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An Integrated Fuzzy Analytic Network Process and Fuzzy Regression Method for Bitcoin Price Prediction

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ABSTRACT

Predicting the prices of cryptocurrencies is more complicated than that of classical financial assets because they do not seem to have reached the maturity stage of their life. In addition, many known and unknown factors may affect Bitcoin prices; these factors and their importance seem to be changing faster than other financial assets. Therefore, the data used to predict the prices of cryptocurrencies can be considered big data challenging to manage due to their volume, variety, and variability. This study presents an integrated approach to managing the data when predicting the Bitcoin price. We first prepare a list of factors affecting the Bitcoin price. We then use the Fuzzy Analytic Network Process (FANP) to screen these factors and select the most important ones based on the experts' opinions. The selected factors are considered independent variables affecting the Bitcoin price. Next, we extract a fuzzy regression model using the historical data in which the Bitcoin price is considered the dependent variable. Finally, this model is validated with different confidence levels, and the appropriate level is selected to predict the Bitcoin price. The results show that Bitcoin prices fall within the forecasting intervals obtained from the fuzzy regression model for a 99% confidence level. Unlike crisp regression models, the fuzzy regression model used in this study does not predict the Bitcoin price as a crisp value; instead, it predicts the price as an interval value. The contributions of this study are fourfold: (1) identifying the factors affecting the Bitcoin price and investigating their mutual impacts on each other; (2) determining the most influential factors using the FANP method; (3) using fear and greed as essential sentimental independent variables in regression to predict the Bitcoin price; (4) and predicting the Bitcoin price as an interval instead of a crisp value.

1. Introduction

In recent years, cryptocurrencies have attracted much attention in financial markets. These financial assets are internet-based virtual currencies that use cryptographical functions for conducting and processing secure payment transactions [63]. Due to their rapid growth worldwide, cryptocurrencies make the decentralized payment system more valuable [49]. The most famous cryptocurrency, Bitcoin, is a peer-to-peer electronic cash system that allows direct online payments from one party to another without needing a financial institution [6]. Bitcoin has experienced sharp price fluctuations. For example, its price dropped from around $20,

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000 in 2017 to about $3,000 in 2019 and rose to about $69,000 in 2021. These price fluctuations reflect the complexities inherent in predicting cryptocurrencies' prices.

Considering that the investment risk of cryptocurrencies is more significant than traditional financial assets, predicting their prices and fluctuation tendencies is of great importance [56]. Different approaches have been proposed to predict the cryptocurrencies' prices, including genetic programming [22], deep learning [34,35], and machine learning [50]. Statistical time-series methods are common approaches for predicting cryptocurrencies' prices. Some of the methods are vine copula approaches [7], value-at-risk (VaR) analysis [41], univariate and multivariate autoregressive models [12], Markov-switching COGARCH-R-vine model [37], and GARCH-MIDAS framework [18,59].

Most of the methods, even some fuzzy methods, used in the literature predict the prices of financial assets as crisp values. However, we cannot predict the future values of these assets with certainty. In other words, predicting financial asset prices is complex, and their real prices may be more or less than the predicted values. Point estimating a financial asset's price in the future may be very challenging because we do not know to what extent the mismatch between real prices and predicted values is acceptable to investors. Another problem that most methods face when predicting the prices of financial assets arises from the big data that needs to be considered, including the variety of factors affecting the prices, the variability indicating the inconsistency which the data may show at different times, the large volume of data related to each factor, and the speed of data generation; these are more highlighted in crypto markets. It implies that we must use large databases to predict the prices of financial assets. However, these databases are difficult to manage effectively. Moreover, considering all factors and their large databases may confuse analysts and sometimes lead to deviations in forecasting. Therefore, an approach should be used to identify the most important factors affecting the prices of financial assets in each period and only use them when predicting the prices.

This study aims to present an integrated approach to predicting the Bitcoin price. This approach consists of two phases. The first phase uses the Multi-Attribute Decision-Making (MADM) method to obtain the most influential variables based on experts' opinions, and the second phase applies them in a regression model to predict the Bitcoin price. The Analytic Network Process (ANP) is widely used in MADM to capture interdependencies among attributes. It allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence) [27,53]. We also use the Fuzzy ANP (FANP) to cope with the uncertainties and ambiguities in complex decision-making processes, as experts prefer to express their preferences as linguistic or approximate values instead of crisp values.

Predicting the exact value of Bitcoin price is complicated because many factors affect the Bitcoin price; hence, it fluctuates a lot. Also, the point prediction may raise or lower the investors' unreasonable expectations and negatively affect them when the real price does not meet its predicted value. For these reasons, we predict the Bitcoin price as an interval value, not a crisp value. To this end, we use a special fuzzy regression model that uses crisp historical data and approximates regression variables' coefficients as interval values. Unlike classical regression models, which predict a crisp value for Bitcoin price, this fuzzy regression model predicts Bitcoin price as an interval value at the given confidence level. The contributions of the model proposed in this study include:

- using the FANP method to rank the factors affecting the Bitcoin price,
- investigating the mutual impacts of factors on each other,
- using fuzzy regression to predict the Bitcoin price, and
- using fear and greed as essential sentimental variables in regression to predict the Bitcoin price.

The rest of the paper is organized as follows. Section 2 gives the related literature review. Section 3 reviews the FANP and fuzzy regression models. Section 4 presents the research methodology. Section 5 provides a numerical example to illustrate the proposed approach. Section 6 discusses and compares our method with some existing approaches. Section 7 is devoted to conclusions and suggestions for future research.

2. Literature review

2.1. Bitcoin characteristics

Bitcoin is the digital version of a commodity to store value because it is the mining reward, and its supply is restricted [30]. Bitcoin lightning network is an off-chain crypto transaction platform working on payment channels to transact bi-directionally between two parties [49]. The growth of demand for Bitcoin is accelerating day by day. The data from blockchain.com indicates that while the number of wallets was about 32 million at the beginning of 2019, it significantly increased by the end of 2021 to about 80 million. de la Horra et al. [17] show that Bitcoin is a speculative asset in the short term. According to them, speculation does not seem to influence demand for Bitcoin in the long term. Tzouvanas et al. [58] identify the momentum effect in the cryptocurrency market, which is highly significant for short-term trading but disappears over the longer term. Corbet et al. [15] also show that Bitcoin may have diversification benefits for investors with short investment horizons. Due to its sharp price fluctuations, Bitcoin attracts different speculators, including scalpers, swingers, and short-term traders. Speculators purchase a financial asset for later re-sale rather than for use or a temporary sale to later repurchase in the hope of profiting from an intervening price change [25]. Speculators generally pay little attention to fundamental value and instead focus on price movements.

Bitcoin comes with several advantages and disadvantages. The speed and low cost of its transfer, the anonymity of the transference, and the transparency of transactions recorded in the blockchain are some positive aspects of Bitcoin that promote its adoption as cash; on the other hand, it can be used to facilitate trade-based money laundering schemes (Hakim das [23]) illegally. Blockchain, one of the foundations of Bitcoin and its underlying technology, is also recognized as the second internet (Asadi [4]). It enables reliable data structure against tampering even without any authority to rely on, instead of checking consistency with everyone who joins the system [21].

From a financial point of view, Bitcoin is more volatile than traditional financial assets, even in normal market conditions [55], leading to higher returns and risks [43]. Although Bitcoin is a highly complex and risky asset, it still represents an alternative investment instrument with the unique characteristic of high return and a low correlation with financial assets [56]. Investors who hold a conventional asset portfolio may consider Bitcoin an alternative asset to hedge their holdings [51]. Therefore, it is suggested that part of the capital be invested in Bitcoin when forming a portfolio because it can diversify the portfolio.

2.2. Predicting the Bitcoin price

Some studies have been conducted to predict the prices of cryptocurrencies. For example, Ha and Moon [22] use genetic programming to find attractive technical patterns in a cryptocurrency market that consistently finds profitable and frequent signals. Shen et al. [52] propose a simple three-factor pricing model with robust performance to different factor constructions. Their model consists of market, size, and reversal factors. Lucarelli and Borrotti [35] propose a deep Q-learning portfolio management framework that learns asset behaviors and describes the global reward function. They show that this framework is a promising approach for optimizing the dynamic portfolio, particularly for a crypto portfolio. Sebastião and Godinho [50] examine the predictability of cryptocurrencies and the profitability of trading strategies devised by machine learning techniques.

Some studies use statistical and time-series methods to predict the prices of cryptocurrencies. For example, Boako et al. [7] use vine copula approaches to model cryptocurrencies' co-dependence and portfolio VaR. Catania et al. [12] apply a set of crypto-predictors and propose univariate and multivariate autoregressive models for combining these predictors. Walther et al. [59] apply the GARCH-MIDAS framework to identify drivers of cryptocurrency volatility and forecast their daily, weekly, and monthly volatility. Fang et al. [18] use the GARCH-MIDAS model to investigate the impacts of the news-based implied volatility of cryptocurrencies in long-term volatility. Mba and Mwambi [37] present a Markov-switching COGARCH-R-vine model for cryptocurrency portfolio selection in which the optimal portfolio is obtained using the heuristic, derivative-free search algorithm differential evolution. Sung et al. [57] use an autoregressive conditional heteroskedasticity method to predict the prices of cryptocurrencies based on the relevant features affecting them.

