An efficient multi-vehicle multi-criteria mission planning and control system for autonomous underwater vehicles

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Abstract: In their study, Tavana and Bourgeois (2010) proposed a multi-criteria decision analysis model that considered dynamic and episodic phenomenon on the surface of the ocean and provided specific navigation plans. They showed that the transect for an autonomous underwater vehicle (AUV) includes not only the desired horizontal path, but also the depth range that the vehicle will operate in. They argued that the current models should be extended to include considerations that change vertically, that is, with different ocean depths. They also suggested expansion of the current models to cooperative teams of AUVs and showed that working together will allow underwater vehicles to complete tasks that could not be completed by a single vehicle. This study extends their model by: reducing the number of judgements required to generate the navigation scores within the decision region; considering ocean depth and finding the navigation scores for the ocean surface and interior; and developing a model which derives an optimal allocation of multiple vehicles to various locations within the decision region. The proposed framework is an efficient multi-vehicle multi-criteria mission planning and control system that considers ocean phenomenon on the surface and in the interior and provides an optimal allocation of vehicles with respect to the stated *objective* and *subjective* mission goals.

Keywords: AUV; autonomous underwater vehicles; mission planning and control; fuzzy sets; multi-criteria decision analysis; multi-vehicle system; ANP; analytic network process.

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1 Introduction

This study extends the mission planning decision support system (DSS) developed by Tavana and Bourgeois (2010). In their study, they proposed a multi-criteria DSS that provided reasonable and in-context navigation plans for autonomous underwater vehicles (AUVs) by considering dynamic and episodic ocean phenomenon. They used multi-criteria decision analysis, analytic network process (ANP) and fuzzy sets to reduce the vehicle routing solution space and maximise time-on station in adverse environments with respect to the stated objective and subjective mission goals. Their DSS provided mission planners with the navigation points and depths; given the present vehicle state and sampling objectives; and, based on a series of objective and subjective goal factors. The navigation points were represented by a vector of velocity $(v_t = [x_t \;\; y_t \;\; z_t])$ and time (t) measured in minutes since start of mission. Three buoys: x, y and z (latitude, longitude and depth) represented the horizontal and vertical positions of the vehicle. Latitude and longitude were measured in degrees, and depth was measured in meters. They considered a comprehensive set of mission planning and control factors and classified them into environmental and non-environmental factors. Environmental factors included static and dynamic factors. Non-environmental factors were divided into safety, priority and cost. The priorities of the environmental and non-environmental factors were obtained by ANP. For complete treatment see Tavana and Bourgeois (2010), Saaty (2005) and Saaty and Ozdemir (2005). In their model, Tavana and Bourgeois (2010) considered only the current at the surface of the ocean. However, transect for an underwater vehicle includes not only the desired horizontal path, but also the depth range that the vehicle will operate in. In addition, working together will allow underwater vehicles to complete tasks that could not be completed by a single vehicle.

The most important disadvantage of the ANP method is that it does not consider uncertain human judgements. To cope with this problem, the fuzzy ANP method can be used (Yu and Tzeng, 2006). Fuzzy ANP has been the subject of several research projects in location planning (Guneri et al., 2009), research and development project selection (Mohanty et al., 2005), enterprise resource planning (Ayag and Ozdemir, 2007) and risk analysis (Dagdeviren et al., 2008). A series of importance weights and scores is used in our model which may require defuzzification. Defuzzification is the process of producing a quantifiable result in fuzzy logic. Some examples include multiple importance weights or scores provided by multiple decision makers (DMs) or multiple weights or scores provided by a single DM who is unsure about his or her judgement. Defuzzification is the translation of linguistic or fuzzy values into numerical, scalar and crisp representations. The process of condensing the information captured by fuzzy sets into numerical values is similar to that of the transformation of uncertainty-based concepts into certainty-based concepts. Intuitively speaking, the defuzzification process here is similar to an averaging procedure. Special defuzzification methods can be used to increase the numerical efficiency and transparency of the computations.

The research on the conjoint application of fuzzy sets and probability theory reports on several studies including marine and offshore safety assessment (Eleye-Datubo et al., 2008), financial modelling (Muzzioli and Reynaerts, 2007), information systems (Intan and Mukaidono, 2004), auditing (Friedlob and Schleifer, 1999), manufacturing cost estimation (Jahan-Shahi et al., 1999) and water quality management (Benoit, 1994). Many defuzzification techniques have been proposed in the literature. The most commonly used method is the centre of gravity (COG). Other methods include: random choice of maximum, mean of maxima, basic defuzzification distributions, generalised level set defuzzification, semi-linear defuzzification, centre of area and fuzzy clustering defuzzification. Roychowdhury and Pedrycz (2001) and Dubois and Prade (2000) provide excellent reviews of the most commonly used defuzzification methods.

The literature reports on several aggregation functions in multi-criteria decision analysis (Ali and Zhang, 2001; Roychowdhury and Pedrycz, 2001; Runkler, 1996; Van Leekwijk and Kerre, 1999). The selection of a specific aggregation function must be based on the problem characteristics and model requirements. While the selection of an aggregation operation is context dependent, it is recommended to consider the criteria suggested by Klir und Yuan (1995).

The research on multiple vehicle planning and control system reports on several optimisation studies in target assignment where optimum assignment of multiple vehicles to multiple targets is determined by maximising objective functions such as the expected number of targets destroyed (Beard et al., 2002). Literature surveys are available on the subject (Murphey, 1999; Voss, 1999). We use a dynamic approach as suggested by Tavana et al. (2008 and 2009) where the outcome of previous engagements is used in making future assignments. The remainder of the paper is organised as follows. Section 2 presents a detailed description of our models followed by an illustration of a mission planning example in Section 3 and conclusions in Section 4.

2 Models and procedures

To navigate through the environment, we use a grid-based approach for map representation and modelling the environment and partition the geographical map of the area into uniform grid squares referred to as 'cells'. This mapping approach is easy to build and facilitates the computation of the shortest path with the Voronoi graph (Aurenhammer and Klein, 2000; Bailey et al., 2006). The grid-based approach is the most commonly used map representation for AUV navigation (Dissanayake et al., 2001). The size of the cells is dependent on the mission objectives and requirements. The cells within a specific mission operations region are scored according to the environmental and non-environmental factors identified by Tavana and Bourgeois (2010). Different factors require different scoring schemes. All the relevant factors in our model are divided into continuous and discrete factors. Eventually, all scores are transformed into a 0–10 scale for meaningful comparisons. Lower scores are more desirable than higher scores in the transformed scale. Scale transformation adjusts the range of data, rather than the centre of the distribution. This eliminates differences in the range of scores and makes data more comparable across arrays.

2.1 The scoring system

We use COG in our model which is highly popular and is often used as a standard defuzzification method to calculate the centroid of a distribution function. Defining W_{ij} as the weight of factor j defined by DM i $(i = 1, 2, \ldots, I; j = 1, 2, \ldots, J)$ and S_{ij}^{m} as the score of jth factor for cell m provided by the ith DM $(i = 1, 2, \ldots, I; j = 1, 2, \ldots, J; m = 1, 2, \ldots, M)$; we find R^m, the 'navigation index' of the mth cell. With factor weights as membership grades, $\mu_{ij}(S_{ij}^{m}) = W_{ij}$, Equations (1) and (2) are used to aggregate the jth factor weights and scores given by I DMs for cell m:

$$\mathrm{COG}_j^m = \frac{\sum_{i=1}^{I} S_{ij}^m \mu(S_{ij}^m)}{\sum_{i=1}^{I} \mu(S_{ij}^m)} \tag{1}$$

$$R^m = \sum_{j=1}^{J} \mathrm{COG}_j^m \text{ where } 0 \le R^m \le 100 \tag{2}$$

2.2 Solution space reduction model

The first model proposed in this study reduces the number of judgements required to generate the navigation score of cells within the decision region. We first measure the cells on the surface of the ocean and then extend the model to consider depth. The continuous factors are denoted by $\overline{S}$, so that $\overline{S}_{i,j}^{m,n}$ denotes the value of the *j*th continuous factor by the *i*th DM for the cell (x_m, y_n). We assume that there are a total of J_C continuous factors where $j = 1, 2, \ldots, J_C$. The discrete factors are denoted by $\overline{\overline{S}}$, so that $\overline{\overline{S}}_{i,j}^{m,n}$ denotes the value of the *j*th discrete factor for the *i*th DM for the cell (x_m, y_n).

