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An extension of the Electre I method for group decision-making under a fuzzy environment

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ABSTRACT

Many real-world decision problems involve conflicting systems of criteria, uncertainty and imprecise information. Some also involve a group of decision makers (DMs) where a reduction of different individual preferences on a given set to a single collective preference is required. Multi-criteria decision analysis (MCDA) is a widely used decision methodology that can improve the quality of group multiple criteria decisions by making the process more explicit, rational and efficient. One family of MCDA models uses what is known as "outranking relations" to rank a set of actions. The Electre method and its derivatives are prominent outranking methods in MCDA. In this study, we propose an alternative fuzzy outranking method by extending the Electre I method to take into account the uncertain, imprecise and linguistic assessments provided by a group of DMs. The contribution of this paper is fivefold: (1) we address the gap in the Electre literature for problems involving conflicting systems of criteria, uncertainty and imprecise information; (2) we extend the Electre I method to take into account the uncertain, imprecise and linguistic assessments; (3) we define outranking relations by pairwise comparisons and use decision graphs to determine which action is preferable, incomparable or indifferent in the fuzzy environment; (4) we show that contrary to the TOPSIS rankings, the Electre approach reveals more useful information including the incomparability among the actions; and (5) we provide a numerical example to elucidate the details of the proposed method.

1. Introduction

Multi-criteria decision analysis (MCDA) is a general term for methods providing a systematic quantitative approach to support decision making in problems involving multiple criteria and actions [1]. The aim is to help the decision maker (DM) take all important objective and subjective criteria of the problem into consideration using a more explicit, rational and efficient decision process [2,3]. Each of these criteria is used to evaluate any potential action on an appropriate quantitative or qualitative scale [4]. The principle ingredients of MCDA are very simple: at least one DM, two criteria and two actions [5]. The classical MCDA methods can be grouped into three major categories [6–10]:

(i) Outranking is a MCDA approach in which actions are systematically compared to one another on each criterion. The comparisons between the actions lead to numerical results that show the concordance and/or the discordance between the actions. Outranking methods usually involve two steps [11,12]. First, the actions are compared pairwise in order to build an outranking relation. In the second step, this outranking relation is exploited in order to propose a recommendation to the DM. The most widely used method in this group is Electre [12]. Other similar methods have been proposed by Belacel [13], Doumpos and Zopounidis [14] and Perny [15]. Recently, some metaheuristics have been proposed for outranking methods. Belacel et al. [16] used the reduced variable neighborhood search metaheuristic to deduce the parameters of a fuzzy multi-criteria classification method, called PROAFTN, from a set of reference examples. Goletsis et al. [17] used a genetic algorithm for the development of an outranking model in a two-group problem involving ischemic beat classification. The outranking methods as a special subgroup of MCDA methods are particularly suitable for integral decision making through the notion of weak preference and incomparability, which better represent the real decision situation [18,19]. Vincke [20] provides an excellent review of the best known outranking methods; see also Figueira et al. [7] for state-of-art surveys.

(ii) Multi-attribute value theory (MAVT) [21] is a MCDA approach in which the problem is constructed into a hierarchical structure of objectives with the overall goal on the top and the criteria on the lowest level. The actions are measured with

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respect to each criterion, and the criteria are weighted according to their importance. As a result, an overall value for each action is attained. Such methods have been proposed, among others, by Bugera et al. [22], Dembczynski et al. [23], Geldermann et al. [24], Köksalan and Ulu [25], and Zopounidis and Doumpos [26]. See Belton and Stewart [27] and Von Winterfeldt and Edwards [28] for a comprehensive review of the MAVT methods.

(iii) Multiple objective mathematical programming is a MCDA approach in which solution methods based on aspiration points can generate non-dominated solutions using a variety of scalarizing functions [29–31]. These reference point solution methods commonly use a scalarizing function that reaches down from the ideal solution, in a direction specified by the aspiration point. On the contrary, a similar scalarizing function can push out from the nadir point toward a specified aspiration point. These scalarizing functions are similar in structure but diametrically opposed in their reference points [32].

The outranking methods compare all couples of actions and determine which actions are being preferred to the others by systematically comparing them on each criterion. The comparisons between the actions lead to numerical results that show the concordance and/or the discordance between the actions. The outranking relation is the most widely used criteria aggregation method in the MCDA context [10]. An outranking relation is a binary relation S defined on the set of actions A indicating the degree of dominance of one action over another (e.g. [33]). All outranking approaches permit incomparability and intransitivity of preferences [11]; however, methods may differ in the way they formalize mathematical approaches [34].

