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Positive and normative use of fuzzy DEA-BCC models: A critical view on NATO enlargement

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Abstract

Data envelopment analysis (DEA) is a widely used mathematical programming approach for comparing the input and output of a set of comparable decision-making units (DMUs) by evaluating their relative efficiency. The traditional DEA methods require accurate measurement of both the inputs and outputs. However, the real evaluation of the DMUs is often characterized by imprecision and uncertainty in data definitions and measurements. The development of fuzzy DEA (FDEA) with imprecise and ambiguous data has extended the scope of application for efficiency measurement. The purpose of this paper is to develop a fuzzy DEA framework with a BCC model for measuring crisp and interval efficiencies in fuzzy environments. We use an α-level approach to convert the fuzzy Banker, Charnes, and Cooper (BCC) (variable returns to scale) model into an interval programming model. Instead of comparing the equality (or inequality) of the two intervals, we define a variable in the interval to satisfy our constraints and maximize the efficiency value. We present a numerical example to show the similarities and differences between our solution and the solutions obtained from four fuzzy DEA methods in the literature. In addition, a case study for NATO enlargement is presented to illustrate the applicability of the proposed method.

Keywords: Data envelopment analysis; imprecise inputs and outputs; fuzzy mathematical programming; BCC model; NATO enlargement

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming technique widely used for comparing the inputs and outputs of a set of homogeneous decision-making units (DMUs) by evaluating their relative efficiency. DEA generalizes Farrell's (1957) single-input single-output ratio

to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs. While DEA does not provide normative guidance for achieving efficiency, it does inform about magnitude and sources of relative (in)efficiency and thereby enables efficiency management. The conventional DEA methods such as the constant returns-to-scale (RTS) model (CCR; Charnes et al., 1978) and the variable RTS model (BCC; Banker et al., 1984) presume accurate, quantitative, cardinal measurements of both the inputs and outputs. However, the observed values of the input and output data in real-world problems are often imprecise or vague. Imprecise evaluations may be the result of qualitative, incomplete, or unattainable information.

In recent years, many researchers have formulated DEA models to deal with the uncertain input and output data. One way to manipulate uncertain data in DEA is via probability distributions. Nevertheless, probability distributions require either *a priori* predictable regularity or *a posteriori* outcome data, which may be difficult or impossible to construct for settings where the event is unique or deterministic. Furthermore, the assumption of stochasticity is not relevant when the uncertainty results from qualitative data definitions applied to deterministic data. An alternative approach is to represent the imprecise or vague values by membership functions of the fuzzy sets theory. In this study, we propose a fuzzy variable RTS (VRS or BCC) model for evaluating the crisp and interval efficiency of a set of DMUs with fuzzy inputs and outputs. Instead of comparing the equality (or inequality) of the two intervals, we define a variable in the interval to satisfy our constraints and maximize the efficiency value. We compare our solution with the solutions obtained from four fuzzy DEA methods in the literature.

We also present an application based on the enlargement of the North Atlantic Treaty Organization (NATO). Promoting security, stability, and cooperation is the *raison d'être* of the NATO, and these are the aims of its strategy of membership enlargement following the conclusion of the Cold War. The potential NATO membership has inspired some former Warsaw Pact applicant countries to reform their political systems, transform their economies, deal with corruption, rationalize their military expenditure, and improve social justice and human rights. However, the criteria for candidacy for NATO membership and the advancement of the application are not well defined yet there is an interest to extend the alliance in order of potential synergies rather than order of application. We apply the proposed DEA model to evaluate 18 countries for possible NATO membership. The original idea for this application is to determine whether the stated objectives and fuzziness of political preferences indeed govern the actual decision making.

This paper is organized into nine sections. In Section 2, we present a brief review of the literature on fuzzy DEA (FDEA). In Section 3, we introduce the primary BCC model followed by a discussion of the preliminaries for fuzzy sets in Section 4. In Section 5 we show a fuzzy BCC model, and in Section 6 we present the details of the model proposed in this study. In Section 7 we show a numerical example, and in Section 8 we present the NATO enlargement case to demonstrate the applicability of our model. In Section 9, we summarize with our conclusions and future research directions.

2. The review of literature on fuzzy DEA

The fuzzy DEA methods in the literature are categorized into four general groups by Lertworasirikul et al. (2003a) and Hatami-Marbini et al. (2011a): (1) the tolerance approach, (2) the α-level based approach, (3) the fuzzy ranking approach, and (4) the possibility approach. Sengupta (1992)

proposed a fuzzy mathematical programming approach by incorporating fuzzy input and output data into a DEA model and defined tolerance levels for the objective function and constraint violations. Guo and Tanaka (2001, 2008) and León et al. (2003) considered the uncertainties in fuzzy objectives and fuzzy constraints in three similar fuzzy DEA models. Lertworasirikul et al. (2003b) used the credibility approach and replaced the fuzzy variables with expected credits according to the credibility measures in a fuzzy DEA model.

Kao and Liu (2000) used α-level sets and transformed fuzzy input and output data into intervals. Saati et al. (2002) extended the α-level set approach by defining a fuzzy DEA model as a possibilistic-programming problem and transforming it into an interval programming problem. Entani et al. (2002) changed fuzzy input and output data into intervals and further extended the α-level set research. Dia (2004) used a fuzzy aspiration level and a safety α level to transform the fuzzy DEA model into a crisp DEA model. Soleimani-damaneh et al. (2006) addressed some of the limitations of the fuzzy DEA models proposed by Kao and Liu (2000), León et al. (2003) and Lertworasirikul et al. (2003a) and suggested a new fuzzy DEA model for producing crisp efficiencies.

Wang et al. (2009) developed two fuzzy DEA models using fuzzy arithmetic to handle fuzziness in input and output data in DEA. Soleimani-damaneh (2008, 2009) used the fuzzy-signed distance and the fuzzy upper-bound concepts to formulate a fuzzy additive DEA model with fuzzy input–output data. Khodabakhshi et al. (2010) formulated two alternative fuzzy and stochastic additive models to determine RTS in DEA. They developed the fuzzy and stochastic DEA models based on the possibility approach and the concept of chance constraint programming, respectively.

More recently, Liu (2008) and Liu and Chuang (2009) further advanced the α-level set approach and proposed the assurance region approach in fuzzy DEA. Hatami-Marbini et al. (2009) proposed a fuzzy DEA model to assess efficiency scores with fuzzy data based on the ranking fuzzy numbers. Hatami-Marbini and Saati (2009) considered fuzziness in the input and output data as well as the u_0 variable in a fuzzy BCC model. Hatami-Marbini et al. (2010) further developed a four-phase fuzzy DEA framework based on the theory of displaced ideal. Saati et al. (2011) addressed an alleged inconsistency when defining relative efficiency based on interval bounds using a fuzzy DEA model with discretionary and nondiscretionary factors for the constant RTS (CRS) case. Emrouznejad et al. (2011) reformulated the conventional profit Malmquist productivity index (MPI) problem as an imprecise DEA problem and proposed two novel methods for measuring the overall profit MPI when the inputs, outputs, and price vectors were fuzzy or vary in intervals. Hatami-Marbini et al. (2011b) proposed an interactive evaluation process that considered the decision makers' preferences in measuring the relative efficiencies of the DMUs in fuzzy DEA. They constructed a linear programming model with fuzzy parameters and calculated the fuzzy efficiency of the DMUs for different α levels.

3. DEA preliminaries

Charnes et al. (1978) proposed the constant RTS model (CRS or CCR) to evaluate the radial technical efficiency of a given DMU_p. Assume that there are n DMUs to be evaluated, where every DMU_j, $j = 1, 2, \ldots, n$, produces s outputs, y_{rj} $(r = 1, 2, \ldots, s)$, using m inputs, x_{ij} $(i = 1, 2, \ldots, m)$. The CCR model (Charnes et al., 1978) is proposed to evaluate the efficiency of a specific DMU_p. The primal and dual DEA models are given in (1).

