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Solving fuzzy Multidimensional Multiple-Choice Knapsack Problems: The multi-start Partial Bound Enumeration method versus the efficient epsilon-constraint method

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A B S T R A C T

In this paper a new fuzzy Multidimensional Multiple-choice Knapsack Problem (MMKP) is proposed. In the proposed fuzzy MMKP, each item may belong to several groups according to a predefined fuzzy membership value. The total profit and the total cost of the knapsack problem are considered as two conflicting objectives. A mathematical approach and a heuristic algorithm are proposed to solve the fuzzy MMKP. One method is an improved version of a well-known exact multi-objective mathematical programming technique, called the efficient ε-constraint method. The second method is a heuristic algorithm called multi-start Partial-Bound Enumeration (PBE). Both methods are used to comparatively generate a set of non-dominated solutions for the fuzzy MMKP. The performance of the two methods is statistically compared with respect to a set of simulated benchmark cases using different diversity and accuracy metrics.

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1. Introduction

The knapsack problem is best described with a problem where a hitchhiker is searching for a combination of different items that maximizes the total value of all chosen items in his/her knapsack. The multiple objective knapsack problem is a well-known combinatorial optimization problem with a wide range of business applications in capital budgeting [5] and production planning [9,42,44] among others. A good review of the knapsack problem and its associated exact and heuristic algorithms is available in [43].

The Multidimensional Multiple-choice Knapsack Problem (MMKP) (as a generalization of the knapsack problem) is one of the most complex forms of the knapsack problem family. Given a set of knapsacks with limited resources and some disjoint groups of items, the MMKP aims to fill the knapsacks by picking exactly one item from each group, such that the total profit value of the collected items is maximized and none of the resource constraints are violated [12,21,22,29,31,53]. The MMKPs are non-deterministic polynomial-time hard (NP-hard) optimization problems where the computing time grows exponentially with problem size [25]. Several heuristic and meta-heuristic approaches have been proposed to solve the MMKP [2,4,26,52].

One limitation of the conventional 0–1 knapsack problems is the assumption that the knapsack capacity, item weights and profits in the conventional 0–1 knapsack formulations are assumed to be integers. However, many real-world knapsack problems involve imprecise data with vagueness and ambiguity that could not be reasonably set to crisp integers [32]. Furthermore, the quantities of the constraints in many real-world knapsack problems are often dynamic. These conditions drive the need to introduce fuzzy set theory into knapsack models. Fuzzy logic and fuzzy sets can represent ambiguous, uncertain or imprecise information by formalizing inaccuracy in decision making. Fuzzy set algebra developed by Zadeh [58] is the formal body of theory that allows the treatment of imprecise estimates in uncertain environments [see [34] for an overview of fuzzy sets theory and fuzzy mathematics]. Fuzzy sets theory has been well developed and applied to a wide variety of real-world problems [36].

The results obtained from the conventional optimization methods involving deterministic variables exhibit various shortcomings. In particular, the effects of the uncertainty attached to input information is often ignored or taken into account to a limited degree. Fuzzy optimization describes an optimization problem with a fuzzy objective function and fuzzy constraints. Fuzzy optimization methods are widely used to solve real-world optimization problems [see e.g., product-mix decisions [7], asset portfolio management

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[28], vendor selection problem [15], industrial production planning problems [19,55], power plant optimization [16], transportation planning problems [48], water resources management [56], vehicle routing [20], municipal solid waste management [39], and supply chain management [18]].

Different variations of the fuzzy knapsack problem have been proposed in the literature [1,38,45,46,51]. One type of uncertainty has not been considered in the MMKPs. In the conventional fuzzy MMKPs, each item belongs to one and only one group. This assumption may not be realistic in the real-world problems with some uncertainties regarding the membership of the items since one item may belong to more than one group based on some similarity or consistency criteria. In this paper, we model such uncertainties in the MMKPs using fuzzy sets. A new variation of the fuzzy MMKP is proposed and the following two methods are developed to solve the proposed fuzzy MMKP. One method is an improved version of a well-known exact multi-objective mathematical programming technique, called the efficient ε-constraint method. The second method is a heuristic algorithm called multi-start Partial-Bound Enumeration (PBE). Both methods are used to comparatively generate a set of non-dominated solutions for the fuzzy MMKP. The performances of both methods are statistically compared with respect to a set of simulated benchmark cases using different diversity and accuracy metrics.

The MMKP applications have attracted a great deal of interest from the research community since the mid-1990s due to their practical values [50]. The MMKPs frequently encountered in the real-world applications include but are not limited to the following: nurse scheduling [3,27], capital budgeting with bounded multiple-choice constraints [49], quality adaptation and admission control of interactive multimedia systems [35], service level agreement management in telecommunication networks [30], strike force asset allocation management [37], resource allocation [47], warehouses planning in supply chain management [6], employee timetabling [14], open-pit mine production scheduling in the mining industry [42,44], and project selection in the engineering-procurement-construction industry [10], among others [see [8,22,29] for a review of the MMKP algorithms and applications].

The remainder of the paper is organized as follows. In Section 2, we present a brief literature of the concepts of Multi-Objective Decision Making (MODM), the ε-constraint method, and the enumeration algorithm for the optimization problems. The fuzzy MMKP and the solution procedures, including the efficient ε-constraint method and the multi-start PBE method are proposed in Section 3. The experimental results are represented in Section 4. Finally, we end the paper with conclusions and future research directions in Section 5.

2. Literature review

Most real-world decision problems involve multiple and conflicting objectives, sometimes subject to certain constraints. MODM is commonly used to solve these problems characterized by multiple and conflicting objective functions such as maximizing performance while minimizing fuel consumption of a vehicle simultaneously over a feasible set of solutions. AMODM model considers a vector of decision variables, objective functions, and constraints [17,33]. The goal is to optimize the objective functions, while the decision makers (DMs) choose a solution among a set of efficient solutions since MODM problems rarely have a unique solution [59].

Formally, MODM models consider vector of decision variables, objective functions, and constraints. MODM problems rarely have a unique solution and therefore DMs are expected to choose a solution from the set of efficient solutions. Generally, the MODM problem with minimum objective functions can be formulated as follows:

$$(\text{MODM}) \begin{cases} min & f(x) \\ \text{s.t.} & x \in S = \{x \in R^n | g(x) \leq b, x \geq 0\} \end{cases} \tag{1}$$

where $f(x)$ represents k conflicting objective functions, $g(x) \leq b$ represents m constraints, S is the feasible solution space, and x is an n-vector of decision variables, $x \in R^n$ Let us consider the following definitions for the MODM problems [23]:

Definition 1.1. x^* is said to be a *complete optimal solution*, if and only if there exists $x^* \in X$ such that $f_i(x^*) \leq f_i(x)$, $i = 1, \ldots, k$, for all $x \in X$. Also, the *ideal solution*, *superior solution*, or *utopia point* are equivalent terms indicating a complete optimal solution. In general, such a complete optimal solution that simultaneously minimizes all of the objective functions does not always exist when the objective functions conflict with each other.

