# Supplier selection using chance-constrained data envelopment analysis with non-discretionary factors and stochastic data

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**Abstract:** The changing economic conditions have challenged many organisations to search for more efficient and effective ways to manage their supply chain. During recent years supplier selection decisions have received considerable attention in the supply chain management literature. There are four major decisions that are related to the supplier selection process: what product or services to order, from which suppliers, in what quantities and in which time periods? Data envelopment analysis (DEA) has been successfully used to select the most efficient supplier(s) in a supply chain. In this study, we introduce a novel supplier selection model using chance-constrained DEA with non-discretionary factors and stochastic data. We propose a deterministic equivalent of the stochastic non-discretionary model and convert this deterministic problem into a quadratic programming problem. This quadratic programming problem is then solved using algorithms available for this class of problems. We perform sensitivity analysis on the proposed non-discretionary model and present a case study to demonstrate the applicability of the proposed approach and to exhibit the efficacy of the procedures and algorithms.

**Keywords:** supplier selection; SCM; supply chain management; DEA; data envelopment analysis; chance-constrained programming; non-discretionary factors; stochastic data; quadratic programming.

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## 1 Introduction

Supply chain management (SCM) encompasses the entire value chain and addresses materials and supply management from the extraction of raw materials to its end of useful life (Tan, 2001). One of the most important activities in SCM is the purchasing function (Cakravastia and Takahashi, 2004; Chou and Chang, 2008). Purchasing plays a key role in corporate strategy through the selection of suppliers that support the organisation's competitive position. Selecting the most efficient suppliers considerably reduce the purchasing expenditure and enhance corporate competitiveness, which is why numerous experts believe that supplier evaluation and selection is one of the most significant activities in a purchasing department (Haq and Kannan, 2006). Supplier selection is the procedure by which suppliers are reviewed, evaluated and selected to become associated with the company's supply chain.

Shin et al. (2000) argue that several significant factors have caused the recent shift to single sourcing or a reduced supplier centre. Firstly, multiple sourcing prevents suppliers from attaining the economies of scale based on order volume and learning curve effect. Secondly, multiple supplier system can be more costly than a decreased supplier base. For example, running a large number of suppliers for a particular item increases labour and ordering costs associated with managing multiple resource inventories. In the meantime, multiple sourcing may lower the overall quality because of the increased variation in incoming quality among the suppliers. Thirdly, a reduced supplier base can eliminate distrust among the purchasers and suppliers due to the lack of communication. Finally, global rivalry forces companies to find the best suppliers in the world.

Several mathematical programming techniques have been proposed for supplier selection in the literature. Table 1 categorises the papers reviewed in this study with respect to these techniques. Nevertheless, because of the intricacy of the decision-making process involved in supplier selection, all the aforementioned references in Table 1, except for the data envelopment analysis (DEA) model; rely heavily on some sort of procedure for determining the importance weights associated with the performance criteria. These importance weights are generally subjective and it is often difficult for the decision makers to precisely assign numbers to their preferences. This is especially intimidating for the decision makers when the number of performance criteria is increased. Furthermore, these methods do not consider stochastic data in the supplier selection process.

Discretionary models of DEA assume that all inputs and outputs are discretionary, i.e. controlled by the management of each decision-making unit (DMU) and varied at his/her discretion. Thus, failure of a DMU to produce the maximal output level with the minimal input consumption results in a decreased efficiency score. In any realistic situation, however, there may exist exogenously fixed or non-discretionary inputs or outputs that are beyond the control of a DMU's management. Ray (1988) argues that technical inefficiency is simply the result of a failure to incorporate all relevant non-discretionary variables. Instances from the DEA literature include snowfall or weather in evaluating the efficiency of maintenance units, soil characteristics and topography in different farms, number of competitors in the branches of a restaurant chain, age of facilities in different universities and number of transactions (for a purely gratis service) in library performance (Farzipoor Saen, 2005).

Chance-constrained programming (CCP) developed by Charnes and Cooper (1963) is an operations research approach for optimisation under uncertainty when some or all coefficients in a linear programme are random variables distributed in accordance with some probability law. In chance-constrained programming, the optimisation problem is concerned with identification of the value of the decision variables so that the expected loss in the criterion is minimised subject to the requirement that the probability that any given constraint is violated is not allowed to exceed some *a priori* specified level (Olesen, 2006). The stochastic input and output variations in DEA have been studied by Sengupta (1982, 1987, 1990, 1997, 1998, 2000), Land et al. (1993), Olesen and Petersen (1995), Li (1998), Morita and Seiford (1999), Sueyoshi (2000), Huang and Li (2001), Cooper et al. (2004) and Olesen (2006). Talluri et al. (2006) utilised the CCP model proposed by Land et al. (1993) for supplier selection but did not consider non-discretionary factors.

**Table 1** A summary of the supplier selection methods

| *Method* | *References* |
|---|---|
| Analytic hierarchy process | Barbarosoglu and Yazgac (1997), Muralidharan et al. (2002), Kahraman et al. (2003), Chan (2003), Çebi and Bayraktar (2003), Chan and Chan (2004), Wang et al. (2004), Liu and Hai (2005), Chan et al. (2007), Xia and Wu (2007), Hou and Su (2007) and Ng (2008) |
| Fuzzy set theory | Jain et al. (2004), Chen et al. (2006), Sarkar and Mohapatra (2006), Lopez (2007), Amid et al. (2009) and Sanayei et al. (2010) |
| Analytic network process | Sarkis and Talluri (2002), Bayazit (2006) and Gencer and Gürpinar (2007) |
| Mathematical programming | Talluri and Narasimhan (2003, 2005), Talluri (2002), Hong et al. (2005), Karpak et al. (2001), Narasimhan et al. (2006) and Wadhwa and Ravindran (2007) |
| Case-based reasoning | Choy and Lee (2002) and Choy et al. (2002, 2003a,b, 2004, 2005) |
| Data envelopment analysis | Ramanathan (2007), Farzipoor Saen (2006, 2007a,b) and Sevkli et al. (2007) |
| Simple multi-attribute rating technique | Barla (2003) and Huang and Keska (2007) |
| Scoring method and fuzzy expert system | Kwong et al. (2002) |
| Discrete choice analysis experiments | Verma and Pullman (1998) |
| Genetic algorithm | Ding et al. (2005) |
| Weighted linear models | Lamberson et al. (1976) and Timmerman (1986) |
| Vague sets | Zhang et al. (2009) |
| Scatter search | Ebrahim et al. (2009) |

We use the CCP model proposed by Cooper et al. (2004) since it not only has the advantages proposed by Land et al. (1993), but also it opens up possible new routes for ‘sensitivity analysis’. Additionally, it can be solved by a deterministic equivalent.

To the best of our knowledge, there are no references that deal with supplier selection in the presence of both non-discretionary factors and stochastic data. The objective of this paper is to propose a method that allows for supplier evaluation in the presence of both stochastic and non-discretionary factors. In summary, the approach proposed in this study has the following distinctive features:

- the stochastic data and non-discretionary factors are considered simultaneously in the model
- a stochastic model is developed and its deterministic equivalent which is a non-linear programme is derived
- it is shown that the deterministic equivalent of the stochastic non-discretionary factors model can be converted into a quadratic programme
- sensitivity analysis of the proposed model is discussed with respect to changes in the parameters

- the proposed model does not require the decision maker(s) to provide importance weights associated with the decision criteria
- the proposed model is uniquely applied to a supplier selection problem.

This paper is organised into five sections. Section 2 presents the mathematical notations used in our model. In Section 3, we present the details of the proposed method followed by a case study to demonstrate the applicability of the method and to exhibit the efficacy of the procedures and algorithms in Section 4. We close this paper with conclusions and future research directions in Section 5.

## 2 Mathematical notations

| | |
|---|---|
| $j = 1, \ldots, n$ | The collection of suppliers (DMUs) |
| $r = 1, \ldots, s$ | The set of outputs |
| $i = 1, \ldots, m$ | The set of inputs |
| $\text{DMU}_o$ | The DMU under investigation |
| $y_{rj}$ | The mean of the $r$th output of the $j$th DMU |
| $x_{ij}$ | The mean of the $i$th input of the $j$th DMU |
| $y_{ro}$ | The mean of the $r$th output of the $\text{DMU}_o$ |
| $x_{io}$ | The mean of the $i$th input of the $\text{DMU}_o$ |
| $\check{y}_{rj}$ | The $r$th output of the $j$th DMU |
| $\check{x}_{rj}$ | The $i$th input of the $j$th DMU |
| $\check{y}_{ro}$ | The $r$th output of the $\text{DMU}_o$ |
| $\check{x}_{io}$ | The $i$th input of the $\text{DMU}_o$ |
| $\sim$ | Used to identify the inputs and outputs as random variables with a known joint probability distribution |
| $\eta$ | The best possible relative efficiency achieved by $\text{DMU}_o$ |
| $\Phi^{-1}$ | The inverse of the standard normal distribution function |
| $t_r^+$ | The shortfalls of the $r$th output |
| $t_i^-$ | The excesses of the $i$th input |
| $\varepsilon$ | The non-Archimedean positive infinitesimal |
| $\sigma_r^o$ | The standard deviation of the $r$th output |
| $\sigma_i^I$ | The standard deviation of the $i$th input |
| $\alpha$ | The risk that is between zero and 1 |
| $\text{Var } y_{ro}$ | The $r$th output variance of the $\text{DMU}_o$ |
| $\text{Var } x_{io}$ | The $i$th input variance of the $\text{DMU}_o$ |

| | |
|---|---|
| $\xi, \varsigma$ and $\zeta$ | The external slacks |
| $Z$ | The standard normal random variable |
| $\mu$ | [ $\mu_j$ ] vector of DMU loadings, determining the best practice for the DMU$_o$ |
| $E$ | The expected value |

## 3 Proposed method

DEA is a widely used mathematical programming technique for comparing the inputs and outputs of a set of homogenous DMUs by evaluating their relative efficiency. The field of DEA has grown at an exponential rate since the pioneering papers of Farrell (1957) and Charnes et al. (1978). DEA is generally used to measure the relative efficiencies of a set of DMUs producing multiple outputs from multiple inputs.

