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# An interactive MOLP method for identifying target units in output-oriented DEA models: The NATO enlargement problem

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ABSTRACT

Data Envelopment Analysis (DEA) is a mathematical programming technique for identifying efficient Decision Making Units (DMUs) with multiple inputs and multiple outputs. DEA provides a technical efficiency score for each DMU, a technical efficiency reference set with peer DMUs, and a target for the inefficient DMU. The target unit informs the Decision Maker (DM) of the amount (%) by which an inefficient DMU should decrease its inputs and/or increase its outputs to become efficient. However, the conventional DEA models generally do not consider the DM's preference structure in identifying the target units. Several equivalence models between the output-oriented DEA and Multiple Objective Linear Programming (MOLP) models have been proposed in the literature to take the DMs' preferences into consideration. However, these models are not able to identify target units when undesirable outputs are produced with desirable outputs in the production process. In this study we obtain a new link between a BCC model and the weighted minimax reference point of the MOLP formulation that simultaneously and interactively considers the increase in the total desirable outputs and the decrease in the total undesirable outputs. We present a pilot study for the North Atlantic Treaty Organization (NATO) enlargement problem to demonstrate the applicability of the proposed method and exhibit the efficacy of the procedures and algorithms.



## 1. Introduction

Data Envelopment Analysis (DEA), initially introduced by Charnes et al. [9], is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous Decision Making Units (DMUs) by evaluating their relative efficiency. DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs. Although DEA does not provide a precise mechanism for achieving efficiency, it does quantify the magnitude of change required to make the inefficient DMUs efficient and hence contributes to productivity growth. A DMU is considered efficient when no other DMU can either produce the same outputs by consuming fewer inputs, known as the "input-orientated approach", or produce more outputs by consuming the same inputs, known as the "output-orientated approach".

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DEA also provides efficiency scores and reference sets for inefficient DMUs. The efficiency scores are used in practical applications as performance indicators of the DMUs. The reference set for inefficient units consists of efficient units and determines a virtual unit on the efficient surface. The virtual unit can be regarded as a target unit for the inefficient unit. The target unit can inform the Decision Maker (DM) of the amount (%) by which an inefficient DMU should decrease its inputs and/or increase its outputs to become efficient.

The conventional DEA models generally do not consider the DM's preference structure or value judgments. However, various methods have been proposed to incorporate the DM's preference information in DEA. Allen et al. [1] have defined value judgments as "logical constructs, incorporated within an efficiency assessment study, reflecting the DMs' preferences in the process of assessing efficiency". *Target setting* and *weight restriction* models are often used to incorporate the DMs' value judgments in DEA. Golany [20] first proposed an interactive target setting model by combining both DEA and Multiple Objective Linear Programming (MOLP) approaches where the DM is assumed to be able to allocate a set of input levels as resources and to select the most preferred set of output levels from a set of alternative points on the efficient frontier. Selected papers in this stream of research include: Pedraja-Chaparro et al. [38], Thanassoulis et al. [46], Tone [50], Korhonen et al. [29], Aparicio et al. [4], Yang et al. [54], Hosseinzadeh Lotfi et al. [23], Bi et al. [8], Hinojosa and Mármol [21], Jain et al. [27], Esmaeilzadeh and Hadi-Vencheh [16], and Amirteimoori et al. [3].

Thompson et al. [49] first proposed a weight restriction model in DEA by incorporating the relative importance of the inputs and outputs in the model. Similar models have been proposed by Dyson and Thannassoulis [12], Charnes et al. [10], Thompson et al. [48], Thompson et al. [47], Wong and Beasley [51], Podinovski [39], and Asmild et al. [5]. Yang et al. [54] established the equivalence relationship between the output-oriented DEA dual models and the minimax reference point approach of MOLP, showing how a DEA problem can be solved interactively without any prior judgments by transforming it into an MOLP formulation. Similarly, Wong et al. [52] established an equivalence relationship between the output-oriented DEA dual models and the minimax formulations that led to the construction of the three equivalence models namely the super-ideal point model, the ideal point model and the shortest distance model. Ebrahimnejad and Hosseinzadeh Lotfi [13] established an equivalence model between the general combined-oriented CCR model and MOLP that gives the Most Preferred Solution (MPS) for DM with trade-off analysis on both input and output values of DMUs. Hosseinzadeh Lotfi et al. [22,23] provided new links between the output-oriented DEA dual model and the general combined-oriented CCR model with the min-ordering optimization problem in MOLP and showed how a DEA problem can be solved interactively by transforming it into a MOLP formulation. They used the Zionts–Wallenius method [57] to reflect the DMs' preferences in the efficiency assessment process. However these methods are not able to identify target units and MPS when undesirable outputs are produced with desirable outputs in the production process. In this study we obtain a new link between an output-oriented BCC model and the weighted minimax reference point MOLP formulation that simultaneously and interactively considers the increase in the total desirable outputs and the decrease in the total undesirable outputs.

The remainder of this paper is organized as follows: Section 2 provides a brief discussion of the output-oriented BCC model with undesirable outputs. Section 3 introduces a technique for solving MOLP problems known as the "weighted minimax MOLP method". In Section 4, we establish a new link between the output-oriented BCC model in the presence of undesirable outputs and the weighted minimax MOLP formulation. Section 5 introduces an interactive multi objective programming method known as the satisfying trade-off method for reflecting the DM's preferences in the efficiency assessment process. In Section 6 we present a pilot study for the NATO enlargement problem to demonstrate the applicability of the proposed model and exhibit the efficacy of the procedures and algorithms. Finally, Section 7 draws the conclusive remarks and future research directions.

## 2. Output-oriented BCC model with undesirable outputs

The production process for each DMU involves using a set of inputs to produce a set of outputs. Each producer has varying levels of inputs and gives varying levels of outputs. DEA assumes that either making more output with the same input or making the same output with less input is a criterion of efficiency. In the presence of undesirable outputs, DMUs with more good (desirable) outputs and less bad (undesirable) outputs (relative to less input resources) should be recognized as efficient. For example, if there are inefficiencies in the production process, the outputs of wastes and pollutants (which are undesirable) should be reduced to improve the performance [43,31,2,14].

