Data Envelopment Analysis with Fuzzy Parameters: An Interactive Approach

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ABSTRACT

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. In the conventional DEA, all the data assume the form of specific numerical values. However, the observed values of the input and output data in real-life problems are sometimes imprecise or vague. Previous methods have not considered the preferences of the decision makers (DMs) in the evaluation process. This paper proposes an interactive evaluation process for measuring the relative efficiencies of a set of DMUs in fuzzy DEA with consideration of the DMs' preferences. The authors construct a linear programming (LP) model with fuzzy parameters and calculate the fuzzy efficiency of the DMUs for different α levels. Then, the DM identifies his or her most preferred fuzzy goal for each DMU under consideration. A modified Yager index is used to develop a ranking order of the DMUs. This study allows the DMs to use their preferences or value judgments when evaluating the performance of the DMUs.

Keywords: *Data Envelopment Analysis, Efficiency Evaluation, Fuzzy Mathematical Programming, Interactive Solution, Preference Modeling*

INTRODUCTION

The changing economic conditions have challenged many organizations to search for more efficient and effective ways to manage their business operations. Data envelopment analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous decision making units (DMUs) by evaluating their relative efficiency. A DMU is considered efficient when no other DMU can produce more outputs using an equal or lesser amount of inputs. The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs. The traditional DEA methods such as CCR (Charnes et al., 1978) and BCC (Banker et al., 1984) require accurate measurement of both the inputs and outputs. However, the real evaluation of the DMUs often

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exhibit imprecision and great uncertainty. In general, as the system's complexity increases, exact evaluation of data becomes extremely difficult. In addition, the traditional DEA models generally do not consider the decision maker's (DM's) preferences or value judgments. Although a few researchers have considered the DM's preferences, their model requires precise and exact measurement of both the input and output data (Joro et al., 2003; Wong et al., 2009; Yang et al., 2009).

In this study, we propose an interactive evaluation process for measuring the relative efficiencies of a set of DMUs in fuzzy DEA with consideration of the DMs' preferences. We construct a linear programming (LP) model with fuzzy parameters and calculate the fuzzy efficiency of the DMUs for different α levels. Then, the DM identifies his/her most preferred fuzzy goal for each DMU under consideration. A modified Yager index is introduced and used to develop a ranking order of the DMUs. The main thrust of this study is to allow the DMs to use their preferences or value judgments when evaluating the performance of the DMUs. This paper is organized into seven sections. The next section presents a brief review of the existing literature on fuzzy DEA followed by a discussion of fuzzy number rankings. Then, we present an overview of fuzzy DEA. Following this overview, we illustrate the details of the proposed framework followed by a numerical example in order to demonstrate the applicability of the proposed framework and also to exhibit the efficacy of the procedures and algorithms. Finally, we conclude with our conclusions and future research directions.

FUZZY DEA LITERATURE REVIEW

In the conventional DEA, all the data assume the form of specific numerical values. However, the observed values of the input and output data in real-life problems are sometimes imprecise or vague. Imprecise evaluations may be the result of unquantifiable, incomplete and non obtainable information. The "Stochastic approach" and the "fuzzy approach" are two existence approaches for modeling uncertainty in the DEA literature. The stochastic approach involves specifying a probability distribution function (e.g., normal) for the error process (Sengupta, 1992). However, as pointed out by Sengupta (1992), the stochastic approach has two drawbacks associated with modeling the uncertainty in DEA problems:

a. Small sample sizes in DEA make it difficult to use stochastic models, and
b. In stochastic approaches, the DM is required to assume a specific error distribution (e.g., normal or exponential) to derive specific results. However, this assumption may not be realistic because on an *a priori* basis there is very little empirical evidence to choose one type of distribution over another.

Some researchers have proposed various fuzzy methods for dealing with the impreciseness and ambiguity in DEA. Fuzzy set algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments. Sengupta (1992) proposed a fuzzy mathematical programming approach by incorporating fuzzy input and output data into a DEA model and defining tolerance levels for the objective function and constraint violations. Triantis and Girod (1998) proposed a mathematical programming approach by transforming fuzziness into a DEA model using membership function values. Guo and Tanaka (2001), León et al. (2003) and Lertworasirikul et al. (2003a) proposed three similar fuzzy DEA models by considering the uncertainties in fuzzy objectives and fuzzy constraints using the possibility approach. Lertworasirikul et al. (2003b) proposed a fuzzy DEA model using the credibility approach where fuzzy variables were replaced by expected credits according to the credibility measures. Lertworasirikul et al. (2003c) further extended the fuzzy DEA through the possibility and credibility approaches.

