = 1, 2, \ldots, n)$, are given by

1) $h_j^{\lambda} = \{(h_j^{\sigma(t)})^{\lambda} | t = 1, 2, \ldots, l\}$,
2) $\lambda h_j = \{1 - (1 - h_j^{\sigma(t)})^{\lambda} | t = 1, 2, \ldots, l\}$,
3) $h_1 \oplus h_2 = \{h_1^{\sigma(t)} + h_2^{\sigma(t)} - h_1^{\sigma(t)} h_2^{\sigma(t)} | t = 1, 2, \ldots, l\}$,
4) $h_1 \otimes h_2 = \{h_1^{\sigma(t)} h_2^{\sigma(t)} | t = 1, 2, \ldots, l\}$,
5) $\oplus_{j=1}^{n} h_j = \{1 - \Pi_{j=1}^{n}(1 - h_j^{\sigma(t)}) | t = 1, 2, \ldots, l\}$,
6) $\otimes_{j=1}^{n} h_j = \{\Pi_{j=1}^{n} h_j^{\sigma(t)} | t = 1, 2, \ldots, l\}$,

where $h_j^{\sigma(t)}$ represents the tth smallest value in h_j.

Several variants of the definition of HFSs have been introduced in the literature, consisting mainly of HFEs that range from interval-valued elements [28] to fuzzy numbers [12], [36]. We base our approach on the HFNs introduced by [22], whose HFSs consider values defined on $\mathbb{R}$. More precisely, an HFN $\langle a; h(a)\rangle$ is composed by a real value, *a*, and a finite set of values in $[0, 1]$, $h(a) = \{\gamma_1, \gamma_2, \ldots, \gamma_k\}$, describing the different membership degrees.

Our interpretation of the HFEs composing a given HFN focuses on the credibility assigned by the DM to the real value describing the evaluations provided by the experts. HFEs range from zero, indicating a total lack of credibility, to a completely credible report, which is assigned a value of one. As a reference benchmark, a default value of ½ would correspond to an uncertain environment where DMs lack any information about the credibility of the experts.

Definition 4: Let X be a reference set. An HFN, $\tilde{A}^H = \langle a, h(a)\rangle$, in $\mathbb{R}$ consists of $a \in \mathbb{R}$ and a finite set of values in $[0, 1]$, namely, the HFE $h(a)$, describing the membership degrees of $a \in X$.

Definition 5: Let $\tilde{A}^H = \langle a, h(a)\rangle$ and $\tilde{B}^H = \langle b, h(b)\rangle$ be two HFNs, and $\lambda > 0$, then the following conditions hold:

1) $\tilde{A}^H \oplus \tilde{B}^H = \langle a + b, h(a) \cup h(b)\rangle$.
2) $\lambda \tilde{A}^H = \langle \lambda a, h(a)\rangle$.
3) $(\tilde{A}^H)^{\lambda} = \langle a^{\lambda}, h(a)\rangle$.
4) $\tilde{A}^H \otimes \tilde{B}^H = \langle a.b, h(a) \cap h(b)\rangle$, if $h(a) \cap h(b) = \emptyset$, then $h(a) \cap h(b) = \bigcup_{\substack{\gamma_i \in h(a), \\ \gamma_j \in h(b)}} \min\{\gamma_i, \gamma_j\}$.

The score function, $Score(\tilde{A}^H)$, and variance, $Var(\tilde{A}^H)$, of an HFN, $\tilde{A}^H = \langle a, h(a)\rangle$, with membership degrees $h(a) = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$, with $\gamma_i \in [0, 1]$, are defined as follows:

$$\begin{aligned} Score(\tilde{A}^H) &= a \times PA(\gamma_1, \gamma_2, \ldots, \gamma_n) \\ &= a \times \frac{\sum_{j=1}^{n}(1+T(\gamma_j))\gamma_j}{\sum_{j=1}^{n}(1+T(\gamma_j))} \end{aligned} \tag{1}$$

where

$$T(\gamma_j) = \sum_{k=1, k\neq j}^{n} Supp(\gamma_j, \gamma_k),$$

$$Supp(\gamma_j, \gamma_k) = 1 - |\gamma_j - \gamma_k| \tag{2}$$

$$Var(\tilde{A}^H) = a\sqrt{\frac{1}{n-1}\sum_{\substack{i=1, \\ i\neq j}}^{n} (\gamma_j - \gamma_i)^2}. \tag{3}$$

C. Subtracting and Dividing HFNs

The arithmetic operations defined in the current section are based on the relationship existing between HFSs and IFSs [22], [43].

Definition 6: Let $\tilde{A}^H = \langle a, h(a)\rangle$ and $\tilde{B}^H = \langle b, h(b)\rangle$ be HFNs with the same cardinality $|h(a)| = |h(b)|$. Let $\gamma_{(j)}^a$ and $\gamma_{(j)}^b$ be the jth higher value of $h(a)$ and $h(b)$, respectively. We have the unnumbered equation shown at the bottom of the next page, where $\gamma = \begin{cases} \frac{\gamma_{(j)}^a - \gamma_{(j)}^b}{1-\gamma_{(j)}^b} & \text{if } \gamma_{(j)}^a > \gamma_{(j)}^b \\ \frac{\gamma_{(j)}^b - \gamma_{(j)}^a}{1-\gamma_{(j)}^a} & \text{if } \gamma_{(j)}^a < \gamma_{(j)}^b \\ \gamma_{(j)}^a & \text{if } \gamma_{(j)}^a = \gamma_{(j)}^b \end{cases}$; the second unnumbered equation shown at the bottom of the next page, if $a, b \neq 0$, where $\gamma = \begin{cases} \frac{\gamma_{(j)}^a}{\gamma_{(j)}^b} \ if\ \gamma_{(j)}^a \leq \gamma_{(j)}^b \text{ and } \gamma_{(j)}^b \neq 0 \\ 1 \qquad \text{otherwise} \end{cases}$.

Example: Let $\tilde{A}_1^H = \langle 5; \{0.3, 0.5, 0.7\}\rangle$ and $\tilde{A}_2^H = \langle 2; \{0.2, 0.4, 0.6\}\rangle$ be two HFNs. The following results are directly derived from the application of the operations described through the current section:

$\tilde{A}_1^H \oplus \tilde{A}_2^H = \langle 7; \{0.44, 0.7, 0.88\}\rangle$,
$\tilde{A}_1^H \otimes \tilde{A}_2^H = \langle 10; \{0.06, 0.2, 0.42\}\rangle$,
$\tilde{A}_1^H \ominus \tilde{A}_2^H = \langle 3; \{0.125, .0.166, 0.25\}\rangle$, and
$\tilde{A}_1^H \oslash \tilde{A}_2^H = \langle 2.5; \{1, 1, 1\}\rangle$.

Definition 7: Let $a \in \mathbb{R}^+$ and $\tilde{B}^H = \langle b, h(b)\rangle$ be a HFN. We have that

1) $a + \tilde{B}^H = \tilde{B}^H + a = \langle a + b, h(b)\rangle$;
2) $a - \tilde{B}^H =$

$$\begin{cases} \langle a - b, h(b)\rangle & if \ \ a > Score(\tilde{B}^H) \& a > b, \\ \langle b - a, h(b)\rangle & if \ \ a > Score(\tilde{B}^H) \& a < b, \\ -\langle |a - b|, h(b)\rangle & if \ \ a < Score(\tilde{B}^H), \end{cases}$$

$\tilde{B}^H - a =$

$$\begin{cases} -\langle a-b, h(b)\rangle & if \quad a > Score\left(\tilde{B}^H\right) \& a > b, \\ -\langle b-a, h(b)\rangle & if \quad a > Score\left(\tilde{B}^H\right) \& a < b, \\ \langle |b-a|, h(b)\rangle & if \quad a < Score\left(\tilde{B}^H\right), \end{cases}$$

3) $a.\tilde{B}^H = \tilde{B}^H.a = \langle a.b, h(b)\rangle$;

4) $$a \oslash \tilde{B}^H = \begin{cases} \langle {}^{a}\!/_{b}, h(b)\rangle & if \quad a > Score(\tilde{B}^H)\&a > b, \\ \langle {}^{b}\!/_{a}, h(b)\rangle & if \quad a > Score(\tilde{B}^H)\&a < b, \\ \langle {}^{b}\!/_{a}, h(b)\rangle & if \quad a < Score(\tilde{B}^H)\&a > b, \\ \langle {}^{a}\!/_{b}, h(b)\rangle & if \quad a < Score(\tilde{B}^H)\&a < b, \end{cases}$$

$$\tilde{B}^H \oslash a = \begin{cases} \langle {}^{b}\!/_{a}, h(b)\rangle & if \quad a > Score(\tilde{B}^H)\&a > b, \\ \langle {}^{a}\!/_{b}, h(b)\rangle & if \quad a > Score(\tilde{B}^H)\&a < b, \\ \langle {}^{a}\!/_{b}, h(b)\rangle & if \quad a < Score(\tilde{B}^H)\&a > b, \\ \langle {}^{b}\!/_{a}, h(b)\rangle & if \quad a < Score(\tilde{B}^H)\&a < b, \end{cases},$$

with $b \neq 0$. Two important remarks follow regarding the $a \oslash \tilde{B}^H$ and $\tilde{B}^H \oslash a$ operations. Note how the case defined for $a < Score(\tilde{B}^H)\&a > b$, delivering $\langle {}^{b}\!/_{a}, h(b)\rangle$ and $\langle {}^{a}\!/_{b}, h(b)\rangle$, respectively, has been included for completeness and cannot arise within the current framework. The intuition behind this statement relates to the fact that whenever $a > b$ we cannot have $a < Score(\tilde{B}^H)$ since

$PA(\gamma_1, \gamma_2, \ldots, \gamma_n) = \frac{\sum_{j=1}^{n}(1+T(\gamma_j))\gamma_j}{\sum_{j=1}^{n}(1+T(\gamma_j))} \leq 1$ for $\gamma_j \in [0,1]$, $j = 1, \ldots, n$. That is, the way uncertainty has been introduced through the HFEs implies that $Score(\tilde{B}^H) \leq b$, with $Score(\tilde{B}^H) = b$ only with fully credible reports, i.e., $\gamma_j = 1$, $j = 1, \ldots, n$.

