IMA Journal of Management Mathematics (2026) **37**, 61–93
https://doi.org/10.1093/imaman/dpaf033
Advance Access publication on 22 August 2025

Forward-looking beyond expectations: anticipating outcomes in environments with multiattribute alternatives

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[Received on 25 July 2024; accepted on 07 August 2025]

Accepted by: Ali Emrouznejad

The information retrieval behaviour of decision-makers (DMs) dealing with alternatives defined by multiple characteristics is generally determined by the expectations operator. Even when considering decision trees in sequential evaluation environments, the central assumptions imposed state that given a probability function defined on the set of potential realizations from a given alternative, the expected value suffices as a decision-making operator. However, suppose DMs evaluate the combinations of multiple characteristics relative to their expected values when selecting an alternative. Why do not they incorporate these combinatorial possibilities beforehand into their information retrieval process? We analyse the consequences of incorporating this combinatorial behaviour within an online information retrieval environment and illustrate the differences in utility that arise when evaluating sets of alternatives of different cardinality. We simulate 1,000,000 runs for different selection settings determined by the information retrieved by DMs before choosing and computing the utility received from implementing a standard expected utility and a combinatorial forward-looking strategy. We illustrate how this latter strategy pays off if DMs behave according to the postulates defining rational behaviour through the whole decision process. This requirement implies evaluating the entire set of characteristics composing the alternatives before making a decision and considering their corresponding certainty equivalents as a benchmark to evaluate the alternatives. Finally, we use the information retrieval profiles generated to analyse the capacity of an artificial neural network to categorize DMs across the different evaluation and selection settings correctly.

Keywords: multiattribute sequential search; expected utility; decision trees; risk attitude; artificial neural network; uncertainty.

1. Introduction

The standard expected utility approach, widely used in economics and decision sciences, assumes that decision-makers (DMs) are rational and that both present and future events shape their choices. Future events are incorporated through expected values derived from potential realizations of the characteristics defining the available alternatives (Cushman & Morris, 2015; He *et al.*, 2022). A simple example illustrates this basic idea. Consider a DM deciding whether to evaluate an alternative online before purchasing it. The DM lacks any information on the alternative. He only knows the distribution of potential realizations and the fact that the alternative has two types of characteristics, some of which can be observed immediately by reading the snippets and others that would become evident after clicking on the corresponding link (Wang *et al.*, 2018; Gönsch, 2020). Both types of characteristics are independent. That is, the observation of the initial realizations does not condition the expectations of the second characteristic. This assumption can be eliminated and conditional densities defined, but the intuition would become more cumbersome, while the main analytical results described would remain unchanged.

Assume that uncertainty prevails through both sets of characteristics and the DM assigns a uniform density to their respective domains. This assumption implies that the DM knows the expected values derived from every characteristic composing each alternative. Thus, we expect DMs to proceed with evaluating a given alternative whenever the initial realization describing one of its characteristics—or a subset of them—is above the expected value, defining a retrieval sequence considered intuitively correct in the economics and decision-making literature (Etco *et al.*, 2017; Zhang *et al.*, 2020a,b). This type of behaviour also guides DMs through decision trees, where they must account for the risk faced at each node when determining their evaluation paths (De Reyck *et al.*, 2008; Dey, 2012; Di Caprio *et al.*, 2022a).

An important simplification is implicitly imposed on DMs when facing the initial alternative or decision node; namely, expectations do not incorporate the effect of the potential realizations derived from the second evaluation when interacting with the realizations from the first evaluation. That is, the decision made by the DM is determined by the value of the initial realization relative to its expected value. This is the case since whenever the first observation is being retrieved, the second evaluations are assumed to be given by the expected value of the potential realizations. Thus, the behaviour of the DM is completely determined by the realizations of the first characteristic relative to its expected value.

An interesting phenomenon arises whenever the DM proceeds with the second set of characteristics defining the alternative. Given the value of the first realizations, those of the second characteristics eliminate any uncertainty from the evaluation of the alternative, and the DM knows enough to make an informed decision. Thus, it seems plausible to assume that whenever the sum of the realizations of both characteristics is above that of the expected values, the DM proceeds with the alternative being considered. This quality also allows the DM to determine whether evaluating the second characteristics of a given alternative has proven to be the correct decision, as opposed to discarding the alternative and proceeding with a new one.

We focus on this feature of the retrieval and evaluation process. That is, we incorporate the whole set of potential realizations that may be observed at the second evaluation node within the initial decision query faced by the DM. The effects derived from this enhanced forward-looking model are compared with those of the standard expected utility scenario commonly applied in economics and management literature.

1.1. *Literature review*

The literature on information retrieval has grown consistently in recent years, formalizing the mechanisms underlying information-seeking and exploring cognitive strategies and biases in the behaviour of

DMs (Qi *et al.*, 2023, 2024). The areas of analysis are quite heterogeneous and consider a substantial variety of applications. For instance, Gorichanaz (2024) developed a conceptual framework for the introduction of intellectual virtues, enabling good thinking and wise reasoning in online search environments. Ming *et al.* (2021) analysed different industrial applications of information acquisition processes subject to uncertainty arising from several sources. Mashhady *et al.* (2022) designed and applied a decision-making procedure to select change agents within an organization.

The information retrieval behaviour of DMs when dealing with alternatives composed of more than one characteristic is generally based on the expectations operator and the subsequent expected values (Fan *et al.*, 2018; Chen & Liao, 2021; Liang *et al.*, 2021). Even when considering decision trees in sequential retrieval environments, the main assumptions imposed state that given a probability function defined on the set of potential realizations from a given alternative, the expected value suffices as an actual and correct decision-making operator on which DMs can base their choices (Aliev *et al.*, 2016; Tavana *et al.*, 2016). It is also assumed that rational DMs consider their expectations, whose values may be flawed or biased depending on the shape of the probability functions, as a determinant of their behaviour and that such a behavioural pattern is indeed optimal (Baker *et al.*, 2019; Power *et al.*, 2019; Lyu *et al.*, 2024).

In other words, a DM who considers the effect of future realizations when defining his current choices does so using the expected value obtained from a set of potential realizations together with their associated probabilities (Kim *et al.*, 2017; Kruis *et al.*, 2020; Kopa *et al.*, 2022). Whenever learning is allowed, these probabilities are updated at the different steps of the decision process (Soni *et al.*, 2016; Dzyabura & Hauser, 2019).

Decision trees, considered instruments summarizing the optimal behaviour of DMs facing sequential evaluation environments, are indeed solved recursively, assuming that DMs account for the expected value of any set of potential evaluations arising at each decision node (Danielson *et al.*, 2020). These expected values determine the behaviour of DMs, reflecting the risk faced and providing a simplified decision mechanism that allows them to behave accordingly. This is also the case under uncertainty, with uniform densities accounting for the information entropy faced by DMs when acquiring information before making a decision (Tavana *et al.*, 2015; Gu & Wang, 2022).

2. Contribution

We address the following question in detail: What would affect the retrieval behaviour of DMs if they were to incorporate and consider the combinatorial consequences of their current decision on the next one before making the current decision? In other words, an entirely rational DM should be able to compute and compare the consequences of their current decision on the next step of the evaluation process and incorporate the subsequent information to determine their current decision.

Indeed, when evaluating an alternative, the DM should compare each potential combination of the first and second characteristics to their certainty equivalent values and those that arise from discarding the alternative and proceeding through a different retrieval path. We define a formal decision model incorporating the consequences of a forward-looking strategy beyond standard expected utility maximization. That is, DMs consider the interactions between the potential realizations that can be observed after evaluating the first characteristics within the same and across alternatives before acquiring any information. These interactions determine the decisions DMs make and condition their information acquisition behaviour.

We analyse the consequences of incorporating this behaviour within an online sequential information retrieval framework and compute the differences in utility between our forward-looking scenario and

a standard expected utility one after DMs set out to observe a given set of alternatives. We evaluate the observations retrieved and the utility derived from both information acquisition strategies when simulating 1,000,000 runs for each selection set described below.

- First, we assume that DMs select those alternatives whose characteristics improve upon the sum of the utilities derived from the corresponding certainty equivalents.
- Second, we assume that DMs require both the first and second realizations to be higher than the respective forward-looking thresholds and certainty equivalent values.
- Third, we analyse the setting where the selection decision is based on the realization of the first set of characteristics.

The alternatives will be described through two different evaluation backgrounds; each characteristic may be composed of a unique element, based on a heuristic mechanism that averages several features, or the sum of two features and their corresponding convolution. In this regard, the model could be extended to consider three features per characteristic. However, the qualitative results obtained would not be modified, and the complexity of the presentation would increase substantially.

Finally, we analyse the accuracy displayed by an artificial neural network (ANN) when categorizing DMs in terms of their information retrieval behaviour, namely, when considering different numbers of alternatives to evaluate across retrieval decision settings within both expected utility and forward-looking scenarios.

The article proceeds as follows. The next section presents the main notations and assumptions used to define the expected utilities introduced in Section 4. Sections 5 and 6 analyse the threshold values that determine the behaviour of DMs when considering a unique feature or the sum of two per characteristic, respectively. Both scenarios are studied numerically in Section 7, together with the capacity of an ANN to correctly categorize DMs across the different evaluation and selection settings simulated. Section 8 presents several alternative retrieval scenarios and describes the main managerial implications of the model. Section 9 discusses the main results and suggests potential extensions. Section 10 summarizes the main results obtained and suggests additional extensions.

3. Notations and assumptions

The information retrieval model incorporates the subjective beliefs of DMs regarding the uncertainty faced and the subsequent spread of the domain on which the characteristics describing the alternatives are defined. We will also introduce the attitudes of DMs towards risk when determining their retrieval behaviour. The evaluation model describing the basic retrieval incentives of DMs builds on the one developed by Di Caprio & Santos Arteaga (2009). We restate the main assumptions in this section.

Let X_1 and X_2 represent the sets of all potential realizations that may be observed when evaluating the first and second characteristics of an alternative, respectively. We will assume that, for every $k = 1,\ 2$ there exist $0 \leq x_k^m < x_k^M$ such that $X_k = \left[x_k^m,\ x_k^M\right]$. The DMs are endowed with an additive utility function u whose components $u_k : X_k \to \Re$, $k = 1,\ 2$, are continuous and strictly increasing utility functions, such that $\forall \left(x_1, x_2\right) \in X_1 \times X_2$, $u\left(x_1, x_2\right) = u_1\left(x_1\right) + u_2\left(x_2\right)$. DMs assign a continuous probability density to each X_k, $\eta_k : X_k \to [0,\ 1]$, $k = 1,\ 2$, whose support, $S\left(\eta_k\right)$, consists of the set $\left\{x_k \in X_k : \mu_k\left(x_k\right) \neq 0\right\}$.

The densities $\eta_k\left(x_k\right)$, $k = 1,\ 2$, describe the subjective beliefs of DMs regarding the probability that a randomly selected alternative displays a value of $x_k \in X_k$ as its k-th characteristic. We will assume that the densities η_1 and η_2 are independent, though correlations across characteristics could be easily introduced in the model.

DMs consider the k-th certainty equivalent defined through η_k and u_k as the benchmark determining their information retrieval behaviour. In particular, the certainty equivalent induced by η_k and u_k, denoted by ce_k, $k = 1,\ 2$, is the value in X_k providing the DM with the same utility as the expected one derived from η_k and u_k. That is, for each $k = 1,\ 2$, $ce_k = u_k^{-1}\left(E_k\right)$, where E_k corresponds to the expected value of u_k. Thus, $u\left(ce_k\right) = u\left(u_k^{-1}\left(E_k\right)\right) = E_k$. The continuity and strict increasingness of u_k guarantee the existence and uniqueness of ce_k, respectively.

We must highlight two important remarks developed in the following sections. First, X_1 and X_2 could be interpreted as sets of product features classified within two main categories of characteristics. DMs could be assumed to add the value of the different features within each set and generate an average to which a uniform density is assigned. This heuristic simplifies the computational requirements imposed on DMs, particularly when evaluating alternatives encompassing multiple features. It will, therefore, constitute the main framework of analysis. On the other hand, we could introduce the convolution of several random variables defining the features that compose each category of characteristics. We will illustrate how the resulting formal complexities do not modify the main results obtained while imposing substantial computational requirements on the DMs.

Finally, the structure of the information retrieval environment implies that incorporating an additional category of characteristics to the analysis would require operating within a three-dimensional environment, a pattern that is maintained as additional categories are incorporated by DMs when evaluating the alternatives.

