

AN INTERACTIVE MOLP METHOD FOR SOLVING OUTPUT-ORIENTED DEA PROBLEMS WITH UNDESIRABLE FACTORS

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ABSTRACT. Data Envelopment Analysis (DEA) and Multiple Objective Linear Programming (MOLP) are widely used for performance assessment in organizations. Although DEA and MOLP are similar in structure, DEA is used to assess and analyze past performance and MOLP is used to predict future performance. Several equivalence models between output-oriented DEA models and MOLP models have been proposed in the literature. However these models are not applicable to performance evaluation problems with undesirable outputs. We propose an interactive method for solving output-oriented DEA models with undesirable outputs. We show that the output-oriented BCC model of Seiford and Zhu [47] can be equivalently stated as the maximization of the minimum of several objectives over the production possibility set, which in turn is a scalarization of a multi-objective linear program. We then employ the well-known Zionts-Wallenius procedure to solve the multi-objective optimization problem. We present an example to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures and algorithms.

1. Introduction. Data envelopment analysis (DEA) is a widely used non parametric method for comparing the inputs and outputs of a set of decision making units (DMUs). DEA, initially introduced by Charnes et al. [9], utilizes linear programming (LP) to determine the relative efficiencies of these functionally similar

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DMUs. A score of one is assigned to the frontier (efficient) units. The frontier units are those with maximum output levels for given input levels or with minimum input levels for given output levels. In the “input-orientated” DEA, a DMU is considered efficient when no other DMU can produce the same outputs by consuming fewer inputs. In contrast, in the “output-orientated” DEA, a DMU is considered efficient when no other DMU can produce more outputs by consuming the same inputs. The conventional DEA generally does not consider a decision maker’s (DM’s) preference structure or value judgments. In recent years, various methods have been proposed to take the DM’s preference information into consideration. Allen et al. [1] have defined value judgments as “logical constructs, incorporated within an efficiency assessment study, reflecting the DMs’ preferences in the process of assessing efficiency”.

Two streams of research known as “target setting” and “weight restriction” are often used to take the DMs’ value judgments into consideration in the DEA models. The first interactive target setting model was introduced by Golany [21]. He combined DEA with multiple objective linear programming (MOLP) and used the DM to allocate a set of input levels as resources and selected the most preferred set of output levels from a set of viable points on the efficient frontier. Thanassoulis et al. [50], Pedraja-Chaparro et al. [41], Tone [54], Jess et al. [29], Korhonen et al. [31], Aparicio et al. [3], Yang et al. [58], Hosseinzadeh Lotfi et al. [23, 24], Bi et al. [7], Ebrahimnejad [12], Ebrahimnejad et al. [13], Hinojosa and Mirmol [22], Jain et al. [28], Mahdiloo et al. [38], Saati et al. [46] and Amirteimoori and Kordrostami [2] have proposed various target setting DEA models in the performance evaluation literature. The first weight restriction DEA model was proposed by Thompson et al. [53]. Dyson and Thanassoulis [11], Charnes et al. [10], Wong and Beasley [56], Thompson et al. [51, 52], Podinovski [42], and Asmild et al. [4] have proposed various weight restriction models for performance measurement in DEA.

Yang et al. [58] established a new link between MOLP and the output-oriented DEA dual model, which is a radial model and projects all DMUs on to the efficient frontier by solving n LP problems. Malekmohammadi et al. [39] suggested a super-ideal model which is identical to a target model that considers both the decrease in total input consumption and the increase in total output production. The proposed model is non-radial; moreover, all of the inputs and outputs are simultaneously projected on to the efficient frontier by solving one LP problem. Yang et al. [59] investigated a hybrid minimax reference point-DEA approach to incorporate the value judgements of both branch managers and head-office directors and to search for the most preferred solution (MPS) along the efficient frontier for each bank branch. Yang et al. [61] explored graphical and analytical methods and procedures for generating and analysing data envelopes and efficient frontiers for multiple input and multiple output DEA models using the DEA-oriented interactive minimax reference point approach. This computational investigation leads to the definition of new efficiency measures as instrumental to help conduct trade-off analysis for setting realistic performance targets. Yang and Xu [60] proved that under certain conditions minimax reference point models are identical to input-oriented dual DEA models for performance assessment. This equivalency link leads to the development of an interactive minimax reference point approach for hybrid efficiency and trade-off Analyses, with the DM’s preferences taken into account interactively. Hosseinzadeh Lotfi et al. [23] have also proposed an equivalence model

between DEA and MOLP. They showed how a DEA problem can be solved interactively through a MOLP transformation. They used the method proposed by Zionts-Wallenius [64] and considered the DMs preferences in the performance measurement process. However their method could not solve problems with undesirable output. Recently, Ebrahimnejad and Tavana [15] first provided a new link between the output-oriented DEA model in the presence of undesirable outputs through a MOLP model with a weighted minimax reference point. They then used the satisfying trade-off method proposed by Nakayama and Sawaragi [40] to help the DM search a target unit corresponding to each inefficient unit. In this study, we propose a simpler and more efficient model by investigating an equivalence model and proposing an interactive trade-off analysis procedure in the MOLP for integrating the DEA-oriented performance assessment and target setting while taking into consideration the DMs preferences. We simultaneously and interactively consider the increase in the total desirable outputs and the decrease in the total undesirable outputs.

The remainder of this paper is organized as follows: Section 2 provides a brief discussion of the output-oriented BCC model in the presence of undesirable outputs. Section 3 introduces a technique for solving MOLP problems known as “min-ordering optimization problem”. In Section 4, we establish a new link between the output-oriented BCC model in the presence of undesirable outputs and the min-ordering optimization problem. Section 5 introduces an interactive multi objective programming method known as the Zionts-Wallenius method for reflecting the DM’s preferences in the efficiency assessment process. In Section 6 we present an example from the literature to demonstrate the applicability of the proposed model and exhibit the efficiency of the procedures and algorithms. Finally, Section 7 draws the conclusive remarks and future research directions.

2. Output-oriented BCC model in the presence of undesirable outputs.

The production process in DEA uses a set of inputs to produce a set of outputs. Each producer uses varying levels of inputs and produces varying levels of outputs. According to the efficiency criterion in DEA, either making more output with the same input or making the same output with less input is more efficient. In performance measurement problems with undesirable outputs, DMUs with more desirable outputs and less undesirable outputs (relative to less input resources) are considered efficient. For example, if there are inefficiencies in the production process, the undesirable outputs of wastes and pollutants should be reduced to improve the efficiency of a DMU (Seiford and Zhu [47]; Lewis and Sexton [33]).

Various transformation methods have been proposed to deal with desirable and undesirable outputs in DEA (see Liu and Sumaila [34]). Lu and Lo [37] have classified the alternatives for dealing with undesirable outputs in DEA into three different categories. The methods in the first category simply ignore the undesirable outputs. The methods in the second category either treat the undesirable outputs in terms of a non-linear DEA model or treat them as outputs and adjust the distance measurement to restrict the expansion of the undesirable outputs (Fare et al. [18]). The methods in the third category either treat the undesirable outputs as inputs or apply a monotone decreasing transformation (Lovell et al. [35]). Seiford and Zhu [47] have suggested a simple and interesting method by multiplying the undesirable outputs by (-1) and then using a translation vector to turn the negative undesirable outputs into positive desirable outputs. Fare and Grosskopf [17] have proposed a

more superior approach to Seiford and Zhu's [47] method by adopting a directional distance function and estimating the DMUs' efficiencies based on weak disposability of undesirable outputs.

Zhou et al. [65] integrated DEA efficiency measures with environmental DEA technology in an environmental performance measurement system. They studied the environmental DEA technologies that exhibited non-increasing returns to scale and variant returns to scale (VRS) and proposed pure measures for different scenarios and mixed measures under VRS. Yang and Pollitt [57] used different DEA-based efficiency measurement models and examined the impact of uncontrollable variables together with undesirable outputs based on a research sample of Chinese coal-fired power plants. You and Yan [62] proposed a ratio model to evaluate the undesirable as well as the desirable outputs simultaneously. They used their model to investigate the impact of production pollutants while conducting the efficiency evaluation in the textile industry. Huang et al. [27] introduced an undesirable DEA model for providing accurate and reasonable efficiency score that not only dealt with undesirable output efficiently but also considered the relationship between undesirable outputs. In the proposed model, all the undesirable outputs were divided into three categories and then the desirable and undesirable outputs were combined through a transformation mechanism. They also empirically tested their proposed model in 15 Chinese commercial banks.

The recent applications of DEA models with desirable and undesirable outputs can be seen in the agriculture and dairy industry (Reinhard et al. [43]; Reinhard and Thijssen [44]; Shaik et al. [48]), paper mills (Fare et al. [18]; Hua et al. [26]; Seiford and Zhu [47]), electric utilities (Fare et al. [19]; Korhonen and Luptacik [30]), cement manufacturing (Kumar Mandal and Madheswaran [32]; Riccardi et al. [45]), the oil and gas industry (Sueyoshi and Goto [49]), airports (Yu [63]; Lozano and Gutierrez [36]), health care (Hu et al. [25]), and commercial banks (Barros et al. [6], Ebrahimnejad et al. [14]).