Many studies focused on predicting the Bitcoin price as the leading cryptocurrency whose price fluctuations significantly impact other cryptocurrencies. They applied different methods for this purpose. For example, Liu et al. [34] utilized a deep learning method to predict the Bitcoin price. Their method consists of 40 determinants affecting Bitcoin price and considering aspects of the cryptocurrency market, public attention, and the macroeconomic environment. Rajabi et al. [44] used the learnable window size method and the multi-day trend to predict the next day's Bitcoin price. Khurana et al. [28] implemented an artificial intelligence trader robot that predicts Bitcoin prices in different time frames. Li and Du [33] used k-order transaction subgraphs, machine learning, and blockchain transaction pattern data to predict Bitcoin price. Zhong et al. [65] used long short-term memory and a relationwise graph attention network for prediction.

Predicting the Bitcoin price using statistical methods is a common approach in literature. We can mention the study by Ranjan et al. [45], which reviewed several statistical methods. This study showed that the logistic regression method can predict daily Bitcoin prices with an accuracy rate of about 65%. Maiti [36] used the discrete threshold regression model to investigate the relationship between energy consumption and the Bitcoin price. They found that the energy consumption of Bitcoin has no significant impact on its price. Based on the data of other cryptocurrencies, Yi et al. [64] predicted Bitcoin price using the scaled principal component analysis approach in heterogeneous autoregressive. Saheed et al. [48] presented six regression models to predict the Bitcoin price. The other methods for Bitcoin price prediction are continuous time series [29] and quantile regressions [26].

The sentimental factors affect the prices of cryptocurrencies significantly and may increase the risk of investing in this market. Some researchers predicted the Bitcoin price using different methods by considering the effects of sentimental factors. For example, Nasir et al. [41] applied a VaR framework, a copulas approach, and non-parametric drawings and analyzed the predictability of Bitcoin volume and returns based on Google search values. Critien et al. [16] used the Twitter sentiment to predict the Bitcoin price. Zou and Herremans [66] also used daily Twitter data with a text-based convolutional neural network to predict the Bitcoin price. Siu [54] used three major parametric nonlinear time series models to forecast and evaluate Bitcoin risk. Frohmann et al. [20] used time series and sentiment prediction to predict the daily Bitcoin price. These examples emphasize that predicting the Bitcoin price based only on past prices has low accuracy. For this reason, Rathore et al. [46] provided a methodology for predicting the future Bitcoin price that does not rely solely on past data due to seasonality in historical data.

2.3. Research gap

Many studies have been conducted to predict the prices of financial assets using different methods. They typically determine a crisp value to predict a financial asset's price. Therefore, the prediction values may differ greatly from the real prices. Our research uses an approach that predicts the Bitcoin price as an interval instead of a crisp value. This approach also considers the error level for the predicted interval values. Table 1 compares our approach with some existing approaches.

We consider some new factors in our model for the first time, including the fear and greed factors represented as "Bitcoin sentimental analysis" in our model. This factor affects Bitcoin prices based on several studies. However, our study is the only one that considers it in a fuzzy regression model to predict the Bitcoin price more accurately.

Table 1
Comparison of research contributions with some existing approaches.

Research	Financial asset	Affecting variables	Date Range	Prediction methods	Predicted values
[63]	Bitcoin, Ethereum, Bitcoin Cash, Litecoin, Monero and Dash		July 23, 2017 - July 23, 2019	Noise-Assisted Multivariate Empirical Mode Decomposition (NA-MEMD) and Wilcoxon signed-rank	Crisp
[30]	Bitcoin	US Dollar Index, gold, and the stock market	October 1, 2013 - June 30, 2019	Conditional autoregressive Value at Risk (VaR)	Crisp
[56]	42 cryptocurrencies	The stock market, exchange rate, and oil	January 1, 2018 - June 30, 2018	Gradient Boosting Decision Tree (GBDT) and Light Gradient Boosting Machine (LightGBM)	Crisp
[52]	1786 cryptocurrencies		April 2013 - March 2019	Three-factor pricing model	Crisp
[34]	Bitcoin	Stock markets, exchange rate, US Dollar index, oil	July 2013 - December 2019	Stacked denoising autoencoders (SDAE) model	Crisp
[41]	Bitcoin	Google searches and Bitcoin trading volume	weekly dataset from 2013 to 2017	VaR framework, a copulas approach, and non-parametric drawings	Crisp
[12]	Bitcoin, Litecoin, Ripple, and Ethereum	Stock markets, Commodities, Interest rates, and CDS	August 8, 2015 - December 28, 2017	Univariate dynamic linear and multivariate autoregressive models	Crisp
[18]	Five cryptocurrencies	News-based Implied Volatility (NVIX)	May 2013 - May 2019	GARCH-MIDAS model	Crisp
[8]	12 cryptocurrencies		August 8, 2015 - February 28, 2019	AR-GJR-GARCH Models	Crisp
[19]	Bitcoin	Bitcoin Hashrate	August 1, 2016 - February 29, 2020	bivariate vector-autoregression	Crisp
[42]	Bitcoin	Bitcoins in circulation, transaction volume, hash rate and mining difficulty, exchange rates, gold	January 2013 - May 2017	Bayesian structural time series	Crisp
[2]	Bitcoin	VIX	September 2017 - February 2020	Regression	Crisp
[32]	Cryptocurrencies	VIX, S&P 500	January 1, 2016 - December 31, 2020	Linear regression	Crisp
This investigation	Bitcoin	17 factors classified into four categories	January 2021 - November 2021	Fuzzy analytic network process)FANP(and fuzzy regression	Interval

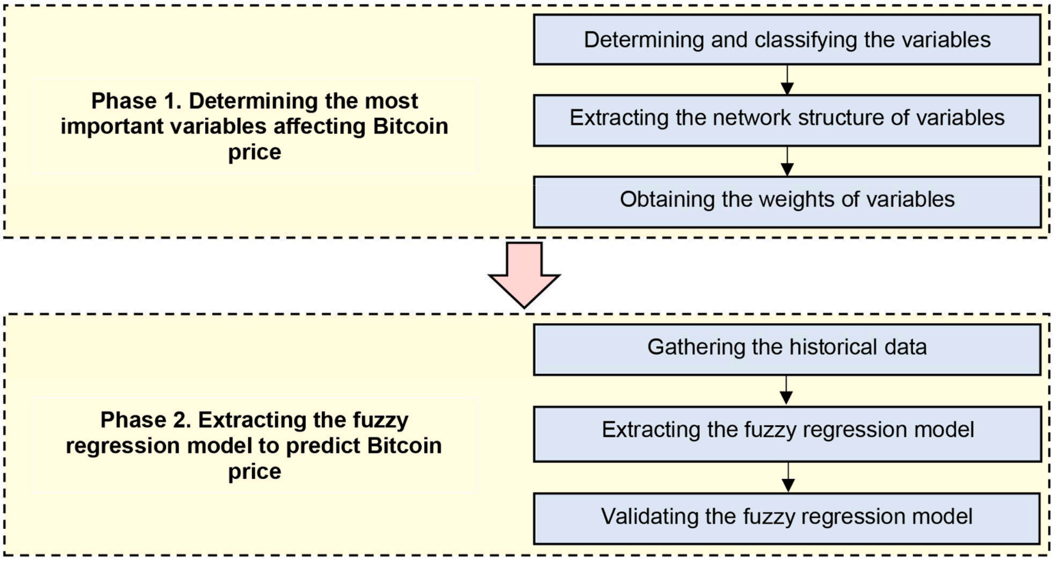

Fig. 1. The scheme of the proposed framework In the following, we describe the phases in Fig. 1 and their related steps.

3. Background

This section reviews the Fuzzy Geometric Mean Method (FGMM) proposed to extract the crisp local weights from a Fuzzy Comparison Matrix (FCM). Then, we review the fuzzy regression method.

3.1. The FGMM

FANP integrates the local weights extracted from several FCMs to obtain the global weights of alternatives. There are some classical methods in literature to extract the local weights from an FCM. However, these methods suffer from essential shortcomings and may even rank the alternatives incorrectly. For example, the initial methods proposed by Laarhoven and Pedrycz [31] and Buckley [9] are not popular due to their computational complexity. Also, the extent analysis method, the most well-known method proposed by Chang [13], provides the least accurate results among all weight-extracting methods [1]. This is because this method misapplies the degree of possibility relation to weigh the elements in an FCM, while it is a relation to rank triangular fuzzy numbers [61]. The other well-known method for weight extracting is the fuzzy preference programming presented by Mikhailov [38]. We cannot rely on this method because it extracts multiple sets of local weights from the same FCM [60].