The discrete factors are measured either in a binary representation or a three-state representation. For example, a binary representation is used to measure the man-made boundaries of a cell; a '1' indicates 'presence' of man-made boundaries and a '0' indicates 'absence' of man-made boundaries. A similar representation is used to indicate the presence or absence of natural boundaries in a cell. A three-state representation is used to measure ocean currents; a '–1' indicates 'assisting currents', a '0' indicates 'no currents' and a '+1' indicates 'conflicting currents'. $\overline{\overline{S}}_{i,j}^{m,n}$, the scores for the discrete factors are calculated the same way as before.

The continuous factors are measured using a 0–10 scaled grading scheme; perceived safety, transit distance and cost, and time off-station are examples of factors that are measured with scaled grading. To reduce the number of judgement points, the ocean space is divided into a lattice of squares, such that the distance of the length and width of each square is denoted by *d*. As shown in Figure 1, the number of cells contained in each square depends on the value of *d*.

$\overline{S}_{i,j}^{m,n}$, the scores for the continuous factors are calculated differently to reduce the number of judgements made by the DMs. The general point of the lattice is (x_k, y_l), where, $x_k = x_0 + kd$, $y_l = y_0 + ld$, $d =$ length and width of each square and (x_0, y_0) is the initial reference point. Figure 2 shows how a particular cell, (x_m, y_n), would be contained in a particular square whose corner points are $(x_k, y_l), (x_k, y_{l+1})$, (x_{k+1}, y_{l+1}) and (x_{k+1}, y_l).

The score for each lattice point in Figure 1 is calculated as before $(\overline{S}_{i,j}^{k,l})$. Now let (x_m, y_n) be an arbitrary point. The procedure for calculating $\overline{S}_{i,j}^{m,n}$ is as follows. First, we find the square of the lattice that contains the cell (x_m, y_n). The corner points of the square are $(x_k, y_l), (x_k, y_{l+1}), (x_{k+1}, y_{l+1})$ and (x_{k+1}, y_l); where, $x_k \leq x_m \leq x_{k+1}$ and $y_l \leq y_n \leq y_{l+1}$. Note that $x_{k+1} = x_{k+d}$ and $y_{l+1} = y_{l+d}$. We calculate $\overline{S}_{i,j}^{m,n}$ as follows:

$$
\begin{aligned}
\overline{S}_{i,j}^{m,n} &= \frac{(x_m - x_k)}{d}\left[\frac{(y_{l+1} - y_n)}{d}\overline{S}_{i,j}^{k,l+1} + \frac{(y_n - y_l)}{d}\overline{S}_{i,j}^{k,l}\right] \\
&\quad + \frac{(x_{k+1} - x_m)}{d}\left[\frac{(y_{l+1} - y_n)}{d}\overline{S}_{i,j}^{k+1,l+1} + \frac{(y_n - y_l)}{d}\overline{S}_{i,j}^{k+1,l}\right] \\
\overline{S}_{i,j}^{m,n} &= \frac{(x_m - x_k)}{d^2}\left[(y_{l+1} - y_n)\,\overline{S}_{i,j}^{k,l+1} + (y_n - y_l)\,\overline{S}_{i,j}^{k,l}\right] \\
&\quad + \frac{(x_{k+1} - x_m)}{d^2}\left[(y_{l+1} - y_n)\,\overline{S}_{i,j}^{k+1,l+1} + (y_n - y_l)\,\overline{S}_{i,j}^{k+1,l}\right]
\end{aligned} \tag{3}
$$

where $\overline{S}_{i,j}^{k,l}, \overline{S}_{i,j}^{k,l+1}, \overline{S}_{i,j}^{k+1,l}$ and $\overline{S}_{i,j}^{k+1,l+1}$ represent the scores of the corner points of the square.

Figure 1 The lattice structure for the ocean surface (see online version for colours)

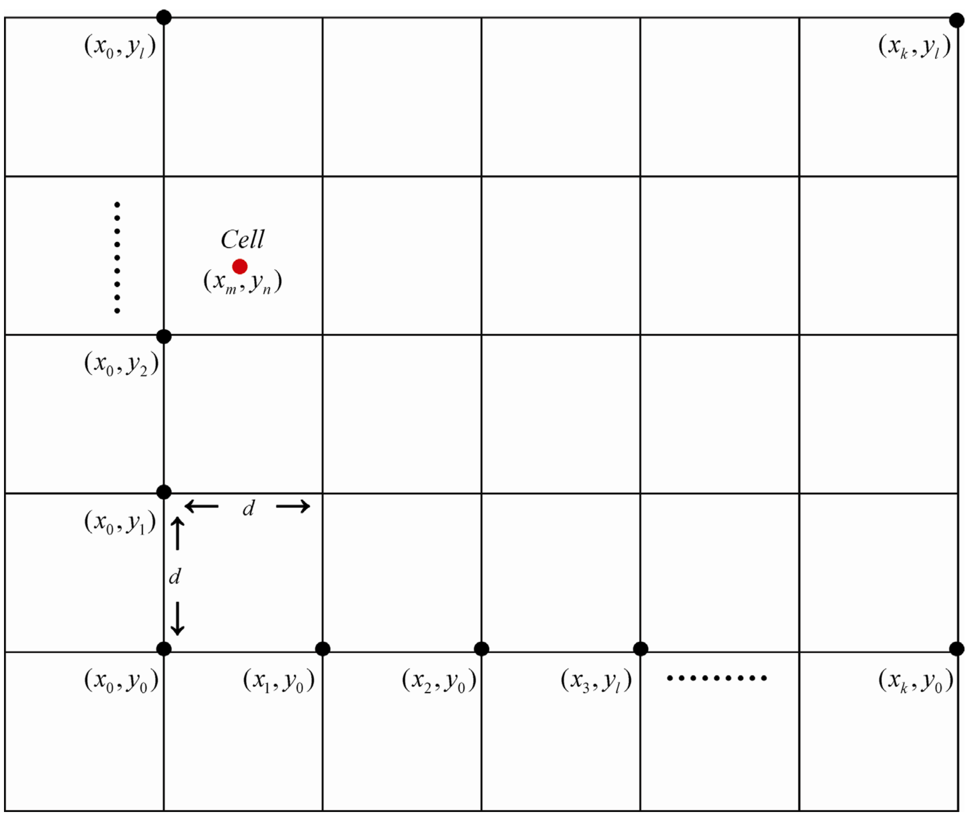
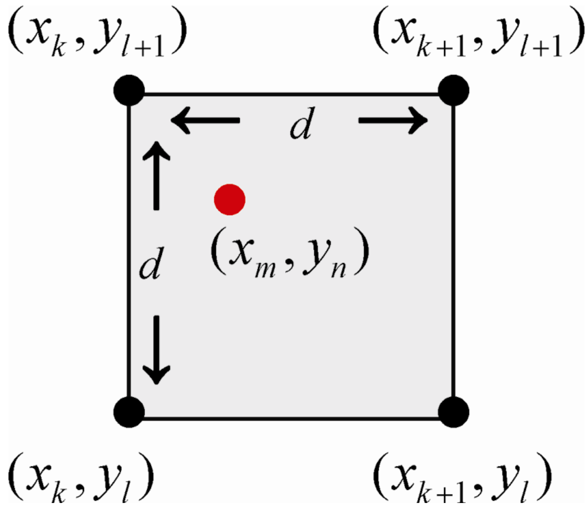