In the recent years, DEA has been used to measure the efficiency of DMUs in many different settings, such as efficiency and effectiveness in operating management (Parkan, 2006), SCM (Wong et al., 2008), the farming industry (Mulwa et al., 2009), the banking industry (Azadeh et al., 2010a,b; Cooper et al., 2008), the investment banking industry (Emrouznejad and Thanassoulis, 2010; Ho, 2007), the healthcare industry (Dharmapala, 2009) and the hotel industry (Cheng et al., 2010).

Suppose that the output variables are partitioned into subsets of discretionary ($D$) and non-discretionary ($N$) variables

$$0 = \{1, 2, \ldots, s\} = O_D \cup O_N, \quad O_D \cap O_N = \Phi$$

The formulation for the output-oriented non-discretionary model with non-discretionary variables is given by Banker and Morey (1986).

$$\begin{aligned}
&\text{Max } \eta + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r \varepsilon D} t_r^+\right) \\
&\text{s.t.} \quad \check{x}_{io} = \sum_{j=1}^{n} \check{x}_{ij}\, \mu_j + t_i^-, \qquad i = 1, \ldots, m \\
&\eta\, \check{y}_{ro} = \sum_{j=1}^{n} \check{y}_{rj}\, \mu_j - t_r^+, \qquad r \in D \\
&\check{y}_{ro} = \sum_{j=1}^{n} \check{y}_{rj} \mu_j - t_r^+, \qquad r \in N \\
&t_r^+ = 0, \qquad r \in N \\
&\mu_j \geq 0, \qquad j = 1, \ldots, n \\
&t_i^- \geq 0, \qquad i = 1, \ldots, m \\
&t_r^+ \geq 0, \qquad r = 1, \ldots, s
\end{aligned} \tag{1}$$

Definition 1 (Stochastic efficiency): *$DMU_o$ is DEA stochastic efficient if and only if the following two conditions are both satisfied:*

*(i)* $\eta^* = 1$

*(ii)* $t_i^{-*} = t_r^{+*} = 0, \quad \forall i, r.$

Note that '*' refers to optimal value. Charnes et al. (1991) have shown that the performance of the DMUs can be separated into the following four classes: *E*, *E′*, *F*, and *N*; where *E* is a set of efficient DMUs which are also maximum points and *E′* is a set of efficient DMUs which are not maximum points. Both *E* and *E′* satisfy Definition 1. F is a set of points on the frontier which are not efficient because they satisfy (i) but not (ii) in Definition 1. Lastly, *N* consists of all points which are not on a frontier and are therefore inefficient.

The stochastic non-discretionary models are developed to accommodate the possible existence of stochastic variability in the data. As we know, the typical DEA models cannot accommodate stochastic variations in the input and output data, hence, DEA efficiency measurement may be sensitive to such variations. For instance, a DMU which is measured as efficient relative to the other DMUs, might turn inefficient if such random variations are considered. In what follows, a stochastic version of the output-oriented non-discretionary Model (1) is presented to allow for the possibility of stochastic alterations in the input and output data.

We suppose that all the inputs and outputs are random variables with multivariate normal distributions and known parameters.

$$\begin{aligned}
&\text{Max} \quad \eta \\
&\text{s.t.} \quad p\left\{\sum_{j=1}^{n} \tilde{x}_{ij}\mu_j \le \tilde{x}_{io}\right\} \ge 1-\alpha, \quad i = 1,\ldots,m \\
&p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j \ge \eta\tilde{y}_{ro}\right\} \ge 1-\alpha, \quad e \in D \\
&p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j \ge \tilde{y}_{ro}\right\} \ge 1-\alpha, \quad r \in N \\
&\sum_{j=1}^{n} \mu_j = 1 \\
&\mu_j \ge 0, \qquad j = 1,\ldots,n
\end{aligned} \tag{2}$$

Definition 2 (DEA efficiency): *$DMU_o$ is DEA efficient if and only if the following two conditions are both satisfied:*

*(i)* $\eta^* = 1$

*(ii) the slack values are zero for the entire optimal solutions.*

The second condition refers to all alternate optima for the reason that the second stage optimisation associated with $\varepsilon > 0$ is not utilised in Model (2). As $j = 0$ is one of the *n*

DMU$_j$, we can always get a solution with $\eta = 1$, $\mu_o = 1$ and $\mu_j = 0\ (j \neq 0)$ and all slacks zero. Nevertheless, this solution does not require to be maximal. It follows that a maximum with $\eta^* > 1$ in Model (2) for any sample of $j = 1, \ldots, n$ observations means that the DMU$_o$ under evaluation is not efficient, this is because when the probability level is specified via $\alpha$, all outputs of DMU$_o$ can be increased to $\eta^* \tilde{y}_{ro} > \tilde{y}_{ro}, r \in D$, via using a convex combination of other DMU*s* which also satisfy

$$p\left\{\sum_{j=1}^{n} \tilde{x}_{ij}\mu_j \leq \tilde{x}_{io}\right\} \geq 1-\alpha, \quad i = 1,\ldots.m \tag{3}$$

Now assume $\varsigma_r > 0$ is the 'external slack' for the *r*th output. The external slacks are those slacks that are outside the brackets. We can select the value of this external slack, which is not stochastic, so it satisfies

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j - \eta\tilde{y}_{ro} \geq 0\right\} = (1-a) + \varsigma_r \tag{4}$$

Then these must exist a positive number $t_r^+ > 0$ such that

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j - \eta\tilde{y}_{ro} \geq t_r^+\right\} = 1-\alpha \tag{5}$$

This positive value of $t_r^+$ permits a further increase in $\tilde{y}_{ro}$ for any set of sample observations devoid of diminishing any other input or output. It is easy to demonstrate that $\varsigma_r = 0$ if and only if $t_r^+ = 0$.

Also, we have

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{ri}\mu_j - \tilde{y}_{ro} \geq 0\right\} = (1-\alpha) + \zeta_r \tag{6}$$

Consequently,

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j - \tilde{y}_{ro} \geq t_r^+\right\} = 1-\alpha \tag{7}$$

In an analogous manner, presume $\xi_i > 0$ represents the external slack for the *i*th input chance-constraint. Let its value satisfy

$$p\left\{\sum_{j=1}^{n} \tilde{x}_{ij}\mu_j - \tilde{x}_{io} \leq 0\right\} = (1-\alpha) + \xi_i \tag{8}$$

Then there must exist a positive number $t_i^- > 0$ such that

$$p\left\{\sum_{j=1}^{n} \tilde{x}_{ij}\mu_j + t_i^- \leq \tilde{x}_{io}\right\} = 1-\alpha \tag{9}$$

Such a positive value of $t_i^-$ permits a decrease in $\tilde{x}_{io}$ for any sample without diminishing any other input or output with respect to the indicated probabilities. It is easy to demonstrate that $\xi = 0$ if and only if $t_i^- = 0$.

Consider the non-Archimedean infinitesimal, $\varepsilon > 0$, so that stochastic efficiencies and inefficiencies can be characterised via the following model, Relations (3)–(9), can replace Relation (3),

$$\text{Max} \quad \eta + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r\in D} t_r^+\right)$$

s.t.

$$p\left\{\sum_{j=1}^{n} \tilde{x}_{ij}\mu_j + t_i^- \le \tilde{x}_{io}\right\} = 1-\alpha, \quad i = 1,\ldots,m$$

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j - \eta\tilde{y}_{ro} \ge t_r^+\right\} = 1-\alpha, \quad r \in D$$

$$p\left\{\sum_{j=1}^{n} \tilde{y}_{rj}\mu_j - \tilde{y}_{ro} \ge t_r^+\right\} = 1-\alpha, \quad r \in N$$

$$\sum_{j=1}^{n} \mu_j = 1$$

$$t_r^+ = 0, \quad r \in N$$

$$\mu_j \ge 0, \quad j = 1,\ldots,n$$

$$t_i^- \ge 0, \quad i = 1,\ldots,m$$

$$t_r^+ \ge 0, \quad r = 1,\ldots,s \tag{10}$$

This leads to the following definition.

