



# A fuzzy multi-criteria decision analysis model for advanced technology assessment at Kennedy Space Center

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The rapid development of computer and information technology has made project evaluation and selection a difficult task at the Kennedy Space Center (KSC) Shuttle Project Engineering Office. Decision Makers (DMs) are required to consider a vast amount of intuitive and analytical information in the decision process. *Fuzzy Euclid* is a Multi-criteria Decision Analysis (MCDA) model that captures the DMs' beliefs through a series of intuitive and analytical methods such as the analytic hierarchy process (AHP) and subjective probability estimation. A defuzzification method is used to obtain crisp values from the subjective judgments provided by multiple DMs. These crisp values are synthesized with Entropy and the theory of displaced ideal to assist the DMs in their selection process by plotting the alternative projects in a four-zone graph based on their Euclidean distance from the 'ideal choice'.

*Journal of the Operational Research Society* (2010) 61, 1459–1470. doi:10.1057/jors.2009.107

Published online 14 October 2009

**Keywords:** multi-criteria decision analysis; fuzzy systems; analytic network process; entropy; theory of displaced ideal

## 1. Introduction

The massive explosion of information and rapid development of technology have focused critical attention on government agencies that support information and technology development. The public is concerned with the governance of these agencies and demands maximum return from investment in technology. Public awareness and pressure has forced Congress to mandate the National Aeronautic and Space Administration (NASA) to be more accountable to the taxpayers. The demand for accountability and the increasing complexity of advanced technology projects has made project assessment and selection an extremely difficult task at NASA.

A large body of scoring, economic and portfolio methods have evolved over the last several decades to assist decision makers (DMs) in project evaluation. Scoring methods use algebraic formulas to produce an overall score for each project (Moore and Baker, 1969; Osawa and Murakami, 2002; Osawa, 2003). Economic methods use financial models to calculate the monetary pay-off of each project (Mehrez, 1988 and Graves and Ringuest, 1991). Portfolio methods evaluate the entire set of projects to identify the most attractive subset (Lootsma *et al*, 1990; Girotra *et al*, 2007; Mojsilović *et al*, 2007; Wang and Hwang, 2007). Cluster analysis, a more

specific portfolio method, groups projects according to their support of the strategic positioning of the firm (Mathieu and Gibson, 1993). Decision analysis methods compare various projects according to their expected value (Hazelrigg and Huband, 1985; Thomas, 1985). Finally, simulation, a more specific decision analysis method, uses random numbers and simulation to generate a large number of problems and picks the best outcome (Mandakovic and Souder, 1985; Abacoumkin and Ballis, 2004; Paisittanand and Olson, 2006).

Most of these methods are used to evaluate research and development projects (Osawa and Murakami, 2002; Osawa, 2003; Girotra *et al*, 2007; Wang and Hwang, 2007), information systems projects (Muralidhar *et al*, 1990; Schniederjans and Santhanam, 1993; Santhanam and Kyparisis, 1995; Paisittanand and Olson, 2006; Mojsilović *et al*, 2007) and capital budgeting projects (Mehrez, 1988; Graves and Ringuest, 1991). Although these methods have made great strides in Multi-criteria Decision Analysis (MCDA), the intuitive models lack a structured framework and the analytical models do not capture intuitive preferences.

Project evaluation problems are group MCDA problems that embrace both qualitative and quantitative criteria. MCDA methods provide a structured framework for information exchange among the group members and thus reducing the unstructured nature of the problem. The obvious obstacle when multiple persons are involved in a group decision problem is the fact that each group member has his/her own perception of the problem that accordingly affects the decision outcome. MCDA frameworks permit group members

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to explore their value system from multiple viewpoints and modify their perceptions by obtaining knowledge of the other group members' preference structure and beliefs. A number of decision methodologies in the group decision-making context have been presented in the MCDA literature. A comprehensive survey can be found in Hwang and Lin (1987). Iz and Gardiner (1993) review formal group decision-making models and describe some examples of conceptual frameworks and actual implementations of group decision-making models. A comprehensive collection of research devoted to synthesis and analysis of group support frameworks and procedures can be found in Jessup and Valacich (1993). When facing such multiple criteria problems, the literature and research show that the following difficulties may be encountered:

- (a) DMs often use verbal expressions and linguistic variables for subjective judgments, which lead to ambiguity in human decision making (Poyhonen *et al*, 1997). Furthermore, the subjective assessment process is intrinsically imprecise and may involve two types of judgments: comparative judgment and absolute judgment (Saaty, 2006).
- (b) DMs often provide imprecise or vague information due to lack of expertise, unavailability of data, or time constraint (Kim and Ahn, 1999).
- (c) Meaningful and robust aggregation of subjective and objective judgments causes problems during the evaluation process (Valls and Torra, 2000).

A decision may not be properly made without fully taking into consideration all criteria in MCDA (Belton and Stewart, 2002; Yang and Xu, 2002). Recently, researchers working on project evaluation and selection have focused on MCDA models to integrate the intuitive preferences of multiple DMs into structured and analytical frameworks (Bailey *et al*, 2003; Costa *et al*, 2003; Hsieh *et al*, 2004; Tavana, 2006; Liesiö *et al*, 2007).

MCDA problems involve the ranking of a finite set of alternatives in terms of a finite number of conflicting decision criteria. More often, decision criteria can be grouped into two contradictory categories, called the 'opportunities' and the 'threats'. Alternatively, opportunities may be called 'benefits' or 'returns' and threats may be called 'costs' or 'risks'. Higher alternative scores are preferred for opportunities and lower alternative scores are preferred for threats. In practice, two aggregation techniques are used to compute two aggregated indexes and evaluate the alternatives when criteria are divided into the opportunities and threats. The first approach is the opportunities to threat ratio approach (Tavana and Banerjee, 1995) and the second is the opportunities minus threat approach (Tavana, 2004). The former approach is a ratio scale and the latter approach is an interval scale.