Figure 2 The corner points for all (x_m, y_n) (see online version for colours)



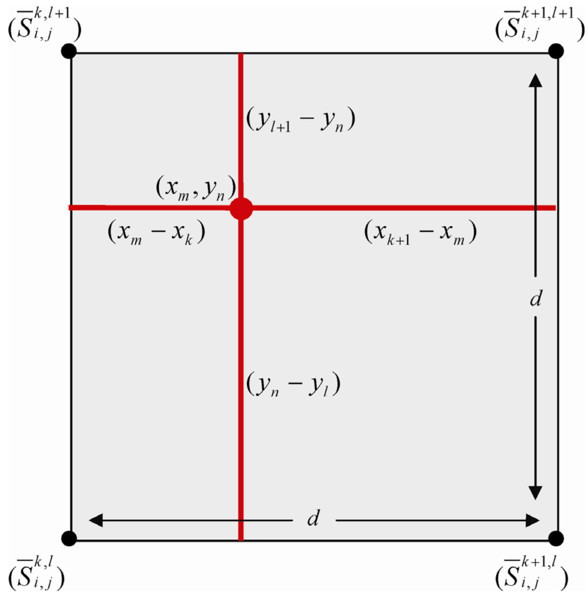
The overall goal is to calculate the score of the cell containing (x_m, y_n) as a weighted sum of the scores of the corner points using the proportional distance of the cell to the corner points as the weight. Equation (3) shows the formula for the score of cell (x_m, y_n) based upon this notion of proportional distance. Figure 3 shows the values for the various distances used in the equation. This model drastically reduces the number of judgements required by the DMs since they only have to make judgements for the scores of the corner points of the cells in the lattice. For example, if we assume that each cell has a width of 1 with $d = 5$, then there would be a total of 25 cells in each square reducing the number of judgements from 25 to 4 (a reduction of more than 80%).

The value of d depends on the amount of variation between the scores of the cells that are within a certain range of each other. For example, one might state that d could be chosen so that the value of the coefficient of variation of the scores of the cells that are within d units of each other (either horizontally or vertically) is less than 30%. A sampling procedure could be utilised to estimate the coefficient of variation of the scores of the cells within certain horizontal and vertical distances of each other. We should note that the total lattice does not have to conform to a complete square or rectangle, but, can be shaped in the form of various rectangles pieced together to reflect the part of the ocean that is considered in the mission planning and control system. The underlying component of these rectangles would be the lattice of squares.

Once the scores of each cell are computed, the overall index for each cell for continuous factor j is calculated by aggregating the scores for all of the DMs. The overall index for point (x_m, y_n) on the jth continuous factor is:

$$\overline{\mathrm{COG}}_j^{m,n} = \frac{\sum_{i=1}^{I} \overline{S}_{i,j}^{m,n}\, \mu\left(\overline{S}_{i,j}^{m,n}\right)}{\mu\left(\overline{S}_{i,j}^{m,n}\right)} \tag{4}$$

Figure 3 Diagram representing Equation (3) (see online version for colours)



Note that the overall index for the discrete scores is computed similarly. The overall index for point (x_m, y_n) on the jth discrete factor is:

$$\overline{\overline{\mathrm{COG}}}_j^{m,n} = \frac{\sum_{i=1}^{I} \overline{\overline{S}}_{i,j}^{m,n} \mu\left(\overline{\overline{S}}_{i,j}^{m,n}\right)}{\mu\left(\overline{\overline{S}}_{i,j}^{m,n}\right)} \tag{5}$$

The overall navigation index for the cell (x_m, y_n) is then computed as the sum of the indices for all the factors (i.e. the continuous factors and the discrete factors). The overall navigation index for point (x_m, y_n) is:

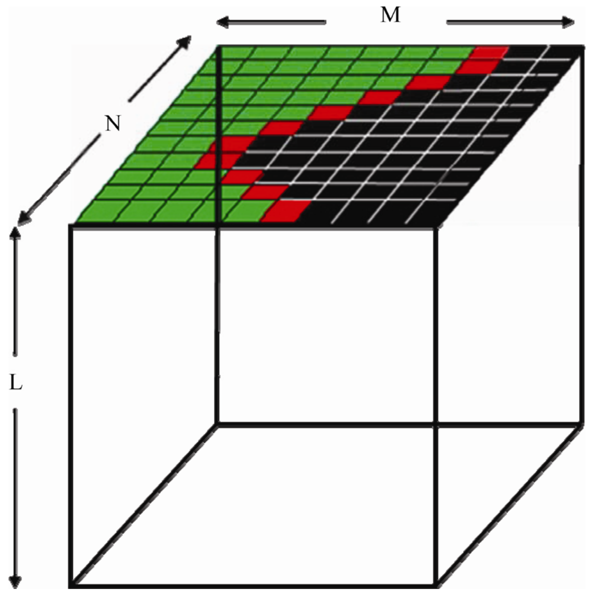
$$R^{m,n} = \sum_{j=1}^{J_C} \overline{\mathrm{COG}}_j^{m,n} + \sum_{j=1}^{J_B} \overline{\overline{\mathrm{COG}}}_j^{m,n} \tag{6}$$

2.3 Depth consideration model

The second model proposed in this study considers ocean depth and finds the score not only for any cell on the ocean surface, but also for any cell within the interior of the ocean. The ocean is divided into a series of cells (*m*, *n*, *l*) as depicted in Figure 4.

Initially, the previously described method is used to find the scores for the cells on the ocean surface as well as the cells on the ocean floor. $\overline{S}_{i,j}^{m,n,1}$ denotes the scores for the cells on the ocean surface. Using the method described earlier, the ocean surface is divided into a lattice of squares, and the DMs would only have to make judgements on the corner points of the squares of the lattice.

Figure 4 The cell structure for the total ocean space (see online version for colours)



A similar method would be used to compute the scores for the cells on the ocean floor. $\overline{S}_{i,j}^{m,n,L}$ denotes the scores for the cells on the ocean floor. The ocean floor is divided into a lattice of squares and the DMs would only have to make judgements on the corner points of the squares of the lattice. Once the scores for the cells on the ocean surface and the ocean floor are computed, the score for an arbitrary cell in the interior of the ocean is computed as a linear function of the corresponding cells on the ocean surface and the ocean floor using the procedure. Let $\overline{S}_{i,j}^{m,n,l}$ denotes the score of the *j*th continuous factor for the *i*th DM for cell (m, n, l) ($m = 1, \ldots, M$; $n = 1, \ldots, N$; $l = 1, \ldots, L$). Note that $\{(m,n,1); m = 1, \ldots, M; n = 1, \ldots, N\}$ consists of the cells on the ocean surface. Furthermore, let $\overline{S}_{i,j}^{m,n,L}$ denotes the score of the *j*th continuous factor for the *i*th DM for cell (m, n, L). Note that $\{(m,n,L); m = 1,\ldots,M; n = 1,\ldots,N\}$ consists of the cells on the ocean floor. $\overline{S}_{i,j}^{m,n,1}$ and $\overline{S}_{i,j}^{m,n,L}$ are calculated as before where a lattice of squares are created for the ocean surface and floor. $\overline{S}_{i,j}^{m,n,l}$ is calculated as a linear function of l:

$$\overline{S}_{i,j}^{m,n,l} = S_{m,n,1} + \frac{(l-1)}{(L-1)}\left(S_{m,n,L} - S_{m,n,1}\right) \tag{7}$$

Let $\overline{\overline{S}}_{i,j}^{m,n,l}$ be a constant binary factor. The constant binary factors are the ones that do not change as we move into the interior of the ocean (i.e. the value of this factor for an arbitrary cell is the same as the corresponding cell on the ocean surface directly above it). Assume that there are a total of J_{CB} constant binary factors ($j = 1, \ldots, J_{\text{CB}}$). Then,

$\overline{\overline{S}}_{i,j}^{m,n,l} = \overline{\overline{S}}_{i,j}^{m,n,1}$ for all $l = 1, \ldots, L$. $\overline{\overline{S}}_{i,j}^{m,n,1}$ is given as before for each cell (m, n, 1) on the surface of the ocean.