Kao and Liu (2000b) transformed fuzzy input and output data into intervals by using α-level sets. The α-level set approach was extended by Saati et al. (2002), who defined the fuzzy DEA model as a possibilistic-programming problem and transformed it into an interval programming. Hatami-Marbini et al. (2009a) proposed a four-phase fuzzy DEA framework based on the theory of displaced ideal. Entani et al. (2002) extended the α-level set research by changing fuzzy input and output data into intervals. Dia (2004) proposed a fuzzy DEA model where a fuzzy aspiration level and a safety α-level were used to transform the fuzzy DEA model into a crisp DEA. Wang et al. (2005) also used the α-level set approach to change fuzzy data into intervals. Saati and Memariani (2005) further extended the α-level set approach so that all DMUs could be evaluated by using a common set of weights under a given α-level set. Soleimani-damaneh et al. (2006) addressed some of the limitations of the fuzzy DEA models proposed by Kao and Liu (2000a), León et al. (2003) and Lertworasirikul et al. (2003a) and suggested a fuzzy DEA model to produce crisp efficiencies. Liu (2008) and Liu and Chuang (2009) also extended the α-level set approach by proposing the assurance region approach in the fuzzy DEA model. Hatami-Marbini and Saati (2009) improved a fuzzy BCC (Banker et al., 1984) model which considered fuzziness in the input and output data as well as the decision variable.

The thrust of this study is to allow the DM to use his or her preferences or value judgments when evaluating the performance of the DMUs. In the DEA framework proposed in this study, the DM defines uncertain data by means of language statements and determines a preferred fuzzy goal based on the obtained fuzzy efficiencies of the DMUs for different levels. In other words, the DM can interactively impact the ranking of the DMUs with his/her most preferred goal. Although all the efficiencies of the DMUs obtained from the fuzzy DEA model are mathematically acceptable, a DM can utilize his/her preferences in relation to the conflicting objectives and select his/her most preferred DMU. Finally, the fuzzy efficiency value and the fuzzy goal stated by the DM for each DMU are aggregated and used to rank the DMUs on the basis of their efficiencies. The interactive fuzzy DEA approach proposed in this study is based on the recently developed fuzzy mathematical programming approach by Jiménez et al. (2007). Jiménez et al.'s method breakdowns in some cases because of the Yager index (Yager, 1979) used in their method. We propose a modification to the Jiménez et al.'s (2007) method to overcome this problem.

THE FUZZY NUMBER RANKINGS

A fuzzy number $\tilde{A} = (a, b, c, d)$, is called a generalized trapezoidal fuzzy number with membership function $\mu_{\tilde{A}}$ which has the following properties (Dubois & Prade, 1978):

A. $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0, 1],
B. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a]$,
C. μ_{A} is strictly increasing on $[a, b]$,
D. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$,
E. $\mu_{\tilde{A}}$ is strictly decreasing on $[c, d]$,
F. $\mu_{\tilde{A}}(x) = 0$ for all $x \in [d, +\infty)$.

If $b = c$, then $\tilde{A}$ is called a generalized triangular fuzzy number. If $a = b = c = d$, then $\tilde{A}$ is called a real number. The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ can be defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} f_a(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ g_a(x), & c \le x \le d, \\ 0, & Otherwise, \end{cases} \tag{1}$$

where $f_a : [a,b] \to [0,1]$ and $g_a : [c,d] \to [0,1]$.

The inverse functions of f_a and g_a, denoted as f_a^{-1} and g_a^{-1}, exist. Since

$f_a : [a,b] \to [0,1]$ is continuous and strictly increasing, $f_a^{-1} : [0,1] \to [a,b]$ is also continuous and strictly increasing. Similarly, since $g_a : [c,d] \to [0,1]$ is continuous and strictly decreasing, $g_a^{-1} : [c,d] \to [0,1]$ is also continuous and strictly increasing. That is, both $\int_0^1 f_a^{-1} dy$ and $\int_0^1 g_a^{-1} dy$ exist (Liou & Wang, 1992).

