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General and multiplicative non-parametric corporate performance models with interval ratio data

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ABSTRACT

The increasing intensity of global competition has led organizations to utilize various types of performance measurement tools for improving the quality of their products and services. Data envelopment analysis (DEA) is a methodology for evaluating and measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. All the data in the conventional DEA with input and/or output ratios assumes the form of crisp numbers. However, the observed values of data in real-world problems are sometimes expressed as interval ratios. In this paper, we propose two new models: general and multiplicative non-parametric ratio models for DEA problems with interval data. The contributions of this paper are fourfold: (1) we consider input and output data expressed as interval ratios in DEA; (2) we address the gap in DEA literature for problems not suitable or difficult to model with crisp values; (3) we propose two new DEA models for evaluating the relative efficiencies of DMUs with interval ratios, and (4) we present a case study involving 20 banks with three interval ratios to demonstrate the applicability and efficacy of the proposed models where the traditional indicators are mostly financial ratios.

1. Introduction

Data envelopment analysis (DEA) is a widely used mathematical programming technique for comparing the inputs and outputs of a set of homogenous decision making units (DMUs) by evaluating their relative efficiency. Due to its solid underlying mathematical basis and wide applications to real-world problems, much effort has been devoted to the DEA models since the pioneering work of Charnes et al. [1,2]. Their approach, stemming from the work of Farrell [3], does not require specification of a functional form for the frontier of performance possibilities. It provides a single measure of efficiency and obviates the need to assign pre-specified weights to either even when dealing with multiple inputs and outputs. The frequently used DEA models are the CCR model, named after Charnes et al. [1] under the assumption of constant returns-to-scale (CRS) and the BCC model, named after Banker et al. [4], under the assumption of variable returns-to-scale (VRS).

The conventional DEA methods require accurate measurement of both the input and output data. However, the observed values of the inputs and outputs in real-world problems are sometimes defined as interval ratios instead of crisp numbers. In

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this paper, we propose two new DEA models, a general non-parametric corporate performance (GNCP) model and a multiplicative non-parametric corporate performance (MNCP) model for evaluating the relative efficiencies of DMUs with interval ratios.

The analysis of financial ratios has traditionally been used to measure the financial state of a firm through a comparison of its ratios with those of other firms operating in the same sector known as ''industry average''. Using such information, it is possible to forecast bankruptcies, proceed with mergers and acquisitions, or cease business operations in a territory. All these aspects make it interesting to characterise and study such ratios. One of the models most commonly used to describe this evolution is the partial adjustment model proposed by Lev [5] and analyzed by Lee and Wu [6], David and Peles [7] and Wu and Ho [8] amongst others [9].

Financial ratio heterogeneity stems from diverse statistical relationships between corporate financial data and companies' ratings. It arises when financial statement structures differ between the subgroups in the sample, because of common distinguishing factors. For instance, industry sectors can differ substantially in terms of balance-sheet structure (e.g., liquidity, fixed asset intensity, profitability standards). At the regional level, different interest rate environments, tax regimes, wage levels, and access to capital markets are prominent drivers of typical profitability levels and capital structures. These factors affect corporate ratings accordingly. For instance, Standard and Poor imposes caps on the ratings of otherwise financially sound companies, if they face high industry or country risk [9–12].

Whittington [13] identified two main uses of financial ratios: positive and normative. The normative use compares a firm's ratios with some standard value, usually some location measure for the industry such as the mean or median [14]. In their positive guise, ratios are used for predictive purpose, most commonly the prediction of business failure. The pioneering works are those of Beaver [15,16] and Altman [17] who used various forms of discriminant analysis. The technique has subsequently enjoyed widespread popularity amongst those seeking to predict failure [10]. However, attempts to estimate probabilities of failure instead of classifications have usually been considered to yield disappointing results [18].

Emrouznejad and Anouze [19] have recently shown that the standard DEA may produce incorrect results in the presence of a ratio variable. The main two problems of using ratio as an input and/or an output in DEA, and similarly in the GNCP model, are the convexity and proportionality properties of the production possibility set. Emrouznejad and Cabanda [20] and Emrouznejad et al. [21] proposed a multiplicative model for non-parametric corporate performance (MNCP) which was used to develop the information and communication technology-opportunity index.

This paper is organized as follows. Section 2 describes the general non-parametric ratio analysis model. Section 3 presents the details of the multiplicative ratio model. Section 4 presents an overview of the interval DEA models and in section 5 we review the BCC model with interval data. In Section 6, we present the general non-parametric and the multiplicative ratio models with interval data proposed in this study. In Section 7 a case study involving 20 banks with three interval ratios is used to demonstrate the applicability and efficacy of the proposed models. In Section 8 we present our conclusions and future research directions.

2. General non-parametric corporate performance model with ratio data

The principles underlying DEA can be applied to financial statement analysis under certain assumptions. Smith [22] has attempted to dissect a single financial ratio using DEA. However, it is possible to develop a much more general formulation. In line with the generally accepted notion that corporate performance is multidimensional in nature, the models proposed here are based on the assumption that there are a variety of indicators of corporate performance, none of which alone tells a complete story.