Let $\overline{\overline{\overline{S}}}_{i,j}^{m,n,l}$ be a non-constant binary factor.

Assume that there are a total of J_{NCB} non-constant binary factors $(j = 1, \ldots, J_{\text{NCB}})$. The non-constant binary factors are the ones that can change as we move into the interior of the ocean. We assume only one change can occur from a given cell on the ocean surface down to the corresponding cell on the ocean floor. For each m, n, there exists $L_{m,n}^{*}$ such that $\overline{\overline{\overline{S}}}_{i,j}^{m,n,l} = \overline{\overline{\overline{S}}}_{i,j}^{m,n,1}$ for $l \le L_{m,n}^{*}$ and $\overline{\overline{\overline{S}}}_{i,j}^{m,n,l} = 1 - \overline{\overline{\overline{S}}}_{i,j}^{m,n,1}$ for $l > L_{m,n}^{*}$ (Figure 5). The concept of constant and non-constant binary factors can be generalised to the case of factors that have a three-state representation

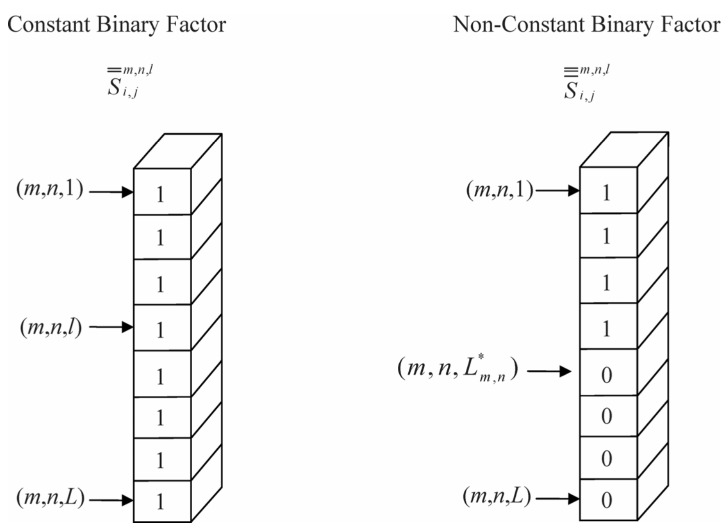
$\overline{\text{COG}}_{j}^{m,n,l}$ is the aggregate score for (m, n, l) for the jth continuous factssor:

$$\overline{\text{COG}}_{j}^{m,n,l} = \frac{\sum_{i=1}^{I} \overline{S}_{i,j}^{m,n,l}\, \mu\left(\overline{S}_{i,j}^{m,n,l}\right)}{\sum_{i=1}^{I} \mu\left(\overline{S}_{i,j}^{m,n,l}\right)} \tag{8}$$

The aggregate score for (m, n, l) for the jth constant binary factor is:

$$\overline{\overline{\text{COG}}}_{j}^{m,n,l} = \frac{\sum_{i=1}^{I} \overline{\overline{S}}_{i,j}^{m,n,l}\, \mu\left(\overline{\overline{S}}_{i,j}^{m,n,l}\right)}{\sum_{i=1}^{I} \mu\left(\overline{\overline{S}}_{i,j}^{m,n,l}\right)} \tag{9}$$

Figure 5 The cell structure for Constant and non-constant binary factors



The aggregate score for (m, n, l) for the jth non-constant binary factor is:

$$\overline{\overline{\overline{\mathrm{COG}}}}_{j}^{m,n,l} = \frac{\sum_{i=1}^{I} \overline{\overline{S}}_{i,j}^{m,n,l} \; \mu\left(\overline{\overline{S}}_{i,j}^{m,n,l}\right)}{\sum_{i=1}^{I} \mu\left(\overline{\overline{S}}_{i,j}^{m,n,l}\right)} \tag{10}$$

The navigation index for (m, n, l) is:

$$R^{m,n,l} = \sum_{j=1}^{J_{\mathrm{C}}} \overline{\mathrm{COG}}_{j}^{m,n,l} + \sum_{j=1}^{J_{\mathrm{CB}}} \overline{\overline{\mathrm{COG}}}_{j}^{m,n,l} + \sum_{j=1}^{J_{\mathrm{NCB}}} \overline{\overline{\overline{\mathrm{COG}}}}_{j}^{m,n,l} \tag{11}$$

$R^{m,n,l}$ is the sum of the aggregate scores for the total of the continuous factors, constant binary factors and non-constant binary factors.

2.4 Multiple vehicle model

Next, we develop a model which derives an optimal allocation of M vehicles to various parts of the ocean. The ocean is divided into a collection of n parts, and each part contains a group of cells. Each part is assumed to have three dimensions: length, width and depth. The objective of this model is to find the allocation that maximises the total value (TV). TV is assumed to be equal to the sum of the values of the individual parts of the ocean. The value of an individual part is equal to the utility of assigning i vehicles to that part times the navigation index for that part.

The following shows the formula for the navigation index for part j when v_j vehicles are assigned to part j. The navigation index is basically the sum of the navigation indices of all the cells in that part to which a vehicle has been assigned. It is assumed that the v_j vehicles are assigned to the cells in part j that have the highest navigation indices.

Let us divide the ocean floor into n parts P_j where $j = 1, \ldots, n$. Assuming that M is the total number of vehicles that should be allocated and v_j is the number of vehicles allocated to part j, $\sum_{j=1}^{n} v_j = M$. Further defining $R_{v_{j,j}}^{m}$ as the overall navigation index for part j when v_j vehicles are allocated to part j, and $R_{v_{j,j}}^{m}$ as the highest possible sum of the navigation indices of v_j vehicles allocated to part j, we find the navigation index for part j when v_j vehicles are assigned to part j as:

$$R_{v_{j,j}}^{m} = \sum_{i=1}^{v_j} R_{i,j}^{m} \tag{12}$$

where $R_{1,j}^{m}$ is the highest navigation index of all the cells in part j, $R_{2,j}^{m}$ is the second highest navigation index of all the cells in part j and $R_{i,j}^{m}$ is the ith highest navigation index of all the cells in part j.

Let us further define $U_{i,j}$ as the utility of having i vehicles assigned to part j; the utility of having i vehicles assigned to part j is an increasing function of i and the utility measures the overall value of having different number of vehicles in various parts of the ocean. The TV is calculated as:

$$\mathrm{TV} = \sum_{j=1}^{n} U_{v_{j,j}} R_{v_{j,j}} \tag{13}$$

where $U_{v_{j,j}} R_{v_{j,j}}$ represents the value of part *j* when v_j vehicles are assigned to part *j*. Equation (13) provides the TV of assigning v_j vehicles to part *j*. Notice that the TV is the sum of the values of the individual parts. The overall problem is to find the optimal allocation of the vehicles to the parts in order to maximise the TV. In other words, the problem is to find the set $v_1, \ldots, v_n$ which maximises:

$$\mathrm{TV} = \sum_{j=1}^{n} U_{v_{j,j}} R^{n}_{v_{j,j}} \tag{14}$$

such that $\sum_{j=1}^{n} v_j = M$.