Consider now the importance of credibility and its effect on the evaluations provided by the experts. Whenever the credibility of the evaluations is sufficiently low, leading to the case $a > Score(\tilde{B}^H)\&a < b$, the pattern of evaluation is reversed. This feature is particularly relevant when averaging the evaluations received, leading to a decrease in the real part of the corresponding HFN compared to other alternatives. In this regard, dealing with the fact that TOPSIS allows for the existence of positive and negative criteria requires additional modifications that will be introduced in the next section.

V. TOPSIS and Credibility

We start by describing the basics of TOPSIS before extending its main structure to incorporate credibility and regret. As an MCDM technique, TOPSIS is used to rank a series of alternatives according to several criteria through the evaluations received from one or more experts. TOPSIS computes two ideal reference points per criterion, positive and negative, and calculates the relative distance between the characteristics defining each alternative and both values. The subsequent ranking delivered by this technique reflects the relative importance assigned to each alternative by the experts. The main steps defining the implementation of TOPSIS follow.

The m alternatives evaluated are denoted by $A_1, A_2, \ldots, A_m$, while the n criteria applied to perform the evaluations are given by $C_1, C_2, \ldots, C_n$. The performance evaluation of alternative A_i, $i = 1, \ldots, m$, with respect to criterion, C_j, $j = 1, \ldots, n$, is denoted by x_{ij} and summarized through a decision matrix defined as follows:

	C_1	C_2	…	C_n
A_1	x_{11}	x_{12}	…	x_{1n}
A_2	x_{21}	x_{22}	…	x_{2n}
⋮	⋮	⋮		⋮
A_m	x_{m1}	x_{m2}	…	x_{mn}

$$W = [w_1, w_2, \ldots, w_n]$$

The terms w_j, $j = 1, \ldots, n$, represent the importance assigned by the DM or the experts to each criterion j. Given the

$$\tilde{A}^H \ominus \tilde{B}^H = \begin{cases} -\langle b-a, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) < Score\left(\tilde{B}^H\right) \& a < b, \\ \langle b-a, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) > Score\left(\tilde{B}^H\right) \& a < b, \\ \langle a-b, \bigcup_{\gamma^a_{(i)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) > Score\left(\tilde{B}^H\right) \& a > b, \\ -\langle a-b, \bigcup_{\gamma^a_{(i)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) < Score\left(\tilde{B}^H\right) \& a > b; \end{cases}$$

$$\tilde{A}^H \oslash \tilde{B}^H = \begin{cases} \langle {}^{a}\!/_{b}, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^a_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) < Score\left(\tilde{B}^H\right) \& a < b, \\ \langle {}^{b}\!/_{a}, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) > Score\left(\tilde{B}^H\right) \& a < b, \\ \langle {}^{a}\!/_{b}, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) > Score\left(\tilde{B}^H\right) \& a > b, \\ \langle {}^{b}\!/_{a}, \bigcup_{\gamma^a_{(j)} \in h(a), \gamma^b_{(j)} \in h(b)} \{\gamma\}\rangle & if \quad Score\left(\tilde{A}^H\right) < Score\left(\tilde{B}^H\right) \& a > b; \end{cases}$$

values provided in the decision matrix and the fact that criteria can be either positive or negative, the set of alternatives is ranked as follows.

Step 1: The decision matrix is normalized to allow for the criteria to be directly compared. The evaluations received by the different alternatives are normalized via

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_i x_{ij}^2}},\ \ i = 1, \ldots, m,\ \ j = 1, \ldots, n. \quad (4)$$

Step 2: The weights describing the relative importance of each criterion are used to generate the weighted normalized decision matrix by multiplying each column of the decision matrix by the corresponding weight

$$v_{ij} = w_j r_{ij},\ \ j = 1, \ldots, n. \quad (5)$$

Step 3: The criteria used to evaluate the alternatives may be positive (representing a beneficial quality) or negative (representing a cost). The ideal positive (best), v_i^+, and negative (worst), v_i^-, values are identified for each criterion. The vectors summarizing the ideal values obtained are defined as follows:

$$A^+ = \left(\nu_1^+, \ldots, \nu_n^+\right) \quad (6)$$

with

$$v_i^+ = \max_i\ \{(v_{ij}) \mid j\ \in\ \textit{positive criterion}\} \quad (7)$$

$$v_i^+ = \min_i \{(v_{ij}) \mid j\ \in\ \textit{negative criterion}\} \quad (8)$$

when considering the best potential values, and

$$A^- = \left(\nu_1^-, \ldots, \nu_n^-\right) \quad (9)$$

where

$$v_i^- = \min_i \{(v_{ij}) \mid j\ \in\ \textit{positive criterion}\} \quad (10)$$

$$v_i^- = \max_i\ \{(v_{ij}) \mid j\ \in\ \textit{negative criterion}\} \quad (11)$$

when dealing with the worst potential values.

Step 4: The distance between the evaluation assigned to each alternative and the positive and negative ideal values is computed as follows:

$$d_i^+ = \left[\sum_{j=1}^{n}\left(\nu_j^+ - \nu_{ij}\right)^2\right]^{1/2},\quad i = 1, \ldots, m \quad (12)$$

$$d_i^- = \left[\sum_{j=1}^{n}\left(\nu_j^- - \nu_{ij}\right)^2\right]^{1/2},\quad i = 1, \ldots, m. \quad (13)$$

Step 5: Given the d_i^+ and d_i^- values computed for each A_i, the alternatives are ranked according to their relative proximity to the ideal solution

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-}\quad i = 1, \ldots, m. \quad (14)$$

R_i describes the relative distance between alternative i and the negative ideal solution, with higher values corresponding to those alternatives farther from the potentially worst one. As a result, a value of $R_i = 1$ represents the best possible score while a value of $R_i = 0$ corresponds to the worst one.

A. *Incorporating HFNs Into TOPSIS*

The introduction of HFNs within a TOPSIS environment is not immediate or straightforward. Indeed, HFNs must be adapted to the main computational requirements of TOPSIS before the resulting technique can be applied.

Consider the score function, $Score(\tilde{A}^H)$, of an HFN, $\tilde{A}^H = \langle a, h(a)\rangle$, with membership degrees $h(a) = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}$ and $\gamma_i \in [0, 1]$. To simplify the presentation, assume that each expert is assigned a unique credibility score. This simplification implies that when the DM receives an evaluation, the corresponding score is given by $Score(\tilde{A}_h^H) = \langle a^h, h(a^h)\rangle = a^h \times \gamma_h$, determined by the credibility assigned by the DM to each expert h. Appendix AI describes the behavior of the score function when several credibility degrees are assigned to each expert.

Assume that each alternative is evaluated by $h = 1, \ldots, n+1$ experts. Then, the sum of all their evaluations, together with their corresponding credibility, is given by $\oplus_{h=1}^{n+1}\tilde{A}_h^H = \langle\sum_{h=1}^{n+1} a^h, \bigcup_{h=1}^{n+1} h(a^h)\rangle$. Note that this operation must be performed for each characteristic of each alternative per decision criterion. Without loss of generality, assume that there are two types of experts, denoted by $h = 1, 2$. More precisely, assume that there is one Type-1 expert and n Type-2 experts. The latter are identical and share both evaluations and credibility.

Consider the different evaluations provided by each type of expert, namely a^1 and a^2. Define the average of the evaluations presented as the value assigned by the DM to the characteristic of an alternative under a given criterion, $\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1} = \langle\sum_{h=1}^{n+1}\frac{a^h}{n+1}, \bigcup_{h=1}^{n+1} h(a^h)\rangle$. We assume that $\sum_{h=1}^{n+1} a^h > n+1$ and $\sum_{h=1}^{n+1} a^h \times \bigcup_{h=1}^{n+1} h(a^h) > n+1$, to define a standard average reference setting per set of evaluations.

Given the definition of score presented in (1) and (2) and considering the evaluations provided by different experts, we have that

$$Score\left(\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1}\right) = \sum_{h=1}^{n+1}\frac{a^h}{n+1}\times\left[\oplus_{h=1}^{n+1}\gamma_h\right] = \frac{a^1 + na^2}{n+1} \times\left[1 - (1-\gamma_1)\times\Pi_{h=2}^{n+1}(1-\gamma_h)\right] \quad (15)$$

which, given the fact that the n Type-2 experts are identical, simplifies to

$$Score\left(\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1}\right) = \frac{a^1 + na^2}{n+1}\times\left[1-(1-\gamma_1)\times(1-\gamma_2)^n\right]. \quad (16)$$

Note that $\lim_{n\to\infty}\left(Score\left(\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1}\right)\right) = a^2$, which eliminates the effect from the evaluation provided by the Type-1 expert, a^1. This result will become essential when describing

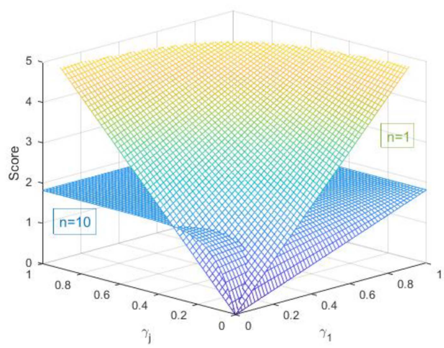

Fig. 1. Multiple experts with identical preferences and score modifications.

the manipulation possibilities inherent to the credibility scores of experts.