The problem of assessing the performance of DMU_p can be addressed by using linear programming. Assume that DMU_j, $j = 1, \ldots, n$, is to be evaluated according to m ratios, r_{ij} $(i = 1, \ldots, m)$. Let us further assume that the ratios are ordered such that a higher level is preferred to a lower level. Fernandez-Castro and Smith [18] presented the following model known as the general non-parametric corporate performance (GNCP) for measuring the efficiency of DMUs with ratio data:

$$\begin{aligned} \max \quad & z_p \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j r_{ij} \geqslant z_p r_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geqslant 0, \quad \forall j \end{aligned} \tag{1}$$

where $\lambda_j (j = 1, \ldots, n)$ is the weight placed on each DMU for constructing the efficient facet of DMU_p. Note that in model (1) there is no input. That is, the non-zero λs indicate the DMUs which make up the best performance composite DMU relative to DMU_p, so it is the target for DMU_p. The final non-trivial constraint is added to ensure that, in searching for an optimal solution, the method uses only interpolation of the observations, and does not seek to extrapolate the observed behaviour outside of the observed domain [14]. Note also that model (1) is developed on the assumption of variable returns-to-scale (see [23]). The optimal value of model (1) is z_p^* and the DMU_p is efficient if z_p^* =1 and the slacks are zero in model (1); otherwise, $z_p^* > 1$, and the DMU_p is inefficient. Fernandez-Castro and Smith [18] present a detailed explanation of this model.

3. Multiplicative ratio model

The GNCP model proposed by Fernandez-Castro and Smith [18] has two shortcomings: the convexity and the proportionality properties that were identified by Emrouznejad and Cabanda [20] (see also [24] and [21]). Emrouznejad and Cabanda [20] have proposed the following model known as the multiplicative non-parametric corporate performance model (MNCP) to deal with these shortcomings:

$$\begin{aligned} \max \quad & h_p \\ s.t. \quad & \prod_{j=1}^{n} r_{ij}^{\lambda_j} \geqslant h_p r_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geqslant 0, \quad \forall j. \end{aligned} \tag{2}$$

The following alteration is applied to model (2) to obtain a linear programming model [25]:

$$h_p r_{ip} = e^{-s_i} \prod_{j=1}^{n} r_{ip} \lambda_j, \quad \forall i. \tag{3}$$

Furthermore, we obtain the following model by substituting $h_p \exp(\varepsilon \sum_{i=1}^{m} s_i)$ for the objective function in model (2) where $s_i \geqslant 0$ and ε are the slacks and the non-Archimedean infinitesimal, respectively:

$$\begin{aligned} \max \quad & g_p + \varepsilon \sum_{i=1}^{m} s_i \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \rho_{ij} - s_i = g_p + \rho_{ip}, \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & s_i \geqslant 0, \lambda_j \geqslant 0, \quad \forall i, j. \end{aligned} \tag{4}$$

where $g_p = log(h_p)$ and $\rho_{ij} = log(r_{ij})$*.

Model (4) is used to maximize the output and the efficiency of the DMU_p which is equal to $e^{g_p^*}$ in model (2) where g_p^* is obtained from model (4). Note that DMU_p is efficient if $e^{g_p^*} = 1$; otherwise, $e^{g_p^*} > 1$, and the DMU_p is inefficient.

4. Overview of the interval DEA models

The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs. The conventional DEA methods such as CCR [1] and BCC [4] require accurate measurement of both the inputs and outputs. However, the observed value of the input and output data in real-world situations are often inexact, incomplete, vague, ambiguous or imprecise. Some researchers have proposed various methods for dealing with the imprecise and ambiguous data in DEA [26]. Generally, the uncertainty in DEA has been investigated in the literature in three ways including the stochastic approach, the fuzzy approach and the interval approach.

The stochastic approach involves specifying a probability distribution function (e.g., normal) for the error process [27]. However, as pointed out by Sengupta [27], the stochastic approach has two drawbacks associated with modelling the uncertainty in DEA problems:

(a) Small sample sizes in DEA make it difficult to use stochastic models.
(b) In stochastic approaches, the DM is required to assume a specific error distribution (e.g., normal or exponential) to derive specific results. However, this assumption may not be realistic because in an *a priori* basis there is very little empirical evidence to choose one type of distribution over another.

Recently researchers have proposed various fuzzy methods for dealing with the impreciseness and ambiguity in DEA. Fuzzy sets algebra developed by Zadeh [28] is the formal body of theory that allows for the treatment of imprecise estimates in uncertain environments. Sengupta [27] proposed a fuzzy mathematical programming approach by incorporating fuzzy input and output data into a DEA model and defining tolerance levels for the objective function and constraint violations. Since the

* Note that the log transformation we used here is natural logarithm (i.e. which is the logarithm to the base e), however in general the log-transform could be base on any number (e.g. $\log_{10}$).

original study by Sengupta [27], there has been a continuous interest and increased development in fuzzy DEA literature [50]. The tolerance approach [27,29], the α-level based approach [30–33], the fuzzy ranking approach [34,35], and the possibility approach [36,37]; are the most cited fuzzy approaches in DEA literature.