Definition 3: *$DMU_o$ is stochastic efficient if and only if the following two conditions are both satisfied:*

*(i)* $\eta^* = 1$

*(ii)* $t_r^{+*} = t_i^{-*} = 0 \quad \forall i, r.$

This definition aligns more closely with Definition 1 since $\varepsilon > 0$ in the objective function of Model (10) makes it unnecessary to refer to 'all optimal solutions', as in Definition 2. Although it has the same form as Definition 1, it differs in that probabilistic components are considered. For example, as determined by the selection of $\alpha$, there is a possibility that $DMU_o$ will not be efficient even when the assumptions of Definition 3 are satisfied.

Following Cooper et al. (1996), we suppose that the inputs and outputs are random variables with a multivariate normal distribution and known parameters. We also limit our concentration to the class of zero-order decision rules. The selection of multivariate normal distributions and zero-order rules is less restrictive than might appear on the surface. Conversions are obtainable for bringing other kinds of distributions into approximately normal form as it was done in Charnes et al. (1958) when they found it necessary to treat extremely skewed distributions with log-normal estimates (see Charnes and Cooper, 1961; Charnes et al., 1958).

For the second constraint in Model (10), we have

$$p\left\{\frac{\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}-E\left(\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}\right)}{\sqrt{\operatorname{var}\left\{\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}\right\}}}\right.$$
$$\left.\leq\frac{t_r^{+}-E\left(\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}\right)}{\sqrt{\operatorname{var}\left\{\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}\right\}}}\right\}=\alpha$$

We should note that the conversion process has been discussed for constraint 2 in Model (10) and the same process could be repeated for constraints 1 and 3.

For the sake of simplicity we indicate $\sqrt{\operatorname{var}\{\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}\}}$ by $\sigma_r^o(\eta_o,\mu)$. Hence,

$$p\left\{\frac{\sum_{j=1}^{n}\tilde{y}_{rj}\mu_j-\eta_o\tilde{y}_{ro}-\sum_{j=1}^{n}y_{rj}\mu_j+\eta_o y_{ro}}{\sigma_r^o(\eta_o,\mu)}\leq\frac{t_r^{+}-\sum_{j=1}^{n}y_{rj}\mu_j+\eta_o y_{ro}}{\sigma_r^o(\eta_o,\mu)}\right\}=\alpha$$

In other words

$$p\left\{Z\leq\frac{t_r^{+}-\sum_{j=1}^{n}y_{rj}\mu_j+\eta_o y_{ro}}{\sigma_r^o(\eta_o,\mu)}\right\}=\alpha$$

where $Z$ is a normal standard variable, and we have,

$$\Phi\left\{\frac{t_r^{+}-\sum_{j=1}^{n}y_{rj}\mu_j+\eta_o y_{ro}}{\sigma_r^o(\eta_o,\mu)}\right\}=\alpha$$

or

$$\eta_o y_{ro}-\sum_{j=1}^{n}y_{rj}\mu_j+t_r^{+}-\Phi^{-1}(\alpha)\sigma_r^o(\eta_o,\mu)=0$$

The deterministic equivalents for Model (10) are

$$
\begin{aligned}
&\text{Max} \quad \eta + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r\in D} t_r^+\right) \\
&\text{s.t.} \ \sum_{j=1}^{n} x_{ij}\mu_j + t_i^- - \Phi^{-1}(\alpha)\sigma_i^I(\mu) = x_{io}, \quad i = 1,\ldots,m \\
&\eta y_{ro} - \sum_{j=1}^{n} y_{rj}\mu_j + t_r^+ - \Phi^{-1}(\alpha)\delta_r^o(\eta,\mu) = 0, \quad r \in D \\
&\sum_{j=1}^{n} y_{rj}\mu_j - t_r^+ - \Phi^{-1}(\alpha)\delta_r^o(\mu) - y_{ro} = 0, \quad r \in N \\
&t_r^+ = 0, \quad r \in N \\
&\mu_j \ge 0, \quad j = 1,\ldots,n \\
&t_i^- \ge 0, \quad i = 1,\ldots,m \\
&t_r^+ \ge 0, \quad r = 1,\ldots,s
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\sigma_i^I(\mu)^2 &= \text{Var}\left\{\sum_{j=1}^{n} \mu_j \tilde{x}_{ij} - \tilde{x}_{io}\right\} = \text{Var}\left\{\sum_{j=1}^{n} \mu_j x_{ij} + (\mu_o - 1)x_{io}\right\} \\
&= \text{Var}\left(\sum_{j=1}^{n} \mu_j x_{ij}\right) + \text{Var}\left((\mu_o - 1)x_{io}\right) + 2\text{Cov}\left(\sum_{j=1}^{n} \mu_j x_{ij}, (\mu_o - 1)x_{io}\right)
\end{aligned}
$$

Therefore,

$$
\sigma_i^I(\mu)^2 = \sum_{j\ne 0}\sum_{k\ne 0} \mu_i\mu_k \text{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{ik}\right) + 2(\mu_o - 1)\sum_{j\ne 0} \mu_j \text{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{ij}\right) + (\mu_o - 1)^2 \text{Var}\left(\tilde{x}_{io}\right),
$$

and

$$
\begin{aligned}
\left(\sigma_i^o(\mu)\right)^2 &= \sum_{j\ne 0}\sum_{k\ne 0} \mu_i\mu_k \text{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{ik}\right) + 2(\mu_o - 1)\sum_{j\ne 0} \mu_j \text{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{io}\right) \\
&\quad + (\mu_o - \eta)^2 \text{Var}\left(\tilde{x}_{io}\right) \\
\left(\sigma_r^D(\eta,\mu)\right)^2 &= \sum_{i\ne 0}\sum_{j\ne 0} \mu_i\mu_j \text{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{rj}\right) + 2(\mu_o - \eta)\sum_{i\ne 0} \mu_j \text{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{ro}\right) \\
&\quad + (\mu_o - 1)^2 \text{Var}\left(\tilde{y}_{ro}\right) \\
\left(\sigma_r^N(\mu)\right)^2 &= \sum_{i\ne 0}\sum_{j\ne 0} \mu_i\mu_k \text{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{rj}\right) + 2(\mu_o - \eta)\sum_{j\ne 0} \mu_j \text{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{ro}\right) \\
&\quad + (\mu_o - \eta)^2 \text{Var}\left(\tilde{x}_{io}\right)
\end{aligned}
$$

It is obvious, from the forms of $\sigma_i^o(\lambda)$, $\sigma_i^D(\lambda)$ and $\sigma_r^N(\lambda)$ that Model (23) is a non-linear programme. We demonstrate that this non-linear programme can be transformed to a quadratic programme. Assume that $w_i^I, w_i^D$ and $w_r^N$ are non-negative variables. Replacing $w_i^I, w_i^D$ and $w_r^N$, respectively, by $\sigma_j^I(\lambda), \sigma_r^D(\lambda)$ *and* $\sigma_r^N(\lambda)$ and adding the following quadratic equality constraints

$$\left(w_i^I\right)^2 = \left(\sigma_i^I(\mu)\right)^2, \left(w_r^D\right)^2 = \left(\sigma_r^D(\mu)\right)^2, (w_r^N)^2 = \left(\sigma_r^N(\mu)\right)^2$$

Hence, Model (23) is transformed to a quadratic programming problem.

$$\left(w_i^I\right)^2 = \sum_{j\neq 0}\sum_{k\neq 0}\mu_i\mu_k \operatorname{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{ik}\right) - 2\sum_{j\neq 0}\mu_j \operatorname{Cov}\left(\tilde{x}_{ij}, \tilde{x}_{io}\right) + \operatorname{Var}\left(\tilde{x}_{io}\right)$$

$$\left(w_r^D\right)^2 = \sum_{i\neq 0}\sum_{j\neq 0}\mu_i\mu_k \operatorname{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{rj}\right) - 2\eta\sum_{i\neq 0}\mu_j \operatorname{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{ro}\right) + \operatorname{Var}(\tilde{y}_{ro})$$

$$\left(w_r^N\right)^2 = \sum_{i\neq 0}\sum_{j\neq 0}\mu_i\mu_j \operatorname{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{rj}\right) - 2\sum_{i\neq 0}\mu_j \operatorname{Cov}\left(\tilde{y}_{ri}, \tilde{y}_{ro}\right) + \operatorname{Var}\left(\tilde{y}_{ro}\right)$$

his problem, which is free of chance elements, is the preferred 'deterministic equivalent' for Model (10) – a term which is justified for the reason that an optimal selection of the variables in Model (11) will also be optimal for Model (10) and, *vice versa*, an optimal solution of Model (10) will also be optimal for Model (11).

Although void of chance elements, Model (11) is a non-linear programming problem – because of the functional forms of $\delta_r^o(\eta, \mu)$ and $\sigma_i^I(\mu)$. Utilising techniques developed in Cooper et al. (1996), it can be converted into a quadratic programming problem and solved using algorithms available for this class of problems.