The first outranking method called Electre I was developed by Roy [35]. Since then, several other outranking methods were developed mainly during the 1970s and 1980s (i.e., Electre II [36], Electre III [37], QUALIFLES [38], ORESTE [39,40], Electre IV [41], MELCHIOR [42], PROMETHEE I and II [43], PRAGMA [44], MAPPACC [44], and TACTIC [45]).

An important advantage of the outranking methods (e.g., Electre methods) is their ability to take ordinal scales into account without converting the original scales into abstract ones with an arbitrary imposed range [46] and at the same time maintain the original verbal meaning (see [47] for an example of a methodology considering purely ordinal scales). Such conversions are used in many MCDA models including: AHP [48,49], MACBETH [50,51], MAUT [21,52], SMART [28,53], TOPSIS [5,54] and methods based on fuzzy integrals [55,56]. A second advantage of the outranking methods is that indifference and preference thresholds can be considered when modeling imperfect knowledge, which is impossible in the previous mentioned methods.

The Electre method and its derivatives such as Electre I, II, III and IV have played a prominent role in the group of outranking methods. The main objective in Electre is the proper utilization of the outranking relations. The outranking methods enable the utilization of incomplete value information and, for example, judgments on ordinal measurement scale (e.g. [33]). The Electre I method is used to construct a partial prioritization and choose a set of promising actions. The Electre II is used for ranking the actions. In Electre III an outranking degree is established, representing an outranking creditability between two actions which makes this method more sophisticated and difficult to interpret. Other variations of the Electre methods include Electre IV, Electre IS and Electre TRI, to mention a few. See Figueira et al. [7] for more details and further members of the Electre family.

The Electre methods have been widely used in civil and environmental engineering [57], optimization of decentralized energy systems [58], electric project selection [59], economic performance assessment [60], energy planning [61], material selection [62], outsourcing contract selection [63] and solid waste management [64], among others.

An important pitfall of the Electre method is the need for precise measurement of the performance ratings and criteria weights [7]. However, in many real-world problems, ratings and weights cannot be measured precisely as some DMs may express their judgments using linguistic terms such as *low*, *medium* and *high* [65–67]. The fuzzy sets theory is ideally suited for handling this ambiguity encountered in solving MCDA problems [68]. Since Zadeh [69] introduced fuzzy set theory, and Bellman and Zadeh [70] described the decision making method in fuzzy environments, an increasing number of studies have dealt with uncertain fuzzy problems by applying fuzzy set theory [71,72]. According to Zadeh [67], it is very difficult for conventional quantification to reasonably express complex situations and it is necessary to use linguistic variables whose values are words or sentences in a natural or artificial language.

The fuzzy outranking methods are developed to deal with the imprecise measurement of the performance ratings and criteria weights [73]. Roy [74] and Siskos et al. [75] effectively used fuzzy outranking relations and introduced the fuzzy concordance and fuzzy discordance relations. In fuzzy outranking methods, fuzzy numbers are compared based on α-cuts [76], possibility and necessity measures [77–79] and the comparison of areas fuzzy numbers [80,81].

Wang [78] proposed a fuzzy outranking approach to select the critical design requirements for product development in an imprecise and uncertain design environment. Güngör and Arikan [81] used an outranking approach to model the imprecise preference structure in a project selection problem. Büyüközkan and Feyzioglu [82] applied the outranking concept into the pseudo-order fuzzy preference model to discriminate the set of actions. There are several other fuzzy outranking approaches in the literature (e.g., [18,83–88], among others). See Roy [74] for further details on the outranking relation, Bouyssou [89] for outranking methods, Fernandez and Leyva [90] for some recent developments in outranking methods, and Ertay and Kahraman [91] for an interesting comparison of different outranking methods.

Complex business problems, characterized by the presence of conflict of values, require the inclusion of some form of decision making process to deal with the multiple and often opposing perspectives. In this context, the Electre methods with participatory approaches have not fully emerged in the MCDA literature as many outranking methods and applications assume a single DM for simplicity [92]. Due to the unavoidable existence of multiple and often conflicting interest and values in decision making, this reductionist approach is insufficient to tackle many contemporary real-world problems.

