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ORIGINAL ARTICLE

A fuzzy bi-objective mixed-integer programming method for solving supply chain network design problems under ambiguous and vague conditions

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Abstract Supply chain (SC) network design problems are complex problems with multi-layer levels and dynamic relationships which involve a considerable amount of uncertainty concerning customer demand, facility capacity, or lead times, among others. A large number of optimization methods (i.e., fuzzy mathematical programming, stochastic programming, and interval mathematical programming) have been proposed to cope with the uncertainties in SC network design problems. We propose a fuzzy bi-objective mixed-integer linear programming (MILP) model to enhance the material flow in dual-channel, multi-item, and multi-objective SCs with multiple echelons under both ambiguous and vague conditions, concurrently. We use a computationally efficient ranking method to resolve the ambiguity of the parameters and propose two methods for resolving the vagueness of the objective functions in the proposed fuzzy MILP model. The preferences of the decision makers (DMs) on the priority of the fuzzy goals are represented with crisp importance weights in the first method and fuzzy preference relations in the second method. The fuzzy preference relations in the second method present a unique practical application of type-II fuzzy sets. The performance of the two methods is compared using comprehensive statistical analysis. The results show the perspicuous dominance of the method which uses fuzzy preference relations (i.e., type-II fuzzy sets). We present a case study in the food industry to demonstrate the applicability of the proposed model and exhibit the efficacy of the procedures and algorithms. To the best of our knowledge, a concurrent interpretation of both ambiguous and vague uncertainties, which is applicable to many real-life problems, is novel and has not been reported in the literature.

Keywords Supply chain network · Mathematical programming · Ambiguity · Vagueness · Fuzzy sets · Fuzzy preference relations

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1 Introduction

Supply chain (SC) network design problems play an essential role in modern SC management and have attracted a great deal of the attention from the research community. A SC is a network of firms and organizations that produces value in the form of products and services [15]. Network design and network flow problems are among the most common problems in SC management. A well-structured SC can improve efficacy and efficiency in organizations and ultimately enhance competitive advantage. Most SC network design and network flow problems are undertaken to meet customer requirements and/or reduce the SC-related costs in organizations [22].

SC network design and network flow problems are complex problems with multi-layer levels and dynamic relationships among different levels [6]. They also involve a considerable amount of uncertainty concerning customer demand, facility capacity, and lead times among others [30, 57]. A large number of optimization methods (i.e., fuzzy mathematical programming, stochastic programming, and interval mathematical programming) have been proposed to cope with various uncertainties in SCs [47]. Interval programming is used when the available data are insufficient to create distribution functions or membership functions. Stochastic programming methods deal with programming with random input information. These methods model uncertainties with probability distributions constructed from historical data [56]. There are three major drawbacks in applying stochastic programming: (1) lack of sufficient historical data, (2) lack of reliable historical data, and (3) presence of large number of scenarios, which generally leads to high computationally complexities in scenario-based stochastic programming [60]. Fuzzy set theory [87] and possibility theory provide a framework for coping with vague and ambiguous uncertainties [19].

Several researchers have studied uncertain procurement planning activities in the tactical SC literature. Degraeve and Roodhooft [18] considered the *activity-based costing* (ABC) and the *total cost of ownership* (TCO) concepts and proposed a typical hierarchical structure for purchasing decision containing the supplier level activities, the order level activities, and the unit level activities. ABC [68] and TCO [18] are often used to construct a comprehensive total cost function in SCs. The ABC is used for supplier selection and leads to an objective function that minimizes the TCO associated with the purchasing decision and ultimately improves the objectivity in the selection process. The TCO is a comprehensive financial estimation approach which measures all the costs and benefits associated with the purchasing-related activities within a company's value chain [79]. Although most SC design and planning studies focus on the cost function in SCs, both time and cost factors play an important role in the real-world SC management and procurement planning activities.

This research addresses several gaps in the SC design literature. First, the distribution and manufacturing levels have attracted significantly more attention than the suppliers in the SC design literature. Second, very few studies have simultaneously considered the ambiguity of the parameters and the vagueness of the objective functions in the real-life SC design problems. Third, the just-in-time (JIT) strategies including pull systems and zero lead times have rarely been considered in the SC design problems in the literature. Fourth, the shelf-life of the items in the warehouses also has also been widely ignored in the literature. Fifth, several real-life scenarios including multi-item, multi-echelon, multi-period, and multi-objective problems have not been considered concurrently in the previous SC design research.

In this paper, we propose a new fuzzy bi-objective mixed-integer linear programming (MILP) for effectively designing and managing the flow of materials in dual-channel, multi-item, multi-echelon, multi-period SCs under vague and ambiguous uncertainties. We minimize the total inventory costs and maximize the total purchasing value in a JIT environment. We determine how much raw materials should be purchased from each supplier and how much raw materials should be transferred among various facilities. We also determine the amount of bulk inventory, compute the amount of materials delivered to manufacturer, and identify which suppliers should be employed in each period.

The fuzzy bi-objective MILP method proposed in this study is intended to resolve both ambiguous and vague uncertainties. The ambiguous uncertainties are resolved in the first phase of the procedure using a computationally efficient ranking method. The vague uncertainties in the objective functions are resolved in the second phase of the procedure using two different methods. Both methods used to resolve the vagueness in the objective functions are grounded in fuzzy goal programming. In the first goal programming method, the minimum achievement levels of the fuzzy goals and the weighted sum of the achievement levels of the fuzzy goals are maximized. In the second goal programming method, a linear combination of the weighted sum of the achievement levels of the fuzzy goals and the weighted sum of the preferences of the achievement levels of the fuzzy goals are maximized. The performance of the two methods in resolving the vagueness in the objective functions is compared using a comprehensive statistical analysis.

In summary, the contribution of the fuzzy bi-objective MILP model proposed in this study is sixfold: (1) the proposed MILP model is novel and to the best of our knowledge a concurrent interpretation of both ambiguous and vague uncertainties has not been reported in the literature; (2) the proposed method is applicable to many real-life problems with both ambiguous and vague uncertainties; (3) a computationally efficient ranking method is proposed to resolve the ambiguity of the parameters; (4) two methods are proposed to resolve the vagueness of the objective functions in the proposed fuzzy MILP model; (5) the preferences of the decision makers (DMs) on the priorities of the fuzzy goals are represented with crisp importance weights in the first method and fuzzy preference relations in the second method; and (6) the fuzzy preference relations in the second method present a unique practical application of type-II fuzzy sets.

The remainder of this paper is organized as follows. In Section 2, we present the relevant literature on SC network design methods. In Section 3, we define the problem and present the sets, indices, parameters, and decision variables used in the proposed method. In Section 4, we illustrate the hybrid method proposed in this study. In Section 5, we present a real-world case study and discuss our computational results, statistical analysis, and the comparison metrics. In Section 6, we conclude our paper and present some future research directions.

2 Literature review

In this section, we present a review of the SC network design literature followed by a review of the literature on the uncertainty in mathematical modeling and preference presentation.

2.1 SC network design

Several studies have been dedicated to the field of SC network design in recent years. Croom et al. [16] analyzed a large number of publications on SC management and classified the literature according to two criteria: content-oriented and methodology-oriented. Zanjirani Farahani and Elahipanah [88] proposed a bi-objective model for the distribution network of a three-echelon SC, with two objective functions: minimizing costs and minimizing the sum of backorders and surpluses of products in all periods. They considered delivery lead times and capacity constraints in a multi-period, multi-product, and multi-channel network. They also applied a hybrid non-dominated sorting genetic algorithm model to solve a real-size mixed-integer linear programming problem. Nikolopoulou and Ierapetritou [53] reviewed some of the relevant research on sustainable chemical processes and SC design focusing on three main areas: sustainable SCs with respect to energy efficiency and waste management, environmentally sustainable SCs, and sustainable water management. Pishvaee et al. [59] proposed a mixed-integer linear programming model to minimize the transportation and fixed opening costs in a multi-stage reverse logistics network. Since such network design problems belong to the class of NP-hard problems, they applied a simulated annealing algorithm with special neighborhood search mechanisms to find the near optimal solution. They also compared the associated numerical results through exact solutions in a set of problems to present the high-quality performance of the applied simulated annealing algorithm. Vlajic et al. [83] showed a need for an integrated framework to support the analysis and design of robust food SCs and presented such a framework. They defined the concept of robustness and classify SC disturbances, sources of food SC vulnerability, and adequate redesign principles and strategies to achieve robust SC performances. To test and illustrate the applicability of their framework, they applied their research framework to a meat SC.

In this study, we extend the current SC design classification schemes and propose a comprehensive framework with six distinct classes of problem definition, constraints, outputs, objective functions, solution approach, and industrial applications as shown in Table 1.

In Table 2, we use the classification scheme proposed in Table 1 and present a detailed description of the existing SC design and planning studies in the literature.

Table 1 SC network design (classification codes)

Classification description			Classification code
Problem definition	Planning level	Strategic	St
		Tactical	Ta
		Operational	Op
	Product	Single product	SPr
		Multi product	MPr
		Single source	SS
	SC levels	Procurement	Su
		Production	Pr
		Distribution	Dis
	Uncertainty	Deterministic	Cr
		Nondeterministic	U
	Periods	Single period	SP
		Multi-period	MP
	Raw materials	Single	SRM
		Multiple	MRM
	No. of objectives	Single objective	SOb
		Multi-objective	MOb
	Model structure	Linear	L
		Nonlinear	NL
	Logistics network	Forward	F
		Reverse-integrated	I/L
Constrains	Total demand satisfaction		DS
	Facilities capacity		FC
	Time and interval allowed for delivery		TG
	Number of facilities to be opened		UN
	Time windows		TW
	Level service		LS
Outputs	Facility location		FL
	Production amount		PQ
	Transportation amount		TQ
	Transportation mode		TM
	Allocation		DD
	Inventory		In
	Routing		Ro
Objective functions	Min cost/max profit		C/P
	Max service level		CLC
	Balance between facilities		B
	Max total value of purchasing		Ro
	Max robust		VP
Solution approach	Exact		Ex
	Heuristic		Hu
	Meta-heuristic		MHu
Industrial application	Without case study		Y-IA
	With case study		N-IA

Table 2 SC network design (definitions)

Reference	Attributes of the problem													
	Planning level	Product	SC levels	Uncertainty	Period	Raw material	No. of objectives	Model structure	Logistics network	Constraints	Outputs	Objectives	Solution approach	industrial application
Eskigun et al. [24]	St,Op	MPr,SS	Dis	Cr	SP	–	SOb	NLP	F	DS,FC	FL,TM	C/P,CLC	Hu	Y-IA
Chen and Lee [12]	Ta	MPr	Pro,Dis	U	MP	–	MOb	NLP	F	DS,FC	PQ,TQ,In	C/P,CLC,B,RO	Hu	N-IA
Syarif et al. [75]	St,Op	SPr	Sup,Pro,Dis	Cr	SP	SRM	SOb	LP	F	DS,FC,UN	PQ,TQ,FL	C/P	MHu	N-IA
Wang et al. [84]	Ta	MPr	Pro,Dis	Cr	MP	–	MOb	LP	F	FC,TG	PQ,TQ	C/P,CLC	Ex	N-IA
Jayaraman and Pirkul [35]	St,Op	MPr,SS	Sup,Pro,Dis	Cr	SP	MRM	SOb	LP	F	DS,FC,UN,LS	FL,PQ,TQ,DD	C/P	Hu	Y-IA
Amiri [1]	St	SPr	Dis	U	SP	–	SOb	LP	F	DS,FC,UN	FL,TQ	C/P	Hu	N-IA
Peidro et al. [58]	Ta	MPr	Sup,Pro,Dis	U	MP	MRM	SOb	LP	F	DS,FC	PQ,TQ,In	C/P	Ex	Y-IA
Liang [46]	Ta	MPr	Pro,Dis	U	MP	–	MOb	LP	F	DS,FC	PQ,TQ,In	C/P,CLC	Hu	Y-IA
Zanjirani Farahani and Elahipanah [88]	Ta,Op	MPr	Dis	Cr	MP	–	MOb	LP	F	FC,TG	TQ	C/P,CLC	MHu	N-IA
Bidhandi et al. [8]	St,Ta	MPr	Sup,Pro,Dis	Cr	SP	MRM	SOb	LP	F	DS,FC	FL,TQ	C/P	Ex	N-IA
Lee et al. [45]	Ta	SPr	Dis	Cr	MP	–	SOb	LP	F	DS,FC	In,TQ	C/P	Ex	N-IA
Tsiakis and Papageorgiou [81]	St,Ta,Op	MPr	Pro,Dis	Cr	SP	–	SOb	LP	F	DS,FC	FL,PQ,TQ	C/P	Ex	Y-IA
Torabi and Hassini [79]	Ta	MPr	Sup,Pro,Dis	U	MP	MRM	MOb	LP	F	DS,FC,TG	PQ,TQ,In	C/P,VP	Hu	Y-IA
Lu and Bostel [48]	St,Ta	SPr	Pro,Dis	Cr	SP	–	SOb	LP	I/L	DS	FL,PQ,TQ	C/P	Hu	N-IA
Pishvaee and Torabi [60]	St,Ta	SPr	Pro,Dis	U	MP	–	MOb	LP	I/L	DC,FC	FL,PQ,TQ	C/P,CLC	Hu	N-IA
Selim and Ozkarahan [71]	St,Ta	SPr	Dis	U	SP	–	MOb	LP	F	DC,FC	FL,TQ	C/P,CLC	Hu	N-IA
Eskigun et al. [25]	St,Ta	SPr	Dis	Cr	SP	–	SOb	NLP	F	DS,TG	FL,TQ,TM,Ro	C/P,CLC	Hu	Y-IA
Torabi and Hassini [80]	Ta	MPr	Sup,Pro,Dis	U	MP	MRM	MOb	LP	F	DS,FC,TG	PQ,TQ,In	C/P,VP,CLC	Hu	Y-IA
Bidhandi and Mohd Yusuff [7]	St,Ta	MPr	Sup,Pro,Dis	U	SP	MRM	SOb	LP	F	DS,FC	FL,DD,TQ	C/P	Hu	N-IA
Wang et al. [85]	St,Ta	MPr	Dis	Cr	MP	–	MOb	LP	F	DS,FC	FL,TQ	C/P,(*)[a]	Ex	Y-IA
ElMaraghy and Majety [23]	St,Ta	SPr,SS	Sup,Pro,Dis	Cr	MP	SRM	SOb	LP	F	DS,FC	FL,PQ,TM,TQ,In	C/P	Ex	Y-IA
Pishvaee et al. [59]	St,Ta	SPr	-	Cr	SP	–	SOb	LP	I/L	FC	FL,TQ	C/P	MHu	N-IA
Sadjady and Davoudpour [70]	St,Ta	MPr	Dis	Cr	SP	–	SOb	LP	F	DS,FC,UN	FL,TM,DD	C/P	Ex	N-IA
Paksoy et al. [55]	Ta	MPr	Pro,Dis	U	SP	–	MOb	LP	F	DC,FC	PQ	C/P	Hu	Y-IA
This study	St,Ta	MPr	Sup	U	MP	MRM	MOb	LP	F	UC,TG	TQ,TM,IN	C/P,VP,CLC	Hu	Y-IA

[a] CO_2 emission

2.2 Uncertainty in mathematical modeling

Ambiguity and vagueness are two of the most common forms of uncertainty in real-world problems. Ambiguous data are uncertain because they are subject to multiple interpretations while vague data are uncertain because they lack detail or precision. Fuzzy set and possibility theory can be used to tackle uncertainty and account for local specifications of preferences in optimization problems [19]. Fuzzy mathematical programming, often used to deal with such uncertainties in optimization problems, is classified into the following three categories [33, 34]:

- Fuzzy mathematical programming with vagueness (also known as flexible programming) is applied to decision-making problems with fuzzy goals and constraints [5, 76, 89, 90]. The fuzzy goals and constraints represent the flexibility of the target values of the objective functions and the elasticity of the constraints. In these models, fuzzy goals represent the flexibility of the target values of the objective functions, and fuzzy constraints represent the elasticity of the constraints.
- Fuzzy mathematical programming with ambiguity (also known as possibilistic programming) treats ambiguous coefficients of objective functions and constraints but does not treat fuzzy goals and constraints [20]. Dubois and Prade [21] have introduced four inequality indices between fuzzy numbers in mathematical programming problems with fuzzy coefficients based on possibility theory.
- Fuzzy mathematical programming with vagueness and ambiguity (also known as robust programming) treats ambiguous coefficients as well as vague DM's preference [52]. Various formulations have been proposed for this type of fuzzy mathematical programming because it is the most generalized one [31, 32, 50, 63, 64].