Consider a set of n DMUs, where DMU_j has a production plan (x_j, y_j) with $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ inputs and $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$ outputs. Suppose that x_{ij} ($i = 1, 2, \dots, m$) is the i th input consumed by DMU_j and y_{rj} ($r = 1, 2, \dots, s$) is r th output produced by DMU_j . The relative efficiency of DMU_p can be obtained by using the following LP model, so-called output-oriented BCC dual model [5]:

$$\begin{aligned}
 & \max \beta_p \\
 & \text{s.t.} \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, 2, \dots, m \\
 & \quad \sum_{j=1}^n \lambda_j y_{rj} \geq \beta_p y_{rp}, \quad r = 1, 2, \dots, s \\
 & \quad \sum_{j=1}^n \lambda_j = 1 \\
 & \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{1}$$

In this model, an efficiency score, $\frac{1}{\beta_p}$, is produced for DMU_p by maximizing all the inputs radially (proportionally). The objective value of model (1) lies within $\beta_p \geq 1$. If $\beta_p > 1$, then DMU_p is not efficient and the parameter β_p indicates the extent by which DMU_p has to increase outputs to become efficient. However,

model (1) does not consider the undesirable output in the performance measurement and is not capable of finding accurate and reasonable efficiency results if DMUs simultaneously produce desirable and undesirable outputs.

There is no unique and widely accepted method for handling DEA problems with coexisting desirable and undesirable outputs in the literature. Two main methods are non-linear monotonic decreasing transformation approach and linear monotonic decreasing transformation approach. The former approach suggested by Golany and Roll [21] converts an undesirable output into a normal output by a monotonic decreasing function u . In this approach the undesirable output is modeled by $u(y_{rj}^b) = \frac{1}{y_{rj}^b}$ where y_{rj}^b is the undesirable output r for DMU_j .

The later approach suggested by Seiford and Zhu [43] first multiplies each undesirable output by -1 and then finds a proper translation vector to change all negative undesirable outputs into positive values. However, in the non-linear transformation approach the scale and intervals of the original data is lost and the reciprocal of a zero value does not exist. Therefore, we employ Seiford and Zhu's [47] approach in the MOLP method proposed in this study.

Suppose we have n DMUs and each DMU_j produces s_1 desirable outputs y_{rj}^g ($r = 1, 2, \dots, s_1$) and s_2 undesirable outputs y_{tj}^b ($t = 1, 2, \dots, s_2$) using m inputs. The following output-oriented BCC model suggested by Seiford and Zhu [47] evaluates the efficiency of DMU_p in the presence of undesirable outputs:

$$\begin{aligned}
 & \max \beta_p \\
 & \text{s.t.} \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, 2, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj}^g \geq \beta_p y_{rp}^g, \quad r = 1, 2, \dots, s_1 \\
 & \sum_{j=1}^n \lambda_j \bar{y}_{tj}^b \geq \beta_p \bar{y}_{tp}^b, \quad t = 1, 2, \dots, s_2 \\
 & \sum_{j=1}^n \lambda_j = 1 \\
 & \lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{2}$$

where $\bar{y}_{tj}^b = -y_{tj}^b + w_t > 0$.

We define a reference set $E_p = \{j | \lambda_j^* > 0, j = 1, 2, \dots, n\}$ for each inefficient DMU_p , where $\lambda_j^* = (\lambda_{1j}^*, \lambda_{2j}^*, \dots, \lambda_{nj}^*)$ is the optimal solution of model (2). The following point on the efficient frontier, regarded as a target unit for the inefficient unit DMU_p , is then used to evaluate the performance of this DMU:

$$\left(\sum_{j \in E_p} \lambda_j^* x_j, \sum_{j \in E_p} \lambda_j^* y_j^g, \sum_{j \in E_p} \lambda_j^* y_j^b \right) \tag{3}$$

However, this target unit has not taken into consideration the DM's preference structure and an interactive MOLP can be used to address this shortcoming.

3. Min-ordering optimization problem. We propose using the MOLP model (4) to optimize a vector of linear functions in the presence of linear constraints:

$$\begin{aligned} \max f(\lambda) &= [g_1(\lambda), \dots, g_r(\lambda), \dots, g_{s_1}(\lambda, h_1(\lambda)), \dots, h_t(\lambda), \dots, h_{s_2}(\lambda)] \\ \text{s.t. } \lambda &\in \Lambda. \end{aligned} \quad (4)$$

The goal of this model is to maximize $s_1 + s_2 (\geq 2)$ conflicting objective functions with $g_r : \Lambda \rightarrow R (r = 1, 2, \dots, s_1)$ and $h_t : \Lambda \rightarrow R (t = 1, 2, \dots, s_2)$. The decision variables $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$ belong to the non-empty feasible region Λ . The objective vectors in the objective space R^n include the objective values $f(\lambda)$. The image of the feasible region, $Z = f(\lambda)$, is called a “feasible objective region”. Generally, the primary goal in MOLP is to find the Pareto optimal solutions and assist DMs in selecting the most preferred solution because there are no solutions in the MOLP problems that can simultaneously optimize all the objective functions. Consequently, a solution represented by a point in the decision space is a strictly Pareto optimal if it is not possible to move it within the feasible region and improving the objective function value without deteriorating at least one of the other objectives in the model. However, a solution represented by a point in the decision variable space is a weak Pareto optimal solution if it is not possible to move it within the feasible region and improve all the objective functions. Therefore, we have the following definitions:

Definition 3.1. A feasible solution $\hat{\lambda} \in \Lambda$ is called a strict Pareto optimal solution if there is no feasible solution $\lambda \in \Lambda$ such that $f(\lambda) \geq f(\hat{\lambda})$ and $f(\lambda) \neq f(\hat{\lambda})$.

Definition 3.2. A feasible solution $\hat{\lambda} \in \Lambda$ is called a weak Pareto optimal solution if there is no feasible solution $\lambda \in \Lambda$ such that $f(\lambda) > f(\hat{\lambda})$.

The traditional scalarization methods are used to solve the MOLP problems. This class of methods does not require any assumption or information with regards to the utility functions of the DMs. In this paper, we use a scalarization method known as min-ordering optimization (Ehrgott [16]) for solving the MOLP problem (4). The min-ordering optimization problem corresponding to the MOLP problem (4) can be defined as follows:

$$\begin{aligned} \max \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\} \\ \text{s.t. } \lambda \in \Lambda. \end{aligned} \quad (5)$$

Definition 3.3. A feasible solution $\hat{\lambda} \in \Lambda$ is a min-ordering optimal solution if there is no feasible solution $\lambda \in \Lambda$ such that $\min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\} > \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\hat{\lambda}), h_t(\hat{\lambda})\}$.

The following Theorem indicates that the weak Pareto optimal solution of the MOLP problem (4) can be determined by solving the min-ordering optimization problem (5).

Theorem 3.4. *The optimal solution of the min-ordering optimization problem (5) is the weak Pareto optimal solution of the MOLP problem (4).*

Proof. See Ehrgott [16]. □

We now show that the min-ordering optimization problem (5) can be solved as a single objective LP problem. Let us assume $\psi = \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\}$

and rewrite the min-ordering problem (5) as follows:

$$\begin{aligned} & \max \psi \\ & \text{s.t. } g_r(\lambda) \geq \psi, \quad r = 1, 2, \dots, s_1 \\ & \quad h_t(\lambda) \geq \psi, \quad t = 1, 2, \dots, s_2 \\ & \quad \lambda \in \Lambda. \end{aligned} \quad (6)$$

In the next section we show that the output-oriented BCC model (2) and the min-ordering optimization problem (6) are equivalent under certain conditions.

4. A generalized equivalence model. In the output-oriented DEA models with undesirable outputs, an efficiency score is generated for a DMU by maximizing the desirable outputs and minimizing undesirable outputs with limited inputs. This concept can be represented with a multiple objective optimization problem. The theoretical considerations for combining the DEA and MOLP models are presented next.

Suppose $y_{rp}^g > 0 (r = 1, 2, \dots, s_1)$ and define $g_r(\lambda) (r = 1, 2, \dots, s_1)$ and $h_t(\lambda) (t = 1, 2, \dots, s_2)$ in formulation (2) as follows:

$$g_r(\lambda) = \frac{\sum_{j=1}^n \lambda_j y_{rj}^g}{y_{rp}^g}, \quad r = 1, 2, \dots, s_1 \quad (7)$$

$$h_t(\lambda) = \frac{\sum_{j=1}^n \lambda_j \bar{y}_{tj}^b}{\bar{y}_{tp}^b}, \quad t = 1, 2, \dots, s_2 \quad (8)$$

Therefore, formulation (2) can be written as follows:

$$\begin{aligned} & \max \beta_p \\ & \text{s.t. } g_r(\lambda) \geq \beta_p, \quad r = 1, 2, \dots, s_1 \\ & \quad h_t(\lambda) \geq \beta_p, \quad t = 1, 2, \dots, s_2 \\ & \quad \lambda \in \Lambda_p. \end{aligned} \quad (9)$$

where

$$\Lambda_p = \left\{ \lambda = (\lambda_1, \lambda_2, \dots, \lambda_n) \mid \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, (i = 1, 2, \dots, m), \right. \\ \left. \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, 2, \dots, n \right\}$$

Suppose the feasible region Λ_p in formulation (9) and Λ in formulation (6) are the same. In addition, suppose $\psi = \beta_p$. The equivalence relationship between the output-oriented BCC model (9) and the min-ordering optimization model (6) can be easily established by the following theorem:

Theorem 4.1. *Suppose $y_{rp}^g > 0 (r = 1, 2, \dots, s_1)$. The output-orientated BCC model (9) can be equivalently transformed into the min-ordering optimization model(6) using equations (7) and (8) and equations $\psi = \beta_p$ and $\Lambda_p = \Lambda$.*

In fact, Theorem 4.1 shows that the output-orientated BCC model in the presence of undesirable outputs is actually constructed to locate a specific efficient solution on the efficient frontier of the following generic MOLP formulation for the observed DMU_p :

$$\begin{aligned} \max \quad & \left[\frac{\sum_{j=1}^n \lambda_j y_{1j}^g}{y_{rp}^g}, \dots, \frac{\sum_{j=1}^n \lambda_j y_{s_1j}^g}{y_{s_1p}^g}, \frac{\sum_{j=1}^n \lambda_j \bar{y}_{1j}^b}{y_{1p}^b}, \dots, \frac{\sum_{j=1}^n \lambda_j y_{s_2j}^b}{y_{s_2p}^b} \right] \\ \text{s.t. } & \lambda \in \Lambda. \end{aligned} \quad (10)$$

The generic MOLP problem (10) defines the production possibility set for the observed DMU_p in which there may be more preferred efficient solutions than the DEA efficient solution. The main reason for establishing the equivalence condition between the output-oriented BCC model (9) and the min-ordering optimization model (6) is to use the interactive methods in MOLP to locate the most preferred solution on the efficient frontier for the target setting and resource allocation. Several interactive MOLP models have been proposed in the literature to help a DM search for different solutions along the efficient frontier and choose one solution from the set of Pareto optimal solutions as his/her final solution. At each stage of the process, the current solution is adapted to the preference structure of the DM. The interactive MOLP method is designed to drive the DM towards his/her most preferred solution. Zionts and Wallenius [64] have proposed the well-known and widely used interactive MOLP method explored next.