We now notice that if the prearranged value of $\alpha$ is equivalent to $0.5$, the stochastic inefficiencies and efficiencies can then be attained from Model (1) if this deterministic non-discretionary model is based on mean values of inputs and outputs. A significant result in this case is that the stochastic inefficiencies and efficiencies recognised for each DMU$_o$ are similar as those obtained from this deterministic model formed from merely the input and output means.

### 3.1 The relationship between the stochastic and non-discretionary models

Now suppose that all the inputs and outputs are statistically independent. Letting $\sigma_{ij}^I$ represents the standard deviation of $\tilde{x}_{ij}$ and letting $\sigma_{rj}^o$ represents the standard deviation of $\tilde{y}_{rj}$, we have

$$\sigma_r^o(\mu) = \left[\sum_{j\neq 0}\mu_j^2\left(\sigma_{rj}^o\right)^2 + (\eta - \mu_o)^2\left(\sigma_{ro}^o\right)^2\right]^{1/2} \leq \sum_{j\neq 0}\mu_j\sigma_{ri}^o + (\eta - \mu_o)\sigma_{ro}^o \qquad (12)$$

Since for an optimal solution, $\eta^* \geq 1$ and $\mu_o^* \leq 1,$ we also have $\eta - \mu_o \geq 0,$ thus we likewise obtain

$$\sigma_i^I(\mu) = \left[\sum_{j \neq 0} \mu_j^2 \left(\sigma_{ij}^I\right)^2 + \left(1-\mu_0\right)^2 \left(\sigma_{io}^I\right)^2\right]^{1/2} \leq \sum_{j \neq 0} \mu_j \left(\sigma_{ij}^I\right)^2 + \left(1-\mu_0\right)\sigma_{io}^I \tag{13}$$

We also observe that equality holds in Relation (12), if and only if $\sigma_{rj}^o = 0$ for all $j \neq o$, and equality holds in Relation (13) if and only if $\sigma_{ij}^o = 0$ for all $j \neq o$.

(i) If $0 < \alpha < 0.5,$ we have $\Phi^{-1}(\alpha) < 0$ and hence

$$\sum_{j=1}^n x_{ij}\mu_j - x_{io} - \Phi^{-1}(\alpha)\sigma_i^o(\mu) \leq \sum_{j=1}^n x'_{ij}\mu_j - x'_{io}, \quad i = 1,\ldots,m \tag{14}$$

$$\eta y_{ro} - \sum_{j=1}^n y_{rj}\mu_j - \Phi^{-1}(\alpha)\sigma_r^o(\eta,\mu) \leq \eta y'_{ro} - \sum_{j=1}^n y'_{ri}\mu_j, \quad r \in D \tag{15}$$

$$\sum_{j=1}^n y_{rj}\mu_j - y_{ro} - \Phi^{-1}(\alpha)\sigma_r^o(\mu) \leq \sum_{j=1}^n y'_{rj}\mu_j - y'_{ro}, \quad r \in N \tag{16}$$

(ii) If $1 > \alpha > 0.5,$ we have $\Phi^{-1}(\alpha) > 0$ and hence

$$\sum_{j=1}^n x_{ij}\mu_j - x_{io} - \Phi^{-1}(\alpha)\sigma_i^o(\mu) \geq \sum_{j=1}^n x'_{ij}\mu_j - x'_{io}, \quad i = 1,\ldots,m \tag{17}$$

$$\eta y_{ro} - \sum_{j=1}^n y_{rj}\mu_j - \Phi^{-1}(\alpha)\sigma_r^o(\eta,\mu) \leq \eta y_{ro} - \sum_{j=1}^n y_{ri}\mu_j, \quad r \in D \tag{18}$$

$$\sum_{j=1}^n y_{rj}\mu_j - y_{ro} - \Phi^{-1}(\alpha)\sigma_i^o(\mu) \geq \sum_{j=1}^n y_{ij}\mu_j - y_{io'} \quad r \in N \tag{19}$$

where

$$x'_{io} = x_{io} + \sigma_{io}^I\Phi^{-1}(\alpha), \quad i = 1,\ldots,m \tag{20}$$

$$x'_{ij} = x_{ij} + \sigma_{ij}^I\Phi^{-1}(\alpha), \quad j \neq 0, i = 1,\ldots,m \tag{21}$$

$$y'_{ro} = y_{ro} + \sigma_{ro}^o\Phi^{-1}(\alpha), \quad r \in D \tag{22}$$

$$y'_{rJ} = y_{rj} + \sigma_{rj}^o\Phi^{-1}(\alpha), \quad r \in D \tag{23}$$

$$y'_{ro} = y_{ro} - \sigma_{ro}^o\Phi^{-1}(\alpha), \quad r \in N \tag{24}$$

$$y'_{rJ} = y_{rj} - \sigma_{rj}^o\Phi^{-1}(\alpha), \quad r \in N \tag{25}$$

Lastly, the equality will hold for $r$ and $i$ in Relations (14)–(16) if and only if $\sigma_{ij}^{I} = 0$ and $\sigma_{rj}^{I} = 0$ for $j \neq 0$.

Consider the following linear programming problem

$$
\begin{aligned}
&\text{Max} \quad \eta + \varepsilon\left(\sum_{i=1}^{m} t_i^- + \sum_{r \in D} t_r^+\right) \\
&\text{s.t.} \quad \sum_{j=1}^{n} x'_{ij}\mu_j + t_i^- - x'_{io'} \quad i = 1,\ldots,m \\
&\eta y_{ro} - \sum_{j=1}^{n} y_{rj}\mu_j - t_r^+ =, \quad r \in D \\
&\sum_{j=1}^{n} y'_{rj}\mu_j - t_r^+ = y'_{ro'} \quad r \in N \\
&\sum_{j=1}^{n} \mu_j = 1 \\
&t_r^+ = 0, \quad r \in N \\
&\mu_j \geq 0, \quad j = 1,\ldots,n \\
&t_i^- \geq 0, \quad i = 1,\ldots,m \\
&t_r^+ \geq 0, \quad r = 1,\ldots,s
\end{aligned}
\tag{26}
$$

This is simply the non-discretionary model represented in Model (1) for DMU$_o$ with the adjusted input and output values $(x'_j, y'_j)$ as defined in Relations (20)–(25) for DMU$_j$, $j = 1,\ldots,n$.

### 3.2 Results

We now illustrate the following results.

Theorem 1: For $0 < \alpha < 0.5$:

*(i) if DMU$_o$ is stochastic efficient, then DMU$_o$ is efficient for the adjusted inputs and outputs in the deterministic Model (26)*

*(ii) if DMU$_o$ is inefficient for the adjusted inputs and outputs in Model (26), then DMU$_o$ is stochastic inefficient.*

*Proof*: (i) Assume to the contrary that DMU$_o$ is inefficient for the adjusted inputs and outputs in Model (26). Then we must have (I) $\eta^* > 1$ or (II) $\eta^* = 1$ and $t_r^+ > 0$ or $t_r^- > 0$ for at least one $r$ or $i$ in Model (26). Since a solution for Model (26) is also a solution for Model (11), either case will contradict the supposed stochastic efficiency of DMU$_o$.

(ii) This follows directly from (*i*). □

Theorem 2: For $0.5 < \alpha < 1$:

(i) *if $DMU_o$ is efficient for the adjusted inputs and outputs; in Model (26), then $DMU_o$ is stochastic efficient*

(ii) *if $DMU_o$ is inefficient for the adjusted inputs and outputs in Model (26), then $DMU_o$ is stochastic inefficient.*

*Proof*: This is analogous to the proof of Theorem 1. □

### 3.3 The relations to sensitivity analysis

To perform sensitivity analysis, we suppose that only DMU$_o$ has random variations in its inputs and outputs, i.e.

$$\sigma_{ij}^I = 0, j \neq o, \sigma_{io}^I \neq 0, \sigma_{rj}^o = 0, (j \neq o), \sigma_{ro}^o \neq 0, \forall i, r$$

In this instance, Model (26) is the same as Model (11), where

$$\begin{aligned} x'_{io} &= x_{io} + \sigma_{io}^I \Phi^{-1}(\alpha) \quad i = 1, \dots, m \\ x'_{ij} &= x_{ij}, \quad j \neq 0, \quad i = 1, \dots, m \end{aligned} \tag{27}$$

$$\begin{aligned} y'_{ro} &= y_{ro} - \sigma_{ro}^o \Phi^{-1}(\alpha), \quad r \in D \\ y'_{rJ} &= y_{rj}, \quad j \neq o, \quad r \in D \end{aligned} \tag{28}$$

$$\begin{aligned} y'_{ro} &= y_{ro} - \sigma_{ro}^o \Phi^{-1}(\alpha), \quad r \in D \\ y'_{rJ} &= y_{rJ}, \quad j \neq 0, \quad r \in N \end{aligned} \tag{29}$$

Hence, Model (26) is the deterministic equivalent of stochastic Model (10). This sensitivity analysis is completely deterministic. Our chance-constrained approach can be executed by deterministic equivalents that are similar to those utilised in the sensitivity analysis although the conceptual meanings are dissimilar. The matter originally addressed in the chance-constrained formulation diverges and this introduces components, such as the risk interrelated with $\alpha$, that are not present in this sensitivity analysis.

