Applied Mathematical Modelling 38 (2014) 4388–4395

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers

Ali Ebrahimnejad [a], Madjid Tavana [b,c,*]

[a] Department of Mathematics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran
[b] Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA
[c] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany

ARTICLE INFO

Article history:
Received 28 January 2013
Received in revised form 22 November 2013
Accepted 13 February 2014
Available online 28 February 2014

Keywords:
Fuzzy linear programming
Primal simplex algorithm
Ranking
Trapezoidal fuzzy number

ABSTRACT

Linear programming (LP) is a widely used optimization method for solving real-life problems because of its efficiency. Although precise data are fundamentally indispensable in conventional LP problems, the observed values of the data in real-life problems are often imprecise. Fuzzy sets theory has been extensively used to represent imprecise data in LP by formalizing the inaccuracies inherent in human decision-making. The fuzzy LP (FLP) models in the literature generally either incorporate the imprecisions related to the coefficients of the objective function, the values of the right-hand-side, and/or the elements of the coefficient matrix. We propose a new method for solving FLP problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. We convert the FLP problem into an equivalent crisp LP problem and solve the crisp problem with the standard primal simplex method. We show that the method proposed in this study is simpler and computationally more efficient than two competing FLP methods commonly used in the literature.

1. Introduction

Linear programming (LP) is a mathematical technique for optimal allocation of scarce resources to several competing activities on the basis of given criteria of optimality. Precise data are fundamentally indispensable in conventional LP problems. However, the observed values of the data in real-life problems are often imprecise. Fuzzy sets theory has been used to handle imprecise data in LP by generalizing the notion of membership in a set. Essentially, each element in a fuzzy set is associated with a point-value selected from the unit interval [0,1]. The fundamental challenge in fuzzy LP (FLP) is to construct an optimization model that can produce the optimal solution with imprecise data.

The theory of fuzzy mathematical programming was first proposed by Tanaka et al. Tanaka et al. [1] based on the fuzzy decision framework of Bellman and Zadeh [2]. Zimmerman [3] introduced the first formulation of FLP to address the impreciseness of the parameters in LP problems with fuzzy constraints and objective functions. Zimmerman [3] constructed a crisp

* Corresponding author at: Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA. Tel.: +1 215 951 1129; fax: +1 267 295 2854.

E-mail addresses: aemarzoun@gmail.com (A. Ebrahimnejad), tavana@lasalle.edu (M. Tavana).
URL: http://tavana.us/ (M. Tavana).

http://dx.doi.org/10.1016/j.apm.2014.02.024
0307-904X/

model of the problem and obtained its crisp results using an existing algorithm. He then used Bellman and Zadeh [2] interpretation that a fuzzy decision is a union of goals and constraints and fuzzified the problem by considering subjective constants of admissible deviations for the goal and the constraints. Finally, he defined an equivalent crisp problem using an auxiliary variable that represented the maximization of the minimization of the deviations on the constraints. FLP is by far the most widely used method by practitioners for constrained optimization problems with fuzzy data [4–10].

We propose a simplified new method for solving FLP problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. We show that the optimal solution of the FLP problem can be found simply by solving an equivalent crisp LP problem.

The remainder of this paper is organized as follows. We review the relevant FLP literature in Section 2. In Section 3, we review some necessary concepts and backgrounds on fuzzy arithmetic. We then formulate the FLP problem proposed by Ganesan and Veeramani [11] in Section 4. In Section 5, we present our proposed FLP method. We present our conclusions and future research directions in Section 6.