4. Expected utilities

After evaluating the initial characteristic of an alternative or the features composing the corresponding category, DMs must decide whether to continue retrieving information from the same alternative or discard it and start evaluating a different one. We will impose a basic behavioural assumption on the evaluation behaviour of DMs; that is, they consider the initial characteristic being evaluated relatively more important than the second one. In other words, their limited assimilation capabilities imply that DMs focus on the preferred characteristics when retrieving information from an alternative (Di Caprio *et al.*, 2014). Therefore, the initial characteristics deliver a higher expected utility than the remaining ones that define each alternative.

For instance, consider a DM who must acquire information on a series of products listed in a rank, determined by an initial online search, together with a description of their main characteristics and the corresponding reviews provided by previous DMs. If the products were hotels or houses for rent, the features considered more important by the DM could range from their price to their location or cleanliness. After evaluating one or some of these characteristics, the DM must decide whether to check any additional secondary features, which may range from amenities and the proximity of restaurants or shopping areas to the opinions of other DMs, or evaluate a new alternative from the list.

The information retrieval incentives of DMs are formalized via two real-valued expected utility functions defined on X_1. The sum of the expected utilities defined by $\left(u_1, \eta_1\right)$ and $\left(u_2, \eta_2\right)$, namely, $E_1 + E_2$, constitutes one of the main benchmark values determining the behaviour of these functions.

Consider a DM who has just observed and evaluated the initial characteristic from an alternative, x_1, and is deciding whether to continue evaluating its second characteristic, x_2. In this case, the gain in expected utility over $E_1 + E_2$ is determined by the value of x_1. That is, for each $x_1 \in X_1$, it is defined by

$$\Delta^+\left(x_1\right) = \left\{ x_2 \in X_2 \cap S\left(\eta_2\right) : u_2\left(x_2\right) > E_1 + E_2 - u_1\left(x_1\right) \right\} \tag{1}$$

and

$$\Delta^{-}\left(x_1\right)=\left\{\, x_2 \in X_2 \cap S\left(\eta_2\right) : u_2\left(x_2\right) \leq E_1+E_2-u_1\left(x_1\right)\right\} \tag{2}$$

the set of x_2 values whose combination with those of x_1 provide the DM with a higher or lower utility than that of a randomly chosen alternative, respectively.

The continuation function $C: X_1 \rightarrow \Re$ is defined as follows:

$$C\left(x_1\right) \stackrel{def}{=} \int\limits_{\Delta^{+}(x_1)} \eta_2\left(x_2\right)\left(u_1\left(x_1\right)+u_2\left(x_2\right)\right)\, dx_2+\int\limits_{\Delta^{-}(x_1)} \eta_2\left(x_2\right)\left(E_1+E_2\right)\, dx_2 \tag{3}$$

The continuation function describes the expected utility received from an alternative when observing any potential realization of x_2 for each and every value of x_1. In order to provide some intuition, note that the continuation function evaluated at the certainty equivalent value when considering a risk-neutral utility, $u_k\left(x_k\right)=x_k, k=1,\ 2$, equals

$$C\left(ce_1\right) \stackrel{def}{=} \int_{ce_2}^{x_2^M} \eta_2\left(x_2\right)\left(u_1\left(ce_1\right)+x_2\right)\, dx_2+\int_{x_2^m}^{ce_2} \eta_2\left(x_2\right)\left(E_1+E_2\right)\, dx_2 \tag{4}$$

Given that $u_1\left(ce_1\right)=E_1$ and the definition of the uniform density function—assigned to both characteristics to reflect the uncertainty faced by the DM (Kahneman & Tversky, 2000)—the continuation function simplifies to

$$C\left(ce_1\right) \stackrel{def}{=} \int_{ce_2}^{x_2^M}\left(\frac{1}{x_2^M-x_2^m}\right)\left(E_1+x_2\right)\, dx_2+\int_{x_2^m}^{ce_2}\left(\frac{1}{x_2^M-x_2^m}\right)\left(E_1+E_2\right)\, dx_2 \tag{5}$$

The discard function is determined by the potential improvements that may be realized relative to the evaluation of a given alternative x_1. If the DM discards the current alternative, a new one should be evaluated while remaining uncertain regarding the second characteristic of the initial product. For a DM to discard an alternative, he should receive a higher expected utility from starting to acquire information on a new alternative than from completely evaluating the alternative partially observed. Denote by y_1 the first characteristic of a new alternative different from the one being evaluated, then, for each value $x_1 \in X_1$, we can define

$$\Phi^{+}\left(x_1\right)=\left\{\, y_1 \in X_1 \cap S\left(\eta_1\right) : u_1\left(y_1\right)>\max\left\{u_1\left(x_1\right), E_1\right\}\right\} \tag{6}$$

and

$$\Phi^{-}\left(x_1\right)=\left\{\, y_1 \in X_1 \cap S\left(\eta_1\right) : u_1\left(y_1\right) \leq \max\left\{u_1\left(x_1\right), E_1\right\}\right\} \tag{7}$$

as the sets of y_1 values improving upon or lacking with respect to the maximum between x_1 and the expected utility derived from a random choice, E_1, respectively.

The discard function $D: X_1 \rightarrow \Re$ is defined as follows:

$$D\left(x_1\right) \stackrel{def}{=} \int\limits_{\Phi^{+}(x_1)} \eta_1\left(y_1\right)\left(u_1\left(y_1\right)+E_2\right)\, dy_1+\int\limits_{\Phi^{-}(x_1)} \eta_1\left(y_1\right)\left(\max\left\{u_1\left(x_1\right), E_1\right\}+E_2\right)\, dy_1 \tag{8}$$

The function evaluated at the certainty equivalent value equals

$$D\left(ce_1\right) \overset{def}{=} \int_{ce_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m}\right)\left(u_1\left(y_1\right) + E_2\right)\, dy_1 + \int_{x_1^m}^{ce_1} \left(\frac{1}{x_1^M - x_1^m}\right)\left(E_1 + E_2\right)\, dy_1 \tag{9}$$

This function defines the expected utility derived from discarding the alternative observed and retrieving information on a new one. The DM remains bound by the uncertainty derived from not observing the initial alternative fully and obtaining only partial information about the new alternative.

The continuation and discard functions highlight the fact that the decision made by the DM is determined by the relative values of $u_1\left(x_1\right) + u_2\left(x_2\right)$ and $E_1 + E_2$. Clearly, if the DM would not consider the potential interactions between both characteristics when evaluating the alternatives, the decision would be based on the values of $u_1\left(x_1\right)$ and E_1.

We will return to these definitions through the next section when illustrating the existence and uniqueness of a threshold value—determined by the continuation and discard functions—located below the certainty equivalent defined by $\left(u_1, \eta_1\right)$. Whenever both functions cross, $C\left(x_1^*\right) = D\left(x_1^*\right)$, we obtain a threshold value defining a change in the evaluation behaviour of DMs. We can guarantee that $\exists x_1^* \in X_1$ under standard behavioural assumptions (Di Caprio & Santos Arteaga, 2009). More importantly, we will provide conditions illustrating that, if a threshold point x_1^* exists, it does for a lower value than the certainty equivalent one. This result constitutes one of the main contributions of the paper and validates the numerical results obtained when considering standard distributions of characteristics.

THEOREM 1. Consider evaluation intervals X_1 and X_2 with identical upper limits, that is, $\left[x_1^m, x_1^M\right]$ and $\left[x_2^m, x_2^M\right]$, such that $x_1^M = x_2^M$. Assume complete uncertainty about the realizations of the characteristics, leading DMs to assign a uniform probability function to each evaluation interval. If DMs are risk-neutral, defining identical linear utilities on both characteristic spaces, $x_1^m > x_2^m$ and $x_1^m \leq ce_2$, a unique crossing point between $C\left(x_1\right)$ and $D\left(x_1\right)$ exists for $x_1 < u_1^{-1}\left(E_1\right)$.

A generalization of the conditions stated in the theorem to any utility function requires imposing further formal constraints that do not provide additional intuition regarding the actual behaviour of DMs. It should be highlighted that we are not endowing DMs with memory capacities but focusing on the differences in utility that arise after evaluating a given number of alternatives. We could, however, assume that DMs consider the observed alternative displaying the highest utility as a reference benchmark when determining their information acquisition behaviour.

The certainty equivalents could define the reference values DMs consider until an alternative delivering a higher utility is observed. This latter alternative would provide the reference benchmark for the subsequent evaluations until an alternative delivering a higher utility is observed. This evaluation process can be easily defined within the current retrieval framework. We are, however, focusing on the set of alternatives available to DMs after performing different evaluation processes and the expected utility derived when modifying their retrieval incentives.

The linear utility functions considered through the analysis are given by $u\left(x_i\right) = x_i$, $i = 1, 2$. That is, when defining preferred characteristics we will modify the beliefs of DMs and use them as a self-selection mechanism focusing on those alternatives located above a given minimum instead of adding another parameter, and denote it by $\alpha > 1$, to reflect the higher slope of the corresponding linear utility function, $u\left(x_i\right) = \alpha x_i$. In this case, the relative intensity of preferences would add another factor to the determinants of the results. Thus, we assume that DMs consider alternatives whose initial evaluations

are located at least above a given subjective minimum value. The higher this value, the more important the alternative and the higher the corresponding expected utility.

5. Threshold behaviour

In the current section, we formally illustrate that the crossing points—or thresholds—between the continuation and discard functions of risk-neutral DMs are defined for values lower than the certainty equivalent. This result highlights the main behavioural differences from the current scenario relative to the standard expected utility approach implemented by economists and decision theorists.

Except when indicated, the following simplifying features will be assumed to hold through this section. We will assume that the upper limits of the evaluation intervals are identical, $x_1^M = x_2^M$, while $x_1^m > x_2^m$, implying that $E_1 > E_2$. Moreover, each evaluation interval is assigned a uniform density to account for the existing uncertainty, and identical linear utilities are defined for both characteristic spaces.

We start by analysing the behaviour of the continuation and discard functions at the main reference points defining the evaluation intervals $\left[x_1^m, x_1^M\right]$ and $\left[x_2^m, x_2^M\right]$, which can be summarized as follows:

Lemma 1.

$$C\left(x_1^M\right) > D\left(x_1^M\right).$$

Proof. See proposition 7.4 in Di Caprio & Santos Arteaga (2009). □

Lemma 2.

$$\frac{dC\left(ce_1\right)}{dx_1} = \frac{dD\left(ce_1\right)}{dx_1}.$$

Proof. Note that $\frac{dC(ce_1)}{dx_1} = \frac{du_1(ce_1)}{dx_1}\mu_2\left(\left[ce_2, x_2^M\right]\right)$ and $\frac{dD(ce_1)}{dx_1} = \frac{du_1(ce_1)}{dx_1}\mu_1\left(\left[x_1^m, ce_1\right]\right)$. Both derivatives illustrate the result together with the fact that $ce_i = \frac{x_i^M + x_i^m}{2}$, $i = 1, 2$, which implies $\mu_1\left(\left[x_1^m, ce_1\right]\right) = \mu_2\left(\left[ce_2, x_2^M\right]\right)$. □

Lemma 3. Consider a risk-neutral DM that assigns uniform distributions to the domains, defining the potential realizations of both characteristics. Assume that these domains are identical, that is, $x_1^m = x_2^m$ and $x_1^M = x_2^M$. The $C\left(x_1\right)$ and $D\left(x_1\right)$ functions cross at $x_1 = ce_1$.

Proof. Under risk neutrality, $E_j = \left(\frac{x_j^M + x_j^m}{2}\right)$, $j = 1, 2$. It therefore follows immediately that $\int_{x_j^m}^{E_j} \eta_j\left(x_j\right) dx_j = \frac{E_j - x_j^m}{x_j^M - x_j^m} = \frac{1}{2}, j = 1, 2$. Clearly, the same intuition applies to $\int_{E_j}^{x_j^M} \eta_j\left(x_j\right) dx_j$.