To illustrate the implications of the suppositions, we begin with the following lemma.

Lemma 1:

(i) *For $0 < \alpha < 0.5$: suppose that $(\bar{\eta}, \bar{\mu}, \bar{t}^+, \bar{t}^-)$ is a feasible solution of (26), and is consequently also a feasible solution of Model (10). Consider the transformations*

$$\eta = \bar{\eta}, \quad \mu = \bar{\mu}$$

$$t_i^- = \bar{t}_i^- - (1 - \bar{\mu}_o)\sigma_{io}^I \Phi^{-1}(\alpha) \quad i = 1, \dots, m$$

$$t_r^+ = \bar{t}_r^+ - (\bar{\eta} - \bar{\mu}_o)\sigma_{ro}^o \Phi^{-1}(\alpha), \quad r \in D$$

$$t_r^+ = \bar{t}_r^+ - (\bar{\eta} - \bar{\mu}_o)\sigma_{ro}^o \Phi^{-1}(\alpha), \quad r \in N.$$

*Then $(\eta, \mu, t^+, t-)$ is a feasible solution of Model (1).*

*(ii) For $1 > \alpha > 0.5$: suppose that $(\eta, \mu, t^{+}, t^{-})$ is a feasible solution of Model (1). Consider the transformations*

$$\bar{\eta} = \eta, \quad \bar{\mu} = \mu$$

$$\bar{t}_i^{-} = t_i^{-} + (1 - \bar{\mu}_o)\sigma_{io}^{I}\Phi^{-1}(\alpha), \quad i = 1, \ldots, m$$

$$\bar{t}_r^{+} = t_r^{+} + (\bar{\eta} - \bar{\mu}_o)\sigma_{ro}^{o}\Phi^{-1}(\alpha), \quad r \in D$$

$$\bar{t}_r^{+} = t_r^{+} + (\bar{\eta} - \bar{\mu}_o)\sigma_{ro}^{o}\Phi^{-1}(\alpha), \quad r \in N$$

*We then have $(\bar{\eta}, \bar{\mu}, \bar{t}^{+}, \bar{t}^{-})$ as a feasible solution of Model (26), which is the deterministic equivalent of Model (26) under the suppositions made at the start of this section.*

*Proof*: This is straightforward upon substitution into Models (1) and (26), respectively. □

Also, based on what we have previously discussed for Model (11), we have the following immediate results.

Theorem 3: *For $\alpha = 0.5$: the inefficiency vs. the efficiency of $DMU_o$ in the input–output mean Model (1) is similar as in the stochastic Model (10). Note that the input–output vectors in (1) are mean values of random variables.*

Theorem 4: *For $0 < \alpha < 0.5$:*

*(i) assuming $DMU_o$ is efficient with $DMU_o \in E \cup E'$ in the input–output mean Model (1), then $DMU_o \in E$ in the stochastic Model (10)*

*(ii) assuming $DMU_o \in F$ in the input–output mean Model (1), then $DMU_o \in E$ in the stochastic Model (10)*

*(iii) assuming $DMU_o \in N$ in the input–output mean Model (1), then $DMU_o \in N$ in the stochastic Model (10) if $((\beta_i^{-*})/(-\Phi^{-1}(\alpha))) > \sigma_{io}^{I}$ and $\left((\beta_r^{+*})/(-\Phi^{-1}(\alpha))\right) > \sigma_{ro}^{o}$, where, for $0.5 > \alpha$ we have $\Phi^{-1}(\alpha) < 0$. Here $\sum_{i=1}^{m}\beta_i^{-*} + \sum_{r=1}^{s}\beta_r^{+*}$ is the optimal value of*

$$\begin{aligned}
&Max \quad \sum_{i=1}^{m}\beta_i^{-} + \sum_{r \in D}\beta_r^{+} \\
&s.t. \quad \sum_{i=1}^{m}x_{ij}\mu_j + \beta_i^{-} \le x_{io}, \quad i = 1, \ldots, m \\
&\sum_{j=1}^{n}y_{rj}\mu_j - \beta_j^{+} \ge y_{ro}, \quad r \in D \\
&\sum_{j=1}^{n}y_{rj}\mu_j - \beta_j^{+} \ge y_{ro}, \quad r \in N \\
&\beta_i^{-} \ge 0, \beta_r^{+}, \mu_j \ge 0, \quad r = 1, \ldots, s; i = 1, \ldots m \text{ and } j = 1, \ldots, n
\end{aligned} \tag{30}$$

*Proof*: (i) To prove (*i*) we note that $y_{ro} \le y'_{ro}$ and $x_{io} \le x'_{io}$ from Relations (27)–(29). Therefore, if DMU$_o$ is efficient in Model (1), then DMU$_o$ is also efficient in Model (26). Therefore, DMU$_o$ is stochastic efficient as stated in Definition 3. Consider a feasible solution $(\eta, \mu, t^+, t^-)$ of Model (26) with $\eta = 1, \mu_o = 1$ and $\mu_j = 0 (j \ne 0), t^+ = 0,$ and $t^- = 0.$ It is clearly an optimal solution of Model (26). We now show that this is the unique optimal solution. Assume on the contrary that there is another optimal solution $(\bar{\eta}, \bar{\mu}, \bar{t}^+, \bar{t}^-)$ of (26) with $\bar{\eta} = 1$ and $0 \le \bar{\mu}_o < 1.$ Consider the transformation $(\eta^*, \mu^*, t^{+*}, t^{-*})$ of $(\bar{\eta}, \bar{\mu}, \bar{s}^+, \bar{s}^-)$ in Lemma l(i). This is a feasible solution of Model (1) with $\eta^* = \bar{\eta} = 1.$ However, since $\bar{\mu}_o < 1,$ we have

$$\begin{aligned} t_r^{+*} &= \bar{t}_r^+ - (\bar{\eta} - \bar{\mu}_o)\sigma_{ro}^o \Phi^{-1}(\alpha) \\ &= \bar{t}_r^+ - (1 - \mu_o)\sigma_{ro}^o \Phi^{-1}(\alpha) > \bar{t}_r^+ \ge 0, \quad r \in D \end{aligned}$$

This, nevertheless, contradicts DMU$_o \in E \cup E'$ in Model (1). Consequently, $(\bar{\eta}, \bar{\mu}, \bar{t}^+, \bar{t}^-)$ is the unique optimal solution of Model (26) and DMU$_o \in E$ in the stochastic Model (10).

(ii) To prove (*ii*) we first demonstrate that DMU$_o$ is efficient in Model (26). Assume on the contrary that DMU$_o$ is inefficient in Model (26). Inefficiency for DMU$_o$ amounts to DMU$_o$ being dominated by the other DMUs. In this case, there exists a $\mu \ge 0$ with $\mu_o = 0$ and $\sum_{j \ne 0} \mu_j = 1$ such that

$$\sum_{j \ne 0} x_{ij}\mu_j = \sum_{j \ne 0} x'_{ij}\mu_j \le x'_{io} = x_{io} - \sigma_{io}^o \Phi^{-1}(\alpha) < x_{io}, \quad i = 1, \ldots, m$$

$$\sum_{j \ne 0} y_{rj}\mu_j = \sum_{j \ne 0} y'_{rj}\mu_j \le y'_{ro} = y_{ro} - \sigma_{ro}^o \Phi^{-1}(\alpha) > y_{ro}, \quad r \in D$$

$$\sum_{j \ne 0} y_{rj}\mu_j = \sum_{j \ne 0} y'_{rj}\mu_j \ge y'_{ro} = y_{ro} - \sigma_{ro}^o \Phi^{-1}(\alpha) > y_{ro}, \quad r \in N$$