5. Zionts-Wallenius method. The interactive MOLP process involves the DM(s), the analyst and a model. The analyst is the intermediary between the DM(s) and the model. The analyst obtains an initial solution for the model and presents it to the DM(s) and asks them to evaluate this solution based on their preferences. The analyst then injects the preferences of the DM(s) into the model and generates a new solution. This iterative and interactive process is repeated until a satisfactory solution is obtained. In the interactive MOLP method proposed by Zionts and Wallenius [64] the DM is asked to provide answers to yes and no questions regarding certain trade-offs that he/she likes or dislikes. This type of communication is applicable to problem (10) where the objective functions are concave and the feasible space is a convex set. The overall utility function is assumed to be unknown explicitly to the DM, but is implicitly a linear function and more generally a concave function of the objective functions. The method makes use of such an implicit function on an interactive basis.

The first step of the method is to choose an arbitrary set of positive multipliers or weights and generate a composite objective function or utility function using these multipliers. The composite objective function is then optimized to produce a Pareto optimal solution λ^* to problem (10). The continuation of the procedure is essentially the same as the simplex method except that here the DM chooses a non-basic variable (NBV) to enter the basis at each iteration. From the set of non-basic variables, a subset of Pareto variables is selected (a Pareto variable is one which, when introduced into the basis, cannot increase one objective without decreasing at least one other objective). In the process of finding a set of Pareto variables from the set of non-basic variables, initially, w_{dj} values are computed based on the

information implied by the optimal solution at hand. To do this, the following model is solved for each non-basic variable λ_j :

$$\begin{aligned} &\max \lambda_j \\ &\text{s.t. } \lambda \in \Lambda = \Lambda_p. \end{aligned} \tag{11}$$

Assuming that $\hat{\lambda}$ is the optimal solution of model (11), the values of w_{dj} are estimated as follows:

$$w_{dj} = \frac{f_d(\lambda^*) - f_d(\hat{\lambda})}{\hat{\lambda}_j}, \quad j \in \text{NBV} \tag{12}$$

where

$$f_d(\lambda) = \begin{cases} \frac{\sum_{j=1}^n \lambda_j y_{dj}^g}{y_{dp}^g}, & \text{if } d = r = 1, 2, \dots, s_1 \\ \frac{\sum_{j=1}^n \lambda_j y_{dj}^b}{y_{dp}^b}, & \text{if } d = t = 1, 2, \dots, s_2 \end{cases} \tag{13}$$

The w_{dj} values show the decrease in the objective function f_d due to some specified increase in λ_j . After that, the following model is solved for each non-basic variable λ_k :

$$\begin{aligned} &\max \sum_{d=1}^{s_1+s_2} w_{dk} \delta_d \\ &\text{s.t. } \sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \geq 0, j \notin k, j \in \text{NBV} \\ &\quad \sum_{d=1}^{s_1+s_2} \delta_d = 1 \\ &\quad \delta_d \geq 0, d = 1, 2, \dots, s_1 + s_2. \end{aligned} \tag{14}$$

The variable λ_k is Pareto if the optimal solution of model (14) is negative. In contrast, this variable is not Pareto if the optimal solution of model (14) is non-negative.

Next, for each Pareto variable λ_k a set of trade-offs is defined by which some objectives are increased and others are decreased. For example, considering whether a decrease in the objective functions f_1 of w_{ij} , or a decrease in the objective function f_2 of w_{2j} is desirable or not, are among some of the potential trade-off scenarios. A number trade-offs similar to these two examples are presented to the DM, who is requested to state whether the trade-offs are desirable, undesirable or neither. If the trade-offs are undesirable for all efficient variables, we terminate the procedure and take the δ_d 's as the best set of weights. Otherwise, using the DM's preferences, we construct constraints to restrict the choice of the weights δ_d to be used in finding a new Pareto solution. Then, for each desirable trade-off we construct an inequality of the form (15), for each undesirable trade-off, construct an inequality of the form (16) and for each indifference trade-off, construct an equality of the form (17) as follows:

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \leq -\varepsilon \tag{15}$$

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \geq \varepsilon \tag{16}$$

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d = 0 \tag{17}$$

A feasible solution is established for all the previously constructed constraints of the form (15)-(17), as well as the following set of constraints:

$$\sum_{d=1}^{s_1+s_2} \delta_d = 1, \delta_d \geq \varepsilon, d = 1, 2, \dots, s_1 + s_2 \tag{18}$$

Thus, a new set of consistent multipliers is constructed and the associated Pareto solution is found. The process is then repeated, a new set of trade-offs for the current solution is presented to the DM, and convergence to an overall optimal solution with respect to the DMs implicit utility function is assured.

It is worth noting that there is no such thing as the best interactive method. In this research we apply the method proposed by Zionts and Wallenius [64] to find the most preferred solution. This method guarantees a convergence in a finite number of iterations, since each trade-off is attractive to the DM, and ultimately is less demanding for the DM.

6. Application problem. We used the performance evaluation problem proposed by Fare et al. [18] to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures and algorithms. In this problem, 30 paper mills use fiber, energy, capital and labor as inputs to produce paper, together with four undesirable outputs (pollutants) including biochemical oxygen demand (BOD), total suspended solids (TSS), particulates (PART) and sulfur oxides (SOX) as outputs. Table 1 shows the input and output values used in this example.

Table 1: Input and Output values

| <i>DMU</i> | <i>I</i> ₁ :Fibr | <i>I</i> ₂ :Energy | <i>I</i> ₃ :Capital | <i>I</i> ₄ :Labor | <i>O</i> ₁ :Paper | <i>O</i> ₂ :BOD | <i>O</i> ₃ :TSS | <i>O</i> ₄ :Part | <i>O</i> ₅ :SOX |
|--------------------------|-----------------------------|-------------------------------|--------------------------------|------------------------------|------------------------------|----------------------------|----------------------------|-----------------------------|----------------------------|
| <i>DMU</i> ₁ | 114784 | 2583397 | 104617771 | 2325437 | 126400 | 8125.93 | 2074.67 | 35.14 | 1206.14 |
| <i>DMU</i> ₂ | 144360 | 2693196 | 93222042 | 1258590 | 140000 | 1948.95 | 582.12 | 11.73 | 99.1 |
| <i>DMU</i> ₃ | 33476 | 1297442 | 43125693 | 946425 | 48610 | 5027.53 | 892.34 | 281.75 | 2273.05 |
| <i>DMU</i> ₄ | 101107 | 2026610 | 101389518 | 984721 | 112400 | 3874.19 | 2051.83 | 25.66 | 574.15 |
| <i>DMU</i> ₅ | 56233 | 1240542 | 60412649 | 1452778 | 50404 | 1474.07 | 493.5 | 23.03 | 602.2 |
| <i>DMU</i> ₆ | 32710 | 979727 | 18687200 | 163993 | 33181 | 2708.95 | 190.69 | 3.32 | 62.99 |
| <i>DMU</i> ₇ | 132060 | 4108283 | 141911775 | 1638857 | 191700 | 684.79 | 1838.42 | 254.4 | 12129.65 |
| <i>DMU</i> ₈ | 167581 | 3925382 | 63997349 | 536171 | 126382 | 398.39 | 3844.1 | 1633.84 | 3651.88 |
| <i>DMU</i> ₉ | 73433 | 1294136 | 69009645 | 922255 | 96255 | 857.17 | 407.36 | 38.78 | 87.17 |
| <i>DMU</i> ₁₀ | 88526 | 1066561 | 76110116 | 1083787 | 90904 | 4704.38 | 2709.68 | 65.74 | 1668.86 |
| <i>DMU</i> ₁₁ | 70023 | 2427810 | 59340716 | 1593778 | 86400 | 1874.64 | 1299.04 | 2284.27 | 3255.85 |
| <i>DMU</i> ₁₂ | 101840 | 3755227 | 105794281 | 1138557 | 126500 | 6679.5 | 5511.5 | 2177.77 | 7699.69 |
| <i>DMU</i> ₁₃ | 181960 | 3143984 | 132106333 | 2943591 | 157500 | 8522.75 | 4270.5 | 58.18 | 7242.54 |
| <i>DMU</i> ₁₄ | 165751 | 2277712 | 114003930 | 932654 | 147000 | 2871.85 | 1456.4 | 250.11 | 4102.9 |
| <i>DMU</i> ₁₅ | 18002 | 517173 | 26009657 | 221670 | 18000 | 122.43 | 163.97 | 122.55 | 671.65 |
| <i>DMU</i> ₁₆ | 37543 | 533377 | 20780841 | 424940 | 30000 | 443.04 | 845.21 | 31.13 | 849.43 |
| <i>DMU</i> ₁₇ | 43307 | 1062966 | 46773273 | 692170 | 41300 | 10199.93 | 1357.8 | 285.43 | 3547.98 |
| <i>DMU</i> ₁₈ | 204769 | 4934056 | 94988555 | 3144336 | 146000 | 2779.83 | 2672.09 | 257.81 | 8638.65 |
| <i>DMU</i> ₁₉ | 14743 | 304031 | 18135073 | 315986 | 19462 | 211.65 | 121.27 | 341.72 | 469.3 |
| <i>DMU</i> ₂₀ | 96646 | 2636359 | 102718778 | 1425165 | 101000 | 5420.85 | 9015.5 | 57.24 | 3469.72 |
| <i>DMU</i> ₂₁ | 134856 | 3083519 | 106167654 | 1721720 | 143000 | 13318.19 | 991.13 | 286.62 | 6537.93 |
| <i>DMU</i> ₂₂ | 312910 | 5771544 | 262191752 | 1911426 | 293000 | 1370.43 | 2796.95 | 257.04 | 71.71 |
| <i>DMU</i> ₂₃ | 88889 | 2668211 | 74725292 | 549671 | 100000 | 2634.47 | 2546.63 | 31.83 | 381.88 |
| <i>DMU</i> ₂₄ | 63574 | 574183 | 28979040 | 383460 | 53700 | 1139.25 | 1997.72 | 233.6 | 458.48 |
| <i>DMU</i> ₂₅ | 88550 | 3467369 | 127072960 | 1710987 | 115000 | 936 | 936 | 357.52 | 5853.66 |
| <i>DMU</i> ₂₆ | 181248 | 2823012 | 76068394 | 762071 | 216579 | 593.1 | 889.75 | 40.95 | 4153.82 |
| <i>DMU</i> ₂₇ | 122178 | 1630787 | 46925164 | 256286 | 119986 | 838 | 988.45 | 197.77 | 1970.87 |
| <i>DMU</i> ₂₈ | 165178 | 3795558 | 66950232 | 783639 | 184393 | 455.9 | 1146.56 | 51.03 | 1.26 |
| <i>DMU</i> ₂₉ | 71380 | 1277625 | 38307455 | 350594 | 92615 | 117.56 | 17.38 | 118.19 | 171.19 |
| <i>DMU</i> ₃₀ | 32298 | 676122 | 35833745 | 643351 | 34000 | 86.79 | 115.48 | 2.84 | 1.97 |