Primal CCR model (input oriented)

$$\begin{aligned} \min \quad & Z_p = \theta \\ \text{s.t.} \quad & \theta x_{ip} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0, \quad \forall\, i, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad \forall\, r, \\ & \lambda_j \geq 0, \quad \forall\, j. \end{aligned}$$

Dual CCR model (input oriented)

$$\begin{aligned} \max \quad & \theta_p = \sum_{r=1}^{s} u_r y_{rp} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall\, j \\ & u_r, v_i \geq 0, \quad \forall\, r, i. \end{aligned} \tag{1}$$

Banker et al. (1984) extended the earlier work of Charnes et al. (1978) by proposing the model for variable RTS (VRS or BCC). The BCC and CCR models differ only in that the former includes an additional convexity constraint, $\sum_{j=1}^{n} \lambda_j = 1$, in the primal BCC model and an additional variable, u_0, in the dual BCC model as shown in (2).

Primal BCC model (input oriented)

$$\begin{aligned} \min \quad & w_p = \theta \\ \text{s.t.} \quad & \theta x_{ip} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0, \quad \forall\, i, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \quad \forall\, r, \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \quad \forall\, j. \end{aligned}$$

Dual BCC model (input oriented)

$$\begin{aligned} \max \quad & w_p = \sum_{r=1}^{s} u_r y_{rp} - u_0 \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \leq 0, \quad \forall\, j \\ & u_r, v_i \geq 0, \quad \forall\, r, i. \end{aligned} \tag{2}$$

The u_r and v_i in (1) and (2) are the weights assigned to the rth output and the ith input, respectively. The primal and dual models are referred to as the envelopment and the multiplier, respectively. The multiplier DEA models can be interpreted as the ratio of the weighted sum of outputs to the weighted sum of inputs for every DMU, and this ratio is maximized by assigning weights to the inputs and outputs of DMUs. Hence, the multiplier DEA models are the fractional program problems and they can be converted to linear forms by $\sum_{i=1}^{m} v_i x_{ip} = 1$, which are referred as a normalization constraint. Note that the optimal value of u_0 can be used to characterize the situation for RTS when a DMU_p is efficient in the dual BCC model. The optimum value of the objective function in the CCR and BCC models is unity. Nevertheless, the DMU_p can be inefficient even if the optimum value of the objective function is equal to unity. Banker and Thrall (1992) identify VRS with the sign of u_0. Assuming that (x_0, y_0) is on the efficient frontier, the following conditions identify the situation for RTS for this point:

- If u_0 takes negative values in all optimal solutions to the dual BCC model (2) then locally at DMU_p increasing RTS hold;
- if u_0 takes positive values in all optimal solutions to the dual BCC model (2) then locally at DMU_p decreasing RTS hold; and

- if u_0 takes a zero value in some optimal solutions to the dual BCC model (2) then locally where DMU_p lies or is projected on the efficient boundary CRS hold.

The ratio of the CRS and VRS scores is interpreted as the scale efficiency of the unit.

4. Preliminaries to arithmetic

In this section, we review some basic definitions of fuzzy sets (Dubois and Prade, 1978; Kaufmann and Gupta, 1991, Klir and Yuan, 1995; Zimmermann, 1996):

Definition 1. Let U be a universe set. A fuzzy set $\tilde{A}$ of U is defined by a membership function $\mu_{\tilde{A}}(x) \to [0, 1]$, where $\mu_{\tilde{A}}(x), \forall\, x \in U$, indicates the degree of membership of $\tilde{A}$ to U.

Definition 2. A fuzzy subset $\tilde{A}$ of real number R is convex if and only if

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y)), \quad \forall\, x, \quad y \in R, \quad \forall\, \lambda \in [0,\ 1],$$

where "$\wedge$" denotes the minimum operator.

Definition 3. The γ level of fuzzy set $\tilde{A}$, $\tilde{A}_\gamma$ is the crisp set $\tilde{A}_\gamma = \{x \mid \mu_{\tilde{A}}(x) \geq \gamma\}$. The support of $\tilde{A}$ is the crisp set $Sup(\tilde{A}) = \{x \mid \mu_{\tilde{A}}(x) > 0\}$. $\tilde{A}$ is normal if and only if $Sup_{x\in U}\mu_{\tilde{A}}(x) = 1$, where U is the universal set. We here introduce an alternative notation of $\tilde{A}_\gamma$ as $\tilde{A}_\gamma = [a_L(\gamma), a_R(\gamma)]$, where $a_L(\gamma)$and $a_R(\gamma)$are the lower and upper bounds of the interval $\tilde{A}_\gamma$.

Definition 4. $\tilde{A}$ is a fuzzy number iff $\tilde{A}$ is a normal and convex fuzzy subset of R.

Definition 5. A real fuzzy number $\tilde{A}$ denoted by $\tilde{A} = (a^{m1}, a^{m2}, a^l, a^u, w)$ is described as any fuzzy subset of the real line R with a membership function $\mu_{\tilde{A}}$, which satisfies the following properties:

- $\mu_{\tilde{A}}$ is a semicontinuous mapping from R to the closed interval $[0,\ w]$, $0 \leq w \leq 1$,
- $\mu_{\tilde{A}}(x) = 0$, for all $x \in [-\infty, a^l]$,
- $\mu_{\tilde{A}}$ is increasing on $[a^l,\ a^{m1}]$,
- $\mu_{\tilde{a}}(x) = w$ for all$x \in [a^{m1}, a^{m2}]$, where w is a constant and $0 < w \leq 1$,
- $\mu_{\tilde{A}}$ is decreasing on $[a^{m2}, a^u]$, and
- $\mu_{\tilde{A}}(x) = 0$, for all $x \in [a^u, \infty]$,

where a^l, a^{m1}, a^{m2}, and a^u are real numbers.

Unless elsewhere specified, it is assumed that $\tilde{A}$ is convex and bounded; i.e., $-\infty < a^l,\ a^u < \infty$. If $w = 1$, $\tilde{A}$ is a normal fuzzy number, and if $0 < w < 1$, $\tilde{A}$ is a nonnormal fuzzy number.

The membership function $\mu_{\tilde{A}}$ of $\tilde{A}$ can be expressed as

$$\mu_{\tilde{A}}(x) = \begin{cases} f^L(x), & a^l \le x \le a^{m1}, \\ w, & a^{m1} \le x \le a^{m2}, \\ f^R(x), & a^{m2} \le x \le a^u, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

where $f^L : [a^l, a^{m1}] \to [0,\ w]$ and $f^R : [a^{m2}, a^u] \to [0,\ w]$.

In addition, a fuzzy number $\tilde{A}$ in a parametric form is denoted by $(\underline{a}, \bar{a})$ of a function $\underline{a}(r)$, $\bar{a}(r)$, $0 \le r \le 1$, which satisfies the following requirements:

(1) $\underline{a}(r)$ is a bounded increasing left continuous function,
(2) $\bar{a}(r)$ is a bounded decreasing right continuous function, and
(3) $\underline{a}(r) \le \bar{a}(r)$, where $0 \le r \le 1$.