Peidro et al. [57] have classified the source of uncertainties in SCs into three groups: (1) demand, (2) process/manufacturing, and (3) supply [57]. Uncertainties have also been categorized into environmental uncertainties and system uncertainties [30]. Environmental uncertainties refer to uncertainties in supply/demand while system uncertainties refer to uncertainties in process/manufacturing.

2.3 Uncertainty in SC network design

Uncertainty is the key driver for SC redesign, and identification of the sources of uncertainty supports the selection of the relevant SC redesign strategies [82]. Singh et al. [72] proposed a capacities-based SC network design model by considering demand uncertainty and using two-stage stochastic programming. They developed a two-stage stochastic programming model for a SC network with flexible demands while considering inventory carrying costs and missed opportunity costs. PrasannaVenkatesan and Kumanan [61] proposed a hybrid optimization and simulation approach for designing the SC sourcing strategy. They introduced a multi-objective binary particle swarm algorithm for minimizing the total cost and maximizing the supplier delivery reliability. Selected scenarios from the optimization results were modeled using simulation software to evaluate the robustness of sourcing strategies under price, exchange rate, and demand risks. Singh et al. [73] proposed a model for the multi-stage global SC network problem by incorporating a set of risk factors (i.e., late shipment, exchange rates, quality problems, logistics breakdown, and production risks) along with their expected values, probabilities of occurrence, and associated costs. Different scenarios were considered to demonstrate the applicability of the model. Optimal decisions regarding the facility locations and inter-echelon quantity flows in the global SC were based on initial information for the risk factors.

Kristianto and Zhu [43] proposed a methodology for using an axiomatic approach for assembly planning by designing and integrating assembly into the SC planning process. The effect of fixture layout planning, the accuracy of demand forecast, and the supplier capabilities for providing the required material quality were studied. An optimum SC network was configured by combining the product, assembly, and SC planning. Ramezani et al. [62] proposed a robust design for a multi-product, multi-echelon, closed-loop logistic network model in an uncertain environment. Their model included a general network structure with both forward and reverse processes for various industries such as electronics, digital equipment, and vehicle manufacturing. Peidro et al. [57] presented a review of the literature and a taxonomy of the SC planning methods under uncertainty. Their main objective was to provide the reader with a starting point for modeling SC under uncertainty by applying quantitative approaches. Mirzapour Al-e-Hashem et al. [51] presented a multi-objective model for dealing with a multi-period, multi-product, multi-site aggregate production planning problem in a medium-term planning horizon under uncertainty. They used an efficient algorithm that combined an augmented ε-constraint method and a genetic algorithm to solve the proposed model.

Selim and Ozkarahan [71] proposed a SC distribution network design model for determining the optimum numbers, locations, and capacity levels for plants and warehouses used to deliver products to retailers. A maximal covering approach was used for the service level. Because of the uncertainties in the retailers' demand and the DMs' aspiration levels for the goals, a fuzzy goal programming solution approach was proposed to determine the preferred compromise solution. Fazel Zarandi et al. [28] considered backward parameters in the design of closed-loop SC distribution networks by importing reverse flows into a forward model proposed by Selim and Ozkarahan [71]. Azadeh et al. [3] used computer simulation and a genetic algorithm to select suppliers and new facilities by reducing delivery times and lowering final production costs. They utilized collective information to reduce total cycle time and cost simultaneously. A dynamic model was designed to determine the status of new facilities. A genetic algorithm was used to solve the dynamic model by considering the time and cost reduction as the main objectives in the model.

Bozorgi-Amiri et al. [9] investigated a relief chain design problem in which demands, supplies, and the cost of procurement and transportation were considered as the uncertain parameters. The proposed model considered uncertainty in the locations where demand could rise due to the destruction of facilities caused by disasters. They proposed a mixed-integer non-linear programming model to minimize the sum of the expected total cost and the variance of the total cost. The proposed model was solved by utilizing a particle swarm optimization algorithm. Khalili-Damghani and Tavana [42] proposed a network data envelopment analysis model for measuring the performance of agility in SCs. The uncertainty of the input and output data was modeled with linguistic terms parameterized with fuzzy sets.

2.4 Preference presentation

Chiclana et al. [13, 14] have classified the following four groups of methods for representing the DMs' preferences in decision-making problems: individual preference ordering, fuzzy preference relations, utility preference form, and positive preference relations. Fuzzy preference relations are effectively used to represent DMs' preferences in a wide range of optimization problems [26, 27, 37, 38, 77].

Definition 2.1: A fuzzy relation is a fuzzy set defined as a Cartesian product of crisp sets $U_1, U_2, \ldots, U_n$. More formally, a fuzzy relation R in $U_1 \times U_2 \times \ldots \times U_n$ is defined as the fuzzy set $R = \{((U_1, U_2, \ldots, U_n), \mu_R(U_1, U_2, \ldots, U_n)) | (U_1, U_2, \ldots, U_n) \in U_1 \times U_2 \times \ldots \times U_n\}$ where $\mu_R : U_1 \times U_2 \times \ldots \times U_n \rightarrow [0, 1]$

Khalili-Damghani et al. [41] and Khalili-Damghani and Sadi-Nezhad [40] applied the following notation for representing the DMs' preferences on the priority of the fuzzy goals based on fuzzy preference relations:

Suppose $X = \{x_1, \ldots, x_n\}$, $n \geq 2$ is a finite set of alternatives. The alternatives are classified from best to worst, using DM's preferences. The DMs' fuzzy preferences for a set of alternatives, X, can be represented according to the following definition:

Definition 2.2: A fuzzy preference relation on X is described through $P \subset X \times X$, with a membership function, $\mu_P : X \times X \rightarrow [0, 1]$, where $\mu_P(x_i, x_j) = p_{ij}$ denotes the preference degree of alternative x_i over x_j.

3 Problem statement and formulation

In this section, we present our problem statement and introduce the parameters used in the proposed model.

3.1 Problem statement

In this study, fuzzy sets [87] are used to represent the vague and ambiguous parameters in the proposed model. Let us consider the following assumptions:

- The network under consideration includes suppliers, intermediaries, and one manufacturer (multi-echelon).
- The total cost of SC design and the total amount of purchasing are two fuzzy objectives in the proposed model.
- The ABC and TCO concepts have been considered to calculate the cost of the SC design.
- Multiple items are produced in the SC under consideration.
- Multiple periods in a mid-term planning horizon are considered for modeling purposes.
- The shelf-life and time windows of the products are considered in the SC design problem.
- Inventory shortages and surpluses are allowed at each echelon.
- Inventory holding costs are linear and non-decreasing in the succeeding echelons.
- Lead times of each echelon are assumed equal to zero to help JIT implementation.
- Goods are shipped through a pull system throughout network.
- There are two options (channels) for shipping the item through the network: direct channels in which the items are shipped directly from suppliers to the manufacturer and indirect channels in which items are first shipped from the suppliers to the intermediaries and then shipped from the intermediaries to the manufacturer.
- There are several modes to transmit items between facilities.
- Due to the incompleteness and/or unavailability of the required data over the mid-term decision horizon, most parameters are assumed to have a considerable amount of vagueness and ambiguity in the objective functions and the constraints, respectively.

- Triangular fuzzy numbers are used to model imprecise parameters due to their simplicity and computational efficiency.
- The preferences of the DMs on the priority of the fuzzy goals are modeled using fuzzy preference relations.

3.2 Formulation

The notations used in the proposed model are presented in Table 3.

Considering the above notation, the following mathematical model is proposed:

$$\begin{aligned}\text{Min } \widetilde{Z}_1 = \widetilde{\text{TCO}} = &\left[\sum_j \widetilde{\text{slc}}_j^{11}\cdot z_j^1 + \sum_k \widetilde{\text{slc}}_k^{12}\cdot u_k^1 + \sum_t\sum_j \widetilde{\text{olc}}_j^{11}(t)\cdot m_j^1(t) + \sum_t\sum_k \widetilde{\text{olc}}_k^{12}(t)\cdot n_k^1(t)\right.\\ &+\sum_p\sum_t\sum_j\sum_l\left(\widetilde{c}_{pjl}^{11}(t) + \widetilde{\text{aulc}}_{pjl}^{11}(t)\right)\cdot x_{pjl}^1(t) + \sum_p\sum_t\sum_j\sum_l\left(\widetilde{c}_{pkl}^{12}(t) + \widetilde{\text{aulc}}_{pkl}^{12}(t)\right)\cdot y_{pkl}^1(t)\\ &\left.+\sum_t\sum_p \widetilde{h}_p^1(t)\cdot I_p^1(t)\right] + \left[\sum_k\sum_j \widetilde{\text{slc}}_{jk}^2\cdot z_{jk}^2 + \sum_k\sum_j\sum_t \widetilde{\text{olc}}_{jk}^2\cdot m_{jk}^2(t)\right.\\ &\left.+\sum_k\sum_p\sum_t\sum_j\sum_l\left(\widetilde{c}_{pjkl}^2(t) + \widetilde{\text{aulc}}_{pjkl}^2(t)\right) + (t)\cdot x_{pjkl}^2(t) + \sum_k\sum_t\sum_p \widetilde{h}_{pk}^2(t)\cdot I_{pk}^2(t)\right]\\ &+\left[\sum_p\sum_t \widetilde{\text{csur}}_p^1(t)\cdot \text{sur}_p^1(t) + \sum_p\sum_t \widetilde{\text{cshor}}_p^1(t)\cdot \text{shor}_p^1(t) + \sum_p\sum_k\sum_t \widetilde{\text{csur}}_{pk}^2(t)\cdot \text{sur}_{pk}^2(t)\right.\\ &\left.+\sum_p\sum_k\sum_t \widetilde{\text{cshor}}_{pk}^2(t)\cdot \text{shor}_{pk}^2(t)\right] \end{aligned} \tag{1}$$

$$\text{Max } \widetilde{Z}_2 = \widetilde{\text{TVP}} = \sum_s R_s^1\cdot\left(\sum_p\sum_l\sum_t x_{psl}^1(t) + \sum_p\sum_l\sum_t y_{psl}^1(t)\right) + \sum_j\sum_k R_{jk}^2\cdot\sum_p\sum_l\sum_t x_{pjkl}^2(t) \tag{2}$$

S.T.

$$\sum_l\sum_j x_{pjl}^1(t) + \sum_l\sum_k y_{pkl}^1(t) - \widetilde{d}_p^1(t) = sur_p^1(t) - shor_p^1(t), \quad \forall p,t \tag{3}$$

$$\text{sur}_p^1(t) \le v_p^1(t)\cdot w_p^1(t).\widetilde{d}_p^1(t), \quad \forall p,t \tag{4}$$

$$\text{shor}_p^1(t) \le \left(1 - v_p^1(t)\right)\cdot ww_p^1(t)\cdot\widetilde{d}_p^1(t), \quad \forall p,t \tag{5}$$

$$b_p^1 + \text{sur}_p^1(1) - \text{shor}_p^1(1) + \left(\widetilde{d}_p^1(1) - \widetilde{rn}_p^1(1)\right) = I_p^1(1), \quad \forall p \tag{6}$$

$$I_p^1(t-1) + \text{sur}_p^1(t) - \text{shor}_p^1(t) + \left(\widetilde{d}_p^1(t) - \widetilde{rn}_p^1(t)\right) = I_p^1(t) \quad \forall p, t \in T - \{|1|\} \tag{7}$$

$$\widetilde{\text{Inf}I}_p^1(t) \le I_p^1(t) \le \widetilde{\text{Max}I}_p^1(t), \quad \forall p,t \tag{8}$$

$$\sum_l\sum_j x_{pjkl}^2(t) - \widetilde{d}_{pk}^2(t) = \text{sur}_{pk}^2(t) - \text{shor}_{pk}^2(t), \quad \forall p,t,k \tag{9}$$

$$\text{sur}_{pk}^2(t) \le v_{pk}^2(t)\cdot w_{pk}^2(t)\cdot\widetilde{d}_{pk}^2(t), \quad \forall p,t,k \tag{10}$$

$$\text{shor}_{pk}^2(t) \le \left(1 - v_{pk}^2(t)\right)\cdot ww_{pk}^2(t)\cdot\widetilde{d}_{pk}^2(t), \quad \forall p,t,k \tag{11}$$

$$b_{pk}^2 + \text{sur}_{pk}^2(1) - \text{shor}_{pk}^2(1) + \left(\widetilde{d}_{pk}^2(1) - \widetilde{rn}_{pk}^2(1)\right) = I_{pk}^2(1), \quad \forall p,k \tag{12}$$