Particularly, we are working with a special type of the triangular fuzzy number *(a,m,b)* with a membership function $\mu_{\tilde{A}}$ expressed by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{m-a}, & a \le x \le m, \\ \dfrac{b-x}{b-m}, & m \le x \le b, \\ 0, & \textit{Otherwise}. \end{cases} \tag{2}$$

In practice, many real-world problems require a ranking of fuzzy numbers for decision making. However, the process of ranking fuzzy numbers is not a simple task and has been studied by many researchers (Detyniecki & Yager, 2000; Jiménez, 1996; Liou & Wang, 1992; Wang & Kerre, 2001a, 2001b). Although this problem has been treated by several authors, there is no "golden method" in the literature. Different approaches lead to different solutions. Indeed, some methods are counterintuitive and suffer from lack of discrimination between alternatives. Chen and Hwang (1992) reviewed several existing approaches and pointed out some illogical results arising out of them. Detyniecki and Yager (2000) proposed a weight distribution function for ranking fuzzy numbers based on the idea of associating scalar values to fuzzy numbers. Chu and Tsao (2002) proposed a method to rank fuzzy numbers by employing an area between the centroid and original points to rank fuzzy numbers. Wang and Lee (2008) show the problems with the ranking method proposed by Chu and Tsao (2002) and proposed a revised method. Wang and Kerre (2001a, 2001b) defined a ranking function to map a fuzzy number into a real number and a natural ranking in a subsequent function. They also defined a comparison function that maps two fuzzy numbers to a real number and determines the level of dominance between the two fuzzy numbers. In this study, the fuzzy ranking method of Jiménez (1996) is used to rank the fuzzy objective values and to handle the inequality relations in the model.

For any pair of fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the membership functions are $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$, respectively. The level of fuzzy satisfaction of $\tilde{A}$ over $\tilde{B}$ can be expressed as:

$$\mu_M(\tilde{A},\tilde{B}) = \begin{cases} 0, & E_2^a - E_1^b < 0, \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & E_1^a - E_2^b < 0 < E_2^a - E_1^b, \\ 1, & E_1^a - E_2^b > 0. \end{cases} \tag{3}$$

where $[E_1^a, E_2^a] = [\int_0^1 f_a^{-1}(r)dr, \int_0^1 g_a^{-1}(r)dr]$ and $[E_1^b, E_2^b] = [\int_0^1 f_b^{-1}(r)dr, \int_0^1 g_b^{-1}(r)dr]$ are the expected intervals of $\tilde{A}$ and $\tilde{B}$, respectively. Note that $f_{\cdot}^{-1}(r)$ and $g_{\cdot}^{-1}(r)$, $r \in [0,1]$, are the inverse function of the left and right membership function, respectively. If $\mu_M(\tilde{A},\tilde{B}) \ge \alpha$, with $\alpha \in (0,1]$, then $\tilde{A}$ is said to be bigger than or equal to $\tilde{B}$ at least in a level α, denoted by $(\tilde{A},\tilde{B})_\alpha$, and it is equivalent to:

$$\frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} \ge \alpha \tag{4}$$

or,

$$E_2^a(1-\alpha) + \alpha E_1^a \ge E_1^b(1-\alpha) + \alpha E_2^b \tag{5}$$

The expected interval of an arbitrary fuzzy triangular number $\tilde{V} = (v^a, v^m, v^b)$ is defined as:

$$[E_1^v, E_2^v] = [\frac{v^a + v^m}{2}, \frac{v^m + v^b}{2}] \quad (6)$$

where v^a, v^m and v^b are the left, center and right value of the triangular fuzzy number, respectively.

THE FUZZY DEA MODEL

DEA is a methodology for evaluating and measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. The DEA methodology is based on the economic notion of Pareto optimality, which states that a DMU is considered to be inefficient if some other DMUs can produce at least the same amount of output with less of the same resource input and not more of any other resource. Otherwise, a DMU is considered to be Pareto efficient. Due to its solid underlying mathematical basis and wide applications to real-world problems, much effort has been devoted to the research and development DEA models and frameworks since the pioneering work of Charnes et al. (1978).

Assume that there are n DMUs to be evaluated. The efficiency of DMU_j ($j = 1, 2, \ldots, n$) with m inputs and s outputs is denoted by x_{ij} ($i = 1, 2, \ldots, m$) and y_{rj} ($r = 1, 2, \ldots, s$), respectively. The relative efficiency of the DMU_p can be obtained by using the following linear programming (LP) model (called CCR) proposed by Charnes et al. (1978):