Definition 6. Particularly, a special type of the trapezoidal fuzzy number is determined by quadruples $\tilde{u} = (u^{m1}, u^{m2}, u^l, u^u)$, which membership function can be defined as follows:

$$\mu_{\tilde{u}}(x) = \begin{cases} \dfrac{x - u^l}{u^{m1} - u^l}, & u^l \le x \le u^{m1}, \\ 1, & u^{m1} \le x \le u^{m2}, \\ \dfrac{u^u - x}{u^u - u^{m2}}, & u^{m2} \le x \le u^u, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

The trapezoidal fuzzy number $\tilde{u} = (u^{m1}, u^{m2}, u^l, u^u)$ is reduced to a real number u if $u^{m1} = u^{m2} = u^l = u^u$. Conversely, a real number u can be written as a trapezoidal fuzzy number $\tilde{u} = (u, u, u, u)$. Similarly, the γ-level $\tilde{u} = (u^{m1}, u^{m2}, u^l, u^u)$ can easily be determined as $[\tilde{u}]^\gamma = [\gamma u^{m1} + (1-\gamma)u^l, \gamma u^{m2} + (1-\gamma)u^u]$, where $\gamma \in [0, 1]$. If $u^m = u^{m1} = u^{m2}$, then $\tilde{u} = (u^m, u^l, u^u)$ is called a triangular fuzzy number. A triangular fuzzy number has the following membership function:

$$\mu_{\tilde{u}}(x) = \begin{cases} \dfrac{x - u^l}{u^m - u^l}, & u^l \le x \le u^m, \\ \dfrac{u^u - x}{u^u - u^m}, & u^m \le x \le u^u, \\ 0, & \text{otherwise}. \end{cases} \tag{5}$$

A trapezoidal fuzzy number is widely used for solving practical problems. Hence, for the sake of simplicity and without loss of generality, we assume that all fuzzy numbers used throughout the paper are trapezoidal fuzzy numbers.

Definition 7. Suppose that we have two positive trapezoidal fuzzy numbers $\tilde{A} = (a^{m1}, a^{m2}, a^l, a^u)$ and $\tilde{B} = (b^{m1}, b^{m2}, b^l, b^u)$, then the arithmetic operations of these two trapezoidal fuzzy numbers are defined as follows:

Addition: $\tilde{A}(+)\tilde{B} = (a^{m1} + b^{m1}, a^{m2} + b^{m2}, a^l + b^l, a^u + b^u)$,
Subtraction: $\tilde{A}(-)\tilde{B} = (a^{m1} - b^{m2}, a^{m2} - b^{m1}, a^l - b^u, a^u - b^l)$,
Multiplication: $\tilde{A}(\times)\tilde{B} = (a^{m1}b^{m1}, a^{m2}b^{m2}, a^l b^l, a^u b^u)$, $k\tilde{A} = (ka^l, ka^m, ka^n, ka^u)$, $\forall\, k \in R^+$
Inverse: $(\tilde{B})^{-1} = (\frac{1}{b^{m2}}, \frac{1}{b^{m1}}, \frac{1}{b^u}, \frac{1}{b^l})$,
Division: $\tilde{A}(\div)\tilde{B} = \tilde{A}(\times)\tilde{B}^{-1} = (\frac{a^{m1}}{b^{m2}}, \frac{a^{m2}}{b^{m1}}, \frac{a^l}{b^u}, \frac{a^u}{b^l})$.

Definition 8. Linguistic variables are those variables whose values are not numbers but phrases or sentences expressed in a natural or artificial language. For example, "very low," "low," "medium," "high," or "very high" are linguistic variables because their values are represented by linguistic terms rather than numerical terms. The concept of linguistic variable is useful in dealing with situations that are too complex or too ill-defined to be reasonably described with quantitative values. Linguistic values can also be represented by fuzzy numbers.

Definition 9. The minimum t-norm is usually applied in fuzzy linear programming to assess a linear combination of fuzzy quantities. Therefore, a given set of trapezoidal fuzzy numbers $\tilde{u}_j = (u_j^{m1}, u_j^{m2}, u_j^l, u_j^u)$, $j = 1,2,\ldots,n$ and $\lambda_j \geq 0$, $\sum_{j=1}^n \lambda_j \tilde{u}_j$ are defined as follows:

$$\sum\nolimits_{j=1}^n \lambda_j \tilde{u}_j = \left(\sum\nolimits_{j=1}^n \lambda_j u_j^{m1}, \sum\nolimits_{j=1}^n \lambda_j u_j^{m2}, \sum\nolimits_{j=1}^n \lambda_j u_j^l, \sum\nolimits_{j=1}^n \lambda_j u_j^u\right) \tag{6}$$

where $\sum_{j=1}^n \lambda_j \tilde{u}_j$ denotes the combination $\lambda_1\tilde{u}_1 \oplus \lambda_2\tilde{u}_2 \oplus \cdots \oplus \lambda_n\tilde{u}_n$.

5. A fuzzy BCC model

In this section, we propose an alternative fuzzy BCC model for evaluating the efficiency of a set of DMUs with fuzzy inputs and outputs derived from the α-level approach proposed by Saati et al. (2002). Let us consider n DMUs, each of which uses m different fuzzy inputs to generate s different fuzzy outputs. The typical fuzzy BCC model with fuzzy data can be expressed as

$$\begin{aligned} \max \quad & w_p = \frac{\sum_{r=1}^s u_r \tilde{y}_{rp} - u_0}{\sum_{i=1}^m v_i \tilde{x}_{ip}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^s u_r \tilde{y}_{rj} - u_0}{\sum_{i=1}^m v_i \tilde{x}_{ij}} \leq 1, \quad \forall\, j \\ & u_r, v_i \geq 0, \quad \forall\, r, i \end{aligned} \tag{7}$$

where $\tilde{y}_{rj} = (y_{rj}^{m1}, y_{rj}^{m2}, y_{rj}^{l}, y_{rj}^{u})$ and $\tilde{x}_{ij} = (x_{ij}^{m1}, x_{ij}^{m2}, x_{ij}^{l}, x_{ij}^{u})$, the fuzzy output and fuzzy input values of the jth DMU, respectively, are characterized as trapezoidal fuzzy numbers. Therefore, (7) can be rewritten as follows:

$$\begin{aligned} \max \quad & w_p = \frac{\sum_{r=1}^{s} u_r\left(y_{rp}^{m1}, y_{rp}^{m2}, y_{rp}^{l}, y_{rp}^{u}\right) - u_0}{\sum_{i=1}^{m} v_i\left(x_{ip}^{m1}, x_{ip}^{m2}, x_{ip}^{l}, x_{ip}^{u}\right)} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r\left(y_{rj}^{m1}, y_{rj}^{m2}, y_{rj}^{l}, y_{rj}^{u}\right) - u_0}{\sum_{i=1}^{m} v_i\left(x_{ij}^{m1}, x_{ij}^{m2}, x_{ij}^{l}, x_{ij}^{u}\right)} \le \tilde{1}, \quad \forall\, j \\ & u_r, v_i \ge 0, \quad \forall\, r, i. \end{aligned} \tag{8}$$

The right-hand side of the first constraint in Model (8) must be equal to 1 because of the normalization of the efficiency scores of the DMUs. In the next section, we apply the α-level based approach to define the fuzzy BCC model.

6. The proposed method

In this section, we develop the fuzzy BCC model in order to measure both the crisp and interval efficiency of each DMU under consideration for different α values.