In this study, we extend the Electre I method to take into account the uncertain, imprecise and linguistic assessments provided by a group of DMs. The proposed method is designed for choice problematic rather than for ranking of the actions. The hybrid fuzzy Electre I approach considers the fuzziness in the decision data and group decision-making process. Linguistic variables are used to assess the weights of all criteria and the performance ratings of each action with respect to each criterion. The proposed method allows a group of DMs to make their opinion independently with linguistic terms and use the fuzzy decision matrix and criteria weights to aggregate their opinions. We determine the concordance matrix and the discordance matrix for each action. Consequently, two different fuzzy assessments for each action are obtained and aggregated. Based on the aggregate matrix, we depict a decision graph to determine which action is preferable, incomparable or indifferent. The paper is organized as follows: In Section 2, we review some of the basic definitions of the fuzzy sets and Electre I method. In Section 3, we introduce our method and algorithms. In Section 4, we present a numerical example to elucidate the details of the proposed method and in Section 5 we interpret our results. In Section 6 we present our conclusions and future research directions.

2. Preliminary definitions

In this section, some basic definitions of fuzzy sets and the Electre I method are reviewed [7,71,93,94].

Definition 1. If U is a collection of objects denoted generically by x, then, a fuzzy set $\tilde{A}$ in U is a set of ordered pairs $\tilde{A} = \{(x,\mu_{\tilde{A}}(x)) | x \in U\}$. $\mu_{\tilde{A}}(x)$ is called the membership function or the grade of membership which associates a real number $[0,1]$ with each element x in U.

Definition 2. A fuzzy subset $\tilde{A}$ of the universe set U is convex if $\mu_{\tilde{A}}(\lambda x_1+(1-\lambda)x_2) \geq (\mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2))$, $\forall\, x_1, x_2 \in U$, $\lambda \in [0,1]$, where $\wedge$ denotes the minimum operator.

Definition 3. For any fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the Hamming distance $(\tilde{A},\tilde{B})$ is defined by the formula [95]:

$$d(\tilde{A},\tilde{B}) = \int_R |\mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)|\, dx \tag{2}$$

where R is the set of real numbers.

Definition 4. Preference in Electre I method is modeled by using binary outranking relations, S, whose meaning is "at least as good as." Considering two actions x and y, four situations may arise:

(i) $x\ S\ y$ and not ySx, i.e., $x\ P\ y$ (x is strictly preferred to y),
(ii) $y\ S\ x$ and not xSy, i.e., $y\ P\ x$ (y is strictly preferred to x),
(iii) $xS\ y$ and ySx, i.e., $x\ I\ y$ (x is indifferent to y), and
(iv) Not xSy and not $y\ S\ x$ (x is incomparable to y).

Note that the incomparability preference is a useful relation to account for situations in which DMs are not able to compare two actions.

Definition 5. According to Electre I method, given two actions x and y, an outranking relation is based on two major concepts: the concordance and the discordance. The following statements provide insights into these concepts:

- The concordance concept: For an outranking xSy to be validated, a *sufficient* majority of the criteria should be in favor of this assertion.
- The discordance concept: When the concordance condition holds, none of the criteria in the minority should oppose too strongly to the assertion $x\ S\ y$.

These two circumstances must be implemented for validating the assertion $x\ S\ y$.

3. Fuzzy Electre I method

It is often difficult for a DM to assign precise weights or precise performance ratings in MCDA. The merit of using a fuzzy approach is its contemplation of ambiguity and the imprecision in the decision making process. Therefore, the DM is not required to assign specific weights to the criteria or specific performance ratings to the actions under consideration.

In this section we propose a hybrid and systematic approach for using the Electre I method under a fuzzy environment. Most decision-making problems involve multiple DMs that could be described by means of the following sets:

- A set of K DMs called$E=\{e_1,e_2,\ldots,e_K\}$;
- A set of m actions called$A=\{A_1,A_2,\ldots,A_m\}$;
- A set of n criteria called$C=\{C_1,C_2,\ldots,C_n\}$;
- A set of performance ratings of A_i ($i=1, 2,\ldots, m$) on criteria C_j ($j=1, 2,\ldots,n$) called $X=\{x_{ij} | i=1, 2,\ldots,m,\ j=1, 2,\ldots,n\}$.

Suppose that K DMs participate in the decision-making process, and the fuzzy performance ratings and weights of each DM $e_k(k=1, 2,\ldots,K)$ can be represented with linguistic terms. The linguistic terms used for performance ratings are Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G) and Very Good (VG). The linguistic terms used for criteria weights are Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H) and Very High (VH).

In the proposed method, trapezoidal fuzzy numbers are used to capture and convert the individual DM's fuzzy information and subjective judgments as a group judgment. Hence, fuzzy numbers are generated for aggregating individual performance ratings of an action with respect to a qualitative criterion into a group performance rating for the action. Among the various types of fuzzy numbers, trapezoidal fuzzy numbers are used most often for characterizing linguistic information in practical applications [94,96]. The common use of trapezoidal fuzzy numbers is mainly attributed to their simplicity in both concept and computation.