As illustrated in Equation (5), the continuation function $C\left(x_1\right)$ evaluated at $x_1 = ce_1$ equals

$$\int_{ce_2}^{x_2^M} \left(\frac{1}{x_2^M - x_2^m}\right)\left(E_1 + u_2\left(x_2\right)\right) dx_2 + \int_{x_2^m}^{ce_2} \left(\frac{1}{x_2^M - x_2^m}\right)\left(E_1 + E_2\right) dx_2$$

with $u_j\left(ce_j\right) = E_j, j = 1, 2$. Developing and simplifying this equation we obtain

$$E_1 + \frac{E_2}{2} + \frac{\left(x_2^M\right)^2 - \left(E_2\right)^2}{2\left(x_2^M - x_2^m\right)} \tag{10}$$

The same intuition applies to the discard function $D\left(x_1\right)$ evaluated at $x_1 = ce_1$, which was described in Equation (9)

$$\int_{ce_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m}\right)\left(u_1\left(x_1\right) + E_2\right) dx_1 + \int_{x_1^m}^{ce_1} \left(\frac{1}{x_1^M - x_1^m}\right)\left(E_1 + E_2\right) dx_1$$

Developing and simplifying the above equation we obtain

$$E_2 + \frac{E_1}{2} + \frac{\left(x_1^M\right)^2 - \left(E_1\right)^2}{2\left(x_1^M - x_1^m\right)} \tag{11}$$

Clearly, for $C\left(ce_1\right)$ to be equal to $D\left(ce_1\right)$ we require

$$E_1 + \frac{E_2}{2} + \frac{\left(x_2^M\right)^2 - \left(E_2\right)^2}{2\left(x_2^M - x_2^m\right)} = E_2 + \frac{E_1}{2} + \frac{\left(x_1^M\right)^2 - \left(E_1\right)^2}{2\left(x_1^M - x_1^m\right)} \tag{12}$$

For this equality to hold, we just need the domains to be identical, that is, $x_2^M = x_1^M$ and $x_2^m = x_1^m$, which implies that $E_1 = E_2$. As a result, the above expression simplifies to

$$\left(x_2^M\right)^2 - \left(E_2\right)^2 = \left(x_1^M\right)^2 - \left(E_1\right)^2 \tag{13}$$

which is trivially satisfied, implying that the continuation and discard functions cross at $x_1 = ce_1$. □

We illustrate now how as the importance assigned to the first characteristic increases, that is, as the domain on which this characteristic is defined shrinks while preserving the same upper limit, $C\left(ce_1\right) > D\left(ce_1\right)$. As a result, the next proposition—together with the first two lemmas—implies that if a crossing point between both functions exists, it is located below ce_1.

PROPOSITION 1. $C\left(ce_1\right) > D\left(ce_1\right)$ can be guaranteed if $x_2^m = 0$.

Proof. Given the simplified expression described in Equation (12), we require that as the interval $\left[x_1^m, x_1^M\right]$ shrinks from below, leading to $E_1 > E_2$, the following inequality holds

$$\frac{E_1}{2} + \frac{\left(x_2^M\right)^2 - \left(E_2\right)^2}{2\left(x_2^M - x_2^m\right)} > \frac{E_2}{2} + \frac{\left(x_1^M\right)^2 - \left(E_1\right)^2}{2\left(x_1^M - x_1^m\right)} \tag{14}$$

Even though $E_1 > E_2$, the assumption $x_2^M = x_1^M$ does not suffice to guarantee that the inequality holds since $\frac{\left(x_2^M\right)^2 - (E_2)^2}{2\left(x_2^M - x_2^m\right)} < \frac{\left(x_1^M\right)^2 - (E_1)^2}{2\left(x_1^M - x_1^m\right)}$. Developing the full expression described in Equation (14), we

find that the inequality simplifies to

$$\left(x^M\right)^2\left(3x_2^m+x_1^m\right)+3\left(x_2^m\right)^2\left(x^M-x_1^m\right)+\left(x_1^m\right)^2\left(x_2^m-x^M\right)-4x^Mx_1^mx_2^m>0 \tag{15}$$

Even in the linear case being considered, this inequality cannot be guaranteed without imposing additional assumptions. Thus, besides $x_2^M=x_1^M$, requiring that $x_2^m=0$ suffices for the inequality to hold. That is, we require identical upper interval limits and a reference value of $x_2^m=0$ for $C\left(ce_1\right)>D\left(ce_1\right)$ as the first evaluation interval shrinks. □

Given the results presented in the first two lemmas and the fact that $C\left(ce_1\right)>D\left(ce_1\right)$, if the functions $C\left(x_1\right)$ and $D\left(x_1\right)$ cross, they must do so for a value of x_1 lower than ce_1. This result is validated by the following facts

- $C\left(x_1^M\right)>D\left(x_1^M\right)$, as stated in Lemma 1;
- $\frac{dC(ce_1)}{dx_1}=\frac{dD(ce_1)}{dx_1}$, as stated in Lemma 2;
- $\mu_2\left(\left[u_2^{-1}\left(E_1+E_2-u_1\left(x_1\right)\right),x_2^M\right]\right)$ and $\mu_1\left(\left[x_1^m,x_1\right]\right)$ increase as x_1 increases above ce_1;
- $\frac{dD(x_1^M)}{dx_1}>\frac{dC(x_1^M)}{dx_1}$, since $x_1^M=x_2^M$ and $x_1^m>x_2^m$ imply $\mu_1\left(\left[x_1^m,x_1^M\right]\right)>\mu_2\left(\left[u_2^{-1}\left(E_1+E_2-u_1\left(x_1^M\right)\right),x_2^M\right]\right)$.

An additional step is required to prove the existence and, with it, the uniqueness of a crossing point between both functions; namely, we need $C\left(x_1^m\right)<D\left(x_1^m\right)$, which requires analysing the behaviour of these functions at a final reference point.

PROPOSITION 2. $C\left(x_1^m\right)<D\left(x_1^m\right)$ can be guaranteed if $x_1^m\le ce_2$.
Proof. At x_1^m, the continuation function equals

$$C\left(x_1^m\right)=\int_{ce_1+ce_2-x_1^m}^{x_2^M}\left(\frac{1}{x_2^M-x_2^m}\right)\left(u_1\left(x_1^m\right)+u_2\left(x_2\right)\right)dx_2+\int_{x_2^m}^{ce_1+ce_2-x_1^m}\left(\frac{1}{x_2^M-x_2^m}\right)\left(E_1+E_2\right)dx_2 \tag{16}$$

Note that the integration limits of this function are defined for $x_2>ce_2$ since $ce_1>x_1^m$.
The value of $D\left(x_1\right)$ at x_1^m is given by

$$D\left(x_1^m\right)=\int_{ce_1}^{x_1^M}\left(\frac{1}{x_1^M-x_1^m}\right)\left(u_1\left(x_1\right)+E_2\right)dx_1+\int_{x_1^m}^{ce_1}\left(\frac{1}{x_1^M-x_1^m}\right)\left(E_1+E_2\right)dx_1 \tag{17}$$

Given the linearity and uniform probability assumptions, both functions are identical within their respective $\left[x_2^m,ce_2\right]$ and $\left[x_1^m,ce_1\right]$ domains since they accumulate the same probability mass and the expected utilities are constant values.

Consider now the $x_1^m=ce_2$ case. The identical probability mass contained within the intervals $\left[ce_2,x_2^M\right]$ and $\left[ce_1,x_1^M\right]$ together with the assumption that $x_1^m=ce_2$ simplify the above expressions to

$$C\left(x_1^m\right)\big|_{\left[ce_2,x_2^M\right]}=\int_{ce_1}^{x_2^M}\left(\frac{1}{x_2^M-x_2^m}\right)u_2\left(x_2\right)dx_2+\left(\frac{ce_1-ce_2}{x_2^M-x_2^m}\right)E_1 \tag{18}$$

and

$$D\left(x_1^m\right)\big|_{[ce_1,x_1^M]} = \int_{ce_1}^{x_1^M} \left(\frac{1}{x_1^M - x_1^m}\right) u_1\left(x_1\right) dx_1 \tag{19}$$

After some algebra, we find that the $D\left(x_1^m\right)\big|_{[ce_1,x_1^M]} > C\left(x_1^m\right)\big|_{[ce_2,x_2^M]}$ requirement simplifies to $\left(x_1^M\right)^2 + \left(x_1^m\right)^2 - 2x_1^M x_1^m > 0$, which is equivalent to $\left(x_1^M - x_1^m\right)^2 > 0$.

The $x_1^m < ce_2$ case follows immediately from the previous analysis and the fact that $u_1\left(x_1^m\right) < E_2$ and $ce_1 + ce_2 - x_1^m > ce_1$. □

The previous propositions and lemmas allow us to conclude that a unique crossing point between $C\left(x_1\right)$ and $D\left(x_1\right)$ exists for $x_1 < u_1^{-1}\left(E_1\right)$ within the linear utility environment considered.

Generalizing this result to any utility function within the same evaluation framework, particularly, a concave utility describing risk-averse DMs, requires imposing the following condition in order for $C\left(ce_1\right) > D\left(ce_1\right)$

$$E_1 + \left(\frac{ce_2 - x_2^m}{x_2^M - x_2^m}\right)E_2 + \frac{\int_{ce_2}^{x_2^M} u_2\left(x_2\right) dx_2}{\left(x_2^M - x_2^m\right)} > E_2 + \left(\frac{ce_1 - x_1^m}{x_1^M - x_1^m}\right)E_1 + \frac{\int_{ce_1}^{x_1^M} u_1\left(x_1\right) dx_1}{\left(x_1^M - x_1^m\right)} \tag{20}$$

Clearly, the spread of the distribution of potential realizations conditions the above result, which states that if a crossing point between both functions exists, it is located below the certainty equivalent value. The above condition adds to those required for $C\left(x_1^m\right) < D\left(x_1^m\right)$ as the value of E_1 increases.

Figures 1 and 2 describe the behaviour of $C\left(ce_1\right)$ and $D\left(ce_1\right)$ as the value of x_1^m increases, implying that, in both the risk-neutral and risk-averse scenarios, if a threshold value exists, it is located below ce_1. In case the functions do not cross, we will have $C\left(x_1\right) > D\left(x_1\right)$, $\forall x_1 \in X_1$, providing a similar evaluation framework to the one being analysed through the article. The main qualitative results would remain unchanged, while quantitative modifications would arise as a result of the different crossing points, reinforcing the behavioural patterns obtained.

In order to present the main results intuitively, we will assume that the characteristics considered by the DMs are distributed within the following intervals: $\left[x_1^m, x_1^M\right] = [5, 10]$ and $\left[x_2^m, x_2^M\right] = [0, 10]$. Figures 3 and 4 illustrate the resulting evaluation frameworks for a risk-neutral and a risk-averse DM, respectively.

6. Convolution of two random variables per characteristic

We now consider a more complex retrieval framework where DMs incorporate the convolution that results from the sum of two random variables within each space of characteristics into the analysis. We will assume that DMs must still evaluate two characteristics per alternative. However, in this case, each characteristic consists of the sum of two independent random variables, defining four-dimensional alternatives evaluated through two distinct retrieval stages.

The next set of computations requires defining the convolution of two uniform random variables, which consists of the density assigned to their sum. That is, assume that X_A and X_B are uniformly distributed on their corresponding intervals, $X_A \sim U\left[x_A^m, x_A^M\right]$ and $X_B \sim U\left[x_B^m, x_B^M\right]$, with associated densities.

$$f_{X_A}\left(x_A\right) = \begin{cases} \dfrac{1}{x_A^M - x_A^m} & \text{if } x_A^m \le x_A \le x_A^M \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

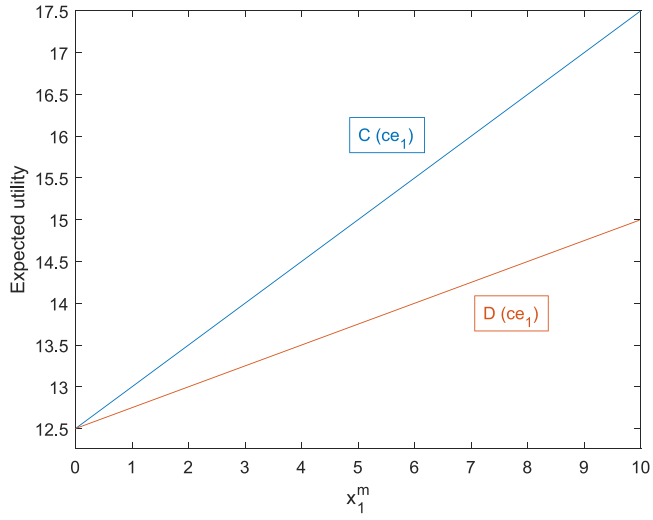


FIG. 1 Continuation and discard functions at the certainty equivalent with $\left[x_2^m, x_2^M\right] = [0, 10]$ and $\left[x_1^m, x_1^M\right] = \left[x_1^m, 10\right]$ for $x_1^m \in [0, 10]$. The $u(x) = x$ case.