Fig. 1 represents (16) and the consequences derived from the introduction of multiple experts with identical preferences. The evaluations provided by the Type-1 and Type-2 experts are given by $a^1 = 10$ and $a^2 = 1$, respectively. The credibility of both experts varies within the [0, 1] domain. Fig. 1 illustrates how the preferences of the Type-2 experts can be imposed by adding multiple experts with the same evaluations independently of their credibility.

The score decreases in the credibility of both experts, while its highest potential value decreases in the number of Type-2 experts added to the analysis. Note how low credibility values can be compensated by including additional experts, leading their aggregate credibility to converge to a value of one. Fig. 1 illustrates this latter effect as the score function with ten Type-2 experts remains above that with a unique Type-2 expert for sufficiently low credibility values.

Clearly, the score function should be adapted to be fully implementable within TOPSIS, since we must consider the existence of negative criteria and the effect of credibility on the resulting scores, which could be incorporated as follows:

$$\begin{aligned} Score(\tilde{A}^H)^- &= a \times PA([1-\gamma_1],[1-\gamma_2],\ldots,[1-\gamma_n]) \\ &= a \times \frac{\sum_{j=1}^{n}(1+T([1-\gamma_j]))[1-\gamma_j]}{\sum_{j=1}^{n}(1+T([1-\gamma_j]))} \end{aligned} \quad (17)$$

where

$$T([1-\gamma_j]) = \sum_{k=1,k\neq j}^{n} Supp([1-\gamma_j],[1-\gamma_k]),$$

$$\begin{aligned} Supp([1-\gamma_j],[1-\gamma_k]) &= 1 - |(1-\gamma_j)-(1-\gamma_k)| \\ &= 1 - |\gamma_k - \gamma_j| \end{aligned} \quad (18)$$

for an HFN, $\tilde{A}^H = \langle a, h(a)\rangle$, with membership degrees $h(a) = \{\gamma_1,\gamma_2,\ldots,\gamma_n\}$ and $\gamma_i \in [0,1]$. The main difference with respect to the score of a positive criterion is determined by the fact that when dealing with a negative criterion an increase in credibility should decrease the score. That is, since lower values are preferred to higher ones, the higher the credibility of the expert the lower the score value assigned to the negative criterion.

However, as it can be intuitively inferred from (17), credibility degrees close to a value of one would lead to scores close to zero, underestimating the importance of the corresponding criteria. Appendix AII explains in detail the modifications required to preserve the cost nature of the criteria while preventing the bias just described.

Definition 8: Let $\tilde{A}_1^H$ and $\tilde{A}_2^H$ be two HFNs. Then

1) $\tilde{A}_1^H \prec \tilde{A}_2^H \left(\tilde{A}_1^H \succ \tilde{A}_2^H\right)$ if

$$Score\left(\tilde{A}_1^H\right) < Score\left(\tilde{A}_2^H\right)\left(Score\left(\tilde{A}_1^H\right) > Score\left(\tilde{A}_2^H\right)\right).$$

2) $\tilde{A}_1^H \prec \tilde{A}_2^H$ $(\tilde{A}_1^H \succ \tilde{A}_2^H)$ if
$Score(\tilde{A}_1^H) = Score(\tilde{A}_2^H)\&$
$Var(\tilde{A}_1^H) > Var(\tilde{A}_2^H)$ $(Var(\tilde{A}_1^H) < Var(\tilde{A}_2^H))$.

An important remark regarding the preference relationship between HFNs follows. Consider two HFNs, $\tilde{A}_1^H$ and $\tilde{A}_2^H$, with identical scores and variances. This fact alone does not allow to conclude that $\tilde{A}_1^H = \tilde{A}_2^H$. Validating this equality requires comparing the corresponding real parts and the maximum and minimum values of their membership sections. The sequential comparison process proceeds as follows: $a^1 > a^2 \Leftrightarrow \tilde{A}_1^H \succ \tilde{A}_2^H$. If $a^1 = a^2$, the membership elements will determine the preferences according to their highest values. Let $\gamma_{i(j)}$ be the jth largest value of $h(a^i)$. The preferred HFN has the higher $\gamma_{i(j)}$ value. However, if $\gamma_{1(j)} = \gamma_{2(j)}$, the $\gamma_{i(j-1)}$ values must be compared, and so on.

The above process defines a complete preference order among HFNs, allowing for the incorporation of the resulting values in the final ranking generated by any MCDM technique, such as TOPSIS. At the same time, the scores trigger a series of potential strategic scenarios that allow for the introduction of expert coalitions designed to impose their evaluations across decision criteria and alternatives over those of credible experts.

B. Dealing With Manipulation

We illustrate how the continuity of the score function allows for coalitions of experts to impose their preferred alternative independently of their credibility. This quality of the score function is desirable when credible experts can discard the evaluations of noncredible ones. However, this feature also works in the opposite sense with a sufficiently large number of noncredible experts being able to discard the evaluations of credible ones. Thus, a basic credibility limit must be imposed on the evaluations used to rank the alternatives.

Proposition 1: There exists a sufficiently large coalition of experts whose preferred alternative can be imposed independently of their reputation.

Proof: Consider the framework defined by two types of experts. Assume that the reputation of the first type is much higher than that of the second, that is, $\gamma_1 >> \gamma_2$. Assume that there are l Type-1 experts, who provide an evaluation of a^1, and $n + 1 - l$ Type-2 experts, providing an evaluation of a^2. Given the definition of the score and considering both evaluations, we have that

$$Score\left(\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1}\right)=\frac{la^1+(n+1-l)a^2}{n+1}$$
$$\times\left[1-(1-\gamma_1)^l\times(1-\gamma_2)^{n+1-l}\right]. \quad (19)$$

Fix the number of Type-1 experts and increase that of Type-2. Clearly, $\lim_{n\to\infty}\left(\oplus_{h=1}^{n+1}\frac{\tilde{A}_h^H}{n+1}\right)=a^2$, eliminating the evaluation of a^1. The result generalizes immediately to multiple types of experts and dimensions of the HFEs.

This result relates to the concept of core in economics and game theory but is applied to an MCDM setting. The DM must therefore define a minimum credibility requirement on the evaluations used to generate the ranking. For instance, a credibility threshold could be imposed per set of evaluations of each characteristic

$$h^{\sigma(t)}\geq\hat{\gamma}^{\sigma(t)} \quad (20)$$

where $h^{\sigma(t)}=\oplus_{h=1}^{k}h_h^{\sigma(t)}=\{1-\Pi_{h=1}^{k}(1-h_h^{\sigma(t)})|t=1,2,\ldots,l\}$, implying that $\oplus_{t=1}^{l}h^{\sigma(t)}\geq\oplus_{t=1}^{l}\hat{\gamma}^{\sigma(t)}$, where $h=1,2,\ldots,k$, is the number of experts, $t=1,2,\ldots,l$, the set of credibility features per expert, and $\hat{\gamma}^{\sigma(t)}$ the threshold per credibility feature when evaluating the characteristics of each alternative. Alternatively, the DM could also impose $h_h^{\sigma(t)}\geq\hat{\gamma}$, for each feature $t=1,2,\ldots,l$, and expert $h=1,2,\ldots,k$, or $\oplus_{t=1}^{l}h_h^{\sigma(t)}\geq\hat{\gamma}$, with $\hat{\gamma}$ referring to the minimum credibility per expert considered by the DM.

Credibility requirements must therefore be imposed on each evaluation of every characteristic per alternative being ranked. Otherwise, the results obtained would lack sufficient reliability. However, a minimum credibility requirement does not suffice to prevent manipulation, requiring an additional limit on the number of experts selected.

Corollary 1: There exists a coalition of experts satisfying the minimum credibility requirements imposed by the DM, i.e., $\oplus_{h=1}^{k}h_h^{\sigma(t)}\geq\hat{\gamma}^{\sigma(t)}$, $\oplus_{t=1}^{l}h_h^{\sigma(t)}\geq\hat{\gamma}$, or $h_h^{\sigma(t)}\geq\hat{\gamma}$, with $t=1,2,\ldots,l$ and $h=1,2,\ldots,k$, whose preferred alternative can be imposed whenever a sufficiently large number of experts forms the coalition.

The DM must therefore limit both the loss in reputation derived from the group of experts selected and their number, i.e., $k<\hat{k}$, with $\hat{k}$ representing the maximum number of experts consulted by the DM. Both types of parameters, $\hat{\gamma}^{\sigma(t)}$ or $\hat{\gamma}$ and $\hat{k}$, must be subjectively defined to prevent manipulation to a reasonable extent defined by the DM.

The intuition on which these results are based follows from the description of the score function, which, after substituting (2) into (1), reads as follows:

$$Score(\tilde{A}^H)=$$
$$a\times\frac{\sum_{j=1}^{n}\left(1+\sum_{k=1,k\neq j}^{n}1-|\gamma_j-\gamma_k|\right)\gamma_j}{\sum_{j=1}^{n}\left(1+\sum_{k=1,k\neq j}^{n}1-|\gamma_j-\gamma_k|\right)}. \quad (21)$$

Intuitively, the score function is designed to punish those experts displaying a lower reputation. That is, consider an expert whose reputation across several subjective characteristics is given by $h(a)=\{\gamma_1,\gamma_2,\ldots,\gamma_n\}$. Equation (21) illustrates how the weight assigned to each credibility score is determined by the relative distance between the credibility scores assigned to the expert

$$\frac{\partial\left(1+\sum_{k=1,k\neq j}^{n}1-|\gamma_j-\gamma_k|\right)}{\partial\left(|\gamma_j-\gamma_k|\right)}<0. \quad (22)$$

Thus, similarly credible evaluations will lead to higher weights within the weighted average defined in (21). At the same time, the relative credibility of the expert determines the value of the score obtained via $\frac{\partial(Score(\tilde{A}^H))}{\partial\gamma_j}>0$.