2. Literature review

There are generally five FLP classifications in the literature:

- Zimmermann [12] has classified FLP problems into two categories: (1) symmetrical problems and (2) non-symmetrical problems. In a symmetrical problem there is no difference between the weight of the objectives and constraints while in the non-symmetrical problems, the objectives and constraints are not equally important and have different weights [13].
- Leung [14] has classified FLP problems into four categories: problems with (1) precise objective and fuzzy constraints; (2) fuzzy objective and precise constraints; (3) fuzzy objective and fuzzy constraints; and (4) robust programming.
- Luhandjula [15] has classified FLP models into three categories: (1) flexible programming; (2) mathematical programming with fuzzy parameters; and (3) fuzzy stochastic programming.
- Inuiguchi et al. [16] have classified FLP models into six categories: (1) flexible programming; (2) possibilistic programming; (3) possibilistic LP using fuzzy max; (4) robust programming; (5) possibilistic programming with fuzzy preference relations; and (6) possibilistic LP with fuzzy goals.
- Kumar et al. [17] have classified FLP problems into two categories: FLP problems with (1) inequality constraints and (2) equality constraints. Some authors [18–20] have proposed different methods for solving FLP problems with inequality constraints where the FLP problem is first converted into a crisp LP problem and then the resulting crisp LP problem is solved to find the fuzzy optimal solution for the original FLP problem. Other authors [21,22] have proposed methods for solving FLP problems with equality constraints which are generally approximate.

In the past four decades, numerous researchers have studied various properties of FLP problems and proposed different models for solving LP problems with fuzzy data. Tanaka et al. [1] first proposed the theory of fuzzy mathematical programming and Zimmerman [3] first formulated and solved the FLP problem. Tanaka and Asai [23] proposed a possibilistic LP formulation where the coefficients of the decision variables were crisp while the decision variables were fuzzy numbers. Verdegay [24] presented the concept of a fuzzy objective based on the fuzzification principle and used this concept to solve FLP problems. Herrera et al. [25] examined the fuzzified version of the mathematical problem assuming that the coefficients are given by fuzzy numbers and the relations in the definition of the feasible set are also fuzzy.

Zhang et al. [26] proposed a FLP with fuzzy numbers for the objective function coefficients. They showed how to convert FLP problems into multi-objective optimization problems with four objective functions. Stanciulescu et al. [27] proposed a FLP model with fuzzy coefficients for the objective function coefficients and the constraints. Their model uses fuzzy decision variables with a joint membership function instead of crisp decision variables and linked the decision variables together to sum them to a constant.

Ganesan and Veeramani [11] proposed a FLP model with symmetric trapezoidal fuzzy numbers. They proved fuzzy analogues for some important LP theorems and derived a solution for the FLP problems without converting them into crisp LP problems. Ebrahimnejad [28] showed that the method proposed by Ganesan and Veeramani [11] stops in a finite number of iterations and proposed a revised version of their method that was more efficient and robust in practice. He also proved the absence of degeneracy and showed that if an FLP problem has a fuzzy feasible solution, it also has a fuzzy basic feasible solution and if an FLP problem has an optimal fuzzy solution, it also has an optimal fuzzy basic solution.

Mahdavi-Amiri and Nasseri [29] proposed a FLP model where a linear ranking function was used to order trapezoidal fuzzy numbers. They established the dual problem of the LP problem with trapezoidal fuzzy variables and deduced some duality results to solve the FLP problem directly with the primal simplex tableau. Mahdavi-Amiri and Nasseri [30] developed some methods for solving the FLP problems by introducing certain auxiliary problems. They applied a linear ranking function to order trapezoidal fuzzy numbers and deduced some duality results by establishing the dual problem of the LP problem with trapezoidal fuzzy variables. Ebrahimnejad et al. [31] introduced a new primal–dual algorithm for solving FLP problems by using the duality results proposed by Mahdavi-Amiri and Nasseri [30]. Ebrahimnejad [32] also generalized the concept of sensitivity analysis in FLP problems by applying a fuzzy simplex algorithm and by using general linear ranking functions on fuzzy numbers.

