

## AN INTERACTIVE MOLP METHOD FOR SOLVING OUTPUT-ORIENTED DEA PROBLEMS WITH UNDESIRABLE FACTORS

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**ABSTRACT.** Data Envelopment Analysis (DEA) and Multiple Objective Linear Programming (MOLP) are widely used for performance assessment in organizations. Although DEA and MOLP are similar in structure, DEA is used to assess and analyze past performance and MOLP is used to predict future performance. Several equivalence models between output-oriented DEA models and MOLP models have been proposed in the literature. However these models are not applicable to performance evaluation problems with undesirable outputs. We propose an interactive method for solving output-oriented DEA models with undesirable outputs. We show that the output-oriented BCC model of Seiford and Zhu [47] can be equivalently stated as the maximization of the minimum of several objectives over the production possibility set, which in turn is a scalarization of a multi-objective linear program. We then employ the well-known Zionts-Wallenius procedure to solve the multi-objective optimization problem. We present an example to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures and algorithms.

**1. Introduction.** Data envelopment analysis (DEA) is a widely used non parametric method for comparing the inputs and outputs of a set of decision making units (DMUs). DEA, initially introduced by Charnes et al. [9], utilizes linear programming (LP) to determine the relative efficiencies of these functionally similar

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DMUs. A score of one is assigned to the frontier (efficient) units. The frontier units are those with maximum output levels for given input levels or with minimum input levels for given output levels. In the “input-orientated” DEA, a DMU is considered efficient when no other DMU can produce the same outputs by consuming fewer inputs. In contrast, in the “output-orientated” DEA, a DMU is considered efficient when no other DMU can produce more outputs by consuming the same inputs. The conventional DEA generally does not consider a decision maker’s (DM’s) preference structure or value judgments. In recent years, various methods have been proposed to take the DM’s preference information into consideration. Allen et al. [1] have defined value judgments as “logical constructs, incorporated within an efficiency assessment study, reflecting the DMs’ preferences in the process of assessing efficiency”.

Two streams of research known as “target setting” and “weight restriction” are often used to take the DMs’ value judgments into consideration in the DEA models. The first interactive target setting model was introduced by Golany [21]. He combined DEA with multiple objective linear programming (MOLP) and used the DM to allocate a set of input levels as resources and selected the most preferred set of output levels from a set of viable points on the efficient frontier. Thanassoulis et al. [50], Pedraja-Chaparro et al. [41], Tone [54], Jess et al. [29], Korhonen et al. [31], Aparicio et al. [3], Yang et al. [58], Hosseinzadeh Lotfi et al. [23, 24], Bi et al. [7], Ebrahimnejad [12], Ebrahimnejad et al. [13], Hinojosa and Mirmol [22], Jain et al. [28], Mahdilo et al. [38], Saati et al. [46] and Amirteimoori and Kordrostami [2] have proposed various target setting DEA models in the performance evaluation literature. The first weight restriction DEA model was proposed by Thompson et al. [53]. Dyson and Thanassoulis [11], Charnes et al. [10], Wong and Beasley [56], Thompson et al. [51, 52], Podinovski [42], and Asmild et al. [4] have proposed various weight restriction models for performance measurement in DEA.

Yang et al. [58] established a new link between MOLP and the output-oriented DEA dual model, which is a radial model and projects all DMUs on to the efficient frontier by solving  $n$  LP problems. Malekmohammadi et al. [39] suggested a super-ideal model which is identical to a target model that considers both the decrease in total input consumption and the increase in total output production. The proposed model is non-radial; moreover, all of the inputs and outputs are simultaneously projected on to the efficient frontier by solving one LP problem. Yang et al. [59] investigated a hybrid minimax reference point-DEA approach to incorporate the value judgements of both branch managers and head-office directors and to search for the most preferred solution (MPS) along the efficient frontier for each bank branch. Yang et al. [61] explored graphical and analytical methods and procedures for generating and analysing data envelopes and efficient frontiers for multiple input and multiple output DEA models using the DEA-oriented interactive minimax reference point approach. This computational investigation leads to the definition of new efficiency measures as instrumental to help conduct trade-off analysis for setting realistic performance targets. Yang and Xu [60] proved that under certain conditions minimax reference point models are identical to input-oriented dual DEA models for performance assessment. This equivalency link leads to the development of an interactive minimax reference point approach for hybrid efficiency and trade-off Analyses, with the DM’s preferences taken into account interactively. Hosseinzadeh Lotfi et al. [23] have also proposed an equivalence model

between DEA and MOLP. They showed how a DEA problem can be solved interactively through a MOLP transformation. They used the method proposed by Zionts-Wallenius [64] and considered the DMs preferences in the performance measurement process. However their method could not solve problems with undesirable output. Recently, Ebrahimnejad and Tavana [15] first provided a new link between the output-oriented DEA model in the presence of undesirable outputs through a MOLP model with a weighted minimax reference point. They then used the satisfying trade-off method proposed by Nakayama and Sawaragi [40] to help the DM search a target unit corresponding to each inefficient unit. In this study, we propose a simpler and more efficient model by investigating an equivalence model and proposing an interactive trade-off analysis procedure in the MOLP for integrating the DEA-oriented performance assessment and target setting while taking into consideration the DMs preferences. We simultaneously and interactively consider the increase in the total desirable outputs and the decrease in the total undesirable outputs.

The remainder of this paper is organized as follows: Section 2 provides a brief discussion of the output-oriented BCC model in the presence of undesirable outputs. Section 3 introduces a technique for solving MOLP problems known as “min-ordering optimization problem”. In Section 4, we establish a new link between the output-oriented BCC model in the presence of undesirable outputs and the min-ordering optimization problem. Section 5 introduces an interactive multi objective programming method known as the Zionts-Wallenius method for reflecting the DM’s preferences in the efficiency assessment process. In Section 6 we present an example from the literature to demonstrate the applicability of the proposed model and exhibit the efficiency of the procedures and algorithms. Finally, Section 7 draws the conclusive remarks and future research directions.

## 2. Output-oriented BCC model in the presence of undesirable outputs.

The production process in DEA uses a set of inputs to produce a set of outputs. Each producer uses varying levels of inputs and produces varying levels of outputs. According to the efficiency criterion in DEA, either making more output with the same input or making the same output with less input is more efficient. In performance measurement problems with undesirable outputs, DMUs with more desirable outputs and less undesirable outputs (relative to less input resources) are considered efficient. For example, if there are inefficiencies in the production process, the undesirable outputs of wastes and pollutants should be reduced to improve the efficiency of a DMU (Seiford and Zhu [47]; Lewis and Sexton [33]).

Various transformation methods have been proposed to deal with desirable and undesirable outputs in DEA (see Liu and Sumaila [34]). Lu and Lo [37] have classified the alternatives for dealing with undesirable outputs in DEA into three different categories. The methods in the first category simply ignore the undesirable outputs. The methods in the second category either treat the undesirable outputs in terms of a non-linear DEA model or treat them as outputs and adjust the distance measurement to restrict the expansion of the undesirable outputs (Fare et al. [18]). The methods in the third category either treat the undesirable outputs as inputs or apply a monotone decreasing transformation (Lovell et al. [35]). Seiford and Zhu [47] have suggested a simple and interesting method by multiplying the undesirable outputs by (-1) and then using a translation vector to turn the negative undesirable outputs into positive desirable outputs. Fare and Grosskopf [17] have proposed a

more superior approach to Seiford and Zhu's [47] method by adopting a directional distance function and estimating the DMUs' efficiencies based on weak disposability of undesirable outputs.

Zhou et al. [65] integrated DEA efficiency measures with environmental DEA technology in an environmental performance measurement system. They studied the environmental DEA technologies that exhibited non-increasing returns to scale and variant returns to scale (VRS) and proposed pure measures for different scenarios and mixed measures under VRS. Yang and Pollitt [57] used different DEA-based efficiency measurement models and examined the impact of uncontrollable variables together with undesirable outputs based on a research sample of Chinese coal-fired power plants. You and Yan [62] proposed a ratio model to evaluate the undesirable as well as the desirable outputs simultaneously. They used their model to investigate the impact of production pollutants while conducting the efficiency evaluation in the textile industry. Huang et al. [27] introduced an undesirable DEA model for providing accurate and reasonable efficiency score that not only dealt with undesirable output efficiently but also considered the relationship between undesirable outputs. In the proposed model, all the undesirable outputs were divided into three categories and then the desirable and undesirable outputs were combined through a transformation mechanism. They also empirically tested their proposed model in 15 Chinese commercial banks.

The recent applications of DEA models with desirable and undesirable outputs can be seen in the agriculture and dairy industry (Reinhard et al. [43]; Reinhard and Thijssen [44]; Shaik et al. [48]), paper mills (Fare et al. [18]; Hua et al. [26]; Seiford and Zhu [47]), electric utilities (Fare et al. [19]; Korhonen and Luptacik [30]), cement manufacturing (Kumar Mandal and Madheswaran [32]; Riccardi et al. [45]), the oil and gas industry (Sueyoshi and Goto [49]), airports (Yu [63]; Lozano and Gutierrez [36]), health care (Hu et al. [25]), and commercial banks (Barros et al. [6], Ebrahimnejad et al. [14]).

Consider a set of  $n$  DMUs, where  $DMU_j$  has a production plan  $(x_j, y_j)$  with  $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})$  inputs and  $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$  outputs. Suppose that  $x_{ij}$  ( $i = 1, 2, \dots, m$ ) is the  $i$ th input consumed by  $DMU_j$  and  $y_{rj}$  ( $r = 1, 2, \dots, s$ ) is  $r$ th output produced by  $DMU_j$ . The relative efficiency of  $DMU_p$  can be obtained by using the following LP model, so-called output-oriented BCC dual model [5]:

$$\begin{aligned}
 & \max \beta_p \\
 & \text{s.t.} \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, 2, \dots, m \\
 & \quad \sum_{j=1}^n \lambda_j y_{rj} \geq \beta_p y_{rp}, \quad r = 1, 2, \dots, s \\
 & \quad \sum_{j=1}^n \lambda_j = 1 \\
 & \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{1}$$

In this model, an efficiency score,  $\frac{1}{\beta_p}$ , is produced for  $DMU_p$  by maximizing all the inputs radially (proportionally). The objective value of model (1) lies within  $\beta_p \geq 1$ . If  $\beta_p > 1$ , then  $DMU_p$  is not efficient and the parameter  $\beta_p$  indicates the extent by which  $DMU_p$  has to increase outputs to become efficient. However,

model (1) does not consider the undesirable output in the performance measurement and is not capable of finding accurate and reasonable efficiency results if DMUs simultaneously produce desirable and undesirable outputs.

There is no unique and widely accepted method for handling DEA problems with coexisting desirable and undesirable outputs in the literature. Two main methods are non-linear monotonic decreasing transformation approach and linear monotonic decreasing transformation approach. The former approach suggested by Golany and Roll [21] converts an undesirable output into a normal output by a monotonic decreasing function  $u$ . In this approach the undesirable output is modeled by  $u(y_{rj}^b) = \frac{1}{y_{rj}^b}$  where  $y_{rj}^b$  is the undesirable output  $r$  for  $DMU_j$ .