Various transformation techniques have been proposed in the literature for dealing with desirable inputs and undesirable outputs (see [32]). Lu and Lo [35] have classified the alternatives for dealing with undesirable outputs in the DEA framework as follows. The first alternative is to simply ignore the undesirable outputs. The second alternative is to either treat the undesirable outputs in terms of a non-linear DEA model or to treat them as outputs and adjust the distance measurement in order to restrict the expansion of the undesirable outputs [18]. The third alternative is either to treat the undesirable outputs as inputs or to apply a monotone decreasing transformation [33]. Seiford and Zhu [43] have suggested an alternative method where they multiply the undesirable outputs by $(-1)$ and then use a translation vector to turn the negative undesirable outputs into positive desirable outputs. Färe and Grosskopf [17] suggested an alternative approach to Seiford and Zhu's [43] method by adopting a directional distance function to estimate the DMUs' efficiencies based on weak disposability of undesirable outputs. You and Yan [55] compared different approaches to incorporate undesirable factors into the DEA model and proposed a ratio model to simultaneously evaluate the undesirable and desirable outputs. They applied their efficiency evaluation model to study the impact of production pollutants in the Chinese textile industry. The results showed that the production output-oriented efficiency evaluation can be significantly altered once the environmental aspects are considered in the model.

We should note that a conventional strategy for dealing with undesirable outputs is to treat them as inputs. However, this approach has some drawbacks. Seiford and Zhu [43] has described the shortcomings of this approach and shown that a DEA model may not reflect the true production process if undesirable outputs are treated as inputs, conceivably, because inputs and outputs are related and all outputs are interconnected. Choosing an input-oriented model may result in neglecting some ecologically relevant slacks [11]. Moreover, an unlimited decrease in undesirables (holding other inputs constant) is not technically possible [53]. In addition, the ratio model proposed by You and Yan [55] computes desirable output and undesirable output as a fraction, where the undesirable output is the denominator and the desirable output is the numerator. In this case, the output value is interpreted as a ratio of the desirable output to the undesirable output. Consequently, the obtained target unit does not inform the DM of the amount (%) by which an inefficient DMU should decrease its undesirable outputs and increase its desirables outputs separately to become efficient. Finally, the proposed approach by Seiford and Zhu [43], which is preferred in this study, applies a linear monotone decreasing transformation. The use of linear transformation preserves the convexity relations and is an attractive feature for a DEA model.

The recent applications of DEA models with desirable and undesirable outputs can be seen in various industries including dairy farms [40,41], electric utilities [19,28], agriculture [44], paper mills [18,25,43], cement manufacturing [30,42], aquaculture [32], airports [56,34,36], petroleum [45], health care [24] and commercial banks [7] among others. Addressing the shortage for efficiency measurement in a real bank management and operational environment, Huang et al. [26] introduced an undesirable DEA model for providing an accurate and reasonable efficiency score that can not only deal with definite output efficiently but also consider the relationship between undesirable outputs. They used the data for 15 Chinese commercial banks and illustrated the advantages of their model with competing models in the literature.

Consider $n$ DMUs under evaluation. Each DMU consumes varying amounts of $m$ different inputs to produce $s$ different outputs. Specifically, $DMU_j$ consumes $X_j = (x_{ij})$ amounts of inputs $(i = 1, 2, \ldots, m)$ and produces $Y_j = (y_{rj})$ amounts of outputs $(r = 1, 2, \ldots, s)$. All data are assumed to be nonnegative but at least one component of every input and output vector is positive, that is; $X_j \geqslant 0, X_j \neq 0$ and $Y_j \geqslant 0$, $Y_j \neq 0$. In addition, assume that $X_p = (x_{1p}, x_{2p}, \ldots, x_{mp})$ and $Y_p = (y_{1p}, y_{2p}, \ldots, y_{rp})$ are the amounts of inputs and outputs for $DMU_p$ (the DMU under evaluation). The following output-oriented BCC model suggested by Banker et al. [6] evaluates the efficiency of $DMU_p$:

$$\begin{aligned} &\max \quad \beta_p \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip} \quad i = 1, 2, \ldots, m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \beta_p y_{rp} \quad r = 1, 2, \ldots, s \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \geqslant 0 \quad j = 1, 2, \ldots, n \end{aligned} \tag{1}$$

This model is an output-oriented BCC model where an efficiency score is generated for a DMU by maximizing outputs with limited inputs and for each observed $DMU_p$ an imaginary composite unit is constructed that outperforms $DMU_p$. Also, $\lambda_j$ represents the proportion for which $DMU_j$ is used to construct the composite unit for $DMU_p$. In model (1), the composite unit consumes at most the same levels of inputs as $DMU_p$ and produces outputs that are at least equal to a proportion $\beta_p$ of the outputs of $DMU_p$ with $\beta_p \geqslant 1$. The inverse of $\beta_p$ is the efficiency score of $DMU_p$. If $\beta_p > 1$, $DMU_p$ is not efficient and the parameter $\beta_p$ indicates the extent by which $DMU_p$ has to increase its outputs to become efficient.

There is no unique model in the literature for handling DEA problems with coexisting desirable and undesirable outputs. In the variable return to scale environment, Seiford and Zhu [43] have proposed a method that first multiplies each undesirable output by $-1$ and then finds a proper translation vector to let all negative undesirable outputs be positive. This approach is very simple and easy to understand. Also, it maintains the current property of production. Therefore, we employ this approach in the MOLP method proposed in this study.

Suppose we have $n$ DMUs and each $DMU_j$ produces $s_1$ desirable outputs, $y^g_{rj} (r = 1, 2, \ldots, s_1)$ and $s_2$ undesirable outputs $y^b_{tj} (r = 1, 2, \ldots, s_2)$ using $m$ inputs $x_{ij} (i = 1, 2, \ldots, m)$. The following extended output-oriented BCC model suggested by Seiford and Zhu [43] evaluates the efficiency of $DMU_p$ in the presence of undesirable outputs:

$$\begin{aligned} &\max \quad \beta_p \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ip} \quad i = 1, 2, \ldots, m \\ &\sum_{j=1}^{n} \lambda_j y^g_{rj} \geqslant \beta_p y^g_{rp} \quad r = 1, 2, \ldots, s_1 \\ &\sum_{j=1}^{n} \lambda_j \bar{y}^b_{tj} \geqslant \beta_p \bar{y}^b_{tp} \quad t = 1, 2, \ldots, s_2 \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \geqslant 0 \quad j = 1, 2, \ldots, n \end{aligned} \tag{2}$$

where $\bar{y}^b_{tj} = \bar{y}^b_{tj} + u_t > 0$.

For each inefficient $DMU_p$, we define a reference set $E_p = \left\{ \lambda^*_j | \lambda^*_j > 0, j = 1, 2, \ldots n \right\}$ where $\lambda^* = (\lambda^*_1, \lambda^*_2, \ldots, \lambda^*_n)$ is the optimal solution of (2). In this case, the following point on the efficient frontier is used to evaluate the performance of $DMU_p$ and can be regarded as a target unit for the inefficient unit $DMU_p$:

$$\left( \sum_{j\in E_p} \lambda_j^* x_j, \quad \sum_{j\in E_p} \lambda_j^* y_r^g, \quad \sum_{j\in E_p} \lambda_j^* y_t^b \right) \tag{3}$$

This target unit usually does not include a DM's preference structure or value judgments. An interactive MOLP could be used to address this deficiency.