Wu [33] derived the optimality conditions for FLP problems by proposing two solution concepts called the "non-dominated solution" in multi-objective programming. In order to solve the multi-objective programming problem with fuzzy coefficients, Wu [34] transformed the problem into a vector optimization problem by applying the embedding theorem and using a suitable linear defuzzification function. Gupta and Mehlawat [35] studied a pair of fuzzy primal–dual LP problems and calculated duality results using an aspiration level approach. Their approach is particularly important for FLP where the primal and dual objective values may not be bounded. Peidro et al. [8] used fuzzy sets and developed a FLP method to model the supply chain uncertainties.

Hosseinzadeh Lotfi et al. [22] considered full FLP problems where all the parameters and variables were triangular fuzzy numbers. They pointed out that there is no method in the literature for finding the fuzzy optimal solution of the generalized FLP problem and proposed a new method to find the fuzzy optimal solution for the full FLP problem with equality constraints. They used the concept of symmetric triangular fuzzy numbers and introduced an approach to defuzzify a general fuzzy quantity. They first approximate the fuzzy triangular numbers to their nearest symmetric triangular numbers and convert every FLP model into two crisp complex LP models. They then use a special ranking for fuzzy numbers to transform their full FLP model into a multi-objective LP model where all the variables and parameters are crisp.

Some authors have proposed using the concept of fuzzy number comparisons with ranking functions to convert the FLP problems into its equivalent crisp linear or nonlinear program which then could be solved with standard solution procedures. Maleki et al. [36] utilized this concept and extended the primal simplex method for solving LP problems with fuzzy cost coefficients. Ebrahimnejad et al. [37] generalized the Maleki et al. [36] method for FLP problems with bounded decision variables. Ebrahimnejad et al. [38] used this method for solving the minimum cost flow problem with fuzzy cost. They found the minimum fuzzy cost of a commodity through a capacitated network by satisfying demands at certain nodes using the available supplies at other nodes. In summary, the FLP models in the literature could be classified into the following six groups:

- *Group 1*: The FLP problems in this group involve fuzzy numbers for the decision variables and the right-hand-side of the constraints (e.g. [30]).
- *Group 2*: The FLP problems in this group involve fuzzy numbers for the coefficients of the decision variables in the objective function (e.g. [33]).
- *Group 3*: The FLP problems in this group involve fuzzy numbers for the coefficients of the decision variables in the constraints and the right-hand-side of the constraints (e.g. [39]).
- *Group 4*: The FLP problems in this group involve fuzzy numbers for the decision variables, the coefficients of the decision variables in the objective function and the right-hand-side of the constraints (e.g. [11]).
- *Group 5*: The FLP problems in this group involve fuzzy numbers for the coefficients of the decision variables in the objective function, the coefficients of the decision variables in the constraints and the right-hand-side of the constraints (e.g. [29,40,41]).
- *Group 6*: The FLP problems in this group, the so-called Fully FLP (FFLP) problems, involve fuzzy numbers for the decision variables, the coefficients of the decision variables in the objective function, the coefficients of the decision variables in the constraints and the right-hand-side of the constraints (e.g. [22]).

Although the FFLP group is the general case of FLP, it may not be suitable for all FLP problems with different assumptions and sources of fuzziness. The FLP model proposed in this study belongs to Group 4 in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers.

3. Preliminaries

In this section, we review some necessary concepts and backgrounds on fuzzy arithmetic:

Definition 3.1 [11]. A fuzzy set $\tilde{A}$ on R is called a symmetric trapezoidal fuzzy number if its membership function is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha \leqslant x \leqslant a^L \\ 1 & a^L \leqslant x \leqslant a^U \\ \dfrac{(a^U+\alpha)-x}{\alpha} & a^U \leqslant x \leqslant a^U+\alpha \\ 0 & \text{else}. \end{cases}$$

We denote a symmetric trapezoidal fuzzy number $\tilde{A}$ by $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and the set of all symmetric trapezoidal fuzzy numbers by $F(R)$.

