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A novel common set of weights method for multi-period efficiency measurement using mean-variance criteria

Seyed Hossein Razavi Hajiagha[a], Hannan Amoozad Mahdiraji[b], Madjid Tavana[c,d,*], Shide Sadat Hashemi[e]

[a] Department of Management, Faculty of Management and Finance, Khatam University, Tehran, Iran
[b] Faculty of Management, University of Tehran, Tehran, Iran
[c] Business Systems and Analytics Department, Distinguished Chair of Business Analytics, La Salle University, Philadelphia, USA
[d] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, Paderborn, Germany
[e] Department of Management, Allameh Tabataba'i University, Tehran, Iran

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ABSTRACT

Data envelopment analysis (DEA) is a popular method for evaluating a set of homogeneous decision-making units (DMUs). One of the main shortcomings of DEA is the weights flexibility where each unit can take its desirable weights. Several methods have been developed for finding a common set of weights (CSWs) and overcoming this drawback. The CSWs methods are used to evaluate the relative efficiency of the DMUs in a single time-period. However, single period DEA models cannot handle organizational units performing in a continuum of time. We propose a novel method for determining the CSWs in a multi-period DEA. Initially, the CSWs problem is formulated as a multi-objective fractional programming problem. Subsequently, a multi-period form of the problem is formulated and the mean efficiency of the DMUs is maximized while their efficiency variances is minimized. A fuzzy set-based approach is used to solve the multi-period CSWs problem. We present a real-world case study to demonstrate applicability and exhibit the efficacy of the proposed method. The results indicate a significant improvement in the discrimination power of the proposed multi-period method.

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. [12] (known as the CCR model) is a well-known framework for evaluating the relative efficiency of a set of homogeneous organizational units, known as decision making units (DMUs). DEA illustrates the efficiency of the process that transforms inputs into outputs by applying an m-dimensional input vector to produce an s-dimensional output vector. DEA does not make any assumption about the system structure [7]. Liu et al. [41] and later Emrouznejad and Yang [21] surveyed several applications of DEA in different domains. In addition, a comprehensive review on the theoretical foundations of DEA can be found in Cooper et al. [16].

Beyond their advantages and strengths, the classic DEA models are confronted with some shortcomings. Dyson et al. [19] has identified the homogeneity assumption, full flexibility of weights, and weight restrictions as the main pitfalls of DEA. Since its initial introduction, researchers have focused on resolving these shortcomings by extending new research directions. Liu et al. [42] have shown that the main research topics in DEA are focused on: (1) bootstrapping and two-stage analysis, (2) undesirable factors, (3) cross-efficiency and ranking, and (4) network DEA, dynamic DEA, and SBM.

The aim of research in cross-efficiency and ranking problems is increasing the discrimination power of classic DEA models in which, DMUs are classified into efficient and inefficient classes. The common set of weights (CSWs) problem is a popular research stream in DEA dealing with full flexibility of weights. DEA models provide a flexible condition for the DMUs to take their desirable weights of inputs and outputs and maximize the relative efficiency of the considered DMU. Solving the DEA model for n DMUs requires a set of n different weight vectors for the DMUs. The efficiency calculated with this weighting scheme will overestimate the real-world efficiency of each DMU. This type of weighting has some problems as described in [17]. To overcome these problems, the CSW seeks to find a set of common weights for the

* Corresponding author at: Business Systems and Analytics Department, Distinguished Chair of Business Analytics, La Salle University, Philadelphia, PA 19141, USA.
E-mail addresses: h.razavi@khatam.ac.ir (S.H. Razavi Hajiagha), h.amoozad@ut.ac.ir (H. Amoozad Mahdiraji), tavana@lasalle.edu (M. Tavana), s.s.hashemi@alumni.atu.ac.ir (S.S. Hashemi).

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DMUs. Roll et al. [54] and Roll and Golany [55] were among the first researchers to work on the CSW problem.

Kao and Hung [35] proposed a compromise solution approach for generating common weights under the DEA framework to rank the DMUs with the same scale. Furthermore, Ramon et al. [50] proposed a DEA approach aimed at deriving a CSWs to be used to obtain the ranking of the DMUs. Additionally, Qi and Guo [49] presented a methodology by combining the CSWs with the Shannon's entropy. Jahanshahloo et al. [31] also defined an ideal line and determined a CSWs for the efficient DMUs and used the new efficiency scores obtained to rank them. In addition, they developed a special line and ranked the efficient DMUs by comparing them with this line. Wang et al. [64] proposed a new methodology based on regression analysis to find CSWs that are easy to estimate and can produce a full ranking of the DMUs. They used the most favorable weights to obtain the DEA efficiencies.

Given the large number of DMUs in real applications, the computational and conceptual complexities are considerable with weights that are potentially zero-valued or incommensurable across units. In this regard, Saati et al. [56] proposed a two-phase algorithm to address this situation. Similarly, [17] proposed an innovative method using a CSWs leading to solving a linear programming problem. Their method determines the efficiency score of all DMUs and ranks them simultaneously. Moreover, Ching et al. [15] proposed a context-dependent DEA model to address the shortcomings resulting from redundant restraints on the weights of an efficient DMU and converted the optimal weight to analyze the influences of redundant restraints on weights. Hosseinzadeh Lotfi et al. [28] proposed an allocation mechanism using common dual weights approach and applied it to allocate the fixed resources to the units. Razavi Hajiagha et al. [51] formulated the CSW problem as a multiple objective fractional programming model and then applied the Dutta et al. [20] method for solving this problem. Wu et al. [65] proposed a method of finding CSWs based on a DMU's satisfaction degree with respect to common weights. Their model contained a min–max model and two approaches were proposed for solving this model.

Considering uncertain situations, Omrani [47] introduced a robust optimization approach to find common weights in DEA models with uncertain data. They considered uncertainty in inputs and outputs and developed a suitable robust counterpart for the DEA model. In addition, Tavana et al. [61] illustrated a CSW model for ranking the DMUs with the stochastic data and the ideal point concept. Their proposed method minimizes the distance between the evaluated DMUs and the ideal DMU. Recently, Dong et al. [18] provided a DEA-based approach for obtaining DMUs' efficiencies by assuming the DMUs as a collection of rational units and maximizing an objective for satisfaction degrees of the DMUs. They also provided a maxim model and two corresponding algorithms for generating the CSWs. Furthermore, Hatami-Marbini et al. [27] introduced an alternative DEA model for centrally imposed resource or output reduction across the reference set. They determined the amount of input and output reduction needed for each DMU to increase the efficiency score of all the DMUs.

Another research direction in DEA is the problem of multi-period efficiency evaluation, known as dynamic DEA. While classical DEA models are considered as cross-sectional or single point evaluation, comparison of DMUs performance over several periods of time can be considerable as time series DEA [11,53] or multi-period (dynamic) DEA. In this context, each individual input or output measure is captured in the form of a time series which reflects the level of that measure in different time periods. Thus, a method is required for dealing with fluctuation. Using the concept of Debreu-Farrell technical efficiency, Park and Park [48] proposed the multi-period DEA model that found the efficiency of DMUs in different periods. They call a DMU fully-efficient if it gains full efficiency in all periods. Sengupta [58] introduced the concept of dynamic DEA to consider inputs and outputs change over time.

Amirteimoori and Kordrostami [5] defined the aggregate efficiency of a DMU as convex combination of its periodic efficiencies and proposed a method for finding the aggregated and periodic efficiencies. Kao [34] proposed a model in which the complement of the system efficiency is a linear combination of period efficiencies. Similarly, Kao and Liu [37] defined the overall efficiency of a DMU as a weighted average of its individual periodic efficiencies. Razavi Hajiagha et al. [52] used the concept of Chebyshev inequality bounds for finding the confidence intervals of inputs and outputs and transformed the multi-period DEA problem into an equivalent interval DEA problem. They evaluated the multi-period efficiency of DMUs in the form of interval efficiencies. Kou et al. [40] extended the idea of Kao [34] to find the multi-period efficiency of a multi-division network. Kordrostami and Jahani Sayyad Noveiri [39] proposed a method based on fuzzy expected value for determining the overall and period efficiencies of DMUs in a multi-period problem. Jahani Sayyad Noveiri et al. [30] proposed a DEA-based procedure to estimate the multi-period efficiency of systems with desirable and undesirable outputs. They defined the overall efficiency of units as a weighted average of the efficiencies of the periods and approximated the efficiency changes between two periods. Multi-period DEA models are applied in commercial banks [37,52], insurance companies [36], international airports [4], universities [24], and regional R&D efficiency [33].

The aim of this paper is to bring together these two fields of study. Considering the above-mentioned researches, it is notable that while a great deal of attention is paid to solve multi-period or dynamic DEA problems, the research on finding CSWs for measuring the efficiency of DMUs performed over several periods of time is very rare. In this study, we develop a mathematical model for determining the best CSWs for DMUs performing in multiple periods of time. The considered quandary, known as multi-period CSWs problem, is useful for those managers who seek a general weighting scheme to evaluate organizational units in a time horizon. To this end; first, the CSWs problem is formulated and after modification, it is extended to determine the multi-period CSWs.

The reminder of the paper is organized as follows. After the introduction, the problem considered in this study is described in Section 2. The mathematical formulation of this problem is presented in Section 3. In Section 4, we propose the solution procedure and in Section 5, we present a real-world case study to demonstrate the applicability and exhibit the efficacy of the proposed model. Finally, the paper is concluded in Section 6.