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r y_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad \forall j, \\ & u_r, v_i \ge 0, \qquad \forall r, i. \end{aligned} \quad (7)$$

where u_r and v_i are the weights assigned to the rth output and ith input, respectively. Note that the DMUs with $\theta_p^* = 1$, is called the efficient unit, and those units with $\theta_p^* \neq 1$ are called inefficient units. In the conventional DEA, all the data assume the form of specific numerical values. However, the observed value of the input and output data are sometimes imprecise or vague. Some researchers have proposed various methods for dealing with the imprecise and ambiguous data in DEA. Cooper et al. (1999) has studied this problem in the context of interval data. However, many real-world problems use linguistic data such as good, fair or poor and cannot be used as interval data. Fuzzy logic and fuzzy sets can represent imprecise or ambiguous data in DEA by formalizing inaccuracy in decision making (Collan et al., 2009). Fuzzy set algebra developed by Zadeh (1965) is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments. In fuzzy DEA, uncertainty is represented in the LP model by using fuzzy coefficients (Zimmermann, 1996; Zadeh, 1978). A generic CCR model with fuzzy coefficients is given as:

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r \tilde{y}_{rp} \\ s.t. \quad & \sum_{i=1}^{m} v_i \tilde{x}_{ip} = 1 \\ & \sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \le 0, \qquad \forall j, \\ & u_r, v_i \ge 0, \qquad \forall r, i. \end{aligned} \quad (8)$$

where u_r and v_i ($r = 1, \ldots, s; i = 1, \ldots, m$) are the crisp decision variables. In the above model, we assume that the right hand sides of the constraints are crisp value because they are similar to the original CCR model used for normalization of the value of the efficiency in the objective function.

The fuzzy DEA model can be used to solve all kinds of fuzzy number shapes. In this study, we use triangular fuzzy numbers to illustrate our approach, though our approach is adaptable to various types of fuzzy numbers. When fuzzy coefficients of model (8) are assumed to be triangular fuzzy numbers and as $\tilde{x}_{ij} = (x_{ij}^a, x_{ij}^m, x_{ij}^b)$ and $\tilde{y}_{rj} = (y_{rj}^a, y_{rj}^m, y_{rj}^b)$, model (8) can be rewritten as:

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r (y_{rp}^a, y_{rp}^m, y_{rp}^b) \\ s.t. \quad & \sum_{i=1}^{m} v_i (x_{ip}^a, x_{ip}^m, x_{ip}^b) = 1, \\ & \sum_{r=1}^{s} u_r (y_{rj}^a, y_{rj}^m, y_{rj}^b) - \\ & \sum_{i=1}^{m} v_i (x_{ij}^a, x_{ij}^m, x_{ij}^b) \leq 0, \qquad \forall j, \\ & u_r, v_i \geq 0, \qquad \forall r, i. \end{aligned} \tag{9}$$

The above model cannot be solved by a standard LP solver because the coefficients in model (9) are fuzzy numbers. The fuzzy DEA model includes several linear fuzzy inequality (equality) constraints, and therefore it requires the comparison of two fuzzy numbers. To solve this problem, a great deal of fuzzy ranking methods has been proposed. These ranking methods include degree of optimality, α-cut, Hamming distance, fuzzy mean and spread, comparison function, centroid index, proportion to the ideal, left and right scores, area measurement, linguistic method, etc. (Chen & Hwang, 1992; Despotis & Smirlis, 2002; Guo, 2009; Guo & Tanaka, 2001; Hatami-Marbini et al., 2009b; Hougaard, 2005; Jahanshahloo et al., 2004; Jiménez et al., 2007; Lai et al., 1992; León et al., 2003; Lertworasirikul et al., 2003a; Liu & Chuang, 2009; Saati et al., 2002; Soleimani-damaneh et al., 2006; Triantis, 2003; Wen & Li, 2009; Wu et al., 2006). None of the proposed frameworks for solving fuzzy DEA models consider the DM's preference structures.

THE INTERACTIVE FUZZY DEA FRAMEWORK

The main thrust of this study is to allow the DMs to use their preferences or value judgments when evaluating the performance of the DMUs. In this section, we present a new interactive framework which considers the DM's preferences in measuring the relative efficiency of the DMUs. We use triangular fuzzy input and output data in the CCR model. Taking equation (5) into consideration, model (9) can be rewritten as:

$$\begin{aligned} \max \quad & \theta_p^{\alpha} = \sum_{r=1}^{s} EV(\tilde{y}_{rp}) u_r \\ s.t. \quad & \sum_{i=1}^{m} [(1-\alpha) E_2^{x_{ip}} + \alpha E_1^{x_{ip}}] v_i = 1, \\ & \sum_{r=1}^{s} [(1-\alpha) E_2^{y_{rj}} + \alpha E_1^{y_{rj}}] u_r - \\ & \sum_{i=1}^{m} [(1-\alpha) E_2^{x_{ij}} + \alpha E_1^{x_{ij}}] v_i \leq 0, \qquad \forall j, \\ & u_r, v_i \geq 0, \qquad \forall r, i. \end{aligned} \tag{10}$$

