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A novel mixed binary linear DEA model for ranking decision-making units with preference information

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ABSTRACT

Several mixed binary linear programming models have been proposed in the literature to rank decision-making units (DMUs) in data envelopment analysis (DEA). However, some of these models fail to consider the decision-makers' preferences. We propose a new mixed binary linear DEA model for finding the most efficient DMU by considering the decision-makers' preferences. The model proposed in this study is motivated by the approach introduced by Toloo and Salahi (2018). We extend their model by introducing additional assurance region type I (ARI) weight restrictions (WRs) based on the decision-makers' preferences. We show that direct addition of assurance region type II (ARII) and absolute WRs in traditional DEA models leads to infeasibility and free production problems, and we prove ARI eliminates these problems. We also show our epsilon-free model is less complicated and requires less effort to determine the best efficient unit compared with the existing epsilon-based models in the literature. We provide two real-life applications to show the applicability and exhibit the efficacy of our model.

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes, Cooper, and Rhodes (1978), is a mathematical model used to evaluate the performance of several decision-making units (DMUs), which consume multiple inputs to produce multiple outputs. The model was initially developed for the constant returns-to-scale situation and was later modified by Banker et al. (1984) to accommodate the case of variable returns-to-scale. In time, DEA has proven to be an excellent tool for efficiency and performance measurement (Charles et al., 2018). A survey performed by Liu et al. (2013) found that DEA has been applied in a variety of fields, with prevalence in banking, healthcare, agriculture and farming, transportation, and education, which taken together makeup about 40% of all application-embedded papers.

The basic DEA models divide DMUs into two groups: efficient and inefficient. The relative efficiency scores of the efficient DMUs are equal to one, and the DMUs in the inefficient group have efficiency scores that are strictly less than one. In some cases, such as the supplier selection problem introduced by Toloo and Nalchigar (2011), we may identify several DMUs as efficient while we may be interested in finding the best (or most efficient) DMU. In this sense, the basic DEA suffers from not being able to rank all the DMUs completely (Liu and Wang, 2018). Adler et al. (2002) classify the DEA methods proposed to rank the performance of the DMUs into six groups including cross-efficiency (Sexton et al. 1986), super-efficiency (Andersen and Petersen, 1993), benchmarking (Torgersen et al., 1996), multivariate statistical techniques (Friedman and Sinuany-Stern, 1997), proportional measures of inefficiency (Bardhan et al., 1996), and multiple-criteria decision methodologies (Sinuany-Stern et al., 2000). Referring to all these methods would result in an overly long presentation, which is beyond the scope of the present paper (the interested readers may refer to the references mentioned above for further details).

Several mixed binary linear programming (MBLP) models have been proposed in the literature to rank DMUs in DEA. However, generally,

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these models fail to consider the decision-makers' preferences. Motivated by the importance of incorporating decision-makers' preferences in DEA, we build upon Toloo and Salahi (2018) and propose a new MBLP model for finding the most efficient DMU. For this purpose, we introduce additional assurance region type I (ARI) weight restrictions (WRs) based on the decision-makers' preferences. We show that direct addition of assurance region type II (ARII) and absolute WRs in traditional DEA models leads to infeasibility and free production problems. We prove that ARI eliminates these problems. We further show our epsilon-free model is less complicated and requires less effort to solve compared with the existing epsilon-based models in the literature.

The rest of this paper is organized as follows. In Section 2, we provide a snapshot of the literature concerned with ranking DMUs in DEA. In Section 3, we review the approach proposed by Toloo and Salahi (2018). In Section 4, we improve upon Toloo and Salahi's (2018) approach by considering the decision-makers' preferences and developing an algorithm to fully rank all efficient DMUs. In Section 5, we provide two real-life applications to validate the proposed model and further discuss managerial implications in Section 6. Conclusions are provided in Section 7.

2. Literature review

To find the best DMU using basic DEA models, such as the approach proposed by Andersen and Petersen (1993), we need to solve at least one model for each DMU. In time, however, research efforts have translated into the development of different MBLP models to find the best DMU by solving only one model. Amin and Toloo (2007) developed a MBLP model to find the best DMU. Their model is epsilon-based, so they also proposed a model to find an appropriate value for the epsilon that prevents the input and output weights from taking the zero value. Amin (2009) further proved that the model proposed by Amin and Toloo (2007) might lead to identifying more than one best DMU. Therefore, he developed a new mixed binary nonlinear programming (MBNLP) DEA model to obtain a single most efficient DMU. On the other hand, the model proposed by Amin and Toloo (2007) is suitable for the situation of constant returns-to-scale. Therefore, Toloo and Nalchigar (2009) extended it to the variable returns-to-scale situation. Foroughi (2011) argued that the model proposed by Amin (2009) might be infeasible in some cases, especially when there are several efficient DMUs. He presented a new MBLP model to identify the best DMU.

Toloo and Nalchigar (2011) extended the model proposed by Amin and Toloo (2007) to contain imprecise data, such as ordinal, interval, and ratio bound data and applied it to the supplier selection problem. Moreover, Asosheh et al. (2010) used the model proposed by Amin and Toloo (2007) to evaluate and rank information technology projects in the presence of imprecise data. In this sense, to consider imprecise data in the DEA model, they employed the approach proposed by Zhu (2003). The interested readers can refer to Ebrahimi and Toloo (2020) and Ebrahimi et al. (2018) for more information regarding the existing approaches for dealing with imprecise data in DEA.

Wang and Jiang (2012) further showed that the model proposed by Foroughi (2011) has several redundant constraints. They proposed new MBLP models to find the best DMU under different returns-to-scale conditions. Toloo (2012) also addressed some of the drawbacks of the previous approaches and presented a new MBLP to select the best DMU in variable returns-to-scale situations. Foroughi (2013) proposed a new algorithm to find all the efficient DMUs. In addition, he also developed a new model to determine the most efficient DMU among efficient DMUs.