The later approach suggested by Seiford and Zhu [43] first multiplies each undesirable output by  $-1$  and then finds a proper translation vector to change all negative undesirable outputs into positive values. However, in the non-linear transformation approach the scale and intervals of the original data is lost and the reciprocal of a zero value does not exist. Therefore, we employ Seiford and Zhu's [47] approach in the MOLP method proposed in this study.

Suppose we have  $n$  DMUs and each  $DMU_j$  produces  $s_1$  desirable outputs  $y_{rj}^g$  ( $r = 1, 2, \dots, s_1$ ) and  $s_2$  undesirable outputs  $y_{tj}^b$  ( $t = 1, 2, \dots, s_2$ ) using  $m$  inputs. The following output-oriented BCC model suggested by Seiford and Zhu [47] evaluates the efficiency of  $DMU_p$  in the presence of undesirable outputs:

$$\begin{aligned}
 & \max \beta_p \\
 & \text{s.t. } \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, \quad i = 1, 2, \dots, m \\
 & \quad \sum_{j=1}^n \lambda_j y_{rj}^g \geq \beta_p y_{rp}^g, \quad r = 1, 2, \dots, s_1 \\
 & \quad \sum_{j=1}^n \lambda_j \bar{y}_{tj}^b \geq \beta_p \bar{y}_{tp}^b, \quad t = 1, 2, \dots, s_2 \\
 & \quad \sum_{j=1}^n \lambda_j = 1 \\
 & \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{2}$$

where  $\bar{y}_{tj}^b = -y_{tj}^b + w_t > 0$ .

We define a reference set  $E_p = \{j | \lambda_j^* > 0, j = 1, 2, \dots, n\}$  for each inefficient  $DMU_p$ , where  $\lambda_j^* = (\lambda_{1j}^*, \lambda_{2j}^*, \dots, \lambda_{nj}^*)$  is the optimal solution of model (2). The following point on the efficient frontier, regarded as a target unit for the inefficient unit  $DMU_p$ , is then used to evaluate the performance of this DMU:

$$\left( \sum_{j \in E_p} \lambda_j^* x_j, \sum_{j \in E_p} \lambda_j^* y_j^g, \sum_{j \in E_p} \lambda_j^* y_j^b \right) \tag{3}$$

However, this target unit has not taken into consideration the DM's preference structure and an interactive MOLP can be used to address this shortcoming.

**3. Min-ordering optimization problem.** We propose using the MOLP model (4) to optimize a vector of linear functions in the presence of linear constraints:

$$\begin{aligned} \max f(\lambda) &= [g_1(\lambda), \dots, g_r(\lambda), \dots, g_{s_1}(\lambda, h_1(\lambda)), \dots, h_t(\lambda), \dots, h_{s_2}(\lambda)] \\ \text{s.t. } \lambda &\in \Lambda. \end{aligned} \quad (4)$$

The goal of this model is to maximize  $s_1 + s_2 (\geq 2)$  conflicting objective functions with  $g_r : \Lambda \rightarrow R (r = 1, 2, \dots, s_1)$  and  $h_t : \Lambda \rightarrow R (t = 1, 2, \dots, s_2)$ . The decision variables  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_n)^T$  belong to the non-empty feasible region  $\Lambda$ . The objective vectors in the objective space  $R^n$  include the objective values  $f(\lambda)$ . The image of the feasible region,  $Z = f(\lambda)$ , is called a “feasible objective region”. Generally, the primary goal in MOLP is to find the Pareto optimal solutions and assist DMs in selecting the most preferred solution because there are no solutions in the MOLP problems that can simultaneously optimize all the objective functions. Consequently, a solution represented by a point in the decision space is a strictly Pareto optimal if it is not possible to move it within the feasible region and improving the objective function value without deteriorating at least one of the other objectives in the model. However, a solution represented by a point in the decision variable space is a weak Pareto optimal solution if it is not possible to move it within the feasible region and improve all the objective functions. Therefore, we have the following definitions:

**Definition 3.1.** A feasible solution  $\hat{\lambda} \in \Lambda$  is called a strict Pareto optimal solution if there is no feasible solution  $\lambda \in \Lambda$  such that  $f(\lambda) \geq f(\hat{\lambda})$  and  $f(\lambda) \neq f(\hat{\lambda})$ .

**Definition 3.2.** A feasible solution  $\hat{\lambda} \in \Lambda$  is called a weak Pareto optimal solution if there is no feasible solution  $\lambda \in \Lambda$  such that  $f(\lambda) > f(\hat{\lambda})$ .

The traditional scalarization methods are used to solve the MOLP problems. This class of methods does not require any assumption or information with regards to the utility functions of the DMs. In this paper, we use a scalarization method known as min-ordering optimization (Ehrgott [16]) for solving the MOLP problem (4). The min-ordering optimization problem corresponding to the MOLP problem (4) can be defined as follows:

$$\begin{aligned} \max \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\} \\ \text{s.t. } \lambda \in \Lambda. \end{aligned} \quad (5)$$

**Definition 3.3.** A feasible solution  $\hat{\lambda} \in \Lambda$  is a min-ordering optimal solution if there is no feasible solution  $\lambda \in \Lambda$  such that  $\min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\} > \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\hat{\lambda}), h_t(\hat{\lambda})\}$ .

The following Theorem indicates that the weak Pareto optimal solution of the MOLP problem (4) can be determined by solving the min-ordering optimization problem (5).

**Theorem 3.4.** *The optimal solution of the min-ordering optimization problem (5) is the weak Pareto optimal solution of the MOLP problem (4).*

*Proof.* See Ehrgott [16]. □

We now show that the min-ordering optimization problem (5) can be solved as a single objective LP problem. Let us assume  $\psi = \min_{1 \leq r \leq s_1, 1 \leq t \leq s_2} \{g_r(\lambda), h_t(\lambda)\}$

and rewrite the min-ordering problem (5) as follows:

$$\begin{aligned} & \max \psi \\ & \text{s.t. } g_r(\lambda) \geq \psi, \quad r = 1, 2, \dots, s_1 \\ & \quad h_t(\lambda) \geq \psi, \quad t = 1, 2, \dots, s_2 \\ & \quad \lambda \in \Lambda. \end{aligned} \quad (6)$$

In the next section we show that the output-oriented BCC model (2) and the min-ordering optimization problem (6) are equivalent under certain conditions.

**4. A generalized equivalence model.** In the output-oriented DEA models with undesirable outputs, an efficiency score is generated for a DMU by maximizing the desirable outputs and minimizing undesirable outputs with limited inputs. This concept can be represented with a multiple objective optimization problem. The theoretical considerations for combining the DEA and MOLP models are presented next.

Suppose  $y_{rp}^g > 0 (r = 1, 2, \dots, s_1)$  and define  $g_r(\lambda) (r = 1, 2, \dots, s_1)$  and  $h_t(\lambda) (t = 1, 2, \dots, s_2)$  in formulation (2) as follows:

$$g_r(\lambda) = \frac{\sum_{j=1}^n \lambda_j y_{rj}^g}{y_{rp}^g}, \quad r = 1, 2, \dots, s_1 \quad (7)$$

$$h_t(\lambda) = \frac{\sum_{j=1}^n \lambda_j \bar{y}_{tj}^b}{\bar{y}_{tp}^b}, \quad t = 1, 2, \dots, s_2 \quad (8)$$

Therefore, formulation (2) can be written as follows:

$$\begin{aligned} & \max \beta_p \\ & \text{s.t. } g_r(\lambda) \geq \beta_p, \quad r = 1, 2, \dots, s_1 \\ & \quad h_t(\lambda) \geq \beta_p, \quad t = 1, 2, \dots, s_2 \\ & \quad \lambda \in \Lambda_p. \end{aligned} \quad (9)$$

where

$$\Lambda_p = \left\{ \lambda = (\lambda_1, \lambda_2, \dots, \lambda_n) \mid \sum_{j=1}^n \lambda_j x_{ij} \leq x_{ip}, (i = 1, 2, \dots, m), \right. \\ \left. \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1, 2, \dots, n \right\}$$

Suppose the feasible region  $\Lambda_p$  in formulation (9) and  $\Lambda$  in formulation (6) are the same. In addition, suppose  $\psi = \beta_p$ . The equivalence relationship between the output-oriented BCC model (9) and the min-ordering optimization model (6) can be easily established by the following theorem:

**Theorem 4.1.** *Suppose  $y_{rp}^g > 0 (r = 1, 2, \dots, s_1)$ . The output-orientated BCC model (9) can be equivalently transformed into the min-ordering optimization model (6) using equations (7) and (8) and equations  $\psi = \beta_p$  and  $\Lambda_p = \Lambda$ .*

In fact, Theorem 4.1 shows that the output-orientated BCC model in the presence of undesirable outputs is actually constructed to locate a specific efficient solution on the efficient frontier of the following generic MOLP formulation for the observed  $DMU_p$ :

$$\begin{aligned} \max \quad & \left[ \frac{\sum_{j=1}^n \lambda_j y_{1j}^g}{y_{rp}^g}, \dots, \frac{\sum_{j=1}^n \lambda_j y_{s_1j}^g}{y_{s_1p}^g}, \frac{\sum_{j=1}^n \lambda_j \bar{y}_{1j}^b}{y_{1p}^b}, \dots, \frac{\sum_{j=1}^n \lambda_j y_{s_2j}^b}{y_{s_2p}^b} \right] \\ \text{s.t. } & \lambda \in \Lambda. \end{aligned} \quad (10)$$

The generic MOLP problem (10) defines the production possibility set for the observed  $DMU_p$  in which there may be more preferred efficient solutions than the DEA efficient solution. The main reason for establishing the equivalence condition between the output-oriented BCC model (9) and the min-ordering optimization model (6) is to use the interactive methods in MOLP to locate the most preferred solution on the efficient frontier for the target setting and resource allocation. Several interactive MOLP models have been proposed in the literature to help a DM search for different solutions along the efficient frontier and choose one solution from the set of Pareto optimal solutions as his/her final solution. At each stage of the process, the current solution is adapted to the preference structure of the DM. The interactive MOLP method is designed to drive the DM towards his/her most preferred solution. Zionts and Wallenius [64] have proposed the well-known and widely used interactive MOLP method explored next.

**5. Zionts-Wallenius method.** The interactive MOLP process involves the DM(s), the analyst and a model. The analyst is the intermediary between the DM(s) and the model. The analyst obtains an initial solution for the model and presents it to the DM(s) and asks them to evaluate this solution based on their preferences. The analyst then injects the preferences of the DM(s) into the model and generates a new solution. This iterative and interactive process is repeated until a satisfactory solution is obtained. In the interactive MOLP method proposed by Zionts and Wallenius [64] the DM is asked to provide answers to yes and no questions regarding certain trade-offs that he/she likes or dislikes. This type of communication is applicable to problem (10) where the objective functions are concave and the feasible space is a convex set. The overall utility function is assumed to be unknown explicitly to the DM, but is implicitly a linear function and more generally a concave function of the objective functions. The method makes use of such an implicit function on an interactive basis.