where $EV(\tilde{y}_{rp}) = \dfrac{E_1^{y_{rp}} + E_2^{y_{rp}}}{2}$ is the expected value of *r*th fuzzy output of the DMU under consideration. Note that $[E_1^{x_{ij}}, E_2^{x_{ij}}]$ and $[E_1^{y_{rj}}, E_2^{y_{rj}}]$ are obtained from equation (3). In addition, u_r^* and v_r^* are the optimal solutions of the DMU$_p$ for each $\alpha \in [0,1]$ in model (10). For a specific value of $\alpha \in [0,1]$, model (10) is solved by the standard LP software. Then, the optimal fuzzy relative efficiency $\tilde{\theta}_p^{\alpha^*}$ of the DMU$_p$ is calculated as:

$$\tilde{\theta}_p^{\alpha^*} = (\theta_p^{a(\alpha)}, \theta_p^{m(\alpha)}, \theta_p^{b(\alpha)}) = \left(\sum_{r=1}^{s} y_{rp}^a u_r^*, \sum_{r=1}^{s} y_{rp}^m u_r^*, \sum_{r=1}^{s} y_{rp}^b u_r^*\right) \tag{11}$$

Although all the efficiencies of the DMUs obtained from the fuzzy DEA model are mathematically acceptable, a DM can utilize his/her preferences in relation to the conflicting objectives and select his/her most preferred DMU. Next, the DM identifies a preferred fuzzy goal based on the fuzzy efficiencies in equation (11) and interactively changes the ranking of the DMUs according to his/her most preferred goal. Hence, it is necessary to define a fuzzy goal $\tilde{G}$ by the DM for each DMU so that equation (11) can be used to find the optimal fuzzy efficiency $\tilde{\theta}_p^{\alpha^*}$ for each α-level. This means that the DM is requested to determine his/her priority fuzzy goal based on the consideration of the optimal fuzzy efficiency of DMU_p ($\tilde{\theta}_p^{\alpha^*}$) for each α-level, denoted as $\tilde{G}_j = (g_j^a, g_j^m, g_j^b)$. The membership function of the fuzzy goal for all DMU, for simplicity, can be defined as seen in Box 1.

For the compatibility of the optimal fuzzy efficiency $\tilde{\theta}_p^{\alpha^*}$ with the DM's preference $\tilde{G}_j$, Jiménez et al. (2007) suggested using the Yager index (Yager, 1979) as follows:

$$K_{\tilde{G}_j}^{\alpha}(z) = \int_{-\infty}^{+\infty} \mu_{\tilde{G}_j}(z).\mu_{\tilde{\theta}_p^{\alpha^*}}(z)dz \Big/ \int_{-\infty}^{+\infty} \mu_{\tilde{\theta}_p^{\alpha^*}}(z)dz \qquad (13)$$

Finally, the balance solution is defined for each α using equation (14) as suggested by Bellman and Zadeh (1970):

$$B_j(\alpha) = \alpha * K_{\tilde{G}_j}^{\alpha}(z) \qquad (14)$$

where * is the algebraic product.

A ranking order of the DMUs can be developed based on the value of the above index for each α.

Jiménez et al. (2007) have claimed that their approach is applicable to a variety of real-world problems. However, we challenge this claim and show that the Yager index (Yager, 1979) used in Jiménez et al.'s method has some drawbacks. In reality, when a fuzzy optimal solution, such as $\tilde{\theta}_p^{\alpha^*}$, is obtained as a crisp value, then the Yager index and the balance level index will be virtually zero. We solve this problem in our framework by using the following variation:

$$\tilde{\theta}_p^{\alpha^*} = (\theta_p^{a(\alpha)} - \varepsilon, \theta_p^{m(\alpha)}, \theta_p^{b(\alpha)} + \varepsilon) \qquad (15)$$

where ε is an infinitesimal positive number. As a theoretical construct, epsilon in the above formula provides conditions for calculating the real value of the Yager index and keeping it away from zero. It means that a crisp value can be replaced by a fuzzy number in the above alteration. In fact, one has to choose an arbitrarily small value for ε. For example, in the numerical example 10^{-7} is chosen for ε. We will illustrate the use of the proposed approach with an example in the next section.