*Fuzzy Euclid* is a MCDA model that captures the DMs' beliefs through a series of intuitive and analytical methods

such as the analytic hierarchy process (AHP) and subjective probabilities. The concept of fuzzy sets is often used to reflect the inherent subjectivity and imprecision involved in the evaluation process (Zadeh, 1965). Fuzzy numbers have been widely used in decision problems where the information available is subjective or imprecise (Zimmermann, 1996). We use a defuzzification method to obtain crisp values from the subjective judgments provided by multiple DMs. These crisp values are synthesized with Entropy and the theory of displaced ideal to assist the DMs in their selection process. Two aggregated opportunity and threat indexes are used to plot the alternative projects in a four-zone graph based on their Euclidean distance from the ideal project.

A decision-making committee of three division chiefs at the Shuttle Project Engineering Office is responsible for the evaluation and selection of advanced-technology projects at the Kennedy Space Center (KSC). The proposed projects are independent and non-additive requests for engineering changes to the space shuttle that are generally initiated by the contractors or different divisions within KSC. *Fuzzy Euclid*, developed at KSC, considers the importance of each project relative to the longevity of the space-shuttle program and enhances the committee's decision quality and confidence. The next section presents a detailed explanation of the mathematical model and procedure followed by a case study and conclusion in Sections 3 and 4.

## 2. Mathematical model and procedure

The evaluation process begins with a preliminary review of  $M$  advanced-technology projects submitted to KSC for funding. The Shuttle Project Engineering Office selects  $I$  divisions to participate in the evaluation process.  $K$  division chiefs, called DMs in this study, are responsible for the evaluation of the advanced-technology projects. Initially, DMs use AHP independently to weight their importance of the participating divisions. Next, the DMs collectively decide what criteria (opportunities and threats) should be considered in the evaluation process. Once the DMs agree on a set of opportunities and threats, they use AHP independently to weigh their importance of the opportunities and threats. Then, the DMs consult with the experts and specialists within their divisions to assign probabilities of occurrence to the opportunities and threats. Next, a defuzzification method is used to obtain crisp values from the subjective judgments and estimates provided by the  $K$  DMs for the  $M$  projects. These crisp values are then synthesized in an MCDA model to produce an overall performance score for each of the  $M$  projects under consideration.

MCDA techniques require the determination of weights that reflect the relative importance of various competing objectives. Several approaches such as point allocation, paired comparisons, trade-off analysis, and regression estimates could be used to specify these weights (Kleindorfer *et al*, 1993). *Fuzzy Euclid* utilizes AHP developed by Saaty (1977, 1990a) to estimate the importance weight of the

opportunities ( $u_{ij}^k$ ) and their divisions ( $w_i^k(U)$ ) for the  $I$  participating divisions and the  $K$  DMs. The advantage of AHP is its capability to elicit judgments and scale them uniquely using a procedure that measures the consistency of these scale values (Shim, 1989; Saaty, 1977; Vaidya and Kumar, 2006; Ho, 2008). The process is simplified by confining the estimates to a series of pairwise comparisons. The measure of inconsistency provided by AHP allows for the examination of inconsistent priorities. One of the advantages of AHP is that it encourages DMs to be consistent in their pairwise comparisons. Saaty (1977) suggests a measure of consistency for the pairwise comparisons. When the consistency ration is unacceptable, the DM is made aware that his or her pairwise comparisons are logically inconsistent, and he or she is encouraged to revise them. A similar procedure is used to find the relative importance weight of the threats ( $t_{ij}^k$ ) and their divisions ( $w_i^k(T)$ ) for the  $I$  divisions and the  $K$  DMs.

Traditionally, AHP is used to estimate the relative importance weight of the criteria and the relative performance of the alternatives in MCDA problems. However, in *Fuzzy Euclid*, we only use AHP to determine the importance weight of the opportunities and threats. Instead of using AHP to find the relative performance of the alternatives (projects) on each criterion, we use subjective probabilities of occurrence to capture these scores. These probabilities are used to identify 'ideal probabilities' and the 'ideal project' discussed later.

*Fuzzy Euclid* is a normative MCDA model with multiple factors representing different dimensions from which the projects are viewed. When the number of factors is large, typically more than a dozen, they may be arranged hierarchically (Saaty, 1977; Triantaphyllou and Mann, 1995; Triantaphyllou, 2000). *Fuzzy Euclid* assumes a hierarchical structure by initially identifying the divisions at NASA who are responsible for the evaluation of the advanced technology projects. Following this identification, each division is asked to identify the relevant factors in their decision-making process and group them into opportunities and threats. This hierarchical structure allows for a systematic grouping of decision factors in large problems. The classification of different factors is undoubtedly the most delicate part of the problem formulation (Bouyssou, 1990) because all different aspects of the problem must be represented while avoiding redundancies. Roy and Bouyssou (1987) have developed a series of operational tests that can be used to check the consistency of this classification. The  $K$  DMs use AHP to estimate their importance weight of the opportunities and threats for the  $I$  divisions.

There has been some criticism of AHP in the operations research literature. Harker and Vargas (1987) show that AHP does have an axiomatic foundation, the cardinal measurement of preferences is fully represented by the eigenvector method, and the principles of hierarchical decomposition and rank reversal are valid. On the other hand, Dyer (1990) has questioned the theoretical basis underlying AHP and argues that it can lead to preference reversals based on the

alternative set being analysed. In response, Saaty (1990b) explains how rank reversal is a positive feature when new reference points are introduced. There are several methods for estimating the local importance weights from pairwise comparison matrices in AHP. We employ the row geometric mean method to determine the local priorities and avoid the controversies associated with rank reversal (Dyer, 1990; Harker and Vargas, 1990; Saaty, 1990b). In this procedure, which Saaty (1990b) calls the approximate method; the local priority of each criterion is obtained by the normalization of the row geometric mean associated with this criterion in the pairwise comparison matrices. The row geometric mean method eliminates the undesired rank reversals caused by the traditional arithmetic mean method (Barzilai and Golany, 1994; Aguarón and Moreno-Jiménez, 2000; Xu, 2000; Escobar *et al*, 2004; Leskinen and Kangas, 2005).