## 3. The weighted minimax formulation

The MOLP model (4) can be used to optimize a vector of linear functions in the presence of linear constraints:

$$\begin{aligned} \max f(\lambda) = [&g_1(\lambda), \ldots, g_r(\lambda), \ldots, g_{s_1}(\lambda), \\ &h_1(\lambda), \ldots, h_t(\lambda), \ldots, h_{s_2}(\lambda)] \\ \text{s.t.}\, &\lambda \in \Lambda \end{aligned} \tag{4}$$

In this model, the goal is to maximize $s_1 + s_2 (\geqslant 2)$ conflicting objective functions with $g_r$: $\Lambda \to R (r = 1, 2, \ldots, s_1)$ and $h_t$: $\Lambda \to R (r = 1, 2, \ldots, s_2)$. The decision variables $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ belong to the non-empty feasible space $\Lambda$. The objective vectors in the objective space $R^n$ consist of objective values $f(\lambda)$ and the image of the feasible space is called a *feasible objective space* $Z = f(\Lambda)$.

Generally, there are no solutions in the MOLP problems that can simultaneously optimize all the objective functions (similar to the single-objective linear programming problems). As a result, the primary goal in MOLP is to find the Pareto optimal solutions and to help select the most preferred solution. In fact, a solution represented by a point in the decision variable space is a strictly Pareto optimal solution if it is not possible to move the point within the feasible region to improve an objective function value without deteriorating at least one of the other objectives. In addition, a solution represented by a point in the decision variable space is a weak Pareto optimal solution if it is not possible to move the point within the feasible region to improve all the objective functions. In other words, we have the following definitions:

**Definition 1.** A feasible solution $\hat{\lambda} \in \Lambda$ is called a *strict* Pareto optimal solution if there is no feasible solution $\lambda \in \Lambda$ such that $f(\lambda) \geqslant f(\hat{\lambda})$ and $f(\lambda) \neq f(\hat{\lambda})$.

**Definition 2.** A feasible solution $\hat{\lambda} \in \Lambda$ is called a *weak* Pareto optimal solution if there is no feasible solution $\lambda \in \Lambda$ such that$f(\lambda) \geqslant f(\hat{\lambda})$.

**Definition 3.** The point $f^* = \left(g_1^*, \ldots, g_r^*, \ldots, g_{s_1}^*, h_1^*, \ldots, h_t^*, \ldots, h_{s_2}^*\right)$ given by $g_r^* = \max_{\lambda\in\Lambda} g_r(\lambda) (r = 1, 2, \ldots, s_1)$ and $h_t^* = \max_{\lambda\in\Lambda} h_t(\lambda) (t = 1, 2, \ldots, s_2)$ is called the ideal point of MOLP (4).

To generate any strict Pareto optimal solution, the MOLP (4) can be written using a weighted minimax approach as follows with $f^* = (g^*, h^*)$ as the ideal point [15]:

$$\begin{aligned} &\min \max_{1\leqslant r\leqslant s_1, 1\leqslant t\leqslant s_2} \{w_r(g_r^* - g_r(\lambda)), v_t(h_t^* - h_r(\lambda))\} \\ &\text{s.t. } \lambda \in \Lambda \end{aligned} \tag{5}$$

We now show that the weighted minimax formulation (5) can be written as a single objective linear programming problem. Let us assume $\phi = \max_{1\leqslant r\leqslant s_1, 1\leqslant t\leqslant s_2} \{w_r(g_r^* - g_r(\lambda)), v_t(h_t^* - h_r(\lambda))\}$ and rewrite the formulation (5) as follows:

$$\begin{aligned} \min \quad & \phi \\ \text{s.t.} \quad & w_r(g_r^* - g_r(\lambda)) \leqslant \phi, \quad r = 1, 2, \ldots, s_1 \\ & v_t(h_t^* - h_r(\lambda)) \leqslant \phi \quad t = 1, 2, \ldots, s_2 \\ & \lambda \in \Lambda \end{aligned} \tag{6}$$

In the next section we show that the output-oriented BCC model (2) and the weighted minimax MOLP formulation (6) are equivalent under certain conditions.

## 4. A generalized equivalence model

In the output-oriented BCC model in the presence of undesirable outputs an efficiency score is generated for a DMU by maximizing the desirable outputs and minimizing undesirable outputs with limited inputs. This concept can be represented by a multiple objective optimization model. The theoretical considerations for combining the MOLP and DEA models are presented next.

The output-oriented BCC model (2) can be equivalently reformulated as follows:

$$\begin{aligned} \max \quad & \beta_p \\ \text{s.t.} \quad & \beta_p y_{rp}^g - \sum_{j=1}^n \lambda_j y_{rj}^g \leqslant 0, \quad r = 1, 2, \ldots, s_1 \\ & \beta_p \bar{y}_{tp}^b - \sum_{j=1}^n \lambda_j \bar{y}_{tj}^b \leqslant 0, \quad t = 1, 2, \ldots, s_2 \\ & \lambda \in \Lambda_p = \left\{ \lambda \left| \sum_{j=1}^n \lambda_j x_{ij} \leqslant x_{ip}, i = 1, 2, \ldots, m, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geqslant 0, \ j = 1, 2, \ldots, n \right. \right\} \end{aligned} \tag{7}$$

Now, we apply certain conditions in order to show that the output-oriented BCC model with undesirable outputs given in (7) is equivalent to the minimax formulation in (6). Suppose in model (6), the $r$th composite desirable output denoted by $g_r(\lambda)$, and the $t$th composite undesirable output denoted by $h_t(\lambda)$, are defined as follows, respectively:

$$g_r(\lambda) = \sum_{j=1}^n \lambda_j y_{rj}^g \quad r = 1, 2, \ldots, s_1 \tag{8}$$

$$h_t(\lambda) = \sum_{j=1}^n \lambda_j \bar{y}_{tj}^b \quad t = 1, 2, \ldots, s_2 \tag{9}$$

In this equivalence analysis, the $r$th composite desirable output and the $t$th composite undesirable output are taken as the objectives for maximization, so there are $s = s_1 + s_1$ objectives in total. The maximum values of the $r$th composite desirable output and the $t$th composite undesirable output denoted by $\bar{g}_{rp} = g_r(\lambda^r)$ and $\bar{h}_{tp} = h_t(\lambda^t)$, respectively, are derived as follows where $\lambda^r$ and $\lambda^t$ are the optimal solutions of the following single objective optimization problems:

$$\max g_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj}^g \quad \text{s.t. } \lambda \in \Lambda_p \tag{10}$$

$$\max h_t(\lambda) = \sum_{j=1}^{n} \lambda_j \bar{y}_{tj}^b \quad \text{s.t. } \lambda \in \Lambda_p \tag{11}$$

Suppose the feasible space $\Lambda_p$ in formulations (7) and $\Lambda$ in formulation (6) are the same. The equivalence relationship between the output-oriented BCC model (7) and the weighted minimax MOLP formulation (6) can be easily established by the following theorem.