Definition 1.2. x^* is said to be a *Pareto optimal solution*, if and only if there does not exist another $x \in X$ such that $f_i(x) \leq f_i(x^*)$ for all i and $f_j(x) \leq f_j(x^*)$ for at least one j. The *Pareto optimal solution* is also named differently by different disciplines: *non-dominated solution*, *non-inferior solution*, *efficient solution*, and *non-dominate solution*.

Definition 1.3. x^* is said to be a *weak Pareto optimal solution*, if and only if there does not exist another $x \in X$ such that $f_i(x) \leq f_i(x^*)$, $i = 1, \ldots, k$.

Here, let X^{CO}, X^P, and X^{WP} denote complete optimal, Pareto optimal, and weak Pareto optimal solution sets, respectively. Then we can easily obtain $X^{CO} \subseteq X^P \subseteq X^{WP}$ from the above definitions.

During the decision making process, some preference information articulation from the DM may be required. In addition, the type and the timing of information play a critical role in the decision making process. Under this consideration, the methods for solving MODM problems have been systematically classified into four classes by Hwang and Masud [23]. In one of the aforementioned classes, when there is a posterior articulation of preference information on the priority of the objective functions, generating non-dominated solutions on the Pareto front of the MODM problem is desirable. The methods in this class deal strictly with the constraints and do not consider the preferences of the DMs before the solution procedure. The desired outcome, however, is to narrow the possible courses of actions. They are also called non-dominated solution generation methods. The ε-constraint method is one of such techniques proposed by Chankong and Haimes [11].

2.1. Epsilon-constraint method

A special kind of MODM problem, including linear programming, produces the entire efficient set. These methods can provide a representative subset of the Pareto set which in most cases is adequate. In the epsilon-constrained method, the DM chooses one objective out of n to be optimized. For an MODM problem with minimization objective functions, the remaining objectives are constraints to be less than or equal to some given target values. In mathematical terms, by selecting $f_j(x)$, $j \in \{1, \ldots, k\}$ as the objective function to be optimized, we have the following problem $P(\varepsilon_j)$, $j \in \{1, \ldots, k\}$:

$$min\{f_j(x), j \in \{1, \ldots, k\}; f_i(x) \leq \varepsilon_i, \forall i \in \{1, \ldots, k\}, i \neq j; x \in S\}. \tag{2}$$

where S is the feasible solution space.

One advantage of the ε-constraint method is that it is able to achieve efficient points in a non-convex Pareto curve. Therefore, the DM can vary the upper bounds ε_i to obtain weak Pareto optima. Clearly, this is also a drawback of this method, i.e., the DM has to choose appropriate upper bounds for the ε_i values. Moreover, the method is not particularly efficient as the number of the objective functions increases. Several research works are dedicated to improving the ε-constraint method [40]. The traditional ε-constraint methods try to obtain the efficient solutions through parametrical variations in the right-hand-side of the constrained objective functions. More formally, the ε-constraint method has three points that need attention in its implementation: (a) the calculation of the range of the objective functions over the efficient set, (b) the guarantee of efficiency of the obtained solution, and (c) the increased solution time for problems with several (more than two) objective functions. Mavrotas [40] has tried to address these three issues with a novel version of the ε-constraint method.

2.2. Efficient epsilon-constraint method

The optimal solution of (2) is guaranteed to be an efficient solution only if the values of the slack or surplus variables of the entire associated $k-1$ objective functions' constraints are equal to zero. Under any other conditions, the solution is not assumed to be efficient. Considering the aforementioned issue, the following slack-based model is proposed is proposed by Mavrotas [40]:

$$\begin{aligned}
\min \quad & f_j(x) + \beta \times (s_1 + \ldots + s_{j-1} + s_{j+1} + \ldots + s_k) \\
\text{s.t.} \quad & \\
& f_i(x) + s_i = \varepsilon_i, \quad \forall i \in \{1, \ldots, k\}, i \neq j. \\
& X \in S. \\
& s_i \in \Re^+ \quad \forall i \in \{1, \ldots, k\}, i \neq j.
\end{aligned} \tag{3}$$

where β is a small number chosen usually between 0.001 and 0.000001. The above formulation of the ε-constraint method produces only efficient solutions. Some consideration about commensurability may be desirable with respect to the objective function. Therefore, the objective function will be $f_j(x) + \beta \times (s_1/r_1 + \ldots + s_{j-1}/r_{j-1} + s_{j+1}/r_{j+1} + \ldots + s_k/r_k)$, where r_i, $i = 1, \ldots, k$, $i \neq j$ represents the range of the objective i which has been calculated from the payoff table of the associated single-objective optimization problem in the original MODM problem. It is clear that the efficient ε-constraint method generates efficient solutions.

2.3. Enumeration algorithm for integer optimization

Different direct enumeration algorithms, including the Partial Bound Enumeration Algorithm (PBEA), have been proposed to solve single objective integer optimization problems. Misra [41] proposed an efficient enumeration algorithm to solve single objective integer programming optimization. Jianping and Xishen [24] proposed a partial bounded enumeration algorithm for single objective reliability design problems. Both algorithms follow fixed sequential general parts and search for an optimal objective function value which lies within the feasibility region restricted by the constraints. Given the notion that the optimum is generally expected to be close to the boundaries restricted by the constraints, one can generate and systematically test a sequence of search points in the feasible region. Consequently, the quality of the bounded solutions is actually better than the solutions in the feasibility region for single objective optimization problems with non-decreasing objective functions. Although their algorithms were not usable for multi-objective cases, the concept motivated us to extend an efficient enumeration algorithm customized for multi-objective 0–1 optimization problems.

3. New fuzzy Multidimensional Multiple-Choice Knapsack Problem

Consider a fuzzy MMKP in which different items are categorized in multiple groups. One item must be selected from each group while satisfying a set of constraints. The problem of simultaneously maximizing the total profit and minimizing the total cost can be formulated as follows:

$$\begin{aligned}
max \quad & \sum_{i=1}^{n} p_i x_i \\
min \quad & \sum_{i=1}^{n} c_i x_i \\
& \sum_{i=1}^{n} w_i^p x_i \le W_p \quad p = 1, \ldots, m \\
& \sum_{i \in G_j} x_i = 1 \quad j = 1, \ldots, k \\
& x_i \in \{0, 1\} \quad i = 1, \ldots, n
\end{aligned} \tag{4}$$

where x_i, $i = 1,\ldots,n$ is equal to one if item i is selected; otherwise, $x_i = 0$. p_i and c_i are the profit and cost of object i, respectively. w_i^p is the weight of the pth dimension of item I and W_p is the total capacity of the pth dimension of knapsack. The items are categorized in $j = 1,2,\ldots,k$ groups. The G_j, $j = 1,\ldots,k$, shows the set of all items which are a member of group j.