Toloo (2014a) presented a new MBLP model to find a single most efficient DMU without explicit inputs and utilized it to find the best professional tennis player among 40 professional tennis players. Toloo (2014b) further developed an epsilon-free MBLP model to find the best DMU. Compared to the previous models, this model is concise, simple, and practical. On the other hand, Toloo and Kresta (2014) developed several DEA models to determine the best DMU without explicit outputs. They applied the models to evaluate the performance of 139 different alternatives for long-term asset financing provided by banks in the Czech Republic and found the best banks.

Toloo (2015) developed a new minimax MBLP model to find the best DMU under a common condition. Lam (2015) developed a new integrated DEA model to determine the most efficient DMU that has an objective similar to the super-efficiency model. Later, Salahi and Toloo (2017) proved that the model proposed by Lam (2015) uses an unsuitable value for the epsilon and so may fail to find the best DMU. In response, they developed a new model to determine the proper value for the epsilon. Toloo and Tavana (2017) developed a novel approach without explicit inputs and outputs to identify a single efficient DMU. Ebrahimi and Khalili (2018) developed a new MBLP imprecise model to determine the best efficient units in the presence of weight restrictions. Like the proposed approach by Amin and Toloo (2007), this model may also produce several best DMUs without finding the most efficient one.

More recently, Toloo and Salahi (2018) developed a new MBNLP DEA model to find the best DMU in two steps. In the first step, a nonlinear model is used to find a proper value for the epsilon. This value is then used in the second step to solve a nonlinear DEA model and find the best DMU. They converted the nonlinear models into equivalent linear DEA models. The authors also showed that their models have greater discriminatory power than the existing models.

At this point, the existing models have largely ignored taking into account the decision-makers' preferences. As a result, the optimal weights obtained through the above-mentioned models may be inconsistent with the decision-makers' views regarding the relative importance of the inputs and outputs. In addition, some input and output weights may take an unusually small or high value in the performance evaluation process. Furthermore, we need to consider weight restrictions in real-life applications to comply with the decision-makers' preferences, as well as produce more realistic results.

In the literature on DEA, different types of weight restrictions (WRs) have been proposed. The most popular type of WRs is linear constraints that can be categorized into three groups (Allen et al. , 1997) including assurance region type I (ARI) developed by Thompson et al. (1986), assurance region type II (ARII) developed by Thompson et al. (1990), and absolute weight restrictions developed by Dyson and Thanassoulis (1988). The interested readers can refer to Thanassoulis et al. (2004) for more information regarding linear WRs.

Applying absolute WRs consists of imposing lower and upper bounds for inputs and outputs to prevent them from being overemphasized or ignored in the assessment. Nevertheless, this approach has its difficulties. For example, switching from an input orientation to an output orientation may yield different efficiency results for DEA models with absolute WRs. This may lead to infeasibility or underestimation of efficiency scores. Applying ARII, on the other hand, consists in imposing restrictions on the ratio between the weights of inputs and outputs and may lead to similar problematic issues as in the case of the absolute WRs. These problems can be avoided; however, when employing ARI, which has become popular due to its practicality and straightforwardness in incorporating value judgments in DEA (Dyson et al., 2001). There are various ways to go about collecting these value judgments, such as using the analytic hierarchy process (AHP). Interesting applications in this sense are the papers by Lee et al. (2012) and Lai et al. (2015), who used DEA and ARI in conjunction with AHP to evaluate the efficiency of photovoltaics firms in Taiwan and of international airports, respectively. Further, Do and Chen (2014) used DEA and ARI with fuzzy DEA to assess the efficiency of universities in Vietnam.

Tracy and Chen (2005) presented a generalized form of WRs that contains all of the above-mentioned linear WRs. They further developed a parametric DEA method to calculate the efficiency scores with this generalized form of WRs. Khalili et al. (2010a) improved the method proposed by Tracy and Chen (2005) by developing a new nonlinear DEA model. They proved that the nonlinear DEA model removes the problems of infeasibility and the underestimation of efficiencies arising from using the ARII and the absolute WRs in the basic DEA model. Khalili et al. (2010b) also discussed the problematic issues in using the ARII in DEA models and proposed a new nonlinear and non-convex DEA model to calculate the exact value of the efficiency scores. They claimed that some nonlinear solvers such as PATHNLP could find the global optimal solution of their model. Podinovski and Bouzdine-Chameeva (2013) argued that infeasibility is only one of several possible problems when using linear WRs in DEA models. They showed that using linear WRs may lead to free and unbounded production, and they developed analytical and computational methods to recognize the existence of the same. Podinovski and Bouzdine-Chameeva (2015) further proposed some new analytical conditions to identify the existence of free and unbounded production in the presence of linear WRs. Podinovski (2016) investigated the optimal weights in DEA models in the presence of linear WRs and showed that these weights might not produce the correct efficiency scores.

We build upon the recent models proposed by Toloo and Salahi (2018) and extend their approach to incorporate weight restrictions that reflect the decision-makers' preferences. From a practical standpoint, the approach proposed in this study considers the decision maker's preferences while this point has not been considered Toloo and Salahi's (2018) method. As a result, the optimal weights obtained by Toloo and Salahi (2018) may be inconsistent with the decision maker's preferences on the importance of the input and output factors. Moreover, a model that includes the decision maker's preferences may lead to different, most efficient DMU. The epsilon-based approach of Toloo and Salahi (2018) consists of two stages, while our epsilon-free approach involves one single stage. While the size of our model and our computational time are similar to Toloo and Salahi's (2018) approach, our model is epsilon-free and requires less effort to determine the best efficient unit. As a result, our single-stage approach is more succinct in comparison with their two-stage approach.