6.1. The crisp efficiency

Using the α-level based approach, the inputs and outputs can be represented by different levels of confidence intervals. This representation results in the following transformation of the fuzzy BCC model:

$$\begin{aligned} \max \quad & w_p = \frac{\sum_{r=1}^{s} u_r\left[\alpha y_{rp}^{m1} + (1-\alpha) y_{rp}^{l}, \alpha y_{rp}^{m2} + (1-\alpha) y_{rp}^{u}\right] - u_0}{\sum_{i=1}^{m} v_i\left[\alpha x_{ip}^{m1} + (1-\alpha) x_{ip}^{l}, \alpha x_{ip}^{m2} + (1-\alpha) x_{ip}^{u}\right]} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r\left[\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}, \alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right] - u_0}{\sum_{i=1}^{m} v_i\left[\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}, \alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right]} \le 1, \quad \forall\, j \\ & u_r, v_i \ge 0, \quad \forall\, r, i. \end{aligned} \tag{9}$$

Model (9) is an interval fractional programming model that cannot be solved by standard optimization methods without further variation. Hence, we transform the interval model (9) into a nonlinear programming model using the following interval alteration variables:

$$\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}, \alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right) = \hat{x}_{ij}, \quad \forall\, i, j,$$

$$\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}, \alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right) = \hat{y}_{rj}, \quad \forall\, r, j.$$

The substitutions of the above interval alteration variables in Model (9) will result in the following nonlinear programming model:

$$\begin{aligned}
\max \quad & w_p = \frac{\sum_{r=1}^{s} u_r \hat{y}_{rp} - u_0}{\sum_{i=1}^{m} v_i \hat{x}_{ip}} \\
\text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r \hat{y}_{rj} - u_0}{\sum_{i=1}^{m} v_i \hat{x}_{ij}} \le 1, \quad \forall\, j \\
& \alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l} \le \hat{x}_{ij} \le \alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}, \quad \forall\, i, j \\
& \alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l} \le \hat{y}_{rj} \le \alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}, \quad \forall\, r, j \\
& u_r, v_i \ge 0, \quad \forall\, r, i.
\end{aligned} \tag{10}$$

In Model (10), the alternation variables $u_r \hat{y}_{rj} = \bar{y}_{rj}$ $(r = 1, 2, \ldots, s,\ j = 1, 2, \ldots, n)$ and $v_i \hat{x}_{ij} = \bar{x}_{ij}$ $(r = 1, 2, \ldots, s,\ j = 1, 2, \ldots, n)$ are introduced to receive the following program:

$$\begin{aligned}
\max \quad & w_p = \frac{\sum_{r=1}^{s} \bar{y}_{rp} - u_0}{\sum_{i=1}^{m} \bar{x}_{ip}} \\
\text{s.t.} \quad & \frac{\sum_{r=1}^{s} \bar{y}_{rj} - u_0}{\sum_{i=1}^{m} \bar{x}_{ij}} \le 1, \quad \forall\, j, \\
& v_i\big(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\big) \le \bar{x}_{ij} \le v_i\big(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\big), \quad \forall\, i, j, \\
& u_r\big(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\big) \le \bar{y}_{rj} \le u_r\big(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\big), \quad \forall\, r, j, \\
& u_r, v_i \ge 0, \quad \forall\, r, i.
\end{aligned} \tag{11}$$

The fractional program problem (11) can be transformed into a linear program using the Charnes–Cooper transformation (Charnes and Cooper, 1962):

$$\begin{aligned}
\max \quad & w_p = \sum_{r=1}^{s} \bar{y}_{rp} - u_0 \\
\text{s.t.} \quad & \sum_{i=1}^{m} \bar{x}_{ip} = 1, \\
& \sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} - u_0 \le 0, \quad \forall\, j, \\
& v_i\big(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\big) \le \bar{x}_{ij} \le v_i\big(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\big), \quad \forall\, i, j, \\
& u_r\big(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\big) \le \bar{y}_{rj} \le u_r\big(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\big), \quad \forall\, r, j, \\
& u_r, v_i \ge 0, \quad \forall\, r, i.
\end{aligned} \tag{12}$$

Model (12) is equivalent to a parametric programming model with $\alpha \in [0,\ 1]$. Therefore, the fuzzy linear programming problem given by (12) is equivalent to a crisp parametric linear programming

problem and the DMU_p is efficient if $w_p^* = 1$ in (12). Using the optimal value of u_0^*, we can determine the situation for RTS when a DMU_p is efficient. Similar to the conventional DEA model of Banker and Thrall (1992), the following conditions characterize the situation for RTS of the efficient DMUs:

- If u_0 takes negative values in all optimal solutions to model (12), then locally at DMU_p increasing RTS (IRS) hold;
- if u_0 takes positive values in all optimal solutions to model (12), then locally at DMU_p decreasing RTS (DRS) hold; and
- if u_0 takes a zero value in some optimal solutions to model (12), then locally where DMU_p lies or is projected on the efficient boundary CRS hold.

6.2. *The interval efficiency*

In this paper, we apply the α-level based approach to determine different levels of confidence intervals for the inputs and outputs that enable us to increase the discriminatory power with minimal loss of information in the fuzzy DEA model. Therefore, we calculate the interval efficiencies by using the following *min–max* and *max–max* models:

$$w_j = [\underline{w}_j, \bar{w}_j] = \begin{bmatrix} \min \ \max \ w_p = \sum_{r=1}^{s} \bar{y}_{rp} - u_0 & \max \ \max \ w_p = \sum_{r=1}^{s} \bar{y}_{rp} - u_0 \\ \text{s.t.} \quad (a) & \text{s.t.} \quad (a) \end{bmatrix}, \tag{13}$$

where the constraint set (a) is defined as follows:

$$\begin{aligned} &\sum_{i=1}^{m} \bar{x}_{ip} = 1, \\ &\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{i=1}^{m} \bar{x}_{ij} - u_0 \le 0, \quad \forall\, j, \\ &v_i\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\right) \le \bar{x}_{ij} \le v_i\left(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right), \qquad \forall\, i, j, \\ &u_r\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\right) \le \bar{y}_{rj} \le u_r\left(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right), \quad \forall\, r, j, \\ &u_r, \quad v_i \ge 0, \quad \forall\, r, i. \end{aligned} \tag{14}$$

In order to run Model (13), we first have to run Model (12) to obtain the optimal solutions (u_r^*, v_i^*, u_0^*) with respect to various α levels. Once the optimal solutions are obtained, we calculate the interval efficiency of DMU_p for $\alpha \in [0,\ 1]$ based on the interval arithmetic operation as follows:

$$\underline{w}_j = \min \left\{ \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right)}, \quad \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\right)}, \right.$$
$$\left. \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right)}, \quad \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\right)} \right\}, \quad \forall\, j$$

$$\bar{w}_j = \max\left\{ \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right)}, \quad \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m1} + (1-\alpha) y_{rj}^{l}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\right)}, \right.$$
$$\left. \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m2} + (1-\alpha) x_{ij}^{u}\right)}, \quad \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rj}^{m2} + (1-\alpha) y_{rj}^{u}\right) - u_0^*}{\sum_{i=1}^{m} v_i^*\left(\alpha x_{ij}^{m1} + (1-\alpha) x_{ij}^{l}\right)} \right\}, \quad \forall\, j \tag{15}$$

We should note that we choose the minimum value of the four components in the min–max case and the maximum value between the four components in the max–max case.

Owing to the positive upper bounds of the interval numbers in the denominator and numerator of (15), the lower and upper bounds of the efficiency score for a given α can be reduced as

$$\begin{aligned} \underline{w}_j &= \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rp}^{m1} + (1-\alpha) y_{rp}^{l}\right) - u_0^*}{\sum_{i=1}^{m} v_i\left(\alpha x_{ip}^{m2} + (1-\alpha) x_{ip}^{u}\right)}, \quad \forall\, j \\ \bar{w}_j &= \frac{\sum_{r=1}^{s} u_r^*\left(\alpha y_{rp}^{m2} + (1-\alpha) y_{rp}^{u}\right) - u_0^*}{\sum_{i=1}^{m} v_i\left(\alpha x_{ip}^{m1} + (1-\alpha) x_{ip}^{l}\right)}, \quad \forall\, j \end{aligned} \tag{16}$$

As a result, by (16) we can achieve the best possible relative interval efficiency of n units under consideration with respect to different α levels.