Box 1.

$$\mu_{\tilde{G}_j}(z) = \begin{cases} 0, & z \le \min_p\{\theta_p^{a(\alpha)}\}, \\ \dfrac{z - \min_p\{\theta_p^{a(\alpha)}\}}{\max_p\{\theta_p^{b(\alpha)}\} - \min_p\{\theta_p^{a(\alpha)}\}}, & \min_p\{\theta_p^{a(\alpha)}\} \le z \le \max_p\{\theta_p^{b(\alpha)}\}, \\ 1, & z \ge \max_p\{\theta_p^{b(\alpha)}\}. \end{cases} \qquad (12)$$

Table 1. The numerical example of Guo and Tanaka (2001)

DMU	Inputs		Outputs	
	1	2	1	2
A	(3.5,4,4.5)	(1.9,2.1,2.3)	(2.4,2.6,2.8)	(3.8, 4.1, 4.4)
B	(2.9,2.9,2.9)	(1.4,1.5,1.6)	(2.2,2.2,2.2)	(3.3, 3.5, 3.7)
C	(4.4,4.9,5.4)	(2.2,2.6,3)	(2.7,3.2,3.7)	(4.3, 5.1, 5.9)
D	(3.4,4.1,4.8)	(2.2,2.3,2.4)	(2.5,2.9,3.3)	(5.5, 5.7, 5.9)
E	(5.9,6.5,7.1)	(3.6,4.1,4.6)	(4.4,5.1,5.8)	(6.5, 7.4, 8.3)

Table 2. The Guo and Tanaka (2001) results

α	A	B	C	D	E
0	(0.66, 0.81, 0.99)	(0.88, 0.98, 1.09)	(0.60, 0.82, 1.12)	(0.71, 0.93, 1.25)	(0.61, 0.79, 1.02)
0.5	(0.75, 0.83, 0.92)	(0.94, 0.97, 1.00)	(0.12, 0.83, 0.14)	(0.85, 0.97, 1.12)	(0.72, 0.82, 0.93)
0.75	(0.77, 0.81, 0.99)	(0.80, 0.98, 1.09)	(0.22, 0.82, 0.30)	(0.71, 0.93, 1.25)	(0.61, 0.79, 1.02)
1	(0.85, 0.85, 0.94)	(1.00, 1.00, 1.00)	(0.86, 0.86, 0.86)	(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)

Table 3. Results of the proposed method

DMU	u_1	u_2	$\tilde{\theta}_p^{\alpha^*}$	Yager index	Balance level index
A	0.321	0	(0.771, 0.835, 0.899)	0.276	0.028
B	0.454	0	(0.999, 0.999, 0.999)	0	0
C	0.257	0	(0.694, 0.823, 0.952)	0.254	0.025
D	0	0.173	(0.951, 0.986, 1.021)	0.560	0.056
E	0.110	0.052	(0.826, 0.950, 1.075)	0.493	0.049

THE NUMERICAL EXAMPLE

The advantages of the proposed framework are illustrated by the numerical example described in (Guo & Tanaka, 2001). The necessary data for this problem are presented in Table 1.

Each DMU consumes two symmetric triangular fuzzy inputs to produce two symmetric triangular fuzzy outputs. The fuzzy efficiencies of the DMUs for different α values using the method proposed by Guo and Tanaka (2001) are presented in Table 2.

Next, we compare the results of our proposed framework with those obtained by Guo and Tanaka (2001). It was shown in Guo and Tanaka (2001) that $S_0 = \{B, C, D, E\}$, $S_{0.5} = \{B, D\}$, $S_{0.75} = \{B, D\}$ and $S_1 = \{B, D, E\}$ are the non-dominated sets for different α values. Note that in their method a DMU is said to be α-possibilistic efficient if the right value of the fuzzy efficiency at that α level is greater than or equal to 1 and the set of all possibilistic efficient DMUs is called the α-possibilistic non-dominated set, denoted by S_α. The fuzzy efficiencies of the DMUs pre-

Table 4. Decision variables for different α values

α	A		B		C		D		E	
	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_2
0.1	0.321	0	0.454	0	0.257	0	0	0.173	0.110	0.052
0.2	0.322	0	0.454	0	0.260	0	0	0.174	0.113	0.052
0.3	0.323	0	0.454	0	0.157	0.066	0	0.174	0.118	0.050
0.4	0.325	0	0.454	0	0.171	0.059	0	0.175	0.127	0.046
0.5	0.329	0	0.310	0.091	0.183	0.054	0	0.175	0.136	0.042
0.6	0.239	0.059	0.130	0.205	0.195	0.048	0.234	0.058	0.199	0
0.7	0.254	0.053	0.122	0.211	0.206	0.043	0.249	0.052	0.202	0
0.8	0.268	0.048	0.115	0.217	0.217	0.039	0.264	0.047	0.204	0
0.9	0.281	0.042	0.109	0.222	0.227	0.034	0.279	0.042	0.207	0
1	0.295	0.036	0.103	0.227	0.238	0.029	0.295	0.036	0.210	0

sented in Table 3 are calculated using model (10) and equation (11) for $\alpha = 0.1$.