2. Problem description

In this section, a description of multi-period CSWs problem is given. Suppose that there are a set of n DMUs, $DMU_j, j = 1, 2, ..., n$, that are evaluated in a time horizon consisting of T time periods. At each period t, $t = 1, 2, ..., T$, DMU_0 receives the input vector $X_0^t = (x_{10}^t, x_{20}^t, ..., x_{m0}^t)$ and produces the output vector $Y_0^t = (y_{10}^t, y_{20}^t, ..., y_{s0}^t)$. This situation is illustrated in Fig. 1.

The relative efficiency of DMU_0 at any time-period t can be assessed using classic DEA models. Solving the individual DMU's model, the relative efficiency of $DMU_0, 0 \in \{1, 2, ..., n\}$ at time-period t is determined. This is a single-period evaluation of relative efficiency, generally called as cross-sectional efficiency. The aim of multi-period DEA models is to determine a single measure of relative efficiency for DMUs that perform like Fig. 1.

On the other hand, solving the above model for each DMU, different weights are obtained for inputs and outputs at each time-period. The aim of the multi-period CSWs problem is to find a CSWs u_r, $r = 1, 2, ..., s$ and v_i, $i = 1, 2, ..., m$ that,

a. Evaluate the relative efficiency of DMUs at a multi-period manner;
b. Determine the common weights of inputs and outputs as a general baseline to evaluate efficiencies;
c. Enhance the comparability of relative efficiencies among DMUs.

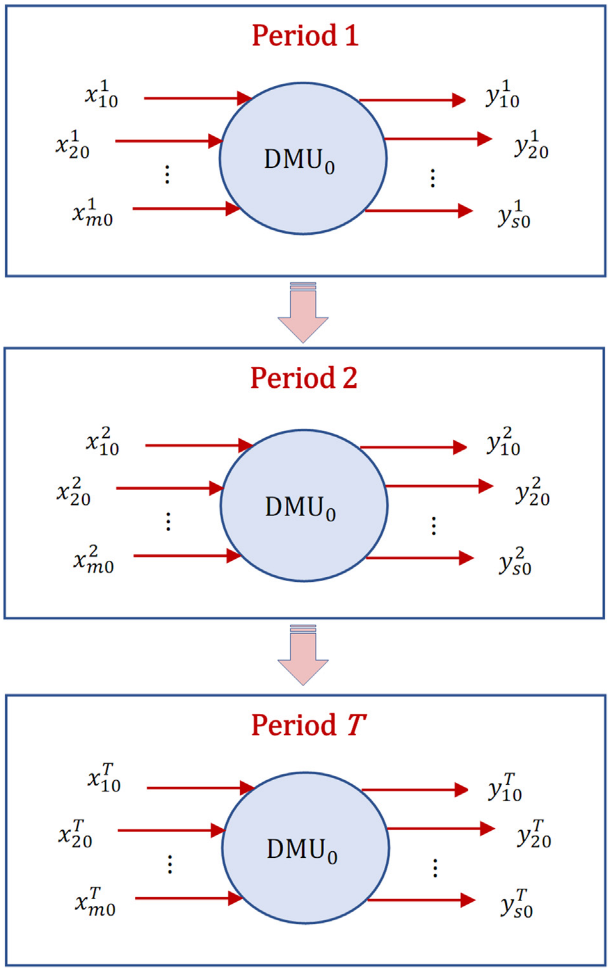

Fig. 1. Multi-period DMU performance measurement.

For any input measure X_{ij}, $i = 1, 2, ..., m$; $j = 1, 2, ..., n$ a mean $\bar{X}_{ij} = \sum_{t=1}^{T} x_{ij}^{t}/T$ and variance $\sigma_{ij}^{2} = \sum_{t=1}^{T} (x_{ij}^{t} - \bar{X}_{ij})^{2}/(T-1)$ is computable. Similarly, for output vector Y_{rj}, $r = 1, 2, ..., s$; $j = 1, 2, ..., n$, the mean $\bar{Y}_{rj} = \sum_{t=1}^{T} y_{rj}^{t}/T$ and variance $\delta_{rj}^{2} = \sum_{t=1}^{T} (y_{rj}^{t} - \bar{Y}_{rj})/T-1$ are calculated.

3. Mathematical formulation

The problem formulation is performed in two stages. Initially, a model is developed to find the CSWs and then a multi-period model is extended. Fig. 2 illustrates an algorithmic scheme of the proposed method.

3.1. Common set of weights modelling

In this stage, a model is developed to find the CSWs. Assuming given time-period t, we consider the aforementioned notation of DMUs and their corresponding inputs and outputs, and develop the following problem, called the input-oriented CCR model:

$$
\begin{aligned}
&E_0^t = \max \frac{\sum_{r=1}^{s} u_r^t y_{r0}^t}{\sum_{i=1}^{m} v_i^t x_{i0}^t}\\
&\frac{\sum_{r=1}^{s} u_r^t y_{rj}^t}{\sum_{i=1}^{m} v_i^t x_{ij}^t} \leqslant 1,\ j = 1, 2, \ldots, n\\
&u_r^t \geqslant 0,\ r = 1, 2, \ldots, s\\
&v_i^t \geqslant 0,\ i = 1, 2, \ldots, m
\end{aligned}
\tag{1}
$$

For each DMU, the relative efficiency of DMU_0, $0 \in \{1, 2, \ldots, n\}$ is determined for a single period (time-period t) using a cross-sectional efficiency model. Solving the above model for each DMU, different values are obtained for u_r^t, $r = 1, 2, ..., s$ and v_i^t, $i = 1, 2, ..., m$. The goal is to find a CSWs; u_r^{tc}, $r = 1, 2, ..., s$ and v_i^{tc}, $i = 1, 2, ..., m$, as real weights of inputs and outputs. Accordingly, the CSWs problem is formulated as a vector maximum problem, with the following objective:

$$
Max\{E_1^{tc}, E_2^{tc}, \ldots, E_n^{tc}\} \tag{2}
$$

where E_j^{tc}, $j = 1, 2, ..., n$, i.e. the relative efficiency of the jth DMU at time-period t by using the common weights, is defined as:

$$
E_j^t = \frac{\sum_{r=1}^{s} u_r^{tc} y_{rj}^t}{\sum_{i=1}^{m} v_i^{tc} x_{ij}^t} \tag{3}
$$

The extended form of Model (2), with its corresponding constraints is constructed as follows:

$$
\begin{aligned}
&E^{Ct} = \max\left\{\frac{\sum_{r=1}^{s} u_r^{tc} y_{r1}^t}{\sum_{i=1}^{m} v_i^{tc} x_{i1}^t}, \frac{\sum_{r=1}^{s} u_r^{tc} y_{r2}^t}{\sum_{i=1}^{m} v_i^{tc} x_{i2}^t}, \ldots, \frac{\sum_{r=1}^{s} u_r^{tc} y_{rn}^t}{\sum_{i=1}^{m} v_i^{tc} x_{in}^t}\right\}\\
&\frac{\sum_{r=1}^{s} u_r^{tc} y_{rj}^t}{\sum_{i=1}^{m} v_i^{tc} x_{ij}^t} \leqslant 1,\ j = 1, 2, \ldots, n\\
&u_r^{tc} \geqslant 0,\ r = 1, 2, \ldots, s\\
&v_r^{tc} \geqslant 0,\ i = 1, 2, \ldots, m
\end{aligned}
\tag{4}
$$

Model (4) is a multi-objective fractional linear programming problem producing the CSWs in each time-period t.

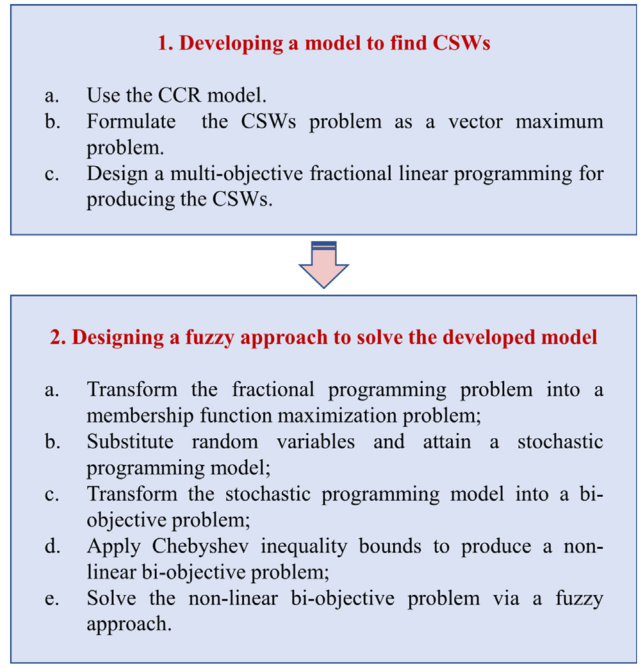

Fig. 2. Algorithmic scheme of multi-period CSWs calculation.

4. Solution approach

In this section, we develop a fuzzy approach for solving the CSWs problem in the multi-objective fractional linear programming problem (4). The proposed approach to solve the above problem is based on the idea of Dutta et al. [20] in solving multi-objective fractional programming problems that later is extended by Razavi Hajiagha et al. [51] to solve the DEA common set of weights problem. In this method, a membership function is developed for both the nominator and denominator of the objectives and then the sum of these membership functions is maximized.

Razavi Hajiagha et al. [51] proposed the above method for finding CSWs of a single-period DEA problem. Comparing the results of their model with three models of Kao and Hung [35] and Makui et al. [67], the proposed method of Razavi Hajiagha et al. [51] illustrated a high level of correlation with the classic CCR model and provided a great discriminant power among the DMUs. Therefore, in this paper, the method of finding CSWs is extended to multi-period conditions.