Definition 3.2 [11]. The arithmetic operations on two symmetric trapezoidal fuzzy numbers $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ are given by:

$$\tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha + \beta, \alpha + \beta),$$

$$\tilde{A} - \tilde{B} = (a^L - b^U, a^U - b^L, \alpha + \beta, \alpha + \beta),$$

$$\tilde{A}\tilde{B} = \left(\left(\frac{a^L + a^U}{2}\right)\left(\frac{b^L + b^U}{2}\right) - t, \left(\frac{a^L + a^U}{2}\right)\left(\frac{b^L + b^U}{2}\right) + t, |a^U\beta + b^U\alpha|, |a^U\beta + b^U\alpha|\right),$$

where

$$t = \frac{t_2 - t_1}{2}, \quad t_1 = \min\left\{a^Lb^L, a^Ub^U, a^Ub^L, a^Ub^U\right\}, \quad t_1 = \max\left\{a^Lb^L, a^Ub^U, a^Ub^L, a^Ub^U\right\}.$$

$$k\tilde{A} = \begin{cases} (ka^L, ka^U, k\alpha, k\alpha) & k \geqslant 0 \\ (ka^U, ka^L, -k\alpha, -k\alpha) & k < 0. \end{cases}$$

Definition 3.3 [11]. Let $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ be two symmetric trapezoidal fuzzy numbers. The relations $\tilde{\leqslant}$ and $\approx$ are defined as follows:

$\tilde{A} \tilde{\leq} \tilde{B}$ if and only if:

(i) $\frac{(a^L-\alpha)+(a^U-\alpha)}{2} < \frac{(b^L-\beta)+(b^U-\beta)}{2}$, that is $\frac{a^L+a^U}{2} < \frac{b^L+b^U}{2}$ (in this case, we may write $\tilde{A} \precsim \tilde{B}$), or
(ii) $\frac{a^L+a^U}{2} = \frac{b^L+b^U}{2}, b^L < a^L, a^U < b^U$ (in this case we say $\tilde{A} \approx \tilde{B}$), or
(iii) $\frac{a^L+a^U}{2} = \frac{b^L+b^U}{2}, b^L = a^L, a^U = b^U, \alpha \leqslant \beta$ (in this case we say $\tilde{A} \approx \tilde{B}$).

4. FLP problem

Ganesan and Veeramani [11] proposed a new model for solving FLP problems in which the elements of the coefficient matrix were represented by real numbers and the rest of the parameters were represented by symmetric trapezoidal fuzzy numbers. This problem can be represented with the following model:

$$\begin{aligned} &\max \tilde{z} \approx \tilde{c}\tilde{x} \\ &s.t. A\tilde{x} \tilde{\leqslant} \tilde{B} \\ &\tilde{x} \geqslant \tilde{0}, \end{aligned} \tag{1}$$

where $\tilde{c} \in F(R)^n, \tilde{B} \in F(R)^m$, and $A \in R^{m\times n}$ are given and $\tilde{x} \in F(R)^n$ is to be determined.

Definition 4.1 (*Fuzzy basic solution*). Suppose $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$ solves $A\tilde{x} \approx \tilde{B}$. If all $\tilde{x}_j \approx (-x_j, x_j, \alpha_j, \alpha_j)$ for some $\tilde{x}_j \tilde{\geqslant} \tilde{0}$ and $\alpha_j \geqslant 0$, then, $\tilde{x}$ is said to be a fuzzy basic solution. If $\tilde{x}_j\neg \approx (-x_j, x_j, \alpha_j, \alpha_j)$ for some $\tilde{x}_j \tilde{\geqslant} \tilde{0}$ and $\alpha_j \geqslant 0$, then, $\tilde{x}$ has some non-zero components, say $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_{k,}, 1 \leqslant k \leqslant m$. Then, $A\tilde{x} \approx \tilde{B}$ can be written as:

$$a_1\tilde{x}_1 + a_2\tilde{x}_2 + \cdots + a_k\tilde{x}_k + a_{k+1}(-x_{k+1}, x_{k+1}, \alpha_{k+1}, \alpha_{k+1}) + \cdots + a_n(-x_n, x_n, \alpha_n, \alpha_n) \approx \tilde{B}.$$

If the columns $a_1, a_2, \ldots, a_k$ corresponding to non-zero components $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_k$ are linear independent, then, $\tilde{x}$ is said to be fuzzy basic solution.