Table 3 Mathematical notations and parameters

Sets	
S	Set of suppliers and intermediaries ($S=J\cup K$)
S_1	Subset of suppliers and intermediaries ($S_1\subseteq S$)
J_1	Subset of suppliers
Indices	
j	Index of suppliers ($j=1, \ldots, J$)
k	Index of intermediaries ($k=1, \ldots, K$)
t	Index of time periods ($t=1, \ldots, T$)
p	Index of items ($p=1, \ldots, P$)
l	Index of transportation modes ($l=1, \ldots, L$)
Decision variables	
$x^1_{pjl}(t)$	Amount of item p transmitted by transportation mode l from supplier j to manufacturer at period t
$x^2_{pjkl}(t)$	Amount of item p transmitted by transportation mode l from supplier j to intermediary k at period t
$y^1_{pkl}(t)$	Amount of item p transmitted by transportation mode l from intermediary k to manufacturer at period t
$I^1_p(t)$	Inventory level of item p in manufacturer at the end of period t
$I^2_{pk}(t)$	Inventory level of item p in intermediary k at the end of period t
$sur^1_p(t)$	Surplus value of item p delivered to manufacturer at period t
$shor^1_p(t)$	Shortage value of item p in manufacturer at period t
$sur^2_{pk}(t)$	Surplus value item p delivered to intermediary k at period t
$shor^2_{pk}(t)$	Shortage value of item p in intermediary k at period t
z^1_j	$\begin{cases} 1, & \text{if manufacturer places an order with supplier } j \text{ over the planning horizon,} \\ 0, & \text{otherwise.} \end{cases}$
u^1_k	$\begin{cases} 1, & \text{if manufacturer places an order with intermediary } k \text{ over the planning horizon,} \\ 0, & \text{otherwise.} \end{cases}$
z^2_{jk}	$\begin{cases} 1, & \text{if intermediary } k \text{ places an order with supplier } j \text{ over the planning horizon,} \\ 0, & \text{otherwise.} \end{cases}$
$m^1_j(t)$	$\begin{cases} 1, & \text{if manufacturer places an order with supplier } j \text{ at period } t \\ 0, & \text{otherwise.} \end{cases}$
$n^1_k(t)$	$\begin{cases} 1, & \text{if manufacturer places an order with intermediary } k \text{ at period } t, \\ 0, & \text{otherwise.} \end{cases}$
$m^2_{jk}(t)$	$\begin{cases} 1, & \text{if intermediary } k \text{ places an order with supplier } j \text{ at period } t, \\ 0, & \text{otherwise.} \end{cases}$
Parameters	
$\widetilde{\text{slc}}^1$	Total fuzzy cost of supplier level imposed to manufacturer over the planning horizon
$\widetilde{slc}^{11}_j$	Total fuzzy cost imposed to supplier j by manufacturer over the planning horizon
$\widetilde{\text{slc}}^{12}_k$	Total fuzzy cost imposed to intermediary j by manufacturer over the planning horizon
$\widetilde{\text{olc}}^1$	Total fuzzy cost of order level imposed to manufacturer over the planning horizon
$\widetilde{\text{olc}}^{11}_j(t)$	Total fuzzy cost imposed to manufacturer by placing an order to supplier j in period t
$\widetilde{\text{olc}}^k 12(t)$	Total fuzzy cost imposed to manufacturer by placing an order to intermediary k in period t
$\widetilde{\text{ulc}}^1$	Unit level fuzzy cost of manufacturer over the planning horizon
$\tilde{c}^{11}_{pjl}(t)$	Purchasing and shipping fuzzy cost per unit of item p from supplier j to manufacturer by transportation mode l at period t
$\tilde{c}^{12}_{pkl}(t)$	Purchasing and shipping fuzzy cost per unit of item p from intermediary k to manufacturer by transportation mode l at period t
$\widetilde{\text{aulc}}^{11}_{pj}(t)$	Additional unit level fuzzy cost of item p bought from supplier j by manufacturer at period t
$\widetilde{\text{aulc}}^{12}_{pk}(t)$	Additional unit level fuzzy costs of item p bought from intermediary k by manufacturer at period t
$\tilde{h}^1_p(t)$	Holding fuzzy cost per unit of item p by manufacturer at period t
$\widetilde{\text{cshor}}^1_p(t)$	Inventory shortage fuzzy penalty per unit of item p delivered to manufacturer at period t
$\widetilde{\text{csur}}^1_p(t)$	Inventory surplus fuzzy penalty per unit of item p delivered to manufacturer at period t
$\widetilde{\text{slc}}^2_k$	Total fuzzy cost imposed to intermediary k by supplier level over the planning horizon

Table 3 (continued)

$\widetilde{\text{slc}}^2_{jk}$	Total fuzzy costs imposed to supplier *j* by intermediary *k* over the planning horizon
$\widetilde{\text{olc}}^2_{k}$	Total fuzzy order level costs of intermediary *k* over the planning horizon
$\widetilde{\text{olc}}^2_{jk}(t)$	Total fuzzy costs imposed to intermediary *k* for placing an order to supplier *j* in period *t*
$\widetilde{\text{ulc}}^2_{k}$	Total unit level fuzzy costs of intermediary *k* over the planning horizon
$\widetilde{c}^2_{pjkl}(t)$	Purchasing and shipping fuzzy cost per unit of item *p* from supplier *j* to intermediary *k* transportation mode *l* at period *t*
$\widetilde{\text{aulc}}^2_{pjk}(t)$	Additional unit level fuzzy costs of item *p* bought from supplier *j* by intermediary *k* at period *t*
$\widetilde{h}^2_{pk}(t)$	Holding fuzzy cost per unit of item *p* by intermediary *k* at period *t*
$\widetilde{\text{cshor}}^2_{pk}(t)$	Inventory shortage fuzzy penalty per unit of item *p* delivered to intermediary *k* at period *t*
$\widetilde{\text{csur}}^2_{pk}(t)$	Inventory surplus fuzzy penalty per unit of item *p* delivered to intermediary *k* at period *t*
R^1_s	Score (weight) of members of the set *S* considering qualitative performance factors evaluated by manufacturer
R^2_{jk}	Score (weight) of supplier *j* considering qualitative performance factors evaluated by intermediary *k*
b^1_p	Beginning inventory of item *p* in manufacturer
b^2_{pk}	Beginning inventory of item *p* in intermediary *k*
$d^1_p(t)$	Demand of item *p* by manufacturer at period *t*
$d^2_{pk}(t)$	Demand of item *p* by intermediary *k* at period *t*
$w^1_p(t)$	Maximum amount of allowed surplus of item *p* in manufacturer at period *t*
$w^2_{pk}(t)$	Maximum amount of allowed surplus of item *p* in intermediary *k* at period *t*
$ww^1_p(t)$	Maximum amount of allowed shortage of item *p* in manufacturer at period *t*
$ww^2_{pk}(t)$	Maximum amount of allowed shortage of item *p* in intermediary *k* at period *t*
$rn^1_p(t)$	Real amount of item *p* needed in manufacturer at period *t*
$rn^2_{pk}(t)$	Real amount of item *p* needed in intermediary *k* at period *t*
$\widetilde{\text{Inf}I}^1_p(t)$	Fuzzy lower bound of inventory of item *p* in manufacturer at period *t*
$\widetilde{\text{Inf}I}^2_{pk}(t)$	Fuzzy lower of inventory of item *p* in intermediary *k* at period *t*
$\widetilde{\text{Max}I}^1_p(t)$	Fuzzy upper bound of inventory of item *p* in manufacturer at period *t*
$\widetilde{\text{Max}I}^2_{pk}(t)$	Fuzzy upper bound for inventory of item *p* in intermediary *k* at period *t*
$\widetilde{f}^1_p$	Fuzzy fraction of total volume of item *p* purchased by manufacturer from special markets
$\widetilde{f}^2_{pk}$	Fuzzy fraction of total volume of item *p* purchased by intermediary *k* from special markets
$\widetilde{q}_{pj}$	Fuzzy average defective rate of item *p* supplied by supplier *j*
$\widetilde{q}_{pk}$	Fuzzy average defective rate of item *p* supplied by intermediary *k*
$\widetilde{\text{tq}}_p$	Fuzzy acceptable defective rate of manufacturer for incoming shipments of item *p*
$\widetilde{\text{sl}}_j$	Fuzzy average service level (the percentage of on-time deliveries) of supplier *j*
$\widetilde{\text{tsl}}$	Fuzzy acceptable service level of manufacturer per period

$$I^2_{pk}(t-1)+\text{sur}^2_{pk}(t)-\text{shor}^2_{pk}(t)+\left(\widetilde{d}^2_{pk}(t)-\widetilde{rn}^2_{pk}(t)\right)=I^2_{pk}(t),\ \forall p,k,t\in T-\{|1|\} \tag{13}$$

$$\widetilde{\text{Inf}I}^2_{pk}(t)\le I^2_{pk}(t)\le \widetilde{\text{Max}I}^2_{pk}(t),\quad \forall p,t,k \tag{14}$$

$$\sum_l y^1_{pkl}(t)\le I^2_{pk}(t),\quad \forall p,t,k \tag{15}$$

$$\sum_l\sum_{j\in s_1}x^1_{pjl}(t)+\sum_l\sum_{k\in s_1}y^1_{pkl}(t)\ge\widetilde{f}^1_p\cdot\left(\sum_l\sum_{j\in s}x^1_{pjl}(t)+\sum_l\sum_{k\in s}y^1_{pkl}(t)\right),\quad \forall p,t \tag{16}$$

$$\sum_l\sum_{j\in J_1}x^2_{pjkl}(t)\ge\widetilde{f}^2_{pk}\cdot\sum_l\sum_j x^2_{pjkl}(t),\quad \forall p,t,k \tag{17}$$

$$\sum_l x^1_{pjl}(t)\le\left(\sum_{\tau\ge t}\widetilde{d}^1_p(\tau)\right).m^1_j(t),\quad \forall p,t,j \tag{18}$$

$$\sum_{l} y^{1}_{pkl}(t) \leq \left(\sum_{\tau \geq t} \widetilde{d}^{1}_{p}(\tau) \right) . n^{1}_{k}(t), \quad \forall p, t, k \tag{19}$$

$$\sum_{l} x^{2}_{pjkl}(t) \leq \left(\sum_{\tau \geq t} \widetilde{d}^{2}_{pk}(\tau) \right) . m^{2}_{jk}(t), \quad \forall p, t, k, j \tag{20}$$

$$I^{1}_{p}(t) \leq \sum_{\tau = t}^{t+1} \widetilde{d}^{1}_{p}(\tau), \quad \forall p, t \in T - \{|T|\} \tag{21}$$

$$\sum_{l}\sum_{j} \widetilde{q}^{11}_{pj} . x^{1}_{pjl}(t) + \sum_{l}\sum_{k} \widetilde{q}^{12}_{pk} . y^{1}_{pkl}(t) \leq \widetilde{tq}_{p} . \left(\sum_{j}\sum_{l} x^{1}_{pjl}(t) + \sum_{k}\sum_{l} y^{1}_{pkl}(t) \right), \quad \forall p, t \tag{22}$$

$$\sum_{p}\sum_{j}\sum_{l} \widetilde{sl}^{11}_{j} . x^{1}_{pjl}(t) + \sum_{p}\sum_{k}\sum_{l} \widetilde{sl}^{12}_{k} . y^{1}_{pkl}(t) \geq \widetilde{tsl} . \left(\sum_{p}\sum_{j}\sum_{l} x^{1}_{pjl}(t) + \sum_{p}\sum_{k}\sum_{l} y^{1}_{pkl}(t) \right), \forall t \tag{23}$$

$$z^{1}_{j} \leq \sum_{t} m^{1}_{j}(t), \quad \forall j \tag{24}$$

$$m^{1}_{j}(t) \leq z^{1}_{j}, \quad \forall t, j \tag{25}$$

$$u^{1}_{k} \leq \sum_{t} n^{1}_{k}(t), \quad \forall k \tag{26}$$

$$n^{1}_{k}(t) \leq u^{1}_{k}, \quad \forall t, k \tag{27}$$

$$z^{2}_{jk} \leq \sum_{t} m^{2}_{jk}(t), \quad \forall k, j \tag{28}$$

$$m^{2}_{jk}(t) \leq z^{2}_{jk}, \quad \forall t, j, k \tag{29}$$

$$x^{1}_{pjl}(t), x^{2}_{pjkl}(t), y^{1}_{pkl}(t), I^{1}_{p}(t), I^{2}_{pk}(t), \mathrm{sur}^{1}_{p}(t), \mathrm{shor}^{1}_{p}(t), \mathrm{sur}^{2}_{pk}(t), \mathrm{shor}^{2}_{pk}(t) \geq 0 \tag{30}$$

$$z^{1}_{j}, z^{2}_{jk}, u^{1}_{k}, m^{1}_{j}(t), m^{2}_{jk}(t), n^{1}_{k}(t), V^{1}_{p}(t), V^{2}_{pk}(t) \in \{0, 1\} \tag{31}$$

The objective function (1) minimizes the total purchasing cost which is comprised of the supplier level costs, the order level costs, and the unit level costs as well as the backorder costs and the surplus costs in all periods. It should be noted that the penalty for the early or late delivery has also been considered in the objective function (1). Objective function (2) maximizes the total purchasing value considering qualitative criteria. Constraint (3) considers the balance of inventory and shows the backordered or excess amounts of each item delivered to the manufacturer in each period. Constraints (4) and (5) assure that, in each period, only one of the early or late delivery cases can occur for a given item delivered to the manufacturer. Constraints (4) and (5) also assure a maximum allowable backordered or excess number of items for each item delivered to the manufacturer in each period. Constraint (6) represents the inventory level for item p at the end of period 1. Constraint (7) represents the inventory level for item p at the end of period t (except for t=1). Constraint (8) imposes a minimum and maximum inventory level for each item in each period.

Constraint (9) shows the backordered or excess number of items delivered to each intermediary in each period. Constraints (10) and (11) assure that, in each period, only one of the early or late delivery cases can occur for a given item delivered to each intermediary. Constraints (10) and (11) also ensure a maximum allowable backorder or excess level for each item delivered to each intermediary in each period. Constraint

(12) represents an inventory level for each item p in each intermediary at the end of period 1. Constraint (13) represents an inventory level for each item p in each intermediary at the end of period t (except for $t=1$). Constraint (14) imposes the minimum and maximum inventory levels for each item in each intermediary in each period.

Constraint (15) assures that the total number of each item supplied by each intermediary in each period does not exceed the inventory level of that intermediary. Some business and trading protocols may force the manufacturer to purchase specific percentage of items from special suppliers. Constraint (16) imposes that at least a specified percentage of the total volume for each item p purchased by the manufacturer in each period should be bought from special suppliers or intermediaries. Constraint (17) imposes that at least a specific percentage of the total volume of item p purchased from each intermediary in each period should be bought from special suppliers.