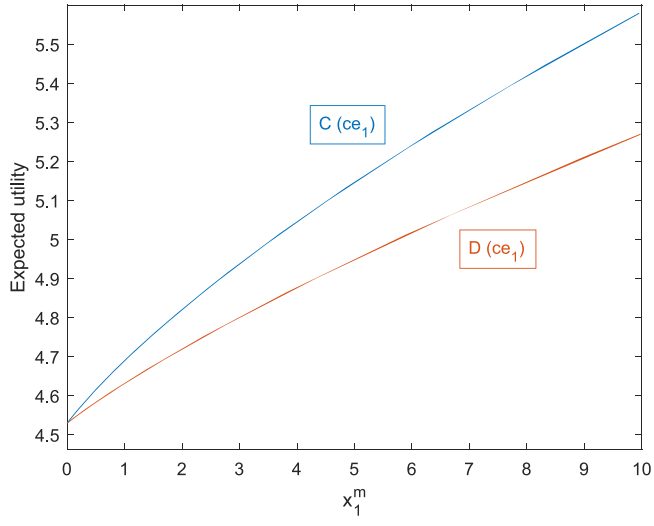


FIG. 2 Continuation and discard functions at the certainty equivalent with $\left[x_2^m, x_2^M\right] = [0, 10]$ and $\left[x_1^m, x_1^M\right] = \left[x_1^m, 10\right]$ for $x_1^m \in [0, 10]$. The $u(x) = \sqrt{x}$ case.

and similarly for $f_{X_B}\left(x_B\right)$. The sum of both variables, $X_1 = X_A + X_B$, is associated with the following density function

$$f_{X_1}\left(x_1\right) = \int_{-\infty}^{\infty} f_{X_A}\left(x_1 - x_B\right) f_{X_B}\left(x_B\right) dx_B \tag{22}$$

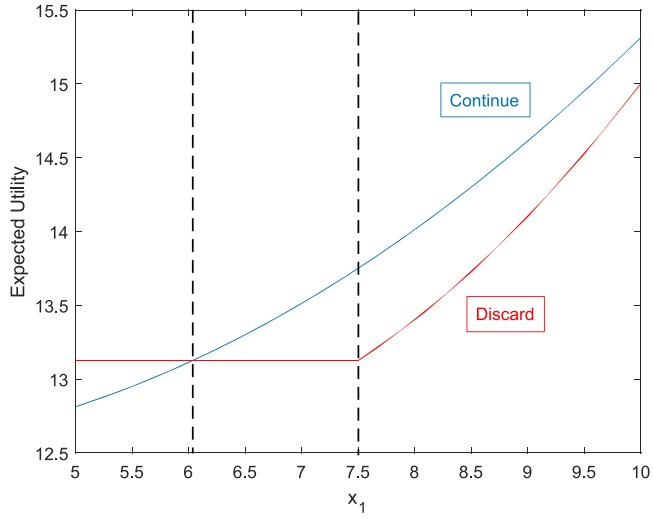


FIG. 3 Threshold values with $u(x) = x$, $\left[x_1^m, x_1^M\right] = [5, 10]$ and $\left[x_2^m, x_2^M\right] = [0, 10]$.

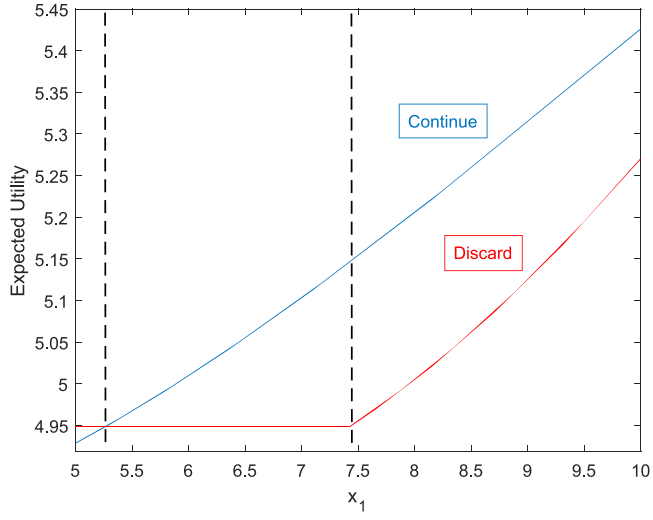


FIG. 4 Threshold values with $u(x) = \sqrt{x}$, $\left[x_1^m, x_1^M\right] = [5, 10]$ and $\left[x_2^m, x_2^M\right] = [0, 10]$.

with $x_1 = x_A + x_B$. Note that when $x_A^M + x_B^m = x_A^m + x_B^M$ the density of the sum of both random variables equals (Killmann & von Collani, 2001)

$$f_{X_1}(x_1) = \begin{cases} 0 & \text{for } x_1 < x_A^m + x_B^m \\ \dfrac{x_1 - \left(x_A^m + x_B^m\right)}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)} & \text{for } x_A^m + x_B^m \leq x_1 < x_B^m + x_A^M = x_A^m + x_B^M \\ \dfrac{\left(x_A^M + x_B^M\right) - x_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)} & \text{for } x_A^m + x_B^M = x_B^m + x_A^M \leq x_1 < x_A^M + x_B^M \\ 0 & \text{for } x_A^M + x_B^M \leq x_1 \end{cases} \tag{23}$$

The same definitions can be applied to X_2, with a convolution function given by $f_{X_2}(x_2)$, derived from the sum of $X_C \sim U\left[x_C^m, x_C^M\right]$ and $X_D \sim U\left[x_D^m, x_D^M\right]$, such that $X_2 = X_C + X_D$. These functions allow us to work within a two dimensional environment that will be used to define a unique threshold between the continuation and discard functions.

We implement both convolutions within our retrieval model, whose utilities have to be categorized based on the value of the certainty equivalent of the first characteristic. That is, given a risk-neutral environment, the continuation function corresponding to realizations of the first characteristic lower than the certainty equivalent value, $x_1 \leq ce_1$, is given by

$$\begin{aligned} C(x_1) \overset{def}{=} & \int_{\Delta^+(x_1)} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1(x_1) + u_2(x_2)\right)\, dx_2 \\ & + \int_{\Delta^-(x_1)} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \\ & + \int_{\Delta^-(x_1)} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \end{aligned} \tag{24}$$

while for values of $x_1 > ce_1$ the function becomes

$$\begin{aligned} C(x_1) \overset{def}{=} & \int_{\Delta^+(x_1)} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1(x_1) + u_2(x_2)\right)\, dx_2 \\ & + \int_{\Delta^+(x_1)} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1(x_1) + u_2(x_2)\right)\, dx_2 \\ & + \int_{\Delta^-(x_1)} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \end{aligned} \tag{25}$$

The previous equations introduce $f_{X_1}(x_1)$ within the corresponding expected utilities, while accounting for the relative value of the realizations defining the characteristics. When incorporating the value of the sets defining the corresponding domains of integration, we obtain the following equations

$$\begin{aligned} C(x_1) \overset{def}{=} & \int_{E_1+E_2-u_1(x_1)}^{x_2^M = x_C^M + x_D^M} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1(x_1) + u_2(x_2)\right)\, dx_2 \\ & + \int_{E_2}^{E_1+E_2-u_1(x_1)} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \\ & + \int_{x_2^m = x_C^m + x_D^m}^{E_2} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \end{aligned} \tag{26}$$

for $x_1 \leq ce_1$ and

$$C\left(x_1\right) \stackrel{def}{=} \int_{E_2}^{x_2^M = x_C^M + x_D^M} \left[\frac{\left(x_C^M + x_D^M\right) - x_2}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1\left(x_1\right) + u_2\left(x_2\right)\right)\, dx_2$$
$$+ \int_{E_1+E_2-u_1(x_1)}^{E_2} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(u_1\left(x_1\right) + u_2\left(x_2\right)\right)\, dx_2$$
$$+ \int_{x_2^m = x_C^m + x_D^m}^{E_1+E_2-u_1(x_1)} \left[\frac{x_2 - \left(x_C^m + x_D^m\right)}{\left(x_C^M - x_C^m\right)\left(x_D^M - x_D^m\right)}\right] \left(E_1 + E_2\right)\, dx_2 \tag{27}$$

for $x_1 > ce_1$. Note that the functions must account for the value of the density relative to the certainty equivalent and the initial realization observed by the DM, together with their effect on the domain of the second characteristic introduced via $\Delta^+\left(x_1\right)$ and $\Delta^-\left(x_1\right)$.

The same intuition applies to the discard function, which for realizations of x_1 located below the certainty equivalent value equals

$$D\left(x_1\right) \stackrel{def}{=} \int_{\Phi^+(x_1)} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(y_1\right) + E_2\right)\, dy_1$$
$$+ \int_{\Phi^-(x_1)} \left[\frac{y_1 - \left(x_A^m + x_B^m\right)}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(E_1 + E_2\right)\, dy_1 \tag{28}$$

while for realizations of x_1 located above the certainty equivalent it becomes

$$D\left(x_1\right) \stackrel{def}{=} \int_{\Phi^+(x_1)} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(y_1\right) + E_2\right)\, dy_1$$
$$+ \int_{\Phi^-(x_1)} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(x_1\right) + E_2\right)\, dy_1$$
$$+ \int_{\Phi^-(x_1)} \left[\frac{y_1 - \left(x_A^m + x_B^m\right)}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(x_1\right) + E_2\right)\, dy_1 \tag{29}$$

As in the continuation case, after introducing the convolution within the expected utilities, we incorporate the value of the corresponding sets defining the domains of integration to obtain the following

equations

$$D\left(x_1\right) \overset{def}{=} \int_{E_1}^{y_1^M = x_A^M + x_B^M} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(y_1\right) + E_2\right)\ dy_1$$
$$+ \int_{y_1^m = x_A^m + x_B^m}^{E_1} \left[\frac{y_1 - \left(x_A^m + x_B^m\right)}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(E_1 + E_2\right)\ dy_1 \tag{30}$$

for $x_1 \leq ce_1$ and

$$D\left(x_1\right) \overset{def}{=} \int_{x_1}^{y_1^M = x_A^M + x_B^M} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(y_1\right) + E_2\right)\ dy_1$$
$$+ \int_{E_1}^{x_1} \left[\frac{\left(x_A^M + x_B^M\right) - y_1}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(x_1\right) + E_2\right)\ dy_1$$
$$+ \int_{y_1^m = x_A^m + x_B^m}^{E_1} \left[\frac{y_1 - \left(x_A^m + x_B^m\right)}{\left(x_A^M - x_A^m\right)\left(x_B^M - x_B^m\right)}\right] \left(u_1\left(x_1\right) + E_2\right)\ dy_1 \tag{31}$$

for $x_1 > ce_1$. Similarly to the continuation case, the functions must account for the value of the density relative to the certainty equivalent and the initial realization observed by the DM, together with their effect on the domain of the first set of characteristics introduced via $\Phi^+\left(x_1\right)$ and $\Phi^-\left(x_1\right)$.

Thus, we can generate the same retrieval framework as the one developed in the previous section by assigning the following intervals to the corresponding random variables.

$$\left[x_A^m, x_A^M\right] = \left[x_B^m, x_B^M\right] = \left[\frac{x_1^m}{2}, \frac{x_1^M}{2}\right];$$
$$\left[x_C^m, x_C^M\right] = \left[x_D^m, x_D^M\right] = \left[\frac{x_2^m}{2}, \frac{x_2^M}{2}\right] \tag{32}$$

Figure 5 illustrates the threshold value obtained within the same numerical setting as the one considered through the initial analysis.

$$\left[x_A^m, x_A^M\right] = \left[x_B^m, x_B^M\right] = [2.5, 5];$$
$$\left[x_C^m, x_C^M\right] = \left[x_D^m, x_D^M\right] = [0, 5] \tag{33}$$

The following section illustrates numerically the main consequences of implementing the evaluation scenarios described and analyses the subsequent differences.

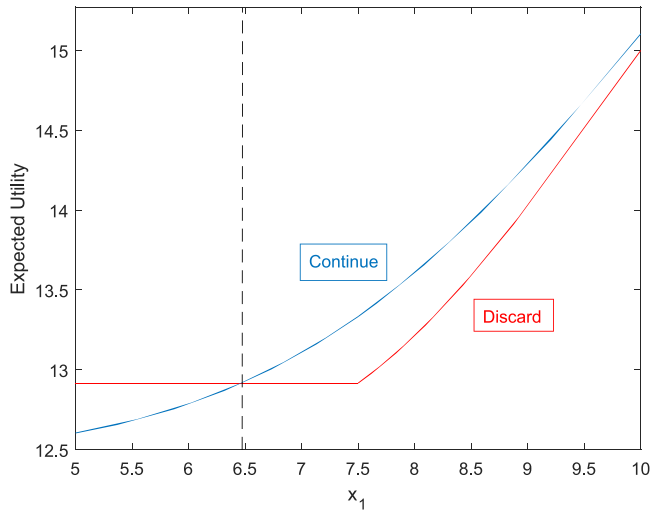


FIG. 5 Convolution scenario with $u(x) = x$, $\left[x_1^m, x_1^M\right] = [5, 10]$ and $\left[x_2^m, x_2^M\right] = [0, 10]$. The cutoff point equals 6.47.