In the sequel, we assume that the fuzzy performance ratings of all the DMs be trapezoidal fuzzy numbers $\tilde{R}_k=(r_k^l,r_k^p,r_k^q,r_k^u)$, $k=1,2,\ldots,K$. Therefore, the aggregated fuzzy performance ratings can be formulated as follows:

$$\tilde{R}=(r^l,r^p,r^q,r^u) \tag{3}$$

where

$$r^l=\min_k\left\{r_k^l\right\},\quad r^p=\frac{1}{K}\sum_{k=1}^{K} r_k^p,\quad r^q=\frac{1}{K}\sum_{k=1}^{K} r_k^q \ \text{ and } \ r^u=\max_k\{r_k^u\} \tag{4}$$

The lower bound (r^l), the most possible values ($r^p$and r^q) and the upper bound (r^u) of the fuzzy group performance rating of the action on the criterion are given by the smallest value, the mean values and the largest value of the individual performance ratings, respectively. As a measure of central tendency, the mean values of all k DMs properly represent the most possible values of a trapezoidal fuzzy number. In addition to its usefulness in aggregating the opinions of the DMs, the mean values are also a significant way of dealing with cases where a group agreement cannot be obtained and the group is not acquiescent to compromise on a judgment [97].

Let the fuzzy performance ratings and importance weights of the kth DM be $\tilde{x}_{ijk}=(x_{ijk}^l,x_{ijk}^p,x_{ijk}^q,x_{ijk}^u)$ and $\tilde{W}_{jk}=(w_{jk}^l,w_{jk}^p,w_{jk}^q,w_{jk}^u)$, respectively ($i=1,2,\ldots,m$, $j=1,2,\ldots,n$ and $k=1,2,\ldots,K$). Hence, the aggregation of the fuzzy ratings $\tilde{x}_{ij}$ on each criterion can be calculated as

$$\tilde{x}_{ij}=(x_{ij}^l,x_{ij}^p,x_{ij}^q,x_{ij}^u),\quad i=1,2,\ldots,m,\quad j=1,2,\ldots,n \tag{5}$$

where

$$x_{ij}^l=\min_k\left\{x_{ijk}^l\right\},\quad x_{ij}^p=\frac{1}{K}\sum_{k=1}^{K} x_{ijk}^p,\quad x_{ij}^q=\frac{1}{K}\sum_{k=1}^{K} x_{ijk}^q \text{ and}$$
$$x_{ij}^u=\max_k\{x_{ijk}^u\},\quad i=1,2,\ldots,m,\quad j=1,2,\ldots,n \tag{6}$$

In addition, the aggregated fuzzy weights $\tilde{W}_j$ ($j=1,2,\ldots,n$) for each criterion can be calculated as

$$\tilde{W}_j=(w_j^l,w_j^p,w_j^q,w_j^u),\quad j=1,2,\ldots,n \tag{7}$$

where

$$w_j^l = \min_k\left\{w_{jk}^l\right\}, \quad w_j^p = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^p, \quad w_j^q = \frac{1}{K}\sum_{k=1}^{K} w_{jk}^q \text{ and}$$
$$w_j^u = \min_k\{w_{jk}^u\}, \quad j = 1,2,\ldots,n. \tag{8}$$

Ultimately, the fuzzy performance ratings and weights can be aggregated as

$$\tilde{U} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \ldots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \ldots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \ldots & \tilde{x}_{mn} \end{bmatrix}, \quad \tilde{W} = [\tilde{W}_1, \tilde{W}_2, \ldots, \tilde{W}_n] \tag{9}$$

To avoid complex operations in the decision process, a linear transformation scale is used to convert the different criteria scales into comparable scales. The normalized fuzzy decision matrix can be represented as

$$\tilde{R} = [\tilde{r}_{ij}]_{m\times n} \tag{10}$$

where

$$\tilde{r}_{ij} = (r_{ij}^l, r_{ij}^p, r_{ij}^q, r_{ij}^u) = \left(\frac{x_{ij}^l}{d_j^*}, \frac{x_{ij}^p}{d_j^*}, \frac{x_{ij}^q}{d_j^*}, \frac{x_{ij}^u}{d_j^*}\right), \quad i = 1,2,\ldots,m; \ j \in \Omega_B$$
$$\tilde{r}_{ij} = (r_{ij}^l, r_{ij}^p, r_{ij}^q, r_{ij}^u) = \left(\frac{a_j^-}{x_{ij}^u}, \frac{a_j^-}{x_{ij}^q}, \frac{a_j^-}{x_{ij}^p}, \frac{a_j^-}{x_{ij}^l}\right), \quad i = 1,2,\ldots,m; \ j \in \Omega_C \tag{11}$$