Remark 4.1. *Consider a system of m simultaneous fuzzy linear equations involving symmetric trapezoidal fuzzy numbers in n unknowns $A\tilde{x} \approx \tilde{B}$. Let B be any matrix formed by m linearly independent rows of A. In this case $\tilde{x} \approx (\tilde{x}_B, \tilde{x}_N) \approx (B^{-1}\tilde{B}, \tilde{0})$ is a fuzzy basic feasible solution.*

Suppose a fuzzy basic feasible solution of problem (1) with basis B is at hand. Let y_j and $\tilde{w}$ be the solutions to $By_j = a_j$ and $\tilde{w}B = \tilde{c}_B$, respectively. Define $\tilde{z}_j \approx \tilde{c}_B y_j = \tilde{c}_B B^{-1} a_j$. Using these notations, Ganesan and Veeramani [11] proved the fuzzy analogues of some important theorems of LP leading to a new method for solving the FLP problem (1) without converting it into a crisp LP model. In the next section we show that it is possible to find the fuzzy solution of the FLP problem (1) with the help of an equivalent crisp LP problem and without ever solving a FLP problem.

5. Proposed FLP method

According to Definition 3.3, we define a rank for each symmetric trapezoidal fuzzy number for comparison purposes. Assuming that $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is a symmetric trapezoidal fuzzy number, then $R(\tilde{A}) = \frac{a^L+a^U}{2}$. This equation allows us to convert the FLP problem (1) into a crisp LP problem. To do this, we substitute the rank order of each fuzzy number for the corresponding fuzzy number in the fuzzy problem under consideration. This leads to an equivalent crisp LP problem which can be solved with a standard method. However, the decision variables in the original problem are fuzzy numbers as well. Therefore, we need a fuzzy solution since the equivalent crisp problem gives a crisp solution. To overcome this concern, we use the relation between the crisp and fuzzy problems to obtain a fuzzy solution for the FLP problem under consideration. Note that the main steps of the fuzzy simplex method proposed by Ganesan and Veeramani [11] and the primal simplex method proposed here are identical. Thus, if B is the optimal basis of the equivalent crisp problem, then it will be the optimal basis of the corresponding FLP problem. In this case, the solution of the crisp problem $\tilde{x} \approx (\tilde{x}_B, \tilde{x}_N) \approx (B^{-1}\tilde{B}, \tilde{0})$ will be the optimal solution of the FLP problem. However, all arithmetic operations in the fuzzy method are performed on the fuzzy numbers while in our proposed method, all arithmetic operations are done on the crisp numbers. As a result, the computational effort is decreased significantly in our method. In the next example, we use the FLP problem of Ganesan and Veeramani [11] to demonstrate the computational simplicity of the method proposed in this study.

Example 4.1. We find the fuzzy optimal solution of the following FLP problem introduced by Ganesan and Veeramani [11]:

$$\begin{aligned}
&\max \tilde{z} \approx (13,15,2,2)\tilde{x}_1 + (12,14,3,3)\tilde{x}_2 + (15,17,2,2)\tilde{x}_3 \\
&s.t. \quad 12\tilde{x}_1 + 13\tilde{x}_2 + 12\tilde{x}_3 \tilde{\leqslant} (475,505,6,6) \\
&\qquad \tilde{x}_1 + 13\tilde{x}_3 \tilde{\leqslant} (460,480,8,8) \\
&\qquad 12\tilde{x}_1 + 15\tilde{x}_2 \tilde{\leqslant} (465,495,5,5) \\
&\qquad \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \tilde{\geqslant} \tilde{0}.
\end{aligned} \tag{2}$$