We can find the optimal $v_1, \ldots, v_n$ by computing the TV for all possible combinations of allocation of vehicles to the parts. The following heuristic is used to solve this problem and reduce the number of computations. We allocate each vehicle one step at a time so that the next vehicle is allocated to give the greatest increase in the TV. The vehicles are allocated in a series of iterations; $j^*(i)$ represents the part that the vehicle will be assigned to at the next iteration $(i+1)$. The formula for $j^*(i)$ is given by Equation (15). The allocations for iterations $(i+1)$ are given by Equation (16). The process is repeated until all of the vehicles are allocated. Assuming that the marginal increase of $U_{v_{j,j}}$ with respect to v_j decreases as v_j increases, let $v_1^i, \ldots, v_n^i$ be the allocation of vehicles to the *n* parts at the *i*th iteration. We begin with $v_i^1 = 0$, $i = 1, \ldots, n$; where no vehicles are allocated in the first iteration. Given that only one more vehicle is assigned to a part at each iteration; at iteration *i*, let $j^*(i)$ represents the part that the vehicle will be assigned to at the next iteration ($i + 1$). The next vehicle will be assigned to the part that gives the greatest next increase in the values $j^*(i)$, and it is the value of $j = 1, \ldots, n$ which maximises:

$$U_{v_j^i+1,j} R_{v_j^i+1,j} - U_{v_j^i,j} R_{v_j^i,j} \tag{15}$$

Assuming that $v_i^1, \ldots, v_n^i$ are the allocation at iteration *i*; then, the allocations at iteration $i + 1$ are calculated as follows:

$$v_j^{i+1} = v_j^i + 1 \text{ if } j = j^*(i) \tag{16}$$

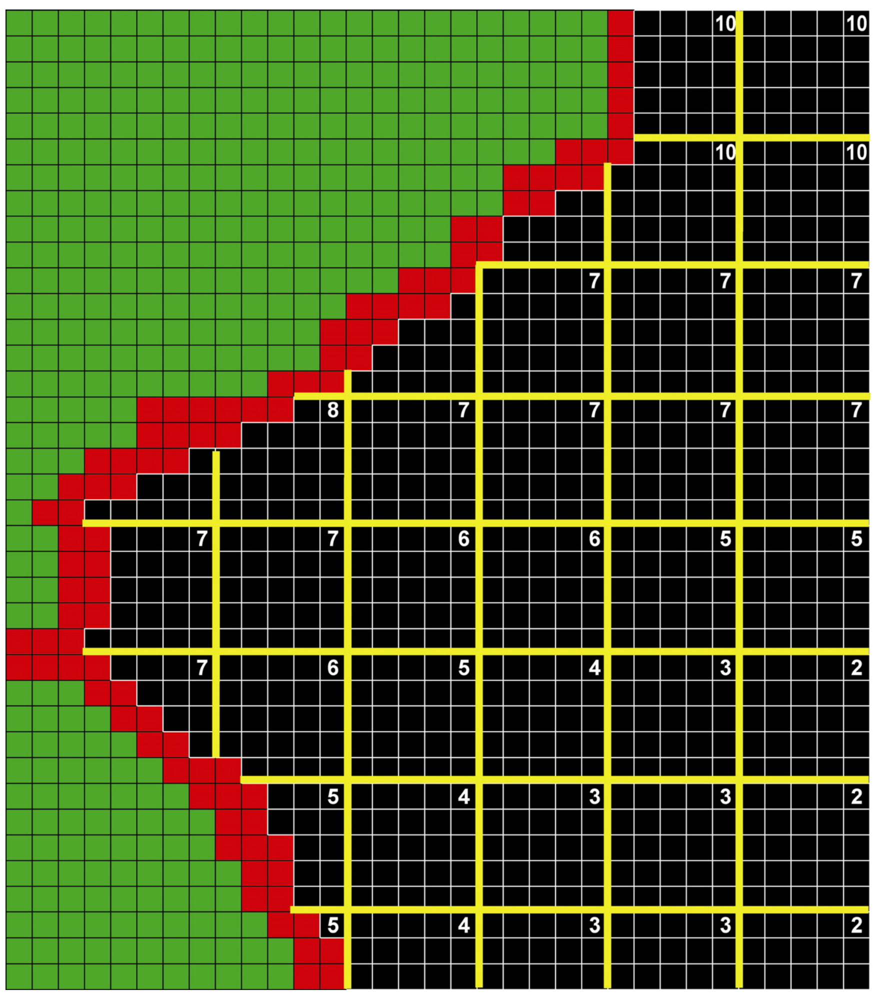
$$v_j^{i+1} = v_j^i \text{ if } j \neq j^*(i)$$

where *i* is increased by 1 ($i = i + 1$) and the process is repeated until all of the vehicles are allocated.

3 Mission planning example

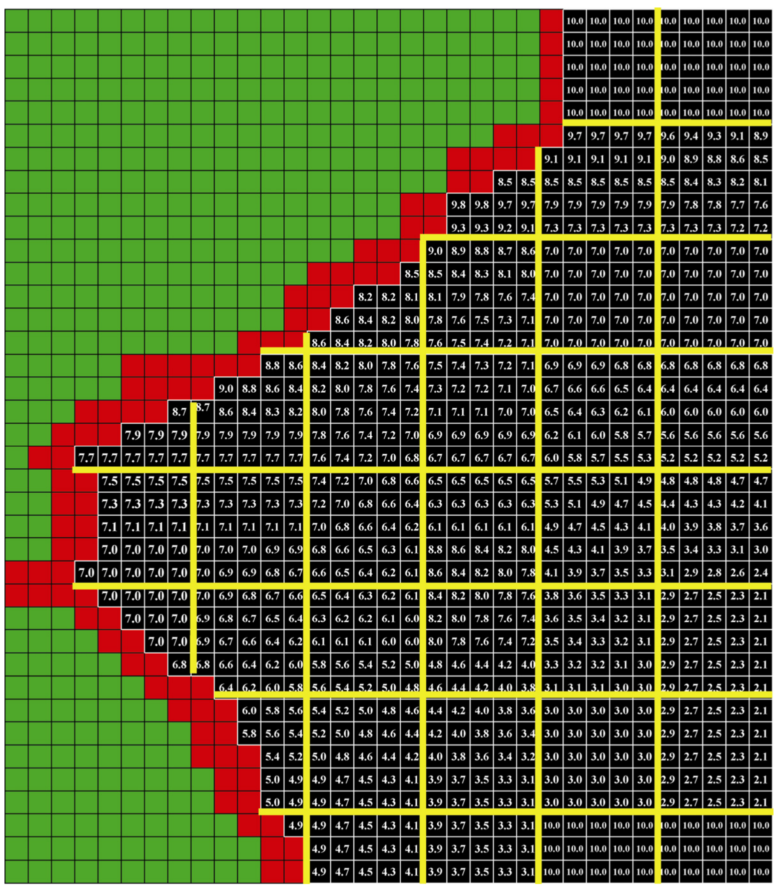
In this section, we describe a numerical example whose main purpose is to demonstrate the computational feasibility of our approach. We begin with models I and II. The total space under consideration is divided into a series of 30 squares lattices. Each square is composed of a length and width of five cells and each square contains 25 cells. Therefore, the total space is comprised of 750 cells. We use our model to reduce the number of judgements for each DM on each factor from 750 to 44. The judgements for all 750 cells can be obtained as a weighted average score of the judgements for the corner points of each cell. We keep the total number of judgements as low as possible by assuming that the corner point of each square is also used as a corner point for all the adjacent squares. As shown in Figures 6 and 7, this process was carried for both the ocean surface and the ocean floor.