7. Numerical results

We introduce an evaluation algorithm that incorporates the values of the different thresholds when determining the set of alternatives selected by the DMs. We maintain the numerical framework described in the previous sections and represented in Figs 3 and 4. As can be inferred from these figures, the realizations of the initial characteristics condition the behaviour of DMs through the search process. The selection of alternatives among those satisfying the initial threshold requirement will be determined by the utilities derived from the first and second characteristics relative to the corresponding certainty equivalent values.

Note that when considering the interactions across characteristics, DMs are willing to evaluate alternatives whose realization of the first characteristic is lower than the certainty equivalent value. As can be intuitively understood from Figs 1 and 2, the results remain qualitatively unchanged when modifying the spread of the domain on which the first characteristic is defined. We will analyse in detail a variety of spread scenarios together with the behaviour of the corresponding threshold values in Section 8.

The evaluation behaviour of DMs has been summarized in the flowchart described in Fig. 6. This flowchart represents the decision process of a DM when evaluating the initial characteristics of an alternative and deciding whether to continue retrieving information from the same alternative or discard it and start evaluating a new one. The reference values defining the risk-neutral environment are given by $ce_1 = 7.5$ and the intersection between the continuation and discard functions at $x_1^* = 6.0355$. In the risk-averse case, the certainty equivalent equals $ce_1 = 7.4292$ and the continuation and discard functions intersect at $x_1^* = 5.2695$.

Table 1 presents several strings of realizations retrieved from the output produced by the algorithm in the risk-neutral case. The algorithm collects the alternatives satisfying the threshold and utility requirements. It computes both the average number of clicks on the second page, which will be denoted by ‘scd’, and the average utility obtained relative to the certainty equivalent benchmark, which will be denoted by ‘utl’. We have normalized the risk-neutral evaluation spaces within the domain $[0, 1]$ to simplify comparisons across realizations and allow for an intuitive interpretation of the results. As a

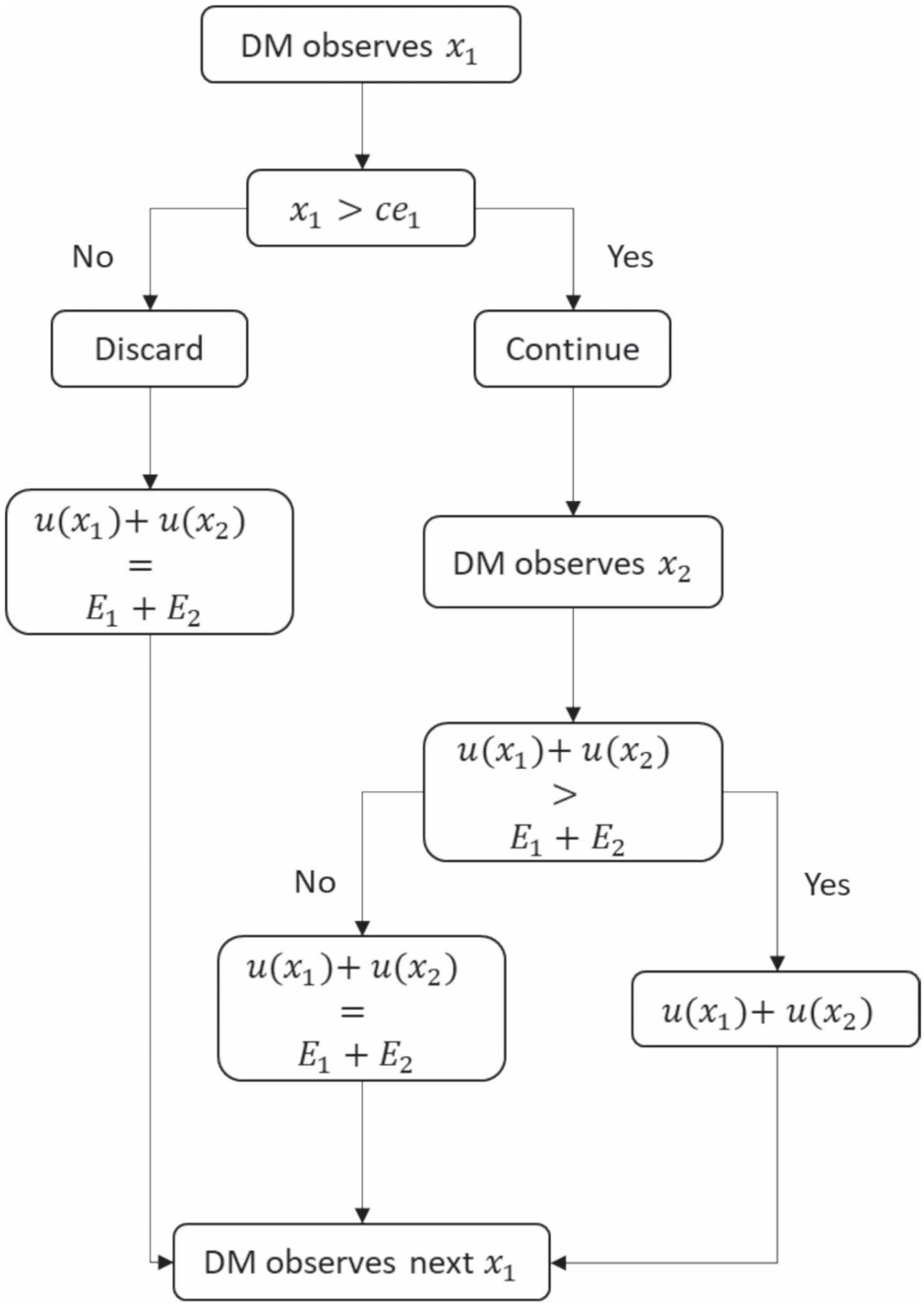


FIG. 6 Basic retrieval loop performed to evaluate the characteristics of an alternative.

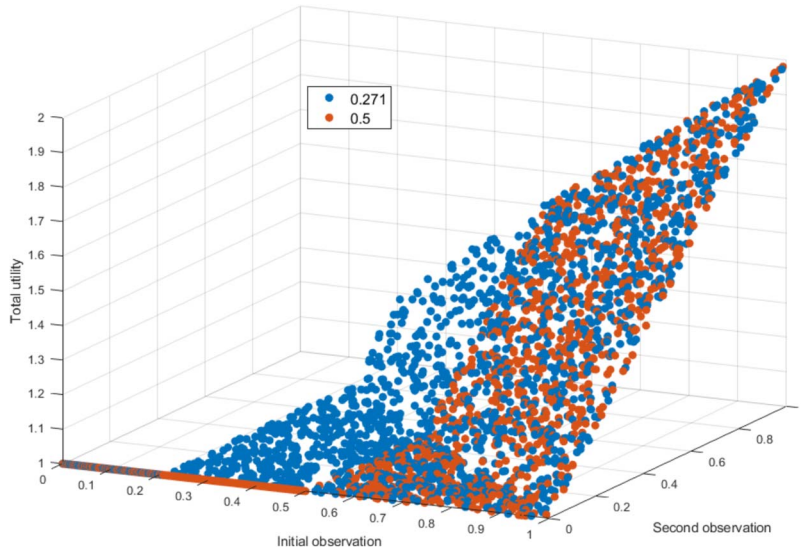


FIG. 7 Utility under risk neutrality: expected versus forward-looking scenarios.

result, the threshold values have been redefined as $x_1^* = 0.271$ for the forward-looking retrieval scenario and $x_1^* = 0.5$ for the expected utility one. We will make use of this feature when comparing utility values to the certainty equivalent alternative, which delivers a reference utility of one.

Certainty equivalents become particularly relevant as reference values when dealing with DMs exhibiting a behaviour different from risk neutrality. Figures 7 and 8 illustrate the realizations and total utilities obtained from 200 runs of the algorithm, each consisting of 10 alternatives evaluated within the risk-neutral and risk-averse environments, respectively. The differences in clicking behaviour between the forward-looking and the expected utility scenario are evident, as well as the patterns exhibited by the resulting utilities. As shown in Tables 2 and 3, the qualitative results obtained are similar when considering risk-neutral and risk-averse DMs.

The above results validate the consistency of the simulations from a retrieval perspective. The main results that follow from the analysis refer to the utility obtained by the different types of DMs across scenarios, how it is determined by their threshold-related behaviour and the capacity of an ANN to categorize DMs based on their retrieval behaviour correctly.

7.1. *Utility differences across scenarios*

The utility obtained per run of the algorithm depends on the number of evaluations the DM sets out to perform, ranging from 1 to 10, as can be observed in the first column of Tables 2 and 3. Each entry within these tables is the result of performing 1 million runs of the algorithm. The average number of times the second page was clicked is calculated by summing the clicks per run and dividing by 1,000,000. The average utility derived from the search process is computed by adding up and averaging the total utility from each algorithm run, then summing the averages and dividing by 1,000,000. Clearly, in all settings, the second page has been clicked more often in the forward-looking scenario. This result is not surprising, given the relative location of the threshold value compared to the expected utility scenario.

TABLE 1 *Algorithm output from the* $u(x_1) + u(x_2) > E_1 + E_2$ *setting*

	Type of Decision—Maker									
	Normalized Threshold = 0.5					**Normalized Threshold = 0.2071**				
	DM1	**DM2**	**DM3**	**DM4**	**DM5**	**DM1**	**DM2**	**DM3**	**DM4**	**DM5**
Initial realization	0.0975	0.0357	0.3171	0.6797	0.1493	0.3596	0.6683	0.7463	0.8076	0.9389
	0.2785	0.8491	0.9502	0.1626	0.2575	0.3829	0.0378	0.4425	0.4906	0.0218
	0.5469	0.6787	0.4387	0.1190	0.8407	0.9720	0.2858	0.1508	0.3650	0.8204
	0.9649	0.7431	0.3816	0.4984	0.8143	0.5053	0.7999	0.5403	0.8569	0.8674
	0.9706	0.6555	0.7655	0.9597	0.9293	0.1044	0.7374	0.7492	0.9515	0.5506
	0.4854	0.7060	0.1869	0.5853	0.1966	0.8884	0.0161	0.7045	0.0065	0.8500
	0.8003	0.2769	0.4898	0.7513	0.2511	0.4537	0.4255	0.6567	0.0419	0.2385
	0.4218	0.0462	0.4456	0.5060	0.6160	0.8508	0.7842	0.4911	0.8709	0.1350
	0.9157	0.0971	0.6463	0.8909	0.3517	0.9119	0.6899	0.9627	0.4425	0.5781
	0.9595	0.8235	0.7547	0.5472	0.8308	0.6160	0.5980	0.2086	0.9397	0.7712
Second page clicked (yes = 1)	0	0	0	1	0	1	1	1	1	1
	0	1	1	0	0	1	0	1	1	0
	1	1	0	0	1	1	1	0	1	1
	1	1	0	0	1	1	1	1	1	1
	1	1	1	1	1	0	1	1	1	1
	0	1	0	1	0	1	0	1	0	1
	1	0	0	1	0	1	1	1	0	1
	0	0	0	1	1	1	1	1	1	0
	1	0	1	1	0	1	1	1	1	1
	1	1	1	1	1	1	1	1	1	1
Second realization	0	0	0	0.6551	0	0.3718	0.3354	0.6939	0.0674	0.5310
	0	0.9340	0.0344	0	0	0.2563	0	0.4191	0.5984	0
	0.9575	0.7577	0	0	0.2543	0.6606	0.9312	0	0.6886	0.2875
	0.1576	0.3922	0	0	0.2435	0.1980	0.3311	0.1964	0.9006	0.8602
	0.9572	0.1712	0.7952	0.3404	0.3500	0	0.5674	0.7157	0.9000	0.4572
	0	0.0318	0	0.2238	0	0.3630	0	0.7363	0	0.3478
	0.1419	0	0	0.2551	0	0.0648	0.0294	0.8911	0	0.0509
	0	0	0	0.6991	0.4733	0.2794	0.9808	0.3418	0.4444	0
	0.7922	0	0.7094	0.9593	0	0.0598	0.7797	0.1249	0.3306	0.9044
	0.6557	0.6948	0.2760	0.1386	0.5853	0.5639	0.2846	0.1550	0.7128	0.7041
Total utility	u1	1	1	1.3348	1	1	1.0037	1.4401	1	1.4699
	1	1.7831	1	1	1	1	1	1	1.0890	1
	1.5044	1.4365	1	1	1.0950	1.6326	1.2170	1	1.0536	1.1079
	1.1225	1.1354	1	1	1.0578	1	1.1309	1	1.7575	1.7276
	1.9278	1	1.5607	1.3001	1.2792	1	1.3049	1.4648	1.8515	1.0079
	1	1	1	1	1	1.2514	1	1.4408	1	1.1978
	1	1	1	1.0064	1	1	1	1.5478	1	1
	1	1	1	1.2050	1.0893	1.1302	1.7650	1	1.3153	1
	1.7079	1	1.3557	1.8502	1	1	1.4696	1.0877	1	1.4825
	1.6152	1.5183	1.0307	1	1.4161	1.1799	1	1	1.6525	1.4753