In summary, the contribution of this paper is fourfold: (1) the model proposed in this study uses ARI to incorporate the decision-makers' preferences on the importance of inputs and outputs weights in the performance evaluation; (2) in contrast to the existing epsilon-based models, our model is epsilon-free and requires less effort to determine the best efficient unit since the current methods usually solve two models to find the best DMU, one to find a suitable value for epsilon and another to find the best DMU. However, our method doesn't need an epsilon model and finds the best DMU by solving one model; (3) we explain the reasons behind the problems in using ARII and the absolute WRs. We further show how these problems can be eliminated with ARI, and; (4) our method does not only find the best DMU, but is also able to fully rank all other efficient DMUs.

3. Toloo and Salahi's (2018) method

Suppose that there are $\mathrm{DMU}_j, j = 1,2,\ldots,n$, with m inputs $x_{ij}, i = 1,2,\ldots,m; \forall j$, and s outputs $y_{rj}, r = 1,2,\ldots s; \forall j$. To find the best (most efficient) DMU, Toloo and Salahi (2018) developed the MBNLP DEA Model (1):

$$
\begin{aligned}
&\max h\\
&\text{s.t.}\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le M I_j - h(1-I_j), \quad j = 1,\ldots,n\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge h I_j - M(1-I_j), \quad j = 1,\ldots,n\\
&\sum_{j=1}^{n} I_j = 1\\
&I_j \in \{0,1\} \quad \forall j\\
&u_r, v_i \ge \varepsilon \quad \forall i, \forall r
\end{aligned} \tag{1}
$$

where M is a large positive number; $I_j(\forall j)$ is a binary variable; and u_r and v_i are the weights of the r^{th} output and i^{th} input, respectively; ε is the non-Archimedean infinitesimal, which prevents the input and output weights from being zero, and h is a non-negative decision variable that should be maximized.

In Model (1), the constraint $\sum_{j=1}^{n} I_j = 1; I_j \in \{0,1\}, \forall j$; implies there is one $p \in \{1,2,\cdots,n\}$ in the optimal solution such that $I_p^* = 1, I_j^* = 0, \forall j \ne p$. Toloo and Salahi (2018) have proved $h^* > 0$. Therefore, by considering the first- and second-type constraints of Model (1), we obtain $0 < h^* \le \sum_{r=1}^{s} u_r^* y_{rp} - \sum_{i=1}^{m} v_i^* x_{ip} \le M$, and $-M \le \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \le -h^* < 0$, $\forall j \ne p$. These equations imply the efficiency score of DMU_p is strictly greater than one, and the efficiency scores of the other DMUs are strictly less than one. Therefore, the solution to Model (1) produces the most efficient DMU_p.

Additionally, Toloo and Salahi (2018) proposed a new nonlinear

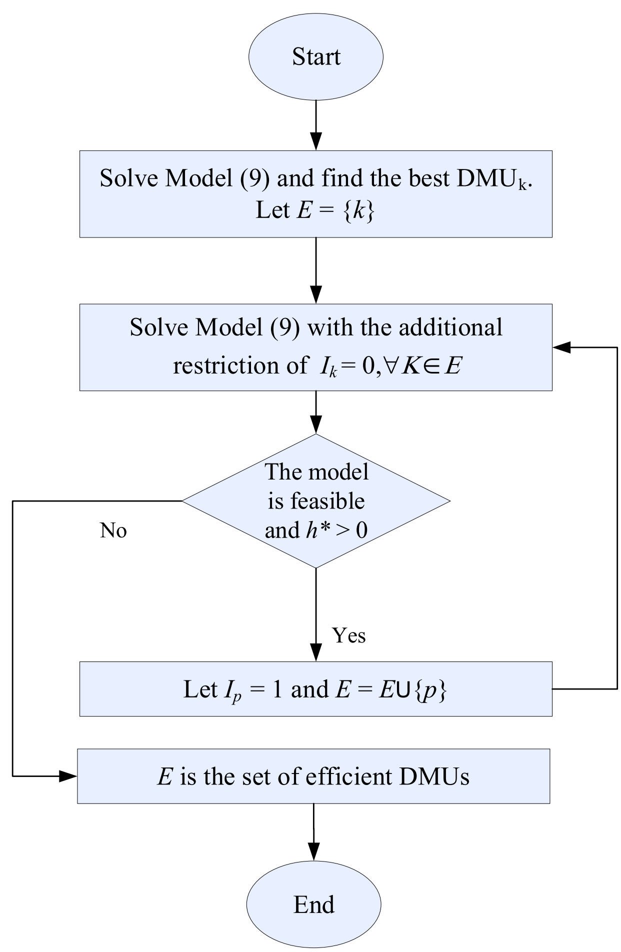

Fig. 1. Flowchart of the proposed algorithm.

DEA model to find an appropriate value for the epsilon in Model (1). It should be noted Model (1) is an MBNLP model due to the existence of the nonlinear terms of $hI_j, j = 1, \dots, n$. Therefore, the authors converted the model into the following MBLP Model (2).

$$\begin{aligned}
&\max h \\
&s.t. \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le MI_j - h + z_j \quad j = 1, \dots, n, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge z_j - M(1 - I_j) \quad j = 1, \dots, n, \\
&\sum_{j=1}^{n} I_j = 1 \\
&z_j \le MI_j \quad j = 1, \dots, n, \\
&z_j \le h \quad j = 1, \dots, n, \\
&h \le z_j + M(1 - I_j) \quad j = 1, \dots, n, \\
&I_j \in \{0, 1\} j = 1, \dots, n, \\
&h, z_j \ge 0 \quad j = 1, \dots, n, \\
&u_r, v_i \ge \varepsilon \quad \forall i, r
\end{aligned} \tag{2}$$

They then developed the following MBLP Model (3) to determine a proper value for the epsilon to use in Model (2).