That is, credible experts rated consistently will display higher weights within the score function, validating their evaluations relative to less credible experts. A similar intuition applies to the addition of several HFNs describing experts' evaluations. However, considering a large number of experts could lead to credibility problems because of Equation (23) shown at the bottom of next page.

In a nutshell, a sufficiently large number of experts would lead to a totally credible evaluation independently of their reputation since $\lim_{n\to\infty}\oplus_{h=1}^{n+1}\gamma_h=\lim_{n\to\infty}\{1-\Pi_{h=1}^{n+1}(1-\gamma_h)\}=1$ for any value of γ_h. This drawback must be accounted for when incorporating multiple experts into the analysis.

Note that the DM is responsible for selecting the experts—and their number—based on their credibility. On the other hand, standard MCDM models assume that the evaluations provided by the experts are either completely truthful or, if some uncertainty exists, it is not due to the strategic reporting of the experts. Uncertainty is inherent to the evaluations and formalized via fuzzy numbers. In this case, selecting the fuzzy numbers and their membership functions constitutes an entirely subjective task assigned to the DM.

The formal requirements imposed on DMs within the current setting are less demanding than those imposed within standard fuzzy MCDM environments. Moreover, they account for the known fact that experts' evaluations may be strategically biased, particularly in complex scenarios dealing with heterogeneous variables, as is the case with sustainable transportation systems.

C. *Credibility Through HFNs*

One of the main qualities of HFNs is their malleability, which allows them to perform the whole set of algebraic operations required to implement TOPSIS. We abuse notation and denote by x_{ij}^h the performance evaluation of alternative A_i, $i=1,\ldots,m$, with respect to criterion, C_j, $j=1,\ldots,n$. Evaluations are provided by experts, $h=1,\ldots,k$, who are also assigned degrees of credibility through the different dimensions composing the corresponding HFEs. That is, the DM retrieves the following evaluation matrix from each expert:

	C_1	C_2	$\ldots$	C_n
A_1	$\langle x_{11}^h, h(x_{11}^h)\rangle$	$\langle x_{12}^h, h(x_{12}^h)\rangle$	$\ldots$	$\langle x_{1n}^h, h(x_{1n}^h)\rangle$
A_2	$\langle x_{21}^h, h(x_{21}^h)\rangle$	$\langle x_{22}^h, h(x_{22}^h)\rangle$	$\ldots$	$\langle x_{2n}^h, h(x_{2n}^h)\rangle$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	$\langle x_{m1}^h, h(x_{m1}^h)\rangle$	$\langle x_{m2}^h, h(x_{m2}^h)\rangle$	$\ldots$	$\langle x_{mn}^h, h(x_{mn}^h)\rangle$

$$W = \left[\left\langle w_1^h, h(w_1^h)\right\rangle, \left\langle w_2^h, h(w_2^h)\right\rangle, \ldots, \left\langle w_n^h, h(w_n^h)\right\rangle\right].$$

Adding the evaluations of the experts per criterion and the corresponding weights gives place to the following matrix, on which TOPSIS can be implemented:

	C_1	C_2	C_n
A_1	$\oplus_{h=1}^k \langle x_{11}^h, h(x_{11}^h)\rangle$	$\oplus_{h=1}^k \langle x_{12}^h, h(x_{12}^h)\rangle$	$\oplus_{h=1}^k \langle x_{1n}^h, h(x_{1n}^h)\rangle$
A_2	$\oplus_{h=1}^k \langle x_{21}^h, h(x_{21}^h)\rangle$	$\oplus_{h=1}^k \langle x_{22}^h, h(x_{22}^h)\rangle$	$\oplus_{h=1}^k \langle x_{2n}^h, h(x_{2n}^h)\rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	$\oplus_{h=1}^k \langle x_{m1}^h, h(x_{m1}^h)\rangle$	$\oplus_{h=1}^k \langle x_{m2}^h, h(x_{m2}^h)\rangle$	$\oplus_{h=1}^k \langle x_{mn}^h, h(x_{mn}^h)\rangle$

$$W = \left[\oplus_{h=1}^k \left\langle w_1^h, h(w_1^h)\right\rangle, \oplus_{h=1}^k \left\langle w_2^h, h(w_2^h)\right\rangle, \ldots, \oplus_{h=1}^k \left\langle w_n^h, h(w_n^h)\right\rangle\right].$$

Step 1: The decision matrix is normalized to allow the criteria to be directly compared. The evaluations received by the different alternatives are normalized via

$$r_{ij} = \frac{\oplus_{h=1}^k \left\langle x_{ij}^h, h(x_{ij}^h)\right\rangle}{\sqrt{\underset{i}{\oplus}\left(\oplus_{h=1}^k \left\langle x_{ij}^h, h(x_{ij}^h)\right\rangle\right)^2}}, \quad i = 1, \ldots, m,\ j = 1, \ldots, n. \tag{24}$$

Step 2: The weights describing the relative importance of each criterion are used to generate the weighted normalized decision matrix by multiplying each column of the decision matrix by the corresponding weight

$$v_{ij} = \oplus_{h=1}^k \left\langle w_j^h, h(w_j^h)\right\rangle \otimes r_{ij},\ j = 1, \ldots, n. \tag{25}$$

Step 3: The criteria used to evaluate the alternatives may be positive (representing a beneficial quality) or negative (representing a cost). The ideal positive (best), v_i^+ and negative (worst), v_i^-, values are identified for each criterion. The vectors summarizing the ideal values obtained are defined as follows:

$$A^+ = \left(\nu_1^+, \ldots, \nu_n^+\right) \tag{26}$$

with

$$v_i^+ = \max_i \ \{score(v_{ij}) \mid j \in \ positive\ criterion\} \tag{27}$$

$$v_i^+ = \min_i \{score(v_{ij}) \mid j \in \ negative\ criterion\} \tag{28}$$

when considering the best potential values, and

$$A^- = \left(\nu_1^-, \ldots, \nu_n^-\right) \tag{29}$$

where

$$v_i^- = \min_i \{score(v_{ij}) \mid j \in \ positive\ criterion\} \tag{30}$$

$$v_i^- = \max_i \ \{score(v_{ij}) \mid j \in \ negative\ criterion\} \tag{31}$$

when dealing with the worst potential values.

Step 4: The distance between the evaluation assigned to each alternative and the positive and negative ideal values is computed as follows:

$$d_i^+ = \left[\oplus_{j=1}^n \left(\nu_j^+ \ominus \nu_{ij}\right)^2\right]^{1/2}, \quad i = 1, \ldots, m \tag{32}$$

$$d_i^- = \left[\oplus_{j=1}^n \left(\nu_j^- \ominus \nu_{ij}\right)^2\right]^{1/2}, \quad i = 1, \ldots, m. \tag{33}$$

Step 5: Given the d_i^+ and d_i^- values computed for each A_i, the final ranking will be based on the relative proximity of each alternative to the ideal solution

$$R_i = d_i^- \oslash \left(d_i^+ \oplus d_i^-\right), \quad i = 1, \ldots, m. \tag{34}$$

Step 6: The alternatives are ranked according to the scores of the different proximity variables, $score(R_i)$, $i = 1, 2, \ldots, m$. Let R_i and R_j be two HFNs describing the relative distance of alternatives i and j from the negative ideal one, respectively. The subsequent ranking is obtained as follows:

1) $R_i \prec R_j \ (R_i \succ R_j)$ if

$$Score(R_i) < Score(R_j) \ \ (Score(R_i) > Score(R_j)).$$

2) $R_i \prec R_j \ (R_i \succ R_j)$ if

$$Score(R_i) = Score(R_j) \ \ \& \\ Var(R_i) > Var(R_j)\ (Var(R_i) < Var(R_j)).$$

In case the scores and variances are identical, we proceed through the corresponding HFEs implementing the comparison process described in Definition 8. The resulting ranking will be complete and strict, with indifference arising among those alternatives with identical R_i HFNs, $i = 1, 2, \ldots, m$.