We first substitute the rank order of each fuzzy number for its corresponding fuzzy number in the above FLP problem to obtain the following crisp problem:

$$\begin{aligned}
&\max z = 14x_1 + 13x_2 + 16x_3 \\
&s.t. \quad 12x_1 + 13x_2 + 12x_3 \leqslant 490 \\
&\qquad 14_1 + 13x_3 \leqslant 470 \\
&\qquad 12x_1 + 15x_2 \leqslant 480 \\
&\qquad x_1, x_2, x_3 \geqslant 0.
\end{aligned} \tag{3}$$

We then construct the standard form of the LP problem (3) as follows where x_4, x_5 and x_6 are the slack variables:

$$\begin{aligned}
&\max z = 14x_1 + 13x_2 + 16x_3 \\
&s.t. \quad 12x_1 + 13x_2 + 12x_3 + x_4 = 490 \\
&\qquad 14x_1 + 13x_3 + x_5 = 470 \\
&\qquad 12x_1 + 15x_2 + x_6 = 480 \\
&\qquad x_1, x_2, x_3, x_4, x_5, x_6 \geqslant 0.
\end{aligned} \tag{4}$$

Problem (4) is a crisp LP problem and can be solved using the standard primal simplex method. Table 1 presents the first primal simplex tableau.

In this table, x_3 is an entering variable and x_5 is a leaving variable. The second simplex tableau presented in Table 2 is derived by pivoting on $y_{23} = 13$.

In this table, x_2 is an entering variable and x_4 is a leaving variable. The third simplex tableau presented in Table 3 is derived by pivoting on $y_{12} = 13$.

Table 3 is the optimal tableau because $z_j - c_j \geqslant 0$ for all non-basic variables. We then use the obtained optimal basis for the crisp Problem (4) to find the fuzzy optimal solution for the FLP Problem (2). Note that:

$$B^{-1} = \begin{bmatrix} \frac{1}{13} & \frac{-12}{169} & 0 \\ 0 & \frac{1}{13} & 0 \\ \frac{-15}{13} & \frac{180}{169} & 1 \end{bmatrix}.$$

Thus, we have:

Table 1
Initial tableau.

Basis	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	−14	−13	−16	0	0	0	0
x_4	12	13	12	1	0	0	490
x_5	14	0	13	0	1	0	470
x_6	12	15	0	0	0	1	480

Table 2
First iteration tableau.

Basis	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	$\frac{42}{13}$	-13	0	0	$\frac{18}{13}$	0	$\frac{7520}{13}$
x_4	$\frac{-12}{13}$	13	0	1	$\frac{-12}{13}$	0	$\frac{730}{13}$
x_3	$\frac{14}{13}$	0	1	0	$\frac{1}{13}$	0	$\frac{470}{13}$
x_6	12	15	0	0	0	1	480

Table 3
Optimal solution tableau.

Basis	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	$\frac{42}{13}$	0	0	1	$\frac{52}{169}$	0	$\frac{112150}{169}$
x_2	$\frac{-12}{169}$	1	0	$\frac{1}{13}$	$\frac{-12}{169}$	0	$\frac{730}{169}$
x_3	$\frac{14}{13}$	0	1	0	$\frac{1}{13}$	0	$\frac{470}{169}$
x_6	$\frac{1848}{169}$	0	0	$\frac{-15}{13}$	$\frac{180}{169}$	1	$\frac{70170}{169}$

$$\tilde{x}_B = \begin{bmatrix} \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_6 \end{bmatrix} \approx B^{-1}\tilde{B} = \begin{bmatrix} \frac{1}{13} & \frac{-12}{169} & 0 \\ 0 & \frac{1}{13} & 0 \\ \frac{-15}{13} & \frac{180}{169} & 1 \end{bmatrix} \begin{bmatrix} (475, 505, 6, 6) \\ (460, 480, 8, 8) \\ (465, 495, 5, 5) \end{bmatrix} = \begin{bmatrix} \left(\frac{405}{169}, \frac{1045}{169}, \frac{174}{169}, \frac{174}{169}\right) \\ \left(\frac{460}{13}, \frac{1480}{13}, \frac{8}{13}, \frac{8}{13}\right) \\ \left(\frac{62910}{169}, \frac{77430}{169}, \frac{3455}{169}, \frac{3455}{169}\right) \end{bmatrix},$$

$$\tilde{x}_N = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_4 \\ \tilde{x}_5 \end{bmatrix} = \begin{bmatrix} \tilde{0} \\ \tilde{0} \\ \tilde{0} \end{bmatrix}.$$