If an order for item p is not placed with supplier j in period t, (i.e., $m_j^1(t)=0$), then constraint (18) will ensure that the number of $x_{pjl}^1(t)$ be equal to zero. If an order for item p is not placed with intermediary k in period t (i.e., $n_k^1(t)=0$), then constraint (19) ensures that the number of $y_{pkl}^1(t)$ be equal to zero. If an order for item p is not placed with supplier j to intermediary k in period t (i.e., $m_{jk}^2(t)=0$), then constraint (20) will ensure that the number of $x_{pjkl}^2(t)$ be equal to zero.

A particularly relevant concern in some industries (e.g., the food processing industry) is that some goods have limited shelf-life and thus can only be kept for a limited period of time. Hence, a constraint should be used to impose that at each time period in the planning horizon, the maximum inventory level for item p should never exceed the demands in the next two periods. Therefore, constraint (21) is considered for each item p in each period t (except for $t=T$) in order to satisfy this concern. Constraint (22), for each item p in each time period t, guarantees a minimum acceptable quality level provided by each supplier to the manufacturer. Constraint (23), which is written for each time period t, guarantees a minimum acceptable level for on time delivery (service level) provided by each supplier to the manufacturer.

Constraint (24), which is written for each supplier j, ensures that the decision variable z_j^1 will be equal to zero, if the proposed models (1)–(31) suggests no buy from supplier j over the planning horizon (i.e., $\sum_t m_j^1(t) = 0$). Constraint (25), which is written for each supplier j and each time period t, will set z_j^1 equal to 1, if during some time period an order is placed with supplier j. Constraints (26)–(29) have a similar description as constraint (25) and are written for other facilities. Finally, constraints (30) and (31) represent the types and ranges of the decision variables.

By solving the proposed model, we can determine the amount of raw materials to buy from each supplier, the amount of raw materials to transfer between the facilities, the amount of bulk inventory, the amount of materials delivered to the manufacturer, and the number of suppliers utilized in each period. It should be noted that the objective functions (1) and (2) in models (1)–(31) have a significant level of vagueness in real-world problems. Moreover, several parameters in the objective functions and the constraints also contain significant levels of ambiguity. Therefore, the solution procedure should effectively handle both the vagueness of the objective functions and the ambiguity of the parameters in the objective functions and the constraints.

4 Proposed method

A solution procedure is proposed to resolve the vagueness and ambiguity in models (1)–(31). In the first phase, the ambiguity of the parameters in models (1)–(31) is resolved, and an equivalent optimization model is proposed. This is done by converting the proposed model into an equivalent auxiliary crisp model based on the fuzzy number ranking method proposed by Jiménez et al. [36]. The vagueness of the objective functions is handled in the second phase.

Two different methods, grounded in fuzzy goal programming (GP), are proposed here to resolve the ambiguity of the parameters. GP is a mathematical programming technique capable of handling multiple objectives with a priori articulation of the preference information [49]. The preference information in GP is provided as a set of target values (aspiration levels) for the objective functions of the DMs [11]. The key idea behind GP is to minimize the unwanted deviations from the goals set by the DMs [74].

The GP models can be classified into three different categories (i.e., non-preemptive, lexicographic, and Chebyshev) based on the achievement function used for combining the unwanted deviations [66, 74]. Romero [65] presented a comprehensive review of the

GP models categorized into 18 areas of application and 12 different variants.

The classical GP considers a set of goals with precise and deterministic aspiration levels. However, in real-life problems, there are many decision-making situations where the DMs are not able to establish the aspiration levels precisely. Several fuzzy GP methods have been developed to deal with such situations [10, 69, 78, 89].

The first method, used to resolve the vagueness in models (36)–(74), is a modified version of the interactive fuzzy possibilistic programming method proposed by Torabi and Hassini [79]. The second method is a fuzzy GP method modified based on the fuzzy preference relation proposed by Khalili-Damghani et al. [41] and Khalili-Damghani and Sadi-Nezhad [40]. Both methods are based on the linear combination of the maximization of minimum achievement level of the fuzzy goals and the maximization of the weighted sum of the achievement level of the fuzzy goals, concurrently. The preferences of the DM on the priority of fuzzy goals are presented using crisp relative importance in the first method and fuzzy preference relations in the second method.

4.1 Resolving ambiguity of the parameters

Several methods have been proposed in the literature to convert the possibilistic mathematical models involving imprecise coefficients in both the objective functions and the constraints to equivalent auxiliary crisp models [2, 33, 36, 86]. Consider the following linear programming problem with fuzzy parameters in both the objective functions and the constraints:

$$\begin{array}{lll} \min & Z = \widetilde{c}x & \\ \text{s.t.} & \widetilde{a}_i x \geq \widetilde{b}_i & i = 1, \ldots, l \\ & \widetilde{a}_i x = \widetilde{b}_i & i = l+1, \ldots, m \\ & x \geq 0 & \end{array} \tag{32}$$

where $x=(x_1,x_2,\ldots,x_n)$ is the crisp decision vector. The uncertain and/or imprecise nature of the parameters used in model (32) requires a comparison between fuzzy numbers. These comparisons must be conducted under the feasibility and optimality conditions [36]. Several methods have been proposed in the literature to solve these problems [44, 67]. As we indicated earlier, the method proposed by Jiménez et al. [36] is commonly used to achieve the equivalent auxiliary crisp model for the fuzzy models (1)–(31). The method proposed by Jiménez et al. [36] has been computationally efficient for solving fuzzy linear problems since it (1) preserves the linearity of the model; (2) does not increase the number of objective functions or inequality constraints; and (3) can be applied to different membership functions [60, 79]. We apply the method proposed by Jiménez et al. [36] and transform model (32) into the crisp equivalent parametric linear programming model (33) as follows:

$$\begin{array}{ll} \min & Z = \mathrm{EV}\left(\widetilde{c}\right)x \\ \text{s.t.} & \left[(1-\alpha)\cdot E_2^{a_i} + \alpha\cdot E_1^{a_i}\right]x \geq \alpha\cdot E_2^{b_i} + (1-\alpha)\cdot E_1^{b_i} \quad , \quad i = 1, \ldots, l \\ & \left[\left(1-\frac{\alpha}{2}\right)\cdot E_2^{a_i} + \frac{\alpha}{2}\cdot E_1^{a_i}\right]x \geq \frac{\alpha}{2}\cdot E_2^{b_i} + \left(1-\frac{\alpha}{2}\right)\cdot E_1^{b_i} \quad , \quad i = l+1, \ldots, m \\ & \left[\frac{\alpha}{2}\cdot E_2^{a_i} + \left(1-\frac{\alpha}{2}\right)\cdot E_1^{a_i}\right]x \leq \left(1-\frac{\alpha}{2}\right)\cdot E_2^{b_i} + \frac{\alpha}{2}\cdot E_1^{b_i} \quad , \quad i = l+1, \ldots, m \\ & x \geq 0 \end{array} \tag{33}$$

where α is the feasibility degree of a decision x, EV is the expected value, and EI is the expected interval of triangular fuzzy number $\widetilde{c}$. EI and EV can be defined as follows [29]:

$$\mathrm{EI}\left(\widetilde{c}\right) = \left[E_1^c, E_2^c\right] = \left[\frac{1}{2}\left(c^p + c^m\right), \frac{1}{2}\left(c^m + c^o\right)\right] \tag{34}$$

$$\mathrm{EV}\left(\widetilde{c}\right) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4} \tag{35}$$

Without loss of generality, triangular fuzzy numbers are used for ambiguous parameters in this paper. Hence, the equivalent auxiliary crisp model of the original models (1)–(31) can be formulated as follows:

$$\begin{aligned}
\text{Min}\quad \widetilde{Z}_1 = \widetilde{\text{TCO}} = & \Bigg[\sum_j \frac{\text{slc}_j^{11-p} + 2\cdot\text{slc}_j^{11-m} + \text{slc}_j^{11-o}}{4}\cdot z_j^1 + \sum_k \frac{\text{slc}_k^{12-p} + 2\cdot\text{slc}_k^{12-m} + \text{slc}_k^{12-o}}{4}\cdot u_k^1 \\
& + \sum_t \sum_j \frac{\text{olc}_j^{11-p}(t) + 2\cdot\text{olc}_j^{11-m}(t) + \text{olc}_j^{11-o}(t)}{4}\cdot m_j^1(t) \\
& + \sum_t \sum_k \frac{\text{olc}_k^{12-p}(t) + 2\cdot\text{olc}_k^{12-m}(t) + \text{olc}_k^{12-o}(t)}{4}\cdot n_k^1(t) \\
& + \sum_p \sum_t \sum_j \sum_l \left(\frac{c_{pjl}^{11-p}(t) + 2\cdot c_{pjl}^{11-m}(t) + c_{pjl}^{11-o}(t)}{4} + \frac{\text{aulc}_{pjl}^{11-p}(t) + 2\cdot\text{aulc}_{pjl}^{11-m}(t) + \text{aulc}_{pjl}^{11-o}(t)}{4}\right)\cdot x_{pjl}^1(t) \\
& + \sum_p \sum_t \sum_k \sum_l \left(\frac{c_{pkl}^{12-p}(t) + 2\cdot c_{pkl}^{12-m}(t) + c_{pkl}^{12-o}(t)}{4} + \frac{\text{aulc}_{pkl}^{12-p}(t) + 2\cdot\text{aulc}_{pkl}^{12-m}(t) + \text{aulc}_{pkl}^{12-o}(t)}{4}\right)\cdot y_{pkl}^1(t) \\
& + \sum_t \sum_p \frac{h_p^{1-p}(t) + 2\cdot h_p^{1-m}(t) + h_p^{1-o}(t)}{4}\cdot I_p^1(t)\Bigg] \\
& + \Bigg[\sum_k \sum_j \frac{\text{slc}_{jk}^{2-p} + 2\cdot\text{slc}_{jk}^{2-m} + \text{slc}_{jk}^{2-o}}{4}\cdot z_{jk}^2 + \sum_k \sum_j \sum_t \text{olc}_{jk}^2(t)\frac{\text{olc}_{jk}^{2-p} + 2\cdot\text{olc}_{jk}^{2-m} + \text{olc}_{jk}^{2-o}}{4}\cdot m_{jk}^2(t) \\
& + \sum_k \sum_p \sum_t \sum_j \sum_l \left(\frac{c_{pjkl}^{2-p}(t) + 2\cdot c_{pjkl}^{2-m}(t) + c_{pjkl}^{2-o}(t)}{4} + \frac{\text{aulc}_{pjkl}^{2-p}(t) + 2\cdot\text{aulc}_{pjkl}^{2-m}(t) + \text{aulc}_{pjkl}^{2-o}(t)}{4}\right)\cdot x_{pjkl}^2(t) \\
& + \sum_k \sum_t \sum_p \frac{h_{pk}^{2-p}(t) + 2\cdot h_{pk}^{2-m}(t) + h_{pk}^{2-o}(t)}{4}\cdot I_{pk}^2(t)\Bigg] \\
& + \Bigg[\sum_p \sum_t \frac{\text{csur}_p^{1-p}(t) + 2\cdot\text{csur}_p^{1-m}(t) + \text{csur}_p^{1-o}(t)}{4}\cdot \text{sur}_p^1(t) \\
& + \sum_p \sum_t \frac{\text{cshor}_p^{1-p}(t) + 2\cdot\text{cshor}_p^{1-m}(t) + \text{cshor}_p^{1-o}(t)}{4}\cdot \text{shor}_p^1(t) \\
& + \sum_p \sum_k \sum_t \frac{\text{csur}_{pk}^{2-p}(t) + 2\cdot\text{csur}_{pk}^{2-m}(t) + \text{csur}_{pk}^{2-o}(t)}{4}\cdot \text{sur}_{pk}^2(t) \\
& + \sum_p \sum_k \sum_t \frac{\text{cshor}_{pk}^{2-p}(t) + 2\cdot\text{cshor}_{pk}^{2-m}(t) + \text{cshor}_{pk}^{2-o}(t)}{4}\cdot \text{shor}_{pk}^2(t)\Bigg]
\end{aligned} \tag{36}$$

$$\text{Max}\quad \widetilde{Z}_2 = \widetilde{\text{TVP}} = \sum_s R_s^1\cdot\left(\sum_p \sum_l \sum_t x_{psl}^1(t) + \sum_p \sum_l \sum_t y_{psl}^1(t)\right) + \sum_j \sum_k R_{jk}^2 \sum_p \sum_l \sum_t x_{pjkl}^2(t) \tag{37}$$

$$\sum_l \sum_j x_{pjl}^1(t) + \sum_l \sum_k y_{pkl}^1(t) - \left(1 - \frac{\alpha}{2}\right) \cdot \frac{d_p^{1-p}(t) + d_p^{1-m}(t)}{2} + \frac{\alpha}{2} \cdot \frac{d_p^{1-m}(t) + d_p^{1-o}(t)}{2} \leq \mathrm{sur}_p^1(t) - \mathrm{shor}_p^1(t) \quad \forall p, t \tag{38}$$

$$\sum_l \sum_j x_{pjl}^1(t) + \sum_l \sum_k y_{pkl}^1(t) - \frac{\alpha}{2} \cdot \frac{d_p^{1-p}(t) + d_p^{1-m}(t)}{2} + \left(1 - \frac{\alpha}{2}\right) \cdot \frac{d_p^{1-m}(t) + d_p^{1-o}(t)}{2} \geq \mathrm{sur}_p^1(t) - \mathrm{shor}_p^1(t) \quad \forall p, t \tag{39}$$

$$\mathrm{sur}_p^1(t) \leq v_p^1(t) \cdot w_p^1(t) \cdot \left(\alpha \cdot \frac{d_p^{1-p}(t) + d_p^{1-m}(t)}{2} + (1-\alpha) \cdot \frac{d_p^{1-m}(t) + d_p^{1-o}(t)}{2}\right), \quad \forall p, t \tag{40}$$

$$\mathrm{shor}_p^1(t) \leq \left(1 - v_p^1(t)\right) \cdot ww_p^1(t) \cdot \left(\alpha \cdot \frac{d_p^{1-p}(t) + d_p^{1-m}(t)}{2} + (1-\alpha) \cdot \frac{d_p^{1-m}(t) + d_p^{1-o}(t)}{2}\right), \forall p, t \tag{41}$$