We conclude by illustrating how the malleability of HFNs allows us to consider further strategic scenarios. For instance,

$$\lim_{n\to\infty} \left[\frac{\sum_{j=1}^n \left(1 + \sum_{k=1, k\neq j}^n 1 - \left| \oplus_{h=1}^{n+1} \gamma_{hj} - \oplus_{h=1}^{n+1} \gamma_{hk}\right|\right) \oplus_{h=1}^{n+1} \gamma_{hj}}{\sum_{j=1}^n \left(1 + \sum_{k=1, k\neq j}^n 1 - \left| \oplus_{h=1}^{n+1} \gamma_{hj} - \oplus_{h=1}^{n+1} \gamma_{hk}\right|\right)}\right] = 1. \tag{23}$$

assume that there are d types of DMs assigning different credibility scores to the experts. We must therefore account for the evaluations of different experts, $h = 1, 2, \ldots, k$, being assigned degrees of credibility along different dimensions by several DMs, $p = 1, 2, \ldots, d$. That is, each DM defines different evaluation matrices based on the credibility assigned to the corresponding experts

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle x_{11}^h, h(x_{11}^h)\right\rangle^p & \left\langle x_{12}^h, h(x_{12}^h)\right\rangle^p & \cdots & \left\langle x_{1n}^h, h(x_{1n}^h)\right\rangle^p \\ A_2 & \left\langle x_{21}^h, h(x_{21}^h)\right\rangle^p & \left\langle x_{22}^h, h(x_{22}^h)\right\rangle^p & \cdots & \left\langle x_{2n}^h, h(x_{2n}^h)\right\rangle^p \\ \vdots & \vdots & \vdots & & \vdots \\ A_m & \left\langle x_{m1}^h, h(x_{m1}^h)\right\rangle^p & \left\langle x_{m2}^h, h(x_{m2}^h)\right\rangle^p & \cdots & \left\langle x_{mn}^h, h(x_{mn}^h)\right\rangle^p \end{array}$$

$$W = \left[\langle w_1^h, h(w_1^h)\rangle^p, \langle w_2^h, h(w_2^h)\rangle^p, \ldots, \langle w_n^h, h(w_n^h)\rangle^p\right].$$

Adding the credibility scores across DMs and the expert evaluations across characteristics, as well as the corresponding weights, gives place to the following evaluation matrix, on which TOPSIS can be implemented

$$\begin{array}{c|ccc} & C_1 & C_2 & C_n \\ \hline A_1 & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{11}^h, h(x_{11}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{12}^h, h(x_{12}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{1n}^h, h(x_{1n}^h)\right\rangle^p \\ A_2 & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{21}^h, h(x_{21}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{22}^h, h(x_{22}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{2n}^h, h(x_{2n}^h)\right\rangle^p \\ \vdots & \vdots & \vdots & \vdots \\ A_m & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{m1}^h, h(x_{m1}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{m2}^h, h(x_{m2}^h)\right\rangle^p & \oplus_{h=1}^k \oplus_{p=1}^d \left\langle x_{mn}^h, h(x_{mn}^h)\right\rangle^p \end{array}$$

$$W = \begin{bmatrix} \oplus_{h=1}^k \oplus_{p=1}^d \left\langle w_1^h, h(w_1^h)\right\rangle^p, \oplus_{h=1}^k \oplus_{p=1}^d \left\langle w_2^h, h(w_2^h)\right\rangle^p, \ldots, \\ \oplus_{h=1}^k \oplus_{p=1}^d \left\langle w_n^h, h(w_n^h)\right\rangle^p \end{bmatrix}$$

We can now proceed with the different steps composing TOPSIS, such as, for instance, the normalization of the evaluation matrix via

$$r_{ij} = \frac{\oplus_{h=1}^k \oplus_{p=1}^d \langle x_{11}^h, h(x_{11}^h)\rangle^p}{\sqrt{\bigoplus_i \left(\oplus_{h=1}^k \oplus_{p=1}^d \langle x_{11}^h, h(x_{11}^h)\rangle^p\right)^2}}, \ i = 1, \ldots, m,$$
$$j = 1, \ldots, n. \tag{35}$$

Furthermore, DMs could also be assigned different degrees of credibility. That is, DMs could allocate their credibility scores to the experts strategically, based on personal affinities or subjective interests. Pushing the limits of the analysis further, we could formalize the credibility of DMs when assigning scores to the evaluations provided by the experts, x_{ij}^{hp}, through

$$\left\langle 1, h(x_{ij}^{hp})\right\rangle \otimes \langle x_{ij}^h, h(x_{ij}^h)\rangle^p. \tag{36}$$

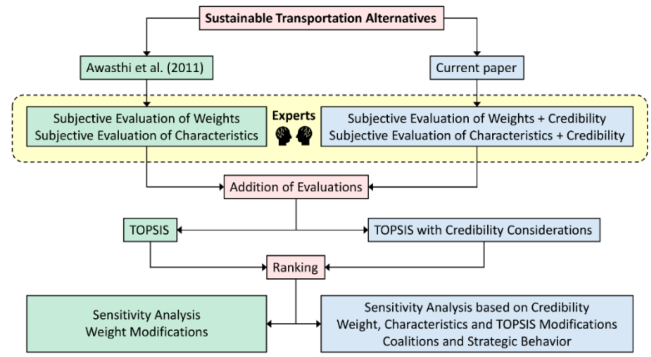

Fig. 2. Evaluation and ranking processes with and without credibility considerations.

These strategic extensions foster the transformation of MCDM frameworks into game-theoretical scenarios where different coalitions may be introduced and analyzed based on the credibility of both experts and DMs.

VI. Case Study

We use a real-life case study to illustrate the main implications derived from the inclusion of credibility when analyzing experts' evaluations and ranking the set of alternatives using an MCDM technique such as TOPSIS. The flexibility of the HFNs introduced by [22] allows for the inclusion of credibility and strategic considerations through other MCDM techniques. However, as we will illustrate through the current case study, the implementation is not immediate, and the characteristics of the HFNs must be carefully incorporated into the structure of the corresponding MCDM technique to preserve consistency through the analysis.

We build on the case analyzed by [6], which was one of the first studies to incorporate the evaluations of several experts within an uncertain MCDM setting. The case study presented by these authors focuses on the development of a sustainable transportation system within a city based on a previous project implemented in La Rochelle, France. The authors consider three potential alternatives, Carsharing (A1), Ridesharing (A2), and Park-n-ride (A3), involving the use of either individual or shared cars or their substitution with public transportation alternatives. Their main contribution consists in incorporating the linguistic opinions of three experts within an uncertain TOPSIS environment formalized via TFNs.

Fig. 2 presents a flowchart highlighting the main differences between the model of [6] and the current one in terms of the credibility considerations that must be implemented through the different steps of the ranking process. In particular, the authors assume the ad hoc existence of a committee composed of three experts, D1, D2, and D3, consulted to help select the most sustainable transportation system to be implemented in the city. There is no information about the committee members, who are assumed identical in terms of their subjective characteristics. This is a common shortcoming of any model incorporating multiple opinions absent credibility considerations. The inclusion

TABLE I
RANKING VALUES ACROSS CREDIBILITY SCENARIOS

[6]	A1	A2	A3
Awasthi et al. (2011)	0.549	0.554	0.545
Individual experts			
Expert 1	0.468	0.447	0.546
Expert 2	0.499	0.501	0.481
Expert 3	0.511	0.537	0.448
Credibility in final step			
(1,1,1)	0.513	0.479	0.550
(0.5,0.5,0.5)	0.513	0.479	0.550
Credibility in all steps			
(0.5,0.5,0.5)	0.522	0.471	0.524

of fuzzy evaluations accounts for the imprecision inherent in the opinions of the experts. However, this is not the only source of uncertainty. When presenting their evaluations, the strategic incentives of experts constitute an essential part of any group decision process.

All tables and figures referring directly to the study case presented by [6] have been relegated to an online appendix section. The results derived from the introduction of credibility considerations within [6] have all been illustrated in the article. In this regard, the set of 24 cost and benefit criteria considered through the evaluation process is described in Table AI. The authors assume that the committee members provide linguistic assessments, based on those presented in Table AII, to rate both the criteria and the three alternatives. The linguistic evaluations assigned to the criteria are described in the first set of columns within Table AIII, while those assigned to the alternatives are presented in Table AIV. Note how the subjectivity inherent to many of the evaluations is evident from the basic description of the criteria, even when strategic considerations are absent.

Awasthi et al. [6] added up all the evaluations received and solved the resulting TOPSIS model based on TFNs. The results obtained by these authors are presented in the second row within Table I. This classification, namely, $A2 \succ A1 \succ A3$, will be used as a benchmark relative to which the effect of credibility modifications will be analyzed.

A. Implementation

The introduction of HFNs within the TOPSIS environment analyzed by [6] requires defuzzifying the evaluations of the experts. To this end, we apply the graded mean integration representation (GMIR) method [9]. Let $\bar{w} = (l, m, u)$ be a TFN corresponding to the linguistic assessments that define the importance of the criteria and alternatives described in Tables AIII and AIV, respectively. The GMIR of the fuzzy weight $\bar{w}$ is defined as follows:

$$R(\bar{w}) = \frac{l + 4m + u}{6}. \tag{37}$$

We apply this defuzzification technique to the linguistic evaluations provided by the experts and reported in the paper of [6]. The results are presented within the second set of columns in Table AIII—when considering the weights—and in Table AV—when accounting for the evaluations of the alternatives. To simplify the presentation, we assign a unique credibility score to each expert, which allows illustrate the results that follow from a complete modification of the credibility assigned to the different experts. That is, the HFEs are composed of a unique element describing the credibility assigned to the evaluation provided by the corresponding expert.

The main contribution of the current article consists in the formalization of the uncertainty derived from the incentives of experts to report strategically and the mechanisms available to compensate for the resulting outcomes within an MCDM setting. In this regard, the case study illustrates the main modifications that must be implemented to incorporate the effect of credibility on the ranking obtained fully. Intuitively, the presentation provided in the previous section illustrates the general cumulative effect of credibility within a TOPSIS setting.

That is, credibility only plays an explicit role in determining the ranking within the final step of the TOPSIS process, after the d_i^+ and d_i^- values have been computed and the R_i values are compared across alternatives $i = 1, \ldots, m$. The implementation described does not consider the specific effect of the credibility scores at each step of the process, which conditions the evaluations of the alternatives, the weights of the criteria, and the relative distances from the ideal solutions.

This simplification becomes a problem when considering many criteria, as is generally the case when analyzing real-life environments since the values of the HFEs composing d_i^+ and d_i^- would converge toward one, eliminating any credibility effect from the analysis. The credibility scores accumulated through the different steps must be added across experts, alternatives, weights, and criteria, leading to the general cumulative credibility assigned through TOPSIS. A sufficiently large number of any of these variables implies that the value of the final credibility score would be equal to one independently of the values assigned throughout the process. In particular,

Corollary 2: A sufficiently large number of criteria would lead to an aggregate credibility of one.