The first step of the method is to choose an arbitrary set of positive multipliers or weights and generate a composite objective function or utility function using these multipliers. The composite objective function is then optimized to produce a Pareto optimal solution  $\lambda^*$  to problem (10). The continuation of the procedure is essentially the same as the simplex method except that here the DM chooses a non-basic variable (NBV) to enter the basis at each iteration. From the set of non-basic variables, a subset of Pareto variables is selected (a Pareto variable is one which, when introduced into the basis, cannot increase one objective without decreasing at least one other objective). In the process of finding a set of Pareto variables from the set of non-basic variables, initially,  $w_{dj}$  values are computed based on the



information implied by the optimal solution at hand. To do this, the following model is solved for each non-basic variable  $\lambda_j$ :

$$\begin{aligned} \max \lambda_j \\ \text{s.t. } \lambda \in \Lambda = \Lambda_p. \end{aligned} \tag{11}$$

Assuming that  $\hat{\lambda}$  is the optimal solution of model (11), the values of  $w_{dj}$  are estimated as follows:

$$w_{dj} = \frac{f_d(\lambda^*) - f_d(\hat{\lambda})}{\hat{\lambda}_j}, \quad j \in \text{NBV} \tag{12}$$

where

$$f_d(\lambda) = \begin{cases} \frac{\sum_{j=1}^n \lambda_j y_{dj}^g}{y_{dp}^g}, & \text{if } d = r = 1, 2, \dots, s_1 \\ \frac{\sum_{j=1}^n \lambda_j y_{dj}^b}{y_{dp}^b}, & \text{if } d = t = 1, 2, \dots, s_2 \end{cases} \tag{13}$$

The  $w_{dj}$  values show the decrease in the objective function  $f_d$  due to some specified increase in  $\lambda_j$ . After that, the following model is solved for each non-basic variable  $\lambda_k$  :

$$\begin{aligned} \max \sum_{d=1}^{s_1+s_2} w_{dk} \delta_d \\ \text{s.t. } \sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \geq 0, j \notin k, j \in \text{NBV} \\ \sum_{d=1}^{s_1+s_2} \delta_d = 1 \\ \delta_d \geq 0, d = 1, 2, \dots, s_1 + s_2. \end{aligned} \tag{14}$$

The variable  $\lambda_k$  is Pareto if the optimal solution of model (14) is negative. In contrast, this variable is not Pareto if the optimal solution of model (14) is non-negative.

Next, for each Pareto variable  $\lambda_k$  a set of trade-offs is defined by which some objectives are increased and others are decreased. For example, considering whether a decrease in the objective functions  $f_1$  of  $w_{ij}$ , or a decrease in the objective function  $f_2$  of  $w_{2j}$  is desirable or not, are among some of the potential trade-off scenarios. A number trade-offs similar to these two examples are presented to the DM, who is requested to state whether the trade-offs are desirable, undesirable or neither. If the trade-offs are undesirable for all efficient variables, we terminate the procedure and take the  $\delta_d$ 's as the best set of weights. Otherwise, using the DM's preferences, we construct constraints to restrict the choice of the weights  $\delta_d$  to be used in finding a new Pareto solution. Then, for each desirable trade-off we construct an inequality of the form (15), for each undesirable trade-off, construct an inequality of the form (16) and for each indifference trade-off, construct an equality of the form (17) as follows:

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \leq -\varepsilon \tag{15}$$

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d \geq \varepsilon \tag{16}$$

$$\sum_{d=1}^{s_1+s_2} w_{dj} \delta_d = 0 \tag{17}$$

A feasible solution is established for all the previously constructed constraints of the form (15)-(17), as well as the following set of constraints:

$$\sum_{d=1}^{s_1+s_2} \delta_d = 1, \delta_d \geq \varepsilon, d = 1, 2, \dots, s_1 + s_2 \tag{18}$$

Thus, a new set of consistent multipliers is constructed and the associated Pareto solution is found. The process is then repeated, a new set of trade-offs for the current solution is presented to the DM, and convergence to an overall optimal solution with respect to the DMs implicit utility function is assured.

It is worth noting that there is no such thing as the best interactive method. In this research we apply the method proposed by Zionts and Wallenius [64] to find the most preferred solution. This method guarantees a convergence in a finite number of iterations, since each trade-off is attractive to the DM, and ultimately is less demanding for the DM.

**6. Application problem.** We used the performance evaluation problem proposed by Fare et al. [18] to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures and algorithms. In this problem, 30 paper mills use fiber, energy, capital and labor as inputs to produce paper, together with four undesirable outputs (pollutants) including biochemical oxygen demand (BOD), total suspended solids (TSS), particulates (PART) and sulfur oxides (SOX) as outputs. Table 1 shows the input and output values used in this example.

**Table 1:** Input and Output values

<i>DMU</i>	<i>I</i> <sub>1</sub> :Fibr	<i>I</i> <sub>2</sub> :Energy	<i>I</i> <sub>3</sub> :Capital	<i>I</i> <sub>4</sub> :Labor	<i>O</i> <sub>1</sub> :Paper	<i>O</i> <sub>2</sub> :BOD	<i>O</i> <sub>3</sub> :TSS	<i>O</i> <sub>4</sub> :Part	<i>O</i> <sub>5</sub> :SOX
<i>DMU</i> <sub>1</sub>	114784	2583397	104617771	2325437	126400	8125.93	2074.67	35.14	1206.14
<i>DMU</i> <sub>2</sub>	144360	2693196	93222042	1258590	140000	1948.95	582.12	11.73	99.1
<i>DMU</i> <sub>3</sub>	33476	1297442	43125693	946425	48610	5027.53	892.34	281.75	2273.05
<i>DMU</i> <sub>4</sub>	101107	2026610	101389518	984721	112400	3874.19	2051.83	25.66	574.15
<i>DMU</i> <sub>5</sub>	56233	1240542	60412649	1452778	50404	1474.07	493.5	23.03	602.2
<i>DMU</i> <sub>6</sub>	32710	979727	18687200	163993	33181	2708.95	190.69	3.32	62.99
<i>DMU</i> <sub>7</sub>	132060	4108283	141911775	1638857	191700	684.79	1838.42	254.4	12129.65
<i>DMU</i> <sub>8</sub>	167581	3925382	63997349	536171	126382	398.39	3844.1	1633.84	3651.88
<i>DMU</i> <sub>9</sub>	73433	1294136	69009645	922255	96255	857.17	407.36	38.78	87.17
<i>DMU</i> <sub>10</sub>	88526	1066561	76110116	1083787	90904	4704.38	2709.68	65.74	1668.86
<i>DMU</i> <sub>11</sub>	70023	2427810	59340716	1593778	86400	1874.64	1299.04	2284.27	3255.85
<i>DMU</i> <sub>12</sub>	101840	3755227	105794281	1138557	126500	6679.5	5511.5	2177.77	7699.69
<i>DMU</i> <sub>13</sub>	181960	3143984	132106333	2943591	157500	8522.75	4270.5	58.18	7242.54
<i>DMU</i> <sub>14</sub>	165751	2277712	114003930	932654	147000	2871.85	1456.4	250.11	4102.9
<i>DMU</i> <sub>15</sub>	18002	517173	26009657	221670	18000	122.43	163.97	122.55	671.65
<i>DMU</i> <sub>16</sub>	37543	533377	20780841	424940	30000	443.04	845.21	31.13	849.43
<i>DMU</i> <sub>17</sub>	43307	1062966	46773273	692170	41300	10199.93	1357.8	285.43	3547.98
<i>DMU</i> <sub>18</sub>	204769	4934056	94988555	3144336	146000	2779.83	2672.09	257.81	8638.65
<i>DMU</i> <sub>19</sub>	14743	304031	18135073	315986	19462	211.65	121.27	341.72	469.3
<i>DMU</i> <sub>20</sub>	96646	2636359	102718778	1425165	101000	5420.85	9015.5	57.24	3469.72
<i>DMU</i> <sub>21</sub>	134856	3083519	106167654	1721720	143000	13318.19	991.13	286.62	6537.93
<i>DMU</i> <sub>22</sub>	312910	5771544	262191752	1911426	293000	1370.43	2796.95	257.04	71.71
<i>DMU</i> <sub>23</sub>	88889	2668211	74725292	549671	100000	2634.47	2546.63	31.83	381.88
<i>DMU</i> <sub>24</sub>	63574	574183	28979040	383460	53700	1139.25	1997.72	233.6	458.48
<i>DMU</i> <sub>25</sub>	88550	3467369	127072960	1710987	115000	936	936	357.52	5853.66
<i>DMU</i> <sub>26</sub>	181248	2823012	76068394	762071	216579	593.1	889.75	40.95	4153.82
<i>DMU</i> <sub>27</sub>	122178	1630787	46925164	256286	119986	838	988.45	197.77	1970.87
<i>DMU</i> <sub>28</sub>	165178	3795558	66950232	783639	184393	455.9	1146.56	51.03	1.26
<i>DMU</i> <sub>29</sub>	71380	1277625	38307455	350594	92615	117.56	17.38	118.19	171.19
<i>DMU</i> <sub>30</sub>	32298	676122	35833745	643351	34000	86.79	115.48	2.84	1.97

**Table 2:** Efficiency scores and observed composite unit

Inefficient units	Efficiency	DMU <sub>2</sub>	DMU <sub>3</sub>	DMU <sub>6</sub>	DMU <sub>7</sub>	DMU <sub>9</sub>	DMU <sub>19</sub>	DMU <sub>26</sub>	DMU <sub>28</sub>	DMU <sub>29</sub>	DMU <sub>30</sub>
DMU <sub>1</sub>	0.99553	0.32318	-	-	-	0.16851	-	0.2641	-	-	0.24421
DMU <sub>4</sub>	0.99918	0.32237	-	-	-	0.3599	-	0.12003	-	-	0.19769
DMU <sub>5</sub>	0.99373	0.15776	-	-	-	-	-	-	-	-	0.84224
DMU <sub>8</sub>	0.99227	-	-	-	-	-	-	-	0.37865	0.62135	-
DMU <sub>11</sub>	0.91676	-	-	-	0.16333	-	0.19895	-	-	-	0.63773
DMU <sub>12</sub>	0.88156	-	0.06216	-	0.53733	0.10268	-	-	-	-	0.29783
DMU <sub>13</sub>	0.98692	0.74423	-	-	-	-	-	0.25577	-	-	-
DMU <sub>14</sub>	0.92485	0.17179	-	-	-	-	-	0.5846	-	-	0.24361
DMU <sub>17</sub>	0.9062	0.06121	-	-	-	-	-	0.02786	-	-	0.91093
DMU <sub>18</sub>	0.90675	0.55091	-	-	-	-	-	0.36249	-	-	0.08661
DMU <sub>20</sub>	0.98511	0.33001	-	-	-	-	-	0.18373	-	-	0.48626
DMU <sub>21</sub>	0.94178	-	-	-	-	-	-	0.47777	-	0.52223	-
DMU <sub>23</sub>	0.99752	-	-	0.36415	-	0.10274	-	0.07534	0.3085	-	0.14928
DMU <sub>25</sub>	0.96587	-	-	-	0.2311	-	-	0.02864	-	0.74026	-

We solved model (2) with  $W = (20000, 10000, 3000, 15000)$  to find the respective efficiency scores. The inefficient units and their references units are given in Table 2. As shown in Table 2, units 1, 4, 5, 8, 11-14, 17, 18, 20, 21, 23 and 25 were inefficient paper mills. We then used formulation (3) for each inefficient unit and found a virtual unit on the efficient frontier that could be regarded as its target unit. The results are presented in Table 3. As shown in Table 3, unit 1 has an