From the second set of these relations we can see that we need $\eta > 1$ to attain the equality that optimality requires for at least some of these $r \in D$ constraints. The remaining constraints are satisfied by an appropriate selection of slacks. Therefore, we have a feasible solution with $\eta > 1$ and possibly non-zero slacks. This contradicts the supposition that DMU$_o \in F$ in Model (1). Thus DMU$_o$ is efficient in Model (26). Consider a feasible solution $(\eta, \mu, t^+, t^-)$ of Model (26) with $\eta = 1, \mu_o = 1$ and $\mu_j = 0 (j \ne 0),$ and $t^+ = 0,$ and $t^- = 0.$ It is obvious that $(\eta, \lambda, t^+, t^-)$ is an optimal solution of Model (26). We want to demonstrate, nevertheless, that it is a unique optimal solution so that DMU$_o \in E$. Assume on the contrary that there is another optimal solution $(\bar{\eta}, \bar{\mu}, \bar{t}^+, \bar{t}^-)$ of Model (26) with $\bar{\eta} = 1, 0 \le \bar{\mu}_o < 1, \bar{t}^+ = 0\ \bar{t}^- = 0.$ Consider the transformation $(\eta^*, \mu^*, t^{+*}, t^-*)$ of $(\bar{\eta}, \bar{\mu}, \bar{t}^+, \bar{t}^-)$ as given in Lemma l(i). This is a feasible solution of Model (1). However, since $\bar{\mu}_o < 1,$ we have

$$\begin{aligned} t_i^{+*} &= \bar{t}_i^+ - (\bar{\eta} - \bar{\mu}_o)\sigma_{io}^I \Phi^{-1}(\alpha) \\ &= \bar{t}_i^+ - (1 - \bar{\mu}_o)\sigma_{io}^I \Phi^{-1}(\alpha) > \bar{t}_i^+ = 0, \quad i = 1, \ldots, m \end{aligned}$$

$$t_r^{+*} = \overline{t}_r^{+} - \left(\overline{\eta} - \overline{\mu}_o\right)\sigma_{ro}^{o}\Phi^{-1}(\alpha)$$
$$= \overline{t}_r^{+} - \left(1 - \overline{\mu}_o\right)\sigma_{ro}^{o}\Phi^{-1}(\alpha) > \overline{t}_i^{+} \geq 0, \quad r \in D$$
$$t_r^{+*} = \overline{t}_r^{+} - \left(\overline{\eta} - \overline{\mu}_o\right)\sigma_{ro}^{o}\Phi^{-1}(\alpha)$$
$$= \overline{t}_r^{+} - \left(1 - \overline{\mu}_o\right)\sigma_{ro}^{o}\Phi^{-1}(\alpha) > \overline{t}_r^{+} \geq 0, \quad N \in D$$

Since

$$x_{io} = \sum_{j=1}^{n} x_{ij}\mu_j^* + t_i^{+*} > \sum_{j=1}^{n} x_{ij}\mu_j^*, \quad i = 1,\ldots,m$$

$$y_{io} = \sum_{j=1}^{n} y_{rj}\mu_j^* + t_r^{+*} < \sum_{j=1}^{n} y_{rj}\mu_j^*, \quad r \in D$$

$$y_{io} = \sum_{j=1}^{n} y_{rj}\mu_j^* + t_r^{+*} < \sum_{j=1}^{n} y_{rj}\mu_j^*, \quad r \in N$$

which contradicts the condition prescribed in (ii) that $\text{DMU}_o \in F$ in Model (1). Thus $\text{DMU}_o \in E$ in Model (26), i.e. $\text{DMU}_o \in E$ in the stochastic Model (10).

(iii) Finally turning to (*iii*), assume that $(\beta^{+*}, \beta^{-*}, \mu^*)$ is an optimal solution of Model (30). As a condition for being the maximum, we must have

$$\sum_{j=1}^{n} x_{ij}\mu_j^* = x_{io} + \beta_i^{-*}, \quad i = 1,\ldots,m$$

$$\sum_{j=1}^{n} y_{rj}\mu_j^* = y_{ro} + \beta_r^{+*}, \quad r \in D$$

$$\sum_{j=1}^{n} y_{rj}\mu_j^* = y_{ro} + \beta_r^{+*}, \quad r \in N$$

By the conditions prescribed in (*iii*) we have $\beta_i^{-*} / (\Phi^{-1}(\alpha))^{>\sigma_{io}^{I}}$ and $\beta_r^{+*} / (\Phi^{-1}(\alpha))^{>\sigma_{ro}^{o}}$.

Therefore, we must also have

$$x'_{io} = x_{io} + \sigma_{io}^{I}\Phi^{-1}(\alpha) > x_{io} - \beta_i^{-*} = \sum_{j \neq 0}^{n} x_{ij}\mu_j^* = x_{io}\mu_o^*$$
$$\geq \sum_{j \neq 0}^{n} x_{ij}\mu_j^* + \left(x_{io} - \sigma_{io}^{I}\Phi^{-1}(\alpha)\right)\mu_o^* = \sum_{j=1}^{n} x'_{ij} + \mu'_j, \quad i = 1,\ldots,m$$
$$y'_{ro} = y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha) < y_{ro} + \beta_r^{+*} = \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + y_{ro}\mu_o^*$$
$$\leq \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + \left(y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha)\right)\mu_o^* = \sum_{j=1}^{n} x'_{ij} + \mu_j^*, \quad i = 1,\ldots,m$$

$$y'_{ro} = y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha) < y_{ro} + \beta_r^{+*} = \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + y_{ro}\mu_o^*$$

$$\leq \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + \left(y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha)\right)\mu_o^* = \sum_{j=1}^{n} y'_{ij} + \mu_j^*, \quad r \in D$$

$$y'_{ro} = y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha) < y_{ro} + \beta_r^{+*} = \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + y_{ro}\mu_o^*$$

$$\leq \sum_{j \neq 0}^{n} y_{rj}\mu_j^* + \left(y_{ro} - \sigma_{ro}^{o}\Phi^{-1}(\alpha)\right)\mu_o^* = \sum_{j=1}^{n} y'_{ij} + \mu_j^*, \quad r \in N$$

This means that $x'_{io}$ and $y'_{ro}$ are dominated by a convex combination of other DMUs. Thus $DMU_o$ is inefficient in Model (26). Consequently, as claimed, $DMU_o \in N$ in the stochastic Model (10) when $DMU_o \in E$ in the mean Model (1). □

Theorem 5: *For* $0.5 < \alpha < 1$:

*(i) assume* $DMU_o \in E$ *in the input–output mean Model (1), then* $DMU_o \in E$ *in the stochastic Model (10) if*

$$\sum_{i=1}^{m} \sigma_{io}^{I} + \sum_{r \in D} \sigma_{ro}^{o} < \frac{\left(\sum_{i=1}^{m} \Phi_i^{-*} + \sum_{r \in D} \Phi_r^{+*}\right)}{\Phi^{-1}(\alpha)} \tag{31}$$

*where* $\sum_{i=1}^{m} \phi_i^{-*} + \sum_{r \in D} \phi_r^{+*}$ *is the optimal value of*

$$\begin{aligned}
& Min \sum_{i=1}^{m} \phi_i^{-} + \sum_{r \in D} \phi_r^{+} \\
& s.t. \sum_{j=0}^{n} x_{ij}\mu_j \leq x_{io} - \phi_i^{-}, \quad i = 1, \ldots, m \\
& \sum_{j \neq 0}^{n} y_{rj}\mu_j \geq y_{ro} + \phi_r^{+}, \quad r \in D \\
& \sum_{y \neq 0}^{n} y_{rj}\mu_j \geq y_{ro} + \phi_r^{+}, \quad r \in N \\
& \sum_{j \neq 0}^{n} \mu_j = 1; \quad \phi_i^{-} \geq 0; \quad \phi_r^{+} \geq 0; \ \mu_j \geq 0 (j \neq o) \\
& r = 1, \ldots, s; \quad j = 1, \ldots, n; \quad i = 1, \ldots, m
\end{aligned} \tag{32}$$

*(ii) assuming that* $DMU_o \in E' \cup F \cup N$ *in the input–output mean Model (1), then* $DMU_o \in N$ *in the stochastic Model (10).*

*Proof*: (i) First we want to demonstrate that $\text{DMU}_o$ is stochastic efficient, as is the case if it is efficient in Model (26). Assume on the contrary $\text{DMU}_o$ is inefficient in Model (26). This implies that there is a $\bar{\mu}$ with $\bar{\mu}_o = 0, \bar{\mu} \geq 0 (j \neq o)$, and $\sum_{j \neq o} \bar{\mu}_j = 1$ such that

$$\sum_{j \neq 0}^{n} x'_{ij} \bar{\mu}_j \leq x'_{io}, \quad i = 1, \ldots, m$$

$$\sum_{j \neq 0} y'_{rj} \bar{\mu}_j \geq y'_{ro'} \quad r = 1, \ldots, D$$

$$\sum_{j \neq 0} y'_{rj} \bar{\mu} \geq y'_{ro'} \quad r = 1, \ldots, N.$$

Therefore, by the definitions of $x'$ and $y'$,

$$\sum_{j \neq 0}^{n} x_{ij} \bar{\mu}_j \leq x_{io} - \sigma_{io}^{I} \Phi^{-1}(\alpha), \quad i = 1, \ldots, m$$

$$\sum_{j \neq 0} y_{rj} \bar{\mu}_j \geq y_{ro} - \sigma_{ro}^{o} \Phi^{-1}(\alpha), \quad r = 1, \ldots, D$$

$$\sum_{j \neq 0} y_{rj} \bar{\mu}_j \geq y_{ro} - \sigma_{ro}^{o} \Phi^{-1}(\alpha), \quad r = 1, \ldots, N$$