The proof follows directly from the limit values described in Corollary 1. Intuitively, when computing R_i , we have that $h_1 \oplus h_2 = h_1^{\sigma(t)} + h_2^{\sigma(t)} - h_1^{\sigma(t)} h_2^{\sigma(t)} > h_1^{\sigma(t)} = h_1$, since $h_2^{\sigma(t)} > h_1^{\sigma(t)} h_2^{\sigma(t)}$, with $h_1^{\sigma(t)} < 1$. Thus, a sufficiently large number of criteria implies that the HFEs defining the final R_i will be equal to one. The resulting scores will be identical to their respective real parts, avoiding any credibility consideration from the ranking whenever its effect is considered only in the final step of the TOPSIS process.

Therefore, we must account for the effect of the credibility scores at each step of the process, which differs from the cumulative credibility determining the evaluation of alternatives at the very last step. HFNs allow to consider multiple credibility dimensions per expert and implement the resulting effects through the different TOPSIS stages. We describe them explicitly within the following steps.

Step 1: We multiply the initial evaluations of the alternatives by the credibility assigned to the corresponding experts per alternative and criterion so that the latter determines the weighted value of each entry composing the decision matrix. The normalization of the decision matrix is therefore based on the following equation:

$$r_{ij} = \frac{\left\langle \sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h}, \oplus_{h=1}^{k}\gamma_{ij}^{h} \right\rangle}{\sqrt{\underset{i}{\oplus}\left(\left\langle \sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h}, \oplus_{h=1}^{k}\gamma_{ij}^{h}\right\rangle\right)^{2}}}, \quad i=1,\ldots,m, \quad j=1,\ldots,n; \tag{38}$$

The HFN $\langle\sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h}, \oplus_{h=1}^{k}\gamma_{ij}^{h}\rangle$ incorporates two different effects, namely, the general credibility per criterion and alternative, $\oplus_{h=1}^{k}\gamma_{ij}^{h}$, and the weighted evaluation $\sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h}$ accounting for the effect that the credibility of each expert has on the corresponding evaluations per criterion and alternative.

Note that the credibility scores and weighted evaluations must be adjusted when dealing with cost criteria. Appendix AII illustrates in detail the implementation of the corresponding adjustments through the different steps of TOPSIS.

Step 2: A similar intuition applies to the weights assigned by the experts and their corresponding credibility scores, both of which determine the weighted value of each normalized evaluation at both local and cumulative levels as follows:

$$v_{ij} = \left\langle \sum_{h=1}^{k} w_{j}^{h}\gamma_{j}^{wh}, \oplus_{h=1}^{k}\gamma_{j}^{wh} \right\rangle \otimes r_{ij}, \quad j=1,\ldots,n. \tag{39}$$

As in the previous step, the HFNs defining the weights are composed of the local weighted credibility assigned to the evaluation of the weights by the experts, $\sum_{h=1}^{k} w_{j}^{h}\gamma_{j}^{wh}$, and the general credibility that will be directly carried into the next step of the process, $\oplus_{h=1}^{k}\gamma_{j}^{wh}$.

Steps 3 and 4: The cumulative credibility of the experts when reaching the third step is given by $[(\oplus_{h=1}^{k}\gamma_{ij}^{h}) \oslash (\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h})] \otimes (\oplus_{h=1}^{k}\gamma_{j}^{wh})$, per alternative, $i=1,\ldots,m$, and criterion, $j=1,\ldots,n$. These HFE values enter locally into the third and fourth steps of TOPSIS as we multiply the normalized weighted evaluation of each alternative per criterion, $[\frac{\sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h}}{\sqrt{\sum_i (\sum_{h=1}^{k} x_{ij}^{h}\gamma_{ij}^{h})^2}}]\sum_{h=1}^{k} w_{j}^{h}\gamma_{j}^{wh}$, by the cumulative weighted credibility in order to obtain the reference minimum and maximum score values—as well as the scores of each alternative per criterion—so that we can compute the corresponding relative distances.

Note that, when considering a unique credibility value per expert, the difference between the real parts of two HFNs is equivalent to the difference between their scores

$$d_i^{+} = \left[\oplus_{j=1}^{n}\left(\begin{array}{l}\left\langle score\left(\nu_j^{+}\right), \left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]^{+}\right\rangle \ominus \\ \left\langle score\left(\nu_{ij}\right), \left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]\right\rangle\end{array}\right)^{2}\right]^{1/2}, \quad i=1,\ldots,m \tag{40}$$

$$d_i^{-} = \left[\oplus_{j=1}^{n}\left(\begin{array}{l}\left\langle score\left(\nu_j^{-}\right), \left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]^{-}\right\rangle \ominus \\ \left\langle score\left(\nu_{ij}\right), \left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]\right\rangle\end{array}\right)^{2}\right]^{1/2}, \quad i=1,\ldots,m. \tag{41}$$

Step 5: The cumulative value of the HFEs composing the R_i HFNs follows from adding the differences between the ν_j^{+}, ν_j^{-}, and ν_{ij} HFNs across criteria per alternative $i=1,\ldots,m$, leading to

$$\oplus_{j=1}^{n}\left(\begin{array}{l}\left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]^{+}\ominus\\\left[\left[\begin{array}{l}\left(\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\oslash\\\left(\oplus_{i=1}^{m}\oplus_{h=1}^{k}\gamma_{ij}^{h}\right)\end{array}\right]\otimes\left(\oplus_{h=1}^{k}\gamma_{j}^{wh}\right)\right]\end{array}\right). \tag{42}$$

Equation (42) defines the cumulative credibility assigned to each d_i^{+} variable, with a similar expression corresponding to the d_i^{-} one. Note that the HFEs assigned to the d_i^{+} and d_i^{-} variables define the general credibility of the system and determine the final ranking through the scores of the R_i variables. Clearly, a large enough number of criteria implies that the cumulative credibility of each d_i^{+} and d_i^{-} variable will converge to one.

All in all, the implementation of HFNs within TOPSIS requires distinguishing between the credibility scores assigned per expert, criterion, weight, and alternative, and the aggregate ones accumulated through the different steps of the ranking process. If not considered before the final step, the cumulative credibility scores may converge to one for all the alternatives, eliminating any credibility effect from the ranking obtained.

B. Analysis of the Results

We now illustrate the ranking scenarios that arise when the credibility scores are introduced—and modified—through the different steps of TOPSIS. We start by deriving the rankings that follow from the individual preferences of each expert. Table I illustrates the rankings obtained when each expert is assigned a credibility of one, highlighting the biases inherent to their individual preferences. Clearly, the ranking preferred by the

TABLE II
VARIATIONS IN THE DISTRIBUTION OF CREDIBILITY AMONG EXPERTS

Credibility distributions $\left(\gamma_{ij}^{1},\gamma_{ij}^{2},\gamma_{ij}^{3}\right),\forall i,j$	**A1**	**A2**	**A3**
First expert			
(0.9,0.5,0.5)	0.541	0.432	0.559
(0.8,0.5,0.5)	0.534	0.440	0.551
(0.7,0.5,0.5)	0.530	0.450	0.542
(0.6,0.5,0.5)	0.526	0.459	0.534
(0.4,0.5,0.5)	0.516	0.486	0.511
(0.3,0.5,0.5)	0.505	0.504	0.494
0.2,0.5,0.5)	0.491	0.526	0.476
(0.1,0.5,0.5)	0.474	0.551	0.458
Second expert			
(0.5,0.9,0.5)	0.488	0.496	0.465
(0.5,0.8,0.5)	0.493	0.488	0.481
(0.5,0.7,0.5)	0.502	0.481	0.495
(0.5,0.6,0.5)	0.512	0.476	0.509
(0.5,0.4,0.5)	0.529	0.465	0.538
(0.5,0.3,0.5)	0.530	0.459	0.550
(0.5,0.2,0.5)	0.528	0.451	0.561
(0.5,0.1,0.5)	0.523	0.445	0.570
Third expert			
(0.5,0.5,0.9)	0.505	0.480	0.524
(0.5,0.5,0.8)	0.515	0.482	0.524
(0.5,0.5,0.7)	0.520	0.481	0.525
(0.5,0.5,0.6)	0.523	0.477	0.525
(0.5,0.5,0.4)	0.517	0.465	0.520
(0.5,0.5,0.3)	0.511	0.463	0.516
(0.5,0.5,0.2)	0.504	0.463	0.514
(0.5,0.5,0.1)	0.499	0.466	0.518

first expert is given by $A3 \succ A1 \succ A2$, while the second and third experts prefer $A2 \succ A1 \succ A3$, coinciding with the ranking derived from the analysis performed by [6]. These rankings provide a reference benchmark to analyze the effects derived from relative modifications in the credibility of each expert. The rankings are illustrated in Fig. A1, where the differences in the intensity of preferences among experts can be observed.

Table I also presents the rankings derived from the implementation framework described in Section V-C, where credibility scores are only considered when ranking the R_i HFNs. Note how assigning a credibility score of either one or ½ to all the experts results in the same ranking. Clearly, aggregating 24 criteria in the final steps of TOPSIS leads to a cumulative credibility sufficiently close to one. For comparison purposes, we have described the case where a credibility of ½ is assigned to all the experts but applied through the whole ranking process.

This last case will be used as the benchmark reference when evaluating changes in the relative credibility assigned to the experts, a set of scenarios analyzed in Table II. We have shaded the relevant results within this table, indicating changes in the rankings caused by the relative credibility assigned to the different experts. Note how when the first expert is assigned a higher credibility relative to the other two, his preferred ranking is consistently implemented $A3 \succ A1 \succ A2$. However, as his credibility decreases below that of the other experts, the $A2 \succ A1 \succ A3$ ranking arises as the preferred one.