Considering the CCR Model (1), suppose a decision maker determines his/her satisfaction regard to input-oriented efficiency of DMU_j according to the following membership function:

$$\mu_{Ij}^{t} = \begin{cases} 0, & \text{if } \theta_l > \sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} \\ \frac{\sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} - \theta_l}{1-\theta_l}, & \text{if } \theta_l \leqslant \sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} \leqslant 1 \end{cases} \quad (5)$$

Eq. (5) shows the input-oriented efficiency is in the range of $[\theta_l, 1]$, where θ_l is a DMU-independent threshold determined by the decision maker. In the case that this efficiency is lower than θ_l, its acceptance by the decision maker is zero. For efficiency scores between θ_l and 1, the acceptance is increased as a monotone increasing function. At the efficiency score of 1, the satisfaction degree will reach 1. It is notable that since input-oriented efficiency is always lower than one, the case of $\sum_{r=1}^{s} u_r^{tc} y_{rj}^{tc} < 1$ is not considered in Eq. (5). The output-oriented efficiency also can be defined in the interval of $[1, \phi_u]$, where ϕ_u is an upper-bound threshold determined by the decision maker. In the case of the CCR model considered here, $\phi_u = 1/\theta_l$, i.e. when the output-oriented efficiency is greater than ϕ_u, its acceptance is zero, while when it is between 1 and ϕ_u, its acceptance is decreased linearly according to Eq. (6):

$$\mu_{Oj}^{t} = \begin{cases} 0, & \text{if } \varphi_u < \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t} \\ \frac{\varphi_u - \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t}}{\varphi_u - 1}, & \text{if } 1 \leqslant \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t} \leqslant \varphi_u \end{cases} \quad (6)$$

The thresholds θ_l and ϕ_u are required to normalize the membership functions and becoming commensurable to proceed with the algorithm. In addition, since the nominators and denominators of the fractions in Eq. (4) are not in one direction, it is necessary to transform them into the above membership functions to put them in one direction and become commensurable, as is required. In Eq. (6), since the output-oriented efficiency is always greater than one, the case of $\sum_{i=1}^{m} v_i^{tc} x_{ij}^{tc} < 1$ is not considered. Hence, the fractional programming problem in Eq. (1) is transformed into the following membership function maximization problem:

$$\begin{aligned} & E_0^t = \max \frac{\sum_{r=1}^{s} u_r^{tc} y_{r0}^{t} - \theta_l}{1-\theta_l} + \frac{\varphi_u - \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t}}{\varphi_u - 1} \\ & s.\ t. \\ & \text{(i) } 0 \leqslant \mu_{Ij}^{t} \leqslant 1, j = 1, 2, \dots, n \\ & \text{(ii) } 0 \leqslant \mu_{Oj}^{t} \leqslant 1, j = 1, 2, \dots, n \\ & \text{(iii) } \sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} - \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t} \leqslant 0, j = 1, 2, \dots, n \\ & u_r^{tc} \geqslant 0,\ r = 1, 2, \dots, s \\ & v_i^{tc} \geqslant 0,\ i = 1, 2, \dots, m \end{aligned} \quad (7)$$

Eq. (7) is a fuzzy approach to solve Eq. (1). The fractional constraints of Eq. (1) are transformed to linear constraints (iii). Two sets of constraints (i) and (ii) are also obtained from the fact that the constructed membership functions, Eqs. (5) and (6), must be greater than zero and lower than one for all DMUs.

If the definition of $\mu_{I_j}^{t}$ in Eq. (5) is replaced in the first constraint of Eq. (7), then $\sum_{r=1}^{s} u_r y_{rj}^{t} \leqslant 1$. Equivalently, for $\mu_{O_j}^{t}$ in Eq. (6), by substituting it in the second constraint of Eq. (7), an inequality of the form $-\sum_{i=1}^{m} v_i x_{ij}^{t} \leqslant -1$ is obtained. Adding these two inequalities, the inequality $\sum_{r=1}^{s} u_r y_{rj}^{t} - \sum_{i=1}^{m} v_i x_{ij}^{t} \leqslant 0$ is obtained. Since the latter inequality is the linear combination of the above inequalities, it is a redundant constraint that can be eliminated [8]. Therefore, two sets of constraints (i) and (ii) imply the constraints of (iii) and thus, the sets of constraints (iii) are redundant. Dutta et al. [20] and Stanco-Minasian and Pop [60] have proved the efficiency of the results obtained by solving the fuzzy equivalent (Eq. (7)) of the fractional programming problems (Eq. (1)). Therefore, solving Eq. (7) is equivalent to solving Eq. (1).

Eq. (7) is an ordinary linear programming problem that can be solved without difficulty. For finding the CSWs using Model (4), the summation of membership functions is maximized as suggested by [66] and Tiwari et al. [62]. It is notable that according to Chen and Tsai [13], the sum of the achievement degrees in a max-sum (additive) model is greater than in the ordinal max–min model of Bellman and Zadeh [9]; thus, this operator is selected in this paper. Consequently, the CSWs problem of model (4) transforms as follows by ignoring the constant values of the objective function:

$$\begin{aligned} & \max \frac{1}{1-\theta_l} \sum_{j=1}^{n} \sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} - \frac{1}{\varphi_u - 1} \sum_{j=1}^{n} \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t} \\ & \text{s.t.} \\ & \sum_{r=1}^{s} u_r^{tc} y_{rj}^{t} \leqslant 1, j = 1, 2, \dots, n \\ & \sum_{i=1}^{m} v_i^{tc} x_{ij}^{t} \geqslant 1, j = 1, 2, \dots, n \\ & u_r^{tc} \geqslant 0,\ r = 1, 2, \dots, s \\ & v_i^{tc} \geqslant 0,\ i = 1, 2, \dots, m \end{aligned} \quad (8)$$

The CSWs for a given time-period t are determined by solving Eq. (8). As mentioned earlier, we seek to find a set of CSWs for a multi-period of times. Considering the time series nature of inputs and outputs in multiple periods, the deliberated problem will be a stochastic optimization problem, i.e. its parameters are determined as time series with unknown distributions. Consider any of the DMUs, e.g. DMU_O, and its ith input measure. This input variable takes different values in each time-period, i.e. $x_{i0}^{1}, x_{i0}^{2}, \dots,$ and x_{i0}^{T}. These values form a time series $\widetilde{x}_{i10} = (x_{i0}^{1}, x_{i0}^{2}, \dots, x_{i0}^{T})$. In fact, $\widetilde{x}_{i0}$ is a random variable with an unknown statistical distribution and its values differ over time periods. The multi-period CSWs problem is formulated as follows by substituting these random variables:

$$\begin{aligned} & \max \frac{1}{1-\theta_l} \sum_{j=1}^{n} \sum_{r=1}^{s} u_r^{c} \widetilde{y}_{rj} - \frac{1}{\varphi_u - 1} \sum_{j=1}^{n} \sum_{i=1}^{m} v_i^{c} \widetilde{x}_{ij} \\ & s.\ t. \\ & \text{(i) } \sum_{r=1}^{s} u_r^{c} \widetilde{y}_{rj} \leqslant 1, j = 1, 2, \dots, n \\ & \text{(ii) } \sum_{i=1}^{m} v_i^{c} \widetilde{x}_{ij} \geqslant 1, j = 1, 2, \dots, n \\ & u_r^{c} \geqslant 0,\ r = 1, 2, \dots, s \\ & v_i^{c} \geqslant 0,\ i = 1, 2, \dots, m \end{aligned} \quad (9)$$

The above problem is a stochastic programming model. Considering the objective function of Eq. (9), this stochastic objective function is transformed into a bi-objective problem of maximizing its mean and simultaneously minimizing its variance. Therefore:

$$\max M=\max \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s}u_r^c\bar{y}_{rj}-\frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m}v_i^c\bar{x}_{ij}$$
$$\min V=\min \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r^c)^2\delta_{rj}^2+\frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i^c)^2\sigma_{ij}^2 \tag{10}$$

The variance relation in Eq. (10) is obtained considering independence of inputs and outputs in the objective function of Eq. (9). Usually, original DEA models assume complete independence of inputs and outputs [26,32]. If the assumption of independence holds, the above relation can be used since the covariance of the variables is zero. However, if the variables are correlated, methods like principal component analysis (PCA) or independent component analysis (ICA) can be used to produce uncorrelated linear combination of original inputs and outputs [1–3,38,32]. The above relation can be used to produce independent inputs and outputs using PCA or ICA.

The stochastic model is then transformed into a bi-objective model using the mean–variance concept taken from the theory of portfolio management developed by Markowitz [44]. Now, consider the stochastic inequality shown in constraint (i) of Eq. (9). There is no easy way to handle this inequality since this constraint is stochastic. In this paper, the stochastic constraints are transformed into linear constraints using the concept of Chebyshev inequality bounds.