Each object i belongs to just one group in Model (4). This relationship can also be represented through a crisp membership function as follows:

$$\mu_{ij} = \begin{cases} 1 & \text{if } i \in G_j \\ 0 & \text{if } i \notin G_j \end{cases} ; \quad i = 1, \ldots, n; j = 1, \ldots, k \tag{5}$$

where μ_{ij} is the membership value of object i in group j. Then, Model (4) can be re-formulated as follows:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{n} p_i x_i \\
\min \quad & \sum_{i=1}^{n} c_i x_i \\
& \sum_{i=1}^{n} w_i^p x_i \le W_p \quad p = 1, \ldots, m \\
& \sum_{i=1}^{n} \mu_{ij} \cdot x_i = 1 \quad j = 1, \ldots, k \\
& x_i \in \{0, 1\} \quad i = 1, \ldots, n
\end{aligned} \tag{6}$$

3.1. Fuzzy MMKP

In some cases DMs may not be able to specify the crisp boundaries of groups (i.e., groups are not completely independent and an item may be fitted in more than one group). In other words, an item can be a member of several groups with different membership values. Therefore, the assumption of dependency of each item to just

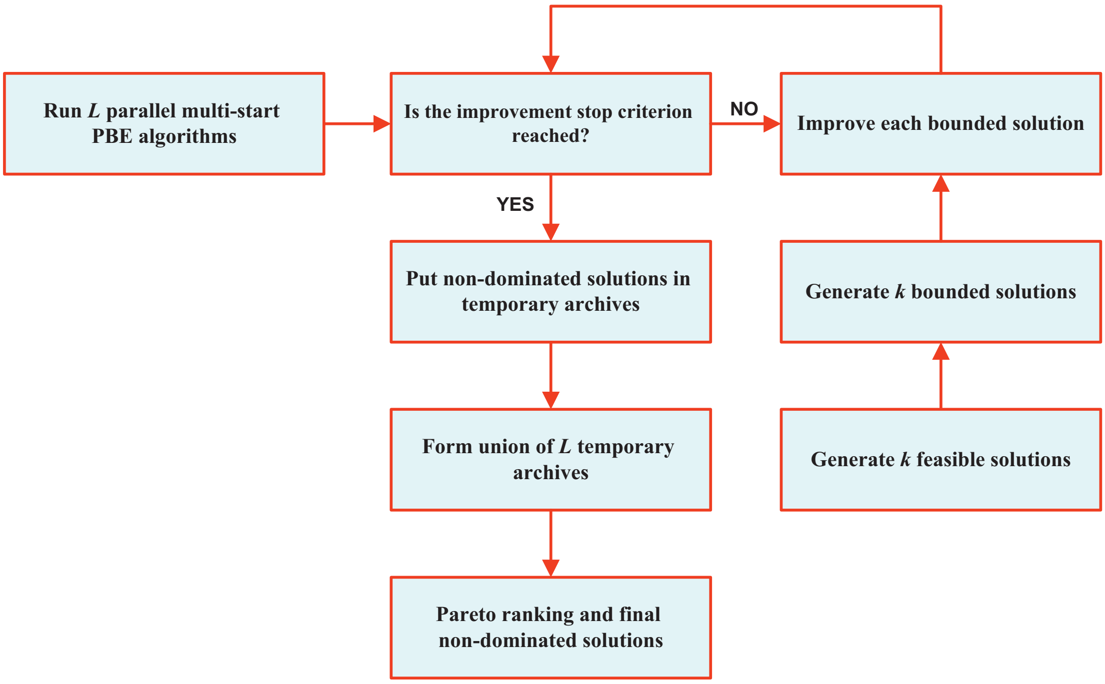


Fig. 1. A schematic view of the proposed multi-start PBE method.

one group may not be realistic and Model (6) can be re-formulated as follows:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} p_i x_i \\
\min \quad & \sum_{i=1}^{n} c_i x_i \\
& \sum_{i=1}^{n} w_i^p x_i \le W_p \quad p = 1, \ldots, m \\
& \sum_{i=1}^{n} \mu_{ij} \cdot x_i \cong 1 \quad j = 1, \ldots, k \\
& x_i \in \{0, 1\} \quad i = 1, \ldots, n
\end{aligned}
\tag{7}
$$

Solving Model (7) may result in some undesirable situations in which the total membership degrees of the selected items in a group is near 1 but the membership degrees of some or all of the selected items in the group are low. In order to prevent this problem, the definition of sufficient relation of an item is supplied as follows:

Definition 3.1. An item has the sufficient relation to a group if its membership degree in that group is equal or greater than a predefined parameter.

Therefore at least one object with sufficient relation should be in the final solution and Model (7) is re-formulated as follows:

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{n} p_i x_i \\
\min \quad & \sum_{i=1}^{n} c_i x_i \\
& \sum_{i=1}^{n} w_i^p x_i \le W_p \quad p = 1, \ldots, m \\
& \sum_{i=1}^{n} \mu_{ij} \cdot x_i \cong 1 \quad j = 1, \ldots, k \\
& \sum_{i \in G_j^\alpha} x_i \ge 1 \quad j = 1, \ldots, k \\
& x_i \in \{0, 1\} \quad i = 1, \ldots n
\end{aligned}
\tag{8}
$$

where G_j^α is the set of items with sufficient relations in the group j and represented as follows:

$$G_j^\alpha = \{i | \mu_{ij} \ge \alpha\} \quad j = 1, \ldots, k \tag{9}$$

Since in the classical multi-dimensional multi-objective multi-choice 0–1 Knapsack problems items belong to one group, the number of selected items in the final solution is equal to the number of groups (i.e., k).[2] However, in Model (8) the objects have fuzzy membership degrees and the number of selected items in the final solution may not be equal to k.

[2] The [0, 1] problem can alternatively be handled by using the groups and ring theory and the fractional matter mentioned in this study could be dealt with via the Fourier transforms.

3.2. Efficient epsilon-constraint method for fuzzy MMKP

In this section, we present the efficient ε-constraint method for fuzzy MMKP which is the direct outcome of the application of the efficient ε-constraint method on Model (8):

$$\begin{aligned} \max \quad & \sum_{i=1}^{n} p_i x_i - \beta \times \left(\frac{S_2}{r}\right) \\ \text{s.t.} \quad & \\ & C_s + S_2 = \varepsilon_2, \quad \varepsilon_2 \in [C_s^-, C_s^+] \\ & X \in S \end{aligned} \tag{10}$$

where r_2 represents the range of the second objective in Model (8) calculated from the payoff table (i.e., using the Nadir and Ideal values of C_s^-, C_s^+). $X \in S$ is the feasible region of Model (8), and β is a small positive number (usually between 0.001 and 0.00001), and S_2 is a slack variable associated with the constraint of the second objective in Model (8).