Figure 6 Top of the ocean for Model 1 (see online version for colours)



The ocean is then assumed to consist of four depth levels: the surface, the floor and two additional levels in between. Once the judgements have been computed for the surface and the floor, a linear function is used to calculate the judgements for the middle cells. Using the method described above, we reduce the number of judgements required by a DM from 3,000 (four levels with 750 cells per level) to 88 (44 for the ocean surface and 44 for the ocean floor). Figures 8–11 show the results for the four levels. This method can be generalised to include any number of cells, any number of levels as well as different square sizes. The original 44 points were assigned as judgements for the lattice points for the ocean surface presented in Figure 6. Using Equation (3), the scores for all 750 cells on the ocean surface were calculated and presented in Figure 7. Similarly, we used the original 44 judgement points and calculated the scores for the ocean floor presented in Figure 8. These scores were used to calculate the scores for the 750 cells on the ocean floor as shown in Figure 9. The values for the depth levels were then calculated using Equation (7). The results for the second level from the top are given in Figure 10, and results for the third level from the top are given in Figure 11.

Figure 7 Safety Scores for DM1 for the top-level (see online version for colours)



Next, we use our numerical example to illustrate the process of assigning different vehicles to different sections of the ocean (Model III). Let us assume that we have a total of ten vehicles that must be assigned to seven sections of the ocean. Figure 12 shows how the ocean space is partitioned into seven sections. We assume that the same partition is applied throughout the ocean depth. As defined above, *U* is the utility of assigning *i* vehicles to section *j*. Table 1 shows the *U* values for this example.

As described earlier in the mathematical model, the problem is to find the optimal allocation of the vehicles to different sections in order to maximise the TV, which is the sum of the values for each section. The value of each section, *V*, is the utility of assigning *i* vehicles to that section times the navigation index for that section. We use the heuristic suggested earlier to find the optimal solution for this example. We begin, by identifying the ten highest navigation indices for the cells of each section as shown in Table 2.

Figure 8 Bottom of the ocean for Model 1 (see online version for colours)

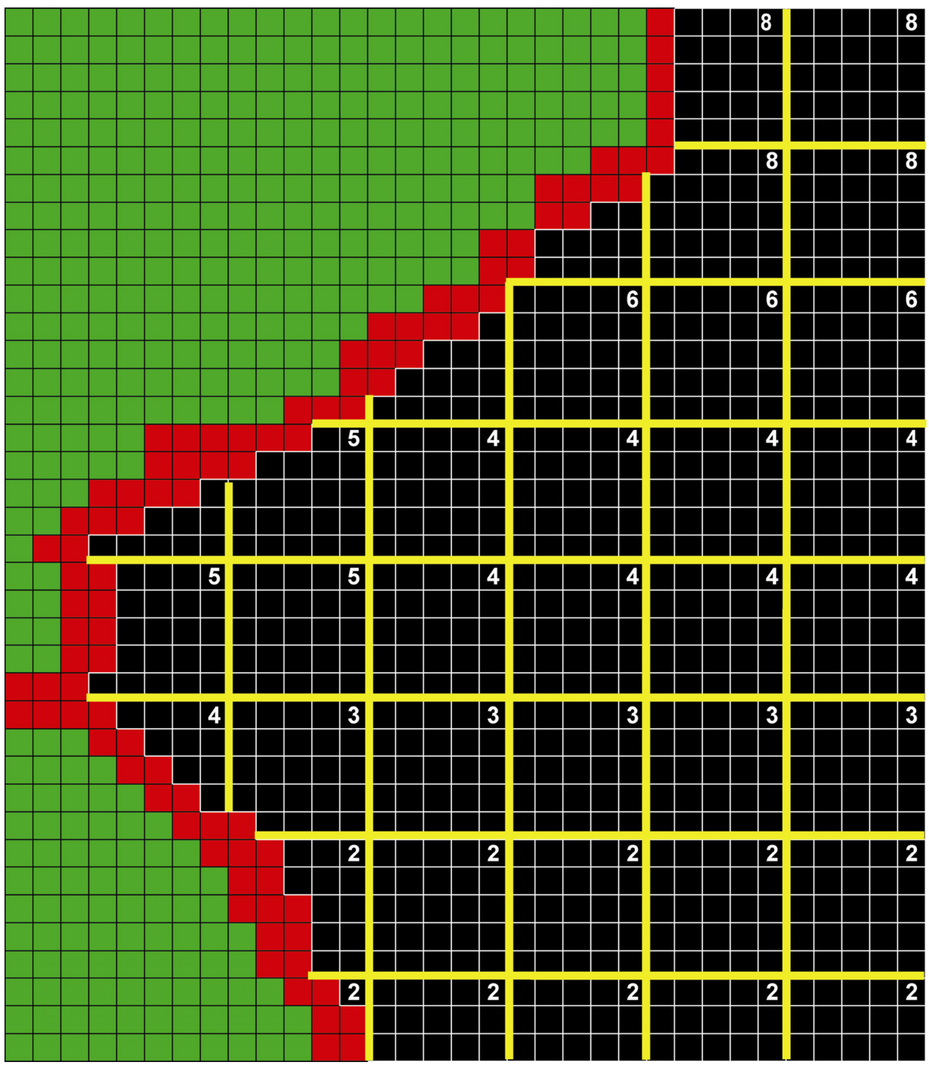


Table 1 Utility of assigning *i* vehicles to section *j*

Vehicle	*Section*						
	I	*II*	*III*	*IV*	*V*	*VI*	*VII*
1	0.50	0.70	0.60	0.30	0.40	0.50	0.80
2	0.60	0.80	0.70	0.40	0.50	0.55	0.85
3	0.70	0.80	0.70	0.50	0.60	0.60	0.90
4	0.75	0.80	0.70	0.55	0.60	0.65	0.95
5	0.80	0.80	0.70	0.60	0.60	0.70	0.95
6	0.85	0.80	0.70	0.60	0.60	0.70	0.95
7	0.90	0.80	0.70	0.60	0.60	0.70	0.95
8	0.90	0.80	0.70	0.60	0.60	0.70	0.95
9	0.90	0.80	0.70	0.60	0.60	0.70	0.95
10	0.90	0.80	0.70	0.60	0.60	0.70	0.95

Figure 9 Safety scores for DM1 for the bottom-level (see online version for colours)