**Table 3:** Target units

Inefficient units	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>	O <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>	O <sub>4</sub>	O <sub>5</sub>
DMU <sub>1</sub>	114784	1999142	70597483	920535.8	126967.3	952.14	519.96	21.83	1144.24
DMU <sub>4</sub>	101107	1806491	71104126	956316.7	112492.4	1025.14	463.9	23.22	562.29
DMU <sub>5</sub>	49976.41	994326.8	44888757	740408.4	50722.1	380.56	189.1	4.24	17.29
DMU <sub>8</sub>	106896.52	2231038	49153016	514566.1	127366.7	245.67	444.94	92.76	106.85
DMU <sub>11</sub>	70023	1546257	51215700	554118.1	94244.81	228.92	335.48	184.91	2183.65
DMU <sub>12</sub>	101840	2801561	97429527	1138557	143495	803.51	1090.32	193.39	6718.91
DMU <sub>13</sub>	153794.9	2726399	88834627	1131595	159586.7	1602.16	660.8	19.2	1136.18
DMU <sub>14</sub>	138625.55	2277712	69214302	818448.6	158945.6	702.69	648.28	26.65	2445.82
DMU <sub>17</sub>	43307	859397.1	40469129	684315.4	45574.95	214.87	165.61	4.45	123.59
DMU <sub>18</sub>	148026.34	2565564	82034093	1025325	158578.9	1296.2	653.22	21.55	1560.48
DMU <sub>20</sub>	96646	1736223	62165711	868198.6	102526.1	794.35	411.73	12.78	796.84
DMU <sub>21</sub>	123871.19	2015958	56348347	547183.7	151840.8	344.76	434.17	81.29	2073.96
DMU <sub>23</sub>	88889	1974258	45629178	549671	100248.8	1272.8	549.27	24.44	345.51
DMU <sub>25</sub>	88550	1976055	63332046	660098.2	119064.2	262.27	463.21	147.46	3048.87

efficient score of 0.99553 implying that it is operating as an inefficient paper mill and also its composite unit on the efficient frontier can be represented as a linear combination of 0.323118 of unit 2, 0.16851 of unit 9, 0.2641 of unit 26 and 0.24421 of unit 30. In this case, based on formulation (3), the following virtual unit was used to evaluate the performance of unit 1 and could be regarded as a target unit of this unit:

$$(I_1, I_2, I_3, I_4) = (114784, 1999142, 70597483, 920535.8)$$

$$(O_1, O_2, O_3, O_4, O_5) = (126967.3, 952.14, 519.96, 21.83, 1144.24)$$

This shows that, for unit 1 to become efficient, the values of inputs should be reduced from 144360, 2693196, 93222042 and 1258590 to 114784, 1999142, 70597483 and 920535.8, respectively, the value of paper (desirable output) should increase from 126400 to 126967.3, and the values of undesirable outputs should decrease from 8125.93, 2074.67, 35.14 and 1206.14 to 952.14, 519.96, 21.83 and 1144.24, respectively. However, the DM has not accepted the DEA composite unit as the most preferred solution for unit 1. Therefore, the approach proposed in this study is needed to search the most preferred solution along the frontier for unit 1. The MOLP problem corresponding to unit 1 is formulated as follows based on model (10):

$$\begin{aligned}
& \max \frac{1}{126400} (126400\lambda_1 + 140000\lambda_2 + 48610\lambda_3 + 112400\lambda_4 + 50404\lambda_5) + \\
& \quad \frac{1}{126400} (33181\lambda_6 + 191700\lambda_7 + 126382\lambda_8 + 96255\lambda_9 + 90904\lambda_{10}) + \\
& \quad \frac{1}{126400} (86400\lambda_{11} + 126500\lambda_{12} + 157500\lambda_{13} + 147000\lambda_{14} + 18000\lambda_{15} + \\
& \quad \frac{1}{126400} (30000\lambda_{16} + 41300\lambda_{17} + 146000\lambda_{18} + 19462\lambda_{19} + 101000\lambda_{20}) + \\
& \quad \frac{1}{126400} (143000\lambda_{21} + 293000\lambda_{22} + 100000\lambda_{23} + 53700\lambda_{24} + 115000\lambda_{25}) + \\
& \quad \frac{1}{126400} (216579\lambda_{26} + 119986\lambda_{27} + 184393\lambda_{28} + 92615\lambda_{29} + 34000\lambda_{30}) \\
& \max \frac{1}{11874.07} (11874.07\lambda_1 + 18051.05\lambda_2 + 14972.47\lambda_3 + 16125.81\lambda_4 + 18525.93\lambda_5) + \\
& \quad \frac{1}{11874.07} (17291\lambda_6 + 19315.21\lambda_7 + 19601.61 + \lambda_8 + 19142.83\lambda_9 + 15295.62\lambda_{10}) + \\
& \quad \frac{1}{11874.07} (18125.36\lambda_{11} + 13320.5\lambda_{12} + 11444.25\lambda_{13} + 17128.15\lambda_{14} + 19877.57\lambda_{15}) + \\
& \quad \frac{1}{11874.07} (19556.96\lambda_{16} + 9800.07\lambda_{17} + 17220.17\lambda_{18} + 19788.35\lambda_{19} + 14579.15\lambda_{20}) + \\
& \quad \frac{1}{11874.07} (6681.81\lambda_{21} + 18629.57\lambda_{22} + 17365.53\lambda_{23} + 18860.75\lambda_{24} + 19064\lambda_{25}) + \\
& \quad \frac{1}{11874.07} (19406.9\lambda_{26} + 19162\lambda_{27} + 19544.1\lambda_{28} + 19882.44\lambda_{29} + 19913.21\lambda_{30}) \\
& \max \frac{1}{7925.33} (7925.33\lambda_1 + 9417.88\lambda_2 + 9107.66\lambda_3 + 7948.17\lambda_4 + 9506.5\lambda_5) + \\
& \quad \frac{1}{7925.33} (9809.31\lambda_6 + 8161.581\lambda_7 + 6155.9\lambda_8 + 9592.64\lambda_9 + 7290.32\lambda_{10}) + \\
& \quad \frac{1}{7925.33} (8700.96\lambda_{11} + 4488.5\lambda_{12} + 5729.5\lambda_{13} + 8543.6\lambda_{14} + 9836.03\lambda_{15}) + \\
& \quad \frac{1}{7925.33} (9154.79\lambda_{16} + 8642.2\lambda_{17} + 7327.91\lambda_{18} + 9878.73\lambda_{19} + 984.5\lambda_{20}) + \\
& \quad \frac{1}{7925.33} (9008.87\lambda_{21} + 7203.05\lambda_{22} + 7453.37\lambda_{23} + 8002.28\lambda_{24} + 9064\lambda_{25}) + \\
& \quad \frac{1}{7925.33} (9110.25\lambda_{26} + 9011.55\lambda_{27} + 8853.44\lambda_{28} + 9982.62\lambda_{29} + 9884.52\lambda_{30}) \\
& \max \frac{1}{2964.86} (2964.86\lambda_1 + 2988.27\lambda_2 + 2718.25\lambda_3 + 2974.34\lambda_4 + 2976.97\lambda_5) + \\
& \quad \frac{1}{2964.86} (2996.68\lambda_6 + 2745.6\lambda_7 + 1366.16\lambda_8 + 2961.22\lambda_9 + 2934.26\lambda_{10}) + \\
& \quad \frac{1}{2964.86} (715.73\lambda_{11} + 822.23\lambda_{12} + 2941.82\lambda_{13} + 2749.89\lambda_{14} + 2877.45\lambda_{15}) + \\
& \quad \frac{1}{2964.86} (2968.87\lambda_{16} + 2714.57\lambda_{17} + 2742.19\lambda_{18} + 2658.28\lambda_{19} + 2942.76\lambda_{20}) + \\
& \quad \frac{1}{2964.86} (2713.38\lambda_{21} + 2742.96\lambda_{22} + 2968.17\lambda_{23} + 2766.4\lambda_{24} + 2642.48\lambda_{25}) + \\
& \quad \frac{1}{2964.86} (2959.05\lambda_{26} + 2802.23\lambda_{27} + 2948.97\lambda_{28} + 2881.81\lambda_{29} + 2997.16\lambda_{30}) \\
& \max \frac{1}{13793.36} (13793.36\lambda_1 + 14900.0\lambda_2 + 12726.95\lambda_3 + 14425.85\lambda_4 + 14397.8\lambda_5) + \\
& \quad \frac{1}{13793.36} (14937.01\lambda_6 + 2870.35\lambda_7 + 11348.12\lambda_8 + 14912.83\lambda_9 + 13331.14\lambda_{10}) + \\
& \quad \frac{1}{13793.36} (11744.15\lambda_{11} + 7300.31\lambda_{12} + 7757.46\lambda_{13} + 10897.1\lambda_{14} + 14328.35\lambda_{15}) + \\
& \quad \frac{1}{13793.36} (14150.57\lambda_{16} + 11452.02\lambda_{17} + 6361.35\lambda_{18} + 14530.7\lambda_{19} + 11530.28\lambda_{20}) + \\
& \quad \frac{1}{13793.36} (8462.07\lambda_{21} + 14928.29\lambda_{22} + 14618.12\lambda_{23} + 14541.52\lambda_{24} + 9146.34\lambda_{25}) + \\
& \quad \frac{1}{13793.36} (10846.18\lambda_{26} + 13029.13\lambda_{27} + 14998.74\lambda_{28} + 14828.81\lambda_{29} + 14998.03\lambda_{30})
\end{aligned}$$