3.3. Proposed multi-start PBE method for the MMKP

Consider a typical multi objective 0–1 optimization problem as follows:

$$\begin{aligned} \max \quad & f_l(x) = f_l(x_1, x_2, \ldots, x_n), \quad l = 1, 2, \ldots, p, \\ \text{s.t.} \quad & \\ & g_j(x) \le b_j \quad j = 1, 2, \ldots, m \\ & x_i \in \{0, 1\}, \quad i = 1, 2, \ldots, n \end{aligned} \tag{11}$$

The steps of the proposed multi-start PBE method for the fuzzy MMKP are depicted in Fig. 1. In addition, the high level pseudo code for the proposed algorithm is presented in Fig. 2.

Using the aforementioned logic, there is no need to check the entire solution space for a 0–1 optimization problem. Only those feasible solutions which are on the boundary of the solution space are candidates for checking. Moreover, using a systematic approach, all the bounded solutions are not required to be checked. So the search procedure is significantly reduced. The concept can also be used for multi-objective integer programming problems with several constraints.

Although the bounded enumeration algorithm can achieve the real Pareto front for an MODM problem, checking the feasibility of a solution against different set of constraints, selecting the non-dominated solutions among the feasible solutions, putting them in a temporary archive of non-dominated solutions in each step of the algorithm and, re-generating an estimation of the true Pareto front for the MODM problem may be computationally expensive. The non-dominated sorting methods have been criticized mainly for their $O(MN^3)$ computational complexity where M is the number of objectives and N is the size of the solution generated in each iteration [13]. Even fast non-dominated sorting methods have a computational complexity of $O(MN^2)$ [13].

The proposed PBEA is customized for a typical MODM problem, called fuzzy MMKP.

The multi-start property is inserted in the proposed algorithm to utilize the capabilities of parallel computations in searching the feasible bounded solutions of the fuzzy MMKP. Several local bounded enumeration algorithms with different bounded solutions are run in the proposed multi-start PBE method to efficiently search the solution space of the fuzzy MMKP. After several iterations of the proposed multi-start PBE method, the archives of the local PBEAs are compared to delete the dominated solutions. This strategy leads into faster convergence to the Pareto front.

```
Find the initial values of the variables
  For i=1 to n do
     Xi ←1
     For j=1 to m do
         If gj(x) > bj Then V ←V+1
     End For
    If V=0 Then Stop and Print the Solutions
  End For
Sort the variables based on their effects on the objective function and constraints
  For i=1 to n do
     For l=1 to p do
        Scoreil ←Benefitil / Costil
        Objective-Sortil ←SORT Scoreil
     End For
   Xi ←0
    Update V as V'
    Calc V-V'
    Violation-Releasei ←SORT (V-V')
  End For
Find the feasible solutions
  Start
    Objective-Sortil
    Violation-Releasei
    Generate K feasible Solution as Xk=(x1k, x2k,…, xnk)
     Solk ←Xk
  End
Find the bounded solutions
  For j=1 to k do
     While V=0 do
         i ←Round (Rand [1, n])
         xij ←1
     End While
  While V>0 do
       Ratioil ←Objective-Sortil/Violation-Releaseil
       Ratioil ←INV SORT Ratioil
       Find the First Non-Zero Component in Ratioil Vector
       Call this Position p
       xij ←0
  End While
Search the solution space systematically in Parallel
  For j=1 to k do
           For l=1 to p do
              OBJjl ←Calc l-th Objective Function of Xk
              X'k ←Perturb Xk basd on violation-release and objective-sort
              OBJ'jl ←Calc l-th Objective Function of X'k
              If OBJ'jl> OBJjl Then OBJjl ←OBJ'jl
           End For
         Non-Dominate ←Non-Dominate SORT (OBJjl)
   End For
```

Fig. 2. The high-level pseudo code for the proposed multi-start PBE method.

4. Experimental results

The comparative experimental results of the proposed multi-start PBE method and the efficient ε-constraint method are discussed in this section.

4.1. Test problems

We simulated three sets of test problems in order to compare the performance of the proposed multi-start PBE method and the efficient ε-constraint method. The simulated test problems were generated using a uniform probability density function for small, medium, and large sizes. The properties of the simulated test problems are presented in Table 1.

4.2. Software–hardware implementation

The proposed ε-constraint method was coded using LINGO 11.0 and MS-Excel 12.0. The multi-start PBE method was coded using

Table 1
The test problems.

Problem	Item	Group	Membership	Profit	Cost	Weight	Capacity
I	5	2	Random between 0 and 1	*U*[1,100]	*U*[100,200]	*U*[1,20]	50
II	20	5	Random between 0 and 1	*U*[1,100]	*U*[100,200]	*U*[1,20]	200
III	100	10	Random between 0 and 1	*U*[1,100]	*U*[100,200]	*U*[1,20]	1000

MATLAB software. All codes were run on a PIV Pentium portable PC with MS-Windows XP Professional, 1 GB of RAM, and 2.0 GHz Core 2 Due CPU.

4.3. Results

We re-generated a reference set (RS) for each test problem since the real Pareto front of the test problems were not known in advance. The RS is the set of non-dominated solutions which were generated through several runs of the two algorithms. A series of pre-screening tests were conducted to achieve the appropriate parameters of the two methods. Table 2 presents the tuned parameters.

We ran each algorithm 50 times for each test problem. The non-dominated solutions for all runs were then selected to re-generate the RS. The RS for the test problems are presented in Fig. 3.

We also retrieved the non-dominated solutions for each procedure separately and the regenerated Pareto front for each algorithm was achieved. Fig. 4 plots the Pareto fronts of each method.

Fig. 5 comparatively plots the RS and the regenerated Pareto front of each algorithm. This figure confirms the closeness of each approach to the RS method.

4.4. Performance comparison

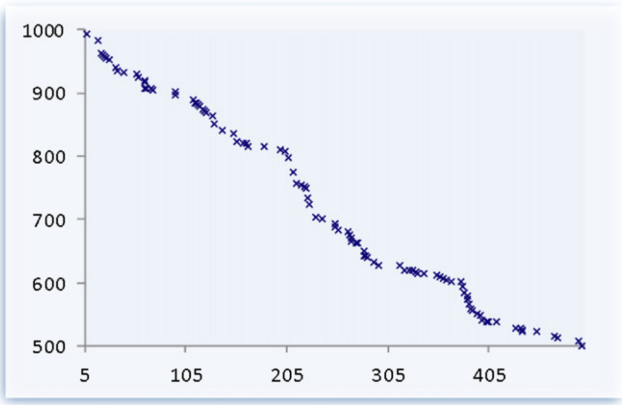
Although the implementation of the two methods on different simulated test problems revealed their ability to generate non-dominated solutions, it was not enough to supply the necessary information to compare the performance of the proposed multi-start PBE method and the efficient ε-constraint method. More specifically, several features of the proposed algorithms should be evaluated. These features are mainly categorized as convergence analysis, accuracy, robustness, function evaluations, stopping criteria and efficiency of the proposed algorithm. We have used several metrics to illustrate the efficiency, accuracy, diversity, and convergence of the proposed algorithms. The performance metrics proposed by Yu and Gen [57] were selected and calculated using the re-generated Pareto fronts.