7. Numeric example

In this section, we use the numerical example proposed by Guo and Tanaka (2001), also used by Lertworasirikul et al. (2003a, 2003b) and Saati et al. (2002), to illustrate the proposed fuzzy BCC model for measuring the crisp efficiency of DMUs and to compare our solution with the solutions obtained from four fuzzy DEA methods in the literature. We consider five DMUs with two fuzzy inputs and two fuzzy outputs presented in Table 1.

Guo and Tanaka (2001) proposed a fuzzy CCR model in which fuzzy constraints (including fuzzy equalities and fuzzy inequalities) were converted into crisp constraints by predefining a possibility level and using the comparison rule for fuzzy numbers. The results of Guo and Tanaka (2001) which are fuzzy efficiencies with different α values, are shown in Table 2.

As shown in Table 2, $S_0 = \{B, C, D, E\}$, $S_{0.5} = \{B, D\}$, $S_{0.75} = \{B, D\}$, and $S_1 = \{B, D, E\}$ are the nondominated sets for different α values. Guo and Tanaka (2001) defines a DMU as α-possibilistic efficient if the maximum value of the fuzzy efficiency at that α level is greater than or equal to 1. The set of all possibilistic efficient DMUs is called the α-possibilistic nondominated set, denoted by S_α. Under the CCR DEA model, Lertworasirikul et al. (2003a) also applied the possibility approach to the numerical example of Guo and Tanaka (2001). Their results are also presented in Table 2. Lertworasirikul et al. (2003a) define a DMU as α-possibilistic efficient if its objective value is greater than or equal to 1 at the specified α level. Hence, as shown in Table 2, DMUs B, D, and E are possibilistically efficient at all possibility levels and DMUs A and C are possibilistically efficient only

Table 1
The numerical example of Guo and Tanaka (2001)

	Inputs		Outputs	
DMU	1	2	1	2
A	(3.5,4,4.5)	(1.9,2.1,2.3)	(2.4,2.6,2.8)	(3.8, 4.1, 4.4)
B	(2.9,2.9,2.9)	(1.4,1.5,1.6)	(2.2,2.2,2.2)	(3.3, 3.5, 3.7)
C	(4.4,4.9,5.4)	(2.2,2.6,3)	(2.7,3.2,3.7)	(4.3, 5.1, 5.9)
D	(3.4,4.1,4.8)	(2.2,2.3,2.4)	(2.5,2.9,3.3)	(5.5, 5.7, 5.9)
E	(5.9,6.5,7.1)	(3.6,4.1,4.6)	(4.4,5.1,5.8)	(6.5, 7.4, 8.3)

Table 2
The comparison of the solutions obtained from four fuzzy DEA methods in the literature

		DMU				
Method	α level	A	B	C	D	E
Guo and Tanaka	0	(0.66, 0.81, 0.99)	(0.88, 0.98, 1.09)	(0.60, 0.82, 1.12)	(0.71, 0.93, 1.25)	(0.61, 0.79, 1.02)
(2001)	0.5	(0.75, 0.83, 0.92)	(0.94, 0.97, 1.00)	(0.12, 0.83, 0.14)	(0.85, 0.97, 1.12)	(0.72, 0.82, 0.93)
CCR model	0.75	(0.77, 0.81, 0.99)	(0.80, 0.98, 1.09)	(0.22, 0.82, 0.30)	(0.71, 0.93, 1.25)	(0.61, 0.79, 1.02)
	1	(0.85, 0.85, 0.94)	(1.00, 1.00, 1.00)	(0.86, 0.86, 0.86)	(1.00, 1.00, 1.00)	(1.00, 1.00, 1.00)
Lertworasirikul	0	1.107	1.238	1.276	1.52	1.296
et al. (2003a)	0.5	0.963	1.112	1.035	1.258	1.159
CCR model	0.75	0.904	1.055	0.932	1.131	1.095
	1	1	0.855	1.000	0.861	1.000
Saati et al. (2002)	0	1	1	1	1	1
CCR model	0.5	0.954	1	1	1	1
	0.75	0.901	1	0.929	1	1
	1	0.855	1	0.862	1	1
Lertworasirikul	0	1.299	1.247	1.699	1.692	∞
et al. (2003b)	0.5	1.062	1.119	1.243	1.300	∞
BCC model	0.75	0.969	1.059	1.074	1.142	∞
	1	0.889	1.000	0.935	1.000	1.000

at some α levels. Saati et al. (2002) used their fuzzy CCR model with an α-level based approach to solve the Guo and Tanaka (2001)'s numerical example, and their efficiency values are presented in Table 2. Lertworasirikul et al. (2003b) also proposed a fuzzy BCC model routed in the possibility approach to solve the example proposed by Guo and Tanaka (2001). Their results presented in Table 2 show that DMUs B, D, and E are possibilistically efficient at all possibility levels, and DMUs A and C are possibilistically efficient only at some possibility levels. Note the similarity between the definition of α-possibilistic efficient in Guo and Tanaka (2001) and Lertworasirikul et al. (2003a).

Next, we use Model (11) proposed in this study and find the efficiencies of the DMUs presented in Table 3. In addition, the optimal values of u_0 for the efficient DMUs are reported in Table 4 to determine the RTS condition. For example, DMU A is efficient when α is equal to 0 and 0.5 and its RTS is IRS since the optimal value of u_0 is positive whereas DMU B is efficient for all α level and its RTS is DRS because of negative value of u_0.

Table 3
The efficiency values of the proposed fuzzy BCC model

	DMU				
α level	A	B	C	D	E
0.0	1	1	1	1	1
0.5	1	1	1	1	1
0.75	0.982	1	1	1	1
1.0	0.918	1	0.96	1	1

Table 4
The optimal value of u_0 for the efficient DMUs

	DMU									
α level	A	RTS	B	RTS	C	RTS	D	RTS	E	RTS
0.0	0.327	IRS	–0.13	DRS	0.834	IRS	0.907	IRS	1	IRS
0.5	0.174	IRS	–0.098	DRS	0.396	IRS	0.756	IRS	1	IRS
0.75			–0.081	DRS	0.128	IRS	0.643	IRS	1	IRS
1.0			–0.519	DRS			0.216	IRS	1	IRS

8. NATO application

The North Atlantic Treaty Organization (NATO) is an alliance of 18 countries from Europe and North America committed to the goals of the North Atlantic Treaty Reference. This treaty was signed in 1949, and its primary goal was to outline the role of NATO as an alliance built to protect the freedom and security of its member countries by political and military means. Expanding the alliance with countries that commit to the principles of the alliance will assist in maintaining the security of its current members. The integration of new NATO members is based on a well-defined process, but the preferences of the decision makers are normally given implicitly and potentially with misleading nominal motivations. In this application, we apply the method to test the consistency of the stated (implied) objectives for new member integration and the actual decisions made. This use of the model can be seen as positive and similar to the approach used in Agrell and Steuer (2000) where the stated criteria order for faculty promotion was compared with the actual outcomes of a DEA model for individual performance.