Cooper et al. [38–40] were among the first to use the interval approach to study the uncertainty in DEA. They transformed a nonlinear programming problem into a linear programming problem equivalent through scale transformations and called the resulting DEA model imprecise DEA (IDEA). The final efficiency scores of the DMUs were derived as deterministic numerical values less than or equal to unity. Zhu [41] has argued that the IDEA method proposed by Cooper et al. [38–40]; (1) significantly adds to the complexity of the DEA model because of the great numbers of data transformations and variable alternations; and (2) their scale transformations also transformed both the precise and imprecise data including preference data and interval data into constraints, leading to a rapid increase in computation burden. Despotis and Smirlis [42] also studied the problem of IDEA and developed an alternative approach for dealing with imprecise data in DEA. Lee et al. [43] further studied IDEA and extended the concept to the additive model. Entani et al. [44] proposed a DEA model with interval efficiencies measured from both the optimistic and the pessimistic viewpoints. Their model was first developed for crisp data and then extended to interval data and fuzzy data.

Wang et al. [45] constructed a linear DEA model without the need of extra variable alternations and used a fixed and unified production frontier to measure the efficiencies of DMUs. Their interval DEA models were developed for measuring the lower and upper bounds of the best relative efficiency of each DMU with interval input and output data and was different from the interval formed by the worst and the best relative efficiencies of each DMU. Kao [46] constructed a pair of two-level mathematical programming models and transformed them into a pair of ordinary one-level linear programs. Solving the associated pairs of linear programs produced the efficiency intervals of all the DMUs.

Smirlis et al. [47] introduced an approach based on interval DEA that allowed the evaluation of the DMUs with missing values along with the other DMUs with available crisp data. The missing values were replaced by intervals in which the unknown values were likely to belong. The constant bounds of the intervals were estimated by using statistical or experiential techniques. For the units with missing values, the proposed models were able to identify an upper and a lower bound of their efficiency scores. Shokouhi et al. [48] proposed an approach based on a robust optimization model in which the input and output parameters were constrained to be within an uncertainty set with additional constraints based on the worst case solution with respect to the uncertainty set. Their robust DEA model maximized efficiency under the assumption of a worst case efficiency defied by the uncertainty set and its supporting constraint. They used a Monte-Carlo simulation to compute the conformity of the rankings. In the next section we present an overview of the BCC model with interval data.

5. Overview of BCC with interval data

Let us continue with our earlier assumption that there are n DMUs to be evaluated where every DMU$_j$, $j = 1, \ldots, n$, produces the same s outputs in different amounts, y_{rj} ($r = 1, \ldots, s$), using the same m inputs, x_{ij} ($i = 1, \ldots, m$), also in different amounts. The primal BCC model [4], expressed as the following linear programming model, can be used to evaluate the output-oriented technical efficiency of the DMU$_p$ (it should be noted that DEA models with ratio data can only work under VRS technology (see [23] and [24]):

$$\begin{aligned} e_p = \max \quad & \theta_p \\ s.t. \quad & x_{ip} \geqslant \sum_{j=1}^{n} \lambda_j x_{ij}, \quad \forall i, \\ & \theta_p y_{rp} \leqslant \sum_{j=1}^{n} \lambda_j y_{rj}, \quad \forall r, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geqslant 0, \quad \forall j. \end{aligned} \tag{5}$$

Let us further consider the uncertainty in the above model by using interval inputs and outputs. A generic BCC model with interval coefficients is given as:

$$\begin{aligned} \tilde{e}_p = \max \quad & \theta_p \\ s.t. \quad & \tilde{x}_{ip} \geqslant \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}, \quad \forall i, \\ & \theta_p \tilde{y}_{rp} \leqslant \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}, \quad \forall r, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geqslant 0, \quad \forall j. \end{aligned} \tag{6}$$

where $\tilde{x}_{ij} \in [x_{ij}^L, x_{ij}^U]$ and $\tilde{y}_{rj} \in [y_{rj}^L, y_{rj}^U]$ are the interval inputs and outputs, respectively. Note that the lower and upper bounds of the interval data are considered as constants and strictly positive. In this case, the efficiency of each DMU can be an interval denoted as $[e_j^L, e_j^U]$. The upper and lower bounds of the interval efficiency of the DMU_p are obtained from the pessimistic and optimistic viewpoints, respectively, using the following pair of linear programming models [42]:

$$\begin{array}{ll}
\text{Optimistic viewpoint} & \text{Pessimistic viewpoint} \\
e_p^L = \max \quad \theta_p & e_p^U = \min \quad \theta_p \\
s.t. \quad x_{ip}^L \geqslant \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij}^U + \lambda_p x_{ip}^L, \quad \forall i, & s.t. \quad x_{ip}^U \geqslant \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j x_{ij}^L + \lambda_p x_{ip}^U, \quad \forall i, \\
\theta_p y_{rp}^U \leqslant \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j y_{rj}^L + \lambda_p y_{rp}^U, \quad \forall r, & \theta_p y_{rp}^L \leqslant \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j y_{rj}^U + \lambda_p y_{rp}^L, \quad \forall r, \\
\sum_{j=1}^{n} \lambda_j = 1 & \sum_{j=1}^{n} \lambda_j = 1 \\
\lambda_j \geqslant 0, \quad \forall j. & \lambda_j \geqslant 0, \quad \forall j.
\end{array} \tag{7}$$