$$b_p^1 + \mathrm{sur}_p^1(1) - \mathrm{shor}_p^1(1) + \left(\left(\left(1 - \frac{\alpha}{2}\right) \cdot \frac{d_p^{1-p}(1) + d_p^{1-m}(1)}{2} + \frac{\alpha}{2} \cdot \frac{d_p^{1-m}(1) + d_p^{1-o}(1)}{2}\right) - \left(\left(1 - \frac{\alpha}{2}\right) \cdot \frac{rn_p^{1-p}(1) + rn_p^{1-m}(1)}{2} + \frac{\alpha}{2} \cdot \frac{rn_p^{1-m}(1) + rn_p^{1-o}(1)}{2}\right)\right) \leq I_p^1(1) \quad \forall p \tag{42}$$

$$b_p^1 + \mathrm{sur}_p^1(1) - \mathrm{shor}_p^1(1) + \left(\left(\frac{\alpha}{2} \cdot \frac{d_p^{1-p}(1) + d_p^{1-m}(1)}{2} + \left(1 - \frac{\alpha}{2}\right) \cdot \frac{d_p^{1-m}(1) + d_p^{1-o}(1)}{2}\right) - \left(\frac{\alpha}{2} \cdot \frac{rn_p^{1-p}(1) + rn_p^{1-m}(1)}{2} + \left(1 - \frac{\alpha}{2}\right) \cdot \frac{rn_p^{1-m}(1) + rn_p^{1-o}(1)}{2}\right)\right) \geq I_p^1(1) \quad \forall p \tag{43}$$

$$I_p^1(t-1) + \mathrm{sur}_p^1(t) - \mathrm{shor}_p^1(t) + \left(\left(\left(1 - \frac{\alpha}{2}\right) \cdot \frac{d_p^{1-p}(t) + d_p^{1-m}(t)}{2} + \frac{\alpha}{2} \cdot \frac{d_p^{1-m}(t) + d_p^{1-o}(t)}{2}\right) - \left(\left(1 - \frac{\alpha}{2}\right) \cdot \frac{rn_p^{1-p}(t) + rn_p^{1-m}(t)}{2} + \frac{\alpha}{2} \cdot \frac{rn_p^{1-m}(t) + rn_p^{1-o}(t)}{2}\right)\right) \leq I_p^1(t) \quad \forall p, t \in T - \{|1|\} \tag{44}$$

$$I_p^1(t-1) + \text{sur}_p^1(t) - \text{shor}_p^1(t) + \left(\left(\frac{\alpha}{2}\cdot\frac{d_p^{1-p}(t)+d_p^{1-m}(t)}{2} + \left(1-\frac{\alpha}{2}\right)\cdot\frac{d_p^{1-m}(t)+d_p^{1-o}(t)}{2}\right) - \left(\frac{\alpha}{2}\cdot\frac{rn_p^{1-p}(t)+rn_p^{1-m}(t)}{2} + \left(1-\frac{\alpha}{2}\right)\cdot\frac{rn_p^{1-m}(t)+rn_p^{1-o}(t)}{2}\right)\right) \geq I_p^1(t) \quad \forall p, t\in T-\{|1|\} \tag{45}$$

$$(1-\alpha)\cdot\frac{InfI_p^{1-p}(t)+InfI_p^{1-m}(t)}{2} + \alpha\cdot\frac{InfI_p^{1-m}(t)+InfI_p^{1-o}(t)}{2} \leq I_p^1(t) \quad \forall p, t \tag{46}$$

$$I_p^1(t) \leq \alpha\cdot\frac{\text{Max}I_p^{1-p}(t)+\text{Max}I_p^{1-m}(t)}{2} + (1-\alpha)\cdot\frac{\text{Max}I_p^{1-m}(t)+\text{Max}I_p^{1-o}(t)}{2} \qquad \forall p, t \tag{47}$$

$$\sum_l\sum_j x_{pjkl}^2(t) - \left(\left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-p}(t)+d_{pk}^{2-m}(t)}{2} + \frac{\alpha}{2}\cdot\frac{d_{pk}^{2-m}(t)+d_{pk}^{2-o}(t)}{2}\right) \leq \text{sur}_{pk}^2(t) - \text{shor}_{pk}^2(t) \quad \forall p, t, k \tag{48}$$

$$\sum_l\sum_j x_{pjkl}^2(t) - \left(\frac{\alpha}{2}\cdot\frac{d_{pk}^{2-p}(t)+d_{pk}^{2-m}(t)}{2} + \left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-m}(t)+d_{pk}^{2-o}(t)}{2}\right) \geq \text{sur}_{pk}^2(t) - \text{shor}_{pk}^2(t) \quad \forall p, t, k \tag{49}$$

$$\text{sur}_{pk}^2(t) \leq v_{pk}^2(t)\cdot w_{pk}^2(t)\cdot\left(\alpha\cdot\frac{d_{pk}^{2-p}(t)+2\cdot d_{pk}^{2-m}(t)}{2} + (1-\alpha)\cdot\frac{d_{pk}^{2-m}(t)+2\cdot d_{pk}^{2-o}(t)}{2}\right) \quad \forall p, t, k \tag{50}$$

$$\text{shor}_{pk}^2(t) \leq \left(1-v_{pk}^2(t)\right)\cdot ww_{pk}^2(t)\cdot\left(\alpha\cdot\frac{d_{pk}^{2-p}(t)+2\cdot d_{pk}^{2-m}(t)}{2} + (1-\alpha)\cdot\frac{d_{pk}^{2-m}(t)+2\cdot d_{pk}^{2-o}(t)}{2}\right) \quad \forall p, t, k \tag{51}$$

$$b_{pk}^{2}+\mathrm{sur}_{pk}^{2}(1)-\mathrm{shor}_{pk}^{2}(1)+\left(\left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-p}(1)+d_{pk}^{2-m}(1)}{2}+\frac{\alpha}{2}\cdot\frac{d_{pk}^{2-m}(1)+d_{pk}^{2-o}(1)}{2}\right)-\left(\left(1-\frac{\alpha}{2}\right)\cdot\frac{rn_{pk}^{2-p}(1)+rn_{pk}^{2-m}(1)}{2}+\frac{\alpha}{2}\cdot\frac{rn_{pk}^{2-m}(1)+rn_{pk}^{2-o}(1)}{2}\right)\leq I_{pk}^{2}(1)\quad\forall p,k \tag{52}$$

$$b_{pk}^{2}+\mathrm{sur}_{pk}^{2}(1)-\mathrm{shor}_{pk}^{2}(1)+\left(\frac{\alpha}{2}\cdot\frac{d_{pk}^{2-p}(1)+d_{pk}^{2-m}(1)}{2}+\left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-m}(1)+d_{pk}^{2-o}(1)}{2}\right)-\left(\alpha\cdot\frac{rn_{pk}^{2-p}(1)+rn_{pk}^{2-m}(1)}{2}+(1-\alpha)\cdot\frac{rn_{pk}^{2-m}(1)+rn_{pk}^{2-o}(1)}{2}\right)\geq I_{pk}^{2}(1)\quad\forall p,k \tag{53}$$

$$I_{pk}^{2}(t-1)+\mathrm{sur}_{pk}^{2}(t)-\mathrm{shor}_{pk}^{2}(t)+\left(\left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-p}(t)+d_{pk}^{2-m}(t)}{2}+\frac{\alpha}{2}\cdot\frac{d_{pk}^{2-m}(t)+d_{pk}^{2-o}(t)}{2}\right)-\left(\left(1-\frac{\alpha}{2}\right)\cdot\frac{rn_{pk}^{2-p}(t)+rn_{pk}^{2-m}(t)}{2}+\frac{\alpha}{2}\cdot\frac{rn_{pk}^{2-m}(t)+rn_{pk}^{2-o}(t)}{2}\right)\leq I_{pk}^{2}(t)\quad\forall p,k,t\in T-\{|1|\} \tag{54}$$

$$I_{pk}^{2}(t-1)+\mathrm{sur}_{pk}^{2}(t)-\mathrm{shor}_{pk}^{2}(t)+\left(\frac{\alpha}{2}\cdot\frac{d_{pk}^{2-p}(t)+d_{pk}^{2-m}(t)}{2}+\left(1-\frac{\alpha}{2}\right)\cdot\frac{d_{pk}^{2-m}(t)+d_{pk}^{2-o}(t)}{2}\right)-\left(\alpha\cdot\frac{rn_{pk}^{2-p}(t)+rn_{pk}^{2-m}(t)}{2}+(1-\alpha)\cdot\frac{rn_{pk}^{2-m}(t)+rn_{pk}^{2-o}(t)}{2}\right)\geq I_{pk}^{2}(t)\quad\forall p,k,t\in T-\{|1|\} \tag{55}$$

$$(1-\alpha).\frac{Inf_{pk}^{2-p}(t)+Inf_{pk}^{2-m}(t)}{2}+\alpha.\frac{Inf_{pk}^{2-p}(t)+Inf_{pk}^{2-m}(t)}{2}\leq I_{pk}^{2}(t)\quad\forall p,t,k \tag{56}$$

$$I_{pk}^{2}(t)\leq\alpha\cdot\frac{\mathrm{Max}I_{pk}^{2-p}(t)+\mathrm{Max}I_{pk}^{2-m}(t)}{2}+(1-\alpha)\cdot\frac{\mathrm{Max}I_{pk}^{2-m}(t)+\mathrm{Max}I_{pk}^{2-o}(t)}{2}\quad\forall p,t,k \tag{57}$$

$$\sum_{l}\sum_{j\in s_1}x_{pjl}^{1}(t)+\sum_{l}\sum_{k\in s_1}y_{pkl}^{1}(t)\geq\left((1-\alpha)\cdot\frac{f_{p}^{1-p}+f_{p}^{1-m}}{2}+\alpha\cdot\frac{f_{p}^{1-m}+f_{p}^{1-o}}{2}\right)\cdot\left(\sum_{l}\sum_{j\in s}x_{pjl}^{1}(t)+\sum_{l}\sum_{k\in s}y_{pkl}^{1}(t)\right)\forall p,t \tag{58}$$

$$\sum_{l}\sum_{j\in J_1}x_{pjkl}^{2}(t)\geq\left((1-\alpha)\cdot\frac{f_{pk}^{2-p}+f_{pk}^{2-m}}{2}+\alpha\cdot\frac{f_{pk}^{2-m}+f_{pk}^{2-o}}{2}\right)\cdot\sum_{l}\sum_{j}x_{pjkl}^{2}(t)\qquad\forall p,k,t \tag{59}$$

$$\sum_{l}x_{pjl}^{1}(t)\leq\left(\sum_{\tau\geq t}\left(\alpha\cdot\frac{d_p^{1-p}(\tau)+d_p^{1-m}(\tau)}{2}+(1-\alpha)\cdot\frac{d_p^{1-m}(\tau)+d_p^{1-o}(\tau)}{2}\right)\right)\cdot\ m_j^{1}(t)\qquad\forall p,j,t \tag{60}$$

$$\sum_{l}y_{pkl}^{1}(t)\leq\left(\sum_{\tau\geq t}\left(\alpha\cdot\frac{d_p^{1-p}(\tau)+d_p^{1-m}(\tau)}{2}+(1-\alpha)\cdot\frac{d_p^{1-m}(\tau)+d_p^{1-o}(\tau)}{2}\right)\right)\cdot\ n_k^{1}(t)\qquad\forall p,k,t \tag{61}$$

$$\left(\sum_{l}x_{pjkl}^{2}(t)\leq\left(\sum_{\tau\geq t}\left(\alpha\cdot\frac{d_p^{1-p}(\tau)+d_p^{1-m}(\tau)}{2}+(1-\alpha)\cdot\frac{d_p^{1-m}(\tau)+d_p^{1-o}(\tau)}{2}\right)\right)\cdot\ m_{jk}^{2}(t)\qquad\forall p,j,k,t \tag{62}$$

$$I_p^{1}(t)\leq\sum_{\tau=t}^{t+1}\left(\alpha\cdot\frac{d_p^{1-p}(\tau)+d_p^{1-m}(\tau)}{2}+(1-\alpha)\cdot\frac{d_p^{1-m}(\tau)+d_p^{1-o}(\tau)}{2}\right)\qquad\forall p,t\in T-\{|T|\} \tag{63}$$

$$\begin{aligned}&\sum_{l}\sum_{j}\left((1-\alpha)\cdot\frac{q_{pj}^{11-p}+q_{pj}^{11-m}}{2}+\alpha\cdot\frac{q_{pj}^{11-m}+q_{pj}^{11-o}}{2}\right)\cdot x_{pjl}^{1}(t)+\sum_{l}\sum_{k}\left((1-\alpha)\cdot\frac{q_{pk}^{12-p}+q_{pk}^{12-m}}{2}+\alpha\cdot\frac{q_{pk}^{12-m}+q_{pk}^{12-o}}{2}\right)\cdot y_{pkl}^{1}(t)\\&\quad\leq\left(\alpha\cdot\frac{Tq_p^{p}+Tq_p^{m}}{2}+(1-\alpha)\cdot\frac{Tq_p^{m}+Tq_p^{o}}{2}\right)\cdot\left(\sum_{j}\sum_{l}x_{pjl}^{1}(t)+\sum_{k}\sum_{l}y_{pkl}^{1}(t)\right)\qquad\forall p,t\end{aligned} \tag{64}$$

$$\begin{aligned}&\sum_{p}\sum_{j}\sum_{l}\left(\alpha\cdot\frac{sl_j^{11-p}+sl_j^{11-m}}{2}+(1-\alpha)\cdot\frac{sl_j^{11-m}+sl_j^{11-o}}{2}\right)\cdot x_{pjl}^{1}(t)\\&\quad+\sum_{p}\sum_{k}\sum_{l}\left(\alpha\cdot\frac{sl_k^{12-p}+sl_k^{12-m}}{2}+(1-\alpha)\cdot\frac{sl_k^{12-m}+sl_k^{12-o}}{2}\right)\cdot y_{pkl}^{1}(t)\\&\quad\geq\left((1-\alpha)\cdot\frac{Tsl^{p}+Tsl^{m}}{2}+\alpha\cdot\frac{Tsl^{m}+Tsl^{o}}{2}\right)\cdot\left(\sum_{p}\sum_{j}\sum_{l}x_{pjl}^{1}(t)+\sum_{p}\sum_{k}\sum_{l}y_{pkl}^{1}(t)\right)\qquad\forall t\end{aligned} \tag{65}$$

$$\sum_{l} y^{1}_{pkl}(t) \leq I^{2}_{pk}(t), \quad \forall p, t, k \tag{66}$$

$$z^{1}_{j} \leq \sum_{t} m^{1}_{j}(t), \quad \forall j \tag{67}$$

$$m^{1}_{j}(t) \leq z^{1}_{j}, \quad \forall t, j \tag{68}$$

$$u^{1}_{k} \leq \sum_{t} n^{1}_{k}(t), \quad \forall k \tag{69}$$

$$n^{1}_{k}(t) \leq u^{1}_{k}, \quad \forall t, k \tag{70}$$

$$z^{2}_{jk} \leq \sum_{t} m^{2}_{jk}(t), \quad \forall k, j \tag{71}$$

$$m^{2}_{jk}(t) \leq z^{2}_{jk}, \quad \forall t, j, k \tag{72}$$

$$x^{1}_{pjl}(t), x^{2}_{pjkl}(t), y^{1}_{pkl}(t), I^{1}_{p}(t), I^{2}_{pk}(t), \mathrm{sur}^{1}_{p}(t), \mathrm{shor}^{1}_{p}(t), \mathrm{sur}^{2}_{pk}(t), \mathrm{shor}^{2}_{pk}(t) \geq 0 \tag{73}$$

$$z^{1}_{j}, z^{2}_{jk}, u^{1}_{k}, m^{1}_{j}(t), m^{2}_{jk}(t), n^{1}_{k}(t), V^{1}_{p}(t), V^{2}_{pk}(t) \in \{0, 1\} \tag{74}$$

The crisp constraints (15) and (24)–(31) are included here similar to constraints (36)–(74) as relations (66)–(74). Constraints (8) and (14) are defined separately as constraints (46)–(47) and (56)–(57) in models (36)–(74), respectively. The new constraints (36)–(74) are precise and have no ambiguity. Next, we resolve the vagueness of the objective functions in models (36)–(74).