where

$$d_j^* = \max_i\{x_{ij}^u\}, \quad j \in \Omega_B$$

$$a_j^- = \min_i\{x_{ij}^l\}, \quad j \in \Omega_C$$

and Ω_B and Ω_C are the benefit and cost criteria index sets, respectively. Then, the weighted normalized fuzzy decision matrix is constructed as

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \ldots & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \ldots & \tilde{v}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \ldots & \tilde{v}_{mn} \end{bmatrix} \tag{12}$$

where $\tilde{v}_{ij} = (v_{ij}^l, v_{ij}^p, v_{ij}^q, v_{ij}^u) = (w_j^l x_{ij}^l, w_j^p x_{ij}^p, w_j^q x_{ij}^q, w_j^u x_{ij}^u), \quad i = 1,2,\ldots,m, \ j = 1,2,\ldots,n.$

Next, the concordance and discordance matrices are calculated using the weighted normalized fuzzy decision matrix($\tilde{V}$) and the pairwise comparison among the actions. Considering two actions A_g and A_f, the concordance set can be defined as $J_C = \{j \mid \tilde{v}_{gj} \geq \tilde{v}_{fj}\}$ where J_C is the index of all criteria belonging to the concordance coalition with the outranking relation $A_g S\, A_f$.

We use the Hamming distance method [95] for comparing any two actions g and f on each criterion. We first determine their least upper bound, $\max(\tilde{v}_{gj}, \tilde{v}_{fj})$, in the lattice. Then, we calculate the Hamming distances $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$ and $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj})$. Therefore, $\tilde{v}_{gj} \geq \tilde{v}_{fj}$ *if and only if* $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}) \geq d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$.

The discordance set can be defined as $J_D = \{j \mid \tilde{v}_{gj} \leq \tilde{v}_{fj}\}$ where J_D is the index of all criteria belonging to the discordance coalition and it is against the assertion "A_g is at least as good as A_f." Similarly, for comparing each criterion of action g and f, the Hamming distance method [95] is used which assumes that $\tilde{v}_{gj} \leq \tilde{v}_{fj}$ *if and only if* $d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj}) \leq d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{gj})$.

The concordance and discordance matrices are determined based on the obtained Hamming distances in the previous step. The concordance matrix for each pairwise comparison of the actions is defined as

$$\tilde{C} = \begin{bmatrix} - & \ldots & \tilde{c}_{1f} & \ldots & \tilde{c}_{1(m-1)} & \tilde{c}_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ \tilde{c}_{g1} & \ldots & \tilde{c}_{gf} & \ldots & \tilde{c}_{g(m-1)} & \tilde{c}_{gm} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ \tilde{c}_{m1} & \ldots & \tilde{c}_{mf} & \ldots & \tilde{c}_{m(m-1)} & - \end{bmatrix} \tag{13}$$

where

$$\tilde{c}_{gf} = (c_{gf}^l, c_{gf}^p, c_{gf}^q, c_{gf}^u) = \sum\nolimits_{j\in J_C} \tilde{W}_j = \left(\sum_{j\in J_C} w_j^l, \sum_{j\in J_C} w_j^p, \sum_{j\in J_C} w_j^q, \sum_{j\in J_C} w_j^u\right) \tag{14}$$

In other words, the elements of concordance matrix are determined as the fuzzy summation of the fuzzy weights of all criteria in the concordance set. The discordance matrix is defined as

$$D = \begin{bmatrix} - & \ldots & d_{1f} & \ldots & d_{1(m-1)} & d_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ d_{g1} & \ldots & d_{gf} & \ldots & d_{g(m-1)} & d_{gm} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ d_{m1} & \ldots & d_{mf} & \ldots & d_{m(m-1)} & - \end{bmatrix} \tag{15}$$

where

$$d_{gf} = \frac{\max_{j\in J_D} |\tilde{v}_{gj} - \tilde{v}_{fj}|}{\max_j |\tilde{v}_{gj} - \tilde{v}_{fj}|} = \frac{\max_{j\in J_D} |d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj})|}{\max_j |d(\max(\tilde{v}_{gj}, \tilde{v}_{fj}), \tilde{v}_{fj})|} \tag{16}$$

Note that there are prominent differences between the elements of $\tilde{C}$ and D. The concordance matrix $\tilde{C}$ reflects weights of the concordance criteria and the asymmetric discordance matrix D reflects most relative differences according to the discordance criteria. Both concordance and discordance indices have to be calculated for every pair of actions (g, f), where $g \neq f$.