Arman et al. [3] reviewed the shortcomings of classical weight-extracting methods and proposed new methods to avoid them. In the following, we review one of these methods, the FGMM, proposed to extract the crisp local weights from a trapezoidal FCM. For this purpose, consider a trapezoidal FCM, $\widetilde{B}$, as

$$\widetilde{B} = \left(\widetilde{b}_{ij}\right)_{n\times n} = \begin{bmatrix} (1,1,1,1) & (l_{12}, m_{12}, m'_{12}, u_{12}) & \dots & (l_{1n}, m_{1n}, m'_{1n}, u_{1n}) \\ (l_{21}, m_{21}, m'_{21}, u_{21}) & (1,1,1,1) & \dots & (l_{2n}, m_{2n}, m'_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, m'_{n1}, u_{n1}) & (l_{n2}, m_{n2}, m'_{n2}, u_{n2}) & \dots & (1,1,1,1) \end{bmatrix} \tag{1}$$

where $\widetilde{b}_{ij} = (l_{ij}, m_{ij}, m'_{ij}, u_{ij}) = \widetilde{b}_{ji}^{-1} = \left(\frac{1}{u_{ji}}, \frac{1}{m'_{ji}}, \frac{1}{m_{ji}}, \frac{1}{l_{ji}}\right), \quad i,j = 1, \dots\dots, n.$ Calculate the geometric mean of each row of the FCM $\widetilde{B}$ as:

$$\widetilde{M}_i = \left(\left(\prod_{j=1}^{n} l_{ij}\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} m_{ij}\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} m'_{ij}\right)^{\frac{1}{n}}, \left(\prod_{j=1}^{n} u_{ij}\right)^{\frac{1}{n}}\right), \quad i = 1, \dots, n. \tag{2}$$

where $\widetilde{M}_i$ is the fuzzy geometric mean of the trapezoidal fuzzy preferences in row i and n is the dimension of matrix $\widetilde{B}$. Then, normalize $\widetilde{M}_i$ as:

Table 2
The factors classified in the BI category.

Factor	Description
Bitcoin trading volume (BV) in dollars	The Bitcoin returns, and corresponding trading volumes jump together [8].
Bitcoin hash rate (BH)	The BH is the number of computations done by Bitcoin miners [19], and its movements are useful in predicting the Bitcoin price [24].
Bitcoin network difficulty (ND)	ND referred to mining difficulty, which describes how hard it is to find a new block [24].
Transactions volume (TV)	TV indicating the number of transferred Bitcoin affects its price [42].

Table 3
The factors classified in the SA category.

Factor	Description
Bitcoin sentimental analysis (BSA)	Bitcoin sentiment significantly predicts the price (Anamika et al., 2021). In this study, we consider the fear and greed index for BSA. This index gathers market volatility, volume, social media, Bitcoin dominance, and trends data. The values of this index are given for different days on https://alternative.me/crypto/fear-and-greed-index.
Bitcoin Google Trends (GT)	The frequency of Google searches positively affects Bitcoin returns [41].
Cboe Volatility Index (VIX)	VIX, one of the most recognized volatility measures, is a calculation designed to measure the US stock market's constant, 30-day expected volatility. As a proxy for fear in the equity market, the VIX index positively influences Bitcoin (Anamika et al., 2021). It controls the variables that might impact Bitcoin returns [32]. It means that when the equity market investors' sentiment is bearish, Bitcoin prices rise (Anamika et al., 2021). VIX can also measure the decreases in Bitcoin prices when the market faces financial uncertainty shocks [14].

$$\widetilde{S}_i = \left(\frac{\left(\prod_{j=1}^{n} l_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{n} u_{ij}\right)^{\frac{1}{n}}}, \frac{\left(\prod_{j=1}^{n} m_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{n} m'_{ij}\right)^{\frac{1}{n}}}, \frac{\left(\prod_{j=1}^{n} m'_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{n} m_{ij}\right)^{\frac{1}{n}}}, \frac{\left(\prod_{j=1}^{n} u_{ij}\right)^{\frac{1}{n}}}{\sum_{k=1}^{n}\left(\prod_{j=1}^{n} l_{ij}\right)^{\frac{1}{n}}} \right), \quad i = 1, \dots, n. \tag{3}$$

where $\widetilde{S}_i$ is the fuzzy local weight of the element corresponding to the row i. $\widetilde{S}_i$ is approximately a trapezoidal fuzzy number (FN) and can be shown as $\widetilde{S}_i = (l_i, m_i, m'_i, u_i)$. To convert $\widetilde{S}_i$ into a crisp value, Arman et al. [3] used the following equation:

$$S_i = \frac{1}{3}\left[(l_i + m_i + m'_i + u_i) - \frac{(m'_i \times u_i) - (l_i \times m_i)}{(m'_i + u_i) - (l_i + m_i)}\right]. \tag{4}$$

where S_i is the crisp weight of element i. Eq. (4), obtained based on the center of gravity method, represents the defuzzification formula for trapezoidal FNs. Note that the sum of S_i(s) is not necessarily equal to 1; therefore, they should be normalized as:

$$W_i = \frac{S_i}{\sum_{t=1}^{n} S_t}, \quad i = 1, \dots, n. \tag{5}$$

where W_i is the crisp weight of element i.

3.2. Fuzzy regression model

Different fuzzy regression models can be found in the literature. Here, we review a fuzzy regression model [10,11]) gave that obtains the interval regression coefficients based on the crisp observed data. We extend the formulas of this model for m independent variables. Consider n data as $(x_{1i}, x_{2i}, \dots, x_{mi}, y_i), i=1,\dots,n$, where x_{ki} is the value of the k^{th} independent variable for i^{th} data and y_i is the value of the dependent variable for i^{th} data. Assume that there is no uncertainty in the values of independent variables. Since predicting the future value of Y_i with certainty is impossible, this model focuses on the mean of Y_i, $E(Y_i)$. Assume that $E(Y_i)$ is a linear function of $x_{ki}(k = 1, .., m)$; the basic regression equation for the mean of Y_i is

$$y_i = a + \sum_{k=1}^{m} b_k x_{ki} \tag{6}$$

Now we estimate the regression coefficient values, i.e., the values of a and b_k ($k=1,\dots, m$). The approach we review first obtains the point estimators for regression coefficients and then uses them to obtain the fuzzy estimators. To obtain the point estimators for regression coefficients, i.e., $\widehat{a}$ and $\widehat{b}_k(k = 1, \dots, m)$, first, we form matrix X as:

$$X = \begin{bmatrix} 1 & x_{11} & x_{21} & \dots & x_{m1} \\ 1 & x_{12} & x_{22} & \dots & x_{m2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_{1n} & x_{2n} & \dots & x_{mn} \end{bmatrix} \tag{7}$$

Table 4
The factors classified in the GI category.

Factor	Description
Dow Jones 30 index (DJ30) US Dollar currency index (DXY) S&P 500 index (S&P 500) Hang Seng Index (HSI) Shanghai Stock Composite Index (SS.) Shenzhen Component Index (SZSE) US Inflation rate (USIR) FTSE China A50 (FTSE)	There are different indices in the literature affecting Bitcoin price. This table presents eight more common indices used by some researchers, such as Sun et al. (2020), Poyser (2019), Choi and Shin (2021), and Leirvik (2021). For example, Choi and Shin (2021) investigated that Bitcoin price increases significantly after a positive inflation shock.

Table 5
The factors classified in the FC category.

Factor	Description
WTI Crude oil futures (CO) USD/CNY (U/C)	There are some FC factors in the literature affecting Bitcoin price. This table presents two factors, CO and U/C, used by researchers like Sun et al. (2020) and Poyser (2019). Some other FC factors in the literature affect Bitcoin price, like gold price (Poyser, 2019). Nevertheless, according to experts, we selected only two factors that have a greater impact on Bitcoin price.

Table 6
Interdependencies of categories.

Category	Bitcoin Internal Factors (BI)	Sentimental and Attractive factors (SA)	Global Indices (GI)	Forex and Commodities (FC)
Bitcoin Internal Factors (BI)	✓	✓		
Sentimental and Attractive factors (SA)		✓	✓	
Global Indices (GI)		✓	✓	✓
Forex and Commodities (FC)			✓	

Then, we obtain the point estimators of regression coefficients as

$$[\widehat{a}, \widehat{b}_1, \widehat{b}_2, \dots, \widehat{b}_m]^T = \left(X^T.X\right)^{-1}.X.y^T \tag{8}$$

where y^T is the transpose of row vector $y=[y_1, y_2, \dots, y_n]$. Therefore, the regression model based on point estimators for coefficients is obtained as follows:

$$\widehat{y}_i = \widehat{a} + \sum\nolimits_{k=1}^{m} \widehat{b}_k.x_{ki} \tag{9}$$

Now, we aim to determine the fuzzy estimators of regression coefficients, i.e., $\overline{a}$ and $\overline{b}_k(k = 1, \dots, m)$. For this purpose, we first estimate the values of dependent variables, $\widehat{y}_i(i = 1, \dots, n)$, using Eq. (9). Then, we calculate a point estimator for the standard deviation

Table 7
Interdependencies of factors.