$$\begin{aligned}
& \text{s.t. } 114784\lambda_1 + 144360\lambda_2 + 33476\lambda_3 + 101107\lambda_4 + 56233\lambda_5 + \\
& 32710\lambda_6 + 132060\lambda_7 + 167581\lambda_8 + 73433\lambda_9 + 88526\lambda_{10} \\
& 170023\lambda_{11} + 101840\lambda_{12} + 181960\lambda_{13} + 165751\lambda_{14} + 18002\lambda_{15} + \\
& 37543\lambda_{16} + 43307\lambda_{17} + 204769\lambda_{18} + 14743\lambda_{19} + 96646\lambda_{20} + \\
& 134856\lambda_{21} + 312910\lambda_{22} + 88889\lambda_{23} + 63574\lambda_{24} + 88550\lambda_{25} + \\
& 181248\lambda_{26} + 122178\lambda_{27} + 1465178\lambda_{28} + 71380\lambda_{29} + 32298\lambda_{30} \leq 114784 \\
& 2583397\lambda_1 + 2693196\lambda_2 + 1297442\lambda_3 + 2026610\lambda_4 + 1240542\lambda_5 + \\
& 979727\lambda_6 + 4108283\lambda_7 + 3925382\lambda_8 + 1294136\lambda_9 + 1066561\lambda_{10} + \\
& 2427810\lambda_{11} + 3755227\lambda_{12} + 3143984\lambda_{13} + 2277712\lambda_{14} + 517173\lambda_{15} + \\
& 533377\lambda_{16} + 1062966\lambda_{17} + 4934056\lambda_{18} + 304031\lambda_{19} + 2636359\lambda_{20} + \\
& 3083519\lambda_{21} + 5771544\lambda_{22} + 2668211\lambda_{23} + 574183\lambda_{24} + 3467369\lambda_{25} + \\
& 2823012\lambda_{26} + 1630787\lambda_{27} + 3795558\lambda_{28} + 1277625\lambda_{29} + 676122\lambda_{30} \leq 2583397 \\
& 104617771\lambda_1 + 93222042\lambda_2 + 43125693\lambda_3 + 101389518\lambda_4 + 60412649\lambda_5 + \\
& 18687200\lambda_6 + 141911775\lambda_7 + 63997349\lambda_8 + 69009645\lambda_9 + 76110116\lambda_{10} + \\
& 59340716\lambda_{11} + 105794281\lambda_{12} + 132106333\lambda_{13} + 114003930\lambda_{14} + 26009657\lambda_{15} + \\
& 20780841\lambda_{16} + 46773273\lambda_{17} + 94988555\lambda_{18} + 18135073\lambda_{19} + 102718778\lambda_{20} + \\
& 106167654\lambda_{21} + 262191752\lambda_{22} + 74725292\lambda_{23} + 28979040\lambda_{24} + 127072960\lambda_{25} + \\
& 76068394\lambda_{26} + 46925163\lambda_{27} + 66950232\lambda_{28} + 38307455\lambda_{29} + 35835745\lambda_{30} \leq 104617771 \\
& 2325437\lambda_1 + 125859042\lambda_2 + 946425\lambda_3 + 984721\lambda_4 + 1452778\lambda_5 + \\
& 163993\lambda_6 + 1638857\lambda_7 + 536171\lambda_8 + 922255\lambda_9 + 1083787\lambda_{10} + \\
& 1593778\lambda_{11} + 1138557\lambda_{12} + 2943591\lambda_{13} + 932654\lambda_{14} + 221670\lambda_{15} + \\
& 424940\lambda_{16} + 692170\lambda_{17} + 3144336\lambda_{18} + 3144336\lambda_{19} + 1425165\lambda_{20} + \\
& 1721720\lambda_{21} + 1911426\lambda_{22} + 549671\lambda_{23} + 383460\lambda_{24} + 1710987\lambda_{25} + \\
& 762071\lambda_{26} + 256286\lambda_{27} + 783639\lambda_{28} + 350594\lambda_{29} + 643351\lambda_{30} \leq 2325437 \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0, j = 1, 2, \dots, n
\end{aligned} \tag{19}$$

The results for using the approach presented in Section 5 for obtaining the most preferred as a target unit for unit 1 are as follows:

**Iteration 1:**

Initially, an arbitrary set of weights  $\delta = (0.2, 0.2, 0.2, 0.2, 0.2)$  was chosen. The composite objective function was then optimized and the optimal basic variables ( $\lambda_{28} = 0.462739, \lambda_{29} = 0.537261$ ) were produced. Hence, the set of non-basic variables were  $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$  and model (11) is solved for each of them. As an instance, the following model is solved for the non-basic variable  $\lambda_3$ :

$$\begin{aligned}
& \max \lambda_3 \\
& \text{s.t. } 114784\lambda_1 + 144360\lambda_2 + 33476\lambda_3 + 101107\lambda_4 + 56233\lambda_5 + \\
& 32710\lambda_6 + 132060\lambda_7 + 167581\lambda_8 + 73433\lambda_9 + 88526\lambda_{10} \\
& 170023\lambda_{11} + 101840\lambda_{12} + 181960\lambda_{13} + 165751\lambda_{14} + 18002\lambda_{15} + \\
& 37543\lambda_{16} + 43307\lambda_{17} + 204769\lambda_{18} + 14743\lambda_{19} + 96646\lambda_{20} + \\
& 134856\lambda_{21} + 312910\lambda_{22} + 88889\lambda_{23} + 63574\lambda_{24} + 88550\lambda_{25} + \\
& 181248\lambda_{26} + 122178\lambda_{27} + 1465178\lambda_{28} + 71380\lambda_{29} + 32298\lambda_{30} \leq 114784 \\
& 2583397\lambda_1 + 2693196\lambda_2 + 1297442\lambda_3 + 2026610\lambda_4 + 1240542\lambda_5 + \\
& 979727\lambda_6 + 4108283\lambda_7 + 3925382\lambda_8 + 1294136\lambda_9 + 1066561\lambda_{10} + \\
& 2427810\lambda_{11} + 3755227\lambda_{12} + 3143984\lambda_{13} + 2277712\lambda_{14} + 517173\lambda_{15} + \\
& 533377\lambda_{16} + 1062966\lambda_{17} + 4934056\lambda_{18} + 304031\lambda_{19} + 2636359\lambda_{20} + \\
& 3083519\lambda_{21} + 5771544\lambda_{22} + 2668211\lambda_{23} + 574183\lambda_{24} + 3467369\lambda_{25} +
\end{aligned}$$

$$\begin{aligned}
& 2823012\lambda_{26} + 1630787\lambda_{27} + 3795558\lambda_{28} + 1277625\lambda_{29} + 676122\lambda_{30} \leq 2583397 \\
& 104617771\lambda_1 + 93222042\lambda_2 + 43125693\lambda_3 + 101389518\lambda_4 + 60412649\lambda_5 + \\
& 18687200\lambda_6 + 141911775\lambda_7 + 63997349\lambda_8 + 69009645\lambda_9 + 76110116\lambda_{10} + \\
& 59340716\lambda_{11} + 105794281\lambda_{12} + 132106333\lambda_{13} + 114003930\lambda_{14} + 26009657\lambda_{15} + \\
& 20780841\lambda_{16} + 46773273\lambda_{17} + 94988555\lambda_{18} + 18135073\lambda_{19} + 102718778\lambda_{20} + \\
& 106167654\lambda_{21} + 262191752\lambda_{22} + 74725292\lambda_{23} + 28979040\lambda_{24} + 127072960\lambda_{25} + \\
& 76068394\lambda_{26} + 46925163\lambda_{27} + 66950232\lambda_{28} + 38307455\lambda_{29} + 35835745\lambda_{30} \leq 104617771 \\
& 2325437\lambda_1 + 125859042\lambda_2 + 946425\lambda_3 + 984721\lambda_4 + 1452778\lambda_5 + \\
& 163993\lambda_6 + 1638857\lambda_7 + 536171\lambda_8 + 922255\lambda_9 + 1083787\lambda_{10} + \\
& 1593778\lambda_{11} + 1138557\lambda_{12} + 2943591\lambda_{13} + 932654\lambda_{14} + 221670\lambda_{15} + \\
& 424940\lambda_{16} + 692170\lambda_{17} + 3144336\lambda_{18} + 3144336\lambda_{19} + 1425165\lambda_{20} + \\
& 1721720\lambda_{21} + 1911426\lambda_{22} + 549671\lambda_{23} + 383460\lambda_{24} + 1710987\lambda_{25} + \\
& 762071\lambda_{26} + 256286\lambda_{27} + 783639\lambda_{28} + 350594\lambda_{29} + 643351\lambda_{30} \leq 2325437 \\
& \sum_{j=1}^n \lambda_j = 1, \\
& \lambda_j \geq 0, j = 1, 2, \dots, n
\end{aligned} \tag{20}$$

Now, with regard to optimal solution of model (20) the values of  $w_{dj}$  are obtained based on formula (12) as follows:

$$w_{13} = 0.684132, w_{23} = -1.2272, w_{33} = -0.88895, w_{43} = 1.562189, w_{53} = -0.84592.$$

Similarly, after solving model (11) for each of non-basic variables  $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$  the  $w_{dj}$  values introduced in Table 4 were computed based on formula (12).

In this case, model (14) is solved for each  $\lambda_j, j = 1, 2, \dots, n, j \neq 28, 29$  and variables  $\lambda_1, \lambda_2, \lambda_9, \lambda_{26}$  and  $\lambda_{30}$  are determined as the efficient variables. As an example the following model is solved corresponding to non-basic variable  $\lambda_3$ :

$$\begin{aligned}
& \max 0.684132\delta_1 - 1.2272\delta_2 - 0.88895\delta_3 + 1.562189\delta_4 - 0.84592\delta_5 \\
& \text{s.t. } 0.068705\delta_1 - 0.96627\delta_2 - 0.73977\delta_3 - 1.479011\delta_4 - 0.92326\delta_5 \geq 0 \\
& 0.23154\delta_1 - 1.96919\delta_2 - 1.21967\delta_3 + 1.938938\delta_4 - 1.29226\delta_5 \geq 0 \\
& 0.179464\delta_1 - 1.32433\delta_2 - 0.74265\delta_3 + 1.475813\delta_4 - 0.96908\delta_5 \geq 0 \\
& 0.669939\delta_1 - 1.52647\delta_2 - 0.93928\delta_3 + 1.474926\delta_4 - 0.96705\delta_5 \geq 0 \\
& 0.806197\delta_1 - 1.42247\delta_2 - 0.97748\delta_3 + 1.468279\delta_4 + 1.00614\delta_5 \geq 0 \\
& 0.164043\delta_1 - 2.68526\delta_2 - 1.42937\delta_3 + 2.611593\delta_4 - 0.78475\delta_5 \geq 0 \\
& 0.607399\delta_1 - 2.57836\delta_2 - 1.09716\delta_3 + 2.94988\delta_4 - 1.32098\delta_5 \geq 0 \\
& 0.307194\delta_1 - 1.57842\delta_2 - 0.95015\delta_3 + 1.480239\delta_4 - 1.00438\delta_5 \geq 0 \\
& 0.349527\delta_1 - 1.25442\delta_2 - 0.65964\delta_3 + 1.489332\delta_4 - 0.88972\delta_5 \geq 0 \\
& 0.38516\delta_1 - 1.49273\delta_2 - 0.83764\delta_3 + 2.237607\delta_4 + 0.77467\delta_5 \geq 0 \\
& 0.538181\delta_1 - 1.9275\delta_2 - 0.81315\delta_3 + 3.015211\delta_4 - 0.95462\delta_5 \geq 0 \\
& 0.43689\delta_1 - 2.02924\delta_2 - 1.12495\delta_3 + 2.54935\delta_4 - 1.14147\delta_5 \geq 0 \\
& 0.371751\delta_1 - 2.24059\delta_2 - 1.32023\delta_3 + 2.357696\delta_4 - 1.21084\delta_5 \geq 0 \\
& 0.9263\delta_1 - 1.6403\delta_2 - 0.98086\delta_3 + 1.508493\delta_4 - 0.96201\delta_5 \geq 0 \\
& 0.831363\delta_1 - 1.6133\delta_2 - 0.8949\delta_3 + 1.477658\delta_4 - 0.94912\delta_5 \geq 0 \\
& 0.741964\delta_1 - 0.7916\delta_2 - 0.83022\delta_3 + 1.56343\delta_4 - 0.75349\delta_5 \geq 0 \\
& 0.856985\delta_1 - 3.10035\delta_2 - 1.68148\delta_3 + 3.186024\delta_4 - 1.39166\delta_5 \geq 0 \\
& 0.914733\delta_1 - 1.63278\delta_2 - 0.98624\delta_3 + 1.582415\delta_4 - 0.97668\delta_5 \geq 0 \\
& 0.290908\delta_1 - 1.23202\delta_2 - 0.113094\delta_3 + 1.523233\delta_4 - 0.78186\delta_5 \geq 0
\end{aligned}$$