4.4.1. Accuracy measures

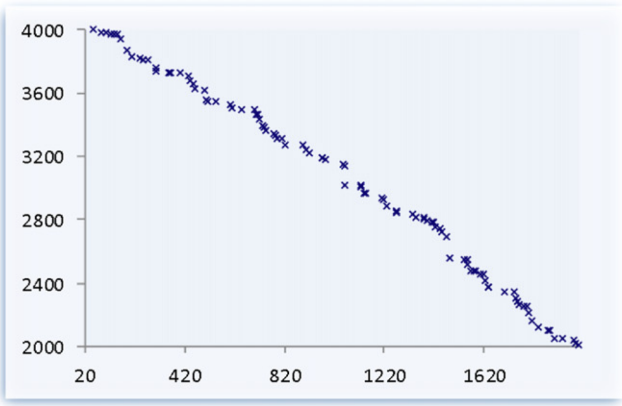
The following accuracy measures were used to compare the proposed multi-start PBE method and the efficient ε-constraint method:

Table 2
The fitted parameters.

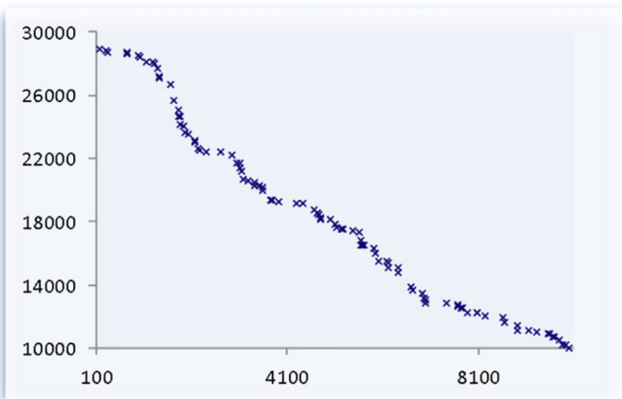
Efficient ε-constraint method		Multi-start PBE method	
Number of runs	50	Number of runs	50
Size of archives	50	Number of initial feasible solutions	20
Step-size of cost	0.01	Number of local PBE algorithms	5
		Temporary size of archive	100
		Maximum number of ierations	200
		Final size of archives	50



(a) Test problem I



(b) Test problem II



(c) Test problem III

Fig. 3. The reference set for the test problems.

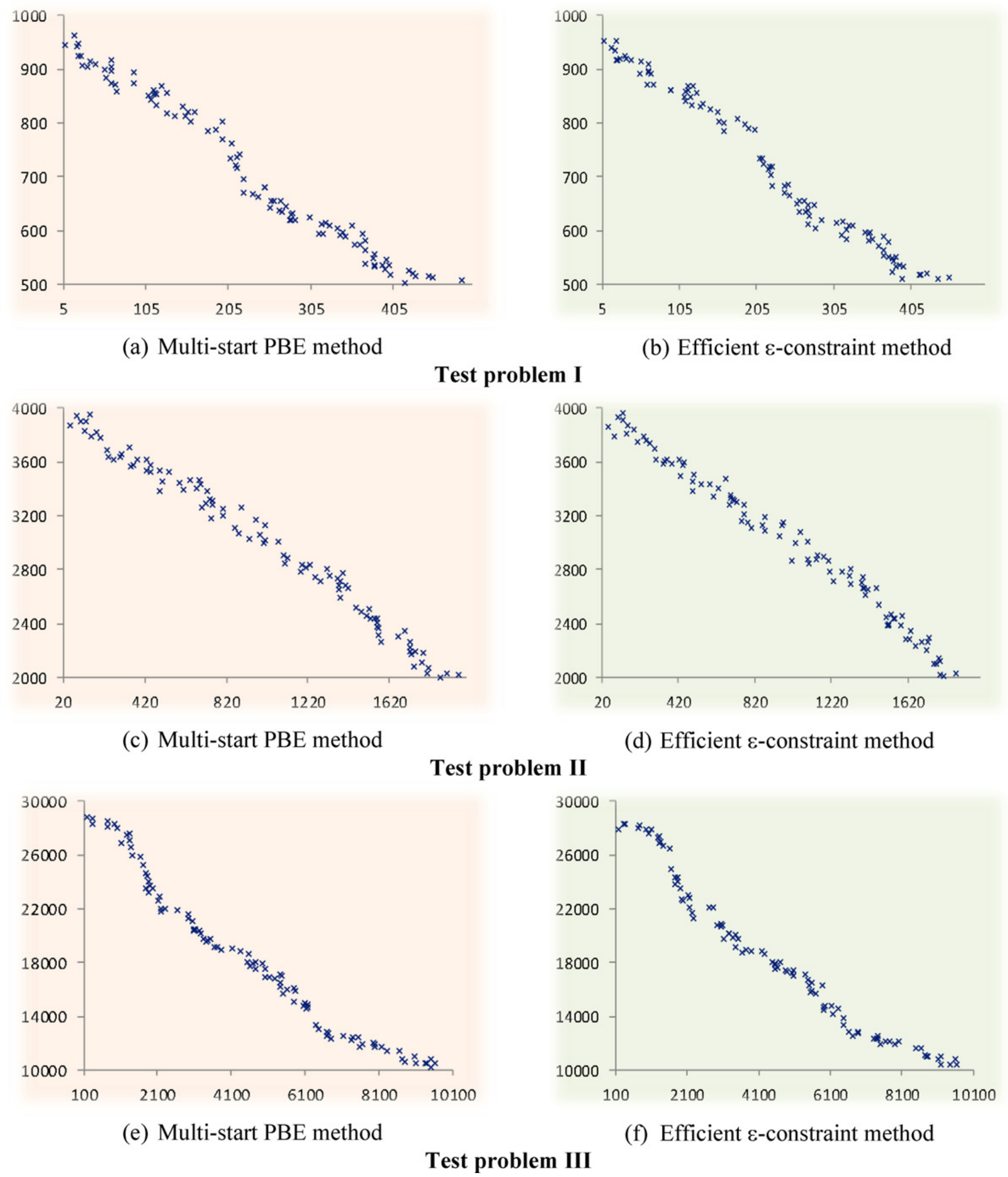


Fig. 4. The re-generated Pareto front of the test problems for the multi-start PBE and the efficient ε-constraint methods.