Next, the  $K$  DMs estimate the subjective probabilities of occurrence of the opportunities ( $p_{ij}^{km}(U)$ ) and threats ( $p_{ij}^{km}(T)$ ) for the  $M$  projects. Subjective probabilities are commonly used in strategic decision making because they require no historical data (observation of regularly occurring events by their long-run frequencies) (De Kluyver and Moskowitz, 1984; Weigelt, 1988; Vickers, 1992; Schoemaker, 1993; Schoemaker and Russo, 1993; Tavana, 1995, 2002). Subjective probabilities can be measured by asking a DM for the odds on an event. If the DMs are familiar with probability concepts, they can be asked directly for the required probability. If not, some sort of measuring instrument is required. Some researchers suggest using verbal phrases such as 'likely,' 'possible,' 'quite certain,' and etc., to elicit the required information and then converting them into numeric probabilities (Budescu and Wallsten, 1985; Brun and Teigen, 1988; Tavana *et al*, 1997). Other commonly used approaches include reasoning (Koriat *et al*, 1980), scenario construction (Schoemaker, 1993) and cross-impact analysis (Stover and Gordon, 1978). In this study, verbal probabilistic phrases were used to elicit numeric probabilities as suggested by Tavana *et al* (1997). Alternatively, the DM may use numeric probabilities instead of the probabilistic phrases. Merkhofer (1987) and Spetzler and Stael von Holstein (1975) review some probability elicitation procedures that are used in practice.

The probabilities associated with the opportunities and threats are assumed to be binomial. Binomial probabilities are commonly used in strategic decision making so that the DM can simplify the problem by analyzing possible outcomes as either occurring or not occurring. For example, Schoemaker (1993) assigns binomial probabilities to factor such as 'Dow Jones Industrial Average falling below 1500 mark by 1990'. Vickers (1992) assigns binomial probabilities to similar factor such as 'Japanese car manufacturers gain at least 30% of the European market share'. Tavana and Banerjee (1995) also assign binomial probabilities to similar factor such as 'Reduction of staff by 2%'. The main motivation for using the binomial probabilities is to reduce the complexity of the model and allow DMs to use event-driven

factors. Next, we use a defuzzification method to obtain crisp values from the subjective judgments and estimates provided by multiple DMs.

A series of weights and probabilities are used in *Fuzzy Euclid* to estimate the importance weight of the selection criteria and their probabilities of occurrence for each alternative. Decision-making theory generally deals with three types of uncertainty: stochastic uncertainty, subjective uncertainty and informational uncertainty. Stochastic uncertainty is treated by probability theory and subjective and informational uncertainties are the target of fuzzy set and fuzzy logic theory.

Although fuzzy logic and probability theory are similar, they are not identical. Probability refers to the likelihood that something is true and fuzzy logic establishes the degree to which something is true. Probability is not a special case of fuzziness, but leads us to consider probability of fuzzy events. Dubois and Prade (1993) provide an analysis of correlation between fuzzy sets and probability theory. They argue that the existence of mathematical objects in probability theory does not suggest that fuzziness is reducible to randomness and it is possible to approach fuzzy sets and possibility theory without any probability considerations. Their study emphasizes on the interpretation multiplicity of probability and fuzzy set theories and shows that fuzzy set theoretic operations can be categorized according to their membership in the upper probability, the one-point coverage of a random set, or a likelihood function.

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo *et al*, 2008), financial modelling (Muzzioli and Reynaerts, 2007), information systems (Intan and Mukaidono, 2004), auditing (Friedlob and Schleifer, 1999), manufacturing cost estimation (Jahan-Shahi *et al*, 1999), and water quality management (Julien, 1994). We use fuzzy logic for project evaluation and selection at NASA and apply a defuzzification process to integrate *I* sets of division weights ( $w_i^k(U)$  and  $w_i^k(T)$ ), factors weights ( $u_{ij}^k$  and  $t_{ij}^k$ ) and subjective probabilities ( $p_{ij}^{km}(U)$  and  $p_{ij}^{km}(T)$ ) into one set of crisp values for the entire group of *K* DMs. Consider fuzzy sets  $A_{ij}^m$  represented by the pairs:

$$A_{ij}^m = \{(p_{ij}^{km}, \mu_{A_{ij}^m}(p_{ij}^{km}))\}, \quad \forall p_{ij}^{km} \in P_{ij}^m \quad (1)$$

where:  $P_{ij}^m$  = The set of DMs' judgments on criterion *j* in the *i*th division given the choice of the *m*th project; ( $i = 1, 2, \dots, I; j = 1, 2, \dots, J_i; m = 1, 2, \dots, M$ );  $p_{ij}^{km}$  = The judgment of the *k*th DM on criterion *j* in the *i*th division given the choice of the *m*th project; ( $i = 1, 2, \dots, I; j = 1, 2, \dots, J_i; k = 1, 2, \dots, K; m = 1, 2, \dots, M$ );  $\mu_{A_{ij}^m}(p_{ij}^{km})$  = The discrete membership function; ( $i = 1, 2, \dots, I; j = 1, 2, \dots, J_i; k = 1, 2, \dots, K; m = 1, 2, \dots, M$ ).

Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar, and crisp representations. The process of condensing the information captured by fuzzy

sets into numerical values is similar to that of transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process in *Fuzzy Euclid* is similar to an averaging procedure. Many defuzzification techniques have been proposed in the literature. The most commonly used method is the Center of Gravity (COG). Other methods include: random choice of maximum, first of maximum, last of maximum, middle of maximum, mean of maxima, basic defuzzification distributions, generalized level set defuzzification, indexed center of gravity, semi-linear defuzzification, fuzzy mean, weighted fuzzy mean, quality method, extended quality method, center of area, extended center of area, constraint decision defuzzification, and fuzzy clustering defuzzification. Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions (Runkler, 1996; Van Leekwijk and Kerre, 1999; Ali and Zhang, 2001; Roychowdhury and Pedrycz, 2001). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. Although the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir and Yuan (1995). We use COG, also referred to as the center of area method, in *Fuzzy Euclid*. This method is highly popular and is often used as a standard defuzzification method. COG calculates the centroid of a possibility distribution function using Equation (2) for discontinuous cases:

$$COG(N) = \frac{\sum_{i=1}^k xi \mu(xi)}{\sum_{i=1}^k \mu(xi)} \quad (2)$$

The procedure for converting the fuzzy numbers in *Fuzzy Euclid* into a set of crisp values can be divided into the following three steps:

1. Evaluation of the membership functions related to the subjective probabilities of occurrence for opportunities ( $\mu_{ij}^k(U)$ ) and threats ( $\mu_{ij}^k(T)$ ):

$$\mu_{ij}^k(U) = w_i^k(U) \cdot u_{ij}^k \quad (3)$$

$$\mu_{ij}^k(T) = w_i^k(T) \cdot t_{ij}^k \quad (4)$$

where:  $w_i^k(U)(w_i^k(T))$  = The *i*th division importance weight for the opportunities (threats) defined for the *k*th DM; ( $i = 1, 2, \dots, I; k = 1, 2, \dots, K$ );  $u_{ij}^k(t_{ij}^k)$  = Importance of the *j*th opportunity (threat) in the *i*th division for the *k*th DM; ( $i = 1, 2, \dots, I^U(I^T); j = 1, 2, \dots, J_i^U(J_i^T); k = 1, 2, \dots, K$ );  $I^U(I^T)$  = The number of divisions for the group of opportunities (threats);  $J_i^U(J_i^T)$  = The number of opportunities (threats) in the *i*th division.

2. Calculation of the overall weighted subjective probabilities of opportunities ( $f_{ij}^m(U)$ ) and threats ( $f_{ij}^m(T)$ ) for the *M* projects as the summed product of the probabilities on their

grades of membership:

$$f_{ij}^m(U) = \sum_{k=1}^K \mu_{ij}^k(U) \cdot p_{ij}^{km}(U) \tag{5}$$

$$f_{ij}^m(T) = \sum_{k=1}^K \mu_{ij}^k(T) \cdot p_{ij}^{km}(T) \tag{6}$$

where:  $p_{ij}^{km}(U)(p_{ij}^{km}(T))$ = Subjective probability of occurrence of the  $j$ th opportunity (threat) for the  $i$ th division given the choice of the  $m$ th project by the  $k$ th DM; ( $m = 1, 2, \dots, M$ ;  $i = 1, 2, \dots, I^U(I^T)$ ;  $j = 1, 2, \dots, J_i^U(J_i^T)$ ;  $k = 1, 2, \dots, K$ ).

- On the final step of the defuzzification process, we divide the overall weighted subjective probabilities of opportunities and threats by their summed membership grades. These calculations result in  $M$  vectors of non-fuzzy values characterizing opportunities and threats for  $M$  projects:

$$COG(U_{ij}^m) = \frac{f_{ij}^m(U)}{\mu_{ij}(U)} \tag{7}$$

$$COG(T_{ij}^m) = \frac{f_{ij}^m(T)}{\mu_{ij}(T)} \tag{8}$$

where  $\mu_{ij}(U)$  and  $\mu_{ij}(T)$  define the membership functions for opportunities and threats with aggregated results for all DMs:

$$\mu_{ij}(U) = \sum_{k=1}^K \mu_{ij}^k(U) \tag{9}$$

$$\mu_{ij}(T) = \sum_{k=1}^K \mu_{ij}^k(T) \tag{10}$$

Next, we find the defuzzified importance weights for the opportunities and threats, as well as of total defuzzified opportunities ( $U^m$ ) and threats ( $T^m$ ) for all projects under consideration:

$$U^m = \sum_{i=1}^{I^U} \sum_{j=1}^{J_i^U} COG(U_{ij}^m) \tag{11}$$

$$T^m = \sum_{i=1}^{I^T} \sum_{j=1}^{J_i^T} COG(T_{ij}^m) \tag{12}$$

Finally, we revise the importance weight of the opportunities and threats determined through the defuzzification process with the entropy concept. Each opportunity or threat is an information source; therefore, the more information an opportunity or threat reveals, the more relevant it is to the decision analysis. The level of entropy  $e(P)$  as a measure of fuzziness, indicates the variance of the assigned preference relation. The concept of entropy, originated in physics and

statistical mechanics, has become increasingly popular in computer science and information theory. Shannon (1948) has defined the entropy of a probability distribution in which the total probability for all elements must add up to 1. However, Luca and Termini (1972) show that this restriction is unnecessary. They define a fuzzy entropy formula on a finite universal set  $X = \{x_1, \dots, x_n\}$  as:

$$e_{LT}(A) = -\beta \sum_{i=1}^n [\mu_A(x_i) \ln \mu_A(x_i) + (1 - \mu_A(x_i)) \ln(1 - \mu_A(x_i))], \quad \beta > 0 \tag{13}$$

where  $\beta > 0$  is a normalization constant,  $\ln$  is the natural logarithm,  $\mu_A(x_i)$  is a membership function for each preference intensity.

An entropy value of 1 indicates that all factors are biased by the maximum fuzziness and a lack of distinction is apparent in the preference relations. The fuzziness of the membership functions has its highest grade at the ‘crossover value’ ( $\mu = 0.5$ ). An entropy value of 0 indicates that the preference relations are definitely credible or definitely non-credible. Maximal distinctness is reached when  $\mu = 0$  and  $\mu = 1$ .

The more different the probabilities of occurrence of an opportunity or threat are, the larger is the contrast intensity of the opportunity or threat, and the greater is the amount of information transmitted by that opportunity or threat. Assuming that vector  $P_{ij}^m(U) = \{p_{ij}^{km}(U)\}$  characterizes the set of weighed probabilities in terms of the  $j$ th opportunity for the  $i$ th division ( $i$ th opportunity) given the choice of the  $m$ th project, the entropy measure of the  $i$ th opportunity is:

$$e(A_{ij}^m(U)) = -\beta \sum_{k=1}^K [\mu(p_{ij}^{km}(U)) \cdot \ln \mu(p_{ij}^{km}(U)) + (1 - \mu(p_{ij}^{km}(U))) \cdot \ln(1 - \mu(p_{ij}^{km}(U)))] \tag{14}$$

where  $0 \leq \mu(p_{ij}^{km}(U)) \leq 1$ , and  $e(A_{ij}^m(U)) \geq 0$ . The smaller  $e(A)$  is, the more information the  $j$ th opportunity transmits, and the larger  $e(A)$  is, the less information it transmits. In addition, the total entropy of opportunities for project  $m$  is defined as  $E_u^m = \sum_{i=1}^{I^U} \sum_{j=1}^{J_i^U} e(A_{ij}^m(U))$ . Similar to the opportunities, the entropy measure of the  $i$ th threat is:

$$e(A_{ij}^m(T)) = -\beta \sum_{k=1}^K [\mu(p_{ij}^{km}(T)) \ln \mu(p_{ij}^{km}(T)) + (1 - \mu(p_{ij}^{km}(T))) \ln(1 - \mu(p_{ij}^{km}(T)))] \tag{15}$$

where  $0 \leq \mu(p_{ij}^{km}(T)) \leq 1$ ,  $e(A_{ij}^m(T)) \geq 0$  and the total entropy of threats for project  $m$  is defined as  $E_T^m = \sum_{i=1}^{I^T} \sum_{j=1}^{J_i^T} e(A_{ij}^m(T))$ .