4.2 Resolving vagueness in the objective functions

We propose two methods for resolving the vagueness in the objective functions.

4.2.1 First Method

Step 1 calculates the pay-off table. We first determine the α-positive ideal solution (α-PIS) and α-negative ideal solution (α-NIS) for each objective function and the α-feasibility level given the minimum acceptable possibility level (α) for the imprecise parameters in the objective functions and the constraints as follows:

We first solve models (36)–(74) for each objective function separately to obtain the α-positive ideal solutions for both objective functions (i.e., $Z_1^{\alpha-\mathrm{PIS}}$ and $Z_2^{\alpha-\mathrm{PIS}}$) and the decision variables $x_1^{\alpha-\mathrm{PIS}}$ and $x_2^{\alpha-\mathrm{PIS}}$.

We then estimate the α-negative ideal solution for each objective function as follows:

$$Z_1^{\alpha-\mathrm{NIS}} = Z_1\left(x_2^{\alpha-\mathrm{PIS}}\right) \quad , \quad Z_2^{\alpha-\mathrm{NIS}} = Z_2\left(x_1^{\alpha-\mathrm{PIS}}\right) \tag{75}$$

Step 2 determines a linear membership function for each objective function. We find a linear membership function for each objective function as follows:

$$\mu_1(x) = \begin{cases} 1 & \text{if} \quad Z_1 \leq Z_1^{\alpha-\mathrm{Pis}} \\ \dfrac{Z_1^{\alpha-\mathrm{Nis}} - Z_1}{Z_1^{\alpha\mathrm{Nis}} - Z_1^{\alpha-\mathrm{Pis}}} & \text{if} \quad Z_1^{\alpha-\mathrm{Pis}} \leq Z_1 \leq Z_1^{\alpha-\mathrm{Nis}} \\ 0 & \text{if} \quad Z_1 > Z_1^{\alpha-\mathrm{Nis}} \end{cases} \tag{76}$$

$$\mu_2(x) = \begin{cases} 1 & \text{if} \quad Z_2 \geq Z_2^{\alpha-\mathrm{Pis}} \\ \dfrac{Z_2 - Z_2^{\alpha-\mathrm{Nis}}}{Z_2^{\alpha-\mathrm{Pis}} - Z_2^{\alpha-\mathrm{Nis}}} & \text{if} \quad Z_2^{\alpha-\mathrm{Nis}} \leq Z_2 \leq Z_2^{\alpha-\mathrm{Pis}} \\ 0 & \text{if} \quad Z_2 < Z_2^{\alpha-\mathrm{Nis}} \end{cases} \tag{77}$$

where $\mu_i(x), i=1,2$ denotes the satisfaction degree of the i-th objective function.

Step 3 converts models (36)–(74) into a single-objective MILP model using the aggregation function [79] as follows:

$$\begin{aligned} &\text{Max} \quad \lambda(x) = \beta\lambda_0(x) + (1-\beta) \times \sum_{i=1}^{2} w_i \mu_{i(x)} \\ &\text{s.t.} \\ &\qquad \lambda_0 \leq \mu_{i(x)}, \quad \forall i, \\ &\qquad x \in F(x), \\ &\qquad \lambda, \lambda_0 \in [0, 1]. \end{aligned} \tag{78}$$

where $F(x)$ denotes the feasible region of the original models (36)–(74) containing all the constraints, w_i represents the relative importance of the i-th fuzzy goals, β is the coefficient of compensation, and $\lambda_0(x) = \underset{i=1,2}{\mathrm{Min}} \{\mu_i(x)\}$ indicates the minimum achievement degree of the fuzzy goal. Notably, the optimal value of the objective function $\lambda(x)$ looks for a compromise solution with a balanced trade-off between the min operator (i.e., $\lambda_0(x)$) and the weighted sum of the achievement level of the fuzzy goals (i.e., $\sum_{i=1}^{2} w_i \mu_{i(x)}$).

Step 4 solves the respective single-objective MILP model (78) given the coefficient of compensation β and relative importance of the fuzzy goals (w_i). If the DM is satisfied with the current solution, the process is terminated; otherwise, we provide another compromise solution by changing the value of β and α (and if necessary the value of w_i) and return to step 1.

4.2.2 Second Method

One of the main features of multi-objective decision-making procedures is their ability to consider the DMs' preferences on the priority of the goals. The hierarchical structure of the DMs' preferences on the priorities of the membership values of the fuzzy goals could be represented through fuzzy relations [41]. The achievement level of different fuzzy goals should follow a hierarchical structure due to the DM's preference on their priorities. We cannot assume crisp relations for plotting the aforementioned hierarchical structure while the achievement level of each goal has also been assumed to be uncertain. Therefore, the fuzzy relation between the aforementioned achievement levels is a proper approach for representing the uncertain hierarchical structure of the DM preferences on the priorities of the achievement levels of the fuzzy goals [40].

Modifying definitions 2.1 and 2.3 for models (36)–(74) results in definitions 4.3 and 4.4 as follows:

Definition 4.1 [40]: Let $G = \{\widetilde{g}_1, \ldots, \widetilde{g}_n\}$ be a finite set of fuzzy goals and $X=\{x_1,\ldots,x_m\}$ be the set of decision variables in models (36)–(74). The membership of the fuzzy goal i is $\mu_i(X) : R^n \rightarrow [0,1], \quad i = 1,..,n.$

Definition 4.2 [40]: Let $G = \{\widetilde{g}_1, \ldots, \widetilde{g}_n\}$ be a finite set of fuzzy goals, $X=\{x_1,\ldots,x_m\}$ be the set of decision variables in models (36)–(74), and $F = \{\mu_1(X), \ldots, \mu_n(X)\}$ be a finite set of membership values for the fuzzy goals in G. The DMs' preferences on F are represented with a fuzzy preference relation, $R \subset F \times F$, with membership function, $\mu_R : F \times F \rightarrow [0, 1]$ where $\mu_R(\mu_i(X), \mu_j(X))$ denotes the membership value of the preference of the membership value of the fuzzy goal i (i.e., $\mu_i(X)$) over the membership value of the fuzzy goal j (i.e., $\mu_j(X)$).

It can be concluded that R is a function of the membership value of the fuzzy goals while the membership values of the fuzzy goals are a function of the decision variables, X, themselves. The hierarchical structure for the DM's preferences on the membership values of the fuzzy goals is modeled through fuzzy relation R. We should note that we use $\mu_i(X)$ instead of μ_i for the sake of simplicity. More formally, μ_R which represents the preference of μ_i on μ_j to some extent is followed by R_i, $i=1,2,\ldots,10$, which is a function of $\mu_i-\mu_j$. We use four different linguistic terms presented in Fig. 1 to represent the DMs' preferences on the achievement level of fuzzy goals.

The details of the linguistic terms and their associated fuzzy relation membership functions are shown in Table 4 and Fig. 2, respectively.

Without loss of generality, the linear membership functions are used to represent the fuzzy preference relations. The membership function of the fuzzy relations between the fuzzy goals can be formulated as follows:

$$\mu_{\widetilde{R}_1} = \begin{cases} 0 & \text{if} \quad -1 \le \mu_i - \mu_j < 0, \\ 1 & \text{if} \quad \mu_i - \mu_j = 0, \\ 0 & \text{if} \quad 0 < \mu_i - \mu_j \le +1. \end{cases} \tag{79}$$

$$\mu_{\widetilde{R}_2} = \begin{cases} 0 & \text{if} \quad -1 \le \mu_i - \mu_j \le -0.5, \\ 2\left(\mu_i - \mu_j + 0.5\right) & \text{if} \quad -0.5 \le \mu_i - \mu_j \le 0, \\ -2\left(\mu_i - \mu_j - 0.5\right) & \text{if} \quad 0 \le \mu_i - \mu_j \le 0.5, \\ 0 & \text{if} \quad 0.5 \le \mu_i - \mu_j \le +1. \end{cases} \tag{80}$$

$$\mu_{\widetilde{R}_3} = \begin{cases} \frac{2}{3}\left(\mu_i - \mu_j + 1\right), & \text{if} \quad -1 \le \mu_i - \mu_j \le 0.5, \\ 1 & \text{if} \quad 0.5 \le \mu_i - \mu_j \le +1. \end{cases} \tag{81}$$

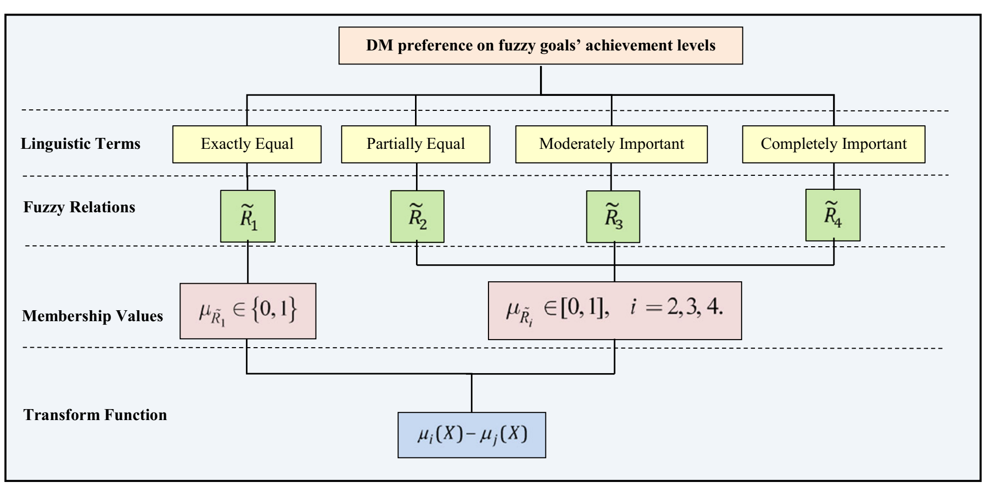


Fig. 1 Hierarchical structure of DM preference on fuzzy goals' achievement levels

Table 4 Linguistic terms and their associated fuzzy relations

Linguistic term	Fuzzy relation
Exactly equal	$\widetilde{R}_1$
Partially equal	$\widetilde{R}_2$
Moderately more important than	$\widetilde{R}_3$
Completely more important than	$\widetilde{R}_4$

$$\mu_{\widetilde{R}_4} = \begin{cases} 0, & \text{if} \quad -1 \le \mu_i - \mu_j \le -0.5, \\ \frac{2}{3}\left(\mu_i - \mu_j + 0.5\right), & \text{if} \quad -0.5 \le \mu_i - \mu_j \le +1. \end{cases} \quad (82)$$

Proposed fuzzy mathematical programming model with fuzzy preference relations Considering the extended fuzzy relation developed in (79)–(82) and the fuzzy GP model proposed by Tiwari et al. [78], the following models (83)–(98), called *goal programming with fuzzy preference relations*, are proposed for solving multi-objective models (36)–(74):

$$\text{Max} \quad \omega = \beta \times \sum_{i=1}^{2} w_i \mu_{z_i} + (1-\beta) \times \sum_{j=1}^{2} \sum_{i=1}^{2} \left(w_{ij} l_{ij} \mu_{\widetilde{R}(i,j)} \right) \quad (83)$$

s.t.