- *Error ratio (ER)*. The ER measures the non-convergence of the two methods toward the real Pareto front. The definition of the ER is as follows:

$$\mathrm{ER} = \frac{\sum_{i=1}^{N} e_i}{N} \tag{12}$$

where N is the number of non-dominated solutions found, and

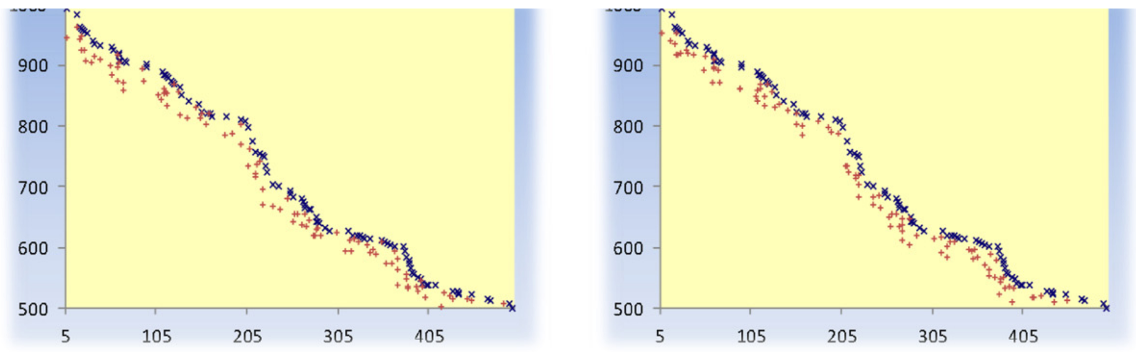
$$e_i = \begin{cases} 0 & \text{if the solution } i \text{ belongs to the Pareto front} \\ 1 & \text{otherwise} \end{cases}$$

The closer this metric is to unity, the less the solution has converged toward the RS. More formally, the higher the *ER* measure is, the lesser the convergence of the associated algorithm is. It is notable that 1-*ER* measure implicitly denotes the convergence of the methods toward the real Pareto front.

- *Generational distance (GD)*. This metric calculates the distance between the RS and the solution set. The definition of this metric is given as follows:

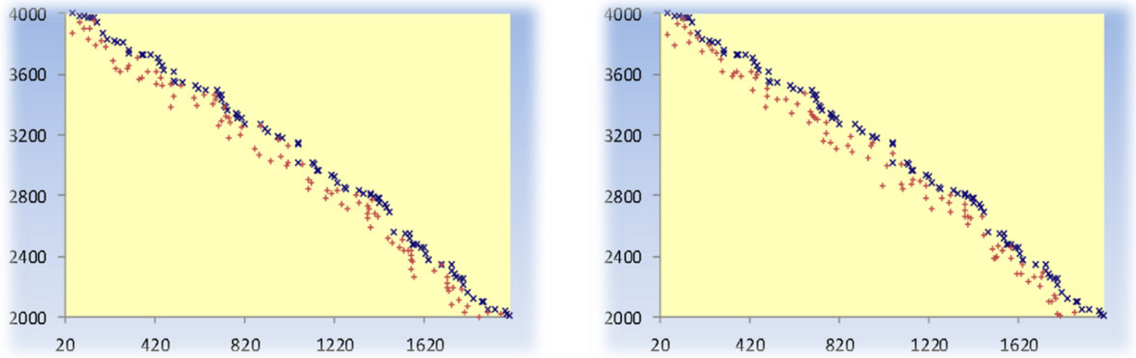
$$\mathrm{GD} = \frac{\sum_{i=1}^{N} d_i}{N} \tag{13}$$

where, $d_i = \min_{p \in PF} \{ \sqrt{\sum_{k=1}^{m} (z_k^i - z_k^p)^2} \}$ is the minimum Euclidean distance between solution i and RS in which $|m|$ is the number of the objective functions.



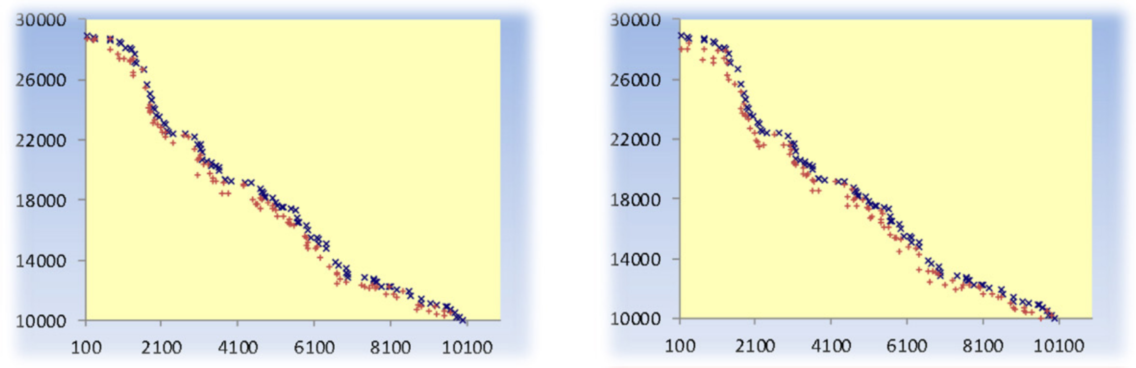
(a) Multi-start PBE methodvs.reference set (b) Efficient ε-constraint method vs. reference set

Test problem I



(c) Multi-start PBE methodvs. reference set (d) Efficient ε-constraint methodvs. reference set

Test problem II



(e) Multi-start PBE methodvs. reference set (f) Efficient ε-constraint method vs. reference set

Test problem III

Fig. 5. The reference set in comparison with the re-generated Pareto front.

4.4.2. Diversity measures

- *Spacing metric (SM)*. The SM measures the uniformity of the spread of the points in the solution set. In order to calculate SM, $\bar{d}$ which is the mean value of all d_i, should be calculated first as follows:

$$\bar{d} = \frac{\sum_{i=1}^{N} d_i}{N} \tag{14}$$

The SP (the standard deviation of the closest distances) is then calculated as follows:

$$\mathrm{SP} = \sqrt{\left(\frac{\sum_{i=1}^{N} (\bar{d} - d_i)^2}{N-1}\right)} \tag{15}$$

- *Diversification metric (DM)*. The DM measures the spread of the solution set as follows:

$$\mathrm{DM} = \left[\sum_{i=1}^{N} \max(\|x_i - y_i\|)\right]^{1/2} \tag{16}$$

where, $\|x_i - y_i\|$ is the Euclidean distance between the non-dominated solution x_i and the non-dominated solution y_i.

The aforementioned metrics as well the CPU run time were calculated for both methods in 10 different runs and the results are represented in Table 3.