$$\begin{aligned}
 &0.13808\delta_1 - 0.88724\delta_2 - 1.09288\delta_3 + 1.911033\delta_4 - 0.75102\delta_5 \geq 0 \\
 &0.562248\delta_1 - 4.76884\delta_2 - 2.60184\delta_3 + 4.687746\delta_4 - 2.93977\delta_5 \geq 0 \\
 &0.311602\delta_1 - 1.4895\delta_2 - 0.71691\delta_3 + 1.536775\delta_4 - 1.01936\delta_5 \geq 0 \\
 &0.643863\delta_1 - 1.55466\delta_2 - 0.74948\delta_3 + 1.545948\delta_4 - 0.97746\delta_5 \geq 0 \\
 &0.513642\delta_1 - 2.205\delta_2 - 1.26592\delta_3 + 2.201429\delta_4 - 0.96511\delta_5 \geq 0 \\
 &-0.03702\delta_1 - 2.68543\delta_2 - 1.54451\delta_3 + 2.532276\delta_4 - 1.35844\delta_5 \geq 0 \\
 &0.187056\delta_1 - 1.70071\delta_2 - 0.94972\delta_3 + 1.650819\delta_4 - 0.94001\delta_5 \geq 0 \\
 &0.799717\delta_1 - 1.6433\delta_2 - 0.98697 - \delta_3 + 1.468117\delta_4 - 1.01056\delta_5 \geq 0 \\
 &\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1 \\
 &\delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \geq 0
 \end{aligned} \tag{21}$$

The DM was then asked to indicate the acceptability of the trade-offs. Since the DM accepts the trade for efficient variables  $\lambda_1, \lambda_2$ , does not accept the trade for efficient variables  $\lambda_9, \lambda_{26}$  and is indifferent to variable  $\lambda_{30}$ , a feasible solution to the following set of constraints was found:

$$\begin{aligned}
 &\sum_{d=1}^5 w_{dj}\delta_d \leq -\varepsilon, \quad j = 1, 2 \\
 &\sum_{d=1}^5 w_{dj}\delta_d \geq \varepsilon, \quad j = 9, 26 \\
 &\sum_{d=1}^5 w_{d,30}\delta_d = 0 \\
 &\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 = 1 \\
 &\delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \geq \varepsilon
 \end{aligned}$$

As a result, the following new set of weights is found:

$$\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$$

**Iteration 2:**

**Table 4:** Initial  $w_{dj}$  values in iteration 1

<i>NBV</i>	$w_{1j}$	$w_{2j}$	$w_{3j}$	$w_{4j}$	$w_{5j}$
$\lambda_1$	0.068705	-0.96627	-0.73977	1.479011	-0.92326
$\lambda_2$	0.23154	-1.96919	-1.21967	1.938938	-1.29226
$\lambda_3$	0.684132	-1.2272	-0.88895	1.562189	-0.84592
$\lambda_4$	0.179464	-1.32433	-0.74265	1.475813	-0.96908
$\lambda_5$	0.669939	-1.52647	-0.93928	1.474926	-0.96705
$\lambda_6$	0.806197	-1.42247	-0.97748	1.468279	-1.00614
$\lambda_7$	0.164043	-2.68526	-1.42937	2.611593	-0.78475
$\lambda_8$	0.607399	-2.57836	-1.09716	2.94988	-1.32098
$\lambda_9$	0.307194	-1.57842	-0.95015	1.480239	-1.00438
$\lambda_{10}$	0.349527	-1.25442	-0.65964	1.489332	-0.88972
$\lambda_{11}$	0.38516	-1.49273	-0.83764	2.237607	-0.77467
$\lambda_{12}$	0.538181	-1.9275	-0.81315	3.015211	-0.95462
$\lambda_{13}$	0.43689	-2.02924	-1.12495	2.54935	-1.14147
$\lambda_{14}$	0.371751	-2.24059	-1.32023	2.357696	-1.21084
$\lambda_{15}$	0.9263	-1.6403	-0.98086	1.508493	-0.96201
$\lambda_{16}$	0.831363	-1.6133	-0.8949	1.477658	-0.94912
$\lambda_{17}$	0.741964	-0.7916	-0.83022	1.56343	-0.75349
$\lambda_{18}$	0.856985	-3.10035	-1.68148	3.186024	-1.39166
$\lambda_{19}$	0.914733	-1.63278	-0.98624	1.582415	-0.97668
$\lambda_{20}$	0.290908	-1.23202	0.113094	1.523233	-0.78186
$\lambda_{21}$	0.13808	-0.88724	-1.09288	1.911033	-0.75102
$\lambda_{22}$	0.562248	-4.76884	-2.60184	4.687746	-2.93977
$\lambda_{23}$	0.311602	-1.4895	-0.71691	1.536775	-1.01936
$\lambda_{24}$	0.643863	-1.55466	-0.74948	1.545948	-0.97746
$\lambda_{25}$	0.513642	-2.205	-1.26592	2.201429	-0.96511
$\lambda_{26}$	-0.03702	-2.68543	-1.54451	2.532276	-1.35844
$\lambda_{27}$	0.187056	-1.70071	-0.94972	1.650819	-0.94001
$\lambda_{30}$	0.799717	-1.6433	-0.98697	1.468117	-1.01056

Using the weights  $\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$ , the composite objective function was optimized and the optimal basic variables were  $(\lambda_9 = 0.549283, \lambda_{28} = 0.450717)$ . However, in this iteration the set of non-basic variables were  $\lambda_j, j = 1, 2, \dots, 30, j \neq 9, 28$  and after solving model (11) for each of them, the  $w_{dj}$  values presented in Table 5 were derived by using formula (12). Model (14) was solved for each  $\lambda_j, j = 1, 2, \dots, 30, j \neq 9, 28$  and the variables  $\lambda_1, \lambda_2, \lambda_4, \lambda_{26}$  and  $\lambda_{30}$  were determined as efficient variables. The DM is then asked to indicate his/her acceptability of the trade-offs. Since all responses for the efficient variables were no, we terminated the procedure and introduced  $\delta = (0.264029138, 0.04897689, 0.000000099, 0.276180101, 0.410813772)$  as the best set of weights for the objective functions in model (10) with the following optimal solutions:  $\lambda_9 = 0.549283$  and  $\lambda_{28} = 0.450717$ . This optimal solution provides the following new target unit for the inefficient unit 1 using formulation (3):

$$(I1, I2, I3, I4) = (114784, 2421569, 68081433, 859778.5)$$

$$(O1, O2, O3, O4, O5) = (135980.3, 676.3109, 740.5298, 44.30128, 48.44893)$$

This composite unit has been produced by taking into consideration the DM's preference information as the most preferred solution for unit 1. As reported in

**Table 5:** The first values of  $w_{dj}$  in iteration 2

<i>NBV</i>	$w_{1j}$	$w_{2j}$	$w_{3j}$	$w_{4j}$	$w_{5j}$
$\lambda_1$	0.075793	-0.91677	-0.64306	0.260708	-0.95983
$\lambda_2$	0.240725	-1.90506	-1.09437	0.360458	-1.33964
$\lambda_3$	0.691222	-1.17771	-0.79225	0.343886	-0.88248
$\lambda_4$	0.186553	-1.27484	-0.64594	0.257511	-1.00565
$\lambda_5$	0.677027	-1.47697	-0.84257	0.256623	-1.00361
$\lambda_6$	0.813285	-1.37297	-0.88078	0.249976	-1.04271
$\lambda_7$	0.175874	-2.60266	-1.26797	0.578251	-0.84578
$\lambda_8$	0.618662	-2.49973	-0.94352	1.014297	-1.37908
$\lambda_{10}$	0.356616	-1.20492	-0.56294	0.271029	-0.92629
$\lambda_{11}$	0.392249	-1.44324	-0.74093	1.019304	-0.81124
$\lambda_{12}$	0.548914	-1.85256	-0.66672	1.170574	-1.00999
$\lambda_{13}$	0.448738	-1.94651	-0.96331	0.512975	-1.2026
$\lambda_{14}$	0.382451	-2.16588	-1.17426	0.518715	-1.26604
$\lambda_{15}$	0.933388	-1.5908	-0.88415	0.29019	-0.99858
$\lambda_{16}$	0.838451	-1.5638	-0.79819	0.259355	-0.98569
$\lambda_{17}$	0.749053	-0.7421	-0.73351	0.345127	-0.79006
$\lambda_{18}$	0.871384	-2.99981	-1.48504	0.711312	-1.46595
$\lambda_{19}$	0.921822	-1.58329	-0.88954	0.364113	-1.01325
$\lambda_{20}$	0.298162	-1.18137	0.212048	0.276622	-0.81927
$\lambda_{21}$	0.146724	-0.82689	-0.97496	0.425419	-0.79562
$\lambda_{22}$	0.583375	-4.62132	-2.31361	1.056657	-3.04876
$\lambda_{23}$	0.318954	-1.43816	-0.61661	0.27314	-1.05729
$\lambda_{24}$	0.650951	-1.50517	-0.65277	0.327645	-1.01403
$\lambda_{25}$	0.523479	-2.13631	-1.13171	0.51065	-1.01586
$\lambda_{26}$	-0.02522	-2.60305	-1.38355	0.504572	-1.41931
$\lambda_{27}$	0.194668	-1.64756	-0.84586	0.342472	-0.97928
$\lambda_{29}$	0.34308	-1.59121	-0.90265	0.288719	-1.03486
$\lambda_{30}$	0.806806	-1.5938	-0.89027	0.249814	-1.04713

Table 3, unit 4 with the efficient score of 0.99918 is another inefficient DMU and its reference set includes units 2, 9, 26 and 30. Thus its projection point (target pattern) on the efficient frontier can be determined as a linear combination of 0.32237 of unit 2, 0.3599 of unit 9, 0.12003 of unit 26 and 0.19769 of unit 30. Therefore, the following target unit, calculated based on formulation (3), can be applied to evaluate the performance of unit 4:

$$(I1, I2, I3, I4) = (101107, 1806491.45, 71104125.72, 956316.68)$$

$$(O1, O2, O3, O4, O5) = (112492.39, 1025.14, 463.9, 23.22, 562.29)$$



However, the DM could not accept this target pattern as the most preferred solution for unit 4. In this case, the interactive method can help the DM to search the most preferred solution on the efficient frontier for unit 4.