Letting $\bar{\phi}_i^- = \sigma_{io}^{I} \Phi^{-1}(\alpha), i = 1, \ldots, m,$ $\bar{\phi}_r^+ = \sigma_{ro}^{o} \Phi^{-1}(\alpha), r = 1, \ldots, D$ and $\bar{\phi}_r^+ = \sigma_{ro}^{o} \Phi^{-1}(\alpha), r = 1, \ldots, N.$ We find that $(\bar{\phi}^+, \bar{\phi}^-, \bar{\mu})$ satisfies Model (32) with $\sum_{i=1}^{m} \bar{\phi}_i^- + \sum_{r \in D} \bar{\phi}_r^+$ $< \sum_{i=1}^{m} \phi_i^{-*} + \sum_{r \in D} \phi_r^{+*}$. This contradicts the assumption that $\sum_{i=1}^{m} \phi_i^{-*} + \sum_{r \in D} \phi_r^{+*}$ is the optimal value of Model (32). Thus we cannot have $\text{DMU}_o$ inefficient. Now, to complete the proof for (*i*), we want to demonstrate that $\text{DMU}_o$ is an extreme point of Model (26). Assume on the contrary it is not an extreme point of Model (26). Then there is a $\hat{\mu}$ with $\hat{\mu}_o = 0, \hat{\mu}_j \geq 0 (j \neq o)$ and $\sum_{j \neq o} \hat{\mu}_j = 1$ such that

$$\sum_{j \neq 0} x_{ij} \hat{\mu}_j = x'_{io} = x_{io} + \sigma_{io}^{o} \Phi^{-1}(\alpha), \quad i = 1, \ldots, m$$

$$\sum_{j \neq 0} y_{rj} \hat{\mu}_j = y'_{ro} = y_{ro} - \sigma_{ro}^{o} \Phi^{-1}(\alpha), \quad r = 1, \ldots, D$$

$$\sum_{j \neq 0} y_{rj} \hat{\mu}_j = y'_{ro} = y_{ro} - \sigma_{ro}^{o} \Phi^{-1}(\alpha), \quad r = 1, \ldots, N$$

See Relations (27)–(29). Let $\hat{\Phi}_i^- = \sigma_{io}^{I} \Phi^{-1}(\alpha), i = 1, \ldots, m,$ $\hat{\phi}_r^+ = \sigma_{ro}^{o} \Phi^{-1}(\alpha), r = 1, \ldots, D$ and $\hat{\phi}_r^+ = \sigma_{ro}^{o} \Phi^{-1}(\alpha), r = 1, \ldots, N.$ We then have $(\hat{\phi}^-, \hat{\phi}^+, \bar{\mu})$ satisfies Model (32) with $\sum_{i=1}^{m} \hat{\Phi}^- + \sum_{r \in D} \hat{\phi}_r^+ < \sum_{i=1}^{m} \phi_r^{-*} \sum_{r \in D} \phi_r^{+*}$ a contradiction of the assumption that $\phi_r^{-*}$ and $\phi_r^{+*}$ are optimal for Model (32).

(ii) Since $\text{DMU}_o \in E' \cup F \cup N$ in the input–output mean Model (1), there exists $\bar{\mu}$ with $\bar{\mu}_o = 0, \bar{\mu}_j \geq 0 (j \neq 0)$ and $\sum_{j \neq 0} \bar{\mu}_j = 1$ such that

$$x_{io} = \sum_{j \neq 0} x_{ij} \bar{\mu}_j = \sum_{j \neq 0} x'_{ij} \bar{\mu}_j < x_{io} + \sigma_{io}^I \Phi^{-1}(\alpha) = x'_{io'} \quad i = 1, \ldots, m$$

$$y_{ro} = \sum_{j \neq 0} x_{rj} \bar{\mu}_j = \sum_{j \neq 0} y'_{rj} \bar{\mu}_j < y_{ro} + \sigma_{ro}^o \Phi^{-1}(\alpha) = y'_{ro'} \quad r = 1, \ldots, D$$

$$y_{ro} = \sum_{j \neq 0} x_{rj} \bar{\mu}_j = \sum_{j \neq 0} y'_{rj} \bar{\mu}_j < y_{ro} - \sigma_{ro}^o \Phi^{-1}(\alpha) = y'_{ro'} \quad r = 1, \ldots, N$$

i.e. $\text{DMU}_o$ is strictly dominated by a DMU which is a convex combination of the other DMUs in Model (26). Hence, $\text{DMU}_o \in N$ in the stochastic Model (10). □

Theorem 6: *For* $0 < \alpha < 0.5$:

*(i) Assuming that* $\text{DMU}_o \in E$ *in the stochastic Model (10), then* $\text{DMU}_o \in E$ *in the input–output mean Model (1) if*

$$\sum_{i=1}^{m} \sigma_{io}^I + \sum_{r \in D} \sigma_{ro}^o < \frac{\left( \sum_{i=1}^{m} \phi_i^{-*} + \sum_{r \in D} \phi_r^{+*} \right)}{\Phi^{-1}(\alpha)} \tag{33}$$

*where* $\sum_{i=1}^{m} \phi_i^{-*} + \sum_{r \in D} \phi_r^{+*}$ *is the optimal value of*

$$\begin{aligned} & Min \sum_{i=1}^{m} \phi_i^- + \sum_{r \in D} \phi_r^+ \\ & \text{s.t.} \sum_{j \neq 0}^{n} x_{ij} \mu_j \leq x'_{io} + \phi'_i, \quad i = 1, \ldots, m \\ & \sum_{j \neq 0}^{n} y_{rj} \geq y'_{ro} - \phi_r^+, \quad r \in D \\ & \sum_{j \neq 0}^{n} y_{rj} \geq y'_{ro} - \phi_r^+, \quad r \in N \\ & \sum_{j \neq 0}^{n} \mu_j = 1; \quad \phi_i^- \geq 0; \quad \phi_r^+ \geq 0; \quad \mu_j \geq 0 (j \neq o) \\ & r = 1, \ldots, s; \; j = 1, \ldots, n; \; i = 1, \ldots, m \end{aligned} \tag{34}$$

*Proof*: This is similar to the proof of Theorem 5 when one replaces $(x_j, y_j)$ by $(x'_j, y'_j)$, and note that $x'_j, y'_j = (x_j, y_j)$ for $j \neq o$. □

Theorem 7: *For* $0.5 < \alpha < 1$:

(i) *assuming that* $DMU_o \in E \cup E'$ *in the stochastic Model (10), then* $DMU_o \cup E$ *in the input–output mean Model (1)*

(ii) *assuming that* $DMU_o \in F$ *in the stochastic Model (10), then* $DMU_o \in E$ *in the input–output mean Model (1)*

(iii) *assume* $DMU_o \in N$ *in the stochastic Model (10), then* $DMU_o \in N$ *in the input–output mean Model (1) if* $\beta_i^{-*} / \Phi^{-1}(\alpha) > \sigma_{io}^I$ *and* $\beta_i^{+*} / \Phi^{-1}(\alpha) > \sigma_{ro}^o$, *where* $\sum_{i=1}^{m} \beta_i^{-*} + \sum_{r \in D} \beta_r^{+*}$ *is the optimal value of the following model*

$$
\begin{aligned}
& Max \quad \sum_{i=1}^{n} \beta_i^- + \sum_{r \in D} \beta_r^+ \\
& s.t. \sum_{j=1}^{n} x'_{io} \mu_j - \beta_i^- \le x'_{io}, \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} y'_{rj} \mu_j - \beta_r^+ \ge y'_{ro}, \quad r \in D \\
& \sum_{j=1}^{n} y'_{rj} \mu_j - \beta_r^+ \ge y'_{ro}, \quad r \in N; \quad \beta_r^+ \ge 0; \quad \beta_r^- \ge 0; \\
& \mu_j \ge 0; \; r = 1, \ldots, s; \; i = 1, \ldots, m; \; j = 1, \ldots, n
\end{aligned} \tag{35}
$$

This is similar to the proof of Theorem 4 when one replaces $(x_j, y_j)$ by $(x'_j, y'_j)$.

Next we present a case study to demonstrate the applicability of the proposed framework and to exhibit the efficacy of the procedures and algorithms.

## 4 Case study

We used the model presented in this study for supplier selection at Infotecx.[1] Infotex was founded in 1988 as a small start-up information technology company with the vision of becoming the industry leader in health informatics. The company is currently one of the largest private software companies in the USA providing information technology solutions in the healthcare industry. Currently, the company has 8 subsidiaries each specialised in different healthcare market segments and 27 satellite companies providing sales, installation and after-sales services. The company taps into its 900 professional staff to reach over 5,000 customers. There are 20 main specialised application developers at Infotecx that supply a variety of health informatics solutions, either starting the development of healthcare information system from the ground up, or taking legacy systems and convert them to fulfil new requirements. The dataset used in this case study includes specifications on 20 suppliers located in 20 different cities across the USA. The performance measures utilised in this study are the number of personnel, average time for serving customers, profit margin and supplier variety. The number of personnel and average time for serving the customers were used as the input variables in the DEA model. Profit margin and supplier variety were considered as the output variables in the DEA model. Moreover, as suggested by Farzipoor Saen (2009), supplier variety is considered as a non-discretionary output, i.e. this factor is exogenously fixed and cannot be increased by suppliers (at least in the short term). Note that the inputs and outputs selected in this paper are not exhaustive by any means. The actual case study included several additional input and output data which are omitted in this paper for simplicity. Table 2 presents the supplier attributes used in this paper.