**Theorem 1.** *Suppose $y_{rp}^g > 0 (r = 1, 2, \ldots, s_1)$. The output-orientated BCC model (7) can be equivalently transformed into the weighted minimax MOLP formulation (6) using Eqs. (8)–(11) and the following equations:*

$$w_r = \frac{1}{y_{rp}^g}, \quad r = 1, 2, \ldots, s_1 \tag{12}$$

$$v_t = \frac{1}{\bar{y}_{tp}^b}, \quad t = 1, 2, \ldots, s_2 \tag{13}$$

$$g_r^* = \frac{F^{\max}}{w_r} = y_{rp}^g F^{\max}, \quad r = 1, 2, \ldots, s_1 \tag{14}$$

$$h_t^* = \frac{F^{\max}}{v_t} = \bar{y}_{tp}^b F^{\max}, \quad t = 1, 2, \ldots, s_2 \tag{15}$$

$$\beta_p = F^{\max} - \phi, \quad \Lambda_p = \Lambda \tag{16}$$

$$F^{\max} = \max_{\substack{1 \leqslant r \leqslant s_1 \\ 1 \leqslant t \leqslant s_2}} \{w_r \bar{g}_{rp}, v_t \bar{h}_{tp}\} = \max_{\substack{1 \leqslant r \leqslant s_1 \\ 1 \leqslant t \leqslant s_2}} \left\{ \frac{\bar{g}_{rp}}{y_{rp}^g}, \frac{\bar{h}_{tp}}{\bar{y}_{tp}^b} \right\} \tag{17}$$

**Proof.** By substituting relations (8) and (12) in the first $s_1$ constraints of (7) and relations (9) and (13) in the second $s_2$ constraints of (7), the output-oriented BCC model with undesirable outputs given in (7) can be rewritten as follows:

$$\begin{aligned} & \max \beta_p \\ \text{s.t. } & \beta_p \frac{1}{w_r} - g_r(\lambda) \leqslant 0, \quad r = 1, 2, \ldots, s_1 \\ & \beta_p \frac{1}{v_t} - h_t(\lambda) \leqslant 0, \quad t = 1, 2, \ldots, s_2 \\ & \lambda \in \Lambda_p \end{aligned} \tag{18}$$

The first $s_1$ constraints in (18) can be equivalently transformed as follows:

$$\begin{aligned} \beta_p \frac{1}{w_r} - g_r(\lambda) \leqslant 0 \iff & -w_r g_r(\lambda) \leqslant -\beta_p \iff F^{\max} - w_r g_r(\lambda) \\ & \leqslant F^{\max} - \beta_p \iff w_r \left( \frac{F^{\max}}{w_r} - g_r(\lambda) \right) \\ & \leqslant F^{\max} - \beta_p \iff w_r (g_r^* - g_r(\lambda)) \leqslant \phi \end{aligned} \tag{19}$$

In a similar way, the second $s_2$ constraints in (18) can be equivalently transformed as follows:

$$\beta_p \frac{1}{v_t} - h_t(\lambda) \leqslant 0 \iff v_r (h_t^* - h_t(\lambda)) \leqslant \phi \tag{20}$$

Also, the objective function of (18) becomes:

$$\begin{aligned} \max \beta_p &= -\min(-\beta_p) = -\min(F^{\max} - \beta_p) \\ &= -\min \phi \end{aligned} \tag{21}$$

It should be noted that for any $\lambda \in \Lambda$ we have:

$$\begin{aligned} \phi &= F^{\max} - \beta_p \\ &\geqslant w_r \bar{g}_{rp} - \beta_p \quad r = 1, 2, \ldots, s_1, \\ &\geqslant w_r g_r(\lambda) - \beta_p \quad r = 1, 2, \ldots, s_1, \\ &\geqslant 0 \end{aligned} \tag{22}$$

and

$$\begin{aligned} \phi &= F^{\max} - \beta_p \\ &\geqslant v_t h_{tp} - \beta_p \quad t = 1, 2, \ldots, s_2, \\ &\geqslant v_t h_t(\lambda) - \beta_p \quad t = 1, 2, \ldots, s_2, \\ &\geqslant 0. \end{aligned} \tag{23}$$

Finally,

$$g_r^* = \frac{F^{\max}}{w_r} \geqslant \frac{w_r \bar{g}_{rp}}{w_r} = \bar{g}_{rp} = \max_{\lambda \in \Lambda_p} g_r(\lambda) \quad r = 1, 2, \ldots, s_1 \tag{24}$$

$$h_t^* = \frac{F^{\max}}{v_t} \geqslant \frac{v_t \bar{h}_{tp}}{v_t} = \bar{h}_{tp} = \max_{\lambda \in \Lambda_p} h_t(\lambda) \quad t = 1, 2, \ldots, s_2 \tag{25}$$

The equivalence model between the output-oriented BCC model (7) and the weighted minimax MOLP formulation of (6) is established, since (19)–(21) hold. □

From Theorem 1, the output-oriented BCC model with undesirable outputs given in (7) can be equivalently rewritten as a weighted minimax MOLP formulation of (6) as follows:

$$\begin{aligned} \min \quad & \phi \\ \text{s.t.} \quad & w_r (g_r^* - g_r(\lambda)) \leqslant \phi, \quad r = 1, 2, \ldots, s_1 \\ & v_t (h_t^* - h_r(\lambda)) \leqslant \phi \quad t = 1, 2, \ldots, s_2 \\ & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ip}, \quad i = 1, 2, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geqslant 0, \quad j = 1, 2, \ldots, n \end{aligned} \tag{26}$$

This means that they share the same decision and objective spaces and have the same optimal solution.