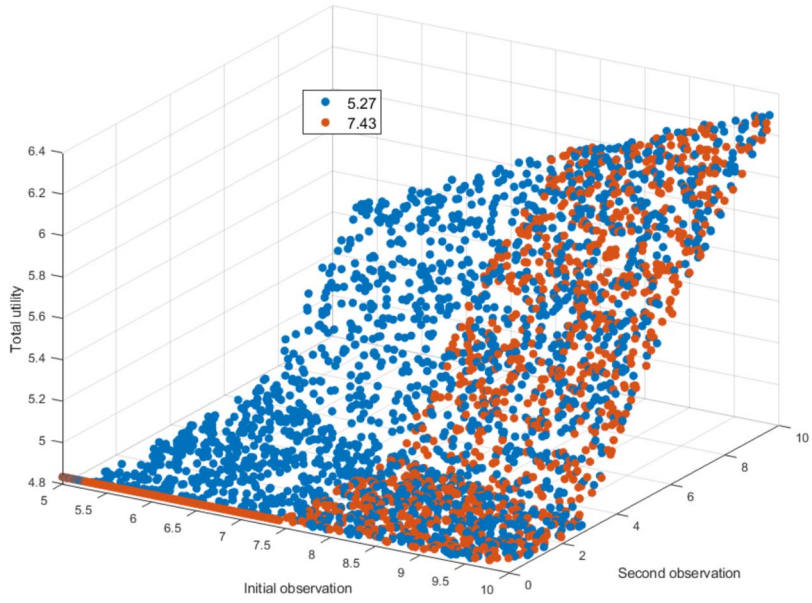


FIG. 8 Utility under risk aversion: expected versus forward-looking scenarios.

TABLE 2 *Risk-neutral utilities: expected versus forward-looking scenarios*

Setting	$u(x_1)+u(x_2) > E_1+E_2$				$x_1 > x_1^*$, $u(x_2) > E_2$ **and** $u(x_1)+u(x_2) > E_1+E_2$				$u(x_1) > E_1$			
Normalized Threshold	**0.5**		**0.2071**		**0.5**		**0.2071**		**0.5**		**0.2071**	
Alternatives	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**
One	0.4996	1.1456	0.7930	1.1655	0.4995	1.1249	0.7933	1.1437	0.4995	1.1248	0.7938	1.0820
Two	1.0001	1.1457	1.5858	1.1651	0.9996	1.1248	1.5855	1.1440	0.9999	1.1249	1.5858	1.0824
Three	1.5007	1.1458	2.3792	1.1652	1.5004	1.1250	2.3792	1.1444	1.4992	1.1247	2.3777	1.0820
Four	2.0015	1.1460	3.1721	1.1653	2.0011	1.1250	3.1718	1.1445	2.0011	1.1252	3.1713	1.0818
Five	2.5004	1.1460	3.9633	1.1651	2.5002	1.1250	3.9647	1.1444	2.5012	1.1249	3.9653	1.0820
Six	2.9979	1.1458	4.7564	1.1650	2.9996	1.1249	4.7575	1.1443	3.0008	1.1251	4.7570	1.0822
Seven	3.4980	1.1457	5.5493	1.1652	3.4985	1.1250	5.5461	1.1442	3.5005	1.1250	5.5500	1.0820
Eight	3.9993	1.1457	6.3443	1.1653	4.0001	1.1249	6.3422	1.1443	4.0023	1.1250	6.3440	1.0819
Nine	4.4987	1.1458	7.1346	1.1651	4.5014	1.1250	7.1366	1.1442	4.5007	1.1250	7.1344	1.0820
Ten	5.0010	1.1459	7.9294	1.1652	4.9992	1.1250	7.9295	1.1442	4.9981	1.1250	7.9286	1.0819

This tendency is indeed exacerbated in the risk-averse case, given the lower search aversion displayed by DMs in Fig. 4.

A more detailed analysis follows from Figs 9 and 10, which describe the differences in both the utility obtained and the number of times the second page was clicked within the expected utility (denoted by 'EU') and forward-looking ('FL') scenarios. Each value represents the average of the 10 entries reported

TABLE 3 *Risk-averse utilities: expected versus forward-looking scenarios*

Setting	$u(x_1)+u(x_2)>E_1+E_2$				$x_1>x_1^*$, $u(x_2)>E_2$ and $u(x_1)+u(x_2)>E_1+E_2$				$u(x_1)>E_1$			
Threshold	**7.4292**		**5.2695**		**7.4292**		**5.2695**		**7.4292**		**5.2695**	
Alternatives	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**	**scd**	**utl**
One	0.5137	5.0652	0.9462	5.1557	0.5147	5.0582	0.9460	5.1481	0.5141	4.9486	0.9463	4.8605
Two	1.0278	5.0650	1.8918	5.1554	1.0269	5.0582	1.8921	5.1489	1.0270	4.9488	1.8919	4.8594
Three	1.5424	5.0650	2.8385	5.1555	1.5425	5.0581	2.8378	5.1486	1.5418	4.9486	2.8380	4.8593
Four	2.0553	5.0651	3.7849	5.1554	2.0569	5.0582	3.7843	5.1488	2.0566	4.9487	3.7848	4.8587
Five	2.5705	5.0650	4.7303	5.1555	2.5694	5.0579	4.7296	5.1486	2.5704	4.9486	4.7302	4.8587
Six	3.0852	5.0651	5.6766	5.1555	3.0859	5.0584	5.6773	5.1486	3.0852	4.9489	5.6771	4.8585
Seven	3.5967	5.0650	6.6224	5.1555	3.5999	5.0584	6.6231	5.1489	3.5982	4.9492	6.6242	4.8583
Eight	4.1115	5.0649	7.5679	5.1554	4.1126	5.0585	7.5682	5.1485	4.1121	4.9486	7.5692	4.8586
Nine	4.6293	5.0651	8.5161	5.1554	4.6237	5.0581	8.5144	5.1485	4.6261	4.9488	8.5145	4.8585
Ten	5.1440	5.0653	9.4604	5.1556	5.1422	5.0583	9.4605	5.1490	5.1392	4.9487	9.4610	4.8584

in the corresponding columns of Tables 2 and 3. The current framework assumes that DMs set out to evaluate a predetermined number of alternatives and select among those providing a satisfying utility level, a decision categorized within three different selection settings.

The highest utility is obtained when the DM evaluates both characteristics and requires the initial one to be above the threshold value and their sum to be higher than the sum of the utility derived from the certainty equivalent values; that is, $u\left(x_1\right)+u\left(x_2\right)>E_1+E_2$. This is followed by the setting where DMs require the initial characteristic to be above the threshold value, the second to be higher than the certainty equivalent of the second characteristic, namely, $x_1>x_1^*$, $u\left(x_2\right)>E_2$ and $u\left(x_1\right)+u\left(x_2\right)>E_1+E_2$. This selection strategy discards alternatives whose total utility is higher than the certainty equivalent but which underperform in the second characteristic, namely, those where the value of the first characteristic compensates for the suboptimal realization of the second. The rejection assigns a certainty equivalent value to the alternative, reflecting the cost of an unsuccessful search. In both these settings, with either risk neutrality or aversion, forward-looking DMs derive a higher utility from the retrieval process, since they are able to select from a larger set of alternatives.

There is, however, a drawback to being more willing to continue acquiring information. Whenever the DM does not verify the second characteristic of the alternative, that is, in the $u\left(x_1\right)>E_1$ setting, the utility derived from the forward-looking scenario is lower than the one obtained from the expected utility scenario. A relationship to impulsive buying could be established here (Zhao *et al.*, 2022), where the decision to select an alternative without validating all the available information leads to lower utility and potential regret. That is, DMs are required to analyse and validate the value of the second characteristic to benefit from a higher willingness to evaluate alternatives. If they behave accordingly, they will obtain a higher utility than in the expected utility scenario, even if they require the second characteristic to be above the certainty equivalent value.

Indeed, the highest utility is obtained within the selection setting where the sum of both characteristics is higher than the sum of the certainty equivalents, even if one of the characteristics is lower. In this case, the utility derived from the forward-looking and expected utility scenarios is the highest, contrasting with

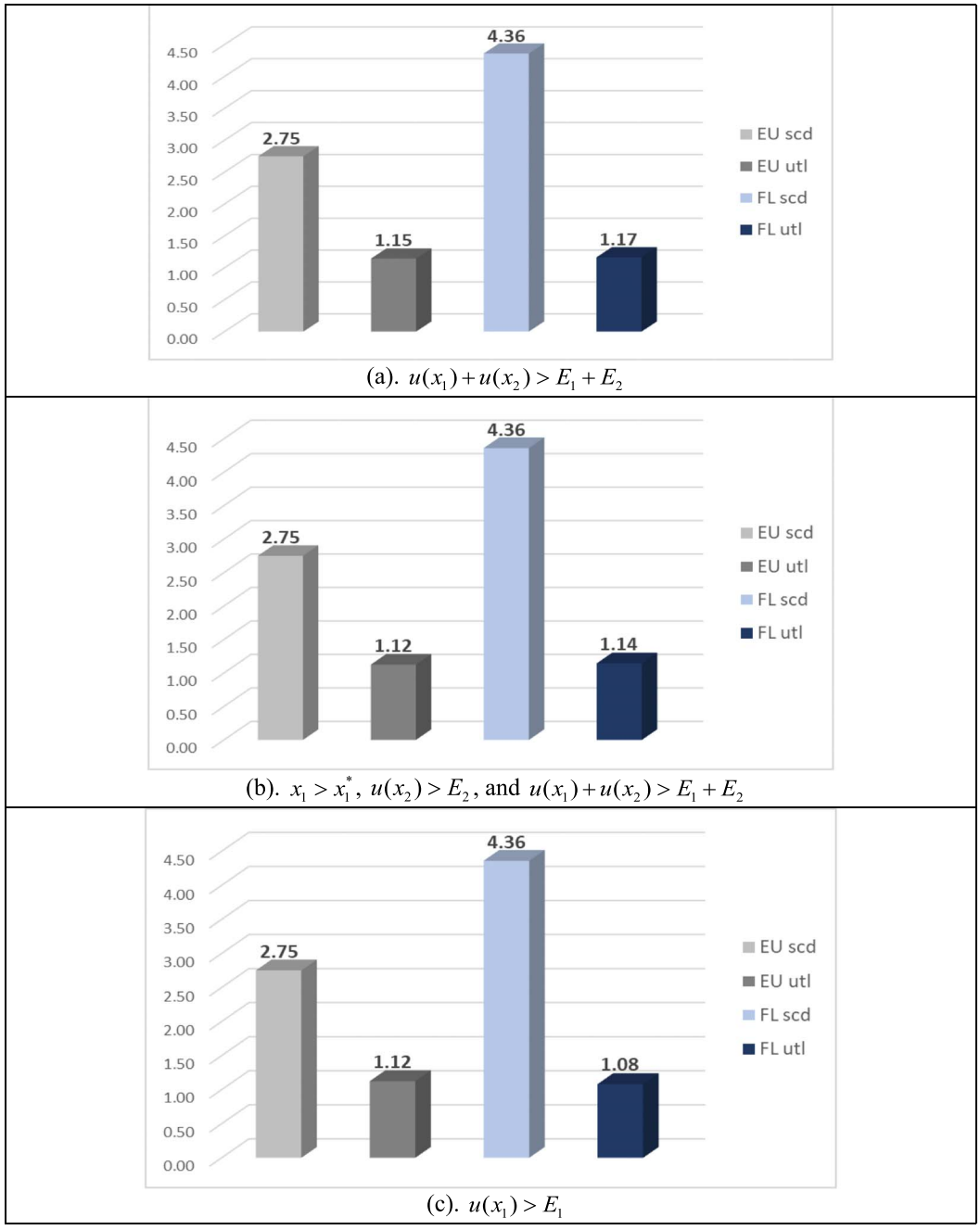
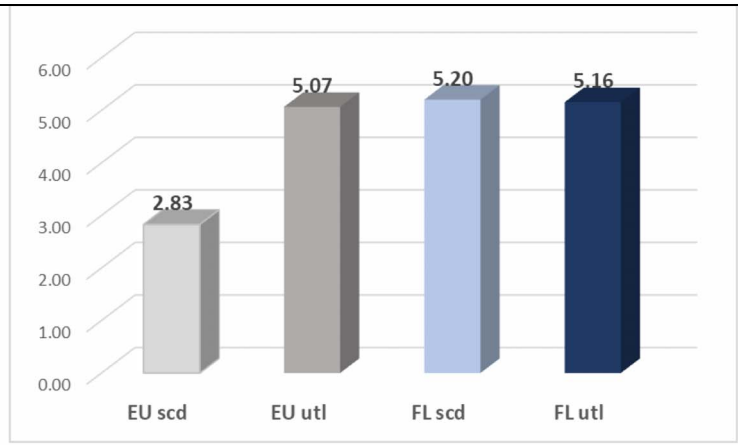
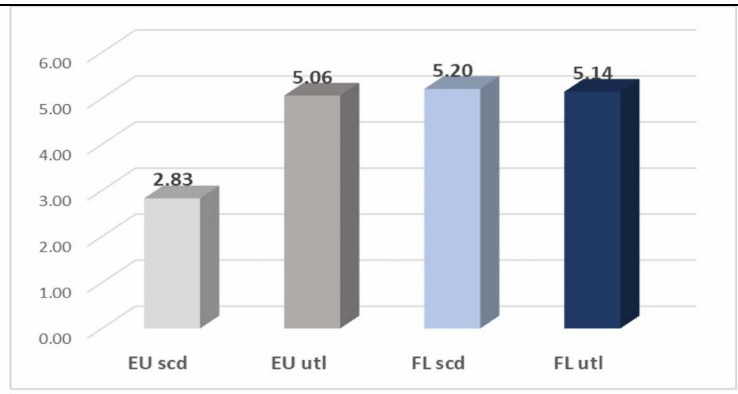