	BV	BH	ND	TV	BSA	GT	VIX	DJ 30	DXY	S&P 500	HIS	SS	SZSE	FTSE	USIR	U/C	CO
BV		✓			✓												
BH			✓														
ND		✓															
TV	✓	✓															
BSA																	
GT					✓												
VIX								✓	✓	✓							
DJ 30							✓		✓	✓		✓					
DXY								✓		✓						✓	✓
S&P 500							✓	✓	✓			✓					
HSI												✓	✓	✓		✓	
SS													✓	✓		✓	
SZSE												✓		✓		✓	
FTSE												✓	✓			✓	
USIR							✓	✓	✓	✓							
U/C									✓		✓	✓	✓	✓			
CO								✓		✓	✓	✓			✓		

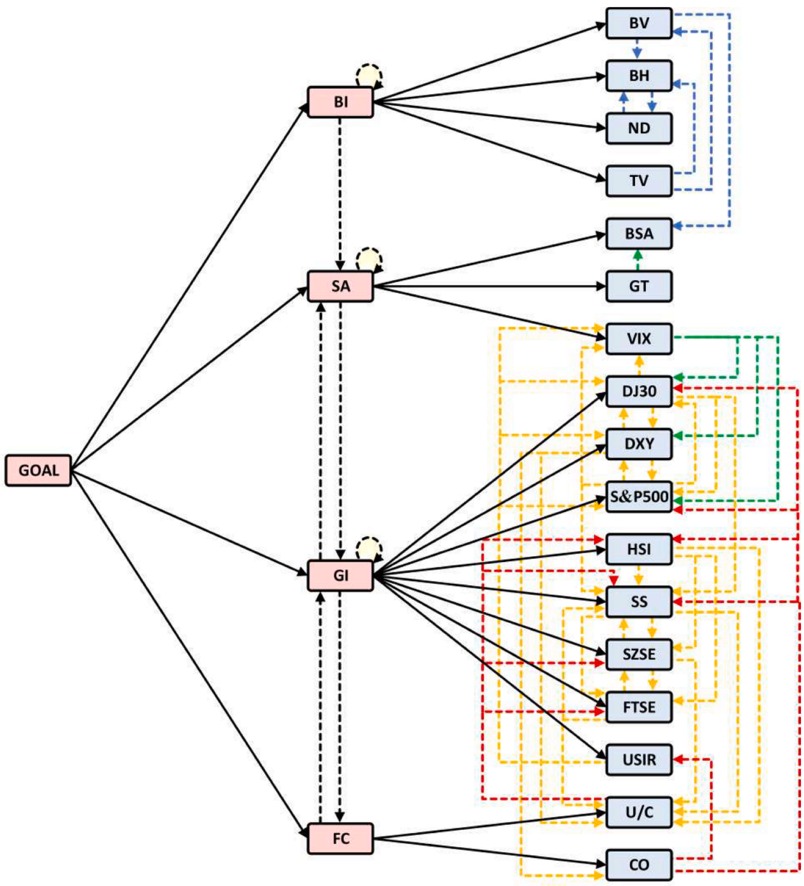

Fig. 2. The network structure of factors affecting Bitcoin price.

of error as:

$$\widehat{\delta} = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \widehat{y}_i)^2}{n-3}} \tag{10}$$

where, n-3 is the degree of freedom. Also, consider a value for λ between 0-1. Here, we consider the value of 0 for λ. Then calculate the values of $L(\lambda)$ and $R(\lambda)$ as:

$$L(\lambda) = [1-\lambda].X_{\left(R, \frac{\beta}{2}, n-3\right)} + \lambda.(n-3) \tag{11}$$

$$R(\lambda) = [1-\lambda].X_{\left(L, \frac{\beta}{2}, n-3\right)} + \lambda.(n-3) \tag{12}$$

where $X_{\left(L, \frac{\beta}{2}, n-3\right)}$ and $X_{\left(R, \frac{\beta}{2}, n-3\right)}$ are the critical values extracted from the chi-square distribution with n-3 degrees of freedom. We must find the confidence intervals for coefficients a and b_k (k=1,…, m) to extract the fuzzy regression. Let

$$A = [a_{ij}] = (X^T.X)^{-1} \tag{13}$$

A is a $(m+1) \times (m+1)$ matrix. The (1-β)100% confidence interval for a is:

$$\left[\widehat{a} - t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{11}},\ \ \widehat{a} + t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{11}} \right] \tag{14}$$

where a_{11} is the first element along the main diagonal of matrix A, and $t_{\frac{\beta}{2}, n-3}$ is the critical value extracted from t-distribution with n-3

Table 8
Linguistic preferences and their equivalent trapezoidal FNs.

Definitions	Row-to-column preference	Column-to-row preference
Equal importance	(1, 1, 1, 1)	(1, 1, 1, 1)
Equal to relatively more important	(1, 1, 2, 3)	(0.33, 0.5, 1, 1)
Relatively more important	(1, 2, 3, 4)	(0.25, 0.33, 0.5, 1)
Relatively important to high importance	(2, 3, 4, 5)	(0.2, 0.25, 0.33, 0.5)
High importance	(3, 4, 5, 6)	(0.17, 0.2, 0.25, 0.33)
High importance to very high importance	(4, 5, 6, 7)	(0.14, 0.17, 0.2, 0.25)
Very high importance	(5, 6, 7, 8)	(0.13, 0.14, 0.17, 0.2)
Very high importance to completely important	(6, 7, 8, 9)	(0.11, 0.13, 0.14, 0.17)
Completely important	(7, 8, 9, 9)	(0.11, 0.11, 0.13, 0.14)

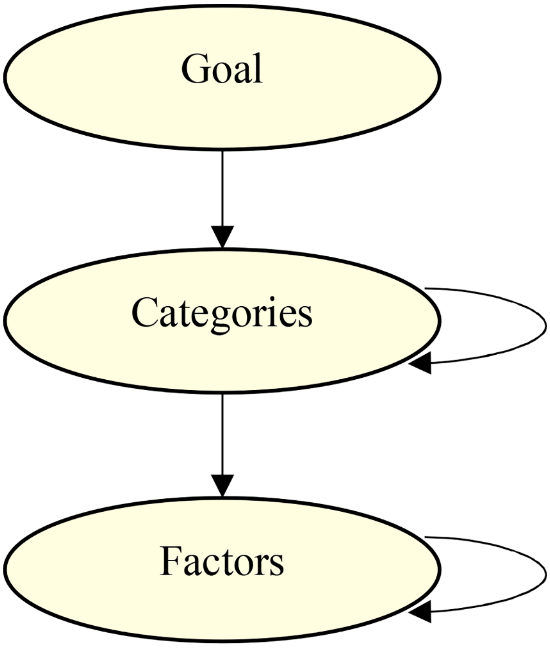

Fig. 3. The general form of the ANP structure.

Table 9
Supermatrix corresponding to Figure 3.

Cluster	Goal	Categories	Factors
Goal	0	0	0
Categories	W_{21}	W_{22}	0
Factors	0	W_{32}	W_{33}

degrees of freedom. Also, the (1-β)100% confidence interval for b_k ($k=1,...,m$) is:

$$\left[\widehat{b}_k - t_{\frac{\beta}{2},n-3}.\widehat{\delta}.\sqrt{a_{(k+1)(k+1)}},\ \ \widehat{b}_k + t_{\frac{\beta}{2},n-3}.\widehat{\delta}.\sqrt{a_{(k+1)(k+1)}}\right] \tag{15}$$

where $a_{(k+1)(k+1)}$ is the element ($k+1$) along the main diagonal of matrix A. Putting these confidence intervals together, one on top of another, results in fuzzy estimators, i.e., $\overline{a}$ and $\overline{b}_k(k=1,...,m)$of a and $b_k(k=1,...,m)$, respectively. As a result, a fuzzy regression model can be obtained based on the fuzzy estimators as:

$$\overline{y}_i = \left[y_i^L, y_i^U\right] = \overline{a} + \sum_{k=1}^{m} \overline{b}_k.x_{ki} \tag{16}$$

where

Table 10
FCM of categories w.r.t. the goal.

Category	BI	SA	GI	FC
BI	(1, 1, 1, 1)	(1, 2, 3, 4)	(1, 1, 2, 3)	(5, 6, 7, 8)
SA		(1, 1, 1, 1)	(1, 1, 1, 1)	(4, 5, 6, 7)
GI			(1, 1, 1, 1)	(6, 7, 8, 9)
FC				(1, 1, 1, 1)

$$y_i^L = \left(\widehat{a} - t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{11}}\right) + \sum_{k=1}^{m}\left(\widehat{b}_k - t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{(k+1)(k+1)}}\right) \tag{17}$$

$$y_i^U = \left(\widehat{a} + t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{11}}\right) + \sum_{k=1}^{m}\left(\widehat{b}_k + t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{(k+1)(k+1)}}\right) \tag{18}$$

$\bar{y}_i = [y_i^L, y_i^U]$ is the fuzzy estimator of $E(Y_i)$ in (1-β)100% confidence interval. Model (18) estimates a fuzzy value for the dependent variable based on the crisp values of independent variables.