Now, we evaluate the value of the concordance matrix elements according to the concordance level. The concordance level, $\tilde{\bar{C}} = (c^l, c^p, c^q, c^u)$, can be defined as the average of the elements in the concordance matrix, represented by $c^l = \sum_{f=1}^{m}\sum_{g=1}^{m} c_{gf}^l / m(m-1)$, $c^p = \sum_{f=1}^{m}\sum_{g=1}^{m} c_{gf}^p / m(m-1)$, $c^q = \sum_{f=1}^{m}\sum_{g=1}^{m} c_{gf}^q / m(m-1)$ and $c^u = \sum_{f=1}^{m}\sum_{g=1}^{m} c_{gf}^u / m(m-1)$. It is most desirable that the DMs achieve a consensus on the definition of the concordance level. If there is a disagreement among the DMs, then, the average value should be used for the definition.

Next, the Boolean matrix B is formed according to the minimum concordance level, $\tilde{\bar{C}}$, as

$$B = \begin{bmatrix} - & \ldots & b_{1f} & \ldots & b_{1(m-1)} & b_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ b_{g1} & \ldots & b_{gf} & \ldots & b_{g(m-1)} & b_{gm} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ b_{m1} & \ldots & b_{mf} & \ldots & b_{m(m-1)} & - \end{bmatrix} \tag{17}$$

where

$$\begin{cases} \tilde{c}_{gf} \geq \tilde{\bar{C}} \Leftrightarrow b_{gf} = 1 \\ \tilde{c}_{gf} < \tilde{\bar{C}} \Leftrightarrow b_{gf} = 0 \end{cases} \tag{18}$$

Similar to Electre I, (18) is used in fuzzy Electre I for achieving a Boolean matrix. Since their parameters are fuzzy sets, in the first relation of (18) $\tilde{c}_{gf}$ is "approximately greater than or equal to $\tilde{\bar{C}}$" and in the second relation $\tilde{c}_{gf}$ is "approximately less than $\tilde{\bar{C}}$." Here, the Hamming distance method [95] is used for comparing $\tilde{c}_{gf}$ and $\tilde{\bar{C}}$. In the matrix B, if $b_{gf}=1$, we say that action g dominates action f.

Similarly, the elements of the discordance matrix are measured by a discordance level. The discordance level, $\overline{D}=\sum_{f=1}^{m}\sum_{g=1}^{m} d_{gf}/m(m-1)$, can be defined as the average of the elements in discordance matrix. The Boolean matrix H is measured by a minimum discordance level as

$$\mathrm{H}=\begin{bmatrix} - & \dots & h_{1f} & \dots & h_{1(m-1)} & h_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ h_{g1} & \dots & h_{gf} & \dots & h_{g(m-1)} & h_{gm} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ h_{m1} & \dots & h_{mf} & \dots & h_{m(m-1)} & - \end{bmatrix} \tag{19}$$

where

$$\begin{cases} d_{gf} < \overline{D} \Leftrightarrow h_{gf}=1 \\ d_{gf} \geq \overline{D} \Leftrightarrow h_{gf}=0 \end{cases} \tag{20}$$

The elements of this matrix measures the power of the discordant coalition, meaning that if its element value surpasses a given level, $\overline{D}$, the assertion is no longer valid. Discordant coalition exerts no power whenever $d_{gf} < \overline{D}$. In other words, the elements of matrix H with the value of 1 show the dominance relations among the actions.

Next, the global matrix Z is calculated by peer to peer multiplication of the elements of the matrices B and H as follows:

$$Z = B \otimes \mathrm{H} \tag{21}$$

where each element (z_{gf}) of matrix Z is obtained as

$$z_{gf} = b_{gf}\ h_{gf}$$

The final step of this procedure consists of exploitation of the above outranking relation (matrix Z) in order to identify as small as possible a subset of actions, from which the best compromise action could be selected. Consequently, it is extremely useful to build a simple graph $G=(V,J)$, where V is the set of vertices and J the set of arcs. For each action, we associate a vertex and for each pair of actions A_g and A_f, an arc exists between them if either A_g is preferred to A_f or A_g is indifferent to A_f. An action A_g outranks A_f if an arc exists between A_g and A_f and the arrow points from A_g to A_f (for this case, $z_{gf}=1$). A_g and A_f are incomparable if no arc exists between A_g and A_f (for this case, $z_{gf}=0$). A_g and A_f are indifferent if an arc exists between A_g and A_f and an arrow exists in both directions (for this case, $z_{gf}=1$ and $z_{fg}=1$). A graphical representation of the binary relations ($>$, $>^{-1}$, $\approx$, ?) is presented in Fig. 1.