In the optimistic viewpoint, the best situation for DMU_p and the worst situation for DMU_j $(j \neq p)$ are used and, therefore, $\tilde{e}_p \geqslant e_p^L$. On the other hand, in the pessimistic viewpoint, the worst situation for DMU_p and the best situation for DMU_j $(j \neq p)$ are used and, therefore, $\tilde{e}_p \leq e_p^U$. With respect to the interval efficiencies of the DMUs, the three following classifications (E^{++}, E^{+} and E^{-}) can be defined for each DMU:

- If the upper efficiency score of a DMU is equal to unity, then, the DMU is classified as E^{++} class.
- If the lower efficiency score of a DMU is equal to unity and its upper efficiency score is greater than unity, then, the DMU is classified as E^{+} class.
- If the lower efficiency score of a DMU is greater than unity, then, the DMU is classified as E^{-} class.

Note that the DMUs that are classified as E^{++} are fully efficient (upper efficient and lower efficient) and the DMUs that are classified as E^{-} are inefficient. DMUs in the E^{+} are only upper efficient.

6. Non-parametric corporate performance model with interval data

This section presents two new models for aggregation ratio data where the ratios are presented in the form of interval data.

6.1. GNCP model with interval data

Let us continue with our earlier assumption of n DMUs with interval ratios. Interval data consideration in model (1) will result in:

$$\begin{aligned}
&\tilde{R}_p = max \quad z_p \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j \tilde{r}_{ij} \geqslant z_p \tilde{r}_{ip}, \quad \forall i, \\
&\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geqslant 0, \quad \forall j.
\end{aligned} \tag{8}$$

where $\tilde{r}_{ij} \in [r_{ij}^L, r_{ij}^U]$ are the interval ratios.

The lower and upper bounds of the efficiency of DMU_p are obtained by solving the following models

$$\begin{array}{ll}
\text{Optimistic viewpoint} & \text{Pessimistic viewpoint} \\
R_p^L = max \quad z_p & R_p^U = max \quad z_p \\
s.t. \quad \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j r_{ij}^L + \lambda_p r_{ip}^U \geqslant z_p r_{ip}^U, \quad \forall i, & s.t. \quad \sum_{\substack{j=1 \\ j \neq p}}^{n} \lambda_j r_{ij}^U + \lambda_p r_{ip}^L \geqslant z_p r_{ip}^L, \quad \forall i, \\
\sum_{j=1}^{n} \lambda_j = 1, & \sum_{j=1}^{n} \lambda_j = 1, \\
\lambda_j \geqslant 0, \quad \forall j. & \lambda_j \geqslant 0, \quad \forall j.
\end{array} \tag{9}$$

The GNCP model with crisp ratios is a special case of models (9). In other words, when the lower and upper bounds are the same for all the data, the bounded intervals are all of zero length, and the two models (9) then coincide for each evaluation.

6.2. MNCP model with interval data

We now propose the MNCP model with interval data. Assume that there are n DMUs to be evaluated with interval ratios. When ratios of model (2) are assumed to be interval data, model (2) can be expressed as:

$$\begin{aligned} &\tilde{K}_p = \max h_p \\ &s.t. \quad \prod_{j=1}^{n} \tilde{r}_{ij}^{\lambda_j} \geqslant h\tilde{r}_{ip}, \quad \forall i, \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1, \\ &\qquad \lambda_j \geqslant 0, \quad \forall j. \end{aligned} \tag{10}$$

where $\tilde{r}_{ij} \in [r_{ij}^L, r_{ij}^U]$ are the interval ratios. We can obtain the upper and lower bounds of the efficiency of DMU_p by solving models (11), respectively:

$$\begin{array}{ll} \text{Optimistic viewpoint} & \text{Pessimistic viewpoint} \\ K_p^L = max \quad h_p & K_p^U = max \quad h_p \\ s.t. \quad \prod_{\substack{j=1\\ j\neq p}}^{n} r_{ij}^{L\lambda_j} + r_{ip}^{U\lambda_j} \geqslant hr_{ip}^{U\lambda_j}, \quad \forall i, & s.t. \quad \prod_{\substack{j=1\\ j\neq p}}^{n} r_{ij}^{U\lambda_j} + r_{ip}^{L\lambda_j} \geqslant hr_{ip}^{L\lambda_j}, \quad \forall i, \\ \qquad \sum_{j=1}^{n} \lambda_j = 1, & \qquad \sum_{j=1}^{n} \lambda_j = 1, \\ \qquad \lambda_j \geqslant 0, \quad \forall j. & \qquad \lambda_j \geqslant 0, \quad \forall j. \end{array} \tag{11}$$