As it is shown in Table 3, when $\alpha = 0.1$, the optimal solutions of B are $u_1 = 0.454$ and $u_2 = 0$ and its optimal fuzzy efficiency is (0.999, 0.999, 0.999). In other words, the optimal value of the objective function is a crisp value and hence the Yager index and balance level index become zero. Therefore, the DM cannot realize real results about some α-level such as B for $\alpha = 0.1$. Therefore, the modified Yager index proposed by equation (5) is recalculated for crisp efficiencies. Ultimately, the modified Yager indices for ten levels are derived and shown in Tables 4, 5, 6, and 7.

In this example, it is also noted that aspiration level (goal) of the DM is defined as follows:

$$\mu_{\tilde{G}}(z) = \begin{cases} 0\ , & z \leq 0.694, \\ \dfrac{z - 0.694}{1.221 - 0.694}, & 0.694 \leq z \leq 1.221, \\ 1\ , & z \geq 1.221 \end{cases}$$

We calculate the balance index value of each DMU for a given α and use them to develop the ranking order presented in Table 7. We should note the similarity between the ranking obtained by our method and the ranking obtained by Guo and Tanaka (2001). This similarity is in spite of the fact that the DM's preferences were considered in our model whereas Guo and Tanka (2001) ignore the DM's preferences. Moreover, Table 7 shows that as the value of α increases, the balance index of each DMU becomes larger. Note that when α=1 and α=0.1, the DM is optimistic (very low risk) and pessimistic (very high risk), respectively.

In general, efficiency evaluation with fuzzy DEA is more complex than the classical DEA because of the inherent fuzziness in the input and output data. The analysis of fuzzy efficiency by balance index under different levels may be very useful to the DMs. Figure 1 presents a graphical presentation of the results where the DM can assess the sensitivity of the DMUs and select a suitable α-level. As it is shown in this figure, DMUs B and E are particularly sensitive to the variable measurements.

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The field of DEA has grown at an exponential rate since the pioneering papers of Farrell (1957) and Charnes et al. (1978). DEA is generally

Table 5. The fuzzy efficiencies of the DMUs for different α values

α	$\tilde{\theta}_A^{\alpha*}$	$\tilde{\theta}_B^{\alpha*}$	$\tilde{\theta}_C^{\alpha*}$	$\tilde{\theta}_D^{\alpha*}$	$\tilde{\theta}_E^{\alpha*}$
0.1	(0.771, 0.835, 0.899)	(0.999, 0.999, 0.999)	(0.694, 0.823, 0.952)	(0.951, 0.986, 1.021)	(0.826, 0.950, 1.075)
0.2	(0.773, 0.837, 0.902)	(0.999, 0.999, 0.999)	(0.701, 0.831, 0.961)	(0.955, 0.989, 1.024)	(0.836, 0.962, 1.088)
0.3	(0.775, 0.839, 0.904)	(0.999, 0.999, 0.999)	(0.709, 0.841, 0.972)	(0.958, 0.993, 1.028)	(0.846, 0.974, 1.102)
0.4	(0.780, 0.845, 0.911)	(0.999, 0.999, 0.999)	(0.718, 0.851, 0.984)	(0.961, 0.996, 1.031)	(0.857, 0.987, 1.118)
0.5	(0.789, 0.855, 0.921)	(0.982, 0.999, 1.018)	(0.726, 0.861, 0.996)	(0.965, 0.999, 1.035)	(0.867, 0.999, 1.132)
0.6	(0.800, 0.866, 0.932)	(0.963, 1.004, 1.045)	(0.735, 0.871, 1.007)	(0.905, 1.011, 1.116)	(0.875, 1.014, 1.153)
0.7	(0.812, 0.879, 0. 945)	(0.966, 1.008, 1. 051)	(0.743, 0.881, 1.019)	(0.912, 1.022, 1.132)	(0.887, 1.028, 1.169)
0.8	(0.823, 0.891, 0. 959)	(0.970, 1.013, 1. 057)	(0.752, 0.891, 1.030)	(0.919, 1.034, 1.149)	(0.899, 1.043, 1.186)
0.9	(0.835, 0.904, 0. 973)	(0.973, 1.018, 1. 062)	(0.760, 0.901, 1.042)	(0.928, 1.048, 1.168)	(0.913, 1.058, 1.203)
1	(0.847, 0.916, 0. 986)	(0.977, 1.023, 1. 068)	(0.769, 0.912, 1.054)	(0.937, 1.063, 1.188)	(0.926, 1.074, 1.221)