Alternatively, since formulation (6) is equivalent to (4), formulation (26) can be rewritten as follows:

$$max\left[\sum_{j=1}^{n}\lambda_j y_{1j}^{g},\ldots,\sum_{j=1}^{n}\lambda_j y_{s_1J}^{g},\sum_{j=1}^{n}\lambda_j \bar{y}_{1j}^{b},\ldots,\sum_{j=1}^{n}\lambda_j \bar{y}_{s_2J}^{b}\right] \quad (27)$$

$$\text{s.t.} \sum_{j=1}^{n}\lambda_j x_{ij} \leqslant x_{ip}, \quad i=1,2,\ldots,m,$$

$$\sum_{j=1}^{n}\lambda_j = 1.$$

$$\lambda_j \geqslant 0, \quad j=1,2,\ldots,n.$$

We note that formulation (26) is equivalent to formulation (7) if $w_r(r=1,2,\ldots,s_1)$ and $v_t(t=1,2,\ldots,s_2)$ are calculated using (12) and (13), respectively, and $g_r^*(r=1,2,\ldots,s_1)$ and $h_t^*(t=1,2,\ldots,s_2)$ in formulation (31) are calculated using (14) and (15), respectively. Likewise, formulation (27) is equivalent to formulation (7). So, the efficiency score of $\text{DMU}_p$ can be generated by solving formulation (26). Therefore, an interactive MOLP method can be used to solve the DEA problem in order to locate the MPS on the efficient frontier for target setting and resource allocation.

Although the objective functions of the MOLP problem (27) are identical for all inefficient DMUs, the feasible space in this problem is different for each DMU and the iterative process given in the next section produces a unique target unit for each inefficient unit.

## 5. Satisfying trade-off method

Generally, an interactive MOLP process involves three components: the DM, the analyst and the model. The analyst is an intermediary between the DM and the model. The interaction process involves the following activities. The analyst obtains an initial solution for the model and presents it to the DM to evaluate this solution based on his or her preferences. The preferences of the DM are fed back into the model by the analyst and a new solution is generated. This process is repeated in an iterative way until a satisfactory solution is obtained.

Several interactive MOLP models have been proposed in the literature to help a DM search for different solutions along the efficient frontier and choose one solution from the set of Pareto optimal solutions as his/her final solution. Nakayama and Sawaragi [37] have proposed a new type of interactive method for MOLP called the satisfying trade-off method. To explain this interactive method, let us consider the general MOLP formulation in the following form:

$$\begin{aligned} \max \quad & f(\lambda) = [f_1(\lambda),\ldots,f_r(\lambda),\ldots,f_s(\lambda)] \\ \text{s.t.} \quad & \lambda \in \Lambda \end{aligned} \quad (28)$$

The first step of the method is to find the reference point of achievement function that does not change within the whole solution process. After that the reference levels of the objectives are provided by the DM at each iteration and incorporated in the weights of the achievement function.

Assume $(\lambda^{k-1}, f^{k-1})$ be the solution of iteration $k-1$. Based on this solution, the DM should classify the objective functions into three categories, namely;

(i) The class of objective functions that should improve more.
(ii) The class of objective functions that has to be maintained.
(iii) The class of objective functions that must be relaxed.

Suppose that $\{f_i, i \in I_I^k\}$ represents the objective functions to be improved where $\{\Delta f_i, i \in I_I^k\}$ are the amounts to be improved, $\{f_i, i \in I_M^k\}$ denotes the objective functions that are to be maintained and $\{f_i, i \in I_R^k\}$ are the objective functions to be relaxed with the amounts $\{\Delta f_i, i \in I_R^k\}$. Thus, the new reference point is provided as follows:

$$\begin{aligned} y_i^k &= f_i^{k-1} + \Delta f_i^k \forall i \in I_I^k \\ y_i^k &= f_i^{k-1} \quad \forall i \in I_M^k \\ y_i^k &= f_i^{k-1} - \Delta f_i^k \quad \forall i \in I_R^k \end{aligned} \quad (29)$$

Based on this new reference point, the values of $u_i^k$ are estimated as follows:

$$u_i^h = \frac{1}{f_i^* - y_i^k} \quad i=1,2,\ldots,s \quad (30)$$

And the following weighted minimax MOLP problem is solved:

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & u_i^h(f_i^* - f_i(\lambda)) \leqslant \theta, i=1,2,\ldots,s \\ & \lambda \in \Lambda, \theta \geqslant 0 \end{aligned} \quad (31)$$

This process is repeated in an interactive way until a satisfactory solution is obtained.

## 6. A pilot study: NATO enlargement problem

NATO's open door policy on enlargement invites European countries that are in a position to advance the principles of the North Atlantic Treaty and contribute to security in the Euro-Atlantic area, to join the alliance. Deciding ideal candidates for the expansion of NATO can be complicated. The integration of non-members in NATO is made in four steps indicating an increasing level of cooperation: Partnership for Peace (PFP), Individual Partnership Action Plan (IPAP), Intensified Dialogue (ID) and Membership Action Plan (MAP). The first level, PFP, was created in 1993 to create a dialog with neutral European and former Warsaw Block member states. The second level, IPAP, was installed in 2002 for eight countries within PFP potentially eligible for NATO membership. The third level of integration, ID, is currently initiated for two countries prior to the final candidacy. Finally, the MAP is currently in action for three countries for which membership is under negotiation. Decisions on enlargement are ultimately made by NATO and its members; however, the North Atlantic Council is NATO's principal decision-making body and is responsible for inviting new members to join the alliance. Decisions to invite new members come as a result of a unanimous vote by current member countries in the final stage. Relationships between members of the alliance are vital and a unanimous vote protects the integrity of the alliance and prevents tension among the member countries.

Each decision to expand is made individually on a case-by case basis and is a result of an agreement that the invited country will add to the security and stability of the alliance. The determination must also allow the alliance to preserve the ability to perform its main function of defense. Countries outside of the alliance are not given a voice in these decisions nor should countries be excluded for consideration due to membership of other groups or organizations.

### 6.1. Potential input–output variables

The specification of the model follows the publicly announced criteria related to economic and social stability, as well as the absence of conflicts with existing or future members or partners of the alliance. Consequently, in this study, revenue or Gross Domestic Product (GDP), Budget (Revenues), and unemployment rate are considered as the output variables while budget expenditures and public debt are considered as the input variables:

#### 6.1.1. Gross Domestic Product (GDP)

GDP is the value of all the goods and services produced within a country in a given year. It is a desirable output measuring a country's economic power.

#### 6.1.2. Budget (Revenues)

This output variable measures the amount of money raised by the state's government. It is a desirable output and a proxy of the capacity for public economic action.

#### 6.1.3. Unemployment rate

A high unemployment rate is an undesirable output indicating high social stability, labor mobilization and potential for public action.

#### 6.1.4. Budget expenditures

This input variable relates to the ability of a country to balance its revenues and expenditures. The threat involved lies in those countries that cannot bring in enough revenues to cover their expenditures. This may be for a number of reasons including but not limited to a lack of natural resources, bad soil or climates for growing crops, lack of education or lack of skilled manpower. Countries that cannot effectively balance their budget typically require economic assistance from other countries.