Table 2: Efficiency scores and observed composite unit

| Inefficient units | Efficiency | DMU ₂ | DMU ₃ | DMU ₆ | DMU ₇ | DMU ₉ | DMU ₁₉ | DMU ₂₆ | DMU ₂₈ | DMU ₂₉ | DMU ₃₀ |
|-------------------|------------|------------------|------------------|------------------|------------------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| DMU ₁ | 0.99553 | 0.32318 | - | - | - | 0.16851 | - | 0.2641 | - | - | 0.24421 |
| DMU ₄ | 0.99918 | 0.32237 | - | - | - | 0.3599 | - | 0.12003 | - | - | 0.19769 |
| DMU ₅ | 0.99373 | 0.15776 | - | - | - | - | - | - | - | - | 0.84224 |
| DMU ₈ | 0.99227 | - | - | - | - | - | - | - | 0.37865 | 0.62135 | - |
| DMU ₁₁ | 0.91676 | - | - | - | 0.16333 | - | 0.19895 | - | - | - | 0.63773 |
| DMU ₁₂ | 0.88156 | - | 0.06216 | - | 0.53733 | 0.10268 | - | - | - | - | 0.29783 |
| DMU ₁₃ | 0.98692 | 0.74423 | - | - | - | - | - | 0.25577 | - | - | - |
| DMU ₁₄ | 0.92485 | 0.17179 | - | - | - | - | - | 0.5846 | - | - | 0.24361 |
| DMU ₁₇ | 0.9062 | 0.06121 | - | - | - | - | - | 0.02786 | - | - | 0.91093 |
| DMU ₁₈ | 0.90675 | 0.55091 | - | - | - | - | - | 0.36249 | - | - | 0.08661 |
| DMU ₂₀ | 0.98511 | 0.33001 | - | - | - | - | - | 0.18373 | - | - | 0.48626 |
| DMU ₂₁ | 0.94178 | - | - | - | - | - | - | 0.47777 | - | 0.52223 | - |
| DMU ₂₃ | 0.99752 | - | - | 0.36415 | - | 0.10274 | - | 0.07534 | 0.3085 | - | 0.14928 |
| DMU ₂₅ | 0.96587 | - | - | - | 0.2311 | - | - | 0.02864 | - | 0.74026 | - |

We solved model (2) with $W = (20000, 10000, 3000, 15000)$ to find the respective efficiency scores. The inefficient units and their references units are given in Table 2. As shown in Table 2, units 1, 4, 5, 8, 11-14, 17, 18, 20, 21, 23 and 25 were inefficient paper mills. We then used formulation (3) for each inefficient unit and found a virtual unit on the efficient frontier that could be regarded as its target unit. The results are presented in Table 3. As shown in Table 3, unit 1 has an

Table 3: Target units

| Inefficient units | I ₁ | I ₂ | I ₃ | I ₄ | O ₁ | O ₂ | O ₃ | O ₄ | O ₅ |
|-------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| DMU ₁ | 114784 | 1999142 | 70597483 | 920535.8 | 126967.3 | 952.14 | 519.96 | 21.83 | 1144.24 |
| DMU ₄ | 101107 | 1806491 | 71104126 | 956316.7 | 112492.4 | 1025.14 | 463.9 | 23.22 | 562.29 |
| DMU ₅ | 49976.41 | 994326.8 | 44888757 | 740408.4 | 50722.1 | 380.56 | 189.1 | 4.24 | 17.29 |
| DMU ₈ | 106896.52 | 2231038 | 49153016 | 514566.1 | 127366.7 | 245.67 | 444.94 | 92.76 | 106.85 |
| DMU ₁₁ | 70023 | 1546257 | 51215700 | 554118.1 | 94244.81 | 228.92 | 335.48 | 184.91 | 2183.65 |
| DMU ₁₂ | 101840 | 2801561 | 97429527 | 1138557 | 143495 | 803.51 | 1090.32 | 193.39 | 6718.91 |
| DMU ₁₃ | 153794.9 | 2726399 | 88834627 | 1131595 | 159586.7 | 1602.16 | 660.8 | 19.2 | 1136.18 |
| DMU ₁₄ | 138625.55 | 2277712 | 69214302 | 818448.6 | 158945.6 | 702.69 | 648.28 | 26.65 | 2445.82 |
| DMU ₁₇ | 43307 | 859397.1 | 40469129 | 684315.4 | 45574.95 | 214.87 | 165.61 | 4.45 | 123.59 |
| DMU ₁₈ | 148026.34 | 2565564 | 82034093 | 1025325 | 158578.9 | 1296.2 | 653.22 | 21.55 | 1560.48 |
| DMU ₂₀ | 96646 | 1736223 | 62165711 | 868198.6 | 102526.1 | 794.35 | 411.73 | 12.78 | 796.84 |
| DMU ₂₁ | 123871.19 | 2015958 | 56348347 | 547183.7 | 151840.8 | 344.76 | 434.17 | 81.29 | 2073.96 |
| DMU ₂₃ | 88889 | 1974258 | 45629178 | 549671 | 100248.8 | 1272.8 | 549.27 | 24.44 | 345.51 |
| DMU ₂₅ | 88550 | 1976055 | 63332046 | 660098.2 | 119064.2 | 262.27 | 463.21 | 147.46 | 3048.87 |

efficient score of 0.99553 implying that it is operating as an inefficient paper mill and also its composite unit on the efficient frontier can be represented as a linear combination of 0.323118 of unit 2, 0.16851 of unit 9, 0.2641 of unit 26 and 0.24421 of unit 30. In this case, based on formulation (3), the following virtual unit was used to evaluate the performance of unit 1 and could be regarded as a target unit of this unit:

$$(I_1, I_2, I_3, I_4) = (114784, 1999142, 70597483, 920535.8)$$

$$(O_1, O_2, O_3, O_4, O_5) = (126967.3, 952.14, 519.96, 21.83, 1144.24)$$

This shows that, for unit 1 to become efficient, the values of inputs should be reduced from 144360, 2693196, 93222042 and 1258590 to 114784, 1999142, 70597483 and 920535.8, respectively, the value of paper (desirable output) should increase from 126400 to 126967.3, and the values of undesirable outputs should decrease from 8125.93, 2074.67, 35.14 and 1206.14 to 952.14, 519.96, 21.83 and 1144.24, respectively. However, the DM has not accepted the DEA composite unit as the most preferred solution for unit 1. Therefore, the approach proposed in this study is needed to search the most preferred solution along the frontier for unit 1. The MOLP problem corresponding to unit 1 is formulated as follows based on model (10):