This intuition is validated when modifying the credibility of the other experts. The second requires a substantial increase in

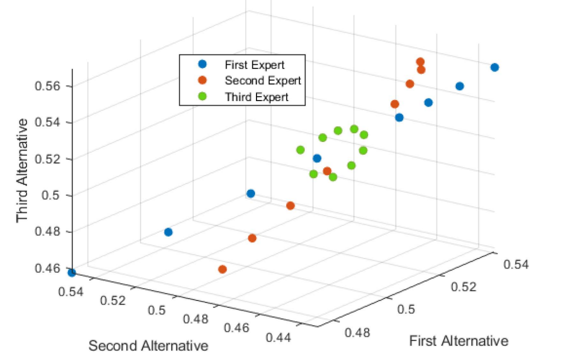

Fig. 3. Ranking variations caused by changes in the distribution of credibility among experts.

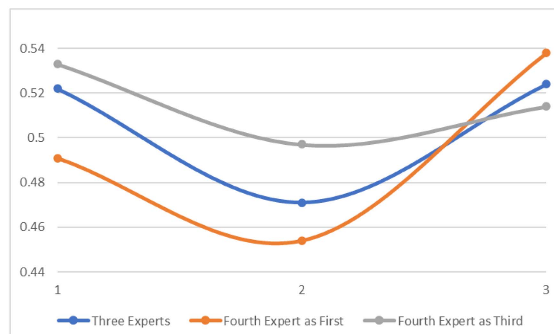

Fig. 4. Ranking variations when strategically adding a fourth expert to the analysis.

credibility to be able to impose his preferred ranking, $A2 \succ A1 \succ A3$, while the third expert is unable to modify the initial ranking and impose his preferred one.

Fig. 3 summarizes the set of rankings generated as the relative credibility of the different experts is modified. Given its strategic importance, the addition of experts with different preferences and credibility scores constitutes one of the main extensions of the model. We analyze the resulting strategic framework in the following section.

C. Sensitivity Analysis

Table AVI describes the rankings obtained when adding a fourth expert to the analysis whose preferences coincide with those of another expert. This extension allows us to analyze the strategic effects derived from the introduction of experts who may form potential coalitions to modify the rankings according to their subjective preferences.

The benchmark scenario is given by the case where all experts are endowed with a credibility of ½. The initial set of modifications introduces a fourth expert with the same preferences and evaluations as the first one. We define a progressive increment of his individual credibility and that of the first expert. As can be inferred from the initial set of rows described in Table AVI, the ranking preferred by the first expert, $A3 \succ A1 \succ A2$, is imposed through the different scenarios considered. The addition to the analysis of an expert with the same evaluations and credibility as the first one validates the initial ranking obtained. This tendency is clearly preserved as the credibility assigned to these experts increases.

On the other hand, when the fourth expert displays the same preferences and evaluations as the third one, we observe a progressive increment in the score of the second alternative as the credibility of the fourth expert increases. This pattern does not suffice to place the second alternative above the first one in the ranking. However, as illustrated in Fig. A2, it provides a clear and intuitive description of how the preferences of the experts determine the ranking obtained as their credibility is modified.

Fig. 4 summarizes the consequences of adding a fourth expert with the same preferences and evaluations as the first and third one, respectively. All experts have been assigned equal credibility of ½, and both scenarios are compared to the benchmark case with three experts. The ability to impose the ranking preferred by the first expert clearly contrasts with the inability to impose that of the third one. Appendix AIII describes the ranking modifications that arise when experts alter their evaluation reports strategically.

VII. Conclusion

We have analyzed the main consequences derived from implementing a strategic evaluation framework within a standard MCDM technique such as TOPSIS, highlighting the capacity of experts to manipulate the rankings delivered. We have also formalized the requirements imposed on DMs to contain the manipulation inherent to the subsequent decision process. DMs must assign a credibility score to each expert and define a minimum credibility value determined by the scores assigned and the number of experts considered. Both these requirements are novel to the current strategic MCDM environment, which has been applied to analyze a sustainable transportation problem involving significant environmental and pecuniary costs.

Among the potential extensions, it could be assumed that the evaluations provided by the experts consist of fuzzy numbers. In this case, standard operations could be defined on the fuzzy part of the corresponding HFNs, with the credibility assessments interacting in the same way as described in the current article.

The structure of HFNs combines the quantitative evaluations received with the credibility assigned to the expert performing the evaluations. The malleability of this type of HFNs allows for their applicability within other standard MCDM techniques such as, for instance, VIKOR or PROMETHEE, as well as different research areas—other than sustainable transportation—involving strategic reports.

References

[1] M. A. Agi, S. Faramarzi-Oghani, and Ö. Hazır, "Game theory-based models in green supply chain management: A review of the literature," *Int. J. Prod. Res.*, vol. 59, no. 15, pp. 4736–4755, 2021, doi: 10.1080/00207543.2020.1770893.

[2] A. Alinezhad and J. Khalili, *New Methods and Applications in Multiple Attribute Decision Making (MADM)*, vol. 277. Cham, Switzerland: Springer, 2019, doi: 10.1007/978-3-030-15009-9.

[3] K. Aljohani and R. G. Thompson, "A multi-criteria spatial evaluation framework to optimise the siting of freight consolidation facilities in inner-city areas," *Transp. Res. Part A Policy Pract.*, vol. 138, pp. 51–69, Aug. 2020, doi: 10.1016/j.tra.2020.05.020.

[4] A. Ashraf, K. Ullah, A. Hussain, and M. Bari, "Interval-valued picture fuzzy Maclaurin symmetric mean operator with application in multiple attribute decision-making," *Rep. Mech. Eng.*, vol. 3, no. 1, pp. 301–317, 2022, doi: 10.31181/rme20020042022a.

[5] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986, doi: 10.1016/S0165-0114(86)80034-3.

[6] A. Awasthi, S. S. Chauhan, and H. Omrani, "Application of fuzzy TOPSIS in evaluating sustainable transportation systems," *Expert Syst. Appl.*, vol. 38, no. 10, pp. 12270–12280, Sep. 2011, doi: 10.1016/j.eswa.2011.04.005.

[7] A. Awasthi, H. Omrani, and P. Gerber, "Investigating ideal-solution based multicriteria decision making techniques for sustainability evaluation of urban mobility projects," *Transp. Res. Part A Policy Pract.*, vol. 116, pp. 247–259, Oct. 2018, doi: 10.1016/j.tra.2018.06.007.

[8] E. Broniewicz and K. Ogrodnik, "A comparative evaluation of multicriteria analysis methods for sustainable transport," *Energies*, vol. 14, no. 16, Aug. 2021, Art. no. 5100, doi: 10.3390/en14165100.

[9] S. H. Chen and C. H. Hsieh, "Representation, ranking, distance, and similarity of LR type fuzzy number and application," *Aust. J. Intell. Process. Syst.*, vol. 6, no. 4, pp. 217–229, 2000.

[10] A. Churkin, J. Bialek, D. Pozo, E. Sauma, and N. Korgin, "Review of cooperative game theory applications in power system expansion planning," *Renewable Sustain. Energy Rev.*, vol. 145, 2021, Art. no. 111056, doi: 10.1016/j.rser.2021.111056.

[11] A. K. Das and C. Granados, "FP-intuitionistic multi fuzzy N-soft set and its induced FP-hesitant N soft set in decision-making," *Decis. Making, Appl. Manage. Eng.*, vol. 5, no. 1, pp. 67–89, 2022, doi: 10.31181/dmame181221045d.

[12] I. Deli, "A TOPSIS method by using generalized trapezoidal hesitant fuzzy numbers and application to a robot selection problem," *J. Intell. Fuzzy Syst.*, vol. 38, no. 1, pp. 779–793, Jan. 2020, doi: 10.3233/JIFS-179448.

[13] D. Di Caprio and F. J. Santos Arteaga, "Strategic diffusion of information and preference manipulation," *Int. J. Strategic Decis. Sci.*, vol. 2, no. 2, pp. 1–19, 2011, doi: 10.4018/ijsds.2011040101.

[14] Y. Dong, Y. Liu, H. Liang, F. Chiclana, and E. Herrera-Viedma, "Strategic weight manipulation in multiple attribute decision making," *Omega*, vol. 75, pp. 154–164, Mar. 2018, doi: 10.1016/j.omega.2017.02.008.

[15] J. Fan, L. Yuanyuan, and W. Xingyuan, "An extended MABAC method for multi-criteria group decision making based on intuitionistic fuzzy rough numbers," *Expert Syst. Appl.*, vol. 127, pp. 241–255, Aug. 2019, doi: 10.1016/j.eswa.2019.03.016.

[16] H. Garg, A. Keikha, and H. M. Nehi, "Multiple-attribute decision making problems using TOPSIS and Choquet integral with hesitant fuzzy number information," *Math. Probl. Eng.*, vol. 2020, 2020, Art. no. 9874951, doi: 10.1155/2020/9874951.

[17] L. Garmendia, G. D. Campo, and J. Recasens, "Partial orderings for hesitant fuzzy sets," *Int. J. Approx. Reasoning*, vol. 84, pp. 159–167, May 2017, doi: 10.1016/j.ijar.2017.02.008.

[18] S. Hajduk, "Multi-criteria analysis in the decision-making approach for the linear ordering of urban transport based on TOPSIS technique," *Energies*, vol. 15, no. 1, 2022, Art. no. 274, doi: 10.3390/en15010274.

[19] T. H. Hsu, C. H. Chen, and W. C. Liao, "A fuzzy MCDM analytic model for building customers' brand attachment preference in car firms," *Int. J. Fuzzy Syst.*, vol. 23, no. 7, pp. 2270–2282, May 2021, doi: 10.1007/s40815-021-01100-2.