4. Methodology

In this section, we present the proposed methodology. Our methodology consists of two phases. The first phase determines the variables affecting Bitcoin price and selects the most important ones. The second phase extracts the fuzzy regression models to predict the Bitcoin price. Our proposed framework is given in Fig. 1.

4.1. Determining the most important variables affecting Bitcoin price

In this phase, the most important variables affecting Bitcoin price are determined. This phase consists of three steps. The first step extracts the variables affecting Bitcoin price from the literature and then classifies them into different categories. These variables may affect each other. The second step extracts the network structure of the interdependencies of variables based on experts' opinions. The

Table 11
The computation required for calculating W_{21}.

Category	The geometric mean of each row using Eq. (2)	Normalization using Eq. (3)	Defuzzification using Eq. (4)	The crisp local weight (W_{21}) using Eq. (5)
BI	(1.49, 1.86, 2.54, 3.13)	(0.29, 0.36, 0.48, 0.57)	0.426	0.417
SA	(1, 1.14, 1.42, 1.63)	(0.16, 0.20, 0.27, 0.36)	0.252	0.246
GI	(1.19, 1.37, 1.68, 1.73)	(0.19, 0.25, 0.34, 0.39)	0.293	0.287
FC	(0.21, 0.23, 0.26, 0.30)	(0.03, 0.04, 0.06, 0.08)	0.052	0.050
Total			1.021	1

Table 12
The global weights of factors.

Factors	Global weights	Rank
BV	0.047	7
BH	0.265	1
ND	0.054	5
TV	0.051	6
BSA	0.175	2
GT	0.028	10
VIX	0.044	8
D.J. 30	0.018	13
DXY	0.091	4
S&P 500	0.015	14
HSI	0.021	12
SS	0.023	11
SZSE	0.094	3
FTSE	0.013	15
USIR	0.012	16
U/C	0.042	9
CO	0.008	17

Table 13
The historical data for extracting the fuzzy regression model.

Week	Date	Bitcoin price	BH	BSA	DXY	SZSE
1	2021-01-03	$32,782.02	146350014.1	93	89.937	14470.68
2	2021-01-10	$38,356.44	151089413.1	94	90.098	15319.29
3	2021-01-17	$35,791.28	151611247.4	79	90.772	15031.7
4	2021-01-24	$32,289.38	146698957.5	70	90.238	15628.73
5	2021-01-31	$33,114.36	150835173.6	78	90.584	14822
6	2021-02-07	$38,903.44	158947631.1	86	91.042	15007.3
7	2021-02-14	$48,717.29	156021049.0	95	90.48	15962.25
8	2021-02-21	$57,539.94	150850010.3	91	90.364	15823.11
9	2021-02-28	$45,137.77	154581775.5	55	90.879	14507.45
10	2021-03-07	$51,206.69	153716875.8	76	91.977	14412.31
11	2021-03-14	$59,302.32	156274408.9	78	91.679	13897.03
12	2021-03-21	$57,523.42	159862383.8	73	91.919	13606
13	2021-03-28	$55,950.75	165991691.2	74	92.766	13769.68
14	2021-04-04	$58,758.56	162156064.8	74	92.929	14122.61
15	2021-04-11	$60,204.96	171868015.6	76	92.163	13813.31
16	2021-04-18	$56,216.19	153847004.5	79	91.556	13720.74
17	2021-04-25	$49,004.25	146198398.7	31	90.859	14351.86
18	2021-05-02	$56,631.08	157685930.7	66	91.28	14438.57
19	2021-05-09	$58,232.32	177526237.2	73	90.233	13933.81
20	2021-05-16	$46,456.06	176179075	20	90.321	14208.78
21	2021-05-23	$34,770.58	145139314.8	14	90.017	14417.46
22	2021-05-30	$35,678.13	149763851.8	10	90.031	14852.88
23	2021-06-06	$35,862.38	149939087.8	17	90.136	14870.91
24	2021-06-13	$39,097.86	136465896.3	23	90.555	14801.24
25	2021-06-20	$35,698.30	125129902.8	21	92.225	14583.67
26	2021-06-27	$34,649.64	99935754	22	91.851	15003.85
27	2021-07-04	$35,287.78	86292418.37	27	92.226	14670.71
28	2021-07-11	$34,240.19	96184636.55	20	92.13	14844.36
29	2021-07-18	$31,796.81	99739876.72	19	92.687	14972.21

third step obtains the weights of these variables and ranks them accordingly. Different MADM methods can be used for this purpose. This study uses ANP because the variables affect each other, and the most appropriate method for these situations is ANP. Experts also express their preferences as linguistic values. Hence, this study uses a fuzzy version of ANP to cope with these uncertainties.

4.2. Extracting the fuzzy regression model to predict Bitcoin price

In this phase, a fuzzy regression model is extracted based on historical data. This model is validated and then used to predict Bitcoin price. This phase consists of three steps. The first step gathers the historical data related to the dependent variable (Bitcoin price) and independent variables (the most important variables affecting Bitcoin price). The second step derives the fuzzy regression model based on crisp historical data using the approach described in Subsection 2.2. The third step evaluates the validation of the fuzzy regression model based on real historical data. This validation assesses to what extent the Bitcoin price obtained from the fuzzy regression model reflects reality. We derive several fuzzy regression models for different levels of confidence. The higher the confidence level, the wider

Table 14
Sample data for evaluating fuzzy regression models.

Week	Date	Bitcoin price	BH	BSA	DXY	SZSE
30	2021-07-25	$35,350.19	99328674.26	27	92.912	15028.57
31	2021-08-01	$39,974.90	108889829	60	92.174	14473.21
32	2021-08-08	$43,798.12	112005260.9	74	92.8	14827.41
33	2021-08-15	$47,047.00	113148142.6	71	92.518	14799.03
34	2021-08-22	$49,321.65	125274213.3	76	93.496	14253.54
35	2021-08-29	$48,829.83	129727331.4	72	92.686	14436.9
36	2021-09-05	$51,753.41	128094799.3	73	92.035	14179.86
37	2021-09-12	$46,063.27	134785023.1	32	92.582	14771.87
38	2021-09-19	$47,260.22	137836460.2	53	93.195	14359.36
39	2021-09-26	$43,208.54	138379012.4	27	93.327	14357.85
40	2021-10-03	$48,199.95	145029457.8	49	94.035	14309.01
41	2021-10-10	$54,771.58	141032003.3	71	94.067	14414.16
42	2021-10-17	$61,553.62	143953931	79	93.937	14415.99
43	2021-10-24	$60,930.84	151210929.2	73	93.642	14492.82
44	2021-10-31	$61,318.96	159871336.2	74	94.123	14451.38
45	2021-11-07	$63,326.99	161379304.2	73	94.32	14462.62
6	2021-11-14	$65,466.84	160273275.8	74	95.128	14705.37
47	2021-11-21	$58,730.48	166066724.2	49	96.031	14752.49

Table 15
Bitcoin price forecasting with different confidence intervals.

Week	99% Upper	99% Lower	90% Upper	90% Lower	80% Upper	80% Lower	60% Upper	60% Lower
30	34293.65	41441.80	32782.36	42729.19	31494.97	46262.11	27962.04	39930.52
31	37051.66	43573.43	35672.81	44748.01	34498.23	47971.34	31274.90	42194.58
32	40735.26	47563.90	39291.52	48793.75	38061.67	52168.75	34686.67	46120.16
33	39813.33	46050.48	38494.65	47174.80	37371.33	50256.46	34288.67	44731.80
34	42711.63	61467.92	46332.62	57846.93	47652.11	56527.44	49201.08	54978.47
35	41817.19	55539.66	44466.38	52890.47	45431.74	51925.10	46565.00	50791.85
36	38452.39	53167.49	41293.21	50326.66	42328.41	49291.47	43543.63	48076.24
37	37720.95	54858.03	41029.35	51549.63	42234.93	50344.06	43650.17	48928.81
38	43825.08	61064.45	47153.23	57736.31	48366.01	56523.53	49789.70	55099.84
39	40784.72	62563.82	44989.28	58359.25	46521.42	56827.11	48320.02	55028.52
40	45061.06	71720.60	50207.82	66573.84	52083.30	64698.37	54284.94	62496.73
41	45943.41	71032.41	50786.97	66188.85	52551.95	64423.86	54623.90	62351.92
42	47019.23	71265.90	51700.17	66584.96	53405.90	64879.23	55408.28	62876.85
43	47141.68	71174.99	51781.43	66535.24	53472.15	64844.52	55456.91	62859.76
44	48756.75	79342.25	54661.44	73437.56	56813.10	71285.90	59338.96	68760.04
45	48768.67	81892.06	55163.31	75497.43	57493.51	73167.23	60228.96	70431.77
46	46916.28	89802.86	55195.75	81523.39	58212.78	78506.36	61754.51	74964.63
47	44158.51	101252.27	55180.75	90230.03	59197.24	86213.54	63912.25	81498.53