Suppose that X is a random variable with mean μ and standard deviation σ, and that its statistical distribution is unknown. Chebyshev inequality bounds state that with a probability of at least $1-1/k^2$, this random variable lies in the interval of $(\mu-k\sigma,\ \mu+k\sigma)$. For $k=1/\sqrt{\alpha}$, an approximation of the $100(1-\alpha)\%$ confidence interval of X can be obtained. Now, for the random variable $\tilde{y}_{rj}$ with mean $\bar{y}_{rj}$ and standard deviation δ_{rj}, its approximation of the $100(1-\alpha)\%$ confidence interval is obtained as $(\bar{y}_{rj}-k\delta_{rj},\ \bar{y}_{rj}+k\delta_{rj})$. This interval contains the random value of $\tilde{y}_{rj}$, and by substituting these confidence intervals in the considered constraint, its interval equivalent is obtained as follows with a probability of at least $100(1-\alpha)\%$:

$$\sum_{r=1}^{s}u_r^c(\bar{y}_{rj}-k\delta_{rj},\ \bar{y}_{rj}+k\delta_{rj})\leqslant 1,\ j=1,2,\ ...,n \tag{11}$$

Now, using interval numbers arithmetic [46], the above inequality is converted into:

$$\left(\sum_{r=1}^{s}u_r^c(\bar{y}_{rj}-k\delta_{rj}),\ \sum_{r=1}^{s}u_r^c(\bar{y}_{rj}+k\delta_{rj})\right)\leqslant 1,\ j=1,2,\ ...,n \tag{12}$$

Ishibuchi and Tanaka [29] has proposed an ordering relation among interval numbers. If $A=[a_1,b_1]$ and $A=[a_2,b_2]$, they state that $A\leqslant B$ if $b_1\leqslant b_2$ and $(a_1+b_1/2)\leqslant(a_2+b_2/2)$. Applying this ordering relation to Eq. (12), the following inequalities can be obtained:

$$\begin{cases}\sum_{r=1}^{s}u_r^c(\bar{y}_{rj}+k\delta_{rj})\leqslant 1\\ \sum_{r=1}^{s}u_r^c\bar{y}_{rj}\leqslant 1\end{cases},\ j=1,2,\ ...,n \tag{13}$$

The constraints (ii) are reduced to the following interval constraints by using a similar reasoning:

$$\begin{cases}\sum_{i=1}^{m}v_i^c(\bar{x}_{ij}-k\sigma_{ij})\geqslant 1\\ \sum_{i=1}^{m}v_i^c\bar{x}_{ij}\geqslant 1\end{cases},\ j=1,2,\ ...,n \tag{14}$$

Applying Eqs. (10), (13), and (14), and using the mean–variance idea of Markowitz [44] that later was extended by Amoozad Mahdiraji et al. [6] to solve multi-objective stochastic programming problems, the stochastic multi-period CSWs problem in Eq. (9) is transformed into the following non-linear bi-objective problem:

$$\max M=\max \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s}u_r^c\bar{y}_{rj}-\frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m}v_i^c\bar{x}_{ij}$$
$$\min V=\min \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r^c)^2\delta_{rj}^2+\frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i^c)^2\sigma_{ij}^2$$
$$s.\ t.$$
$$\begin{cases}(i.\ 1)\ \sum_{r=1}^{s}u_r^c(\bar{y}_{rj}+k\delta_{rj})\leqslant 1\\ (i.\ 2)\ \sum_{r=1}^{s}u_r^c\bar{y}_{rj}\leqslant 1\end{cases},\ j=1,2,...,n$$
$$\begin{cases}(ii.\ 1)\ \sum_{i=1}^{m}v_i^c(\bar{x}_{ij}-k\sigma_{ij})\geqslant 1\\ (ii.\ 2)\ \sum_{i=1}^{m}v_i^c\bar{x}_{ij}\geqslant 1\end{cases},\ j=1,2,...,n$$
$$u_r^c\geqslant 0,\ r=1,2,...,s$$
$$v_i^c\geqslant 0,\ i=1,2,...,m \tag{15}$$

Considering constraints (i.1) and (i.2) in Eq. (15), it is evident that $\sum_{r=1}^{s}u_r^c\bar{y}_{rj}\leqslant\sum_{r=1}^{s}u_r^c(\bar{y}_{rj}+k\delta_{rj})\leqslant 1$. Correspondingly, (i.2) is a subset of (i.1); thus, the (i.2) constraints are redundant. With a similar argument, the (ii.2) constraints are likewise redundant and can be eliminated. In this model, $\bar{x}_{ij}$ and $\bar{y}_{rj}$, along with σ_{ij} and δ_{rj} act as fixed parameters, where

$$\bar{y}_{rj}=\sum_{t=1}^{T}y_{rj}^t/T$$

$$\delta_{rj}=\sum_{t=1}^{T}(y_{rj}^t-\bar{y}_{rj})^2/T-1$$

$$\bar{x}_{ij}=\sum_{t=1}^{T}x_{ij}^t/T,\quad \text{and}$$

$$\sigma_{ij}=\sum_{t=1}^{T}(x_{ij}^t-\bar{x}_{ij})^2/T-1$$

while v_i^c and u_r^c are decision variables. As a result, the model is derived as:

$$\max M=\max \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s}u_r^c\bar{y}_{rj}-\frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m}v_i^c\bar{x}_{ij} \tag{16}$$
$$\min V=\min \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r^c)^2\delta_{rj}^2+\frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i^c)^2\sigma_{ij}^2$$

s.t.

$$S:\begin{cases}\sum_{r=1}^{s}u_r^c(\bar{y}_{rj}+k\delta_{rj})\leqslant 1,\ j=1,2,...,n\\ \sum_{i=1}^{m}v_i^c(\bar{x}_{ij}-k\sigma_{ij})\geqslant 1,\ j=1,2,...,n\\ u_r^c\geqslant 0,\ r=1,2,...,s\\ v_i^c\geqslant 0,\ i=1,2,...,m\end{cases}$$

This non-linear bi-objective problem is also solved via a fuzzy approach. Since M and V are incommensurable criteria, they are transformed into membership functions to allow their summation in a single objective. To find the membership functions of M and V as objectives of the problem; initially, these two problems presented below are solved:

$$M^+=\max M$$
$$s.\ t.$$
$$(u,v)\in S \tag{17a}$$

$$V^-=\max\ V$$
$$s.\ t.$$
$$(u,v)\in S \tag{17b}$$

Model (17a) represents the ideal value of the mean, where greater is

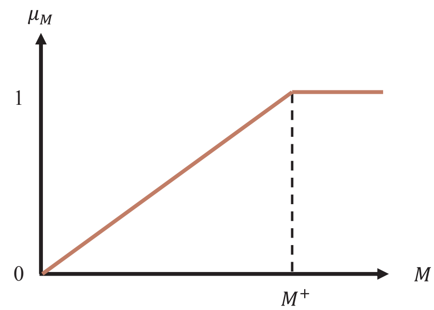

Fig. 3. Membership function of the first objective in the multi-period problem.

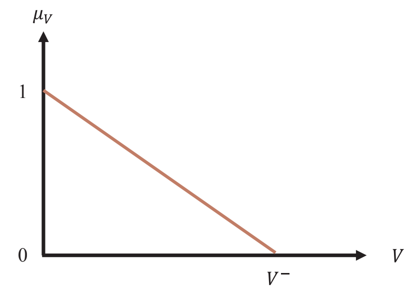

Fig. 4. Membership function of the second objective in the multi-period problem.

better, while Model (17b) represents the nadir ideal value of the variance, where smaller is better. Additionally, M^- which is the nadir ideal value of the mean objective is considered equal to zero, as its worst case, while the ideal value of the variance, i.e. V^+, as its lowest value is considered equal to zero (since variance is always positive). It is notable that since the objective function V is a polynomial of second order and considering the convexity of S (since it is constructed of convex linear constraints), V^+ and V^- will be global optima. Consequently, the M objective function is transformed into the following membership function:

$$\mu_M = \begin{cases} 0, & \text{if } M < 0 \\ \frac{M}{M^+}, & \text{if } 0 \leqslant M \leqslant M^+ \end{cases} \tag{18}$$

This membership function is illustrated in Fig. 3.

In the same manner, the V objective is transformed into the following membership function:

$$\mu_V = \begin{cases} 0, & \text{if } V > V^- \\ \frac{V^- - V}{V^-}, & \text{if } 0 \leqslant V \leqslant V^- \end{cases} \tag{19}$$

This membership function is illustrated in Fig. 4.

Along these lines, the final multi-period CSWs problem model is formulated as follows:

$$\begin{aligned} \max\ & \frac{1}{M^+}\left[\frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r^c \bar{y}_{rj} - \frac{1}{\varphi_u - 1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i^c \bar{x}_{ij}\right] \\ & + \frac{1}{V^-}\left[V^- - \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r^c)^2\delta_{rj}^2 - \frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i^c)^2\sigma_{ij}^2\right] \\ s.\ t\ & \\ & 0 \leqslant \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r^c \bar{y}_{rj} - \frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i^c \bar{x}_{ij} \leqslant M^+ \\ & \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r^c)^2\delta_{rj}^2 + \frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i^c)^2\sigma_{ij}^2 \geqslant 0 \\ & \sum_{r=1}^{s} u_r^c(\bar{y}_{rj} + k\delta_{rj}) \leqslant 1,\ j = 1, 2, \dots, n \\ & \sum_{i=1}^{m} v_i^c(\bar{x}_{ij} - k\sigma_{ij}) \geqslant 1,\ j = 1, 2, \dots, n \\ & u_r^c \geqslant 0,\ \ r = 1, 2, \dots, s \\ & v_i^c \geqslant 0,\ \ i = 1, 2, \dots, m \end{aligned} \tag{20}$$

Where the first and second constraints results from the fact that $\mu_M, \mu_V \leqslant 1$. Note that in the above formulation, it is expected that the lower bounds of inputs take some negative values, i.e. for some $i \in \{1, 2, \dots, m\}$, the values of $(\bar{x}_{ij} - k\sigma_{ij})$ can be negative. In this case, the method of handling negative inputs by their absolute values, proposed by Cheng et al. [14] is applicable.