Fuzzy Euclid is a weighted-sum MCDA model with opportunities and threats as conflicting criteria. Triantaphyllou (2000) has discussed the mathematical properties of

weighted-sum MCDA models. Many weighted-sum models have been developed to help DMs deal with the strategy evaluation process (Leyva-López and Fernández-González, 2003; Gouveia *et al.*, 2008). Triantaphyllou and Baig (2005) have examined the use of four key weighted-sum MCDA methods when benefits and costs (opportunities and threats) are used as conflicting criteria. They compared the simple weighted-sum model, the weighted-product model, and AHP along with some of its variants, including the multiplicative AHP. Their extensive empirical analysis revealed some ranking inconsistencies among the four methods, especially, when the number of alternatives was high. Although, they were not able to show which method results in the ‘correct’ ranking, they did prove multiplicative AHP is immune to ranking inconsistencies.

The weighted-sum scores in *Fuzzy Euclid* are used to compare potential projects among themselves and with the *ideal project*. The concept of ideal choice, an unattainable idea, serving as a norm or rationale facilitating human choice problem is not new (Tavana, 2002). See for example the stimulating work of Schelling (1960), introducing the idea. Subsequently, Festinger (1964) showed that an external, generally non-accessible choice assumes the important role of a point of reference against which choices are measured. Zeleny (1974, 1982) demonstrated how the highest achievable scores on all currently considered decision criteria form this composite ideal choice. As all choices are compared, those closer to the ideal are preferred to those farther away. Zeleny (1982, p 144) shows that the Euclidean measure can be used as a proxy measure of distance.

Using the Euclidean measure suggested by Zeleny (1982), *Fuzzy Euclid* synthesizes the results by determining the ideal opportunity and threat values. The ideal opportunity ( $U^*$ ) is the highest defuzzified importance weight of the opportunities among the set  $U^m$  and the ideal threat ( $T^*$ ) is the lowest defuzzified importance weight of the threats among the set  $T^m$ . The ideal opportunity and threat values form the ideal project. We then find the Euclidean distance of each project from the ideal project. The Euclidean distance is the sum of the quadratic root of squared differences between the ideal and the  $m$ th indices of opportunities and threats. To formulate *Fuzzy Euclid* algebraically, let us assume:

- $D_U^m (D_T^m)$  Total Euclidean distance from the ideal opportunity (threat) for the  $m$ th project; ( $m = 1, 2, \dots, M$ ).
- $D^m$  Overall Euclidean distance of the  $m$ th project; ( $m = 1, 2, \dots, M$ ).
- $\bar{D}$  Mean Euclidean distance.
- $U^m (T^m)$  The total defuzzified opportunity (threat) value of the  $m$ th project; ( $m = 1, 2, \dots, M$ ).
- $U^* (T^*)$  The ideal defuzzified opportunity (threat) value.
- $E_U^* (E_T^*)$  The entropy of the ideal opportunity (threat).

- $DE_U^m (DE_T^m)$  The Euclidean distance from the entropy of the ideal opportunity for the  $m$ th project; ( $m = 1, 2, \dots, M$ ).
- $DE^m$  Overall Euclidean distance of the entropy for the  $m$ th project; ( $m = 1, 2, \dots, M$ ).
- $N_i^U (N_i^T)$  Number of opportunities (threats) for the  $i$ th division ( $i = 1, 2, \dots, I^U (I^T)$ ).

$$D^m = \sqrt{(D_U^m)^2 + (D_T^m)^2} \tag{16}$$

$$\bar{D} = \sum_{m=1}^M D^m / M \tag{17}$$

$$DE^m = \sqrt{(DE_U^m)^2 + (DE_T^m)^2} \tag{18}$$

$$U^* = \text{Max}\{U^m\} \tag{19}$$

$$T^* = \text{Min}\{T^m\} \tag{20}$$

$$E_U^* = \text{Min}\{E_U^m\} \tag{21}$$

$$E_T^* = \text{Min}\{E_T^m\} \tag{22}$$

where

$$D_U^m = U^* - U^m$$

$$D_T^m = T^m - T^*$$

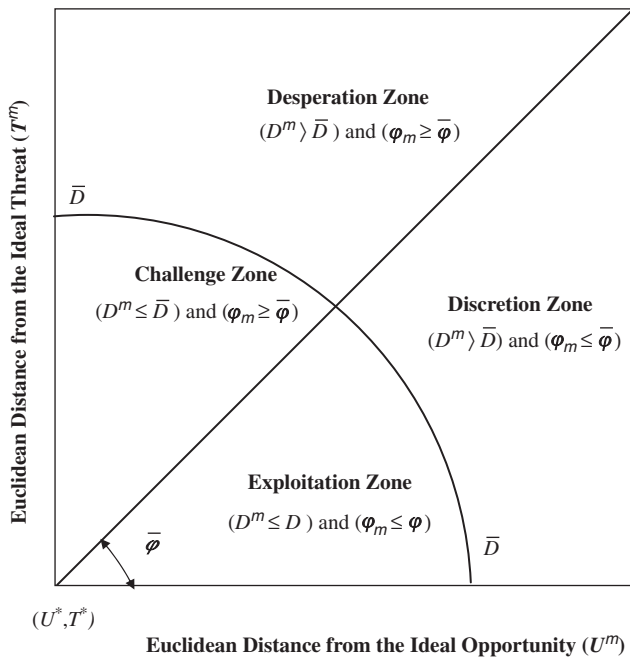
$$DE_U^m = E_U^m - E_U^*$$

$$DE_T^m = E_T^m - E_T^*$$

Next, we plot the alternative projects on a plane using a polar coordinate system (sometimes also referred to as ‘circular coordinates’) in which each point is determined by a distance and an angle. The  $x$ -axis is represented by the total Euclidean distance from the ideal opportunity ( $D_U^m$ ) and the  $y$ -axis is represented by the total Euclidean distance from the ideal threat ( $D_T^m$ ). The position of the point corresponding to project  $m$  with Cartesian coordinates ( $D_U^m, D_T^m$ ) on the graph is determined by its Euclidean distance from the coordinate origin ( $D^m$ ) with an angle of  $\varphi_m$  between vector  $(\overline{D_U^m}, \overline{D_T^m})$  and the  $x$ -axis, where:

$$tg(\varphi_m) = D_T^m / D_U^m \tag{23}$$

We use the mean Euclidean distance ( $\bar{D}$ ) and the angle ( $\bar{\varphi}$ ) to divide the graph into four decision zones. In the case of a tie ( $D_T^m = D_U^m$ ),  $\varphi_m = 45^\circ$  and  $tg(\bar{\varphi}) = 1$ . Projects with smaller  $D^m$  are closer to the ideal project and are preferred to projects with larger  $D^m$ . Furthermore, projects with smaller  $\varphi_m$  and  $DE^m$  are preferred to projects with larger  $\varphi_m$  and  $DE^m$ . Projects with equal  $D^m$  lie on the same circle (sphere). The following assertion is valid for projects lying on the same sphere: with growth of  $\varphi_m$ , the distance to the ideal opportunity decreases ( $U^m \rightarrow \text{min}$ ) and the distance to the ideal threat increases



**Figure 1** The four zones and their characteristics.

( $U^m \rightarrow \max$ ). Therefore, projects with  $\phi_m \leq \bar{\phi}$  are less risky and at the same time have little potential (Figure 1).

We also consider the overall Euclidean distance of the entropy for the  $m$ th project ( $DE^m$ ). Projects with smaller  $DE^m$  (smaller measure of uncertainty) are preferred to those with larger  $DE^m$  (larger measure of uncertainty). With the *ideal project* ( $U^*=0, T^*=0$ ) as the origin, the mean Euclidean distance ( $\bar{D}$ ) and angle ( $\bar{\phi}$ ) divide the graph into *Exploitation*, *Challenge*, *Discretion*, and *Desperation* Zones:

- **Exploitation Zone:** In this zone  $D^m \leq \bar{D}$  and  $\phi_m \leq \bar{\phi}$ . This area represents little threats and a great deal of opportunities. Projects falling into this zone are close to the *ideal project* ( $U^* = 0, T^* = 0$ ) at the origin. These projects are considered very attractive because they have little risk but demonstrate tremendous potentials.
- **Challenge Zone:** In this zone  $D^m \leq \bar{D}$  and  $\phi_m \geq \bar{\phi}$ . This area represents a great deal of threats and a great deal of opportunities. Projects falling into this zone are close to the *ideal project* ( $U^* = 0, T^* = 0$ ) at the origin. These projects are considered challenging because they are very risky but also exhibit tremendous potential. This zone requires full use of the organization’s capabilities and resources.
- **Discretion Zone:** In this zone  $D^m > \bar{D}$  and  $\phi_m \leq \bar{\phi}$ . This area represents little threats and little opportunities. Projects falling into this zone are far from the *ideal project* ( $U^* = 0, T^* = 0$ ) at the origin. These projects are considered discretionary because they are not risky and do not demonstrate meaningful potential. This zone represents the area where the DMs have freedom or power to act or judge on their own.

- **Desperation Zone:** In this zone  $D^m > \bar{D}$  and  $\phi_m \geq \bar{\phi}$ . This area represents a great deal of threats and very little opportunities. Projects falling into this zone are far from the *ideal project* ( $U^* = 0, T^* = 0$ ) at the origin. These projects should be undertaken as a last resort because they are very risky and do not exhibit significant potential.

Final prioritization of projects can be performed using their position in the decision space described above. However, DMs can make corrections in obtained ranking by taking into account the entropy measures of projects. Generally, DMs can make trade-offs between the distance measure and the entropy measure for the final prioritization of projects by specifying subjective threshold values. DMs might be willing to trade-off higher uncertainty levels for lower Euclidean distance or lower uncertainly levels for higher Euclidean distance.

Once the model is developed, sensitivity analyses can be performed to determine the impact on the ranking of projects for changes in various model assumptions. Some sensitivity analyses that are usually of interest are on the weights and probabilities of occurrence. The weights representing the relative importance of the divisions, opportunities, and threats are occasionally a point for discussion among the various DMs. In addition, probabilities of occurrence that reflect the degree of belief that an uncertain event will occur are sometimes a matter of contention.

### 3. A case study<sup>1</sup>

We illustrate the application of *Fuzzy Euclid* to a disguised actual case study at NASA—KSC. In this case, the DMs are a committee of three division chiefs for Safety, Reliability, and Operations considering requests for funding 10 advanced technology projects. The following are the projects and anticipated expenditures: Hubble (\$1,778,000), Photovoltaic (\$1,908,000), Airlock (\$1,515,000), Babaloon (\$1,949,000), Planet-Finder (\$1,266,000), Nebula (\$1,348,000), Solar (\$1,176,000), Truss (\$1,347,000), Centrifuge (\$1,790,000) and Tether (\$961,000). A budget of \$15,038,000 is needed to fund all 10 projects. However, budgetary constraints limit spending to \$10 million.

The process began with an initial meeting of the three DMs. They used Expert Choice (Expert Choice, 2006) to weight the importance of each division. Next, the DMs worked with their divisions to identify a set of opportunities and threats to be used in the evaluation process. Each division held separate meetings and developed their set of opportunities and threats. Then, they used Expert Choice to weight these opportunities and threats. The DMs recorded their consistency ratios and made sure it was below 0.10 as suggested by Saaty (1977).