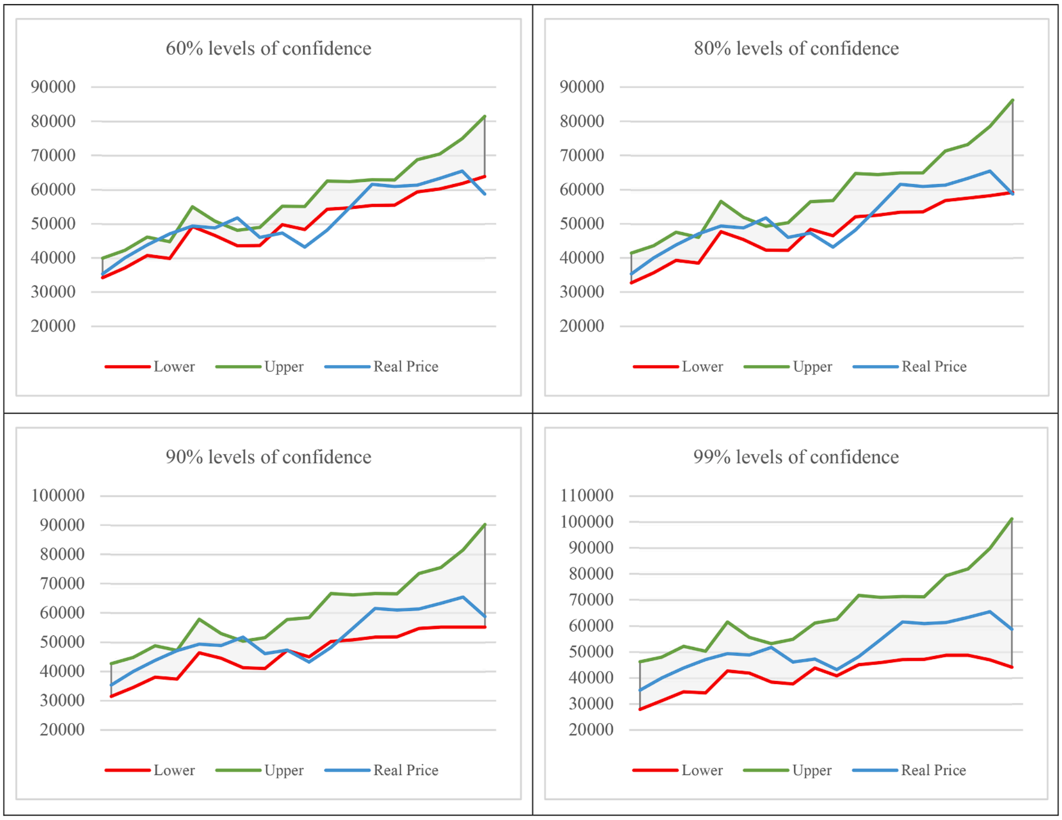

Fig. 4. Forecasting the Bitcoin price schematically.

the interval our model estimates for Bitcoin price, and thus, the more likely it is that the real price will fall within the estimated interval. However, the wider interval may reduce the popularity of the fuzzy regression model. Therefore, we choose the lowest confidence level in which the fuzzy regression model has acceptable validity.

5. Illustrative example

We used the approach proposed in this study to extract the fuzzy regression model for Bitcoin price prediction. To this end, we extracted 17 factors affecting the Bitcoin price from the literature. Then, we considered their conceptual similarities and accordingly classified them into four categories, including Bitcoin Internal (BI) factors, Sentimental and Attractive (SA) factors, Forex and Commodities (FC), and Global Indices (GI). These categories and the related factors in each category are given in Tables 2,3,4,5. These tables also briefly define each factor and describe how that factor affects Bitcoin price.

According to experts, the identified categories are affected by each other. We used the experts' opinions in Table 6 to determine the categories' interdependencies. For example, this table indicates that the FC category affects the GI category.

Experts believe that the factors affecting the Bitcoin price also affect each other. We obtained the interdependencies between factors using the experts' opinions given in Table 7.

Considering Tables 6 and 7, we draw the network structure of the problem in Fig. 2. This figure shows 17 factors affecting the Bitcoin price and the category to which each factor belongs. Fig. 2 also shows the schematical effects of these factors on each other, obtained according to experts.

According to Fig. 2, we formed the corresponding pairwise comparison matrices and arranged a panel of experts to determine the relative preferences in each matrix based on their consensus. The experts filled these matrices by linguistic preferences. We replaced these preferences with their equivalent trapezoidal FNs, given in Table 8.

Then, we used the FANP to obtain the weights of factors. For this purpose, we first obtained the crisp local weights from each fuzzy comparison matrix using the FGMM and then extracted the global weights of factors from the corresponding supermatrix. The general form of the problem and its corresponding supermatrix can be shown in Fig. 3 and Table 9.

Fig. 3 is, in fact, the general form of Fig. 2, indicating the network structure of the problem. The arrows in this figure indicate the problem's network relationships. Therefore, based on its arrows, we can form the supermatrix (Table 9) corresponding to Fig. 3.

Table 9 shows that four intersections in the supermatrix should be filled with local weights denoted as W_{ij}. Each W_{ij} is a submatrix indicating the local weights of elements of cluster *i* with respect to (w.r.t.) the elements of cluster *j*. In the following, we obtain each W_{ij} separately.

$\boldsymbol{W_{21}}$**:** This submatrix, indicating the local weights of categories (the elements of cluster 2) w.r.t. the goal (cluster 1), is formed based on only one trapezoidal FCM shown in Table 10.

We extracted the crisp local weights from this matrix using the FGMM. The required computations for calculating W_{21} are given in Table 11.

$\boldsymbol{W_{22}}$**:** This submatrix, indicating the local weights of categories (the elements of cluster 2) w.r.t. the categories (the elements of cluster 2), is formed based on four trapezoidal FCMs. The crisp local weights are derived from these matrices using the FGMM and form W_{22} as:

	BI	**SA**	**GI**	**FC**
BI	0.5	0	0.096	0
SA	0.5	0.5	0.191	0
GI	0	0.5	0.5	1
FC	0	0	0.404	0

$\boldsymbol{W_{32}}$**:** This submatrix, indicating the local weights of factors (the elements of cluster 3) w.r.t. the categories (the elements of cluster 2), is formed based on four trapezoidal FCMs. The crisp local weights are derived from these matrices using the FGMM and form W_{32} as:

	BI	**SA**	**GI**	**FC**
BV	0.113	0	0	0
BH	0.636	0	0	0
ND	0.13	0	0	0
TV	0.122	0	0	0
BSA	0	0.709	0	0
GT	0	0.114	0	0
VIX	0	0.177	0	0
D.J. 30	0	0	0.062	0
DXY	0	0	0.318	0
S&P 500	0	0	0.051	0
HSI	0	0	0.072	0
SS	0	0	0.081	0
SZSE	0	0	0.327	0
FTSE	0	0	0.046	0
USIR	0	0	0.043	0
U/C	0	0	0	0.841
CO	0	0	0	0.159

$\boldsymbol{W_{33}}$**:** This submatrix, indicating the local weights of factors (the elements of cluster 3) w.r.t. the factors (the elements of cluster 3), is formed based on 17 trapezoidal FCMs. The crisp local weights are derived from these matrices using the FGMM and form W_{33} as:

	BV	BH	ND	TV	BSA	GT	VIX	DJ 30	DXY	S&P 500	HSI	SS	SZSE	FTSE	USIR	U/C	CO
BV	0	0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
BH	0.76	0	1	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0
ND	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BSA	0.24	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
GT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
VIX	0	0	0	0	0	0	0	0.33	0	0.31	0	0	0	0	0.23	0	0
DJ30	0	0	0	0	0	0	0.14	0	0.22	0.42	0	0	0	0	0.33	0	0.29
DXY	0	0	0	0	0	0	0.72	0.10	0	0.21	0	0	0	0	0.11	0.29	0
S&P	0	0	0	0	0	0	0.14	0.51	0.25	0	0	0	0	0	0.33	0	0.29
HSI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.08	0.09
SS	0	0	0	0	0	0	0	0.06	0	0.06	0.28	0	0.63	0.56	0	0.27	0.04
SZSE	0	0	0	0	0	0	0	0	0	0	0.15	0.57	0	0.34	0	0.23	0
FTSE	0	0	0	0	0	0	0	0	0	0	0.49	0.29	0.22	0	0	0.13	0
USIR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.29
U/C	0	0	0	0	0	0	0	0	0.25	0	0.08	0.14	0.15	0.1	0	0	0
CO	0	0	0	0	0	0	0	0	0.28	0	0	0	0	0	0	0	0

By placing the submatrices W_{21}, W_{22}, W_{32}, and W_{33} in Table 9, a 22 × 22 supermatrix is formed. Solving this supermatrix obtains the global weights of factors shown in Table 12.