#### 6.1.5. Public debt

A rising public debt (% of GDP) indicates an increasing debt burden for taxpayers. If the public debt (% of GDP) continues to grow, interest charges on the debt will likely also rise. Debt-servicing charges thus progressively absorb more tax revenues, leaving fewer resources available to fund other program expenditures. At some point, the need to service the outstanding debt compels the government to level program spending, or raise taxes, and/or maintain program spending at current levels and rely increasingly on deficit financing – which further adds to the growing public debt burden. Governments that let debt accumulate over a long period of time risk eroding the living standards of their citizens as debt-servicing costs absorb a progressively larger share of the tax base, "crowding out" program spending.

Table 1 shows the input and output values used in this example.

### 6.2. The solution process

We solved model (2) or (7) with $u = 46.5$ to find the respective efficiency scores. The inefficient units and their references units are given in Table 2.

Before conducting interactive tradeoff analysis, let us first validate the equivalence between the DEA model (2) or (7) and the weighted minimax MOLP formulation (26) developed in the previous sections. The weighted minimax MOLP formulation (26) is run for each DMU, with $w_r$ and $v_t$ assigned by (11) and (13) respectively, and the reference point $f^* = [g^*, h^*] = \left[F^{\max}\left(y^g_{1p}, \ldots, y^g_{s_1p}\right), F^{\max}\left(y^b_{1p}, \ldots, y^b_{s2p}\right)\right]$. The results are shown in Table 3, which shows that the equivalence $\beta_p = F^{\max} - \phi$ holds for each DMU with $\phi$ generated using the weighted minimax MOLP formulation (26), $F^{\max}$ assigned using Eq. (17) and $\beta_p$ obtained using the DEA model (7) for each DMU. The observed composite units for each DMU shown in Table 4 are also the same as the results of Table 2 generated by the DEA model (7).

As shown in Table 4, units 2, 5–7, 9–11, and 15 are found to be inefficient within the reference set of six countries. We then used formulation (3) for each inefficient unit and found a virtual unit on the efficient frontier that could be regarded as its target unit. The results are presented in Table 4.

#### 6.2.1. Target unit for Bosnia and Herzegovina

Now we search a target unit for each inefficient unit. For instance, as shown in Table 2, Unit 5 (Bosnia and Herzegovina) has an efficiency score of 0.9174 implying that it is operating as an inefficient country and also its composite unit on the efficient frontier can be represented as a linear combination of 0.34 of Unit 8 (Georgia), 0.04 of Unit 14 (Russia) and 0.62 of Unit 18 (Ukraine). In this case, based on formulation (3), the following virtual unit is used to evaluate the performance of Unit 5 regarded as a target unit of this inefficient unit:

(I1, I2) = (48.29, 34)
(O1, O2, O3) = (302.33, 52.21, 8.66)

This shows that, for Unit 5 (Bosnia and Herzegovina) to become efficient, the value of the unemployment rate (undesirable output) should decrease from 45.50 to 8.66, and the values of GDP and budget expenditures (desirable outputs) should increase from 14.78 and 48 to 302.33 and 52.21, respectively. However, the DM has not accepted the DEA composite unit as the most preferred solution for BA. Specially, it is difficult for the government of BA to decrease the value of the unemployment rate from 45.50 to 8.66 in a short period of time. Therefore, the approach proposed in this study is needed to search the most preferred

**Table 1**
NATO enlargement case study.

| Non-EU country | | Outputs | | | Inputs | |
|---|---|---|---|---|---|---|
| | | GDP in billion (D-higher is better) | Budget revenues (% of GDP) (D-higher is better) | Unemployment rate (%) (U-lower is better) | Budget expenditures (% of GDP) (D-lower is better) | Public debt (% of GDP) (D-lower is better) |
| $DMU_1$ | Armenia | 17.170 | 9.70 | 7.10 | 10.10 | 25.13 |
| $DMU_2$ | Austria | 322.000 | 55.12 | 4.40 | 55.87 | 59.10 |
| $DMU_3$ | Azerbaijan | 64.660 | 10.45 | 1.00 | 13.26 | 6.70 |
| $DMU_4$ | Belarus | 103.500 | 20.05 | 1.60 | 20.16 | 3.57 |
| $DMU_5$ | Bosnia and Herzegovina | 14.780 | 48.00 | 45.50 | 48.29 | 34.00 |
| $DMU_6$ | Finland | 188.400 | 32.92 | 6.90 | 30.87 | 35.90 |
| $DMU_7$ | FYR Macedonia | 17.350 | 14.46 | 34.90 | 14.33 | 30.80 |
| $DMU_8$ | Georgia | 20.600 | 17.86 | 13.60 | 14.95 | 23.37 |
| $DMU_9$ | Ireland | 191.600 | 48.65 | 4.60 | 48.26 | 24.90 |
| $DMU_{10}$ | Kazakhstan | 168.200 | 14.02 | 7.30 | 15.06 | 7.70 |
| $DMU_{11}$ | Malta | 9.400 | 37.07 | 6.40 | 37.81 | 21.73 |
| $DMU_{12}$ | Moldova | 9.756 | 18.76 | 2.10 | 18.87 | 23.30 |
| $DMU_{13}$ | Montenegro | 5.918 | 118.28 | 14.70 | 757.86 | 38.00 |
| $DMU_{14}$ | Russia | 2097.000 | 14.26 | 6.20 | 12.49 | 5.90 |
| $DMU_{15}$ | Serbia | 77.280 | 12.42 | 18.80 | 12.68 | 37.00 |
| $DMU_{16}$ | Sweden | 338.500 | 73.59 | 6.10 | 68.98 | 41.70 |
| $DMU_{17}$ | Switzerland | 303.200 | 49.67 | 2.80 | 46.67 | 44.20 |
| $DMU_{18}$ | Ukraine | 324.800 | 13.41 | 2.30 | 13.87 | 11.70 |

**Table 2**
Efficiency scores and observed composite unit.

| Inefficient units | $\beta_p$ | Efficiency | $DMU_4$ | $DMU_8$ | $DMU_{14}$ | $DMU_{16}$ | $DMU_{17}$ | $DMU_{18}$ |
|---|---|---|---|---|---|---|---|---|
| $DMU_2$ | 1.02 | 0.9804 | – | – | 0.01 | 0.28 | 0.72 | – |
| $DMU_5$ | 1.09 | 0.9174 | – | 0.34 | 0.04 | 0.62 | – | – |
| $DMU_6$ | 1.02 | 0.9804 | – | – | 0.67 | 0.32 | 0.01 | – |
| $DMU_7$ | 1.17 | 0.8547 | – | 0.75 | 0.25 | – | – | – |
| $DMU_9$ | 1.01 | 0.9901 | 0.44 | – | – | 0.53 | 0.02 | – |
| $DMU_{10}$ | 1.1 | 0.9091 | 0.26 | – | 0.33 | – | – | 0.41 |
| $DMU_{11}$ | 1.07 | 0.9346 | 0.45 | – | 0.09 | 0.28 | 0.18 | – |
| $DMU_{15}$ | 1.17 | 0.8547 | – | 0.08 | 0.92 | – | – | – |