8.1. NATO enlargement problem

NATO's open door policy on enlargement invites European countries that are in a position to advance the principles of the North Atlantic Treaty and contribute to security in the Euro-Atlantic area, to join the alliance. Deciding ideal candidates for the expansion of NATO can be complicated. The integration of nonmembers in NATO is made in four steps, indicating increasing level of cooperation: Partnership for Peace (PfP), Individual Partnership Action Plan (IPAP), Intensified Dialogue (ID), and Membership Action Plan (MAP). The first level of integration, PfP (NATO),

was created in 1993 to create a dialogue with neutral European and former Warsaw Block member states. The second level of integration, IPAP (NATO), was installed in 2002 for eight countries within PfP potentially eligible for NATO membership. The third level of integration, ID, is currently initiated for two countries, prior to the final candidacy. Finally, the MAP (1999) is currently (2012) in action for three countries for which membership is under negotiation. Decisions on enlargement are ultimately made by NATO and its members; however, the North Atlantic Council is NATO's a principal decision-making body and is responsible for inviting new members to join the alliance. Decisions to invite new members come as a result of a unanimous vote by current member countries in the final stage. Relationships between members of the alliance are vital, and a unanimous vote protects the integrity of the alliance and prevents tension among the member countries.

Each decision to expand is made individually on a case-by-case basis and is a result of an agreement that the invited country will add to the security and stability of the alliance. The determination must also allow the alliance to preserve the ability to perform its main function of defense. Countries outside of the alliance are not given a voice in these decisions nor should countries be excluded for consideration due to membership of other groups or organizations.

8.2. Potential input–output variables

The specification of the model follows the publicly announced criteria related to economic and social stability, as well as the absence of conflicts with existing or future members or partners of the alliance. Efficiency measurement in DEA is based on the assumptions that inputs are undesirable (bad) and have to be minimized, and outputs are desirable (good) and have to be maximized. However, there are circumstances in the real world where input variables are desirable or output variables are undesirable. In these cases, the undesirable outputs and desirable inputs should be minimized whereas the desirable outputs and undesirable inputs should be maximized. For example, in this case study, revenue and gross domestic product (GDP) are considered desirable outputs and are maximized whereas expenditures and public or external debts ratios are considered undesirable inputs and are minimized. However, the inflation rate is an undesirable output that must be minimized. Drawing on the nature of this study, the following output variables were used in the proposed DEA model:

8.2.1. Gross domestic product

GDP is the value of all the goods and services produced within a country in a given year. It is a measure of a country's economic power.

8.2.2. Inflation rate

This reciprocal output measures the inverse of annual percentage change in consumer prices compared to the previous year's prices and affects the disposable income in a country.

Table 5
The linguistic variables and their associated trapezoidal fuzzy numbers used for the nonunanimous vote output

Linguistic variable	Trapezoidal fuzzy number
Low	(0, 0, 40, 55)
Medium	(40, 50, 50, 70)
High	(50, 70, 100, 100)

8.2.3. Budget (revenues)

This output variable measures the amount of money raised by the state's government, as a proxy for the capacity for public economic action.

8.2.4. Employment rate

A high employment rate is an indicator of high social stability, labor mobilization, and potential for public action.

8.2.5. Nonunanimous vote

It is very important that all members of NATO vote unanimously to allow a new member because maintaining current relationships with NATO members is vital. If a member of NATO expresses concerns in allowing a specific country into the alliance, the validity of these concerns must be explored. A unanimous agreement must be established to protect the integrity of the alliance and to prevent tension among member countries. As per Article 10 of the North Atlantic Treaty, all members of NATO must vote to add a new member, otherwise that bid for membership is denied. In this study, a nonunanimous vote is a fuzzy output with three possible outcomes: low, medium, and high. In this study, the nonunanimous vote is an imprecise and vague output represented with the following three possible outcomes: low, medium, and high. These linguistic terms are represented with the trapezoidal fuzzy numbers shown in Table 5.

In addition, the following input variables were used in the proposed DEA model.

8.2.6. Budget expenditures

This input variable relates to the ability of a country to balance its revenues and expenditures. The threat involved lies in those countries that cannot bring in enough revenues to cover their expenditures. This may be for a number of reasons including but not limited to a lack of natural resources, bad soil or climates for growing crops, lack of education, or lack of skilled manpower. Countries that cannot effectively balance their budget typically require economic assistance from other countries.

8.2.7. Public debt

A rising public debt (percentage of GDP) indicates an increasing debt burden for taxpayers. If the public debt (percentage of GDP) continues to grow, interest charges on the debt will likely also rise. Debt-servicing charges thus progressively absorb more tax revenues, leaving fewer resources available to fund other program expenditures. At some point, the need to service the outstanding

debt compels the government to level program spending, or raise taxes, and/or maintain program spending at current levels and rely increasingly on deficit financing – which further adds to the growing public debt burden. Governments that let debt accumulate over a long period of time risk eroding the living standards of their citizens as debt-servicing costs absorb a progressively larger share of the tax base, "crowding out" program spending.

8.2.8. External debt

This input variable measures the total public and private debt owed to nonresidents of a country. It can be repaid in foreign currency, goods, or services. A high ratio of external debt is a potential problem as future domestic taxation will be used for external transfer, further deepening recessive tendencies in the economy.

8.2.9. Economic aid

A country unable to finance their budget using internal or external debt may be forced soliciting economic aid from other countries or associations. Such economic aid, paid in kind or as easements of foreign debt payments, is usually negatively linked to the sovereignty of the receiving state toward the sponsors. For these reasons, NATO would more than likely be reluctant to accept an economic aid recipient as a new member.

8.2.10. Military expenditures

The budget share allocated to military expenditure is important because high military expenditure as a percentage of GDP inherently means a smaller allocation of expenditure in other areas. In some cases, countries may be overly concerned with military strength while neglecting other areas of spending that are needed to build a stable economy. Another possible threat with respect to military expenditure is the fact that countries that have high military expenditures tend to do so because they are involved in conflict with other countries. Hence, the factor may be a proxy for the subjective conflict risk associated with the candidate country and the expected level of support that NATO must fulfill in case of membership.

8.2.11. Potential for future international disputes

Candidate countries all carry a potential risk profile with respect to their potential risk for involvement in international disputes. International conflicts have the potential of draining a country financially through military expenses expenditures need to bolster defense, drawing on the resources of the alliance. These disputes can affect relationships with allied countries as well, which could affect the ability to import and export with trading partners. In this study, the potential for future international disputes is an imprecise and vague input represented with the following five possible outcomes: very unlikely, unlikely, maybe, likely, and very likely. These linguistic terms are represented with the trapezoidal fuzzy numbers as shown in Table 6.

Table 6
The linguistic variables and their associated trapezoidal fuzzy numbers used for the potential for future international disputes input

Linguistic variable	Trapezoidal fuzzy number
Very unlikely	(0, 0, 10, 22.5)
Unlikely	(10, 22.5, 32.5, 45)
Maybe	(32.5, 45, 55, 67.5)
Likely	(55, 67.5, 77.5, 90)
Very likely	(77.5, 90, 100, 100)

The inputs and outputs data for the 18 most qualified countries[1] for NATO expansion are summarized in Table 7.

8.3. Results

Using the proposed fuzzy DEA model with the interval efficiency, we evaluated the most qualified countries for NATO expansion using a number of inputs and outputs for 18 prospective countries. Naturally, the production set of this study involves fuzzy, categorical, and range variables for which CRS cannot apply. In consequence, we use a VRS assumption. When the proposed model with crisp efficiency was used to determine the efficiency score of the DMUs for various α levels, the results showed that the efficiency score of most countries is equal to 1. These results indicate that the discriminatory power of this method in this application is weak. In order to deal with this problem and rank the efficient DMUs in the fuzzy environment, we used the formula (16) and calculated the interval efficiency for $\alpha \in \{0, 0.5, 1\}$. We should note that Model (12) differs from the model proposed by Saati et al. (2002). Saati et al. (2002) developed a fuzzy DEA model with triangular fuzzy numbers under CRS assumption. In Model (12), we extend Saati et al.'s (2002) model and propose an alternative fuzzy DEA method with trapezoidal fuzzy numbers under VRS assumption.