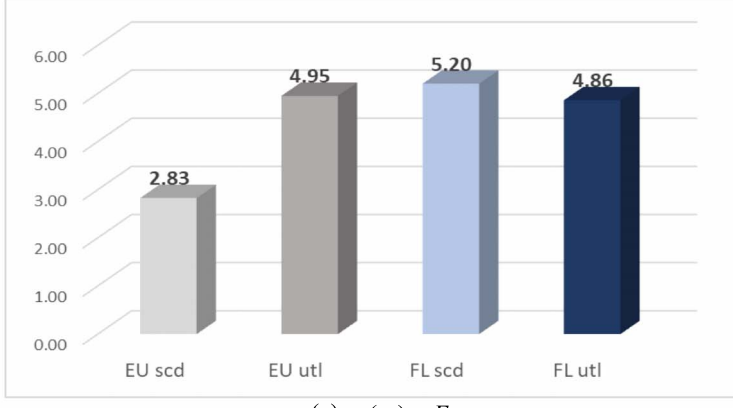
FIG. 9 Average values of the different utility indicators under risk neutrality.



(a). $u(x_1)+u(x_2)>E_1+E_2$



(b). $x_1>x_1^*$, $u(x_2)>E_2$, and $u(x_1)+u(x_2)>E_1+E_2$



(c). $u(x_1)>E_1$

FIG. 10 Average values of the different utility indicators under risk aversion.

the substantial decrease triggered when the second characteristic is not evaluated, particularly within the risk-averse environment.

Obtaining the highest utility requires DMs to operate within the forward-looking scenario, evaluate the second characteristic and be sufficiently flexible to allow for one of the characteristics to be below the certainty equivalent value as long as the sum of both characteristics is above it. Thus, additional rationality pays off as long as the DM continues to interact within the postulates defining rational behaviour, namely, evaluating the second set of characteristics and considering the total certainty equivalent as a benchmark determining the acceptance of an alternative.

The question addressed in the next section relates directly to the capacity of artificial intelligence techniques to differentiate among DMs through their retrieval profiles. If this were the case, firms could target DMs when displaying information based on their purchasing incentives, which are conditioned by both the subjective features defining their information retrieval behaviour and the utility derived from the subsequent search processes.

7.2. *DM categorization through artificial neural networks*

Here, we evaluate the capacity of an ANN to categorize DMs. That is, we use the profiles generated by the retrieval algorithms within each selection setting to train an ANN and evaluate its capacity to categorize DMs. The vectors used as inputs are retrieval profiles with the same structure as those described in Table 1. We generate a total of 18 retrieval categories determined by the risk attitude of DMs, the number of alternatives that they are willing to evaluate and the selection setting considered. This analysis can be easily extended to any machine learning technique to correctly categorize different types of DMs, by defining the corresponding selection settings.

Figures 11 and 12 present the confusion matrices obtained by implementing an ANN in MATLAB within the risk-neutral and risk-averse environments, respectively. In both figures, 70% of the profiles were used for training, 15% for validation and the remaining 15% for testing. The confusion matrices described in these figures encompass the following groups of categories, each of them composed of 100,000 retrieval vectors:

- Categories 1 to 6 correspond to the $u\left(x_1\right)+u\left(x_2\right) > E_1+E_2$ selection setting; the first three describe the cases where DMs evaluate 10, 6 and 2 alternatives within the standard expected utility scenario, while the three remaining categories correspond to the evaluation of 10, 6 and 2 alternatives within the forward-looking scenario.
- Categories 7 to 12 present the same distribution of DMs as the first six categories but within the $x_1 > x_1^*,\ u\left(x_2\right) > E_2$, and $u\left(x_1\right) + u\left(x_2\right) > E_1 + E_2$ selection settings.
- The same description applies to categories 13 to 18 within the $u\left(x_1\right) > E_1$ selection setting.

The number of alternatives evaluated has been selected to illustrate succinctly the differences in the categorization accuracy of the ANN as the number of elements defining the information retrieval profiles of DMs increases. The main results obtained would prevail if the whole set of alternatives described in Tables 2 and 3 were incorporated into the analysis. Note that we are providing the ANN with a considerable amount of information about 1,800,000 DMs distributed across 18 categories, namely, a numerical evaluation of the initial characteristics observed, the subsequent decision to further evaluate the alternative, a numerical evaluation of the second characteristics and the total utility obtained.

The way information is distributed through the retrieval vectors also describes the total number of alternatives evaluated by DMs. That is, the 'Initial realization' section defines the retrieval vector of a DM

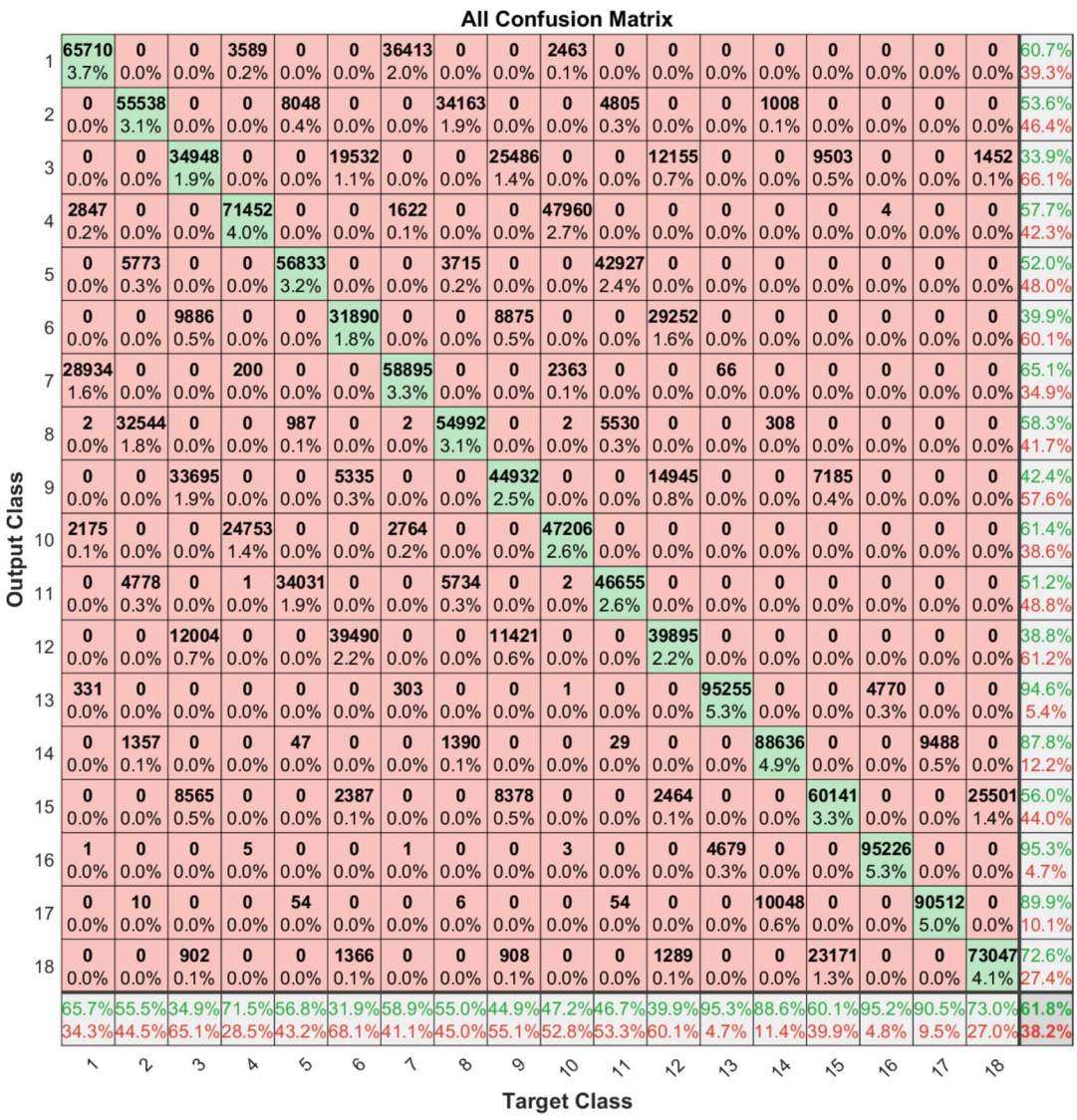


FIG. 11 Confusion matrix delivered by the ANN within the risk-neutral environment.

who evaluates six alternatives, which consist of six initial values different from zero, followed by four zeroes. This quality has immediate consequences for the entries obtained in the 'Second page clicked', 'Second realization' and 'Total utility' subsections of the retrieval vectors. Despite these simplifying features, the ANN underperforms when differentiating among categories with 10, 6 and 2 alternatives across selection settings and between expected utility and forward-looking scenarios. This trait of the ANN prevails in both the risk-neutral and risk-averse environments.

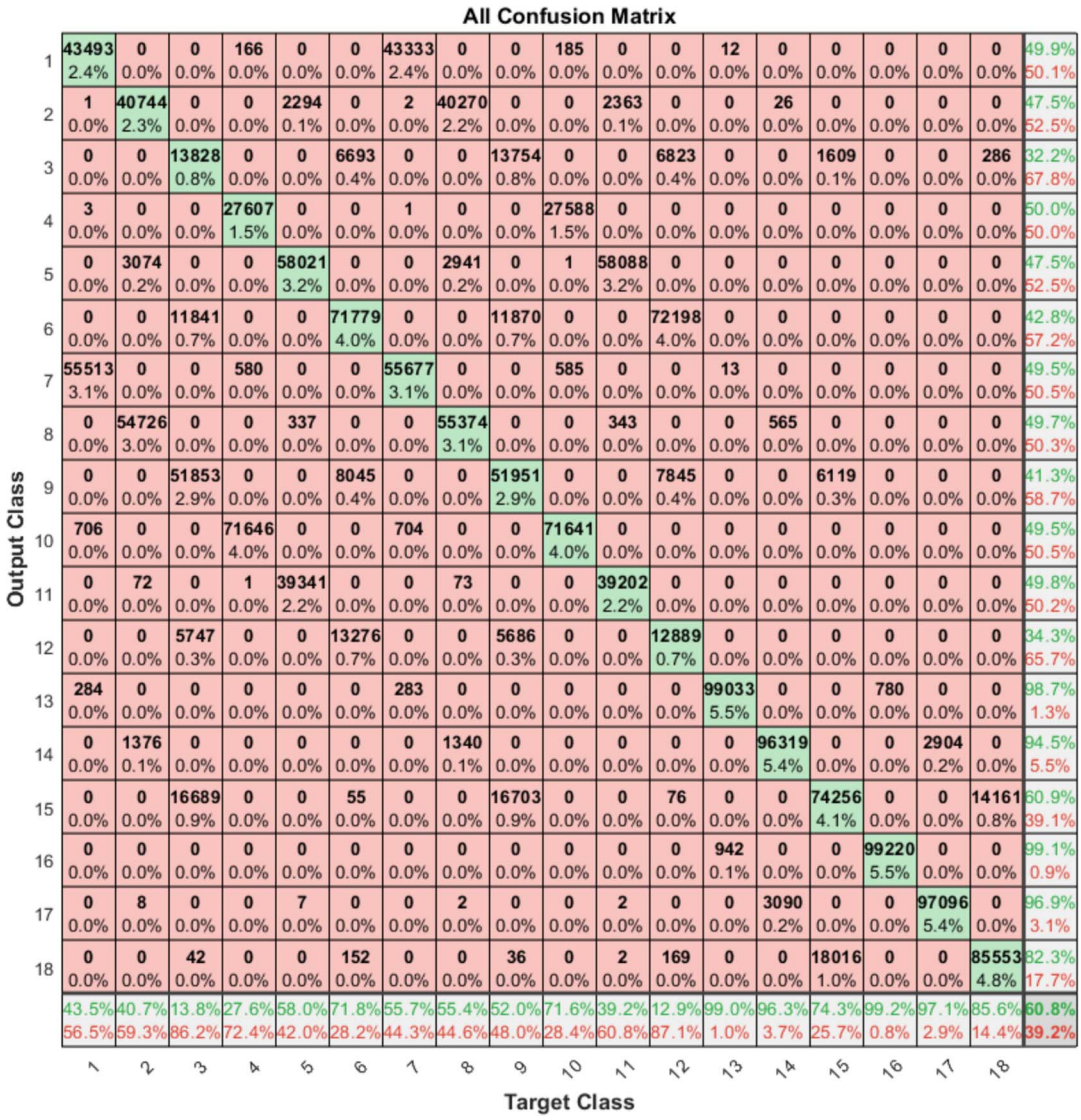


FIG. 12 Confusion matrix delivered by the ANN within the risk-averse environment.