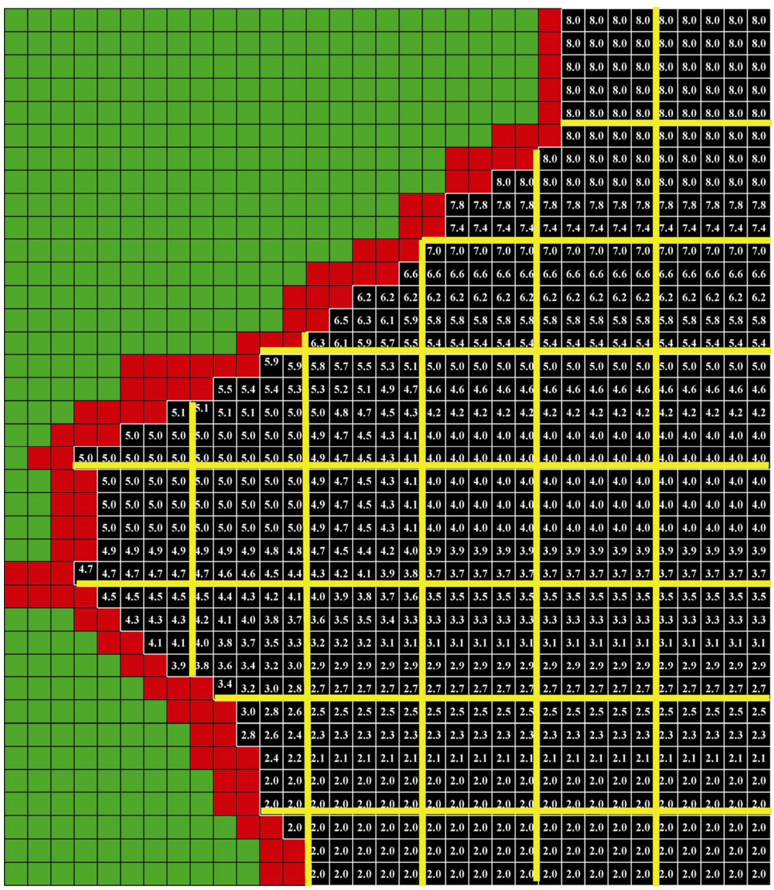
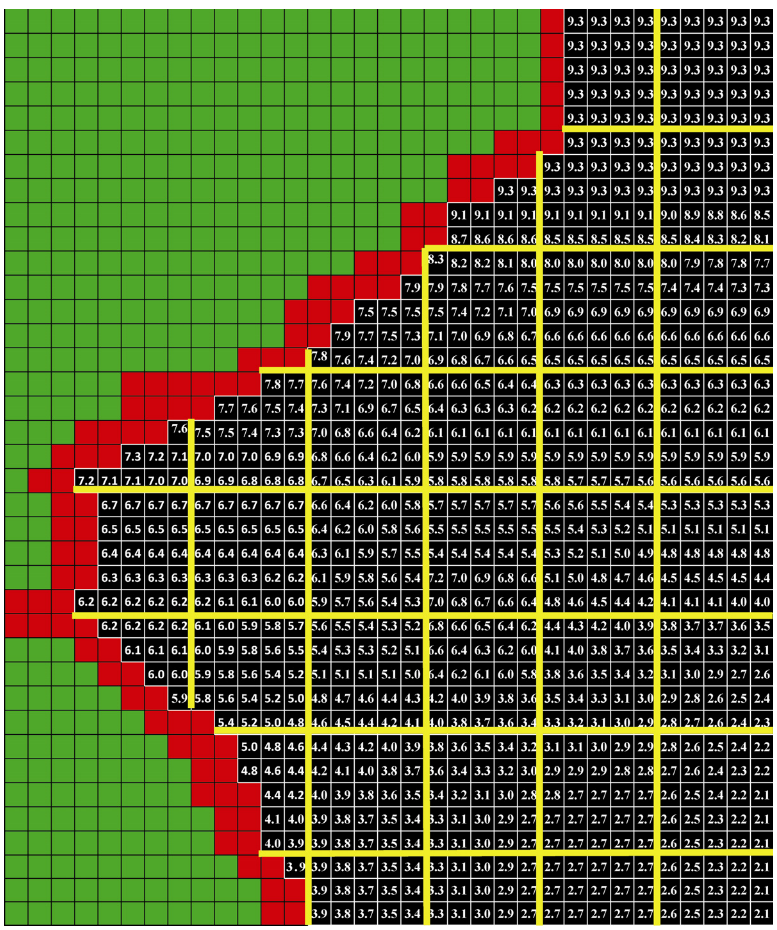


Table 2 Highest navigation indices for each section (highest to lowest)

Rank	*Section*						
	I	*II*	*III*	*IV*	*V*	*VI*	*VII*
1	10.20	45.20	50.30	12.10	60.40	45.10	32.10
2	9.80	42.30	45.20	10.10	58.10	44.30	31.80
3	7.10	41.20	34.20	9.10	56.10	43.20	30.20
4	5.10	40.10	33.20	8.10	55.10	42.30	29.20
5	4.10	39.80	32.10	7.80	53.20	41.70	28.30
6	3.20	28.10	31.20	6.90	52.10	39.80	27.80
7	2.80	26.50	30.30	5.30	51.10	35.10	26.20
8	1.80	23.30	30.20	3.10	49.10	33.10	25.10
9	1.20	22.20	28.20	2.10	47.20	32.30	24.20
10	1.10	21.10	27.10	1.30	46.10	31.30	23.10

Figure 10 Safety scores for DM1 for the second level from the top (see online version for colours)



We assign the first vehicle to the section that has the highest value, which is computed by multiplying the utility of assigning one vehicle to that section times the highest navigation index for that section. The following represents the results of these calculations:

Section I: 0.50 * 10.20 = 5.10

Section II: 0.70 * 45.20 = 31.64

Section III: 0.60 * 50.30 = 30.18

Section IV: 0.30 * 12.10 = 3.63

Section V: 0.40 * 60.40 = 24.16

Section VI: 0.50 * 45.10 = 22.55

Section VII: 0.80 * 32.10 = 25.68

The first vehicle will be assigned to Section II since it has the highest value. The second vehicle will be assigned to the section with the highest possible increase in TV assuming that the first vehicle is assigned to Section II. The following represents the calculations for assigning the second vehicle:

Section I: 0.50 * 10.20 = 5.10

Section II: 0.80 * (45.20 + 42.30) – (0.70 * 45.20) = 38.36

Section III: 0.60 * 50.30 = 30.18

Section IV: 0.30 * 12.10 = 3.63

Section V: 0.40 * 60.40 = 24.16

Section VI: 0.50 * 45.10 = 22.55

Section VII: 0.80 * 32.10 = 25.68

The second vehicle will be assigned to Section II which gives the greatest increase to the overall TV. Table 3 gives the results of the calculations for all of the vehicles.

The final result is to allocate five vehicles to Section II, two vehicles to Section III and three vehicles to Section VII. The results show an interesting interaction between the utility values and the navigation indices. It is possible for the TV to increase as other vehicles are added to a particular section even though the marginal utility of adding other vehicles may not increase. For example, the added value of assigning the first vehicle to Section II is 31.64, whereas the added value of assigning the second vehicle to Section II is 38.36. This points to the interaction effect between the utility and the navigation index, since the *TV* is defined as the product of these two components. Although there are other ways to define the *TV*, this approach shows that utility and the navigation index are two independent measures with possible interactions.

Table 3 Increase in total value

Vehicle	*Section*						
	I	*II*	*III*	*IV*	*V*	*VI*	*VII*
1	5.10	31.64	30.18	3.63	24.16	22.55	25.68
2	5.10	38.36	30.18	3.63	24.16	22.55	25.68
3	5.10	32.96	30.18	3.63	24.16	22.55	25.68
4	5.10	32.08	30.18	3.63	24.16	22.55	25.68
5	5.10	31.60	30.18	3.63	24.16	22.55	25.68
6	5.10	22.48	30.18	3.63	24.16	22.55	25.68
7	5.10	22.48	36.67	3.63	24.16	22.55	25.68
8	5.10	22.48	23.94	3.63	24.16	22.55	25.68
9	5.10	22.48	23.94	3.63	24.16	22.55	28.64
10	5.10	22.48	23.94	3.63	24.16	22.55	30.38

Figure 11 Safety scores for DM1 for the third level from the top (see online version for colours)