**Table 3**
Equivalence between DEA and MOLP.

| DMUs | $\beta_p$ | $DMU_4$ | $DMU_8$ | $DMU_{14}$ | $DMU_{16}$ | $DMU_{17}$ | $DMU_{18}$ | $F^{max}$ | $\phi$ | $F^{max} - \phi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $DMU_1$ | 1 | – | – | – | – | – | – | 1 | 0 | 1 |
| $DMU_2$ | 1.02 | – | – | 0.01 | 0.28 | 0.72 | – | 6.52 | 5.5 | 1.02 |
| $DMU_3$ | 1 | – | – | – | – | – | – | 32.43 | 31.43 | 1 |
| $DMU_4$ | 1 | – | – | – | – | – | – | 1 | 0 | 1 |
| $DMU_5$ | 1.09 | – | 0.34 | 0.04 | 0.62 | – | – | 141.88 | 140.79 | 1.09 |
| $DMU_6$ | 1.02 | – | – | 0.67 | 0.32 | 0.01 | – | 11.13 | 10.11 | 1.02 |
| $DMU_7$ | 1.17 | – | 0.75 | 0.25 | – | – | – | 120.86 | 119.69 | 1.17 |
| $DMU_8$ | 1 | – | – | – | – | – | – | 101.8 | 100.8 | 1 |
| $DMU_9$ | 1.01 | 0.44 | – | – | 0.53 | 0.02 | – | 10.94 | 9.93 | 1 |
| $DMU_{10}$ | 1.1 | 0.26 | – | 0.33 | – | – | 0.41 | 12.47 | 11.37 | 1.1 |
| $DMU_{11}$ | 1.07 | 0.45 | – | 0.09 | 0.28 | 0.18 | – | 223.09 | 222.02 | 1.07 |
| $DMU_{12}$ | 1 | – | – | – | – | – | – | 214.94 | 213.94 | 1 |
| $DMU_{13}$ | 1 | – | – | – | – | – | – | 354.34 | 353.34 | 1 |
| $DMU_{14}$ | 1 | – | – | – | – | – | – | 1 | 0 | 1 |
| $DMU_{15}$ | 1.17 | – | 0.08 | 0.92 | – | – | – | 27.14 | 25.97 | 1.17 |
| $DMU_{16}$ | 1 | – | – | – | – | – | – | 6.19 | 5.19 | 1 |
| $DMU_{17}$ | 1 | – | – | – | – | – | – | 6.92 | 5.92 | 1 |
| $DMU_{18}$ | 1 | – | – | – | – | – | – | 6.46 | 5.46 | 1 |

solution along the frontier for BA. In fact, the following MOLP problem defines the production possibility set for BA, in which there may be more preferred efficient solutions than the DEA efficient solution:

**Table 4**
Target units based on DEA model.

| Inefficient units | I1 | I2 | O1 | O2 | O3 |
|---|---|---|---|---|---|
| $DMU_2$ | 52.55 | 43.21 | 327.07 | 55.99 | 3.74 |
| $DMU_5$ | 48.29 | 34 | 302.33 | 52.21 | 8.66 |
| $DMU_6$ | 30.87 | 17.73 | 1517 | 33.56 | 6.13 |
| $DMU_7$ | 14.33 | 18.97 | 543.92 | 16.95 | 11.73 |
| $DMU_9$ | 46.83 | 24.9 | 233.69 | 49.31 | 4.03 |
| $DMU_{10}$ | 15.06 | 7.69 | 845.12 | 15.42 | 3.39 |
| $DMU_{11}$ | 37.81 | 21.73 | 391.8 | 39.75 | 3.5 |
| $DMU_{15}$ | 12.68 | 7.25 | 1936.63 | 14.54 | 6.77 |

$$\begin{aligned}
\max\ & 17.17\lambda_1 + 322\lambda_2 + 64.66\lambda_3 + 103.5\lambda_4 + 14.78\lambda_5 \\
& + 188.4\lambda_6 + 17.25\lambda_7 + 20.6\lambda_8 + 191.6\lambda_9 + 168.2\lambda_{10} \\
& + 9.4\lambda_{11} + 9.756\lambda_{12} + 5.918\lambda_{13} + 2097\lambda_{14} + 77.28\lambda_{15} \\
& + 338.5\lambda_{16} + 303.2\lambda_{17} + 324.8\lambda_{18}
\end{aligned}$$

$$\begin{aligned}
\max\ & 9.7\lambda_1 + 55.12\lambda_2 + 10.45\lambda_3 + 20.05\lambda_4 + 48\lambda_5 \\
& + 32.92\lambda_6 + 14.46\lambda_7 + 17.86\lambda_8 + 48.65\lambda_9 + 14.02\lambda_{10} \\
& + 37.07\lambda_{11} + 18.76\lambda_{12} + 118.28\lambda_{13} + 14.26\lambda_{14} \\
& + 12.42\lambda_{15} + 73.59\lambda_{16} + 49.67\lambda_{17} + 13.41\lambda_{18}
\end{aligned}$$

$$\begin{aligned}
\max\ & 39.4\lambda_1 + 42.1\lambda_2 + 45.5\lambda_3 + 44.9\lambda_4 + \lambda_5 + 39.6\lambda_6 \\
& + 11.6\lambda_7 + 32.9\lambda_8 + 41.9\lambda_9 + 39.2\lambda_{10} + 40.1\lambda_{11} \\
& + 44.4\lambda_{12} + 31.8\lambda_{13} + 40.3\lambda_{14} + 27.7\lambda_{15} + 40.4\lambda_{16} \\
& + 43.7\lambda_{17} + 44.2\lambda_{18}
\end{aligned}$$

$$\begin{aligned}
\text{s.t. } & 10.1\lambda_1 + 55.87\lambda_2 + 13.26\lambda_3 + 20.16\lambda_4 \\
& + 48.29\lambda_5 + 30.87\lambda_6 + 11.43\lambda_7 + 14.95\lambda_8 \\
& + 41.26\lambda_9 + 15.06\lambda_{10} + 37.81\lambda_{11} + 18.87\lambda_{12} \\
& + 757.86\lambda_{13} + 12.49\lambda_{14} + 12.68\lambda_{15} + 68.98\lambda_{16} \\
& + 436.67\lambda_{17} + 13.87\lambda_{18} \leqslant 48.29 \qquad (32)
\end{aligned}$$