The computational results from using Model (25) with $\alpha = 0.05$ are shown in Table 3. The efficient suppliers are Columbus, Houston, Memphis and San Francisco. These suppliers are efficient because the following two conditions are both satisfied:

1 $\eta^* = 1$

2 $t_i^{-*} = t_r^+ = 0, \quad \forall i, r.$

The visual computational result for this dataset is presented in Figure 1. This figure has two coordinates (DMUs and stochastic efficiency) and shows the efficiency of the 20 DMUs under consideration. As shown in this figure, except for the efficient DMUs, the efficiency scores of all the remaining DMUs are greater than one. The efficiency scores for the efficient DMUs are one and their associated slacks are zero.

This example shows the applicability of the proposed novel supplier selection model using chance-constrained DEA with non-discretionary factors and stochastic data.

**Table 2** Related attributes

| | *Inputs* | | | | *Outputs* | | | |
|---|---|---|---|---|---|---|---|---|
| | *No. of personnel* | | *Average time for serving customers (hr)* | | *Profit margin ($1,000)* | | *Supplier variety* | |
| *Supplier (DMU)* | *Mean* | *Variance* | *Mean* | *Variance* | *Mean* | *Variance* | *Mean* | *Variance* |
| Atlanta | 6 | 0.5 | 70 | 7 | 25 | 3 | 10 | 2 |
| Baltimore | 5 | 1 | 130 | 8 | 17 | 2 | 12 | 3 |
| Boston | 11 | 2 | 125 | 5 | 15 | 1 | 50 | 4 |
| Charlotte | 8 | 1 | 100 | 4 | 25 | 2 | 55 | 5 |
| Chicago | 9 | 1 | 90 | 1 | 30 | 3 | 70 | 7 |
| Columbus | 6 | 2 | 75 | 5 | 50 | 5 | 15 | 5 |
| Dallas | 18 | 1 | 150 | 10 | 14 | 1 | 35 | 5 |
| Denver | 25 | 1 | 280 | 20 | 65 | 0.5 | 42 | 2 |
| Detroit | 12 | 1 | 160 | 10 | 50 | 3 | 60 | 4 |
| Houston | 10 | 1 | 135 | 9 | 40 | 2 | 70 | 9 |
| Los Angeles | 12 | 1 | 120 | 4 | 10 | 4 | 75 | 10 |
| Louisville | 10 | 2 | 95 | 2 | 5 | 1 | 45 | 2 |
| Memphis | 7 | 1 | 70 | 2 | 12 | 2 | 43 | 10 |

**Table 2** Related attributes (continued)

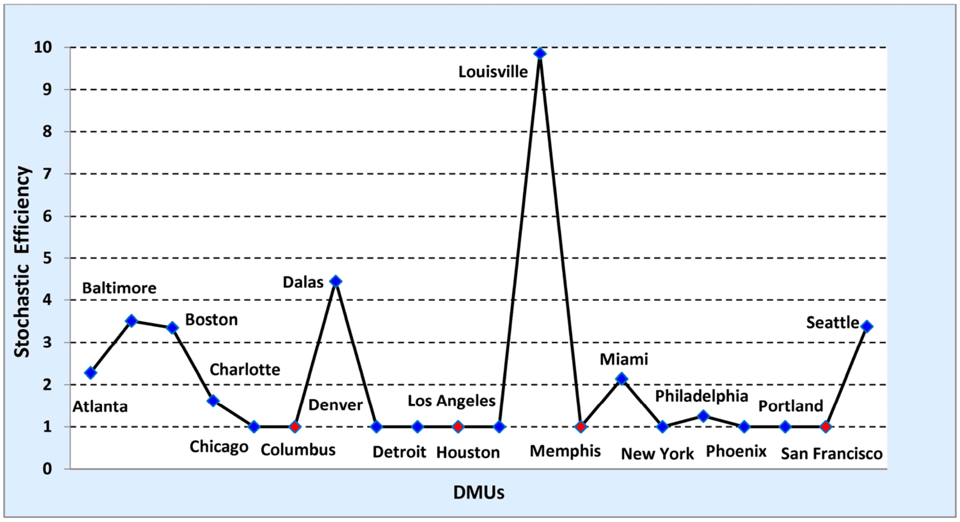
| | *Inputs* | | | | *Outputs* | | | |
|---|---|---|---|---|---|---|---|---|
| | *No. of personnel* | | *Average time for serving customers (hr)* | | *Profit margin ($1,000)* | | *Supplier variety* | |
| *Supplier (DMU)* | *Mean* | *Variance* | *Mean* | *Variance* | *Mean* | *Variance* | *Mean* | *Variance* |
| Miami | 11 | 2 | 140 | 5 | 30 | 1 | 5 | 4 |
| New York | 20 | 3 | 140 | 20 | 80 | 2 | 5 | 2 |
| Philadelphia | 23 | 2 | 150 | 25 | 65 | 4 | 8 | 2 |
| Phoenix | 25 | 3 | 120 | 15 | 78 | 3 | 7 | 2 |
| Portland | 10 | 1 | 70 | 1 | 40 | 2 | 25 | 1 |
| San Francisco | 12 | 1 | 115 | 5 | 5 | 1 | 65 | 4 |
| Seattle | 5 | 2 | 80 | 5 | 17 | 1 | 10 | 3 |

**Table 3** Overall results

| *Supplier (DMU)* | $t_1^-$ | $t_2^-$ | $t_1^+$ | $t_2^+$ | *η (( = 0.05)* |
|---|---|---|---|---|---|
| Atlanta | 2.35 | 0 | 0 | 0 | 2.28 |
| Baltimore | 0 | 43.8 | 0 | 0 | 3.51 |
| Boston | 0 | 1.78 | 0.7 | 0 | 3.35 |
| Charlotte | 0 | 0 | 0 | 0 | 1.62 |
| Chicago | 0.64 | 0 | 0 | 0 | 1 |
| Columbus[a] | 0 | 0 | 0 | 0 | 1 |
| Dallas | 6.95 | 0 | 0 | 0 | 4.45 |
| Denver | 0.20 | 0.19 | 0 | 0 | 1 |
| Detroit | 0.65 | 0.54 | 2.5 | 0 | 1 |
| Houston[a] | 0 | 0 | 0 | 0 | 1 |
| Los Angeles | 0.78 | 0.85 | 0 | 0 | 1 |
| Louisville | 3.06 | 0 | 0 | 0 | 9.85 |
| Memphis[a] | 0 | 0 | 0 | 0 | 1 |
| Miami | 0 | 8.12 | 1.3 | 0 | 2.14 |
| New York | 0.96 | 0.50 | 0.26 | 0 | 1 |
| Philadelphia | 5.8 | 0 | 4.1 | 0 | 1.25 |
| Phoenix | 0.25 | 0 | 0 | 0 | 1 |
| Portland | 0.16 | 0 | 0.7 | 0 | 1 |
| San Francisco[a] | 0 | 0 | 0 | 0 | 1 |
| Seattle | 0 | 0 | 0 | 0 | 3.38 |

[a]Efficient DUMs.

**Figure 1** Visual computation results (see online version for colours)




## 5 Conclusions and future research directions

Over the past few decades, increased rivalry caused by globalisation and rapid technological advances has motivated organisations to improve their SCM efficiency. Nevertheless, many companies have failed to devote adequate resources for managing their SCM activities. Effective SCM requires effective purchasing strategies and effective purchasing is dependent on optimal supplier selection decisions. In today's competitive operating environment, it is very difficult to produce low cost, high quality products without having satisfactory suppliers.

Purchasing materials have long been recognised as a multi-criteria problem. A concurrent consideration of multiple criteria complicates the supplier selection decisions, even for experienced managers. Competing suppliers have different levels of success under multiple criteria. For example, the supplier with the lowest cost in a given market may not have the best delivery performance or product quality. Although multiple criteria supplier selection problems have been addressed in the literature, the emergence of a non-discretionary and stochastic situation has been an obstacle for procurement managers searching for the best suppliers. The contributions of this paper are fourfold:

1 we proposed a chance-constrained DEA model for supplier selection

2 we considered multiple and conflicting criteria in supplier selection decisions

3 we incorporated non-discretionary factors and stochastic data in DEA

4 we presented a case study and demonstrated the applicability of the method to supplier selection decisions.

In this paper, a new approach was proposed to assist the decision makers to determine the most efficient suppliers in the presence of both non-discretionary factors and stochastic data. The problem considered in this study is at the initial stage of investigation and further research can be done based on results of this paper. Some of them are as follows:

1 similar research can be repeated for supplier selection in the presence of both stochastic data and fuzzy data

2 similar research can be conducted for supplier selection in the presence of both undesirable factors and stochastic data

3 similar research can be performed for supplier selection in the presence of both stochastic data and slightly non-homogeneous DMUs

4 this study applied the proposed model to a supplier selection problem.

The proposed model is generic and can be applied to additional problem domains, such as market segmentation decisions, personnel selection decisions and location planning decisions.

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## Note

[1] The name of the company is changed to protect its anonymity.