The Safety division identified seven opportunities and seven threats, the Reliability division identified eight opportunities

<sup>1</sup> All the project names presented in this paper are changed to protect the anonymity of the projects. In addition, the data presented in this study is significantly reduced to allow a meaningful illustration of the model.

**Table 1** The divisions and their opportunities and threats

<i>Opportunities</i>	
	<i>Safety</i>
1.	Ability to decrease ascent catastrophic risk
2.	Ability to decrease orbital and entry/landing catastrophic risk
3.	Ability to detect and eliminate process variability and uncoordinated changes
4.	Supporting protection from exposure to hostile environment
5.	Ability to control in-flight anomalies
6.	Ability to accommodate process deviations
7.	Ability to minimize unsafe inspection discrepancies
	<i>Reliability</i>
1.	Improving mean time to repair
2.	Improving identification/fault isolation
3.	Providing for a simpler system
4.	Improving access for maintenance tasks
5.	Increasing mean time between failures
6.	Reducing support equipment, special tools, and special training requirements
7.	Providing for the use of standard commercial off-the-shelf parts
8.	Providing for equipment interchangeability
	<i>Operations</i>
1.	Meeting the safety, launch, and landing requirements
2.	Meeting the time-sensitive implementation requirements
3.	Meeting the proposed costs
4.	Meeting the proposed schedule
5.	Meeting the advanced technology requirements
6.	Supporting program for near-term requirements
7.	Ability to use less people
8.	Ability to reduce time
9.	Ability to reduce hardware/materials expended during processing
10.	Supporting multi-system configurations
<i>Threats</i>	
	<i>Safety</i>
1.	Possibility of death or serious injury
2.	Possibility of loss of flight hardware, facility, or ground support equipment
3.	Possibility of personal injury and/or flight hardware, facility, or ground support damage
4.	Possibility of a serious violation of safety, health, or environmental federal/state regulation
5.	Possibility of a deminimus violation of safety, health, or environmental federal/state regulation
6.	Possibility of failure propagation to other components or systems
7.	Possibility of critical single failure points
	<i>Reliability</i>
1.	Possibility of cascade failures
2.	Possibility of common cause failures
3.	Possibility of common mode failures
4.	Possibility of dependent failures
5.	Possibility of independent failures
	<i>Operations</i>
1.	Possibility of launchslippage
2.	Possibility of reliance on identified obsolete technology
3.	Possibility of interference in implementation (window of opportunity)
4.	Possibility of flight manifest changes
5.	Possibility of equipment and occupational hazards
6.	Possibility of non-support activity occurrences
7.	Possibility of site-specific restrictions

and five threats, and the Operations division identified 10 opportunities and seven threats to be included in the evaluation process (Table 1).

The importance weight of the three divisions along with the importance weight of the opportunities and threats and

the subjective probabilities of occurrence are all integrated using the defuzzification process described earlier. Table 2 presents the total defuzzified opportunity ( $U^m$ ) and threat values ( $T^m$ ) associated with the 10 projects under consideration.  $U^* = 13.849$  and  $T^* = 4.307$  are the total defuzzified



**Table 2** Project opportunity and threat values and their Euclidean distances

Project	$U^m$	$T^m$	$D_u^m$	$D_t^m$	$D^m$
Airlock	12.215	4.855	1.634	0.548	1.724
Hubble	13.075	5.894	0.774	1.587	1.766
Nebula	12.538	5.545	1.311	1.238	1.803
Planet-Finder	12.068	4.973	1.782	0.666	1.902
Babaloon	12.831	5.989	1.018	1.682	1.966
Centrifuge	11.521	4.788	2.328	0.481	2.378
Solar	11.443	4.307	2.406	0.000	2.406
Photovoltaic	13.849	7.841	0.000	3.534	3.534
Truss	10.289	4.678	3.560	0.371	3.580
Tether	8.670	5.780	5.179	1.473	5.384

**Table 3** Project entropies and their Euclidean distances

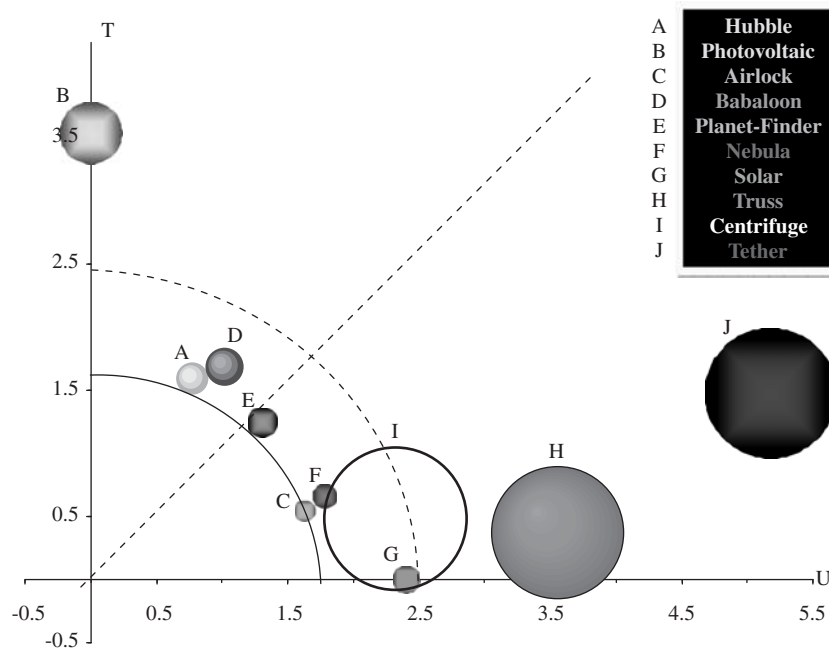
Project	$E_u^m$	$E_t^m$	$DE_u^m$	$DE_t^m$	$DE^m$
Airlock	6.552	2.560	1.134	0.263	1.164
Planet-Finder	6.210	3.234	0.792	0.937	1.227
Solar	6.492	3.133	1.074	0.836	1.361
Nebula	6.692	3.073	1.274	0.776	1.492
Hubble	6.847	3.127	1.429	0.831	1.653
Babaloon	6.963	3.568	1.546	1.271	2.001
Photovoltaic	7.638	4.603	2.221	2.306	3.201
Tether	5.417	3.459	5.417	3.459	6.428
Truss	6.265	2.297	6.265	2.297	6.673
Centrifuge	6.712	2.637	6.712	2.637	7.212

opportunity and threat values for the ideal project. Next, we use Equation (16) to calculate the Euclidean distances ( $D^m$ ) of the 10 projects from the ideal project presented in Table 2.