$$\begin{aligned}
&\varepsilon^* = \max \varepsilon \\
&s.t. \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le MI_j - h + z_j \quad j = 1, \dots, n, \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge z_j - M(1 - I_j) \quad j = 1, \dots, n, \\
&\sum_{j=1}^{n} I_j = 1 \\
&z_j \le MI_j \quad j = 1, \dots, n, \\
&z_j \le h \quad j = 1, \dots, n, \\
&h \le z_j + M(1 - I_j) \quad j = 1, \dots, n, \\
&I_j \in \{0, 1\} j = 1, \dots, n, \\
&\varepsilon - u_r \le 0 \quad \forall i, \\
&\varepsilon - u_r \le 0 \quad \forall r, \\
&\varepsilon - h \le 0 \\
&z_j \ge 0 \quad \forall j, \\
&\varepsilon \ge 0.
\end{aligned} \tag{3}$$

Toloo and Salahi (2018) concluded that Model (2) identifies exactly one DMU as the most efficient DMU with the efficiency score of strictly greater than one and the remaining DMUs with the efficiency scores of strictly less than one. They ranked all the DMUs based on the common set of optimal weights obtained by solving Model (2). The approach proposed by Toloo and Salahi (2018) has two drawbacks:

(i) Two models are required to find the most efficient DMU. Model (3) is solved first to find the maximum value of epsilon to be used in Model (2). Model (2) is solved next to find the most efficient DMU. We extend Model (2) in Section 4 to find the best DMU by solving an MBLP model in the presence of ARI and preference considerations.

(ii) Model (2) does not consider the decision maker's preferences. Consequently, the obtained optimal weights may be inconsistent with the decision maker's preferences on the importance of the inputs and outputs factors. Therefore, the most efficient DMU identified here may not be the best DMU. We will explain this problem further in Section 4 and propose and improvement to Model (2) for eliminating this problem.

4. AR-DEA approach

In conventional DEA models, DMUs are allowed to choose the most favorable weights to achieve their maximum possible efficiency scores. Indeed, in these models, there is no restriction on the weights of the criteria (*i.e.*, inputs and outputs). In other words, the decision-makers' preferences are not considered in these models. Therefore, the optimal weights obtained may be inconsistent with the preferences of the decision-makers concerning the importance of these weights. In addition, some input/output weights may take very small or very large values in the performance evaluation process. Different types of WRs have been proposed in the literature on DEA to address these shortcomings. As explained in Section 2, the most popular WRs are linear weight restrictions that can be categorized into the following three groups:

***Absolute weight restrictions*:** In this type of WR, the decision-maker specifies a lower and an upper bound for each individual weight. In other words, the parameters $\rho_i, \delta_i, \mu_r, \eta_r, \forall i, r$ are estimated to obtain the constraints $\rho_i \le v_i \le \delta_i, \mu_r \le u_r \le \eta_r, \forall i, r$.

***Assurance region type 1* (*ARI*):** This type of WR specifies bounds on the ratios between the weights of inputs or outputs so that the ratio values are within a certain range. Indeed, the decision-makers estimate the parameters $\alpha_i, \beta_i, \theta_r, \lambda_r > 0, \forall i, r$ to obtain the constraints $\alpha_i \le \frac{v_i}{v_{i-1}} \le \beta_i, \theta_r \le \frac{u_r}{u_{r-1}} \le \lambda_r, i = 2, 3, \dots m; r = 2, 3, \dots, s$.

***Assurance region type 2* (*ARII*):** This type of WR imposes restrictions on the ratio between the input and output weights. The decision-makers estimate the parameters $\phi_{ir}, \forall i, \forall r$ to obtain the constraints $v_i \le \phi_{ir} u_r, \forall i, \forall r$.

The above-mentioned WRs can be used in DEA models to incorporate the decision-makers' preferences and to prevent obtaining unusual weights in the performance evaluation process. Also, as previously explained in Section 2, problems such as infeasibility and free and unbounded production may occur by using the absolute WRs and ARII in DEA models. In the following, we will discuss the reason behind the occurrence of such problems and will show these problems could be eliminated by using ARI. Therefore, we will use ARI in our proposed DEA approach to finding the most efficient DMU. It should be noted that ARI has been successfully used in many applications, such as the supplier selection problem and efficiency measurement in bank branches and hospitals, among others (Ebrahimi et al. 2017).

4.1. Linear weight restrictions in DEA

The relative efficiency score of DMU_k can be obtained by solving Models (4) or (5). In other words, the maximin nonlinear Model (5) can be converted into the linear Model (4), and so the optimal objective values of these models are equal. However, directly adding the ARI, ARII, and the absolute WRs to Models (4) and (5) may make these two models unequal. Khalili et al. (2010a, 2010b) discussed this problem and showed that these two models are not equal in the presence of absolute WRs and ARII. Otherwise stated, we need to add WRs to Model (5) and solve it, and not directly add the WRs to Model (4), to obtain the correct efficiency values and prevent the above-mentioned problems. Therefore, the direct addition of WRs to the linear Model (4) is problematic. We present the following theorem to show these two models are equal in the presence of ARI. This allows us to obtain the correct efficiencies values by solving the linear DEA models in the presence of ARI.

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r y_{rk} \\ &\text{s.t.} \\ &\sum_{i=1}^{m} v_i x_{ik} = 1 \\ &\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, \dots, n \\ &u_r, v_i \ge 0 \quad \forall i, r \end{aligned} \tag{4}$$

$$\max_{u,v \ge 0} \left\{ \frac{\frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}}{\max_{\forall j} \left\{ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \right\}} \right\} \tag{5}$$

Theorem. Models (4) and (5) are equivalent in the presence of ARI.