**Iteration 1:**

Having started with weights  $\delta = (0.2, 0.2, 0.2, 0.2)$ , variables  $(\lambda_{26} = 0.034911, \lambda_{28} = 0.276034, \lambda_{29} = 0.689055)$  showed to be optimal basic. After solving model (11) for non basic variables and using formula (12) we obtained the values of  $w_{dj}$  as presented in Table 6. By Solving model (14), new optimal basic variables were

**Table 6:** Initial  $w_{dj}$  values in iteration 1 for unit 4

<i>NBV</i>	$w_{1j}$	$w_{2j}$	$w_{3j}$	$w_{4j}$	$w_{5j}$
$\lambda_1$	1.258187	-2.333264	-2.551751	7.308161	-1.439814
$\lambda_2$	0.30043	-1.645807	-1.544402	4.218496	-0.849073
$\lambda_3$	0.655396	-0.869743	-0.970659	2.864551	-0.423604
$\lambda_4$	0.087869	-0.941264	-0.824777	2.778451	-0.541372
$\lambda_5$	1.091488	-1.668108	-1.624745	4.35782	-0.868376
$\lambda_6$	0.792665	-1.013523	-1.058937	2.77094	-0.576805
$\lambda_7$	0.487759	-2.550998	-2.141869	6.341425	-0.403355
$\lambda_8$	0.971756	-2.444701	-1.77616	6.498911	-0.932781
$\lambda_9$	0.231508	-1.128357	-1.031677	2.782862	-0.575129
$\lambda_{10}$	0.374794	-1.012119	-0.869829	3.126393	-0.53511
$\lambda_{11}$	0.907418	-1.81739	-1.70532	5.594113	-0.78352
$\lambda_{12}$	0.880351	-1.939782	-1.460912	6.396835	-0.597992
$\lambda_{13}$	1.578514	-3.072019	-3.07305	9.402918	-1.455807
$\lambda_{14}$	0.464714	-1.877966	-1.69885	5.01314	-0.707419
$\lambda_{15}$	0.927727	-1.17392	-1.062299	2.811027	-0.534613
$\lambda_{16}$	0.820965	-1.154038	-0.976588	2.78029	-0.522289
$\lambda_{17}$	0.720432	-0.54899	-0.912097	2.865788	-0.335226
$\lambda_{18}$	1.929057	-3.742041	-3.555322	10.214688	-1.49964
$\lambda_{19}$	0.91472	-1.168387	-1.067671	2.884713	-0.54864
$\lambda_{20}$	0.631735	-1.409971	-0.523876	4.297091	-0.650045
$\lambda_{21}$	0.568996	-1.316217	-1.926085	5.405612	-0.645562
$\lambda_{22}$	0.724409	-3.961933	-3.349435	9.930846	-1.92171
$\lambda_{23}$	0.538891	-1.453325	-1.160195	3.854981	-0.759049
$\lambda_{24}$	0.610111	-1.110864	-0.831585	2.848363	-0.54939
$\lambda_{25}$	0.948781	-2.242223	-1.977651	5.569847	-0.685056
$\lambda_{27}$	0.243551	-1.414607	-1.219056	3.540126	-0.578408
$\lambda_{30}$	0.785378	-1.17613	-1.0684	2.770779	-0.581035

$\lambda_2, \lambda_4, \lambda_9, \lambda_{22}, \lambda_{27}$  and  $\lambda_{30}$ . DM accepts the trade for  $\lambda_9, \lambda_{22}, \lambda_{27}$ , rejects the trade for  $\lambda_2, \lambda_{30}$  and is indifferent to variable  $\lambda_4$ . Based on this decision the following model is constructed:

$$\begin{aligned} \sum_{d=1}^5 w_{dj} \delta_d &\leq -\varepsilon, \quad j = 9, 22, 27 \\ \sum_{d=1}^5 w_{dj} \delta_d &\geq \varepsilon, \quad j = 2, 30 \\ \sum_{d=1}^5 w_{d,4} \delta_d &= 0 \\ \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 &= 1 \\ \delta_1, \delta_2, \delta_3, \delta_4, \delta_5 &\geq \varepsilon \end{aligned}$$

By solving the above model, new weights are obtained as

$$\delta = (0.4883066368, 0.3735303166, 0.0000001, 0.1155124839, 0.0226504627).$$

**Iteration 2:**

By using above weights, optimizing the composite objective function resulted in  $(\lambda_7 = 0.167337, \lambda_{26} = 0.17815, \lambda_{28} = 0.654513)$  as optimal basic variables. By solving model (11) for non basic variables and using formula (12), the values of  $w_{dj}$  are derived as shown in Table 7. Model (14) was solved for each non basic variables

**Table 7:**  $w_{dj}$  values in iteration 2 for unit 4

$\lambda_1$	1.469137	-2.28592	-2.4003	10.41463	10.57085
$\lambda_2$	0.420646	-1.61883	-1.4581	5.988813	5.995578
$\lambda_3$	0.735496	-0.85177	-0.91315	4.044116	4.136997
$\lambda_4$	0.16797	-0.92329	-0.76727	3.958016	4.019229
$\lambda_5$	1.217269	-1.63988	-1.53444	6.210085	6.293112
$\lambda_6$	0.872765	-0.99555	-1.00143	3.950505	3.983796
$\lambda_8$	1.140149	-2.40691	-1.65527	8.978692	8.6549
$\lambda_9$	0.311608	-1.11038	-0.97417	3.962427	3.985472
$\lambda_{10}$	0.464562	-0.99197	-0.80538	4.448337	4.575979
$\lambda_{11}$	1.04696	-1.78607	-1.60514	7.649026	7.161474
$\lambda_{12}$	1.040832	-1.90377	-1.3457	8.7601	8.539196
$\lambda_{13}$	1.849793	-3.01114	-2.87829	13.39781	13.98979
$\lambda_{14}$	0.60477	-1.84654	-1.5983	7.075617	7.266821
$\lambda_{15}$	1.007827	-1.15594	-1.00479	3.990592	4.025988
$\lambda_{16}$	0.901066	-1.13606	-0.91908	3.959855	4.038312
$\lambda_{17}$	0.800532	-0.53101	-0.85459	4.045353	4.225375
$\lambda_{18}$	2.221532	-3.67641	-3.34535	14.52171	15.15277
$\lambda_{19}$	0.99482	-1.15041	-1.01016	4.064279	4.011961
$\lambda_{20}$	0.755324	-1.38224	-0.43515	6.117083	6.386662
$\lambda_{21}$	0.722252	-1.28182	-1.81606	7.662468	8.080217
$\lambda_{22}$	1.00095	-3.89987	-3.1509	14.00323	13.82352
$\lambda_{23}$	0.648826	-1.42865	-1.08127	5.473894	5.500218
$\lambda_{24}$	0.690212	-1.09289	-0.77408	4.027928	4.011211
$\lambda_{25}$	1.105221	-2.20712	-1.86534	7.873604	8.222056
$\lambda_{27}$	0.343194	-1.39225	-1.14752	5.00748	5.094884
$\lambda_{29}$	0.549062	-2.31664	-1.91852	8.140228	8.099473
$\lambda_{30}$	0.865479	-1.15815	-1.01089	3.950344	3.979566

and  $\lambda_2, \lambda_4, \lambda_9, \lambda_{28}$  and  $\lambda_{30}$  were determined as efficient variables. At this point, the procedure terminated because DM answered no to all proposed trade-offs. The final weight vector is:

$$\delta = (0.0585696617, 0.3383842365, 0.4125174363, 0.0000001, 0.1905285655)$$

and the optimal solution selected as the most preferred solution for unit 4 is

$$(I1, I2, I3, I4) = (71380, 1277625, 38307455, 350594)$$

$$(O1, O2, O3, O4, O5) = (92615, 117.56, 17.38, 118.19, 171.19)$$

**7. Conclusions and future research directions.** The changing economic conditions have challenged many organizations to search for more effective performance measurement methods. DEA is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous DMUs by evaluating their relative efficiency. Performance measurement in the conventional DEA is based on the assumptions that inputs should be minimized and outputs should be maximized. However, there are circumstances in real-world problems where some output variables should be minimized.

In this paper we obtained an equivalence relation between an output-oriented DEA model with undesirable outputs and a min-ordering optimization model. We showed how a DEA problem can be solved interactively by transforming it into an MOLP formulation. The approach proposed in this study reduced the number of undesirable outputs and resulted in a permissible increase in the number of desirable outputs. The proposed equivalence model utilized an interactive method in the MOLP model and located the most preferred solution along the efficient frontier for each inefficient DMU. We also used the Zionts-Wallenius method to reflect the DMs preferences in the efficiency assessment process. We plan to conduct further research by comparing the results obtained with those that might be obtained with other interactive MOLP methods such as the STEM method [8], the G-D-F method [20], the Wierzbicki method [55], and the STOM method [40]. Our goal is to understand the applicability of different interactive MOLP methods to specific data sets and DM preferences.

We shall also point out two limitations of the equivalency established in this study between the DEA models and the MOLP models which are thought-provoking topics for futures research. In this study all the equivalent relations established between the DEA models and the MOLP models are radial DEA-based models. Since non-radial DEA models usually have a higher discriminating power in evaluating the efficiencies of DMUs, in practice it may be more practical to establish equivalency link between the non-radial DEA models and the MOLP models. Another potential for future research is to extend the proposed model to a cover problems with constant returns-to-scale (CRS).

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#### REFERENCES

- [1] R. Allen, A. Athanassopoulos, R. G. Dyson and E. Thanassoulis, Weights restrictions and value judgements in data envelopment analysis: Evolution, development and future directions, *Annals of Operations Research*, **73** (1997), 13–34.
- [2] A. Amirteimoori and S. Kordrostami, [An alternative clustering approach: a DEA-based procedure](#), *Optimization: A Journal of Mathematical Programming and Operations Research*, **62** (2013), 227–240.
- [3] J. Aparicio, J. L. Ruiz and I. Sirvent, [Closest targets and minimum distance to the Pareto-efficient frontier in DEA](#), *Journal of Productivity Analysis*, **28** (2007), 209–218.
- [4] M. Asmild, J. C. Paradi, D. N. Reese and F. Tam, [Measuring overall efficiency and effectiveness using DEA](#), *European Journal of Operational Research*, **178** (2007), 305–321.
- [5] R. D. Banker, A. Charnes and W. W. Cooper, [Some models for estimating technical and scale inefficiencies in data envelopment analysis](#), *Management Science*, **30** (1984), 1078–1092.
- [6] C. P. Barros, S. Managi and R. Matousek, [The technical efficiency of Japanese banks: Non radial directional performance measurement with undesirable output](#), *Omega*, **40** (2012), 1–8.
- [7] G. Bi, J. Ding, Y. Luo and L. Liang, [Resource allocation and target setting for parallel production system based on DEA](#), *Applied Mathematical Modelling*, **35** (2011), 4270–4280.
- [8] R. Benayoun, J. Montgolfier, J. Tergny and O. Laritchev, [Linear programming with multiple objective functions: Step method \(STEM\)](#), *Mathematical Programming*, **1** (1971), 366–375.
- [9] A. Charnes, W. W. Cooper and E. Rhodes, [Measuring the efficiency of decision making units](#), *European Journal of Operational Research*, **2** (1978), 429–444.
- [10] A. Charnes, W. W. Cooper, Z. M. Huang and D. B. Sun, [Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks](#), *Journal of Econometrics*, **46** (1990), 73–91.
- [11] R. G. Dyson and E. Thanassoulis, [Reducing weight flexibility in data envelopment analysis](#), *Journal of Operational Research Society*, **39** (1988), 563–576.
- [12] A. Ebrahimnejad, [A new link between output-oriented BCC model with fuzzy data in the present of undesirable outputs and MOLP](#), *Fuzzy Information and Engineering*, **3** (2011), 113–125.
- [13] A. Ebrahimnejad, R. Shahverdi, F. Rezai Balf and M. Hatefi, [Finding target units in FDH model by least-distance measure model](#), *Measurement*, **49** (2013), 619–635.
- [14] A. Ebrahimnejad, M. Tavana, F. Hosseinzadeh Lotfi, R. Shahverdi and M. Yousefpour, [A three-stage data envelopment analysis model with application to banking industry](#), *Measurement*, **49** (2014), 308–319.
- [15] A. Ebrahimnejad and M. Tavana, [An interactive MOLP method for identifying target units in Output-Oriented DEA models: The NATO enlargement problem](#), *Measurement*, **52** (2014), 124–134.
- [16] M. Ehrgott, *Multicriteria Optimization*, 2<sup>nd</sup> edition, Springer, Berlin, 2005.
- [17] R. Fare and S. Grosskopf, [Modelling undesirable factors in efficiency evaluation: Comment](#), *European Journal of Operational Research*, **157** (2004), 242–245.