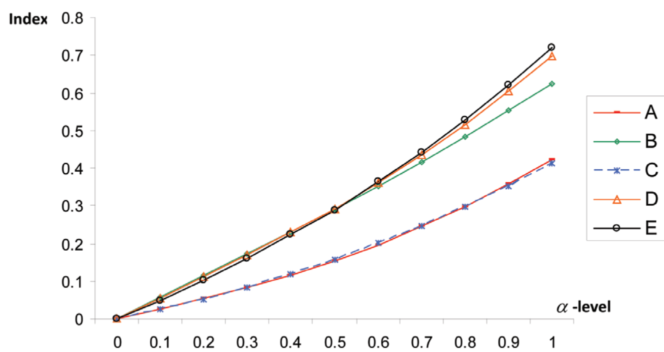
Table 6. The Yager index of the DMUs for different α values

α	$K_{\tilde{G}_A}^{\alpha}$	$K_{\tilde{G}_B}^{\alpha}$	$K_{\tilde{G}_C}^{\alpha}$	$K_{\tilde{G}_D}^{\alpha}$	$K_{\tilde{G}_E}^{\alpha}$
0.1	0.267	0.578	0.244	0.554	0.486
0.2	0.271	0.578	0.259	0.560	0.508
0.3	0.275	0.578	0.278	0.567	0.531
0.4	0.287	0.578	0.297	0.573	0.556
0.5	0.305	0.580	0.316	0.580	0.579
0.6	0.326	0.588	0.335	0.601	0.607
0.7	0.350	0.596	0.354	0.622	0.633
0.8	0.373	0.606	0.373	0.645	0.661
0.9	0.398	0.614	0.392	0.671	0.690
1	0.421	0.623	0.413	0.699	0.720

Table 7. The final balance index and ranking order of the DMUs for different α values

α	$B_A(\alpha)$	Rank	$B_B(\alpha)$	Rank	$B_C(\alpha)$	Rank	$B_D(\alpha)$	Rank	$B_E(\alpha)$	Rank
0.1	0.028	4	0.058	1	0.025	5	0.056	2	0.049	3
0.2	0.056	4	0.117	1	0.054	5	0.113	2	0.103	3
0.3	0.085	5	0.176	1	0.086	4	0.172	2	0.161	3
0.4	0.118	5	0.234	1	0.122	4	0.231	2	0.225	3
0.5	0.157	5	0.293	2	0.162	4	0.293	3	0.293	1
0.6	0.201	5	0.356	2	0.206	4	0.363	3	0.367	1
0.7	0.251	5	0.421	3	0.253	4	0.439	2	0.447	1
0.8	0.305	5	0.488	3	0.305	4	0.520	2	0.533	1
0.9	0.365	4	0.557	3	0.360	5	0.608	2	0.625	1
1	0.429	4	0.628	3	0.420	5	0.703	2	0.724	1

Figure 1. An overall efficiency evaluation of the DMUs



used to measure the relative efficiencies of a set of DMUs producing multiple outputs from multiple inputs. Evaluating the performance of many activities by a traditional DEA approach requires precise input-output data. However, in many empirical studies, inputs and outputs are volatile and complex. To deal with imprecise data, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming problems which are typically solved using some methods for the ranking of fuzzy sets. This study proposes a novel DEA framework which is capable of dealing with imprecise data as well as considering the preferences of the DMs in the evaluation process. The DM has the preference information about the performance of the DMUs which can be easily incorporated into the fuzzy DEA model. We define the balance index to compute the different rankings of the DMUs for different α levels.

The practical aspect of such a fuzzy measure was demonstrated by comparing its results with those of Guo and Tanaka (2001). By solving the same example discussed in Guo and Tanaka (2001), we have shown the similarity between the rankings obtained by the two methods. This similarity is in spite of the fact that the DM's preferences were considered in our model whereas Guo and Tanka (2001) ignore the DM's preferences. The proposed framework is interactive, generic, structured and comprehensive and can be applied to analyze various DMU evaluation problems in fuzzy environments. The application of the fuzzy DEA framework to hierarchical structures is an important area for future research. Many organizational problems tend to exhibit such a profile. The framework developed in this study can potentially lend itself to other areas of study, such as supply chain management. This method and related concepts can be extended to the other topics in DEA such as the BCC (Banker et al., 1984) model. In addition, the principles are also somewhat related to and could be applied to the concepts and structures studied in the network DEA model of Färe and Grosskopf (2000).

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