Table 12 indicates that, according to experts, the most important factors affecting the Bitcoin price are BH, BSA, SZSE, and DXY. We considered these factors the independent variables and extracted the fuzzy regression model accordingly. For this purpose, we first gathered the weekly historical data related to these factors and the Bitcoin price for 29 periods in 2021, shown in Table 13.

Then, we applied the approach reviewed in Subsection 2.2 to extract fuzzy regressions based on the data given in Table 13. These equations were extracted for different confidence intervals, i.e., for 99%, 90%, 80%, and 60%, as below:

Table 16
Factors affecting the Bitcoin price in different studies.

Factors	Choi and Shin (2021)	Sun et al. (2020)	Nasir et al. (2019)	Bouri et al. (2020)	Fantazzini and Kolodin (2020)	Hayes (2017)	Poyser (2019)	Anamika et al. (2021)	Leirvik (2021)	Corbet et al. (2018)	This study
Bitcoin trading volume				✓							
Bitcoin hash rate					✓	✓					✓
Bitcoin network difficulty						✓					
Transactions volume							✓				
Bitcoin sentimental analysis								✓			✓
Bitcoin Google Trends			✓								
Cboe Volatility Index								✓	✓	✓	
Dow Jones 30 index		✓					✓				
The US Dollar currency index		✓					✓				✓
S&P 500 index		✓					✓		✓		
Hang Seng Index		✓									
Shanghai Stock Composite index		✓									
Shenzhen Component index		✓									✓
The US Inflation rate	✓	✓									
FTSE China A50		✓					✓				
WTI Crude oil futures		✓					✓				
USD/CNY		✓					✓				

$$y_{(\beta=0.01)} = [-951981.85,\ 147330.78] + [3.98E^{-05},\ 0.0005].x_1 + [-86.08,\ 206.35].x_2 + [-79.17,\ 9760.66].x_3 + [-10.40,\ 5.1571].x_4$$

$$y_{(\beta=0.10)} = [-739753.99,\ -64897.08] + [0.0001,\ 0.0004].x_1 + [-29.63,\ 149.89].x_2 + [1820.46,\ 7861.03].x_3 + [-7.3977,\ 2.1535].x_4$$

$$y_{(\beta=0.20)} = [-662418.39,\ -142232.67] + [0.0002,\ 0.0004].x_1 + [-9.0566,\ 129.319].x_2 + [2512.68,\ 7168.81].x_3 + [-6.3032,\ 1.0589].x_4$$

$$y_{(\beta=0.40)} = [-571633.13,\ -233017.94] + [0.0002,\ 0.0003].x_1 + [15.093,\ 105.169].x_2 + [3325.29,\ 6356.198].x_3 + [-5.0182,\ -0.2259].x_4$$

To illustrate how these equations were extracted using crisp data, the computations required to extract the regression model for a 99% confidence interval are given in Appendix 1. The lower the confidence interval, the narrower the price forecast interval for Bitcoin, which may be more applicable. However, the lower confidence interval may suffer from unacceptable forecast errors. The lower confidence interval may prevent many real Bitcoin prices from falling within the narrower forecast intervals. Therefore, we chose a low confidence interval with acceptable forecasting errors. For this purpose, we evaluated the validity of different regression models using weekly historical data for the 30$^{\text{th}}$ to 47$^{\text{th}}$ weeks in 2021, given in Table 14.

We use the data in Table 14 to predict the Bitcoin prices as interval values. These predictions are given in Table 15 for different confidence levels.

Fig. 4 schematically shows the results of Table 15. The blue line in this figure represents the real Bitcoin prices, while the red and green lines represent the minimum and maximum prices predicted by our approach. Fig. 4 shows that the higher confidence level leads to a wider interval for Bitcoin price prediction. This, on the one hand, increases the probability that the real Bitcoin price will fall within the predicted interval. On the other hand, it may decrease the model's popularity due to the wide interval it predicts for Bitcoin price. The appropriate confidence level for Bitcoin price prediction may vary for different investors.

Table 15 and Fig. 4 show that the real Bitcoin prices for all periods fall within the forecasting intervals obtained from the regression model for a 99% confidence level. However, these intervals are wider than those obtained from other regression models and may be less applicable. On the other hand, the regression models for 60% and 80% confidence levels suffer from forecasting errors in 6 of the 18 periods, i.e., for periods 33, 36, 38, 39, 40, and 47, indicating a forecast error rate equal to 33% that is not acceptable to experts. According to experts, the most appropriate regression model is obtained for a 90% confidence level because its forecasting intervals are narrower than those obtained by the regression model for a 99% confidence level, and its forecasting errors are acceptable. This model fails in Bitcoin price forecasting only in 3 of the 18 periods, i.e., for periods 36, 39, and 40, indicating a forecast error rate of 16%.

6. Discussion

Many studies use fuzzy sets to overcome uncertainties arising from financial problems. For example, Naranjo and Santos [40] propose an intelligent decision tool for stock market investors that uses fuzzy Japanese candlesticks. Sadeghi et al. [47] forecast and classify the future trend in Forex markets using a combined technique based on an ensemble multi-class support vector machine and fuzzy NSGA-II. Xie et al. [62] integrate a neuro-fuzzy system with the Hammerstein-Wiener model, forming an indivisible five-layer network for stock price prediction. Mittal and Nagpal [39] present a regression-based mechanism to evaluate the fundamental health of the stock. This mechanism operates on a fuzzy rule base to provide the requisite advice based on the stock health index.

Fuzzy sets have also been used to predict the Bitcoin price because the data used for this purpose may be ambiguous and vague. Some studies used fuzzy concepts to overcome these uncertainties. For example, Atsalakis et al. [5] propose a computational intelligence technique that uses a hybrid neuro-fuzzy controller to forecast daily Bitcoin price trends.

We used a completely different fuzzy approach for predicting the Bitcoin price compared to existing fuzzy approaches. These approaches consider the uncertain input data and predict the prices as crisp values accordingly. In contrast, our approach considers the crisp input data and predicts the Bitcoin prices as interval values. In other words, the existing fuzzy approaches consider uncertainties to be the nature of input data, while our approach considers uncertainty to be the nature of a predicting procedure. It is worth saying that the studies do not consider all the factors affecting the Bitcoin price, as shown in Table 16. This table gives some studies on Bitcoin price and the factors each study has considered.

According to Table 16, many factors may affect the Bitcoin price. This table also shows that each study considers only some factors affecting the Bitcoin price, not all of them, for two reasons. The first reason is related to the knowledge and experiences of experts in different studies. For example, Li and Du [33] considered the blockchain transaction volume as an essential factor affecting the Bitcoin price, while we ignored this factor based on the experts' preference we extracted from the FANP method.

The second reason arises from the fact that the importance of each factor may vary over time. This means that a factor affecting the Bitcoin price significantly at a specific time does not necessarily have the same effect at other times. Hence, we cannot express certainty that the factors given in a study are always the most important ones. Therefore, we conclude that selecting the most important factors affecting the Bitcoin price in a specific period should be the initial purpose of each study for Bitcoin price prediction. To this end, we first used the FANP method and selected four important factors from our experts' opinions, as shown in Table 16. We used FANP to choose the most important factors because some affect each other, and the appropriate tool for ranking the factors in these situations is FANP. Then, we formed a fuzzy regression model based on these factors for predicting the Bitcoin price.

Some studies predict the Bitcoin prices by considering the aspects related to its price, such as its past trading prices and trading volume [29], its lagged realized volatility [26], its opening price, its day high and low prices, and its market capitalization [46]. Bitcoin price prediction based only on past prices is generally similar to the technical analysis applied to cryptocurrencies and may be appropriate for a very short-term future. For example, Kim and Byun [29] predict the Bitcoin price minute-by-minute. Table 16 shows that researchers consider more variables for Bitcoin price prediction for extended periods. For this reason, we considered factors like the US and China indexes in this study.

Another main difference between our study and similar ones is the research tool. While some studies use tools like discrete threshold regression [36] and quantile regressions [26] to predict the Bitcoin price as a crisp value, we used a fuzzy regression that predicts the Bitcoin price as an interval in a specified confidence level.

7. Conclusion

Bitcoin price forecasting is one of the most controversial financial issues because many known and unknown factors influence it. Therefore, large databases must be used to predict the Bitcoin price more accurately. However, these databases are difficult to manage. Hence, some studies apply approaches to use the most important parts of these databases. In this study, we applied the FANP to select the most important factors affecting Bitcoin price. Then, we derived a regression model based on these factors to predict the Bitcoin price as an interval value. Predicting the Bitcoin price as an interval instead of a crisp value is the advantage of our approach.

Nevertheless, it may suffer from a weakness. If the historical values of the independent variables come with intensive fluctuations, the Bitcoin price may be predicted in a wider interval. This may reduce the popularity of our approach. However, intensive changes in historical data may significantly impact the Bitcoin price predicted by the other methods. Therefore, they may also suffer from the same weakness.