- [18] R. Fare, S. Grosskopf, C. A. K. Lovell and C. Pasurka, [Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach](#), *The Review of Economics and Statistics*, **71** (1989), 90–98.
- [19] R. Fare, S. Grosskopf and C. Pasurka, [Accounting for air pollution emissions in measures of state manufacturing productivity growth](#), *Journal of Regional Science*, **41** (2001), 381–409.
- [20] A. M. Geoffrion, J. S. Dyer and A. Feinberg, An interactive approach for multi-criterion optimization with an application to the operation of an academic department, Part I, *Management Science*, **19** (1972), 357–368.
- [21] B. Golany, [An interactive MOLP procedure for the extension of DEA to effectiveness analysis](#), *Journal of the Operational Research Society*, **39** (1988), 725–734.
- [22] M. A. Hinojosa and A. M. Mármol, [Axial solutions for multiple objective linear problems. An application to target setting in DEA models with preferences](#), *Omega*, **39** (2011), 159–167.
- [23] F. Hosseinzadeh Lotfi, G. R. Jahanshaloo, A. Ebrahimnejad, M. Soltanifar and S. M. Manosourzadeh, [Target setting in the general combined-oriented CCR model using an interactive MOLP method](#), *Journal of Computational and Applied Mathematics*, **234** (2010), 1–9.
- [24] F. Hosseinzadeh Lotfi, G. R. Jahanshaloo, M. Soltanifar, A. Ebrahimnejad and S. M. Manosourzadeh, [Relationship between MOLP and DEA based on output-orientated CCR dual model](#), *Expert Systems with Applications*, **37** (2010), 4331–4336.
- [25] H. H. Hu, Q. Qi and C. H. Yang, [Evaluation of China's regional hospital efficiency: DEA approach with undesirable output](#), *Journal of the Operational Research Society*, **63** (2011), 715–725.
- [26] Z. S. Hua, Y. W. Bian and L. Liang, [Eco-efficiency analysis of paper mills along the Huai River: An extended DEA approach](#), *Omega*, **35** (2007), 578–587.
- [27] X. Huang, D. Hu and Z. Zhou, [Measuring efficiency in Chinese commercial banks using a DEA model with undesirable output](#), *International Journal of Information and Decision Sciences*, **5** (2013), 140–153.
- [28] S. Jain, K. P. Triantis and S. Liu, [Manufacturing performance measurement and target setting: A data envelopment analysis approach](#), *European Journal of Operational Research*, **214** (2011), 616–626.
- [29] A. Jess, H. Th. Jongen, L. Neralic and O. Stein, [A semi-infinite programming model in data envelopment analysis](#), *European Journal of Operational Research*, **49** (2001), 369–385.
- [30] P. J. Korhonen and M. Luptacik, [Eco-efficiency analysis of power plants: an extension of data envelopment analysis](#), *European Journal of Operational Research*, **154** (2004), 437–446.
- [31] P. Korhonen, S. Stenfors and M. Syrjänen, Multiple objective approach as an alternative to radial projection in DEA, *Journal of Productivity Analysis*, **20** (2003), 305–321.
- [32] S. Kumar Mandal and S. Madheswaran, Environmental efficiency of the Indian cement industry: An interstate analysis, *Energy Policy*, **38** (2010), 1108–1118.
- [33] H. F. Lewis and T. R. Sexton, [Data envelopment analysis with reverse inputs and outputs](#), *Journal of Productivity Analysis*, **21** (2004), 113–132.
- [34] Y. Liu and U. R. Sumaila, Estimating pollution abatement costs of salmon aquaculture: A joint production approach, *Land Economics*, **86** (2010), 569–584.
- [35] C. A. K. Lovell, J. T. Pastor and J. A. Turner, Measuring macroeconomic performance in the OECD: A comparison of European and non-European countries, *European Journal of Operational Research*, **87** (1995), 507–518.
- [36] S. Lozano and E. Gutierrez, [Efficiency analysis and target setting of Spanish airports](#), *Netw. Spat. Econ.*, **11** (2011), 139–157.
- [37] W. M. Lu and S. F. Lo, [A closer look at the economic-environmental disparities for regional development in China](#), *European Journal of Operational Research*, **183** (2007), 882–894.
- [38] M. Mahdilo, A. Noorzadeh and R. Farzipoor Saen, [Developing a new data envelopment analysis model for customer value analysis](#), *Journal of Industrial and Management Optimization*, **7** (2011), 531–558.
- [39] N. Malekmohammadi, F. Hosseinzadeh Lotfi and A. B. Jaafar, [Target setting in data envelopment analysis using MOLP](#), *Applied Mathematical Modelling*, **35** (2011), 328–338.
- [40] H. Nakayama and Y. Sawaragi, [Satisficing trade-off method for multiobjective programming. An interactive decision analysis](#), in *Lecture notes in economics and mathematical systems* (eds. M. Grauer and A.P. Wierzbicki), Laxenburg: Springer, **229** (1984), 113–122.
- [41] F. Pedraja-Chaparro, J. Salinas-Jimenez and P. Smith, On the role of weight restrictions in data envelopment analysis, *Journal of Productivity Analysis*, **8** (1997), 215–230.

- [42] V. V. Podinovski, [Suitability and redundancy of non-homogeneous weight restrictions for measuring the relative efficiency in DEA](#), *European Journal of Operational Research*, **154** (2004), 380–395.
- [43] S. Reinhard, C. A. K. Lovell and G. Thijssen, [Environmental efficiency with multiple environmentally detrimental variables; estimated with SFA and DEA](#), *European Journal of Operational Research*, **121** (2000), 287–303.
- [44] S. Reinhard and G. Thijssen, [Nitrogen efficiency of Dutch dairy farms: A shadow cost system approach](#), *European Review of Agricultural Economics*, **27** (2000), 167–186.
- [45] R. Riccardi, G. Oggioni and R. Toninelli, [Efficiency analysis of world cement industry in presence of undesirable output: Application of data envelopment analysis and directional distance function](#), *Energy Policy*, **44** (2012), 140–152.
- [46] S. Saati, A. Hatami-Marbini, P. J. Agrell and M. Tavana, [A common set of weight approach using an ideal decision making unit in data envelopment analysis](#), *Journal of Industrial and Management Optimization*, **8** (2012), 623–637.
- [47] M. Seiford and J. Zhu, [Modeling undesirable factors in efficiency evaluation](#), *European Journal of Operational Research*, **142** (2002), 16–20.
- [48] S. Shaik, G. A. Helmers and M. R. Langemeier, [Direct and indirect shadow price and cost estimates of nitrogen pollution abatement](#), *Journal of Agricultural and Resource Economics*, **27** (2002), 420–432.
- [49] T. Sueyoshi and M. Goto, [Data envelopment analysis for environmental assessment: Comparison between public and private ownership in petroleum industry](#), *European Journal of Operational Research*, **216** (2012), 668–678.
- [50] E. Thanassoulis, A. Boussofiane and R. G. Dyson, [Exploring output quality targets in the provision of perinatal care in England using data envelopment analysis](#), *European Journal of Operational Research*, **80** (1995), 588–607.
- [51] R. G. Thompson, P. S. Dharmapala and R. M. Thrall, [Linked-cone DEA profit ratios and technical efficiency with application to Illinois coal mines](#), *International Journal of Production Economics*, **39** (1995), 99–115.
- [52] R. G. Thompson, L. N. Langemeier, C. T. Lee and R. M. Thrall, [The role of multiplier bounds in efficiency analysis with application to Kansas farming](#), *Journal of Econometrics*, **46** (1990), 93–108.
- [53] R. G. Thompson, F. D. Singleton Jr, R. M. Thrall and B. A. Smith, [Comparative site evaluations for locating a high-energy physics lab in Texas](#), *Interfaces*, **16** (1986), 35–49.
- [54] K. Tone, [On returns to scale under weight restrictions in data envelopment analysis](#), *Journal of Productivity Analysis*, **16** (2001), 31–47.
- [55] A. P. Wierzbick, [The use of reference objectives in multiobjective optimization](#), in *Multiple criteria decision making theory and application. Lecture notes in economics and mathematics systems* (eds. G. Fandel and T. Gal), Heidelberg: Springer, (1980), 468–486.
- [56] Y. H. B. Wong and J. E. Beasley, [Restricting weight flexibility in data envelopment analysis](#), *Journal of the Operational Research Society*, **41** (1990), 829–835.
- [57] H. Yang and M. Pollitt, [Incorporating both undesirable outputs and uncontrollable variables into DEA: The performance of Chinese coal-fired power plants](#), *European Journal of Operational Research*, **197** (2009), 1095–1105.
- [58] J. B. Yang, B. Y. H. Wong, D. L. Xu and T. J. Stewart, [Integrating DEA-oriented performance assessment and target setting using interactive MOLP methods](#), *European Journal of Operational Research*, **195** (2009), 205–222.
- [59] J. B. Yang, B. Y. H. Wong, D. L. Xu, X. B. Liu and R. E. Steuer, [Integrated bank performance assessment and management planning using hybrid minimax reference point DEA approach](#), *European Journal of Operational Research*, **207** (2010), 1506–1518.
- [60] J. B. Yang and D. L. Xu, [Interactive minimax optimisation for integrated performance analysis and resource planning](#), *Computers and Operations Research*, **46** (2014), 78–90.
- [61] J. B. Yang, D. L. Xu and S. Yang, [Integrated efficiency and trade-off analyses using a DEA-oriented interactive minimax reference point approach](#), *Computers and Operations Research*, **39** (2012), 1062–1073.
- [62] S. You and H. Yan, [A new approach in modelling undesirable output in DEA model](#), *Journal of the Operational Research Society*, **62** (2011), 2146–2156.
- [63] M. M. Yu, [Measuring physical efficiency of domestic airports in Taiwan with undesirable outputs and environmental factors](#), *Journal of Air Transport Management*, **10** (2004), 295–303.

- [64] S. Zionts and J. Wallenius, [An interactive programming method for solving the multiple criteria problem](#), *Management Science*, **22** (1976), 652–663.
- [65] P. Zhou, P. W. Ang and K. L. Poh, [Measuring environmental performance under different environmental DEA technologies](#), *Energy Economics*, **30** (2008), 1–14.

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