In summary, the fuzzy Electre I method proposed here can be described in 14 steps depicted in Fig. 2:

Step 1: Form a group of DMs and determine the evaluation criteria.
Step 2: Determine the performance ratings ($\tilde{x}_{ijk}$, $i=1,2,\dots,m$, $j=1,2,\dots,n$, $k=1,2,\dots,K$) for actions with respect to criteria by the kth DM using predetermined linguistic variables.
Step 3: Choose criteria importance ($\tilde{W}_{jk}, j=1,2,\dots,n, k=1,2,\dots,K$) by the kth DM using predetermined linguistic variables.
Step 4: Convert linguistic evaluations into trapezoidal fuzzy numbers.
Step 5: Aggregate the performance ratings of the DMs ($\tilde{x}_{ij}$) and the criteria importance of the DMs ($\tilde{W}_j$) using Eqs. (5) and (7), respectively.
Step 6: Construct the fuzzy decision matrix ($\tilde{U}$).
Step 7: Construct the normalized fuzzy decision matrix ($\tilde{R}$).

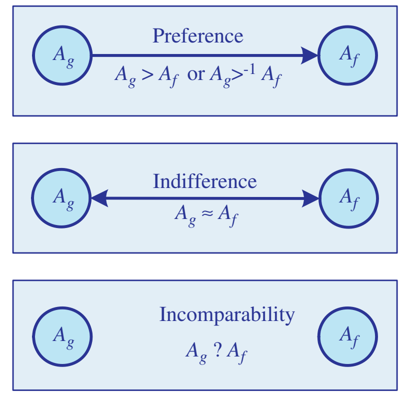

Fig. 1. A graphical representation of the binary relations ($>$, $>^{-1}$, $\approx$, ?) used in the decision graph.

Step 8: Construct the weighted normalized fuzzy decision matrix ($\tilde{V}$).
Step 9: Identify the distance between actions A_g and A_f related to criterion j using the Hamming distance method.
Step 10: Construct the concordance matrix ($\tilde{C}$).
Step 11: Construct the discordance matrix (D).
Step 12: Identify the Boolean matrices B and H based on the minimum concordance level ($\tilde{\bar{C}}$) and the minimum discordance level ($\overline{D}$), respectively.
Step 13: Construct the global matrix Z.
Step 14: Depict a decision graph.

4. Numerical example

In this section, we consider the numerical example used by Chen et al. [98] to demonstrate the details of the proposed fuzzy Electre I method. In Chen et al.'s [98] example, a high-technology manufacturing company desires to select a suitable material supplier among five candidates, A_1, A_2, A_3, A_4 and A_5, who are evaluated by a committee of three DMs against five benefit criteria, namely, profitability of supplier (C_1), relationship closeness (C_2), technology capability (C_3), conformance quality (C_4) and conflict resolution (C_5). Supplier selection in industry is a cross-functional, group MCDA problem, frequently solved by a non-programmed decision making process [99]. The hierarchical structure of this decision-making problem is shown in Fig. 3.

The importance weights of the five criteria are described using the following linguistic terms: very low, low, medium low, medium, medium high, high and very high; which are shown in Fig. 4 and defined in Table 1.

The performance ratings (i.e. criteria values) are characterized by the following linguistic terms: very poor, poor, medium poor, fair, medium good, good and very good; which are shown in Fig. 5 and defined in Table 2.

The three DMs express their opinions on the importance weights of the five criteria and the ratings of each candidate with respect to the five criteria independently. The proposed method is applied to solve this problem and the computational procedure is summarized as follows:

***Steps* 1–3:** The three DMs use the linguistic variables shown in Figs. 4 and 5 to determine the importance weights of criteria and rating of actions under various criteria. These importance weights and ratings are shown in Tables 3 and 4, respectively.