$$\begin{aligned}
& \max \frac{1}{126400} (126400\lambda_1 + 140000\lambda_2 + 48610\lambda_3 + 112400\lambda_4 + 50404\lambda_5) + \\
& \quad \frac{1}{126400} (33181\lambda_6 + 191700\lambda_7 + 126382\lambda_8 + 96255\lambda_9 + 90904\lambda_{10}) + \\
& \quad \frac{1}{126400} (86400\lambda_{11} + 126500\lambda_{12} + 157500\lambda_{13} + 147000\lambda_{14} + 18000\lambda_{15} + \\
& \quad \frac{1}{126400} (30000\lambda_{16} + 41300\lambda_{17} + 146000\lambda_{18} + 19462\lambda_{19} + 101000\lambda_{20}) + \\
& \quad \frac{1}{126400} (143000\lambda_{21} + 293000\lambda_{22} + 100000\lambda_{23} + 53700\lambda_{24} + 115000\lambda_{25}) + \\
& \quad \frac{1}{126400} (216579\lambda_{26} + 119986\lambda_{27} + 184393\lambda_{28} + 92615\lambda_{29} + 34000\lambda_{30}) \\
& \max \frac{1}{11874.07} (11874.07\lambda_1 + 18051.05\lambda_2 + 14972.47\lambda_3 + 16125.81\lambda_4 + 18525.93\lambda_5) + \\
& \quad \frac{1}{11874.07} (17291\lambda_6 + 19315.21\lambda_7 + 19601.61 + \lambda_8 + 19142.83\lambda_9 + 15295.62\lambda_{10}) + \\
& \quad \frac{1}{11874.07} (18125.36\lambda_{11} + 13320.5\lambda_{12} + 11444.25\lambda_{13} + 17128.15\lambda_{14} + 19877.57\lambda_{15}) + \\
& \quad \frac{1}{11874.07} (19556.96\lambda_{16} + 9800.07\lambda_{17} + 17220.17\lambda_{18} + 19788.35\lambda_{19} + 14579.15\lambda_{20}) + \\
& \quad \frac{1}{11874.07} (6681.81\lambda_{21} + 18629.57\lambda_{22} + 17365.53\lambda_{23} + 18860.75\lambda_{24} + 19064\lambda_{25}) + \\
& \quad \frac{1}{11874.07} (19406.9\lambda_{26} + 19162\lambda_{27} + 19544.1\lambda_{28} + 19882.44\lambda_{29} + 19913.21\lambda_{30}) \\
& \max \frac{1}{7925.33} (7925.33\lambda_1 + 9417.88\lambda_2 + 9107.66\lambda_3 + 7948.17\lambda_4 + 9506.5\lambda_5) + \\
& \quad \frac{1}{7925.33} (9809.31\lambda_6 + 8161.581\lambda_7 + 6155.9\lambda_8 + 9592.64\lambda_9 + 7290.32\lambda_{10}) + \\
& \quad \frac{1}{7925.33} (8700.96\lambda_{11} + 4488.5\lambda_{12} + 5729.5\lambda_{13} + 8543.6\lambda_{14} + 9836.03\lambda_{15}) + \\
& \quad \frac{1}{7925.33} (9154.79\lambda_{16} + 8642.2\lambda_{17} + 7327.91\lambda_{18} + 9878.73\lambda_{19} + 984.5\lambda_{20}) + \\
& \quad \frac{1}{7925.33} (9008.87\lambda_{21} + 7203.05\lambda_{22} + 7453.37\lambda_{23} + 8002.28\lambda_{24} + 9064\lambda_{25}) + \\
& \quad \frac{1}{7925.33} (9110.25\lambda_{26} + 9011.55\lambda_{27} + 8853.44\lambda_{28} + 9982.62\lambda_{29} + 9884.52\lambda_{30}) \\
& \max \frac{1}{2964.86} (2964.86\lambda_1 + 2988.27\lambda_2 + 2718.25\lambda_3 + 2974.34\lambda_4 + 2976.97\lambda_5) + \\
& \quad \frac{1}{2964.86} (2996.68\lambda_6 + 2745.6\lambda_7 + 1366.16\lambda_8 + 2961.22\lambda_9 + 2934.26\lambda_{10}) + \\
& \quad \frac{1}{2964.86} (715.73\lambda_{11} + 822.23\lambda_{12} + 2941.82\lambda_{13} + 2749.89\lambda_{14} + 2877.45\lambda_{15}) + \\
& \quad \frac{1}{2964.86} (2968.87\lambda_{16} + 2714.57\lambda_{17} + 2742.19\lambda_{18} + 2658.28\lambda_{19} + 2942.76\lambda_{20}) + \\
& \quad \frac{1}{2964.86} (2713.38\lambda_{21} + 2742.96\lambda_{22} + 2968.17\lambda_{23} + 2766.4\lambda_{24} + 2642.48\lambda_{25}) + \\
& \quad \frac{1}{2964.86} (2959.05\lambda_{26} + 2802.23\lambda_{27} + 2948.97\lambda_{28} + 2881.81\lambda_{29} + 2997.16\lambda_{30}) \\
& \max \frac{1}{13793.36} (13793.36\lambda_1 + 14900.0\lambda_2 + 12726.95\lambda_3 + 14425.85\lambda_4 + 14397.8\lambda_5) + \\
& \quad \frac{1}{13793.36} (14937.01\lambda_6 + 2870.35\lambda_7 + 11348.12\lambda_8 + 14912.83\lambda_9 + 13331.14\lambda_{10}) + \\
& \quad \frac{1}{13793.36} (11744.15\lambda_{11} + 7300.31\lambda_{12} + 7757.46\lambda_{13} + 10897.1\lambda_{14} + 14328.35\lambda_{15}) + \\
& \quad \frac{1}{13793.36} (14150.57\lambda_{16} + 11452.02\lambda_{17} + 6361.35\lambda_{18} + 14530.7\lambda_{19} + 11530.28\lambda_{20}) + \\
& \quad \frac{1}{13793.36} (8462.07\lambda_{21} + 14928.29\lambda_{22} + 14618.12\lambda_{23} + 14541.52\lambda_{24} + 9146.34\lambda_{25}) + \\
& \quad \frac{1}{13793.36} (10846.18\lambda_{26} + 13029.13\lambda_{27} + 14998.74\lambda_{28} + 14828.81\lambda_{29} + 14998.03\lambda_{30})
\end{aligned}$$

$$\begin{aligned}
& \text{s.t. } 114784\lambda_1 + 144360\lambda_2 + 33476\lambda_3 + 101107\lambda_4 + 56233\lambda_5 + \\
& 32710\lambda_6 + 132060\lambda_7 + 167581\lambda_8 + 73433\lambda_9 + 88526\lambda_{10} \\
& 170023\lambda_{11} + 101840\lambda_{12} + 181960\lambda_{13} + 165751\lambda_{14} + 18002\lambda_{15} + \\
& 37543\lambda_{16} + 43307\lambda_{17} + 204769\lambda_{18} + 14743\lambda_{19} + 96646\lambda_{20} + \\
& 134856\lambda_{21} + 312910\lambda_{22} + 88889\lambda_{23} + 63574\lambda_{24} + 88550\lambda_{25} + \\
& 181248\lambda_{26} + 122178\lambda_{27} + 1465178\lambda_{28} + 71380\lambda_{29} + 32298\lambda_{30} \leq 114784 \\
& 2583397\lambda_1 + 2693196\lambda_2 + 1297442\lambda_3 + 2026610\lambda_4 + 1240542\lambda_5 + \\
& 979727\lambda_6 + 4108283\lambda_7 + 3925382\lambda_8 + 1294136\lambda_9 + 1066561\lambda_{10} + \\
& 2427810\lambda_{11} + 3755227\lambda_{12} + 3143984\lambda_{13} + 2277712\lambda_{14} + 517173\lambda_{15} + \\
& 533377\lambda_{16} + 1062966\lambda_{17} + 4934056\lambda_{18} + 304031\lambda_{19} + 2636359\lambda_{20} + \\
& 3083519\lambda_{21} + 5771544\lambda_{22} + 2668211\lambda_{23} + 574183\lambda_{24} + 3467369\lambda_{25} + \\
& 2823012\lambda_{26} + 1630787\lambda_{27} + 3795558\lambda_{28} + 1277625\lambda_{29} + 676122\lambda_{30} \leq 2583397 \\
& 104617771\lambda_1 + 93222042\lambda_2 + 43125693\lambda_3 + 101389518\lambda_4 + 60412649\lambda_5 + \\
& 18687200\lambda_6 + 141911775\lambda_7 + 63997349\lambda_8 + 69009645\lambda_9 + 76110116\lambda_{10} + \\
& 59340716\lambda_{11} + 105794281\lambda_{12} + 132106333\lambda_{13} + 114003930\lambda_{14} + 26009657\lambda_{15} + \\
& 20780841\lambda_{16} + 46773273\lambda_{17} + 94988555\lambda_{18} + 18135073\lambda_{19} + 102718778\lambda_{20} + \\
& 106167654\lambda_{21} + 262191752\lambda_{22} + 74725292\lambda_{23} + 28979040\lambda_{24} + 127072960\lambda_{25} + \\
& 76068394\lambda_{26} + 46925163\lambda_{27} + 66950232\lambda_{28} + 38307455\lambda_{29} + 35835745\lambda_{30} \leq 104617771 \\
& 2325437\lambda_1 + 125859042\lambda_2 + 946425\lambda_3 + 984721\lambda_4 + 1452778\lambda_5 + \\
& 163993\lambda_6 + 1638857\lambda_7 + 536171\lambda_8 + 922255\lambda_9 + 1083787\lambda_{10} + \\
& 1593778\lambda_{11} + 1138557\lambda_{12} + 2943591\lambda_{13} + 932654\lambda_{14} + 221670\lambda_{15} + \\
& 424940\lambda_{16} + 692170\lambda_{17} + 3144336\lambda_{18} + 3144336\lambda_{19} + 1425165\lambda_{20} + \\
& 1721720\lambda_{21} + 1911426\lambda_{22} + 549671\lambda_{23} + 383460\lambda_{24} + 1710987\lambda_{25} + \\
& 762071\lambda_{26} + 256286\lambda_{27} + 783639\lambda_{28} + 350594\lambda_{29} + 643351\lambda_{30} \leq 2325437 \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0, j = 1, 2, \dots, n
\end{aligned} \tag{19}$$

The results for using the approach presented in Section 5 for obtaining the most preferred as a target unit for unit 1 are as follows:

Iteration 1:

Initially, an arbitrary set of weights $\delta = (0.2, 0.2, 0.2, 0.2, 0.2)$ was chosen. The composite objective function was then optimized and the optimal basic variables ($\lambda_{28} = 0.462739, \lambda_{29} = 0.537261$) were produced. Hence, the set of non-basic variables were $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$ and model (11) is solved for each of them. As an instance, the following model is solved for the non-basic variable λ_3 :

$$\begin{aligned}
& \max \lambda_3 \\
& \text{s.t. } 114784\lambda_1 + 144360\lambda_2 + 33476\lambda_3 + 101107\lambda_4 + 56233\lambda_5 + \\
& 32710\lambda_6 + 132060\lambda_7 + 167581\lambda_8 + 73433\lambda_9 + 88526\lambda_{10} \\
& 170023\lambda_{11} + 101840\lambda_{12} + 181960\lambda_{13} + 165751\lambda_{14} + 18002\lambda_{15} + \\
& 37543\lambda_{16} + 43307\lambda_{17} + 204769\lambda_{18} + 14743\lambda_{19} + 96646\lambda_{20} + \\
& 134856\lambda_{21} + 312910\lambda_{22} + 88889\lambda_{23} + 63574\lambda_{24} + 88550\lambda_{25} + \\
& 181248\lambda_{26} + 122178\lambda_{27} + 1465178\lambda_{28} + 71380\lambda_{29} + 32298\lambda_{30} \leq 114784 \\
& 2583397\lambda_1 + 2693196\lambda_2 + 1297442\lambda_3 + 2026610\lambda_4 + 1240542\lambda_5 + \\
& 979727\lambda_6 + 4108283\lambda_7 + 3925382\lambda_8 + 1294136\lambda_9 + 1066561\lambda_{10} + \\
& 2427810\lambda_{11} + 3755227\lambda_{12} + 3143984\lambda_{13} + 2277712\lambda_{14} + 517173\lambda_{15} + \\
& 533377\lambda_{16} + 1062966\lambda_{17} + 4934056\lambda_{18} + 304031\lambda_{19} + 2636359\lambda_{20} + \\
& 3083519\lambda_{21} + 5771544\lambda_{22} + 2668211\lambda_{23} + 574183\lambda_{24} + 3467369\lambda_{25} +
\end{aligned}$$

$$\begin{aligned}
& 2823012\lambda_{26} + 1630787\lambda_{27} + 3795558\lambda_{28} + 1277625\lambda_{29} + 676122\lambda_{30} \leq 2583397 \\
& 104617771\lambda_1 + 93222042\lambda_2 + 43125693\lambda_3 + 101389518\lambda_4 + 60412649\lambda_5 + \\
& 18687200\lambda_6 + 141911775\lambda_7 + 63997349\lambda_8 + 69009645\lambda_9 + 76110116\lambda_{10} + \\
& 59340716\lambda_{11} + 105794281\lambda_{12} + 132106333\lambda_{13} + 114003930\lambda_{14} + 26009657\lambda_{15} + \\
& 20780841\lambda_{16} + 46773273\lambda_{17} + 94988555\lambda_{18} + 18135073\lambda_{19} + 102718778\lambda_{20} + \\
& 106167654\lambda_{21} + 262191752\lambda_{22} + 74725292\lambda_{23} + 28979040\lambda_{24} + 127072960\lambda_{25} + \\
& 76068394\lambda_{26} + 46925163\lambda_{27} + 66950232\lambda_{28} + 38307455\lambda_{29} + 35835745\lambda_{30} \leq 104617771 \\
& 2325437\lambda_1 + 125859042\lambda_2 + 946425\lambda_3 + 984721\lambda_4 + 1452778\lambda_5 + \\
& 163993\lambda_6 + 1638857\lambda_7 + 536171\lambda_8 + 922255\lambda_9 + 1083787\lambda_{10} + \\
& 1593778\lambda_{11} + 1138557\lambda_{12} + 2943591\lambda_{13} + 932654\lambda_{14} + 221670\lambda_{15} + \\
& 424940\lambda_{16} + 692170\lambda_{17} + 3144336\lambda_{18} + 3144336\lambda_{19} + 1425165\lambda_{20} + \\
& 1721720\lambda_{21} + 1911426\lambda_{22} + 549671\lambda_{23} + 383460\lambda_{24} + 1710987\lambda_{25} + \\
& 762071\lambda_{26} + 256286\lambda_{27} + 783639\lambda_{28} + 350594\lambda_{29} + 643351\lambda_{30} \leq 2325437 \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0, j = 1, 2, \dots, n
\end{aligned} \tag{20}$$

Now, with regard to optimal solution of model (20) the values of w_{d3} are obtained based on formula (12) as follows:

$$w_{13} = 0.684132, w_{23} = -1.2272, w_{33} = -0.88895, w_{43} = 1.562189, w_{53} = -0.84592.$$

Similarly, after solving model (11) for each of non-basic variables $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$ the w_{dj} values introduced in Table 4 were computed based on formula (12).

In this case, model (14) is solved for each $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$ and variables $\lambda_1, \lambda_2, \lambda_9, \lambda_{26}$ and λ_{30} are determined as the efficient variables. As an example the following model is solved corresponding to non-basic variable λ_3 :

$$\begin{aligned}
& \max 0.684132\delta_1 - 1.2272\delta_2 - 0.88895\delta_3 + 1.562189\delta_4 - 0.84592\delta_5 \\
& \text{s.t. } 0.068705\delta_1 - 0.96627\delta_2 - 0.73977\delta_3 - 1.479011\delta_4 - 0.92326\delta_5 \geq 0 \\
& 0.23154\delta_1 - 1.96919\delta_2 - 1.21967\delta_3 + 1.938938\delta_4 - 1.29226\delta_5 \geq 0 \\
& 0.179464\delta_1 - 1.32433\delta_2 - 0.74265\delta_3 + 1.475813\delta_4 - 0.96908\delta_5 \geq 0 \\
& 0.669939\delta_1 - 1.52647\delta_2 - 0.93928\delta_3 + 1.474926\delta_4 - 0.96705\delta_5 \geq 0 \\
& 0.806197\delta_1 - 1.42247\delta_2 - 0.97748\delta_3 + 1.468279\delta_4 + 1.00614\delta_5 \geq 0 \\
& 0.164043\delta_1 - 2.68526\delta_2 - 1.42937\delta_3 + 2.611593\delta_4 - 0.78475\delta_5 \geq 0 \\
& 0.607399\delta_1 - 2.57836\delta_2 - 1.09716\delta_3 + 2.94988\delta_4 - 1.32098\delta_5 \geq 0 \\
& 0.307194\delta_1 - 1.57842\delta_2 - 0.95015\delta_3 + 1.480239\delta_4 - 1.00438\delta_5 \geq 0 \\
& 0.349527\delta_1 - 1.25442\delta_2 - 0.65964\delta_3 + 1.489332\delta_4 - 0.88972\delta_5 \geq 0 \\
& 0.38516\delta_1 - 1.49273\delta_2 - 0.83764\delta_3 + 2.237607\delta_4 + 0.77467\delta_5 \geq 0 \\
& 0.538181\delta_1 - 1.9275\delta_2 - 0.81315\delta_3 + 3.015211\delta_4 - 0.95462\delta_5 \geq 0 \\
& 0.43689\delta_1 - 2.02924\delta_2 - 1.12495\delta_3 + 2.54935\delta_4 - 1.14147\delta_5 \geq 0 \\
& 0.371751\delta_1 - 2.24059\delta_2 - 1.32023\delta_3 + 2.357696\delta_4 - 1.21084\delta_5 \geq 0 \\
& 0.9263\delta_1 - 1.6403\delta_2 - 0.98086\delta_3 + 1.508493\delta_4 - 0.96201\delta_5 \geq 0 \\
& 0.831363\delta_1 - 1.6133\delta_2 - 0.8949\delta_3 + 1.477658\delta_4 - 0.94912\delta_5 \geq 0 \\
& 0.741964\delta_1 - 0.7916\delta_2 - 0.83022\delta_3 + 1.56343\delta_4 - 0.75349\delta_5 \geq 0 \\
& 0.856985\delta_1 - 3.10035\delta_2 - 1.68148\delta_3 + 3.186024\delta_4 - 1.39166\delta_5 \geq 0 \\
& 0.914733\delta_1 - 1.63278\delta_2 - 0.98624\delta_3 + 1.582415\delta_4 - 0.97668\delta_5 \geq 0 \\
& 0.290908\delta_1 - 1.23202\delta_2 - 0.113094\delta_3 + 1.523233\delta_4 - 0.78186\delta_5 \geq 0
\end{aligned}$$

$$\begin{aligned}
 &0.13808\delta_1 - 0.88724\delta_2 - 1.09288\delta_3 + 1.911033\delta_4 - 0.75102\delta_5 \geq 0 \\
 &0.562248\delta_1 - 4.76884\delta_2 - 2.60184\delta_3 + 4.687746\delta_4 - 2.93977\delta_5 \geq 0 \\
 &0.311602\delta_1 - 1.4895\delta_2 - 0.71691\delta_3 + 1.536775\delta_4 - 1.01936\delta_5 \geq 0 \\
 &0.643863\delta_1 - 1.55466\delta_2 - 0.74948\delta_3 + 1.545948\delta_4 - 0.97746\delta_5 \geq 0 \\
 &0.513642\delta_1 - 2.205\delta_2 - 1.26592\delta_3 + 2.201429\delta_4 - 0.96511\delta_5 \geq 0 \\
 &- 0.03702\delta_1 - 2.68543\delta_2 - 1.54451\delta_3 + 2.532276\delta_4 - 1.35844\delta_5 \geq 0 \\
 &0.187056\delta_1 - 1.70071\delta_2 - 0.94972\delta_3 + 1.650819\delta_4 - 0.94001\delta_5 \geq 0 \\
 &0.799717\delta_1 - 1.6433\delta_2 - 0.98697 - \delta_3 + 1.468117\delta_4 - 1.01056\delta_5 \geq 0 \\
 &\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1 \\
 &\delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \geq 0
 \end{aligned} \tag{21}$$