As shown here, the fuzzy optimal solution of the FLP method proposed in this study is equivalent to the optimal solution derived from Ganesan and Veeramani [11] method. However, the method proposed in this study is by far simpler and computationally more efficient than the FLP method proposed [11].

Kumar et al. [17] has also proposed an alternative method called Mehar's Method for solving the same problem. Using the linearity property of the ranking function, Kumar et al. [17] convert the FLP problem into a crisp LP problem and then solve the crisp problem with a standard method. However, their crisp LP problem has more constraints and variables compared to the FLP method proposed in this study.

A reduction in the number of constraints and variables leads to a reduction in the complexity of the LP problems solved by the simplex algorithm and almost all of the interior point methods such as Khachian's ellipsoid algorithm and Karmarkar's projective algorithm [42]. Bazaraa et al. [42] have shown that the LP problem (1) with m constraints and n variables can be solved using Khachian's ellipsoid algorithm and Karmarkar's projective algorithm within an effort of $O[n^4L]$ (according to Khachian's algorithm) and $O[(m \times n)^6 L]$ (according to Karmarkar's algorithm); where L is the number of binary bits required to record all the data in the problem and is known as the input length of the LP problem.

According to Mehar's method [17], the FLP problem (1) is converted into the following crisp LP problem assuming $\tilde{c}_j = (c_j, d_j, \beta_j, \beta_j)$ and $\tilde{x}_j = (x_j, y_j, \alpha_j, \alpha_j)$ and $\tilde{B}_i = (t_i, s_i, \delta_i, \delta_i)$:

$$\begin{aligned} &\max z = \sum_{j=1}^{n}\left(\tfrac{c_j+d_j}{4}\right)x_j + \sum_{j=1}^{n}\left(\tfrac{c_j+d_j}{4}\right)y_j \\ &s.t. \quad \sum_{j=1}^{n} a_{ij}x_j + \sum_{j=1}^{n} a_{ij}y_j \leqslant \tfrac{t_i+s_i}{2}, \quad i = 1, 2, \ldots, m, \\ &\qquad x_j + y_j \geqslant 0, \quad j = 1, 2, \ldots, n, \\ &\qquad x_j \leqslant y_j, \qquad j = 1, 2, \ldots, n, \\ &\qquad \alpha_j \geqslant 0, \qquad j = 1, 2, \ldots, n.. \end{aligned} \tag{5}$$

Let us compare the number of constraints and variables in Models (1) with the number of constraints and variables in Model (5). Model (5) has $m + 2n$ constraints and $4n$ variables (without considering slacks), while Model (1) has m constraints and n variables. This shows that Model (5) has $2n$ constraints and $3n$ variables more than Model (1), and hence utilizing Model (1) is computationally more efficient than Model (5).

For example, according to Mehar's method, the FLP problem (2) is converted into the following crisp LP problem assuming $\tilde{x}_1 = (x_1, y_1, \alpha_1, \alpha_1), \tilde{x}_2 = (x_2, y_2, \alpha_2, \alpha_2)$ and $\tilde{x}_3 = (x_3, y_3, \alpha_3, \alpha_3)$:

$$\begin{aligned}
\max z &= 7x_1 + 7y_1 + \tfrac{13}{2}x_2 + \tfrac{13}{2}y_2 + 8x_3 + 8y_3 \\
s.t. \quad & 12x_1 + 12y_1 + 13x_2 + 13y_2 + 12x_3 + 12y_3 \leqslant 490 \\
& 14x_1 + 14y_1 + 12x_3 + 12y_3 \leqslant 470 \\
& 12x_1 + 14y_1 + 15x_2 + 15y_2 \leqslant 480 \\
& x_1 + y_1 \geqslant 0 \\
& x_2 + y_2 \geqslant 0 \\
& x_3 + y_3 \geqslant 0 \\
& x_1 \leqslant y_1 \\
& x_2 \leqslant y_2 \\
& x_3 \leqslant y_3 \\
& \alpha_1, \alpha_2, \alpha_3 \geqslant 0
\end{aligned}$$