$$\mu_{z_1} \le \frac{Z_1^+ - Z_1(X)}{Z_1^+ - Z_1^-} \quad (84)$$

$$\mu_{z_2} \le \frac{Z_2(X) - Z_1^-}{Z_2^- - Z_2^+} \quad (85)$$

$$1 \ge \mu_{\widetilde{R}_1(i,j)}, \forall l_{ij} = 1, \widetilde{R}(i,j) = \widetilde{R}_1. \quad (86)$$

$$2\left(\mu_{z_1} - \mu_{z_2} + 0.5\right) + M \times y \ge \mu_{\widetilde{R}_2(i,j)}, \quad \forall l_{ij} = 1, \widetilde{R}(i,j) = \widetilde{R}_2 \quad (87)$$

$$-2\left(\mu_{z_1} - \mu_{z_2} - 0.5\right) - M \times y(1-z) \le \mu_{\widetilde{R}_2(i,j)}, \quad \forall l_{ij} = 1, \widetilde{R}(i,j) = \widetilde{R}_2 \quad (88)$$

$$2\left(\mu_{z_1} - \mu_{z_2} + 1\right) \ge \mu_{\widetilde{R}_3(i,j)}, \quad \forall l_{ij} = 1, \widetilde{R}(i,j) = \widetilde{R}_3. \quad (89)$$

$$\frac{2}{3}\left(\mu_{z_1} - \mu_{z_2} + 1\right) \ge \mu_{\widetilde{R}_4(i,j)}, \quad \forall l_{ij} = 1, \widetilde{R}(i,j) = \widetilde{R}_4. \quad (90)$$

$$0 \le \mu_{z_1} \le 1 \quad (91)$$

$$0 \le \mu_{z_2} \le 1 \quad (92)$$

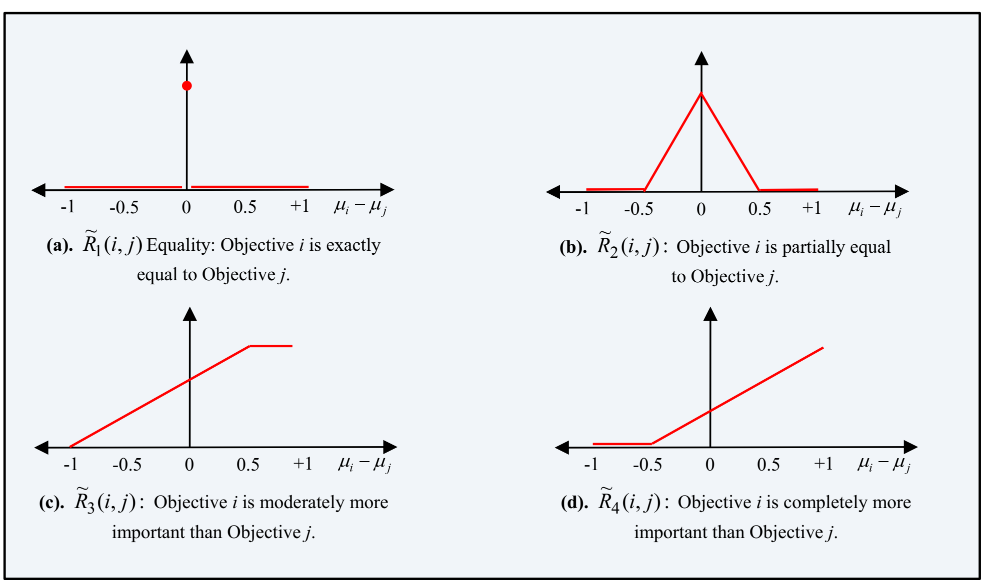


Fig. 2 **a**–**d** Linear fuzzy relation membership functions

$$0 \leq \mu_{\widetilde{R}(i,j)} \leq 1, \quad \forall l_{ij} = 1, i, j = 1, 2, i \neq j \tag{93}$$

$$l_{ij} \in \{0, 1\}, i, j = 1, 2, i \neq j \tag{94}$$

$$0 \leq \beta \leq 1 \tag{95}$$

$$\sum_{i=1}^{2} w_i = 1 \tag{96}$$

$$X \in F(x) \tag{97}$$

$$y \in \{0, 1\} \tag{98}$$

The parameters used in models (83)–(98) are defined as follows:

β, $0 \leq \beta \leq 1$	The coefficient of compensation is the parameter assigned to fine-tune the convex combination of the weighted additive achievement degrees of the fuzzy goals and the weighted sum of the DM's preferences on the membership values of the fuzzy goals
$\mu_{z_i}, i = 1, 2.$	The membership value of the i-th goal in models (36)–(74)
w_i, i=1,2.	The relative importance of the satisfaction level for the i-th fuzzy goal in models (36)–(74)
l_{ij},i,j∈{1,2},i≠j	A binary variable which is equal to 1 if an importance relation has been defined between the membership values of $\widetilde{Z}_1$ and $\widetilde{Z}_2$ and is equal to 0 otherwise
w_{ij}, $0 \leq w_{ij} \leq 1$	The relative importance of the DM's preference on the satisfaction levels of the fuzzy goals i and fuzzy goals j
$\widetilde{R}_k(i,j) = \widetilde{R}_k$, $k = 1, 2, 3, 4.$	The fuzzy relation type k defined between the satisfaction levels of the fuzzy goals of objective functions in models (36)–(74) (i.e., $\widetilde{Z}_1$ and $\widetilde{Z}_2$)
$\mu_{\widetilde{R}_k(i,j)}, k = 1, 2, 3, 4.$	The membership value of the fuzzy relation type k defined between the membership values of the fuzzy goals $\widetilde{Z}_1$ and $\widetilde{Z}_2$
Z_1^+	The maximum value of the first objective function when models (36)–(74) are solved as a single-objective problem
Z_1^-	The minimum value of the first objective function when models (36)–(74) are solved as a single-objective problem
Z_2^+	The minimum value of the second objective function when models (36)–(74) are solved as a single-objective problem
Z_2^-	The maximum value of the second objective function when models (36)–(74) are solved as a single-objective problem
y	A binary variable that enforces just one of the constraints (87)–(88) to be active simultaneously
M	A very large constant value
X	The vector of the decision variables in the models (36)–(74)
$F(x)$	The feasible solution space in the models (36)–(74)

We should note that i and j indices in models (83)–(98) refer to the first and second objective functions in models (36)–(74), respectively. In addition,

- Objective function (83) maximizes a convex combination of the weighted additive membership values of the fuzzy goals and the weighted sum of the uncertain DM's preferences on the membership values of the fuzzy goals, simultaneously.
- Inequalities (84)–(85) have been written for the first and the second fuzzy goals (i.e., $\widetilde{Z}_1$ and $\widetilde{Z}_2$), respectively.
- In constraints (86)–(90), only one of them is held at the same time. This reduces the complexity of models (83)–(98) as well as the complexity of the overall procedure, and only one judgment about the DM's preferences on the membership values of these two goals is required.
- Relations (91)–(93) guarantee that the lower and the upper bound of the membership values for the fuzzy goals and the fuzzy relations, respectively.
- Relation (94) holds the binary properties of variable l_{ij},i,j∈{1,2},i≠j.
- Relation (95) controls the eligible values of parameter β.
- Equation (96) is used to control the sum of the weights as the parameters w_1 and w_2 help the DMs fine-tune the weight of the membership values of the fuzzy goals in the first segment of the objective function.
- Relation (97) represents the constraints in the models (36)–(74).
- Relation (98) represents the binary properties of variable y.

The parametric nature of the proposed model can help the DMs generate arbitrary solutions with desirable trade-offs

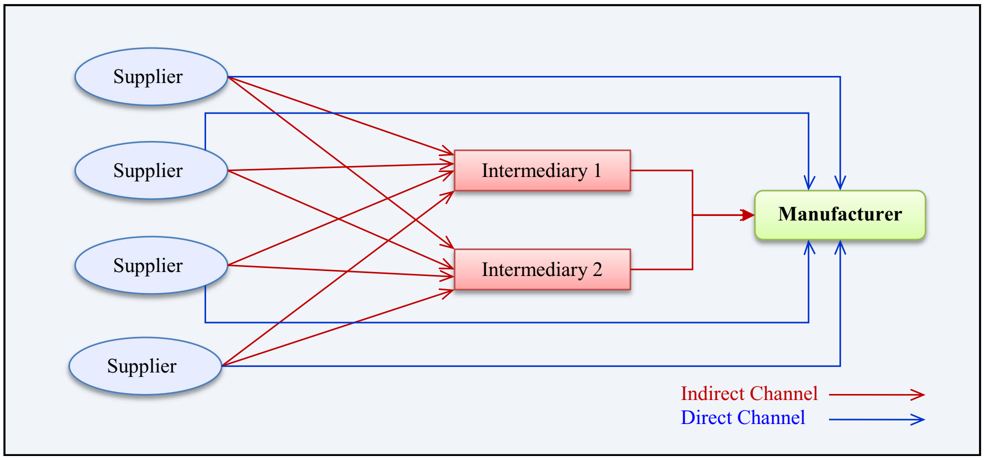


Fig. 3 Dual-channel SC network of tuna fish at Seafood Depot

between the weighted additive membership values of the fuzzy goals and the weighted sum of the DM's preferences on the membership values of the fuzzy goals. As parameter β in (83) increases towards unity, the weighted additive membership values of the fuzzy goals are weighted more, and the procedure tends to generate solutions which satisfy higher satisfaction level for the fuzzy goals. In this case, the DM's preferences on the priority of the membership values of these two fuzzy goals (i.e., $\widetilde{Z}_1$ and $\widetilde{Z}_2$) are weighted less, and consequently, the generated solutions are less relevant in terms of satisfying these preferences. Decreasing parameter β in (83) toward zero will result in an opposite scenario.

5 Case study and computational experiments

The model proposed in this study was used at Seafood Depot[1], a wholesale seafood producer in Southern Iran. The company has five vessels for catching skipjack and works with fisheries and tuna fish packers in Sri Lanka, Vietnam, and Indonesia. Tuna fish is the most popular product sold by Seafood Depot. The company offers two types of tuna fish, *Thunnus albacares* and *Katsuwonus pelamis*, for wholesale. The SC network for these two types of tuna fish is depicted in Fig. 3.

Supplier 1 is located in the southwestern city of *Chabahar*, and supplier 2 is located in the southern city of *Bandar-Abbas*. Suppliers 3 and 4 mainly import tuna fish from various Persian Gulf countries. The intermediaries are located in the northern city of *Tehran*. The primary mode of distribution in this SC network is comprised of a dual-channel with direct (from suppliers to Seafood Depot) and indirect (from suppliers to the intermediaries and then to Seafood Depot) channels of distribution.

The planning horizon considered in this study was 1 year which was divided into six 3-month periods for both methods, and the longest material acquisition lead time was 1 day. Five experienced Seafood Depot operations managers were chosen to participate in this study. Initially, they used the Delphi method and selected (w_1, w_2)=(0.7, 0.3) as the relative importance of the achievement levels of the fuzzy goals for both methods. The Delphi method was developed at the RAND Corporation to obtain the most reliable consensus of opinion from a group of knowledgeable individuals [17]. It is a structured group interaction technique that proceeds through multiple rounds of opinion collection and anonymous feedback. Okoli and Pawloski [54] and Keeney et al. [39] provide excellent reviews of Delphi method. The five operations managers completed four rounds of Delphi which involved an open-ended question about these two weights followed by anonymous feedback. After seeing the results from the previous round, the operations managers were asked to reconsider their opinions. There was a convergence of opinions after four rounds, and a stabilized group opinion emerged.

The step size for changing the coefficient of the compensation was assumed to be equal in both methods. The method proposed by Jiménez et al. [36] is used to resolve the ambiguity of models (1)–(31) by achieving the equivalent auxiliary crisp models (36)–(74). Both methods were coded in GAMS 22.9.2 optimization software. The importance weights for the goals were assumed equal for the objective functions in both methods to make a fair and comparable comparison.

5.1 Results for the first method

Model (78) was run for different β values. Table 5 presents the values of the objective functions, the achievement level of each fuzzy goal, the weighted sum of the achievement levels of the fuzzy goals, the satisfaction degree of the fuzzy preference relation of the fuzzy goals, and the run times.

Table 5 shows that the average achievement levels for the first (μ_1) and the second (μ_2) fuzzy goals are equal to 0.973 and 0.827, respectively. In addition, the average mean weighted achievement level of the fuzzy goals $\left(\sum_{i=1}^{2} w_i \mu_i\right)$

[1] The name is changed to protect the anonymity of the producer.

Table 5 First method results

β	Z_1	Z_2	μ_1	μ_2	$\sum_{i=1}^{2} w_i\mu_i$	$\sum_{j=1}^{2}\sum_{i=1}^{2}\left(w_{ij} l_{ij} \mu_{\widetilde{R}(i,j)}\right)$	Time (s)
0	3,550,367,000	2,762,532	0.996	0.847	0.9513	0.432667	21.204
0.1	3,537,888,000	2,745,149	0.996	0.821	0.9435	0.45	21.93
0.2	3,513,444,000	2,745,149	0.882	0.834	0.8676	0.365333	22.006
0.3	3,513,444,000	2,743,081	0.855	0.817	0.8436	0.358667	21.767
0.4	3,513,444,000	2,745,149	0.997	0.821	0.9442	0.450667	21.509
0.5	3,513,444,000	2,745,149	0.997	0.821	0.9442	0.450667	21.886
0.6	3,513,444,000	2,745,149	0.996	0.821	0.9435	0.45	22.091
0.7	3,520,720,000	2,745,149	0.997	0.821	0.9442	0.450667	21.611
0.8	3,519,885,000	2,750,476	0.997	0.829	0.9466	0.445333	21.604
0.9	3,523,683,000	2,752,310	0.997	0.832	0.9475	0.443333	21.868
1	3,499,187,000	2,752,236	0.998	0.831	0.9479	0.444667	22.105
Mean	3,519,904,545	2,748,321	0.973455	0.826818	0.929464	0.431091	21.7800

is equal to 0.929. Furthermore, the fuzzy relation of achievement level for the fuzzy goals is satisfied (since the average $\sum_{j=1}^{2}\sum_{i=1}^{2}\left(w_{ij} l_{ij} \mu_{\widetilde{R}(i,j)}\right)$ for all runs is 0.431). Finally, the average run time for the first method for all runs is equal to 21.7 s.

5.2 Results for the second method

The selected values for the relative importance of the achievement levels of the fuzzy goals, i.e., $(w_1, w_2)=(0.7, 0.3)$, imply that the first fuzzy goal is more important than the second fuzzy goal. Therefore, the DM preference on the achievement level of the goals is defined using $\widetilde{R}_4$ (i.e., *completely important*). This allows a fair comparison between the two methods. Then, models (83)–(98) were run for different λ values. The values of the objective functions, the achievement level of each fuzzy goal, the weighted sum of the achievement levels of the fuzzy goals, the satisfaction degree of the fuzzy preference relations for the fuzzy goals, and the run times are summarized in Table 6.

Table 6 shows the average achievement level for the first (μ_1), and the second (μ_2) fuzzy goals are equal to 0.963 and 0.828, respectively. In addition, the average mean weighted achievement level of the fuzzy goals $\left(\sum_{i=1}^{2} w_i\mu_i\right)$ is equal to 0.923. Furthermore, the fuzzy relation of achievement level for the fuzzy goals is satisfied (since the average $\sum_{j=1}^{2}\sum_{i=1}^{2}$

Table 6 Second method results

β	Z_1	Z_2	μ_1	μ_2	$\sum_{i=1}^{2} w_i\mu_i$	$\sum_{j=1}^{2}\sum_{i=1}^{2}\left(w_{ij} l_{ij} \mu_{\widetilde{R}(i,j)}\right)$	Time (s)
0	3,549,067,377	2,776,516	0.998	0.851987	0.954196	0.437342	21.204
0.1	3,537,458,017	2,736,934	0.940187	0.826281	0.906015	0.415937	21.929
0.2	3,486,364,505	2,768,971	0.895651	0.835634	0.877646	0.380011	22.006
0.3	3,512,164,859	2,720,071	0.810185	0.807955	0.809516	0.341486	21.767
0.4	3,524,094,983	2,746,052	0.998	0.818617	0.944185	0.459589	21.509
0.5	3,524,831,826	2,760,386	0.998	0.816716	0.943615	0.460856	21.887
0.6	3,497,516,955	2,750,742	0.998	0.823068	0.94552	0.456622	22.090
0.7	3,492,225,567	2,745,833	0.998	0.827951	0.946985	0.453366	21.611
0.8	3,506,085,783	2,732,008	0.998	0.841561	0.951068	0.444293	21.604
0.9	3,519,297,773	2,769,645	0.962638	0.834459	0.924184	0.425453	21.868
1	3,486,831,841	2,773,860	0.998	0.822899	0.94547	0.456734	22.105
Mean	3,512,358,135	2,752,820	0.963151	0.827921	0.922582	0.430154	21.780

$\left(w_{ij} l_{ij} \mu_{\tilde{R}(i,j)}\right)$ for all runs is 0.430). Finally, the average run time for the first method for all runs is equal to 21.8 s.