Proof. Suppose $t = \max_{\forall j} \left\{ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \right\}$; in this case, in the presence of ARI, Model (5) will be equal to the following Model (6):

$$\begin{aligned} &\max \frac{\sum_{r=1}^{s} u_r y_{rk}}{t \sum_{i=1}^{m} v_i x_{ik}} \\ &\text{s.t.} \\ &\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le t, \quad \forall j \\ &\alpha_i \le \frac{v_i}{v_{i-1}} \le \beta_i, \quad i = 2, 3, \dots m \\ &\theta_r \le \frac{u_r}{u_{r-1}} \le \lambda_r \quad r = 2, 3, \dots, s \\ &u_r, v_i \ge 0 \quad , \forall i, r \end{aligned} \tag{6}$$

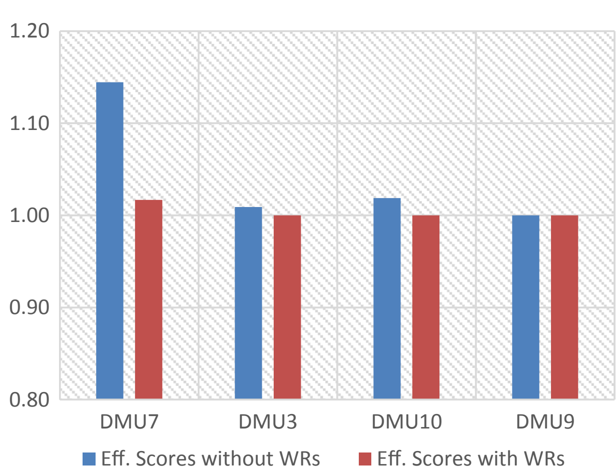

Fig. 2. Efficiency scores of the top four DMUs in the presence and absence of WRs.

By defining $t^{'} = \left(t \sum_{i=1}^{m} v_i x_{ik}\right)^{-1}$, we obtain:

$$\begin{aligned} &\max \sum_{r=1}^{s} t^{'} u_r y_{rk} \\ &s.t. \\ &\sum_{i=1}^{m} t t^{'} v_i x_{ik} = 1 \\ &\sum_{r=1}^{s} t^{'} u_r y_{rj} - \sum_{i=1}^{m} t t^{'} v_i x_{ij} \le 0, \quad j = 1, 2, \dots, n \\ &\alpha_i \le \frac{v_i}{v_{i-1}} \le \beta_i, \quad i = 2, 3, \dots m \\ &\theta_r \le \frac{u_r}{u_{r-1}} \le \lambda_r, \quad r = 2, 3, \dots, s \\ &u_r, v_i \ge 0, \quad \forall i, r \end{aligned} \tag{7}$$

Now, by defining $u^{'}_r = t^{'} u_r \& v^{'}_i = t t^{'} v_i$, we conclude that Model (7) is equivalent to Model (8).

$$\begin{aligned} &\max \sum_{r=1}^{s} u^{'}_r y_{rk} \\ &s.t. \\ &\sum_{i=1}^{m} v^{'}_i x_{ik} = 1 \\ &\sum_{r=1}^{s} u^{'}_r y_{rj} - \sum_{i=1}^{m} v^{'}_i x_{ij} \le 0 \quad , j = 1, \dots, n \\ &\alpha_i \le \frac{v^{'}_i}{v^{'}_{i-1}} \le \beta_i, \quad i = 2, 3, \dots m \\ &\theta_r \le \frac{u^{'}_r}{u^{'}_{r-1}} \le \lambda_r, \quad r = 2, 3, \dots, s \\ &u^{'}_r, v^{'}_i \ge 0 \quad , \forall i, r \end{aligned} \tag{8}$$

As can be seen, Model (8) is equal to Model (4) in the presence of ARI. Consequently, Models (4) and (5) are equal in the presence of ARI. □

4.2. Proposed approach

As we mentioned in the previous section, the MBLP Model (2) can be used to identify the most efficient DMU. We build upon this model and formulate Model (9) to find the most efficient DMU by considering the decision-makers' preferences:

$$h^* = \max h$$

$$s.t.$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le M I_j - h + z_j, \quad j = 1, ..., n$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge z_j - M(1 - I_j), \quad j = 1, ..., n$$

$$\sum_{j=1}^{n} I_j = 1$$

$$z_j \le M I_j \quad, j = 1, ..., n$$

$$z_j \le h \quad, j = 1, ..., n$$

$$h \le z_j + M(1 - I_j), \quad j = 1, ..., n \tag{9}$$

$$\alpha_i \le \frac{v_i}{v_{i-1}} \le \beta_i, \quad i = 2, 3, ..., m$$

$$\theta_r \le \frac{u_r}{u_{r-1}} \le \lambda_r, \quad r = 2, 3, ..., s$$

$$I_j \in \{0, 1\} \quad \forall j$$

$$u_r, v_i, h, z_j \ge 0, \quad \forall r, i, j$$

We can prove $h^* > 0$ in Model (9), similar to Theorem 1 in Toloo and Salahi (2018). The main aim of Model (9) is to find the positive weights u^* and v^*, such that the efficiency score of only one DMU will be strictly greater than one and the rest strictly less than one. These weights are obtained by maximizing h^*, which is defined as the largest distance between the most efficient DMU and the other DMUs, as follows (suppose $I_k^* = 1, I_j^* = 0, \forall j \ne k$):

$$-M \le \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \le -h^* < 0 < h^* \le \sum_{r=1}^{s} u_r^* y_{rk} - \sum_{i=1}^{m} v_i^* x_{ik} \le M, \forall j \ne k.$$

In other words, $[-h^*, h^*]$ is the minimum possible interval between the first two top-ranking DMUs, and so DMU_k is the best DMU. Therefore, by solving Model (9), we obtain only one DMU with the efficiency score of strictly greater than one, while the rest of the DMUs will have efficiency scores of less than one. The following highlights some important properties of Model (9):

(i) The unrealistic weights dispersion in DEA models may occur when some units are identified as efficient DMUs because of having very small or extreme input or output weights. Imposing weight restrictions (WRs) is a technique for eliminating this problem (Bal et al. 2010). Model (9) uses ARI to reduce the flexibility of weights. Therefore, this model yields a more homogeneous input and output weight dispersion with respect to Model (2). We refer readers to Hatami-Marbini and Toloo (2017) for a more detailed discussion of weight dispersion. In Section 5, we present two applications to demonstrate the improvement in the dispersion of the weight by using the new Model (9).