As a result, the number of functional constraints is increased and the increased number of constraints directly effects the computational time of the simplex method. Furthermore, after solving the crisp problem (5), Kumar et al. [17] obtain the fuzzy optimal solution as $\tilde{x}_1 = (0,0,0,0), \tilde{x}_2 = (\frac{730}{169}, \frac{730}{169}, 0, 0)$ and $\tilde{x}_3 = (\frac{470}{13}, \frac{470}{13}, 0, 0)$. Although the fuzzy optimal solution obtained by all three methods are equivalent (based on Definition 3.3), in the fuzzy optimal solution obtained by Mehar's method [43], the left and the right spread of each fuzzy decision variable are zero, and the starting and end points of the cores are equal. It is somewhat unusual that the right-hand-side in Mehar's solution is fuzzy while the obtained decision variables are crisp. In addition, the crisp solution derived from Mehar's method lack flexibility needed for real-life implementation. Finally, there is no need to define the decision variables as fuzzy variables in Mehar's method since the final solution is ultimately crisp. In summary, our method produces a fuzzy optimal solution that is equivalent to the competing methods of Ganesan and Veeramani [11] and Kumar and Kaur [43] but it is simpler and computationally more efficient than these two methods.

6. Conclusions and future research directions

A large number of LP models with different levels of sophistication have been proposed in the literature. However, some of these models have limited real-life applications because the conventional LP models generally assume crisp data for the coefficients of the objective function, the values of the right-hand-side, and the elements of the coefficient matrix. Contrary to the conventional LP methods, we consider imprecise data in the real-life LP problems and develop an alternative FLP method that is simple and yet addresses these shortfalls in the existing models in the literature.

In the FLP method proposed in this study, the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. The optimal solution of the FLP problem is simply found by solving an equivalent crisp LP problem. The FLP problem is converted into a crisp equivalent LP problem and the crisp LP problem is solved with the standard primal simplex method. We showed that the method proposed in this study requires less arithmetic operations as opposed to the FLP method proposed by Ganesan and Veeramani [11]. In addition, the proposed method produces a fuzzy solution by solving an equivalent crisp problem without increasing the number of constraints and variables of the original problem as opposed to Mehar's method proposed by Kumar and Kaur [43].

Future research could focus on the comparison of the results obtained from the method proposed in this study with those that could be obtained with other competing methods. In addition, based on Definition 3.3, we defined a ranking for each symmetric trapezoidal fuzzy number for comparison purposes. If $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is a symmetric trapezoidal fuzzy number, its ranking is defined as $\Re(\tilde{A}) = \frac{a^L + a^U}{2}$ (from the decision maker's point of view). It is obvious that if we have $\Re(\tilde{A}) = \Re(\tilde{B})$, we can't guarantee the equality of $\tilde{A} = \tilde{B}$. Therefore, further research on introducing a new ranking method for solving FLP problems satisfying this property (space) is an interesting stream of future research. Also, the proposed method is not applicable to FLP problems with non-symmetric trapezoidal fuzzy numbers. The generalization of the proposed method to overcome this shortcoming is left to future research in FLP problems with non-symmetric trapezoidal fuzzy numbers. Finally, we point out that the FLP method proposed in this study does not consider fuzzy cost coefficients and a fuzzy constraint matrix. Developing a full fuzzy version of the proposed method and overcoming this limitation is an interesting stream of future research.

Acknowledgments

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

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