Table 8 presents the results of Model (16) for the NATO application. This table also presents the overall ranking of each country under different α levels in terms of the central point of the intervals.

In addition, the average ranking scores for each country with respect to different α levels is presented in Fig. 1 provides some additional insight. In particular, note that smaller intervals indicate less preferential uncertainty, subject to the assumption that the given model specification reflects the real criteria. According to the model outcome in Fig. 1, countries such as Finland, Russia, Georgia, and Malta would be the strongest contenders for integration under $\alpha = 0.5$.

In order to test the consistency of the stated criteria and consequently the validity of the socioeconomic model proposed, we now introduce the data for the real NATO integration process. For all countries in the data, a proxy is set to the sum of the integration processes passed (PfP, IPAP, ID, and MAP), i.e., an integer between 1 and 4. The outcomes, ordered after this proxy, are presented in Fig. 2.

[1]Microstates Monaco, San Marino, Andorra, and Liechtenstein are removed, as is Cyprus, subject to an unresolved conflict among two NATO members (Greece and Turkey).

Table 7
The input–output data used in the NATO enlargement case study

	Outputs					Inputs					
Non-EU country	GDP in billions dollars	Inflation rate (%)	Budget revenues (% of GDP)	Unemployment rate (%)	Nonunanimous vote	Budget expenditures (% of GDP)	Public debt (% of GDP)	External debt (% of GDP)	Economic aids (% of GDP)	Military expenditures (% of GDP)	Potential for future international disputes
Armenia (AM)	17.170	4.40	9.70	7.10	Medium	10.10	25.13	7.99	1.05	6.5	Likely
Austria (AT)	322.000	2.20	55.12	4.40	High	55.87	59.10	233.70	0.00	0.9	Unlikely
Azerbaijan (AZ)	64.660	16.70	10.45	1.00	Medium	13.26	6.70	2.78	0.34	2.6	Likely
Belarus (BY)	103.500	8.40	20.05	1.60	Medium	20.16	3.57	7.10	0.05	1.4	Likely
Bosnia and Herzegovina (BA)	14.780	4.71	48.00	45.50	Low	48.29	34.00	45.56	3.69	4.5	Likely
Finland (FI)	188.400	2.50	32.92	6.90	High	30.87	35.90	143.84	0.00	2	Very unlikely
FYR Macedonia (MK)	17.350	2.30	14.46	34.90	Low	14.33	30.80	22.86	1.33	6	Possible
Georgia (GE)	20.600	9.30	17.86	13.60	High	14.95	23.37	21.84	1.50	0.59	Likely
Ireland (IE)	191.600	4.90	48.65	4.60	High	48.26	24.90	960.86	0.00	0.9	Unlikely
Kazakhstan (KZ)	168.200	10.80	14.02	7.30	Medium	15.06	7.70	57.29	0.14	0.9	Likely
Malta (MT)	9.400	1.30	37.07	6.40	Medium	37.81	21.73	2.01	0.06	0.7	Very unlikely
Moldova (MD)	9.756	12.30	18.76	2.10	Medium	18.87	23.30	33.83	1.97	0.4	Possible
Montenegro (ME)	5.918	3.40	118.28	14.70	Low	757.86	38.00	10.98	0.00	1.79	Unlikely
Russia (RU)	2,097.000	9.00	14.26	6.20	High	12.49	5.90	17.00	0.05	3.9	Very likely
Serbia (RR)	77.280	6.80	12.42	18.80	Medium	12.68	37.00	33.95	2.59	1.66	Likely
Sweden (SE)	338.500	2.20	73.59	6.10	High	68.98	41.70	176.72	0.00	1.6	Very unlikely
Switzerland (CH)	303.200	0.70	49.67	2.80	High	46.67	44.20	441.95	0.00	1.5	Very unlikely
Ukraine (UK)	324.800	12.80	13.41	2.30	High	13.87	11.70	21.26	0.13	1.4	Likely

Table 8
The results for Model (12) and the ranking of each country for different α levels

	$\alpha = 0$		$\alpha = 0.5$		$\alpha = 1$	
Non-EU country	Interval score	Rank	Interval score	Rank	Interval score	Rank
AM	[0.4812, 1.0000]	17	[0.4668, 1.0000]	17	[0.9208, 1.0000]	12
AT	[0.4184, 1.0328]	18	[0.5330, 1.3632]	12	[0.5418, 1.0000]	18
AZ	[0.7116, 1.5454]	8	[0.4527, 1.2584]	15	[0.8865, 1.0000]	14
BY	[0.6343, 1.4059]	13	[0.5029, 1.3920]	13	[0.9330, 1.0000]	11
BA	[0.7743, 1.4447]	10	[0.8706, 1.7792]	8	[0.9883, 1.0000]	7
FI	[0.8253, 1.7747]	2	[1.0000, 2.1888]	1	[1.0000, 1.0188]	6
MK	[0.7224, 1.5863]	6	[0.7012, 1.8157]	9	[1.0000, 1.0301]	3
GE	[0.7911, 1.5569]	4	[1.0000, 1.9905]	3	[0.9968, 1.0260]	5
IE	[0.6870, 1.6320]	5	[0.8197, 1.9707]	6	[0.9726, 1.0000]	8
KZ	[0.8107, 1.6238]	3	[0.7426, 1.3952]	11	[0.9577, 1.0000]	10
MT	[1.0000, 2.2582]	1	[1.0000, 1.9384]	4	[1.0000, 1.0260]	4
MD	[0.6879, 1.3775]	12	[0.5321, 1.0000]	16	[0.9578, 1.0000]	9
ME	[0.6630, 1.0000]	15	[0.0713, 1.0497]	18	[0.7967, 1.0000]	16
RU	[1.0000, 1.2589]	7	[1.0000, 2.0289]	2	[0.8840, 1.1591]	2
RR	[0.7063, 1.0000]	14	[0.7398, 1.0677]	14	[0.9158, 1.0000]	13
SE	[0.8027, 1.2768]	11	[0.8512, 1.8329]	7	[1.0000, 1.2763]	1
CH	[0.5783, 1.0000]	16	[0.6490, 1.4978]	10	[0.7403, 1.0000]	17
UK	[0.8646, 1.3653]	9	[0.8611, 2.0304]	5	[0.8031, 1.0000]	15

Some contradictions are apparent, notably the close integration of FYR Macedonia and the reversed order of preference between the top-level PfP countries (Malta, Finland, Ireland, Russia, and Sweden) and the weaker IPAP countries (Kazakhstan, Azerbaijan, Moldova, and Armenia). The hypothesis of the socioeconomic stability paradigm is rejected using a rank-order correlation in Table 9, where no statistically valid correlation can be found between the model FDEA results and the integration process outcomes. Indeed, the correlation coefficients for the central FDEA scores are even negative, suggesting that the logic may be completely inversed.