The DM was then asked to indicate the acceptability of the trade-offs. Since the DM accepts the trade for efficient variables λ_1, λ_2 , does not accept the trade for efficient variables λ_9, λ_{26} and is indifferent to variable λ_{30} , a feasible solution to the following set of constraints was found:

$$\begin{aligned}
 &\sum_{d=1}^5 w_{dj}\delta_d \leq -\varepsilon, \quad j = 1, 2 \\
 &\sum_{d=1}^5 w_{dj}\delta_d \geq \varepsilon, \quad j = 9, 26 \\
 &\sum_{d=1}^5 w_{d,30}\delta_d = 0 \\
 &\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1 \\
 &\delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \geq \varepsilon
 \end{aligned}$$

As a result, the following new set of weights is found:

$$\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$$

Iteration 2:

Table 4: Initial w_{dj} values in iteration 1

| <i>NBV</i> | w_{1j} | w_{2j} | w_{3j} | w_{4j} | w_{5j} |
|----------------|----------|----------|----------|----------|----------|
| λ_1 | 0.068705 | -0.96627 | -0.73977 | 1.479011 | -0.92326 |
| λ_2 | 0.23154 | -1.96919 | -1.21967 | 1.938938 | -1.29226 |
| λ_3 | 0.684132 | -1.2272 | -0.88895 | 1.562189 | -0.84592 |
| λ_4 | 0.179464 | -1.32433 | -0.74265 | 1.475813 | -0.96908 |
| λ_5 | 0.669939 | -1.52647 | -0.93928 | 1.474926 | -0.96705 |
| λ_6 | 0.806197 | -1.42247 | -0.97748 | 1.468279 | -1.00614 |
| λ_7 | 0.164043 | -2.68526 | -1.42937 | 2.611593 | -0.78475 |
| λ_8 | 0.607399 | -2.57836 | -1.09716 | 2.94988 | -1.32098 |
| λ_9 | 0.307194 | -1.57842 | -0.95015 | 1.480239 | -1.00438 |
| λ_{10} | 0.349527 | -1.25442 | -0.65964 | 1.489332 | -0.88972 |
| λ_{11} | 0.38516 | -1.49273 | -0.83764 | 2.237607 | -0.77467 |
| λ_{12} | 0.538181 | -1.9275 | -0.81315 | 3.015211 | -0.95462 |
| λ_{13} | 0.43689 | -2.02924 | -1.12495 | 2.54935 | -1.14147 |
| λ_{14} | 0.371751 | -2.24059 | -1.32023 | 2.357696 | -1.21084 |
| λ_{15} | 0.9263 | -1.6403 | -0.98086 | 1.508493 | -0.96201 |
| λ_{16} | 0.831363 | -1.6133 | -0.8949 | 1.477658 | -0.94912 |
| λ_{17} | 0.741964 | -0.7916 | -0.83022 | 1.56343 | -0.75349 |
| λ_{18} | 0.856985 | -3.10035 | -1.68148 | 3.186024 | -1.39166 |
| λ_{19} | 0.914733 | -1.63278 | -0.98624 | 1.582415 | -0.97668 |
| λ_{20} | 0.290908 | -1.23202 | 0.113094 | 1.523233 | -0.78186 |
| λ_{21} | 0.13808 | -0.88724 | -1.09288 | 1.911033 | -0.75102 |
| λ_{22} | 0.562248 | -4.76884 | -2.60184 | 4.687746 | -2.93977 |
| λ_{23} | 0.311602 | -1.4895 | -0.71691 | 1.536775 | -1.01936 |
| λ_{24} | 0.643863 | -1.55466 | -0.74948 | 1.545948 | -0.97746 |
| λ_{25} | 0.513642 | -2.205 | -1.26592 | 2.201429 | -0.96511 |
| λ_{26} | -0.03702 | -2.68543 | -1.54451 | 2.532276 | -1.35844 |
| λ_{27} | 0.187056 | -1.70071 | -0.94972 | 1.650819 | -0.94001 |
| λ_{30} | 0.799717 | -1.6433 | -0.98697 | 1.468117 | -1.01056 |

Using the weights $\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$, the composite objective function was optimized and the optimal basic variables were $(\lambda_9 = 0.549283, \lambda_{28} = 0.450717)$. However, in this iteration the set of non-basic variables were $\lambda_j, j = 1, 2, \dots, 30, j \neq 9, 28$ and after solving model (11) for each of them, the w_{dj} values presented in Table 5 were derived by using formula (12). Model (14) was solved for each $\lambda_j, j = 1, 2, \dots, 30, j \neq 9, 28$ and the variables $\lambda_1, \lambda_2, \lambda_4, \lambda_{26}$ and λ_{30} were determined as efficient variables. The DM is then asked to indicate his/her acceptability of the trade-offs. Since all responses for the efficient variables were no, we terminated the procedure and introduced $\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$ as the best set of weights for the objective functions in model (10) with the following optimal solutions: $\lambda_9 = 0.549283$ and $\lambda_{28} = 0.450717$. This optimal solution provides the following new target unit for the inefficient unit 1 using formulation (3):

$$(I1, I2, I3, I4) = (114784, 2421569, 68081433, 859778.5)$$

$$(O1, O2, O3, O4, O5) = (135980.3, 676.3109, 740.5298, 44.30128, 48.44893)$$

This composite unit has been produced by taking into consideration the DM's preference information as the most preferred solution for unit 1. As reported in

Table 5: The first values of w_{dj} in iteration 2

| <i>NBV</i> | w_{1j} | w_{2j} | w_{3j} | w_{4j} | w_{5j} |
|----------------|----------|----------|----------|----------|----------|
| λ_1 | 0.075793 | -0.91677 | -0.64306 | 0.260708 | -0.95983 |
| λ_2 | 0.240725 | -1.90506 | -1.09437 | 0.360458 | -1.33964 |
| λ_3 | 0.691222 | -1.17771 | -0.79225 | 0.343886 | -0.88248 |
| λ_4 | 0.186553 | -1.27484 | -0.64594 | 0.257511 | -1.00565 |
| λ_5 | 0.677027 | -1.47697 | -0.84257 | 0.256623 | -1.00361 |
| λ_6 | 0.813285 | -1.37297 | -0.88078 | 0.249976 | -1.04271 |
| λ_7 | 0.175874 | -2.60266 | -1.26797 | 0.578251 | -0.84578 |
| λ_8 | 0.618662 | -2.49973 | -0.94352 | 1.014297 | -1.37908 |
| λ_{10} | 0.356616 | -1.20492 | -0.56294 | 0.271029 | -0.92629 |
| λ_{11} | 0.392249 | -1.44324 | -0.74093 | 1.019304 | -0.81124 |
| λ_{12} | 0.548914 | -1.85256 | -0.66672 | 1.170574 | -1.00999 |
| λ_{13} | 0.448738 | -1.94651 | -0.96331 | 0.512975 | -1.2026 |
| λ_{14} | 0.382451 | -2.16588 | -1.17426 | 0.518715 | -1.26604 |
| λ_{15} | 0.933388 | -1.5908 | -0.88415 | 0.29019 | -0.99858 |
| λ_{16} | 0.838451 | -1.5638 | -0.79819 | 0.259355 | -0.98569 |
| λ_{17} | 0.749053 | -0.7421 | -0.73351 | 0.345127 | -0.79006 |
| λ_{18} | 0.871384 | -2.99981 | -1.48504 | 0.711312 | -1.46595 |
| λ_{19} | 0.921822 | -1.58329 | -0.88954 | 0.364113 | -1.01325 |
| λ_{20} | 0.298162 | -1.18137 | 0.212048 | 0.276622 | -0.81927 |
| λ_{21} | 0.146724 | -0.82689 | -0.97496 | 0.425419 | -0.79562 |
| λ_{22} | 0.583375 | -4.62132 | -2.31361 | 1.056657 | -3.04876 |
| λ_{23} | 0.318954 | -1.43816 | -0.61661 | 0.27314 | -1.05729 |
| λ_{24} | 0.650951 | -1.50517 | -0.65277 | 0.327645 | -1.01403 |
| λ_{25} | 0.523479 | -2.13631 | -1.13171 | 0.51065 | -1.01586 |
| λ_{26} | -0.02522 | -2.60305 | -1.38355 | 0.504572 | -1.41931 |
| λ_{27} | 0.194668 | -1.64756 | -0.84586 | 0.342472 | -0.97928 |
| λ_{29} | 0.34308 | -1.59121 | -0.90265 | 0.288719 | -1.03486 |
| λ_{30} | 0.806806 | -1.5938 | -0.89027 | 0.249814 | -1.04713 |

Table 3, unit 4 with the efficient score of 0.99918 is another inefficient DMU and its reference set includes units 2, 9, 26 and 30. Thus its projection point (target pattern) on the efficient frontier can be determined as a linear combination of 0.32237 of unit 2, 0.3599 of unit 9, 0.12003 of unit 26 and 0.19769 of unit 30. Therefore, the following target unit, calculated based on formulation (3), can be applied to evaluate the performance of unit 4:

$$(I1, I2, I3, I4) = (101107, 1806491.45, 71104125.72, 956316.68)$$

$$(O1, O2, O3, O4, O5) = (112492.39, 1025.14, 463.9, 23.22, 562.29)$$

However, the DM could not accept this target pattern as the most preferred solution for unit 4. In this case, the interactive method can help the DM to search the most preferred solution on the efficient frontier for unit 4.