(ii) The proposed epsilon-based approach by Toloo and Salahi (2018) solves two models to determine the best efficient unit. However, our epsilon-free approach is less complicated and requires less effort to determine the most efficient unit.

(iii) The optimal solution of Model (9) implies $[-h^*, h^*]$ is the minimum possible interval between the first two top-ranking DMUs. In other words, we have $2h^* = (\sum_{r=1}^{s} u_r^* y_{rk} - \sum_{i=1}^{m} v_i^* x_{ik}) - \max_{\forall j \ne k}\{\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij}\}$. Therefore, the distance between the efficiency scores of the first two top-ranking DMUs is not necessarily equal to $2h^*$.

(iv) The feasible region of Model (9) is smaller than or equal to Model (2). As a result, the optimal objective value of Model (9) is less than or equal to the optimal objective value of Model (2).

It should be noted that considering $\alpha_i, \beta_i, \theta_r, \lambda_r > 0, \forall i, r$ with the weight restrictions $\alpha_i \le \frac{v_i}{v_{i-1}} \le \beta_i, i = 2, 3, ...m$, $\theta_r \le \frac{u_r}{u_{r-1}} \le \lambda_r, r = 2, 3, ..., s$, implies that in the optimal solution of Model (9) we have $v_i^*, u_r^* \ne 0, \forall r, i$. Thus, the constraints $u_r, v_i \ge 0, \forall r, i$, yields $v_i^*, u_r^* > 0, \forall r, i$. Therefore, in contrast to Model (2) and other existing MBLP models to find the best unit, our developed model (9) is an epsilon free model. As a result, we don't need to develop a new model such as Model (3) for finding a suitable value for epsilon.

The approach proposed in this study has two distinct advantages over the existing methods:

(i) The proposed Model (9) is epsilon-free (no other model is needed), is less complicated, and requires less effort to determine the most efficient unit, and
(ii) Model (9) finds the most efficient unit by considering the decision maker's preferences.

In the following, we present a simple algorithm for the scenario in which the decision-maker wants to find and rank all efficient DMUs using Model (9). Several similar algorithms have been proposed in the DEA literature to find and rank the efficient DMUs (e.g., Toloo, 2014c). Nevertheless, while most of these algorithms focus on the models in the absence of WRs, the following method depicted in the flowchart presented in Fig. 1 is offered to find the most efficient DMUs in the presence of WRs:

a. Solve Model (9) and assume that DMU_k is identified as the most efficient DMU. In other words, suppose $I_k = 1$, and let $E = \{k\}$.
b. Solve Model (9) with the additional restriction $I_k = 0, \forall k \in E$. If the new model is feasible and the optimal objective function is positive, then suppose $I_p = 1$, and go to the next step. Otherwise, stop; E is the set of efficient DMUs.
c. Let $E = E \cup \{p\}$, and go to the second step.

In the first iteration of the above algorithm, the second most efficient DMU is determined, if such DMU exists. By continuing the algorithm, all of the most efficient DMUs can be identified and ranked.

Note: If one wants to rank all the DMUs with a common set of optimal weights (similarly to Toloo and Salahi (2018)), the optimal weights of Model (9) can be used to rank all the DMUs.

5. Numerical illustrations

Example 1. In this example, we consider a case of efficiency measurement in the banking sector. Bank efficiency has been a dominant subject of empirical literature over the past two decades (Sousa de Abreu

Table 1
Input and output data of 14 banks.

DMUs	Inputs					Outputs			
	Input 1	Input 2	Input 3	Input 4	Input 5	Output 1	Output 2	Output 3	Output 4
1	400	18	33,600	2596	745	30,696	11,135	14	554
2	217	5	111,706	4958	566	86,967	16,813	634	1700
3	10,760	658	920,403	93,190	18,259	688,624	489,103	15,412	37,717
4	7801	322	937,174	73,930	16,087	629,622	479,516	8747	32,697
5	296	13	8985	1296	601	7502	5611	19	215
6	72	1	33,614	464	173	2940	1762	15	131
7	59	36	18,561	726	347	17,174	6465	211	536
8	3346	260	135,474	34,486	5276	97,063	101,898	3943	11,026
9	293	10	128,425	913	1034	92,579	19,216	468	5139
10	407	3	85,087	7233	1333	62,085	39,330	487	3686
11	8758	399	786,836	100,577	13,511	579,067	451,547	8834	35,972
12	365	18	31,300	2774	1138	20,274	2528	128	1046
13	2927	125	197,628	18,151	57,112	144,143	150,138	2829	8563
14	2004	98	318,909	38,937	13,804	195,120	192,046	2740	8891

et al., 2019). DEA is perhaps the most widely used method for performance measurement in the banking sector (Fethi & Pasiouras, 2010). In this example, we examine the particular case of the banking sector in the Czech Republic. For empirical analyses of the efficiency in the Czech banking sector, the interested reader is referred to Weill (2003), Stavárek and Poloučk (2004), Bonin et al. (2005), Matoušek and Taci (2005), Fries and Taci (2005), Jablonský (2012), Staníčková and Skokan (2012), Stavárek and Řepková (2012), and Řepková (2014), among others.

We consider fourteen banks that are active in the Czech Republic. These banks consume five inputs (number of employees, number of branches, assets, equity, and expenses) to produce four outputs (deposits, loans, non-interest income, and interest income). For additional information regarding the data, interested readers can refer to Toloo and Tichý (2015). The data for these banks can be seen in Table 1.

First, we apply the approach proposed by Toloo and Salahi (2018) to find the most efficient DMU. Solving Model (3) with $M = 100$ yields $\varepsilon^* = 0.0032$.