The above problem is to maximize a non-linear programming problem consisting of non-linear variables $(u_r^c)^2$ and $(v_i^c)^2$. To prove the concavity of the objective function, considering the maximization objective, and the vector of variables $(u_1^c, \dots, u_r^c, v_1^c, \dots, v_m^c)$ respectively, the Hessian matrix of the objective function can be calculated as:

$$H = \begin{bmatrix} -\frac{2\delta_{1j}^2}{(1-\theta_l)^2} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{2\delta_{sj}^2}{(1-\theta_l)^2} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -\frac{2\sigma_{1j}^2}{(\varphi_u-1)^2} & \cdots & \vdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & -\frac{2\sigma_{mj}^2}{(\varphi_u-1)^2} \end{bmatrix}$$

This is a diagonal matrix with negative diagonal elements and therefore is a negative definite matrix. Thus, the objective function is concave. Since the constraints of the problem are linear, their convexity is straightforward. Therefore, Eq. (20) is to maximize a concave function over a convex feasible space and its global optimum can be determined easily using optimization packages such as Lingo or MATLAB. Furthermore, the following theorem illustrates that the model in Eq. (20) is unit invariant.

Theorem1.. *Rescaling the inputs and outputs do not change the optimal solution of the model in Eq. (20).*

Proof.. *See* Appendix A. □

Since the inputs and outputs random variables are transformed into interval numbers, using confidence intervals, the resulted efficiency scores are therefore interval numbers. According to the ordering relations of Ishibuchi and Tanaka [29], for two interval numbers $E_1 = [\underline{E}_1, \bar{E}_1]$ and $E_2 = [\underline{E}_2, \bar{E}_2]$, $E_1 < E_2$ if $\bar{E}_1 < \bar{E}_2$ and $E_{1c} < E_{2c}$, where E_{1c} is the mean value of E_1. Consequently, after finding the optimal weights u_r^c and v_i^c, the mean relative efficiency of DMU_0 is evaluated as:

$$E_{0c} = \frac{\sum_{r=1}^{s} u_r^c \bar{y}_{r0}}{\sum_{i=1}^{m} v_i^c \bar{x}_{i0}} \tag{21}$$

And its upper bound efficiency is calculated as:

$$\bar{E}_0 = \frac{\sum_{r=1}^{s} u_r^c(\bar{y}_{r0} + k\delta_{r0})}{\sum_{i=1}^{m} v_i^c(\bar{x}_{i0} - k\sigma_{i0})} \tag{22}$$

Now, the multi-period CSWs based efficiency of DMUs can be compared and ranked based on Eqs. (21) and (22). It is worth noting here that considering the randomness of the CSWs multi-period problem, due to inputs and outputs variance, it is logical to find a non-crisp solution for this problem, since reaching a crisp solution with uncertain data seems unrealistic.

This can be summarized in the following steps to facilitate application of the above described procedure for evaluating the multi-period efficiency of the DMUs based on a CSWs:

- **Step 1.** Identify decision making units, input and output measures, time periods in which the DMUs are evaluated, and threshold values of θ_l and ϕ_u.

- **Step 2.** If inputs and outputs are rationally independent, then go to step 4, otherwise if there are signs of dependence among inputs and/or outputs, go to Step 3.
- **Step 3.** Using PCA or ICA, transform the dependent inputs and/or outputs into a set of independent components in each time-period and go to Step 4.
- **Step 4.** Compute the mean and variance matrices of inputs and outputs for the DMUs over the considered time periods.
- **Step 5.** Formulate and solve problems in Eqs. (17a) and (17b) to determine the optimal values of M^{+} and V^{-}.
- **Step 6.** Formulate and solve the problem in Eq. (20) to find the CSWs $(v_1^c, v_2^c, ..., v_m^c, u_1^c, u_2^c, ..., u_s^c)$.
- **Step 7.** Determine the mean and upper bound relative efficiency of the DMUs, using Eqs. (21) and (22).

5. Case study

HCI bank is a medium-size bank founded in Iran in 2008. As its competitors are mainly well-established banks, HCI needs to operate more efficiently (i.e. reduce its expenses and increase its productivity) to compete. DEA has been used widely in the banking industry to measure efficiency [41,21]. In this study, we consider a six-month time horizon and apply the guidelines proposed by Berger and Humphrey [10] and [43] and select personnel costs (I_1), general and administrative costs (I_2), account-related costs (I_3), and rental expenses (I_4) as inputs and sum of deposits (O_1), loans (O_2), securities (O_3), and branch income (O_4) as outputs as shown in Table 1.

Considering a six-month period, for four inputs and four outputs at 125 branches, a database with 6804 records is constructed to capture monthly transactions. It is necessary to determine $M+$ and $V-$ using Eqs. (17a) and (17b) when applying Model (20). The senior managers estimated $\theta_l = 0.3$ (i.e. the desirability of the input-oriented efficiency score below 0.3 is 0) and therefore, $\varnothing_u = 3.33$ (i.e. the desirability of the output-oriented efficiency score above 3.33 is 0). These values were determined through in-depth discussions with senior management and the amount of 0.3 was obtained as they believed that there is no justification for a branch to act with an efficiency below threshold value. However, the model can be deployed with any other value for θ_l. Solving models (17a) and (17b), the values are obtained as:

$$M^{-} = 0 \quad \text{and} \quad M^{+} = 66.5339$$

$$V^{-} = 1.263402 \quad \text{and} \quad V^{+} = 0$$

The values of M^{-} and M^{+} are found easily using the MATLAB Linprog command, while V^{-} and V^{+} are determined using the MATLAB Optimtool toolbox. Next, the problem in Eq. (20) is formulated and solved using the sequential quadratic programming approach of the MATLAB Optimtool. To avoid some inputs or outputs being dominated by other, the following assurance region type constraints [11] are used:

$$\frac{u_r}{u_p} \leqslant 3,\ r,\ p = 1, 2, 3, 4,\ r \neq p$$

and

$$\frac{v_i}{v_k} \leqslant 3,\ i,\ k = 1, 2, 3, 4,\ i \neq k$$

These constraints are added to the constraints in Model (20) and the obtained CSWs are summarized in Table 2.

Table 1
Input and output measures.

Inputs	Outputs
Personnel costs (I_1)	Sum of deposits (O_1)
General and administrative costs (I_2)	Loans (O_2)
Account-related costs (I_3)	Securities (O_3)
Rental expenses (I_4)	Branch income (O_4)

Table 2
Input and output measures for the CSWs.

Inputs		Outputs	
I_1	$687(10^{-6})$	O_1	$1.32(10^{-6})$
I_2	$1710(10^{-6})$	O_2	$2.70(10^{-6})$
I_3	$569(10^{-6})$	O_3	$4.37(10^{-6})$
I_4	$569(10^{-6})$	O_4	$0.901(10^{-6})$

Applying these weights in models (21) and (22), the multi-period CSWs based mean and upper bound relative efficiency of DMUs are illustrated in Table 3. For instance, considering the mean input–output vector of DMU1099 as $(\bar{x}_{11099}, \bar{x}_{21099}, \bar{x}_{31099}, \bar{x}_{41099}) = (599, 121, 15957, 0)$ and $(\bar{y}_{11099}, \bar{y}_{21099}, \bar{y}_{31099}, \bar{y}_{41099}) = (732604, 652260, 241243, 61146)$, the mean efficiency is calculated as:

$$E_{1c} = \frac{[1.32(732604) + 2.70(652260) + 4.37(241243) + 0.901(61146)]10^{-6}}{[6.87(599) + 17.10(121) + 5.69(15957) + 5.69(0)]10^{-4}} = \frac{3.84}{9.69} = 0.40$$

Similarly, considering the input vector $(\bar{x}_{11099} - 3.16\sigma_{11099}, \bar{x}_{21099} - 3.16\sigma_{21099}, \bar{x}_{31099} - 3.16\sigma_{31099}, \bar{x}_{41099} - 3.16\sigma_{41099}) = (396, 195, 17501, 0)$ and the output vector $(\bar{y}_{11099} + 3.16\delta_{11099}, \bar{y}_{21099} + 3.16\delta_{21099}, \bar{y}_{31099} + 3.16\delta_{31099}, \bar{y}_{41099} + 3.16\delta_{41099}) = (1285416, 786765, 637349, 211443)$, the upper bound efficiency is calculated as:

$$\bar{E}_1 = \frac{[1.32(1285416) + 2.70(786765) + 4.37(2637349) + 0.901(211443)]10^{-6}}{[6.87(396) + 17.1(195) + 5.69(17501) + 5.69(0)]10^{-4}} = \frac{6.8}{10.6} = 0.64$$

Table 3 also present the results of the aggregated and network connected models of Kao and Liu [37]. Since none of the existing methods can determine the CSWs in multi-period problems, required comparisons are examined between the proposed method with some existing multi-period models to exhibit the improvement of the discrimination power.

Improvement in the discrimination power of the CSWs model can be seen by comparing the proposed CSW's based results with the aggregated and network connected models. The aggregated model includes 7 DMUs with efficiency of 1, while this number is observed by 16 DMUs in the network connected model. Admittedly, the aggregated model classifies 5.6% of the DMUs as efficient without any discrimination between them. Compared with the network connected model, 12.8% of the DMUS are classified as efficient (representing a weak discrimination power). On the other hand, there is only a single DMU with an upper bound efficiency of 1 by the proposed method, i.e. only 0.8% of the DMUs are *indiscriminable*.