By applying the alternations mentioned before, model (10) can be formulated as:

$$\begin{aligned} &H_p = \max \quad g_p + \varepsilon \sum_{i=1}^{m} s_i \\ &s.t. \quad \sum_{j=1}^{n} \lambda_j \tilde{\rho}_{ij} - s_i = g_p + \tilde{\rho}_{ip}, \quad \forall i, \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1, \\ &\qquad s_i \geqslant 0, \lambda_j \geqslant 0, \quad \forall i,j. \end{aligned} \tag{12}$$

where $\tilde{\rho}_{ij} \in [\rho_{ij}^L, \rho_{ij}^U]$ are the interval ratios. In order to measure the upper and lower bounds of the efficiency of DMU_p, the following pair of models are constructed:

$$\begin{array}{ll} \text{Optimistic viewpoint} & \text{Pessimistic viewpoint} \\ H_p^L = max \quad g_p + \varepsilon \sum_{i=1}^{m} s_i & H_p^U = max \quad g_p + \varepsilon \sum_{i=1}^{m} s_i \\ s.t. \quad \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j \rho_{ij}^L + \lambda_p \rho_{ip}^U - s_i = g_p + \rho_{ip}^U, \quad \forall i, & s.t. \quad \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j \rho_{ij}^L + \lambda_p \rho_{ip}^U - s_i = g_p + \rho_{ip}^U, \quad \forall i, \\ \qquad \sum_{j=1}^{n} \lambda_j = 1, & \qquad \sum_{j=1}^{n} \lambda_j = 1, \\ \qquad s_i \geqslant 0, \lambda_j \geq 0, \quad \forall i,j, & \qquad s_i \geqslant 0, \lambda_j \geq 0, \quad \forall i,j, \end{array} \tag{13}$$

where $\rho_{ij}^L = log(r_{ij}^L)$ and $\rho_{ij}^U = log(r_{ij}^U)$.

Note that H_p^{L*} and H_p^{U*} are the optimal objective values of the above models, and furthermore the interval efficiency of models (11) for DMU_p is $[K_p^L, K_p^U]$, in which K_p^L and K_p^U are equal to $e^{g_j^{l*}}$ and $e^{g_j^{u*}}$, respectively. Note also that g_j^{u*} and g_j^{l*} are optimal solution values obtained from the pessimistic viewpoint and optimistic viewpoint models (13), respectively.

The ratios data in real-life problems are often values between 0 and 1. The models (13) cannot be used directly if the ratio values are less than 1 because the log-transform of these values are negative and we require positive values for models (13). Emrouznejad and Cabanda [20] proposed scaling the data before running the model in order to avoid the negative values when using the log-transform. Hence, the same transformation can be used here if necessary.

Similar to interval DEA models, models (11) for all the evaluated units provide bounded intervals of efficiency scores $[K_j^{L*}, K_j^{U*}]$, $j = 1, \ldots, n$ which can be classified according to one of the following three groups:

- *The Fully Efficient Group:* This group consists of all the DMUs which are efficient in their pessimistic viewpoint (and clearly in their optimistic viewpoint), that is, $E^{++} = \{\mathrm{DMU}_j : K_j^U = 1\}$.
- *The Upper Efficient Group:* This group consists of all the DMUs which are efficient in their optimistic viewpoint but inefficient in their pessimistic viewpoint, that is, $E^{+} = \{\mathrm{DMU}_j : K_j^L = 1, K_j^U > 1\}$.
- *The Inefficient Group:* This group consists of all the DMUs which are inefficient in their optimistic viewpoint (and clearly in their pessimistic viewpoint), that is $E^{-} = \{\mathrm{DMU}_j : K_j^L > 1\}$.

7. Illustration with an application of aggregating ratios in banks

In this section, we apply the proposed methods to an example involving a set of regional banks. Let us assume that each of the 20 banks under evaluation in this example use three interval ratios given in Table 1. The three variables are r_1 = customer satisfaction (CS), r_2 = return on equity (ROE), and r_3 = return on assets (ROA). The aim is to aggregate these ratios and rank the 20 banks accordingly. We applied the general non-parametric ratio model (9) presented in this study and obtained the upper and lower bounds of the interval efficiencies and respective classifications presented in Table 1. All the calculations are done using the SAS optimization software (see [49]).

As shown in the last column of Table 1, Bank_2 is efficient in its optimistic and pessimistic viewpoints. Hence, Bank_2 is placed in the fully efficient (E^{++}) group. Bank_{10} and Bank_{12} are placed in the upper efficient (E^{+}) group because their lower efficiency scores are equal to the unity and their upper efficiency scores are greater than unity. Other banks are classified into the inefficient (E^{-}) group because their lower efficiency scores are greater than unity.