Solving Model (2) gives the following optimal solution that implies DMU_{10} is the most efficient DMU:

$$h^* = 0.0033$$

$$I_{10}^* = 1, I_j^* = 0, \forall j \neq 10$$

$$u_1^* = u_2^* = u_4^* = 0.0032, u_3^* = 0.0053$$

$$v_1^* = 0.0107, v_2^* = 0.3229, v_3^* = 0.0033, v_4^* = 0.0062, v_5^* = 0.0032$$

Now, we apply our approach to find and rank the most efficient DMUs in the presence of WRs. Let us assume that the decision-makers' preferences regarding the weights are as follows:

$$0.5 \leq \frac{v_2}{v_1} \leq 1; 2 \leq \frac{v_3}{v_2} \leq 3; 5 \leq \frac{v_4}{v_3} \leq 6; 1.5 \leq \frac{v_5}{v_4} \leq 2; 2 \leq \frac{u_2}{u_1} \leq 3; 1 \leq \frac{u_3}{u_2} \leq 2; 2 \leq \frac{u_4}{u_3} \leq 4.$$

These weight restrictions mean that the number of branches is at least 0.5 times as important as the number of employees and at most one time as important as the number of employees. Also, the assets are at least two times as important as the number of branches and at most three times as important as the number of branches. Similarly, other constraints can be interpreted.

As can be seen, the optimal weights obtained by solving the model of Toloo and Salahi (2018) are inconsistent with the decision-makers' views regarding the relative importance of inputs and outputs. Indeed, the ranking of efficient units without considering the decision-making preferences is theoretically correct; however, it is unacceptable in practice, since, the optimal weights do not satisfy the decision-maker preferences. In the following, we find and rank all efficient units by considering the above weight restrictions. Solving Model (9) with $M = 100$ yields the following optimal solution:

$$h^* = 0.0002$$

$$I_7^* = 1, I_j^* = 0, \forall j \neq 7$$

$$u_1^* = 0.0002, u_2^* = u_3^* = 0.0006, u_4^* = 0.0012$$

$$v_1^* = v_2^* = 0.0002, v_3^* = 0.0003, v_4^* = 0.0016, v_5^* = 0.0024$$

The above-obtained optimal solution implies that the efficiency score of DMU_7 is strictly greater than one, with the rest of the DMUs having efficiency scores that are less than one. Therefore, DMU_7 is the most efficient DMU. As we expected, the best unit is changed by applying the weight restrictions.

Now, we apply the algorithm proposed in the previous section to find and rank the other efficient DMUs in the presence and absence of WRs. To find the second most efficient DMU in the presence of WRs, we add the constraint $I_7 = 0$ to Model (9) and solve it, which gives $I_3^* = 1, h^* = 0.0002$. This optimal solution implies that DMU_3 is a second-most, efficient DMU. Solving Model (9) with the additional restriction $I_3 + I_7 = 0$ gives $I_{10}^* = 1, h^* = 0.0002$, which means that DMU_{10} is the third most efficient DMU. Solving Model (9) with the additional restriction $I_3 + I_7 + I_{10} = 0$ shows further that DMU_9 is the fourth most efficient DMU with the optimal solution $I_9^* = 1, h^* = 0.0001$. Lastly, solving Model (9) with the additional restriction $I_3 + I_7 + I_9 + I_{10} = 0$ gives $h^* = 0$. This optimal solution implies that there are no other most efficient DMUs. In summary, according to Model (2), all the banks are efficient except for banks 6 and 12.

We investigate the input and output weights dispersion by using the coefficient of variation (CV). The CV is the ratio of the standard deviation to the mean. The higher the CV, the greater the level of dispersion around the mean. We use the following formulas to calculate the CV (Hatami-Marbini and Toloo, 2017):

$$\mu = \frac{\sum_{r=1}^{s} u_r^* + \sum_{i=1}^{m} v_i^*}{m+s}, \sigma^2 = \frac{\sum_{r=1}^{s} (u_r^* - \mu)^2 + \sum_{i=1}^{m} (v_i^* - \mu)^2}{m+s}, CV = \frac{\sigma}{\mu}$$

Our calculations indicate that the CV for the optimal weights in Model (2) and Model (9) is equal to 2.49 and 0.89, respectively. In other words, the optimal weights in Model (9) are more dispersed than the optimal weights in Model (2).

Fig. 2 shows the efficiency scores of the top four banks in the presence and absence of WRs. Based on the above discussion and Fig. 1, we can now draw the following conclusions:

- In Cooper et al. (2007), when the number of DMUs (14 banks) is not sufficiently high compared to the number of criteria (9 criteria), the discriminatory power of DEA models weakens. However, our proposed model reduces the number of efficient DMUs from 12 to 4, signifying the discriminatory power of our method.
- As shown in Fig. 2, considering WRs leads to a reduction in the efficiency scores. Indeed, the DEA model, in the absence of WRs, has more flexibility to choose optimal weights. This flexibility is limited when considering the decision-makers' preferences. Also, the optimal weights obtained are consistent with the decision-makers' views concerning the importance of weights.

Example 2. In this example, we consider a real-life problem in the Iranian Space Industry (ISI). The ISI is involved in peaceful research activities under the Ministry of Communication and Information Technology. The department of aerospace engineering at the Amirkabir University of Technology (AUT) is a research site to design, manufacture and test small satellites for the ISI. Each satellite has several subsystems: structure, power, propulsion, communications, thermal, telemetry, and command. AUT outsources the thermal unit, and 21 contractors are considered for manufacturing this unit. AUT is considering the following four factors to evaluate the contractors.

- **Total Cost (x_1):** The cost of designing, constructing, integrating, and testing the thermal unit in 000's dollars (input factor).
- **Distance (x_2):** The distance of the contractor from AUT in 000's km (input factor). Thermal units have a number of interfaces with the other satellite subsystems. This requires repeated testing at AUT, and close proximity to AUT will accelerate the project completion time.
- **Experience (y_1):** The number of products a contractor has produced for previous satellite launches (output factor). This factor shows the experience and ability of the contractor to design and construct a quality component.
- **On-time delivery (y_2):** The number of on-time deliveries by a contractor (output factor).

Table 2 shows the total cost, distance, experience, and on-time delivery for the 21 contractors under consideration.