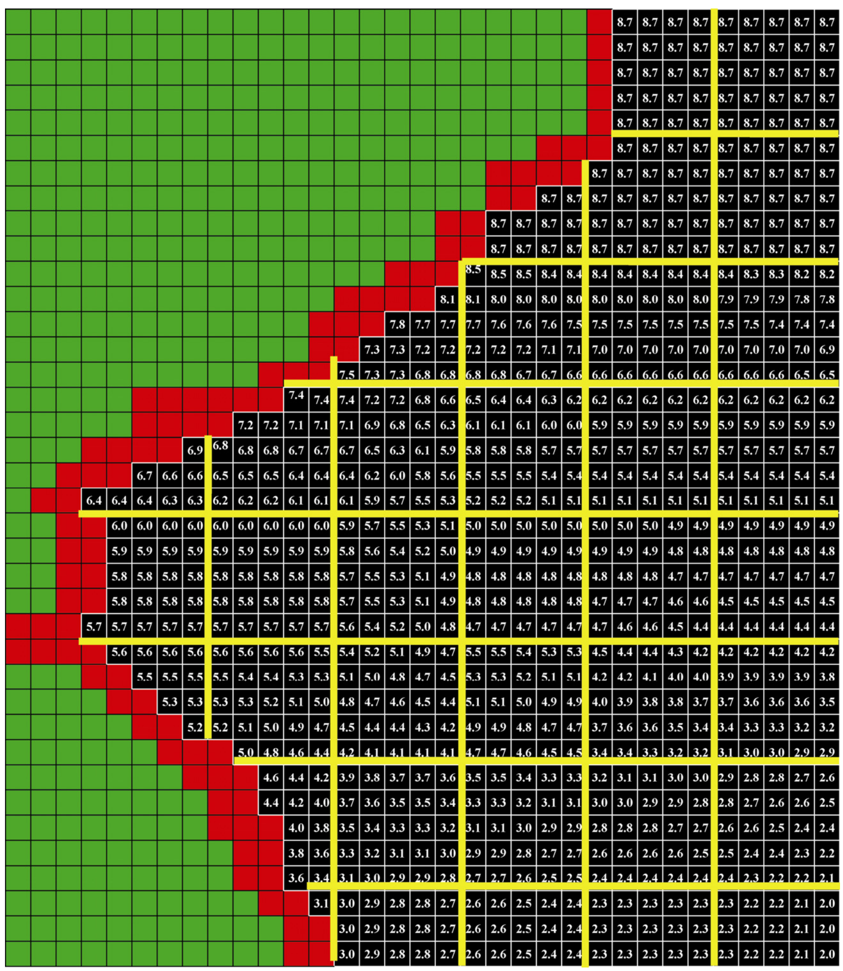
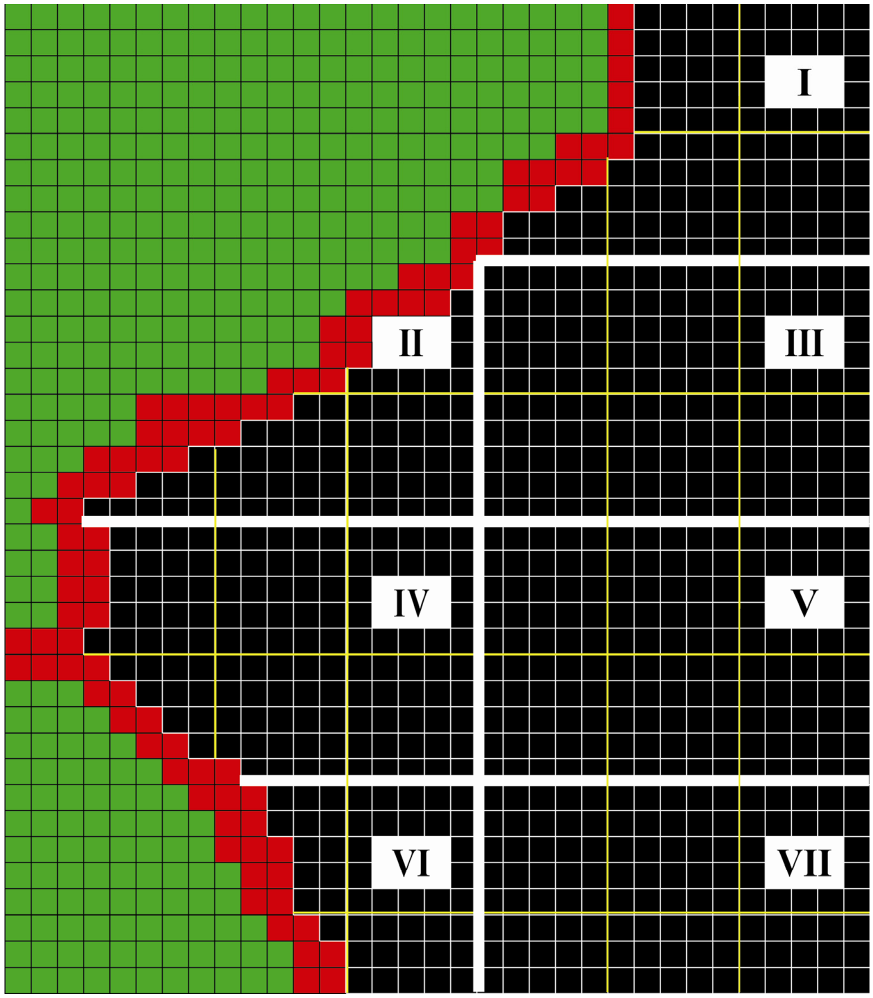


Figure 12 Model 3 (see online version for colours)



4 Conclusions

The growing use of AUVs is continually demonstrating new commercial and military applications. The military uses AUVs for undersea environmental sensing and mapping, intelligence, surveillance, reconnaissance and global war on terror. The oil and gas industry use AUVs to make detailed maps of the seafloor. Mining companies, unlike oil and gas companies, lacked the technology to search the ocean floor and haul their bounty to the surface. Now, the global boom in commodity prices has encouraged mining companies to utilise AUVs for undersea mining. Other applications of underwater vehicles include: search for fish, surveying underwater pipes and cables, search and rescue operations, coastal drug control, tracking of pollutant sources and ocean floor studies prior to sewage outlet construction.

The current static ‘human-centric’ mission planning and control system rely heavily on the execution of pre-planned actions limiting their effectiveness for measuring dynamic and episodic ocean phenomenon. Throughout the mission, AUVs receive new directions from the human operator on the surface or ashore that change the original mission. The plethora of data that must be considered, much of which varies with time as well as position, puts a tremendous amount of cognitive burden on operators. While on the surface, there is very little or no time for real-time data processing by a human operator. Current formal techniques consider constrained or simplified environment assumptions and typically fall short of satisfying necessary operational capabilities. The DSS proposed in this study is dynamic and considers realistic environments with real-world operational constraints and vehicle characteristics.

Tavana and Bourgeois (2010) showed that the transect for an underwater vehicle includes not only the desired horizontal path, but also the depth range that the vehicle will operate in. They argued that the current models should be extended to include considerations that change vertically, that is, with different ocean depths. They also suggested expansion of the current methods to cooperative teams of AUVs and showed that working together will allow underwater vehicles to complete tasks that could not be completed by a single vehicle. This study has extended their model by

1 reducing the number of judgements required to generate the navigation scores within the decision region

2 considering ocean depth and finding the navigation scores for the ocean surface and interior

3 developing a model which derives an optimal allocation of multiple vehicles to various locations within the decision region.

The proposed system integrates the measurement process with the decision and control process. The measurement process provides feasibility by using the factor scores to calculate the navigation indices which are the basis for assigning an individual vehicle to a particular cell, and the utility values which are the overall values for assigning multiple vehicles to the various parts of the ocean. These two measurement processes are then combined to form the basis of finding the optimal allocation of vehicles to different parts of the ocean. The proposed method suggests a group assignment that could be changed throughout the mission, as mission progress information is received. For instance, if the goals change during the mission, some vehicles may leave the original area to enter new territories while other vehicles may pick-up the remaining tasks left incomplete by departing vehicles. Cooperative AUVs would also allow a human operator to retask the group as a whole, rather than each vehicle individually, making it easier to manage the vehicles.

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Disclaimer

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