The computation time of the multi-start PBE method is slightly lower than the computation time of the efficient ε-constraint method. Moreover, the results in Table 3 also reveal that:

Table 3
The results of the quality and diversity metrics for the test problems.

Run	Efficient ε-constraint method					Multi-start PBE method				
	Accuracy		Diversity		CPU Time (s)	Accuracy		Diversity		CPU Time (s)
	ER	GD	SM	DM		ER	GD	SM	DM	
Test problem I										
1	0.21	3.93	7.89	64.28	66.46	0.25	3.4	5.78	60.49	77.92
2	0.25	3.74	5.36	61.08	76.99	0.29	3.79	5.36	62.43	57.13
3	0.28	3.43	5.07	61.19	62.02	0.21	3.31	5.07	64.6	65.48
4	0.22	3.46	4.77	64.58	63.51	0.29	3.22	4.77	61.36	63.37
5	0.28	3.3	5.08	60.99	74.49	0.3	3.67	5.08	64.12	75.27
6	0.23	3.68	9.36	60.12	77.2	0.23	3.25	9.36	61.32	76.07
7	0.23	3.32	4.24	60.35	69.65	0.25	3.46	4.24	62.92	68.74
8	0.3	3.66	3.07	63.73	63.23	0.22	3.47	3.07	63.11	72.26
9	0.28	3.69	3.07	62.34	61.94	0.26	3.85	3.07	61.22	69.78
10	0.22	3.54	7.62	64.45	67.43	0.23	3.59	7.62	61.76	56.31
Mean	**0.25**	**3.58**	**5.55**	**62.31**	**68.29**	**0.25**	**3.50**	**5.34**	**62.33**	**68.23**
Std. Dev.	**0.03**	**0.20**	**2.09**	**1.79**	**6.03**	**0.03**	**0.22**	**1.93**	**1.34**	**7.60**
Test problem II										
1	0.26	3.89	8.19	75.89	115.31	0.29	3.48	5.07	76.49	113.8
2	0.3	3.65	5.36	71.28	110.27	0.2	3.91	5.36	77.03	105.76
3	0.23	3.31	5.07	74.66	101.95	0.28	3.73	5.07	70.08	106.01
4	0.23	3.18	4.77	70.63	115.72	0.24	3.75	4.77	77.83	119.83
5	0.22	3.65	5.08	75.1	119.53	0.29	3.28	5.08	72.09	108.31
6	0.26	3.67	9.36	78.98	117.11	0.29	3.25	9.36	75.36	112.17
7	0.26	3.29	4.24	77.78	107.18	0.3	3.16	4.24	71.98	114.73
8	0.3	3.19	3.07	72.89	111.09	0.3	3.3	3.07	75.37	103.71
9	0.2	3.55	3.07	72.82	110.47	0.26	3.15	3.07	71.15	112.08
10	0.29	3.74	7.62	70.28	113.71	0.28	3.79	7.62	76.16	114.94
Mean	**0.26**	**3.51**	**5.58**	**74.03**	**112.23**	**0.27**	**3.48**	**5.27**	**74.35**	**111.13**
Std. Dev.	**0.03**	**0.25**	**2.13**	**2.97**	**5.17**	**0.03**	**0.29**	**1.93**	**2.76**	**5.06**
Test problem III										
1	0.35	3.1	10.31	102.42	315.07	0.26	3.46	8.04	106.24	292.5
2	0.33	3.73	7.62	107.18	316.52	0.3	3.23	10.27	108.35	309.03
3	0.27	3.71	7.25	101.84	301.86	0.25	3.66	8.33	104.99	287.19
4	0.3	3.57	8.82	102.15	314.7	0.34	3.89	10.42	101.59	309.85
5	0.33	3.4	9.98	101.61	316.89	0.29	3.84	8.88	104.84	286.61
6	0.35	3.83	9.37	101.78	302.26	0.31	3.19	7.6	109.31	315.32
7	0.31	3.11	7.62	103.41	310.53	0.32	3.6	10.13	109.11	295.49
8	0.28	3.23	10.18	107.42	305.1	0.26	3.39	8.07	106.77	296.81
9	0.27	3.67	9.41	101.45	313.3	0.29	3.11	7.88	105.02	307.1
10	0.27	3.56	10.31	106.71	317.44	0.29	3.02	7.56	103.81	292.78
Mean	**0.31**	**3.49**	**9.09**	**103.60**	**311.37**	**0.29**	**3.44**	**8.72**	**106.00**	**299.27**
Std. Dev.	**0.03**	**0.27**	**1.20**	**2.49**	**6.11**	**0.03**	**0.30**	**1.14**	**2.46**	**10.22**

- For small-size test problems, the average value of the ER metric is approximately equal for both methods. For medium size test problems, the average value of the ER metric in the efficient ε-constraint method is slightly better than the ER metric in the multi-start PBE method. For large size test problems, the average value of the ER metric in the multi-start PBE method is better than the efficient ε-constraint method.
- For small size test problems, the average value of the GD metric is approximately equal in both procedures. For medium size test problems, the average value of GD metric in the multi-start PBE method seems to be slightly better. For large size test problems, the average value of the GD metric in the multi-start PBE method is also better.
- The multi-start PBE method re-generates non-dominated solutions which have less average values for the SM metric in all test problems. This observation reveals that the generated solutions using the multi-start PBE method are more uniformly distributed throughout the RS in comparison with the efficient ε-constraint method.

In general, the average value of the DM in the multi-start PBE method is encouraging in comparison with the efficient ε-constraint method for all test problems. More formally, the multi-start PBE method is capable of finding scattered non-dominated solutions.

Although the aforementioned observations confirm the relative preference of the proposed multi-start PBE method, the meaningful dominance, equality, or infirmity should be validated through proper statistical analysis.

4.4.3. Statistical analysis

Statistical analysis was used to study the behavior of the distributions of the population of metrics. The Kolmogorov–Smirnov test was used on the results of different metrics to determine whether the normal distribution is fitted. The result of the Kolmogorov–Smirnov test is presented in Table 4.

Table 4 shows that there is not enough evidence to reject the null hypothesis of the Kolmogorov–Smirnov test. The null hypothesis claims that the population of these metrics follows a normal distribution. Hence, a parametric statistical test can be used to check whether there are meaningful differences between the means of the metrics.

We tested whether the variance of the samples is equal. The *F*-test and the robust Levene's tests for small samples were conducted to check the equality of the variances in the two proposed methods. The results of the *F*-test and the Levene's-test for the equality of variances of metrics are presented in Table 5.

The *P*-values were greater than the significance level for all cases. Consequently, we failed to reject the null hypothesis of the equal variances. That is, these results do not provide enough

Table 4
The results of the K–S test for the normality of the metrics.