Applying the proposed method by Toloo and Salahi (2018) identifies Contractor 15 as the most efficient contractor. The optimal objective solution of Model (2) is as follows:

$$I_{15}^{*}=1, I_j^{*}=0, \forall j \neq 15$$

$$u_1^{*}=11.666, u_2^{*}=2.897, v_1^{*}=2.964, v_2^{*}=2.680$$

Now, we apply our proposed Model (9) to find the most efficient contractor. The project manager considers the following relation between the inputs and output weights.

$$1.5 \leq \frac{v_2}{v_1} \leq 2; 1.5 \leq \frac{u_2}{u_1} \leq 3$$

These weight restrictions indicate distance is at least 1.5 times more important than the total cost and, at most, two times more important than the total cost. In addition, the number of on-time deliveries is at least 1.5 times more important than experience and at most three times more important than experience.

As can be seen, the optimal weights obtained by Model (2) are inconsistent with the decision-makers' views regarding the relative importance of inputs and outputs. Therefore, by considering the above weight restriction, contractor 15 may not be the best DMU. We apply our developed approach to find and rank all efficient contractors. Solving model (9) gives the following optimal solution:

$$h^{*}=0.465, I_{11}^{*}=1, I_j^{*}=0, \forall j \neq 11$$

$$u_1^{*}=6.927, u_2^{*}=12.872, v_1^{*}=2.948, v_2^{*}=5.896$$

In contrast to the proposed method by Toloo and Salahi (2018), our developed approach implies that contractor 11 is the best unit by considering the decision-maker preferences. Solving Model (9) with the additional restriction $I_{11}=0$ gives $h^{*}=0.350, I_{15}^{*}=1, I_j^{*}=0, \forall j \neq 15$, which implies contractor 11 is the second-best DMU. Solving Model (9) with the additional restriction $I_{11}+I_{15}=0$ gives $h^{*}=0$, which means there is not another efficient unit. The CV for the optimal weights in Model (2) and Model (9) is equal to 0.76 and 0.50, respectively, which shows the optimal weights in Model (9) are more dispersed than the optimal weights in Model (2).

Table 2
Input and output data for 21 satellite thermal unit contractors.

Contractors	Inputs		Outputs	
	x_1	x_2	y_1	y_2
1	55	3.4	11	3
2	41	3.7	8	5
3	60	2.7	10	2
4	38	1.4	9	3
5	37	1.6	2	4
6	44	3.5	5	5
7	50	3.9	12	2
8	48	1.8	7	4
9	53	2.7	9	2
10	47	3.8	3	5
11	52	3.3	12	7
12	60	2.4	6	8
13	49	5.1	10	1
14	52	1.2	5	2
15	45	3.6	11	6
16	59	4.9	9	4
17	46	4.3	7	4
18	54	1.5	10	7
19	49	2.3	8	6
20	44	3.6	9	5
21	51	2.7	3	9

6. Discussion and managerial implications

DEA is an excellent management science tool for performance evaluation, but it has traditionally suffered from the difficulty of incorporating the decision-makers' preferences in the analysis. On the other hand, decision-making processes are often characterized by complexity and uncertainty. Most of the decision-making problems in organizations are typically characterized by multiple interests, objectives, and perspectives, as well as different types of information (Wang et al., 2009). Due to this complexity, the consideration and involvement of multiple decision-makers' perspectives and preferences are acknowledged and recommended (Koksalmis and Kabak, 2019).

Most of the time; however, relevant stakeholders are making decisions based on optimization models that do not consider the intervention of the decision-makers. Overall, incorporating the decision-makers' preferences in the modeling phase empowers decision-makers and paves the way towards more meaningful participatory management practices in organizations. In this regard, the preference structure model refines the results of the original DEA model and 'eliminates' apparently efficient DMUs (Zhu, 1996).

The results of the two illustrative examples demonstrate the applicability of our proposed model in the identification of the most efficient bank and most efficient contractor, respectively. From a managerial perspective, our approach is characterized by its ease of application for practicing managers and other relevant stakeholders and less complexity. As shown, the merit of our approach lies in the possibility of incorporating the decision-makers' preferences in the model and in the capacity of the model to identify the most efficient DMU that reflects the decision-makers' diverse set of objectives and interests. The model is applicable and easily implementable for solving practical problems in any domain of interest.

7. Conclusions

In this study, we analyzed the models proposed by Toloo and Salahi (2018) to find the most efficient DMU. We showed that their models and other existing approaches fail to consider the decision-makers' preferences in the performance evaluation process. In this sense, we improved upon Toloo and Salahi (2018) to include the assurance region type 1 (ARI) and consider the decision-makers' preferences concerning the importance of criteria (inputs and outputs). We explained the reasons for the infeasibility, free unbounded production, and other problems that arise when using absolute WRs and ARII in DEA models. We further showed that these problems do not occur when using ARI instead. The epsilon-free model proposed in this study is less complicated and requires less effort to determine the best efficient unit compared with the existing epsilon-based models in the literature. In addition, the decision maker's preferences are considered in finding the most efficient unit.

We have provided two real-life numerical examples to illustrate the reliability and applicability of our proposed model. Findings indicate that incorporating additional weight restrictions consistent with the decision-makers' preferences can result in the identification of a different most efficient DMU. Future research could extend the approach proposed in this paper to contain imprecise data, such as bounded data, ordinal data, as well as fuzzy data, reflecting to a greater extent the context of real-life applications.

CRediT authorship contribution statement

Bohlool Ebrahimi: Conceptualization, Formal analysis, Methodology, Investigation, Validation, Data curation. **Madjid Tavana:** Conceptualization, Writing - review & editing, Visualization, Methodology, Project administration, Supervision. **Mehdi Toloo:** Conceptualization, Formal analysis, Methodology, Validation, Investigation. **Vincent Charles:** Conceptualization, Formal analysis, Methodology, Validation, Writing - review & editing.

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