Comparing the results of Tables 5 and 6 reveals that there is no meaningful difference between the average mean weighted achievement level of fuzzy goals in the first method (0.929) and the second method (0.923). However, the achievement level of the first fuzzy goal (0.973) in the first method is slightly better than the achievement level of the first fuzzy goal (0.963) in the second method, while there is no meaningful difference between the achievement levels of the second fuzzy goal in the two methods. In addition, there is no meaningful difference between the CPU times in the two methods. Therefore, we can conclude that the second method contains all properties of the first method, while the first method does a slightly better job meeting the fuzzy relation of achievement level of the fuzzy goals. In the next subsection, we provide the analysis of variance (ANOVA) results to test this slight superiority of the first method statistically.

5.3 Statistical comparison and ANOVA results

The values of the objective functions, the achievement level of each fuzzy goal, the weighted sum of the achievement levels of the fuzzy goals, the satisfaction degree of the fuzzy preference relation, and the run times are plotted in Fig. 4.

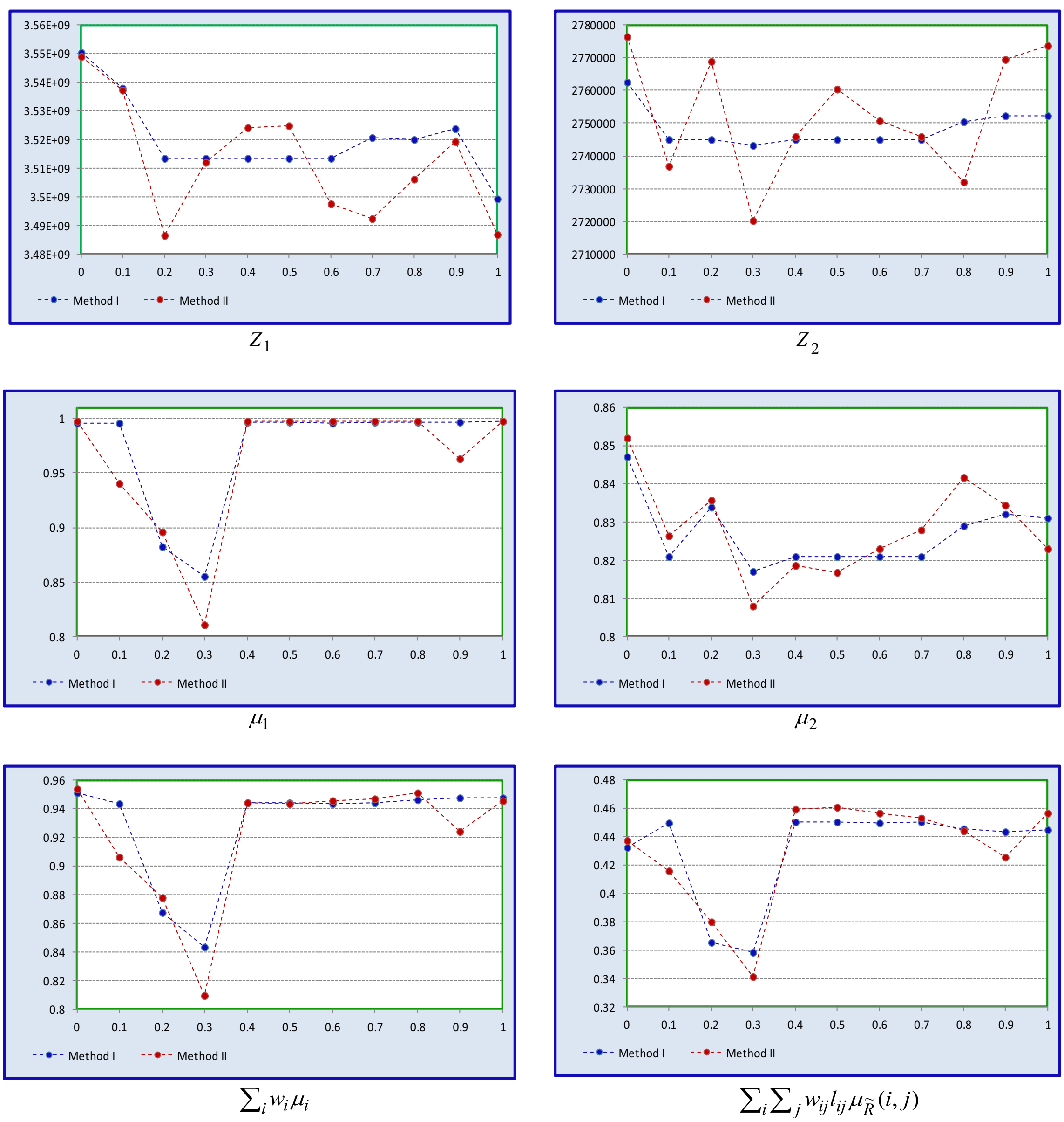
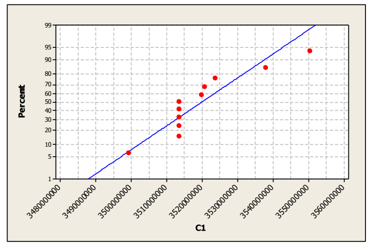
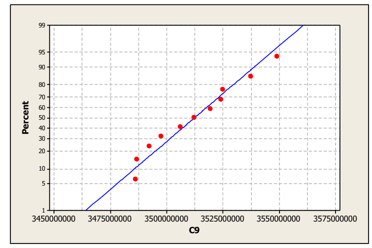


Fig. 4 Comparison metrics of methods for resolving vagueness in the objective functions

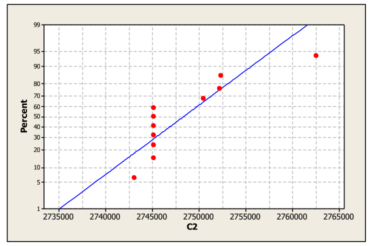
Fig. 5 Results of the Kolmogorov-Smirnov test for each metric for the two methods



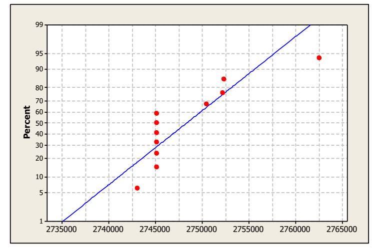
Z_1 for first method



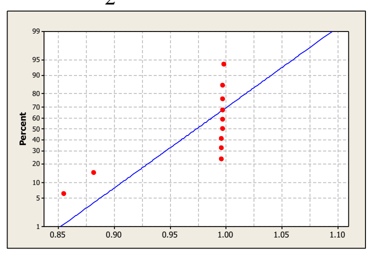
Z_1 for second method



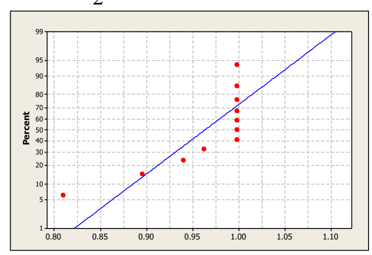
Z_2 for first method



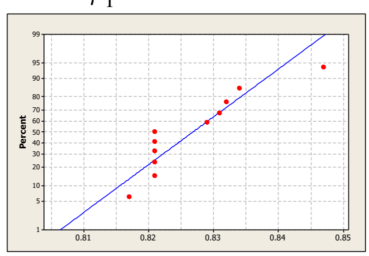
Z_2 for second method



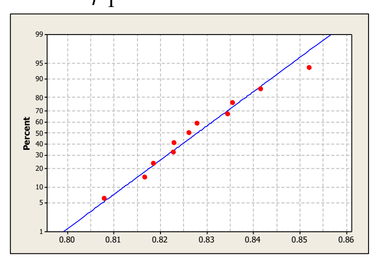
μ_1 for first method



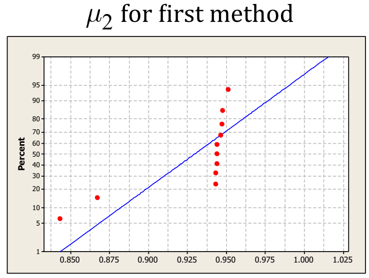
μ_1 for second method



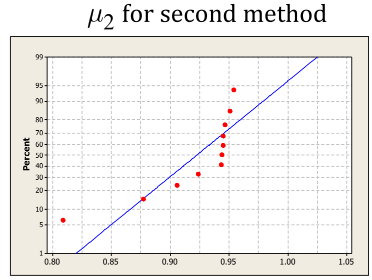
μ_2 for first method



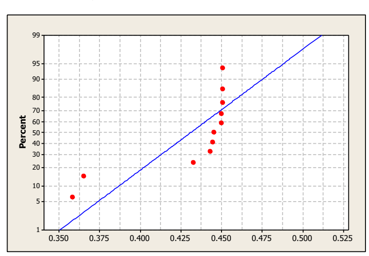
μ_2 for second method



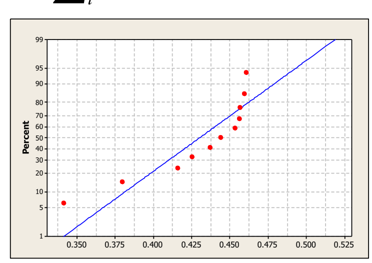
$\sum_i w_i \mu_i$ for first method



$\sum_i w_i \mu_i$ for second method



$\sum_i \sum_j w_{ij} l_{ij} \mu_R(i,j)$ for first method



$\sum_i \sum_j w_{ij} l_{ij} \mu_R(i,j)$ for second method

Although Fig. 4 represents the perspicuous dominance of the second method according to the aforementioned metrics, statistical analysis was used to investigate the meaningful difference between the two methods. The Kolmogorov-Smirnov normality test was accomplished to verify if the samples were normally distributed. Figure 5 shows the results of the Kolmogorov-Smirnov test.

As shown in Fig. 3, the samples were properly fitted with the normal distribution. Hence, the first condition of the ANOVA (there is meaningful differences between mean of performance of two methods) is met. The results of the ANOVA for the aforementioned metrics are represented in Table 7.

As is shown in Table 7, there is enough evidence to reject the hypothesis of equal means for the achievement level of the fuzzy goals, the weighted sum of the achievement levels of the fuzzy goals, and the satisfaction degree of the fuzzy preference relations for the fuzzy goals. We concluded that the achieved mean values of the aforementioned metrics in the second method are superior to the same measures in the first method for resolving the vagueness of the objective functions. Clearly, the p values are less than the significant level (i.e., 0.05) for these metrics. The results of the ANOVA test and the results provided in Table 7 and Fig. 4 indicate that the performance of the second method is significantly better than the performance of the first method for resolving the vagueness in the objective functions.

6 Conclusions and future research directions

Network design and network flow problems are among the most common problems in SC management. Although many attempts have been made to solve and optimize SC

Table 7 Analysis of variance

Source	Degree of freedom	Sum of square	Mean square	F	p value
I. First metric: Z_1					
Method II	4	3.13216E+14	3.13216E+14	1.01	0.327
Error	6	6.21552E+15	3.10776E+14		
Total	10	6.52873E+15			
S=17,628,835; R-Sq=4.80 %; R-Sq(adj)=0.04 %					
II. Second metric: Z_2					
Method II	4	111,323,208	111,323,208	0.59	0.452
Error	6	3,787,988,125	189,399,406		
Total	10	3,899,311,332			
S=13,762; R-Sq=2.85 %; R-Sq(adj)=0.00 %					
III. Third metric: μ_1					
Method II	4	0.0272919	0.0068230	14,328.23	0.000
Error	6	0.0000029	0.0000005		
Total	10	0.0272947			
S=0.0006901; R-Sq=99.99 %; R-Sq(adj)=99.98 %					
IV. Fourth metric: μ_2					
Method II	4	0.030291	0.0070146	91.43	0.000
Error	6	0.000015	0.000072		
Total	10	0.030306			
S=0.03085; R-Sq=92.22 %; R-Sq(adj.)=90.28 %					
V. Fifth metric: $\sum_i w_i \mu_i$					
Method II	1	0.4222	0.4222	9.02	0.014
Error	9	0.3793	0.0421		
Total	10	0.8015			
S=0.2053; R-Sq=52.68 %; R-Sq(adj.)=47.42 %					
VI. Sixth metric: $\sum_i \sum_j w_{ij} l_{ij} \mu \widetilde{R}(i, j)$					
Method II	1	0.000348	0.000348	6.78	0.023
Error	9	0.001871	0.000208		
Total	10	0.002219			
S=0.01442; R-Sq=15.69 %; R-Sq(adj.)=6.32 %					

network design problems, the majority of them are based on deterministic approaches [4]. However, most real-life SC design problems are characterized by numerous sources of uncertainties including the ambiguity in the parameters and the vagueness in the objective functions. We proposed a new fuzzy bi-objective MILP model to enhance the material flow in dual-channel, multi-item, and multi-objective SCs with multiple echelons under ambiguous and vague conditions. We minimized the total inventory costs and maximized the total purchasing value in a JIT environment. We determined how much raw materials should be purchased from each supplier and how much raw materials should be transferred among various facilities. We also determined the amount of bulk inventory, computed the amount of materials delivered to manufacturer, and identified which suppliers should be employed in each period.

The fuzzy bi-objective MILP method proposed in this study resolved both ambiguous and vague uncertainties. The ambiguous uncertainties were resolved in the first phase of the procedure using a computationally efficient ranking method. The vague uncertainties in the objective functions were resolved in the second phase of the procedure using two different methods. The preference of the DM on the priority of the fuzzy goals was presented with crisp values in the first method and with fuzzy preference relations in the second method. The performance of the two methods was compared. The second method performed significantly better than the first method according to a comprehensive statistical analysis. We also presented a case study in the food industry and demonstrated the applicability of the proposed model.

For future research, the model can be expanded to include distribution centers and retailers. Another valuable future research direction is to investigate the feasibility and practicality of different solution procedures based on heuristic and meta-heuristic algorithms for solving more complicated instances of the proposed model in real-life problems.

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