The entropy was calculated to evaluate the level of uncertainty in the DMs' estimations. The entropies for the opportunities ( $E_u^m$ ) and threats ( $E_t^m$ ) are shown in Table 3.  $E_u^* = 5.417$  and  $E_t^* = 2.297$  are the ideal entropy of the opportunities and threats for the ideal project. Next, we use Equation (18) to calculate the Euclidean distances of the entropies ( $DE^m$ ) of the 10 projects from the ideal project presented in Table 3.

A Graphical representation of the results is shown in Figure 2. Project Airlock has the best Euclidean distance as it lies on the orbit, which is the closest to the ideal project. In addition, Airlock has the smallest entropy indicating the DMs agreement regarding this project. Project Photovoltaic has very strong threats and minimal opportunities and should be excluded from the consideration. Projects Centrifuge, Truss and Tether do not have a very high threat, their opportunities are far from the ideal and their large entropy indicates the DMs contradictions concerning these projects. Projects Truss and Tether both lie in the discretion zone. Using the classification scheme introduced earlier, we identified the position of each project in the four zones. Projects Airlock, Planet-Finder and Solar, and a major part of Nebula and Centrifuge lie in the *Exploitation Zone*. Hubble, Babaloon and a minor part of Nebula lie in the *Challenge Zone*. Truss, Tether and a minor part of Centrifuge lie in the *Discretion Zone*. Photovoltaic lies in the *Desperation Zone*.

Table 4 further shows the sorted results for the 10 projects based on their Euclidean distance from the ideal. Given the \$10 million spending limit; Airlock, Hubble, Nebula, Planet-Finder, Babaloon and Centrifuge could be considered for funding. However, although the priorities of Centrifuge and



**Figure 2** A graphical representation of the project scores and entropies.

**Table 4** Project priorities and cumulative costs

Project	$D^m$	Priority	Zone		Cost	Cumulative Cost
			Major	Minor		
Airlock	1.724	1	Exploitation	—	1,778,000	1,778,000
Hubble	1.766	2	Challenge	—	1,908,000	3,686,000
Nebula	1.803	3	Exploitation	Challenge	1,515,000	5,201,000
Planet-Finder	1.902	4	Exploitation	—	1,949,000	7,150,000
Babaloon	1.966	5	Challenge	—	1,266,000	8,416,000
Centrifuge	2.378	6	Exploitation	Discretion	1,348,000	9,764,000
Solar	2.406	7	Exploitation	—	1,176,000	10,940,000
Photovoltaic	3.534	8	Desperation	—	1,347,000	12,287,000
Truss	3.580	9	Discretion	—	1,790,000	14,077,000
Tether	5.384	10	Discretion	—	961,000	15,038,000

Solar are close, six and seven respectively, the entropy for Solar is considerably less than the entropy for Centrifuge. This indicates a higher level of consistency in the DMs opinion for Solar compared with Centrifuge. In addition, Centrifuge lies in both the *Exploitation* and *Discretion* Zones while the entire Solar lies in *Exploitation* Zone. Considering this additional information, we recommended to replace Centrifuge with Solar. Ultimately, projects Airlock, Hubble, Nebula, Planet-Finder, Babaloon and Solar were selected for funding at KSC.

#### 4. Conclusions

Global competition and the rapid development of computer and information technology have made strategic decision making more complex than ever. *Fuzzy Euclid* is a MCDA model that uses AHP, subjective probabilities, defuzzification, entropy, and the theory of displaced ideal to reduce these complexities by decomposing the project evaluation process into manageable steps. This decomposition is achieved without overly simplifying the evaluation process.

*Fuzzy Euclid* promotes consistent and systematic project evaluation and selection throughout the organization. Judgments captured as separate importance weights and probabilities of occurrence are used uniformly across all projects in the evaluation process. In the absence of separate value judgments, it is difficult to apply a set of importance weights and probabilities of occurrence consistently among the opportunities and threats when evaluating projects. *Fuzzy Euclid* provides a consistent combination of all assessments among all the projects. Whether the assessments faithfully represent real world circumstances depends on the competence and degree of effort the DMs exert in making the assessments.

*Fuzzy Euclid* is also useful in examining how sensitive the overall Euclidean scores are to changes in the portfolio of selected projects. *Fuzzy Euclid* also addresses questions about the sensitivity of the portfolio of selected projects to changes in the relative importance of the organizations, the relative importance of the opportunities and threats, and the probabilities of occurrence.

*Fuzzy Euclid* is not intended to replace human judgment in project evaluation and selection at KSC. In fact, human judgment is the core input in the process. *Fuzzy Euclid* helps the DMs to think systematically about complex project selection problems and improves the quality of their decisions. It is almost impossible to obtain objective data on the complex advanced-technology projects because of inherent uncertainties. However, experienced DMs can often make fairly accurate estimates of values. *Fuzzy Euclid* decomposes the project evaluation process into manageable steps and integrates the results to arrive at a solution consistent with managerial goals and objectives. This decomposition encourages DMs to carefully consider the elements of uncertainty.

Using a structured framework like *Fuzzy Euclid* does not imply a deterministic approach to project evaluation and selection. Although *Fuzzy Euclid* enables DMs to crystallize their thoughts and organize their beliefs, it should be used very carefully. Managerial judgment is an integral component of *Fuzzy Euclid*; therefore, the effectiveness of the model relies heavily on the DM's cognitive abilities to provide sound judgments. As with any decision analysis model, the researchers and practicing managers must be aware of the limitations of subjective estimates and use them carefully.

*Acknowledgements*—The authors thank the anonymous reviewers and the editor for their insightful comments and suggestions. This research was supported in part by NASA grants NGT-52605 and NGT-60002. The authors thank the Shuttle Project Engineering Office at Kennedy Space Center for their support and guidance.

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Received June 2008;  
accepted June 2009 after one revision