However, we believe that the multiplicative model (11) (or its equivalent model (13)) is more suitable for aggregating ratios with interval data. Hence, we used the proposed multiplicative ratio models (11) and obtained the interval efficiencies and respective classifications shown in Table 2.

As shown in the last column of Table 2, Bank_2 is efficient in its optimistic and pessimistic viewpoints. Hence, $E^{++} = \{\mathrm{Bank}_2\}$. Bank_{10}, Bank_{12}, and Bank_{19} are efficient in their best condition but inefficient in their worst condition. Therefore, $E^{+} = \{\mathrm{Bank}_{10}, \mathrm{Bank}_{12}, \mathrm{Bank}_{19}\}$ and the remaining banks are in E^{-}.

8. Conclusions and future research directions

Financial ratios are commonly used to evaluate performance in financial institutions. Fernandez-Castro and Smith [18] introduced the General Non-Parametric Corporate Performance model (GNCP). Emrouznejad and Cabanda [20] extended their model to the multiplicative non-parametric corporate performance (MNCP) model to overcome the convexity issue in DEA. In this paper, we proposed two new imprecise DEA models consisting of GNCP model with interval data and MNCP with interval data. An application with 20 DMUs is used to illustrate the results. Further research is needed to investigate other properties of the proposed models such as duality, weights and returns to scale as well as peers when the data is in

Table 1
Interval ratio data, interval efficiencies, and classification for the 20 banks when using the GNCP model with interval data.

Bank	CS		ROE		ROA		Efficiencies and classification		
	r_{1j}^L	r_{1j}^U	r_{2j}^L	r_{2j}^U	r_{3j}^L	r_{3j}^U	R_j^L	R_j^U	Group
1	14.84	15.4	23.1	26.1	3.44	4	1.04112	1.49231	E^-
2	14.17	19.2	36.2	41.8	5.9	6.9	1.00000	1.00000	E^{++}
3	11.91	12.6	31.1	33.2	3.71	4.2	1.09039	1.34409	E^-
4	9.93	13.5	24.7	34.9	3.3	3.46	1.03724	1.69233	E^-
5	12.7	13.6	24.2	26.06	3.1	3.54	1.14025	1.60539	E^-
6	13.2	15	20.22	22.31	2.6	3.34	1.10387	1.69520	E^-
7	9.5	13.64	13.42	24.2	1.83	2.3	1.16131	2.41721	E^-
8	11.5	11.9	22.04	26.4	2.53	3.1	1.24735	1.76835	E^-
9	9.24	11.8	27.83	32	2.57	3.7	1.13135	1.49993	E^-
10	8.4	9.1	23.4	37.44	2.1	3.15	1.00000	1.78635	E^+
11	11.36	12.1	32.3	33.43	3.8	3.9	1.08295	1.29416	E^-
12	18.8	29.13	13.76	17.1	3.2	4.01	1.00000	1.02135	E^+
13	7.44	13.7	17.9	33.3	2.4	2.47	1.05186	2.33536	E^-
14	10.74	11.7	23.7	28.55	2.8	3.07	1.23016	1.76367	E^-
15	13.8	14	20.6	32.1	2.9	4.41	1.04932	1.63052	E^-
16	9.4	10.2	28.84	31.4	2.71	3.2	1.15287	1.44949	E^-
17	8.7	9.69	18.3	22.12	1.6	2.14	1.51814	2.24215	E^-
18	14.5	14.98	16.7	22.03	2.4	3.3	1.10828	1.69722	E^-
19	13.4	14.6	20.04	33.4	2.68	4.9	1.00695	1.67813	E^-
20	7.57	10.8	12.9	14.61	1.1	1.4	1.56642	2.82247	E^-

Table 2
Interval ratio data, interval efficiencies, and classification for the 20 banks. when using the MNCP model with interval data.

Bank	CS		ROE		ROA		Efficiencies and classification		
	ρ_{1j}^{L}	ρ_{1j}^{U}	ρ_{2j}^{L}	ρ_{2j}^{U}	ρ_{3j}^{L}	ρ_{3j}^{U}	K_j^L	K_j^U	Group
1	1.17	1.18	1.36	1.41	0.53	0.60	1.00786	1.17041	E^-
2	1.15	1.28	1.55	1.62	0.77	0.83	1.00000	1.00000	E^{++}
3	1.07	1.10	1.49	1.52	0.56	0.62	1.03050	1.13882	E^-
4	0.99	1.13	1.39	1.54	0.51	0.53	1.01010	1.25945	E^-
5	1.10	1.13	1.38	1.41	0.49	0.54	1.04789	1.22011	E^-
6	1.12	1.17	1.30	1.34	0.41	0.52	1.03157	1.23426	E^-
7	0.97	1.13	1.12	1.38	0.26	0.36	1.05474	1.44781	E^-
8	1.06	1.07	1.34	1.42	0.40	0.49	1.09541	1.26984	E^-
9	0.96	1.07	1.44	1.50	0.40	0.56	1.05130	1.19732	E^-
10	0.92	0.95	1.36	1.57	0.32	0.49	1.00000	1.29702	E^+
11	1.05	1.08	1.50	1.52	0.57	0.59	1.03050	1.12752	E^-
12	1.27	1.46	1.13	1.23	0.50	0.60	1.00000	1.01010	E^+
13	0.87	1.13	1.25	1.52	0.38	0.39	1.02249	1.44781	E^-
14	1.03	1.06	1.37	1.45	0.44	0.48	1.09661	1.28403	E^-
15	1.13	1.14	1.31	1.50	0.46	0.64	1.01906	1.22205	E^-
16	0.97	1.00	1.45	1.49	0.43	0.50	1.06191	1.18540	E^-
17	0.93	0.99	1.26	1.34	0.20	0.33	1.18666	1.42653	E^-
18	1.16	1.17	1.22	1.34	0.38	0.51	1.03157	1.28684	E^-
19	1.12	1.16	1.30	1.52	0.42	0.69	1.00000	1.27535	E^+
20	0.87	1.03	1.11	1.16	0.04	0.14	1.19732	1.58730	E^-