	Efficient ε-constraint method					Multi-start PBE method				
	Accuracy		Diversity		CPU Time (s)	Accuracy		Diversity		CPU Time (s)
	ER	GD	SM	DM		ER	GD	SM	DM	
Test problem I										
Mean	0.25	3.58	5.55	62.31	68.29	0.25	3.50	5.34	62.33	68.23
Std. Dev.	0.03	0.2	2.09	1.79	6.03	0.03	0.22	1.93	1.34	7.6
N	10	10	10	10	10	10	10	10	10	10
K–S	0.18	0.16	0.24	0.24	0.19	0.13	0.16	0.21	0.17	0.13
P-value	0.15	0.15	0.11	0.119	0.15	0.15	0.15	0.15	0.15	0.15
Test problem II										
Mean	0.26	3.51	5.58	74.03	112.23	0.27	3.48	5.27	74.35	111.13
Std. Dev.	0.03	0.25	2.13	2.97	5.17	0.03	0.29	1.93	2.76	5.06
N	10	10	10	10	10	10	10	10	10	10
K–S	0.14	0.19	0.24	0.15	0.15	0.24	0.23	0.28	0.24	0.17
P-value	0.15	0.15	0.10	0.15	0.15	0.11	0.12	0.05	0.09	0.15
Test problem III										
Mean	0.31	3.50	9.09	103.60	311.37	0.29	3.44	8.72	106.00	299.27
Std. Dev.	0.03	0.27	1.20	2.49	6.11	0.03	0.3	1.14	2.46	10.22
N	10	10	10	10	10	10	10	10	10	10
K–S	0.19	0.20	0.19	0.28	0.22	0.11	0.15	0.23	0.16	0.20
P-value	0.15	0.15	0.15	0.05	0.15	0.15	0.15	0.12	0.15	0.15

Table 5
The results of the *F*-test and the Levene's-test for the equality of the variances of the metrics.

		Accuracy		Diversity		CPU Time (s)
		ER	GD	SM	DM	
Test problem I						
F-test	Statistic	1.04	0.83	1.17	1.77	0.63
	P-value	0.95	0.79	0.81	0.41	0.50
Levene's-test	Statistic	0.15	0.02	0.11	1.62	0.44
	P-value	0.70	0.88	0.74	0.22	0.51
Test problem II						
F-test	Statistic	1.20	0.75	1.22	1.16	1.04
	P-value	0.79	0.67	0.77	0.83	0.95
Levene's-test	Statistic	0.33	0.43	0.23	0.10	0.00
	P-value	0.57	0.52	0.64	0.76	0.95
Test problem III						
F-test	Statistic	1.32	0.77	1.11	1.02	0.36
	P-value	0.68	0.70	0.88	0.97	0.14
Levene's-test	Statistic	0.94	0.32	0.03	0.02	2.24
	P-value	0.35	0.58	0.86	0.88	0.15

evidence to claim that the two populations of the metrics have unequal variances. Thus, it is reasonable to assume equal variances when using a two-sample *T*-test.

In order to test the equality of the means of the metrics for the procedures, a two-sample *T*-test with equal variances was carried out. The results of the two-sample *T*-test are presented in Table 6.

We then use the pooled standard deviations since we previously found no evidence for the variances being unequal. The pooled standard deviation is used to calculate the test statistic. Since the *P*-value is greater than the commonly chosen α-levels (i.e., 0.05), there is no evidence for a difference in the means of the metrics for the proposed procedures.

The results show that the proposed multi-start PBE method is able to generate qualified non-dominated solutions just as the efficient ε-constraint method does. Moreover, the CPU Time of the proposed multi-start PBE method is noticeably less than the CPU

Table 6
The results of the two-sample *T*-test for the equality of the means of the metrics.

	Accuracy		Diversity		CPU Time (s)
	ER	GD	SM	DM	
Test problem I					
T-value	−0.21	0.79	0.23	−0.03	0.02
P-value	0.836	0.441	0.817	0.976	0.985
Test problem II					
T-value	−1.21	0.26	0.34	−0.25	0.48
P-value	0.241	0.794	0.735	0.804	0.636
Test problem III					
T-value	1.09	0.41	0.71	−2.18	3.21
P-value	0.289	0.689	0.489	0.043	0.005

Time for the efficient ε-constraint method for large size test problems.

5. Conclusion and future research directions

In the conventional fuzzy MMKPs, each item belongs to one and only one group (equivalence relation). This assumption may not be realistic in the real-world problems with some uncertainties regarding the membership of the items since one item may belong to more than one group based on some similarity or consistency criteria. The issue of equivalence complicates the application of any meta-heuristic to real-world problems, especially in problems with dependent variables. There are alternative methods for specifying algorithms with respect to abstract representations, making them independent of any actual representation or problem domain. These methods can also define a procedure for generating a concrete representation from an explicit characterization of a problem domain. This allows arbitrary algorithms to be applied to arbitrary problems yielding well-specified search strategies suitable for real-world implementation (see e.g., [54]). Nevertheless, we used fuzzy sets to model such uncertainties in the new MMKP proposed in this study.

Two methods of efficient ε-constraint and multi-start PBE were developed to solve the fuzzy MMKP. Both methods were used to comparatively generate a set of non-dominated solutions for the fuzzy MMKP. The performances of both methods were statistically compared on a set of simulated benchmark cases using different diversity and accuracy metrics. The efficient ε-constraint method utilized a lexicographic payoff table to improve the process of determining a range of objective functions over an efficient set of test problems. In the efficient ε-constraint method, as lack-based objective function was considered to guarantee the efficiency of the obtained solutions and a fast-searching procedure was used to quickly identify the infeasible regions of the solution space. A fast-ranking method was utilized in the multi-start PBE method to recognize the non-dominated solutions.

Several small, medium, and large simulated test problems were generated using a uniform probability density function. A series of 2-D reference sets were generated for all benchmark cases. The performance of the two methods was compared statistically using the diversity and the accuracy metrics. The Kolmogorov–Smirnov test was performed to check whether the distribution of the metrics follows a normal probability density function. The equality of variance for the metric populations was tested using the *F*-test and the robust Levene's test for small samples. Finally, a two-sample *T*-test was performed to check the statistical differences between the metric means for both methods. The proposed multi-start PBE method produced solutions as diverse and as accurate as the solutions produced by the efficient ε-constraint method but in a significantly shorter computation time.

The proposed fuzzy MMKP can be used to handle a wide range of management and engineering applications such as investment problems, capital budgeting, portfolio selection, and project selection. The dynamic effect of the planning horizon can be established and analyzed to study the new fuzzy MMKP in multi-period planning horizons. The proposed multi-start PBE method can also be customized for other multi-objective mathematical programming problems with monotone objective functions. A comparative study between the fuzzy MMKP model proposed in this study and the competing meta-heuristics methods in the literature is also another potential future research direction.

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