We observe that the accuracy of the ANN increases when categorizing DMs who belong to the last six categories, namely, those composing the $u\left(x_1\right) > E_1$ selection setting. In this case, rows 21–30 of the retrieval vectors within the expected utility and forward-looking scenarios are given by zeros since DMs do not evaluate the second set of characteristics of the alternatives. We have, however, described the incentives of DMs to click on the second page through rows 11–20 of the corresponding retrieval vectors. This is the case in both the risk-neutral and risk-averse environments, which display similar categorization structures within this particular selection setting.

Inaccuracies become more evident when considering the first two selection settings, where the capacity of the ANN to categorize DMs correctly increases with the number of alternatives evaluated within the risk-neutral environment. Similar results are obtained for risk-averse DMs, though this trend is reversed in the forward-looking scenario of the $u\left(x_1\right)+u\left(x_2\right) > E_1+E_2$ selection setting. The accuracy of the ANN also weakens when categorizing DMs who evaluate the same number of alternatives across the first two selection settings. For instance, when DMs evaluate six alternatives, the ANN displays substantial categorization inaccuracies across selection settings in both the expected utility and forward-looking scenarios within risk-neutral and risk-averse environments.

The same intuition applies when DMs evaluate a similar number of alternatives, with the accuracy of the ANN and machine learning techniques generally depending on the way information is distributed through the retrieval vectors (Di Caprio & Santos-Arteaga, 2022, 2024). As noted above, we have placed the realizations from the alternatives evaluated in the initial entrances of the retrieval vectors. However, different distributions of the evaluations through the vector would affect the accuracy of the ANN when categorizing DMs who evaluate similar numbers of alternatives (Di Caprio *et al.*, 2022b; Santos-Arteaga *et al.*, 2024).

We conclude by noting that the results obtained would be analogous if any standard machine learning technique were implemented to analyse the different categories of retrieval profiles generated.

8. Alternative scenarios and managerial implications

We now define a series of alternative numerical scenarios designed to illustrate the modifications induced by the retrieval incentives of DMs as their subjective preferences change.

The first set of simulations—presented in the upper cells of Fig. 13—illustrates the increase in the willingness to observe the initial alternative triggered by an increase in the intensity of preference of the first group of characteristics relative to the second one. As the width of the interval where the realizations of the first characteristics are contained widens and the second-order stochastic dominance decreases, DMs become increasingly willing to discard the alternative observed and start evaluating a new one. Similar utility levels derived from both sets of characteristics foster the search incentives of DMs. On the other hand, as the first characteristic becomes increasingly more critical, the DM will thoroughly evaluate the initial alternative with an increasing probability.

The second set of simulations—presented in the lower cells of Fig. 13—provides a similar intuition regarding the relative risk aversion of DMs. As DMs become increasingly risk-averse, their incentives to start acquiring information on a new alternative, discarding the one observed initially, decrease. Risk aversion, coupled with a higher expected utility derived from the first alternative, leads DMs to continue evaluating the alternative observed initially instead of discarding it and proceeding with a new one. As a result, DMs will consider alternatives with relatively low realizations of the first characteristic that, if not evaluated carefully when observing the second set of characteristics, may provide a lower utility than the expected value, leading DMs to regret their choices.

These numerical results, together with the analyses performed in the previous sections, have several managerial implications that should be carefully considered by both the DMs and the agents providing information.

The formal framework presented can be interpreted from the perspective of DMs selecting an alternative, for instance, a product in a particular market or a project within a firm, or that of the providers of alternatives, product suppliers, or project developers. Various extended information retrieval and evaluation processes can be formalized, allowing for updates of the density functions and different types of interactions across characteristics and variables. From an applied perspective, the model is sufficiently

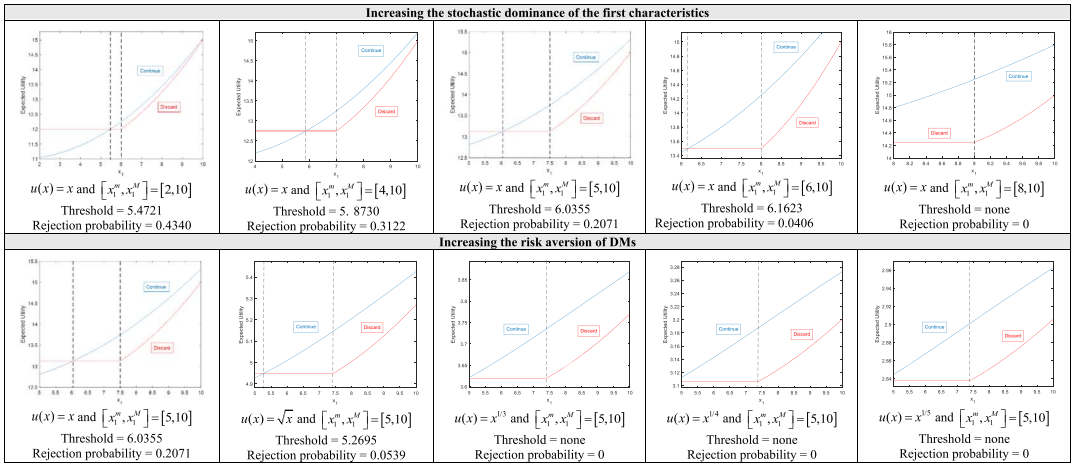


FIG. 13 Alternative scenarios and resulting threshold values. *Note:* In all cases, the second group of characteristics is defined within the interval $\left[x_2^m,\ x_2^M\right] = [0, 10]$.

flexible to formalize the consequences derived from the signalling strategies of information providers when describing the main features of a product or project to a series of potential consumers or partners.

This strategic quality of the model implies that DMs could define information retrieval and decision strategies based on the set of potential intertemporal combinations across characteristics and alternatives. For instance, the densities could be determined by Bayesian updating functions based on signals related to or independent of the initial observation retrieved. The intuition applies directly to the selection of signalling strategies among the suppliers of alternatives and could be developed in subsequent extensions of the current analytical framework.

9. Discussion and potential extensions

An immediate extension of the model could aim at increasing the dimension of the retrieval process by incorporating a third group of characteristics. This extension would require defining and formalizing a more complex evaluation scenario considering potential combinations and intertemporal interactions across groups of characteristics and alternatives within a three-dimensional environment (Di Caprio *et al.*, 2022a). An additional refinement would involve defining the convolution of three different sets of characteristics. Even though both extensions are plausible and can be defined and evaluated, the formal complexity that can be imposed on the retrieval process of DMs remains limited by their evaluation and information-processing capacities. In this regard, Chanda & Ray (2023) studied the conditions determining whether higher comprehensiveness is beneficial or detrimental to the quality of a strategic decision.

Another immediate extension of the model would consist of the addition of costs to the expected utilities defined by DMs. Two types of costs could be considered. First, computing the potential combinatorial outcomes may impose a cognitive cost on the DM relative to the expected utility scenario (Qi *et al.*, 2024). Second, the evaluation requirements of the different selection settings entail spending time or resources retrieving information to verify the characteristics of the alternatives. The relative gains

in utility can determine the capacity of DMs to face these costs to the expected utility scenario described in Tables 2 and 3.

As was previously emphasized, retrieving information regarding the preferences of DMs constitutes an essential requirement to improve the accuracy of the categorization process defined by the ANN. Thus, designing evaluation scenarios with specific selection questions or tasks designed to infer DMs' preferences or main behavioural qualities should constitute a future line of research. However, the subjectivity inherent in DMs' evaluation and selection process complicates the inference procedure substantially. The importance of emotions as determinants of the incentives of DMs to acquire information has been validated in different contexts, ranging from the emergence of random events (Sun *et al.*, 2022) to the privacy concerns existing within user communities in digital environments (Saura *et al.*, 2023).

On the other hand, Zhu & Chao (2023) observed a direct significant influence of experience on the loyalty levels of website users. These consistent relationships can be studied using the current framework to identify specific behavioural qualities of DMs and their information retrieval processes, which could help enhance the categorization capacities of ANNs and machine learning techniques. This would constitute a relevant extension of the model, given the substantial importance of demand forecasting among the main applications of machine learning techniques in supply chain management (Babai *et al.*, 2025). In this regard, the sequential information retrieval and evaluation framework introduced in this article could also be adapted and implemented in dynamic multiple criteria decision-making environments where DMs must deal with the uncertainty inherent to industrial processes and supply chains (Alfzari *et al.*, 2025; Khan *et al.*, 2025).

Finally, Strzelecki & Miklosik (2024) computed a recent estimate of click-through rates in organic search results, based on actual clicks and impressions data, and showed that they have decreased compared to previously reported values. The current model can be easily adapted to investigate the potential causes of this phenomenon, defining a variety of heterogeneous retrieval profiles determined by the corresponding behavioural assumptions imposed on DMs.

10. Conclusions

This article builds on the idea that a DM who considers the effect of future evaluations when defining their current choices uses the values obtained from a set of potential realizations and their associated probabilities at every step of the information retrieval process. The resulting set of expected values and the equivalent corresponding certainty benchmarks determine the behaviour of DMs, reflecting the uncertainty faced and providing a consistent decision mechanism that allows DMs to behave accordingly.

That is, an entirely rational DM selecting alternatives that deliver a utility higher than the certainty equivalent should be able to foresee potential combinations arising across the different characteristics when acquiring information. These combinations should determine the subsequent retrieval strategy and decide what alternative to select.

We have illustrated formally and numerically the utilities and retrieval patterns generated by DMs based on their willingness to acquire information and highlighted the subsequent requirements for verifying the second set of characteristics. We have analysed whether it pays to be rational given the different types of DMs considered, their risk attitudes and the relative complexity of their information retrieval processes.

Furthermore, we have observed that the incentives to evaluate a given alternative independently of the potential characteristics others may display increase with the relative importance of the initial characteristic and the level of relative risk aversion. Both these features are determined by the subjective

preferences of DMs, allowing the use of the strategic scenario of the article to analyse the current fake news and credibility crises.

We have also illustrated the low accuracy displayed by an ANN when categorizing DMs across selection settings and information retrieval scenarios. In this regard, the retrieval profiles of DMs could be further enhanced to incorporate various evaluation strategies determined by the interactions taking place through the decision trees defining the corresponding search processes. That is, the categorization capacity of the ANN could be further tested in future research by accounting for a more varied set of retrieval and evaluation profiles among DMs.

For instance, in our evaluation environment, DMs check a given number of alternatives, ranging from 1 to 10, and select the preferred one among those satisfying the predetermined criteria. An immediate extension of the current framework would incorporate memory into the retrieval process, requiring DMs to either interrupt the search after finding several satisfactory alternatives or update the utility functions based on the realizations observed, the potential ones remaining and the subsequent expected payoffs.

Funding

None to disclose.

Data availability

The MATLAB code used to generate the retrieval profiles of DMs and train the ANN is available from the authors upon request. The series of retrieval profiles generated are also available from the authors upon request.

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