the form of ratio interval. Moreover, according to the recent survey investigated by Hatami-Marbini et al. [50], the development of DEA with fuzzy data has evolved significantly to efficiency measurement in real-life problems over the years. Researchers interested could adopt similar procedure as in this paper and develop fuzzy GNCP and fuzzy MNCP models.

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References

[1] A. Charnes, W.W. Cooper, E.L. Rhodes, Measuring the efficiency of decision making units, Eur. J. Oper. Res. 2 (6) (1978) 429–444.
[2] A. Charnes, W.W. Cooper, E.L. Rhodes, Short communication: measuring the efficiency of decision making units, Eur. J. Oper. Res. 3 (4) (1979) 339.
[3] M.J. Farrell, The measurement of productive efficiency, J. Roy. Stat. Soc. 120 (3) (1957) 253–281.
[4] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, Manage. Sci. 30 (9) (1984) 1078–1092.
[5] B. Lev, Industry averages as targets for financial ratios, J. Account. Res. 7 (2) (1969) 290–299.
[6] C.F. Lee, C. Wu, Expectation formation and financial ratio adjustment processes, Account. Rev. 63 (2) (1988) 292–306.
[7] H.Z. David, Y.C. Peles, Measuring equilibrating forces of financial ratios, Account. Rev. 68 (4) (1993) 725–747.
[8] C. Wu, S.K. Ho, Financial ratio adjustment: industry-wide effects or strategic management, Rev. Quant. Finan. Acc. 9 (1) (1997) 71–88.
[9] J.L. Gallizo, F. Jiménez, M. Salvado, Adjusting financial ratios: a Bayesian analysis of the Spanish manufacturing sector, Omega 30 (3) (2002) 185–195.
[10] O.M. Joy, O.J. Tollefson, On the financial applications of discriminant analysis, J. Financial Quant. Anal. 10 (5) (1957) 723–739.
[11] M. Niemann, J.H. Schmidt, M. Neukirchen, Improving performance of corporate rating prediction models by reducing financial ratio heterogeneity, J. Bank. Finan. 32 (3) (2008) 434–446.
[12] Poor. Standard, Corporate Ratings Criteria, The McGraw-Hill Companies, New York, 2008.
[13] G. Whittington, Some basic properties of accounting ratios, J. Business Finan. Account. 7 (2) (1980) 219–223.
[14] P. Barnes, The analysis and use of financial ratios: a review article, J. Business Finan. Account. 14 (4) (1987) 449–461.
[15] W.H. Beaver, Financial ratios as predictors of failure, J. Account. Res. 4 (Supplement) (1966) 71–111.
[16] W.H. Beaver, Alternative accounting measures as predictors of failure, The Account. Rev. 43 (1) (1968) 113–122.
[17] E.L. Altman, Financial ratios, discriminant analysis and the prediction of corporate failure, J. Finan. 23 (4) (1968) 589–609.
[18] A. Fernandez-Castro, P. Smith, Towards a general non-parametric model of corporate performance, Omega 22 (3) (1994) 237–249.
[19] A. Emrouznejad, A.L. Anouze, DEA/C&R: DEA with classification and regression tree: a case of banking efficiency, Exp. Syst. 27 (4) (2010) 231–246.
[20] A. Emrouznejad, E. Cabanda, An aggregate measure of financial ratios using a multiplicative DEA model, Int. J. Finan. Services Manage. 4 (2) (2010) 114–126.
[21] A. Emrouznejad, E. Cabanda, R. Gholami, An alternative measure of the ICT-opportunity index, Inform. Manage. 47 (4) (2010) 246–254.
[22] P. Smith, Data envelopment analysis applied to financial statements, Omega 18 (2) (1990) 131–138.
[23] B. Hollingsworth, P. Smith, Use of ratios in data envelopment analysis, Appl. Econ. Lett. 10 (11) (2003) 733–735.
[24] A. Emrouznejad, G.R. Amin, DEA models for ratio data: convexity consideration, Appl. Math. Model. 33 (1) (2009) 486–498.
[25] R.D. Banker, W.W. Cooper, L.M. Seiford, R.M. Thrall, J. Zhu, Returns to scale in different DEA models, Eur. J. Oper. Res. 154 (2004) 345–362.
[26] A. Emrouznejad, B. Parker, G. Tavares, Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA, J. Socio-Econ. Plan. Sci. 42 (3) (2008) 151–157.
[27] J.K. Sengupta, A fuzzy systems approach in Data Envelopment Analysis, Comput. Math. Appl. 24 (8–9) (1992) 259–266.
[28] L.A. Zadeh, Fuzzy sets, Inform. Control. 8 (3) (1965) 338–353.
[29] J.K. Sengupta, Measuring efficiency by a fuzzy statistical approach, Fuzzy Sets Syst. 46 (1) (1992) 73–80.
[30] C. Kao, S.T. Liu, Fuzzy efficiency measures in data envelopment analysis, Fuzzy Sets Syst. 113 (2000) 427–437.