Iteration 1:

Having started with weights $\delta = (0.2, 0.2, 0.2, 0.2)$, variables $(\lambda_{26} = 0.034911, \lambda_{28} = 0.276034, \lambda_{29} = 0.689055)$ showed to be optimal basic. After solving model (11) for non basic variables and using formula (12) we obtained the values of w_{dj} as presented in Table 6. By Solving model (14), new optimal basic variables were

Table 6: Initial w_{dj} values in iteration 1 for unit 4

| <i>NBV</i> | w_{1j} | w_{2j} | w_{3j} | w_{4j} | w_{5j} |
|----------------|----------|-----------|-----------|-----------|-----------|
| λ_1 | 1.258187 | -2.333264 | -2.551751 | 7.308161 | -1.439814 |
| λ_2 | 0.30043 | -1.645807 | -1.544402 | 4.218496 | -0.849073 |
| λ_3 | 0.655396 | -0.869743 | -0.970659 | 2.864551 | -0.423604 |
| λ_4 | 0.087869 | -0.941264 | -0.824777 | 2.778451 | -0.541372 |
| λ_5 | 1.091488 | -1.668108 | -1.624745 | 4.35782 | -0.868376 |
| λ_6 | 0.792665 | -1.013523 | -1.058937 | 2.77094 | -0.576805 |
| λ_7 | 0.487759 | -2.550998 | -2.141869 | 6.341425 | -0.403355 |
| λ_8 | 0.971756 | -2.444701 | -1.77616 | 6.498911 | -0.932781 |
| λ_9 | 0.231508 | -1.128357 | -1.031677 | 2.782862 | -0.575129 |
| λ_{10} | 0.374794 | -1.012119 | -0.869829 | 3.126393 | -0.53511 |
| λ_{11} | 0.907418 | -1.81739 | -1.70532 | 5.594113 | -0.78352 |
| λ_{12} | 0.880351 | -1.939782 | -1.460912 | 6.396835 | -0.597992 |
| λ_{13} | 1.578514 | -3.072019 | -3.07305 | 9.402918 | -1.455807 |
| λ_{14} | 0.464714 | -1.877966 | -1.69885 | 5.01314 | -0.707419 |
| λ_{15} | 0.927727 | -1.17392 | -1.062299 | 2.811027 | -0.534613 |
| λ_{16} | 0.820965 | -1.154038 | -0.976588 | 2.78029 | -0.522289 |
| λ_{17} | 0.720432 | -0.54899 | -0.912097 | 2.865788 | -0.335226 |
| λ_{18} | 1.929057 | -3.742041 | -3.555322 | 10.214688 | -1.49964 |
| λ_{19} | 0.91472 | -1.168387 | -1.067671 | 2.884713 | -0.54864 |
| λ_{20} | 0.631735 | -1.409971 | -0.523876 | 4.297091 | -0.650045 |
| λ_{21} | 0.568996 | -1.316217 | -1.926085 | 5.405612 | -0.645562 |
| λ_{22} | 0.724409 | -3.961933 | -3.349435 | 9.930846 | -1.92171 |
| λ_{23} | 0.538891 | -1.453325 | -1.160195 | 3.854981 | -0.759049 |
| λ_{24} | 0.610111 | -1.110864 | -0.831585 | 2.848363 | -0.54939 |
| λ_{25} | 0.948781 | -2.242223 | -1.977651 | 5.569847 | -0.685056 |
| λ_{27} | 0.243551 | -1.414607 | -1.219056 | 3.540126 | -0.578408 |
| λ_{30} | 0.785378 | -1.17613 | -1.0684 | 2.770779 | -0.581035 |

$\lambda_2, \lambda_4, \lambda_9, \lambda_{22}, \lambda_{27}$ and λ_{30} . DM accepts the trade for $\lambda_9, \lambda_{22}, \lambda_{27}$, rejects the trade for λ_2, λ_{30} and is indifferent to variable λ_4 . Based on this decision the following model is constructed:

$$\begin{aligned} \sum_{d=1}^5 w_{dj} \delta_d &\leq -\varepsilon, \quad j = 9, 22, 27 \\ \sum_{d=1}^5 w_{dj} \delta_d &\geq \varepsilon, \quad j = 2, 30 \\ \sum_{d=1}^5 w_{d,4} \delta_d &= 0 \\ \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 &= 1 \\ \delta_1, \delta_2, \delta_3, \delta_4, \delta_5 &\geq \varepsilon \end{aligned}$$

By solving the above model, new weights are obtained as

$$\delta = (0.4883066368, 0.3735303166, 0.0000001, 0.1155124839, 0.0226504627).$$

Iteration 2:

By using above weights, optimizing the composite objective function resulted in $\lambda_7 = 0.167337, \lambda_{26} = 0.17815, \lambda_{28} = 0.654513)$ as optimal basic variables. By solving model (11) for non basic variables and using formula (12), the values of w_{dj} are derived as shown in Table 7. Model (14) was solved for each non basic variables

Table 7: w_{dj} values in iteration 2 for unit 4

| | | | | | |
|----------------|----------|----------|----------|----------|----------|
| λ_1 | 1.469137 | -2.28592 | -2.4003 | 10.41463 | 10.57085 |
| λ_2 | 0.420646 | -1.61883 | -1.4581 | 5.988813 | 5.995578 |
| λ_3 | 0.735496 | -0.85177 | -0.91315 | 4.044116 | 4.136997 |
| λ_4 | 0.16797 | -0.92329 | -0.76727 | 3.958016 | 4.019229 |
| λ_5 | 1.217269 | -1.63988 | -1.53444 | 6.210085 | 6.293112 |
| λ_6 | 0.872765 | -0.99555 | -1.00143 | 3.950505 | 3.983796 |
| λ_8 | 1.140149 | -2.40691 | -1.65527 | 8.978692 | 8.6549 |
| λ_9 | 0.311608 | -1.11038 | -0.97417 | 3.962427 | 3.985472 |
| λ_{10} | 0.464562 | -0.99197 | -0.80538 | 4.448337 | 4.575979 |
| λ_{11} | 1.04696 | -1.78607 | -1.60514 | 7.649026 | 7.161474 |
| λ_{12} | 1.040832 | -1.90377 | -1.3457 | 8.7601 | 8.539196 |
| λ_{13} | 1.849793 | -3.01114 | -2.87829 | 13.39781 | 13.98979 |
| λ_{14} | 0.60477 | -1.84654 | -1.5983 | 7.075617 | 7.266821 |
| λ_{15} | 1.007827 | -1.15594 | -1.00479 | 3.990592 | 4.025988 |
| λ_{16} | 0.901066 | -1.13606 | -0.91908 | 3.959855 | 4.038312 |
| λ_{17} | 0.800532 | -0.53101 | -0.85459 | 4.045353 | 4.225375 |
| λ_{18} | 2.221532 | -3.67641 | -3.34535 | 14.52171 | 15.15277 |
| λ_{19} | 0.99482 | -1.15041 | -1.01016 | 4.064279 | 4.011961 |
| λ_{20} | 0.755324 | -1.38224 | -0.43515 | 6.117083 | 6.386662 |
| λ_{21} | 0.722252 | -1.28182 | -1.81606 | 7.662468 | 8.080217 |
| λ_{22} | 1.00095 | -3.89987 | -3.1509 | 14.00323 | 13.82352 |
| λ_{23} | 0.648826 | -1.42865 | -1.08127 | 5.473894 | 5.500218 |
| λ_{24} | 0.690212 | -1.09289 | -0.77408 | 4.027928 | 4.011211 |
| λ_{25} | 1.105221 | -2.20712 | -1.86534 | 7.873604 | 8.222056 |
| λ_{27} | 0.343194 | -1.39225 | -1.14752 | 5.00748 | 5.094884 |
| λ_{29} | 0.549062 | -2.31664 | -1.91852 | 8.140228 | 8.099473 |
| λ_{30} | 0.865479 | -1.15815 | -1.01089 | 3.950344 | 3.979566 |

and $\lambda_2, \lambda_4, \lambda_9, \lambda_{28}$ and λ_{30} were determined as efficient variables. At this point, the procedure terminated because DM answered no to all proposed trade-offs. The final weight vector is:

$$\delta = (0.0585696617, 0.3383842365, 0.4125174363, 0.0000001, 0.1905285655)$$

and the optimal solution selected as the most preferred solution for unit 4 is

$$(I1, I2, I3, I4) = (71380, 1277625, 38307455, 350594)$$

$$(O1, O2, O3, O4, O5) = (92615, 117.56, 17.38, 118.19, 171.19)$$

7. Conclusions and future research directions. The changing economic conditions have challenged many organizations to search for more effective performance measurement methods. DEA is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous DMUs by evaluating their relative efficiency. Performance measurement in the conventional DEA is based on the assumptions that inputs should be minimized and outputs should be maximized. However, there are circumstances in real-world problems where some output variables should be minimized.

In this paper we obtained an equivalence relation between an output-oriented DEA model with undesirable outputs and a min-ordering optimization model. We showed how a DEA problem can be solved interactively by transforming it into an MOLP formulation. The approach proposed in this study reduced the number of undesirable outputs and resulted in a permissible increase in the number of desirable outputs. The proposed equivalence model utilized an interactive method in the MOLP model and located the most preferred solution along the efficient frontier for each inefficient DMU. We also used the Zionts-Wallenius method to reflect the DMs preferences in the efficiency assessment process. We plan to conduct further research by comparing the results obtained with those that might be obtained with other interactive MOLP methods such as the STEM method [8], the G-D-F method [20], the Wierzbicki method [55], and the STOM method [40]. Our goal is to understand the applicability of different interactive MOLP methods to specific data sets and DM preferences.

We shall also point out two limitations of the equivalency established in this study between the DEA models and the MOLP models which are thought-provoking topics for futures research. In this study all the equivalent relations established between the DEA models and the MOLP models are radial DEA-based models. Since non-radial DEA models usually have a higher discriminating power in evaluating the efficiencies of DMUs, in practice it may be more practical to establish equivalency link between the non-radial DEA models and the MOLP models. Another potential for future research is to extend the proposed model to a cover problems with constant returns-to-scale (CRS).

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