Considering the Bitcoin life cycle, it does not seem to be in its maturity stage; it seems to be going through its growth stage. Therefore, the factors affecting the Bitcoin price and their importance may vary quickly. It means that Bitcoin price forecasting is not a static but dynamic approach. It implies that the factors used in this study should be updated over time. Future research can identify the most critical factors affecting the Bitcoin price in different periods to analyze why the importance of some factors varies over time. Also, the regression model only considers factors determined by experts and ignores the impacts of important events such as seasonal effects, governmental policies, and the Coronavirus pandemic. These events may fluctuate the Bitcoin price sharply and distort the prices predicted by regression models significantly. Investigating the effects of these events on the values predicted by regression models can also be another topic for future research.

This study applied a particular type of fuzzy regression that uses crisp data and predicts Bitcoin prices as interval values. However, most data are vague, and the crisp data may not reflect their uncertainties well. For example, this model considers a crisp value for Bitcoin price in a day that cannot represent its fluctuations on that day. Therefore, it is suggested that future studies use different types of fuzzy regression models considering fuzzy data instead of crisp data. Since the nature of some data is more complicated, we suggest that future studies develop predicting models for the other types of fuzzy sets, including type-2, intuitionistic, or hesitant fuzzy data. In this study, we used fuzzy regression to predict the Bitcoin price as an interval. Predicting the Bitcoin price as an interval instead of a crisp value can be more attractive to many investors and analysts. Therefore, we suggest that future studies develop other methods, like statistical methods, neural networks, and deep learning, to predict Bitcoin prices as interval values.

Declaration of Competing Interest

The above authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix 1

The fuzzy regression model for 99% interval confidence is derived based on the data given in Table 13. The required steps are given as follows. The matrix X is formed as

Unit column	BH	BSA	DXY	SZSE
1	146350014.1	93	89.937	14470.68
1	151089413.1	94	90.098	15319.29
1	151611247.4	79	90.772	15031.7
1	146698957.5	70	90.238	15628.73
1	150835173.6	78	90.584	14822
1	158947631.1	86	91.042	15007.3
1	156021049	95	90.48	15962.25
1	150850010.3	91	90.364	15823.11
1	154581775.5	55	90.879	14507.45
1	153716875.8	76	91.977	14412.31
1	156274408.9	78	91.679	13897.03

(continued on next page)

(*continued*)

1	159862383.8	73	91.919	13606
1	165991691.2	74	92.766	13769.68
1	162156064.8	74	92.929	14122.61
1	171868015.6	76	92.163	13813.31
1	153847004.5	79	91.556	13720.74
1	146198398.7	31	90.859	14351.86
1	157685930.7	66	91.28	14438.57
1	177526237.2	73	90.233	13933.81
1	176179075	20	90.321	14208.78
1	145139314.8	14	90.017	14417.46
1	149763851.8	10	90.031	14852.88
1	149939087.8	17	90.136	14870.91
1	136465896.3	23	90.555	14801.24
1	125129902.8	21	92.225	14583.67
1	99935754	22	91.851	15003.85
1	86292418.37	27	92.226	14670.71
1	96184636.55	20	92.13	14844.36
1	99739876.72	19	92.687	14972.21

The transpose of X is multiplied by X (i.e., $X^T.X$), and the following matrix is obtained:

29	4236882097	1634	2643.934	423864.5
4236882097	6.34108E+17	2.492E+11	3.86072E+11	6.17978E+13
1634	2.492E+11	117170	148924.642	23895207.96
2643.934	3.86072E+11	148924.642	241073.0383	38637088.32
423864.5	6.17978E+13	23895207.96	38637088.32	6205646628

The matrix $(X^T.X)^{-1}$, called matrix A, is obtained as:

984.36	-2.95E-07	0.11005988	-8.6701	-0.0107
-2.95E-07	1.82E-16	-7.35794E-11	2.37E-09	3.86E-12
0.1101	-7.36E-11	6.96E-05	-0.0009	-1.56E-06
-8.6701	2.37E-09	-0.0009	0.0788	8.09E-05
-0.0107	3.86E-12	-1.56E-06	8.09E-05	1.97-07

The point estimators of regression coefficients are obtained as $[\hat{a}, \hat{b}_1, \hat{b}_2, ..., \hat{b}_m]^T = (X^T.X)^{-1}.X.y^T$; the results are shown:

The crisp regression model	**$y=a+b_1x_1+b_2x_2+b_3x_3+b_4x_4$**
The point estimator of a	-402325.5337
The point estimator of b_1	0.000276427
The point estimator of b_2	60.1313839
The point estimator of b_3	4840.744068
The point estimator of b_4	-2.622093871

Therefore, the regression model based on point estimators for coefficients is obtained as follows: $y=-402325.5337+0.0003x_1+60.1314x_2+4840.7441x_3-2.6221x_4$.

This model estimates the values of dependent variables and then calculates a point estimator for standard deviation using Eq. (10). The required computations are given in the following table.

Week	***Y***	**Estimation of *Y***	**Error power 2**
1	32782.02	41140.37	69862004.63
2	38356.44	41064.83	7335352.812
3	35791.28	44323.85	72804814.12
4	32289.38	38274.35	35819905.05
5	33114.36	43688.99	111822744.6
6	38903.44	48143.73	85382986.83
7	48717.29	42651.46	36794299.6
8	57539.94	40784.83	280733759.1
9	45137.77	45594.43	208536.9302

(*continued on next page*)

(*continued*)

10	51206.69	52182.71	952612.0125
11	59302.32	52918.51	40752973.17
12	57523.42	55534.56	3955568.163
13	55950.75	60959.93	25091837.19
14	58758.56	59763.28	1009459.123
15	60204.96	59671.19	284905.052
16	56216.19	52174.48	16335403.91
17	49004.25	42145.04	47048814.58
18	56631.08	49235.69	54691709.27
19	58232.32	51396.29	46731299.59
20	46456.06	47541.92	1179098.02
21	34770.58	36582.13	3281702.043
22	35678.13	36546.01	753213.5608
23	35862.38	37476.37	2604964.623
24	39097.86	36323.75	7695679.803
25	35698.3	41724.44	36314360.55
26	34649.64	31908.03	7516445.953
27	35287.78	31126.09	17319633.9
28	34240.19	32519.62	2960371.925
29	31796.81	35803.31	16052052.87
Total			**1033296509**

The degree of freedom is 29-3=26. The variance of the error is calculated by dividing 1033296509 by 26, i.e., 39742173.43; therefore, the standard deviation of equals to 6034.14. Then, the values of $L(\lambda)$ and $R(\lambda)$ are calculated using Eqs. (12) (13). β=0.01, therefore:

$$X_{\left(L, \frac{\beta}{2}, n-3\right)} = X_{\left(L, \frac{0.0005}{2}, 26\right)} = 11.1603$$

$$X_{\left(R, \frac{\beta}{2}, n-3\right)} = X_{\left(R, \frac{0.0005}{2}, 26\right)} = 48.2899$$

Therefore, the values of $L(\lambda)$ and $R(\lambda)$ are obtained equal to 48.2899 and 11.1603, respectively. Note that we let the value of λ equal to 0. Now, the (1-β)100% confidence interval for regression coefficients are obtained. $t_{\frac{\beta}{2}, n-3} = t_{0.005,26} = 2.779$ and the values of a_{ii} are obtained from matrix A. The (1-β)100% confidence interval for a is obtained using Eq. (14) as

$$\left[\widehat{a} \pm t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{11}}\right] = [-402325.53 \pm \left(2.779 \times 6031.14 \times \sqrt{984.36}\right)]$$

Also, (1-β)100% confidence intervals for b_1, b_2, b_3, and b_4 are obtained using Eq. (15) as

$$\left[\widehat{b}_1 \pm t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{22}}\right] = \left[0.0003 \pm \left(2.779 \times 6031.14 \times \sqrt{1.82E^{-16}}\right)\right]$$

$$\left[\widehat{b}_2 \pm t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{33}}\right] = \left[60.13 \pm \left(2.779 \times 6031.14 \times \sqrt{6.96E^{-05}}\right)\right]$$

$$\left[\widehat{b}_3 \pm t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{44}}\right] = \left[4840.74 \pm \left(2.779 \times 6031.14 \times \sqrt{0.0788}\right)\right]$$

$$\left[\widehat{b}_4 \pm t_{\frac{\beta}{2}, n-3}.\widehat{\delta}.\sqrt{a_{55}}\right] = \left[-2.62 \pm \left(2.779 \times 6031.14 \times \sqrt{1.97E^{-07}}\right)\right]$$

As a result, the fuzzy regression model for (1-β)100% confidence intervals is obtained as $y_{(\beta=0.01)} = [-951981.85,\ 147330.78] + [3.98E^{-05},\ 0.0005]x_1 + [-86.08,\ 206.35].x_2$
$+[-79.17,\ 9760.66].x_3 + [-10.40,\ 5.1571].x_4$

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