Another feature of the proposed method is that there is a DMU with an upper efficiency of 1 (fully efficient), which is a general requirement for CSWs problems as argued by Roll et al. [54] and Golany and Yu [25].

Fig. 5 illustrates the scatter diagram of mean efficiencies in the proposed CSWs model with the results of multi-period efficiencies. Fig. 5a compares the mean and upper bound efficiencies with the aggregated model, while Fig. 5b illustrates the comparison with the network connected model.

This figure reveals a decrease in the mean efficiencies. Hence, it can be argued that the proposed CSWs model dramatically increases the discrimination power of the multi-period efficiency appraisal. Usually, the aim of CSWs is to find a CSWs to evaluate the DMUs' efficiencies. These CSWs will increase the differentiation of the DMUs' efficiencies. The low efficiency scores of DMUs in this case may be due to the weak performance of DMUs in inputs and outputs and the strict approach of CSWs in evaluating the DMUs' efficiencies with respect to ordinal multi-period DEA models which improve the model's discrimination among the DMUs. Table 4 summarizes the Spearman rank correlations among

Table 3
Multi-period CSWs based relative efficiencies.

DMU	Proposed CSWs based efficiencies		Kao and Liu [37]	
	E_{0c}	$\bar{E}_0$	Aggregated model	Connected network model
1099	0.40	0.64	1.00	1.00
1101	0.06	0.09	1.00	1.00
1102	0.45	1.00	1.00	1.00
1103	0.29	0.59	0.94	1.00
1104	0.05	0.11	0.33	0.45
1105	0.12	0.15	0.37	0.58
1106	0.14	0.15	0.45	0.87
1107	0.10	0.14	0.32	0.45
1108	0.06	0.24	0.53	1.00
1109	0.05	0.06	0.22	0.24
1110	0.11	0.14	0.34	0.64
1111	0.17	0.36	0.56	0.84
1112	0.44	1.00	1.00	1.00
1113	0.06	0.18	0.24	0.35
1201	0.07	0.10	0.30	0.41
1202	0.11	0.17	0.44	0.49
1203	0.07	0.08	0.68	0.99
1204	0.14	0.18	0.99	1.00
1206	0.11	0.16	0.42	0.61
1207	0.06	0.09	0.20	0.22
1208	0.14	0.20	0.35	0.78
1209	0.14	0.26	0.36	0.51
1210	0.32	0.59	1.00	1.00
1211	0.05	0.12	0.29	0.33
1212	0.03	0.12	0.17	1.00
1213	0.01	0.03	0.10	0.13
1214	0.00	0.01	1.00	0.00
1301	0.05	0.10	0.23	0.31
1302	0.10	0.15	0.28	0.96
1303	0.06	0.08	0.27	0.48
1304	0.06	0.07	0.23	0.38
1305	0.11	0.12	0.41	0.76
1306	0.06	0.06	0.23	0.76
1307	0.09	0.08	0.36	1.00
1308	0.05	0.08	0.18	0.20
1309	0.09	0.07	0.32	1.00
1310	0.09	0.07	0.32	0.88
1311	0.08	0.07	0.31	0.65
1312	0.07	0.10	0.28	0.40
1313	0.03	0.10	0.15	0.15
1401	0.04	0.07	0.18	0.20
1402	0.08	0.11	0.24	0.37
1403	0.04	0.05	0.16	0.29
1404	0.05	0.07	0.18	0.23
1405	0.08	0.14	0.25	0.44
1408	0.04	0.10	0.33	0.38
1501	0.09	0.11	0.30	0.45
1502	0.07	0.12	0.17	0.29
1503	0.08	0.09	0.27	0.40
1504	0.09	0.33	0.28	0.41
1505	0.06	0.10	0.35	0.41
1506	0.12	0.12	0.34	0.38
1507	0.10	0.14	0.21	0.23
1508	0.05	0.07	0.44	0.64
1509	0.10	0.12	0.28	0.47
1601	0.08	0.12	0.23	0.24
1602	0.10	0.13	0.28	0.65
1603	0.15	0.17	0.32	0.48
1604	0.15	0.33	0.30	0.53
1605	0.10	0.14	0.47	0.91
1606	0.05	0.08	0.44	0.47
1406	0.08	0.26	0.44	0.54
1407	0.11	0.18	0.21	0.25
1701	0.11	0.19	0.60	1.00
1702	0.07	0.08	0.19	0.75
1703	0.10	0.13	0.30	0.60
1704	0.08	0.10	0.29	0.55
1705	0.07	0.08	0.24	0.49
1706	0.03	0.04	0.15	0.59
1707	0.08	0.05	0.25	0.45

Table 3 (*continued*)

DMU	Proposed CSWs based efficiencies		Kao and Liu [37]	
	E_{0c}	$\bar{E}_0$	Aggregated model	Connected network model
1708	0.09	0.12	0.29	0.43
1709	0.13	0.16	0.38	0.78
1710	0.02	0.03	0.11	0.51
1711	0.03	0.07	0.20	0.27
1712	0.01	0.02	0.06	0.07
1713	0.02	0.04	0.12	0.17
1801	0.10	0.14	0.26	0.48
1802	0.12	0.14	0.49	0.56
1803	0.05	0.07	0.26	0.47
1804	0.25	0.20	0.69	1.00
1805	0.10	0.13	0.31	0.54
1806	0.09	0.14	0.36	0.72
1807	0.09	0.12	0.34	0.92
1808	0.05	0.04	0.25	1.00
1809	0.08	0.11	0.28	0.92
1901	0.07	0.11	0.28	0.37
1902	0.05	0.10	0.23	0.26
1903	0.12	0.18	0.45	0.50
1904	0.09	0.12	0.31	0.34
1905	0.05	0.06	0.22	0.38
1906	0.14	0.18	0.48	0.65
1907	0.07	0.09	0.32	0.63
1908	0.31	0.47	1.00	1.00
1909	0.13	0.28	0.50	0.68
1910	0.08	0.11	0.26	0.29
1911	0.07	0.10	0.26	0.28
1912	0.07	0.12	0.38	0.45
2001	0.04	0.05	0.16	0.22
2002	0.08	0.09	0.28	0.52
2003	0.08	0.10	0.27	0.40
2004	0.08	0.11	0.27	0.40
2005	0.04	0.08	0.17	0.21
2006	0.04	0.17	0.22	0.48
2007	0.05	0.10	0.21	0.24
2009	0.04	0.06	0.18	0.22
2010	0.05	0.03	0.25	0.50
2011	0.06	0.07	0.26	0.44
2012	0.04	0.06	0.17	0.22
2101	0.08	0.12	0.36	0.42
2102	0.11	0.13	0.33	0.55
2103	0.13	0.21	0.46	0.71
2104	0.09	0.14	0.32	0.40
2105	0.11	0.19	0.37	0.56
2106	0.06	0.11	0.25	0.30
2107	0.05	0.08	0.22	0.27
2108	0.10	0.18	0.32	0.43
2201	0.05	0.06	0.82	1.00
2202	0.09	0.11	0.28	0.36
2203	0.06	0.09	0.23	0.30
2204	0.05	0.08	0.17	0.28
2205	0.05	0.07	0.23	0.26
2206	0.07	0.08	0.46	0.53
2207	0.09	0.09	0.35	0.73
2208	0.08	0.11	0.26	0.36
2209	0.14	0.21	0.36	0.56

different efficiencies reported in Table 3. The third column shows the obtained *p*-values of significance of correlations.

As shown in Table 4, all pairwise rank correlations are significant at a significance level of 95%. On the other hand, as it is expected, the greatest correlation of about 81% is between the mean and upper-bound efficiencies in the proposed method. Then, there is a correlation of 70.3% between the aggregated and connected network models of Kao and Liu [37]. The correlations in Table 4 show both the mean and upper-bound efficiencies have greater correlation with the aggregated model, which is more evidence for greater discrimination power in the aggregated model. For more explanation on results, Table 5 illustrates the hypothesis tested on significance of differences among mean