9. Conclusions and future research directions

The development of fuzzy DEA with imprecise and ambiguous data has evolved the scope of its application to efficiency measurement in real-life problems. In this study, we developed a fuzzy DEA framework with a BCC model and used an α-level approach to convert the fuzzy BCC model into an interval programming model. Instead of comparing the equality (or inequality) of the two intervals, we defined a variable in the interval to satisfy our constraints and maximize the efficiency value. We presented a numerical example to show the similarities and differences between our solution and the solutions obtained from four fuzzy DEA methods in the literature. In addition, the proposed model was applied to a NATO enlargement problem. The addition of new members into NATO is a strategic issue that has profound economic and political effects on both the entering and existing members of NATO. The NATO enlargement problem is a complex multicriteria problem that embraces qualitative and quantitative data. Potential applicant states must

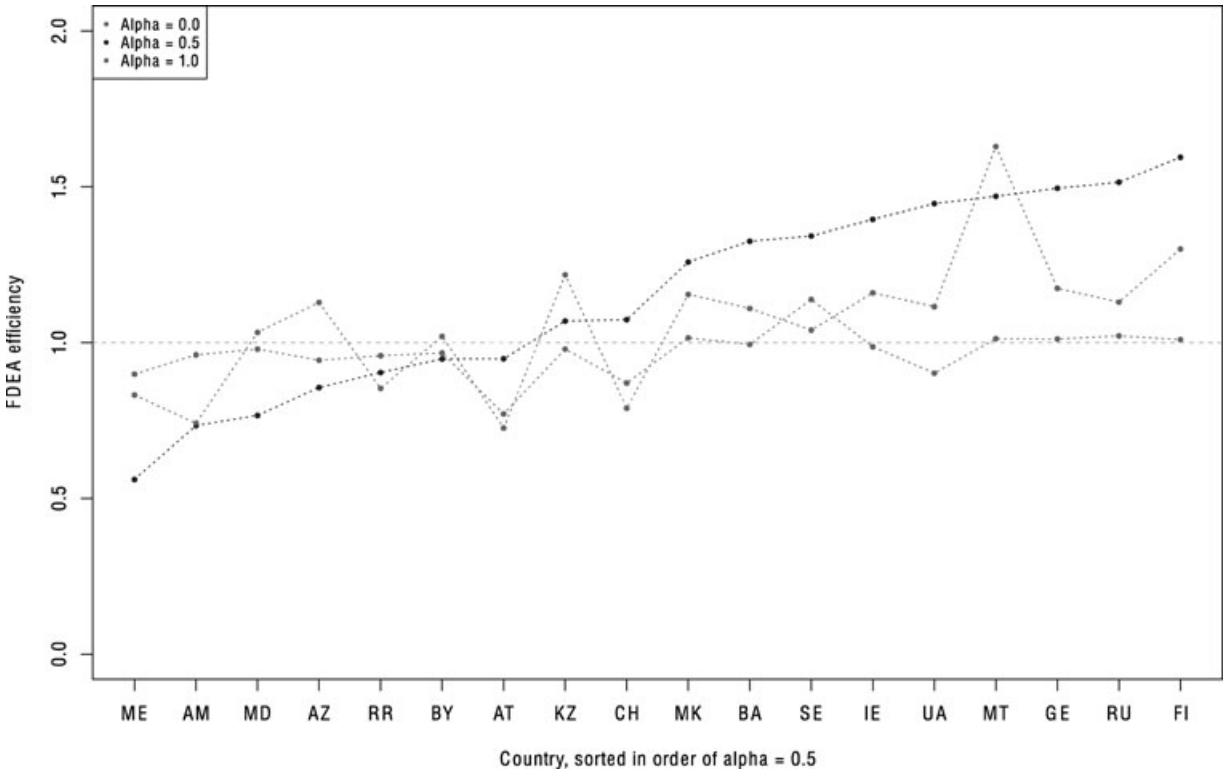

Fig. 1. Central points (FDEA) for α 0, 0.5, and 1.

conform to a large number of quantitative and qualitative entry criteria established by NATO. In political discussions, the enlargement process has sometimes been depicted as favoring socially and economically stable states, with the only political uncertainty related to the issue of intramember conflicts (e.g., Cyprus). We use our model to test the consistency of this conjecture with the real NATO enlargement process. Using a plausible set of socioeconomic indicators and a fuzzy variable for the political consensus, we calculate a fuzzy DEA VRS frontier. Analysis of the obtained results confirms the hypothesis that decisions are not based on socioeconomic stability. In fact, even excluding statutory neutral countries such as Ireland Sweden, and, Switzerland, the outcome correlates poorly with the real ascension. In detail, the order of integration of some former Soviet republics compared to the former Yugoslavian republics clearly suggests that other mechanisms are at play.

In a more general setting, our model lends itself both to normative approaches, in so far that the decision maker has clearly identified objectives, as well as to the positive approach demonstrated. Further research may explore how positive-normative interaction and modeling building may be used to specify the potentially fuzzy variables that do influence outcomes and those that safely can be replaced with crisp proxies.

Table 9
Pearson correlation coefficients between the FDEA scores and the NATO actual integration policy proxies

α levels	FDEA score	Correlation coefficients
0	Low	0.017989
	High	–0.039411
	Center	–0.023369
0.5	Low	–0.250710
	High	–0.047373
	Center	–0.130058
1	Low	0.121417
	High	–0.255082
	Center	–0.024981

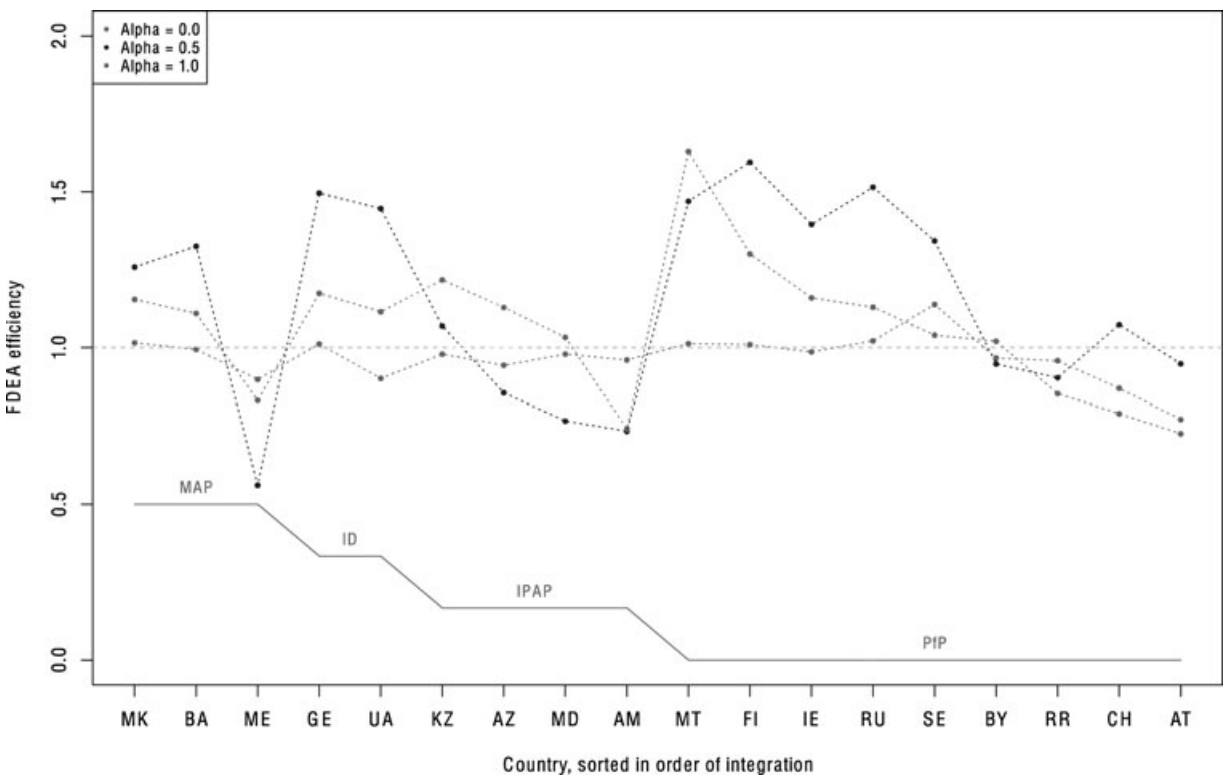

Fig. 2. FDEA central points versus NATO integration proxies.

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Disclaimer

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the United States Department of Defense.

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