$$\begin{aligned}
& 25.13\lambda_1 + 59.1\lambda_2 + 6.7\lambda_3 + 3.57\lambda_4 + 34\lambda_5 + 35.9\lambda_6 \\
& + 30.8\lambda_7 + 23.37\lambda_8 + 24.9\lambda_9 + 7.7\lambda_{10} + 21.73\lambda_{11} \\
& + 23.3\lambda_{12} + 38\lambda_{13} + 4.9\lambda_{14} + 37\lambda_{15} + 41.7\lambda_{16} \\
& + 44.2\lambda_{17} + 11.7\lambda_{18} \leqslant 34
\end{aligned}$$

$$\sum_{j=1}^{18}\lambda_j = 1,$$

$$\lambda_j \geqslant 0, j = 1, 2, \ldots, 18$$

The results of using the approach presented in Section 5 for obtaining the most preferred as a target unit for Bosnia and Herzegovina are as follows:

*6.2.1.1. Iteration 1.* The government of BA as the DM accepts to decrease the value of GDP from 302.33 to 200 and at the same time to increase the values of the unemployment rate and budget expenditures from 8.66 and 52.21 to 20 and 150, respectively. Based on these values, the new reference set is achieved using Eq. (29). Then, the new values of weights $w_r$ and $v_t$ are estimated by (30). In this case, model (26) is solved using these new weights. This optimal solution provides the following new target unit for the inefficient Unit 5 (Bosnia and Herzegovina) using formulation (3):

(I1, I2) = (692.82, 35.2)
(O1, O2, O3) = (188.38, 109.2, 13.96)

DM still does not agree with this target unit. Thus, the process is repeated until a satisfactory solution is obtained.

*6.2.1.2. Iteration 2.* The value of GDP is acceptable for DM. But the DM does not agree with the value of the unemployment rate. The DM accepts to decrease the value of budget expenditures from 109.2 to 180 and at the same time the unemployment rate is increased to 18. Based on these values, the new reference set is achieved using Eq. (29). Then, the new values of weights $w_r$ and $v_t$ are estimated by (30). Now, model (26) runs using these new weights. This optimal solution gives the following new target unit for the inefficient Unit 5 (Bosnia and Herzegovina) using formulation (3):

(I1, I2) = (699.98, 35.51)
(O1, O2, O3) = (168.29, 110.2, 14.04)

Now the DM accepts this target unit and then the interactive process terminates.

*6.2.2. Target unit for FYR Macedonia*

Similarly, it is possible to search for a target unit for the inefficient Unit 7 (FYR Macedonia). As shown in Table 2, Unit 7 has an efficiency score of 0.8547 implying that it is operating as an inefficient country. In this case, its composite unit on the efficient frontier can be represented as a linear combination of 0.75 of Unit 8 (Georgia) and 0.25 of Unit 14 (Russia). Thus, with regards to formulation (3), the following virtual unit could be regarded as a target unit for the inefficient Unit 7.

(I1, I2) = (14.33, 18.97)
(O1, O2, O3) = (543.92, 16.95, 11.73)

This shows that, for Unit 7 (FYR Macedonia) to become efficient, the value of the unemployment rate (undesirable output) should decrease from 34.9 to 11.73, and the values of GDP and budget expenditures (desirable outputs) should increase from 17.35 and 14.46 to 543.92 and 16.95, respectively. However, the DM has not accepted the DEA composite unit as the target unit for BA. Specially, it is difficult for the government of BA to increase the value of GDP from 17.35 to 543.92 in a short period of time (or even in the foreseeable future). Therefore, the proposed iterative process is needed to search for the most preferred solution along the frontier for Unit 7. The obtained results are detailed as follows:

*6.2.2.1. Iteration 1.* The government of FYR Macedoniaas the DM accepts to decrease the value of GDP from 543.92 to 350 and at the same time to increase the values of the unemployment rate and budget expenditures from 11.73 and 16.95 to 21.5 and 140, respectively. Based on these values, the new reference set is achieved using Eq. (29). Then, the new values of weights $w_r$ and $v_t$ are estimated by (30). In this case, model (26) is solved using these new weights. This optimal solution provides the following new target unit for the inefficient Unit 7 (FYR Macedonia):

(I1, I2) = (649.48, 33.33)
(O1, O2, O3) = (309.96, 103.16, 13.46)

DM still is not satisfied with this target unit. Thus, the process is repeated until a satisfactory solution is obtained.

*6.2.2.2. Iteration 2.* Suppose that the government of FYR Macedonia has the option of making a trade-off in the values of outputs, whereby the values of GDP can be decreased from 309.6 to 250 by simultaneously increasing the values of the unemployment rate and budget expenditures from 13.46 and 103.16 to 26.5 and 180, respectively. Based on these values, the new reference set is achieved using Eq. (29). Then, the new values of weights $w_r$ and $v_t$ are calculated by (30). Now, model (26) runs using these new weights. This optimal solution gives the following new target unit for the inefficient Unit 7 (FYR Macedonia):

(I1, I2) = (700.16, 35.52)
(O1, O2, O3) = (167.80, 110.23, 14.04).

Now, the DM agrees with this target unit and thus the interactive process terminates.

## 7. Conclusions and future research directions

The changing economic conditions have challenged many organizations to search for more effective performance measurement methods. DEA is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous DMUs by evaluating their relative efficiency. Performance measurement in the conventional DEA is based on the assumptions that inputs should be minimized and outputs should be maximized. However, there are circumstances in real-world problems where some output variables should be minimized.

In this paper we provided a new ink between the output-oriented BCC model in the presence of undesirable outputs and the weighted minimax reference point MOLP model. We then showed how a DEA problem can be solved interactively by transforming it into an MOLP formulation. This approach results in the reduction in the total undesirable output and a permissible increase in the total desirable output. The proposed equivalence model provided the basis for applying interactive methods in MOLP to solve DEA problems and further locate the most preferred solution along the efficient frontier for each inefficient DMU. We proposed the satisfying trade-off method to reflect the DMs' preferences in efficiency assessment.

Future research will concentrate on the comparison of results obtained with those that might be obtained with other interactive MOLP methods. It would be interesting to study which method best may fit the data set and the DM's preferences. We shall also point out a limitation of the model proposed in this study which is a thought-provoking topic for futures research. The equivalency relation used here only considers an output-oriented DEA model, which is a radial model and focuses heavily on the output changes. Finding a new equivalency link that can simultaneously consider both changes in the total input consumption and the total output production based on a radial method is a valuable extension of the model proposed in this study.

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