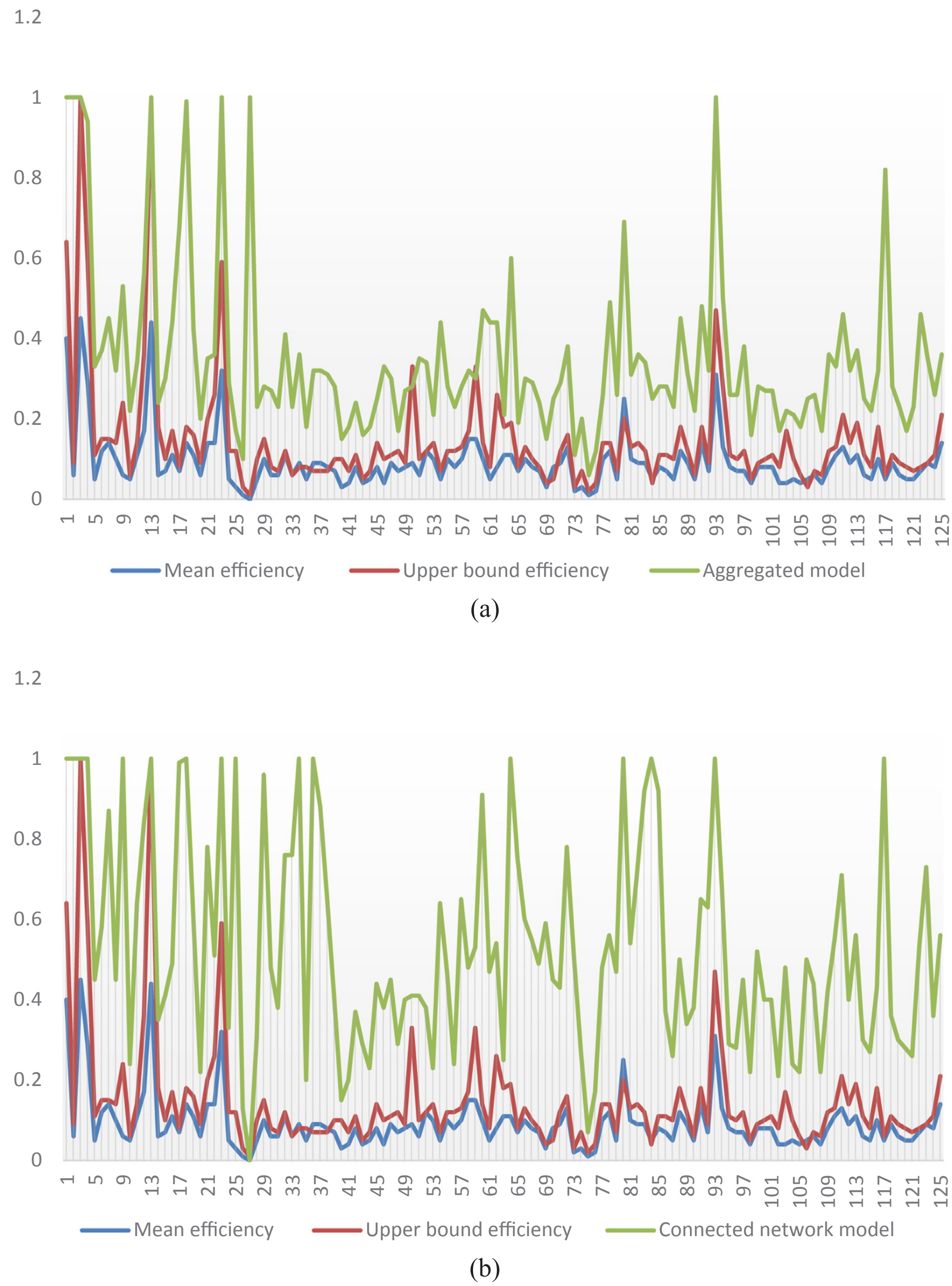

Fig. 5. Scatter diagram of mean efficiencies.

Table 4
Spearman rank correlation among results.

Pairs compared	Spearman rank correlation	Significance (P-value)
$E_{0c} \sim \bar{E}_0$	0.809	3.58E−30
E_{0c} ~ Aggregated model	0.678	3.52E−18
E_{0c} ~ Connected network	0.579	1.44E−12
$\bar{E}_0$ ~ Aggregated model	0.588	5.81E−13
$\bar{E}_0$ ~ Connected network	0.431	5.36E−7
Aggregated model ~ Connected network	0.703	6.75E−20

efficiencies of different methods.

The results show significant differences between the means of performance efficiency scores obtained by the paired methods in all rows. Also, the upper and lower boundaries establishing "the intervals of the differences" show negative figures. The interval related to E_{0c} − Connected method results (−0.49273, −0.38439) suggests that the efficiencies obtained by the network connected model are 38–49 percent above the efficiencies obtained by the mean efficiency scores of the proposed method. The network connected model also obtains efficiencies about 32–44 percent above the upper-bound efficiencies. This can be interpreted as the optimistic behavior of the network connected model, as it might overestimate the efficiencies. These differences are lower for the aggregated model.

Considering Tables 4 and 5, it can be argued that the proposed method has an acceptable correlation with other methods, while it produces more warily estimation of efficiencies as it is more compatible with the uncertainty of multi-period problems.

The final point regarding the results is about the low values of efficiency for some DMUs. The obtained results mainly illustrate the pattern of inputs and outputs. Consider the trends of the inputs and outputs for DMU1121 in Fig. 6. Fig. 6a–d are related to I_1, I_2, I_3, and I_4,

Table 5
Paired *t*-test of mean differences among different methods.

Pairs	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	99% Confidence Interval of the Difference				
				Lower	Upper			
$E_{0c}-\bar{E}_0$	−0.05384	0.08588	0.00768	−0.07393	−0.03375	−7.009	124	0.000
E_{0c}−Aggregated	−0.26040	0.16964	0.01517	−0.30009	−0.22071	−17.162	124	0.000
E_{0c}−Connected	−0.43856	0.23150	0.02071	−0.49273	−0.38439	−21.180	124	0.000
$\bar{E}_0$−Aggregated	−0.20656	0.15941	0.01426	−0.24386	−0.16926	−14.487	124	0.000
$\bar{E}_0$−Connected	−0.38472	0.23686	0.02119	−0.44014	−0.32930	−18.160	124	0.000
Aggregated – Connected	−0.17816	0.21050	0.01883	−0.22741	−0.12891	−9.463	124	0.000

and Fig. 6e–h are related to O_1, O_2, O_3, and O_4, respectively.

The cases above are illustrated based on data related to DMU 1211 whose efficiency is calculated about 0.05. Concentrating on the inputs, it is inferred that the increasing slopes of the inputs range from 50 to 856; while, the increasing slopes of the outputs range from 0.39 to 0.51 suggests diversity in the ascending trend. Next, the above figure is repeated for data related to DMU 1102 with an efficiency score of 0.45. In this figure again, Fig. 7a–d are related to inputs and Fig. 7e–h are related to outputs.

As shown in Fig. 7, the slopes of the outputs differ from 44,914 to 407,234 compared with slopes of the inputs (32–1895). This hundreds of times difference completely emphasizes the effect of the proposed

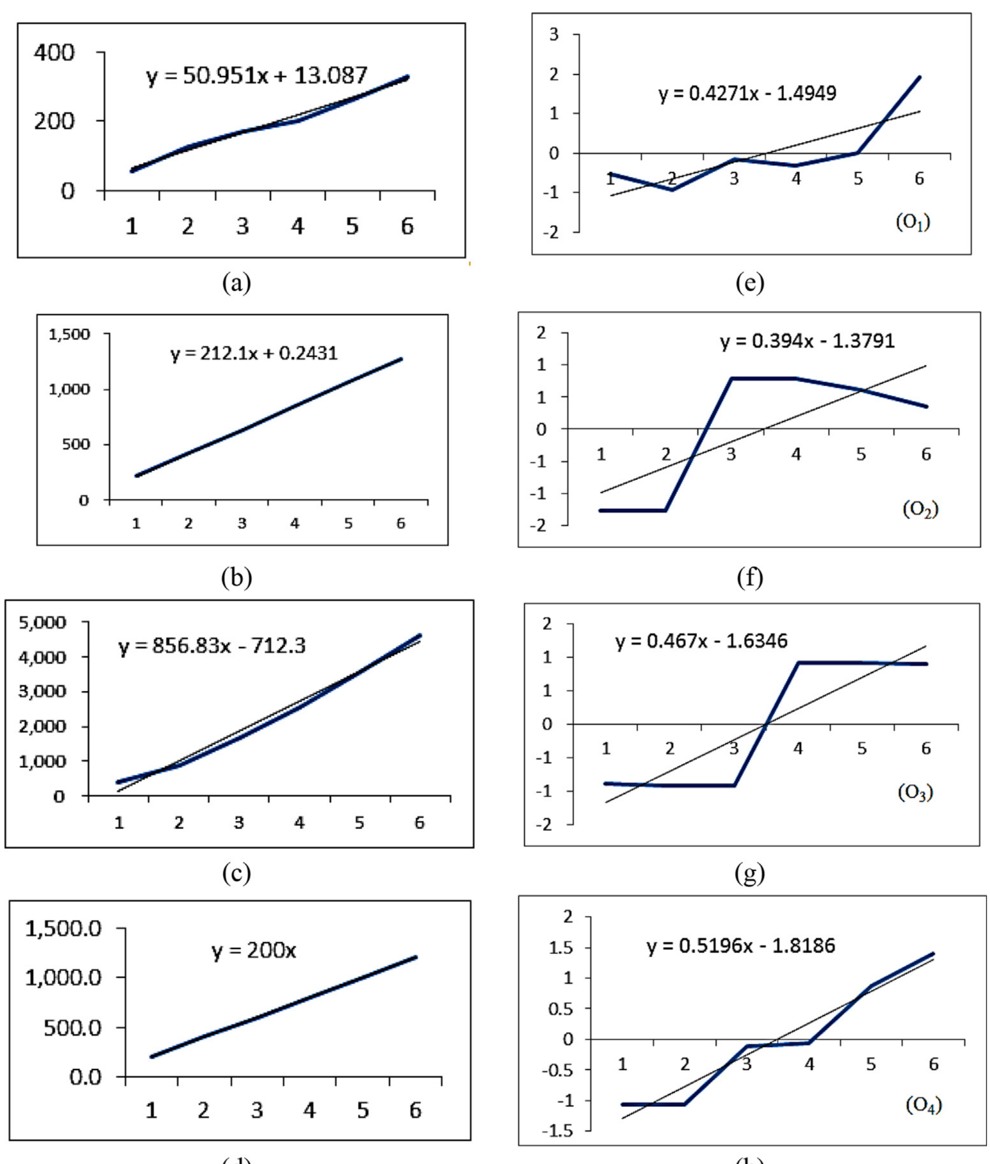

Fig. 6. Inputs/outputs trend for DMU1121 during the considered time period.