[20] W. Huang, B. Shuai, Y. Sun, Y. Wang, and E. Antwi, "Using entropy-TOPSIS method to evaluate urban rail transit system operation performance: The China case," *Transp. Res. Part A Policy Pract.*, vol. 111, pp. 292–303, May 2018, doi: 10.1016/j.tra.2018.03.025.

[21] A. Johansson, N. Berggren, and T. Nilsson, "Intolerance predicts climate skepticism," *Energy Econ.*, vol. 105, Jan. 2022, Art. no. 105719, doi: 10.1016/j.eneco.2021.105719.

[22] A. Keikha, "Archimedean t-norm and t-conorm-based aggregation operators of HFNs, with the approach of improving education," *Int. J. Fuzzy Syst.*, vol. 24, pp. 310–321, 2022, doi: 10.1007/s40815-021-01137-3.

[23] B. Kizielewicz, J. Więckowski, A. Shekhovtsov, J. Wątróbski, R. Depczyński, and W. Sałabun, "Study towards the time-based MCDA ranking analysis—A supplier selection case study," *Facta Univ. Ser.: Mech. Eng.*, vol. 19, no. 3, pp. 381–399, 2021, doi: 10.22190/FUME210130048K.

[24] D. K. Kushwaha, D. Panchal, and A. Sachdeva, “Risk analysis of cutting system under intuitionistic fuzzy environment,” *Rep. Mech. Eng.*, vol. 1, no. 1, pp. 162–173, 2020, doi: 10.31181/rme200101162k.

[25] S. Lalotra and S. Singh, “Knowledge measure of hesitant fuzzy set and its application in multi attribute decision making,” *Comput. Appl. Math.*, vol. 39, Feb. 2020, Art. no. 86, doi: 10.1007/s40314-020-1095-y.

[26] M. E. L. Lambas, N. Giuffrida, M. Ignaccolo, and G. Inturri, “Comparison between bus rapid transit and light-rail transit systems: A multi-criteria decision analysis approach,” *Urban Transp. XXIII*, vol. 176, pp. 143–154, 2017, doi: 10.2495/UT170131.

[27] H. Liao, X. Wu, A. Keikha, and A. Hafezalkotob, “Power average-based score function and extension rule of hesitant fuzzy set and the hesitant power average operators,” *J. Intell. Fuzzy Syst.*, vol. 35, no. 3, pp. 3873–3882, Oct. 2018, doi: 10.3233/JIFS-18794.

[28] X. Liu, Y. Ju, and S. Yang, “Some generalized interval valued hesitant uncertain linguistic aggregation operators and their applications to multiple attribute group decision making,” *Soft Comput.*, vol. 20, no. 2, pp. 495–510, Feb. 2016, doi: 10.1007/s00500-014-1518-z.

[29] H. Liu, L. Wang, Z. Li, and Y. Hu, “Improving risk evaluation in FMEA with cloud model and hierarchical TOPSIS method,” *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 1, pp. 84–95, Jan. 2019, doi: 10.1109/TFUZZ.2018.2861719.

[30] S. Narayanamoorthy, S. Geetha, R. Rakkiyappan, and Y. H. Joo, “Interval-valued intuitionistic hesitant fuzzy entropy based VIKOR method for industrial robots selection,” *Expert Syst. Appl.*, vol. 121, pp. 28–37, May 2019, doi: 10.1016/j.eswa.2018.12.015.

[31] M. Ortiz-Barrios, C. Miranda-De la Hoz, P. López-Meza, A. Petrillo, and F. De Felice, “A case of food supply chain management with AHP, DEMATEL, and TOPSIS,” *J. Multi-Crit. Decis. Anal.*, vol. 27, no. 1–2, pp. 104–128, 2020, doi: 10.1002/mcda.1693.

[32] D. Pamučar, M. Mihajlović, R. Obradović, and P. Atanasković, “Novel approach to group multi-criteria decision making based on interval rough numbers: Hybrid DEMATEL-ANP-MAIRCA model,” *Expert Syst. Appl.*, vol. 88, pp. 58–80, Dec. 2017, doi: 10.1016/j.eswa.2017.06.037.

[33] D. Pamučar, I. Petrović, and G. Ćirović, “Modification of the best-worst and MABAC methods: A novel approach based on interval-valued fuzzy-rough numbers,” *Expert Syst. Appl.*, vol. 91, pp. 89–106, Jan. 2018, doi: 10.1016/j.eswa.2017.08.042.

[34] K. R. Ramakrishnan and S. Chakraborty, “A cloud TOPSIS model for green supplier selection,” *Facta Univ. Ser.: Mech. Eng.*, vol. 18, no. 3, pp. 375–397, 2020, doi: 10.22190/FUME200307036R.

[35] M. Ranjbar and S. Effati, “Symmetric and right-hand-side hesitant fuzzy linear programming,” *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 2, pp. 215–227, Feb. 2020, doi: 10.1109/TFUZZ.2019.2902109.

[36] M. Ranjbar, S. M. Miri, and S. Effati, “Hesitant fuzzy numbers with (α, k)-cuts in compact intervals and applications,” *Expert Syst. Appl.*, vol. 151, Aug. 2020, Art. no. 113363, doi: 10.1016/j.eswa.2020.113363.

[37] R. Salas, M. J. Perez-Villadoniga, J. Prieto-Rodriguez, and A. Russo, “Were traffic restrictions in Madrid effective at reducing NO_2 levels?,” *Transp. Res. D: Transp. Environ.*, vol. 91, Feb. 2021, Art. no. 102689, doi: 10.1016/j.trd.2020.102689.

[38] F. Samaie, S. Javadi, H. Meyar-Naimi, and H. Feshki-Farahani, “Environmental sustainability policy on plug-in hybrid electric vehicle penetration utilizing TOPSIS and game theory,” *Clean Technol. Environ. Policy*, vol. 22, pp. 787–801, May 2020, doi: 10.1007/s10098-020-01821-2.

[39] H. Schmid-Petri and M. Bürger, “The effect of misinformation and inoculation: Replication of an experiment on the effect of false experts in the context of climate change communication,” *Public Understanding Sci.*, vol. 31, no. 2, pp. 152–167, 2022, doi: 10.1177/09636625211024550.

[40] Z. Shen, Q. Zhao, and Q. Fang, “Analysis of green traffic development in Zhoushan based on entropy weight TOPSIS,” *Sustainability*, vol. 13, no. 14, Jul. 2021, Art. no. 8109, doi: 10.3390/su13148109.

[41] G. Sobhani, N. Imtiyaz, S. Azam, and M. Hossain, “A framework for analyzing the competitiveness of unconventional modes of transportation in developing cities,” *Transp. Res. Part A Policy Pract.*, vol. 137, pp. 504–518, Jul. 2020, doi: 10.1016/j.tra.2019.02.001.

[42] J. Tarriño-Ortiz, J. A. Soria-Lara, J. Gómez, and J. M. Vassallo, “Public acceptability of low emission zones: The case of ‘Madrid’ central,” *Sustainability*, vol. 13, no. 6, Mar. 2021, Art. no. 3251, doi: 10.3390/su13063251.

[43] V. Torra, “Hesitant fuzzy sets,” *Int. J. Intell. Syst.*, vol. 25, pp. 529–539, 2010, doi: 10.1002/int.20418.

[44] V. Torra and Y. Narukawa, “On hesitant fuzzy sets and decision,” in *Proc. 18th IEEE Int. Conf. Fuzzy Syst.*, 2009, pp. 1378–1382, doi: 10.1109/FUZZY.2009.5276884.

[45] K. M. Treen, H. T. P. Williams, and S. J. O’Neill, “Online misinformation about climate change,” *WIREs Clim. Change*, vol. 11, no. 5, Sep./Oct. 2020, Art. no. e665, doi: 10.1002/wcc.665.

[46] V. G. Venkatesh, A. Zhang, E. Deakins, S. Luthra, and S. Mangla, “A fuzzy AHP-TOPSIS approach to supply partner selection in continuous aid humanitarian supply chains,” *Ann. Oper. Res.*, vol. 283, pp. 1517–1550, Dec. 2019, doi: 10.1007/s10479-018-2981-1.

[47] D. Vukasović, D. Gligović, S. Terzić, Ž. Stević, and P. Macura, “A novel fuzzy MCDM model for inventory management in order to increase business efficiency,” *Technol. Econ. Develop. Econ.*, vol. 27, no. 2, pp. 386–401, Apr. 2021, doi: 10.3846/tede.2021.14427.

[48] R. Yager, “The power average operator,” *IEEE Trans. Syst. Man Cybern.: Syst.*, vol. 31, no. 6, pp. 724–731, Nov. 2001, doi: 10.1109/3468.983429.

[49] R. Yager, “On ordered weighted averaging aggregation operators in multicriteria decision making,” *IEEE Trans. Syst. Man Cybern.: Syst.*, vol. 18, no. 1, pp. 183–190, Jan./Feb. 1988, doi: 10.1109/21.87068.

[50] L. A. Zadeh, “The concept of a linguistic variable and its application to approximate reasoning-1,” *Inf. Sci.*, vol. 8, no. 3, pp. 199–249, 1975, doi: 10.1016/0020-0255(75)90036-5.

[51] L. Zhang, X. Zhang, S. Yuan, and K. Wang, “Economic, social, and ecological impact evaluation of traffic network in Beijing-Tianjin-Hebei Urban Agglomeration based on the entropy weight TOPSIS method,” *Sustainability*, vol. 13, no. 4, Feb. 2021, Art. no. 1862, doi: 10.3390/su13041862.