[31] C. Kao, S.T. Liu, Data envelopment analysis with missing data: An application to University libraries in Taiwan, J. Oper. Res. Soc. 51 (2000) 897–905.
[32] A. Hatami-Marbini, S. Saati, Stability of RTS of efficient DMUs in DEA with fuzzy under fuzzy U_0 data, Appl. Math. Sci. 3 (44) (2009) 2157–2166.
[33] A. Hatami-Marbini, S. Saati, A. Makui, An application of fuzzy numbers ranking in performance analysis, J. Appl. Sci. 9 (9) (2009) 1770–1775.
[34] P. Guo, H. Tanaka, Fuzzy DEA: a perceptual evaluation method, Fuzzy Sets Syst. 119 (1) (2001) 149–160.
[35] A. Hatami-Marbini, S. Saati, M. Tavana, An ideal-seeking fuzzy data envelopment analysis framework, Appl. Soft Comput. 10 (4) (2010) 1062–1070.
[36] S. Lertworasirikul, S.C. Fang, H.L.W. Nuttle, J.A. Joines, Fuzzy BCC model for data envelopment analysis, Fuzzy Optim. Decis. Making 2 (4) (2003) 337–358.
[37] S. Lertworasirikul, F. Shu-Cherng, J.A. Joines, H.L.W. Nuttle, Fuzzy data envelopment analysis (DEA): A possibility approach, Fuzzy Sets Syst. 139 (2) (2003) 379–394.
[38] W.W. Cooper, K.S. Park, G. Yu, IDEA and AR-IDEA: models for dealing with imprecise data in DEA, Manage. Sci. 45 (1999) 597–607.
[39] W.W. Cooper, K.S. Park, G. Yu, An illustrative application of IDEA (imprecise data envelopment analysis) to a Korean mobile telecommunication company, Oper. Res. 49 (2001) 807–820.
[40] W.W. Cooper, K.S. Park, G. Yu, IDEA (imprecise data envelopment analysis) with CMDs (column maximum decision making units), J. Oper. Res. Soc. 52 (2001) 176–181.
[41] J. Zhu, Imprecise data envelopment analysis (IDEA): a review and improvement with an application, Eur. J. Oper. Res. 144 (2003) 513–529.
[42] D.K. Despotis, Y.G. Smirlis, Data envelopment analysis with imprecise data, Eur. J. Oper. Res. 140 (2002) 24–36.
[43] Y.K. Lee, K.S. Park, S.H. Kim, Identification of inefficiencies in an additive model based IDEA (imprecise data envelopment analysis), Comput. Oper. Res. 29 (2002) 1661–1676.
[44] T. Entani, Y. Maeda, H. Tanaka, Dual models of interval DEA and its extension to interval data, Eur. J. Oper. Res. 136 (2002) 32–45.
[45] Y.M. Wang, R. Greatbanks, J.B. Yang, Interval efficiency assessment using data envelopment analysis, Fuzzy Sets Syst. 153 (2005) 347–370.
[46] C. Kao, Interval efficiency measures in data envelopment analysis with imprecise data, Eur. J. Oper. Res. 174 (2006) 1087–1099.
[47] Y.G. Smirlis, E.K. Maragos, D.K. Despotis, Data envelopment analysis with missing values: an interval DEA approach, Appl. Math. Comput. 177 (1) (2006) 1–10.
[48] A.H. Shokouhi, A. Hatami-Marbini, M. Tavana, S. Saati, A robust optimization approach for imprecise data envelopment analysis, Comput. Ind. Eng. 59 (3) (2010) 387–397.
[49] A. Emrouznejad, Measurement efficiency and productivity in SAS/OR, Comput. Oper. Res. 32 (2005) 1665–1683.
[50] A. Hatami-Marbini, A. Emrouznejad, M. Tavana, A taxonomy and review of the fuzzy data envelopment analysis literature: two decades in the making, Eur. J. Oper. Res. 214 (2011) 457–472.