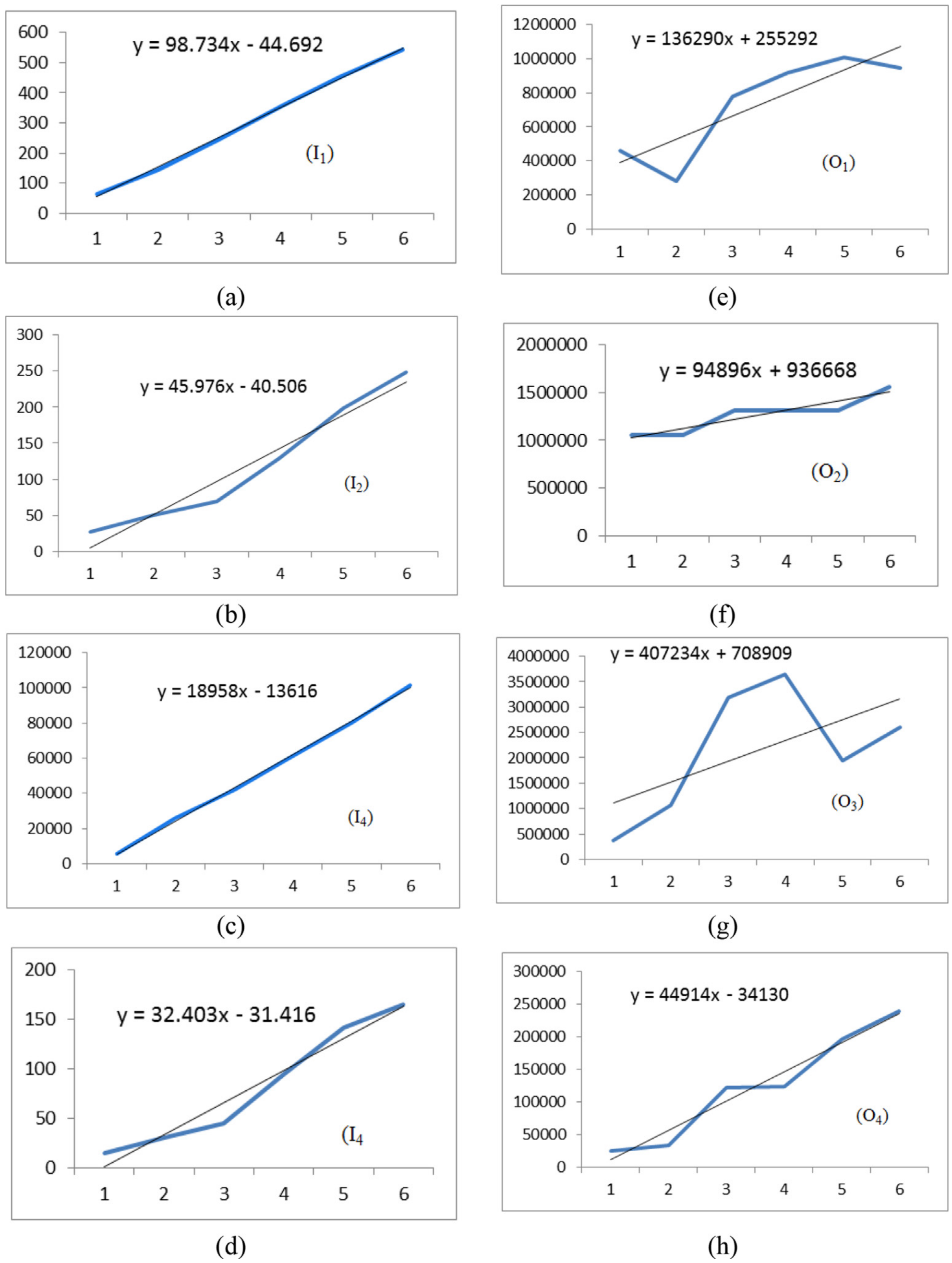

Fig. 7. Inputs/outputs trend for DMU1102 during the considered time period.

method, especially concerning the branches of the HCI Bank with a poor performance.

6. Conclusion

Most managers in large companies must deal with the challenging problem of performance evaluation from time to time. As Simons [59] believed, a good performance evaluation system transforms information, is established based on rational and formal disciplines, usable by managers, and determines directions for performance improvement. DEA is an accepted method for approximating the best production frontier and evaluating the business units according to their distance from this frontier. Beyond its acceptability, DEA is primarily applied in a single time-period, while organizations operate over several continuous time periods. Several researchers have proposed different methods for multi-period DEA which often produce efficiency scores with low discrimination power. With the goal of enhancing the discrimination power of DEA models in a multi-period mode, we propose a method for finding CSWs for DMUs performing over several time periods. To achieve this goal, initially the CSWs problem is formulated as a multi-objective fractional programming. Then, this formulation is extended to several time periods. Following the mean–variance criteria of Markowitz, the problem is formulated to maximize the mean efficiency of all DMUs over the time horizon and simultaneously minimize its variance. Afterwards, the constraints of the problem are handled by using the notion of confidence intervals and interval numbers ranking order. Application of the proposed method is examined in a case of 125 bank branches that are evaluated in six months' time periods. The obtained results show that the proposed model increases the discrimination power of the multi-period DEA model.

This result is clear considering the obtained results that only one of the DMUs reaches a full efficiency in its upper bound. The main features of the proposed method can be summarized as (1) using mean–variance logic to find CSWs in a multi-period DEA problem, (2) using a fuzzy membership based approach for solving the obtained bi-objective non-linear CSWs model, (3) increasing the discrimination power of the model compared with the ordinal multi-period method, (4) the proposed method determines an interval for the efficiency of DMUs that is much more consistent with the uncertainty of a multi-period efficiency appraisal problem. These findings can also be extended to the case of fuzzy multi-period DEA problems, where some inputs and outputs are defined as fuzzy numbers and extending the model by finding CSWs when the DMUs are designed as a network of activities.

Conflict of interest

The authors declare no conflict of interest.

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Appendix A

Proof of theorem 1.

To prove this theorem let us first transform the nonlinear problem of Eq. (20) into an equivalent linear problem. Let $(u_r^c)^2 = U_r^c$ and $(v_i^c)^2 = V_i^c$ The linearized model is obtained as:

$$\max \frac{1}{M^+}\left[\frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r^c \bar{y}_{rj} - \frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i^c \bar{x}_{ij}\right]$$

$$+\frac{1}{V^-}\left[V^- - \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s} U_r^c \delta_{rj}^2 - \frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m} V_i^c \sigma_{ij}^2\right]$$

s.t.

$$0 \leqslant \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r^c \bar{y}_{rj} - \frac{1}{\varphi_u-1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i^c \bar{x}_{ij} \leqslant M^+$$

$$\frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s} U_r^c \delta_{rj}^2 + \frac{1}{(\varphi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m} V_i^c \sigma_{ij}^2 \geqslant 0$$

$$\sum_{r=1}^{s} u_r^c(\bar{y}_{rj} + k\delta_{rj}) \leqslant 1,\ j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} v_i^c(\bar{x}_{ij} - k\sigma_{ij}) \geqslant 1,\ j = 1, 2, \ldots, n$$

$$u_r^c \geqslant 0,\ r = 1, 2, \ldots, s$$

$$v_i^c \geqslant 0,\ i = 1, 2, \ldots, m$$

The dual model of above linear model can be obtained as:

$$\min M^+\theta_1 + V^-\varphi_1$$

s.t.

$$(\theta_1-\theta_2)\left(\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}_{rj}\right) + \sum_{j=1}^{n}\lambda_j(\bar{y}_{rj} + k\delta_{rj}) \geqslant \frac{1}{M^+}\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}_{rj},\ r = 1, 2, \ldots, s$$

$$(\theta_1-\theta_2)\left(\frac{1}{\varphi_u-1}\sum_{j=1}^{n}\bar{x}_{ij}\right) + \sum_{j=1}^{n}\lambda_j(\bar{x}_{ij} - k\sigma_{ij}) \leqslant \frac{1}{M^+}\frac{1}{\varphi_u-1}\sum_{j=1}^{n}\bar{x}_{ij},\ i = 1, 2, \ldots, m$$

$$(\varphi_2-\varphi_1) \leqslant \frac{1}{V^-}$$

$$\theta_1, \varphi_1, \theta_2, \varphi_2 \geqslant 0$$

Now, suppose that an output is rescaled, e.g. $y_r' \to a_r y_r$. Substituting this rescaled variable in the first constraint, if follows that;

$$(\theta_1-\theta_2)\left(\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}'_{rj}\right) + \sum_{j=1}^{n}\lambda_j(\bar{y}'_{rj} + k\delta'_{rj}) \geqslant \frac{1}{M^+}\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}'_{rj}$$

Since $\bar{y}_r' = a_r\bar{y}_r$ and $\delta_r' = a_r\delta_r$, then

$$(\theta_1-\theta_2)\left(\frac{1}{1-\theta_l}\sum_{j=1}^{n}a\bar{y}_{rj}\right) + \sum_{j=1}^{n}\lambda_j a(\bar{y}_{rj} + k\delta_{rj}) \geqslant a\frac{1}{M^+}\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}'_{rj}$$

This inequality can be simplified as:

$$(\theta_1-\theta_2)\left(\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}_{rj}\right)+\sum_{j=1}^{n}\lambda_j(\bar{y}_{rj}+k\delta_{rj})\geqslant\frac{1}{M^+}\frac{1}{1-\theta_l}\sum_{j=1}^{n}\bar{y}'_{rj}$$

This indicates the initial inequality. Correspondingly, this feature is satisfied when an input is rescaled, e.g. $x_i'\to b_i x_r$. Hence, the model is unit invariant.

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