***Steps* 4–6:** The linguistic evaluations shown in Tables 3 and 4 are transformed into trapezoidal fuzzy numbers. Then, the criteria weights and the DMs' ratings are used to get the aggregated fuzzy

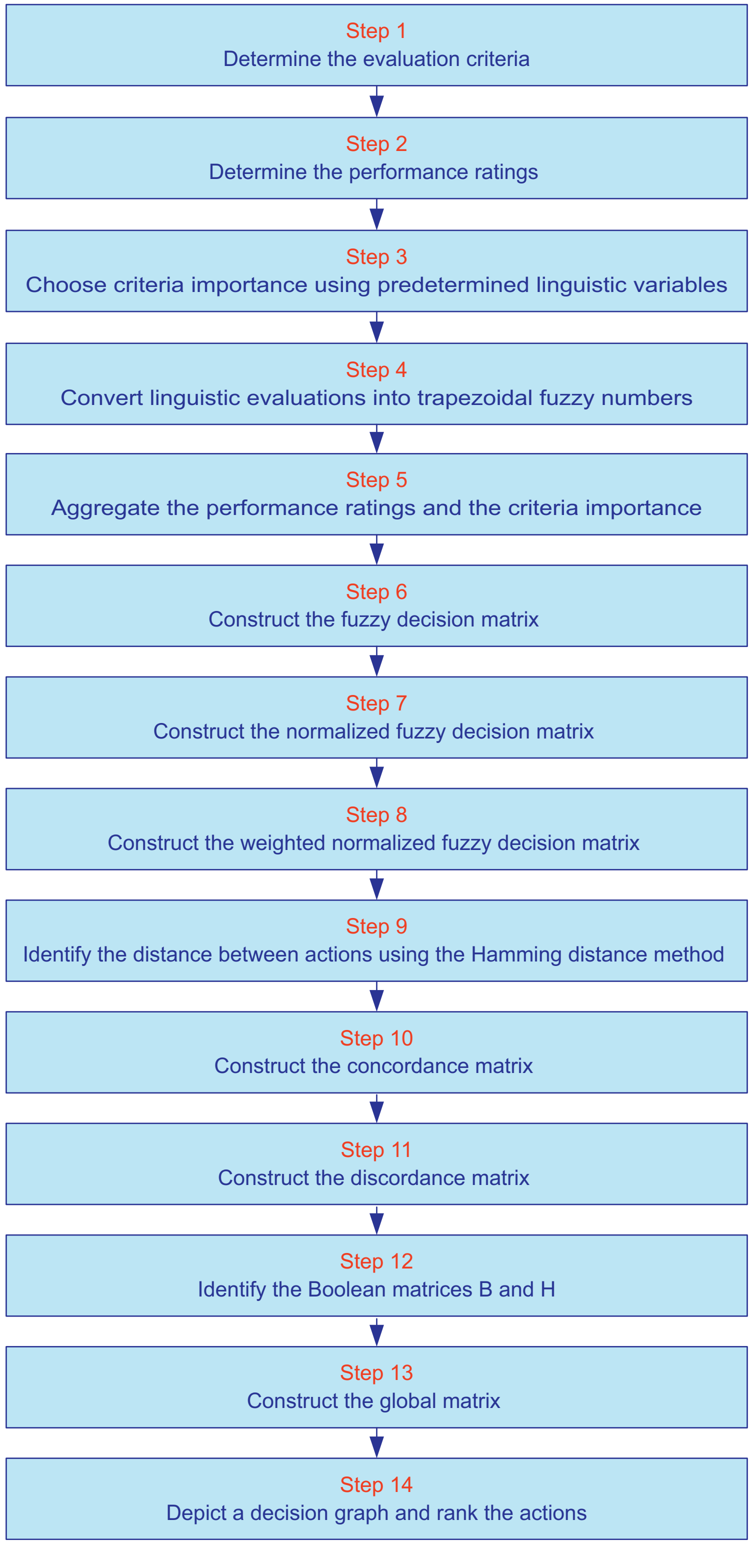

Fig. 2. The fuzzy Electre I procedure.

weight of criteria and fuzzy ratings of supplier with respect to each criterion. Consequently, we can construct the fuzzy decision matrix and determine the fuzzy weight of each criterion, as shown in Table 5.

Step 7: The fuzzy decision matrix developed in Step 6 may include elements with different scales or units. Therefore, it is necessary to normalize all the elements in the fuzzy decision matrix using Eq. (11). The normalized fuzzy decision matrix is presented in Table 6.

***Step* 8:** Next, we obtain the weighted normalized fuzzy decision matrix presented in Table 7.

***Step* 9:** In this step, we use the weighted normalized fuzzy decision matrix to construct Table 8. This table shows the distance between two actions g and f with respect to each criterion

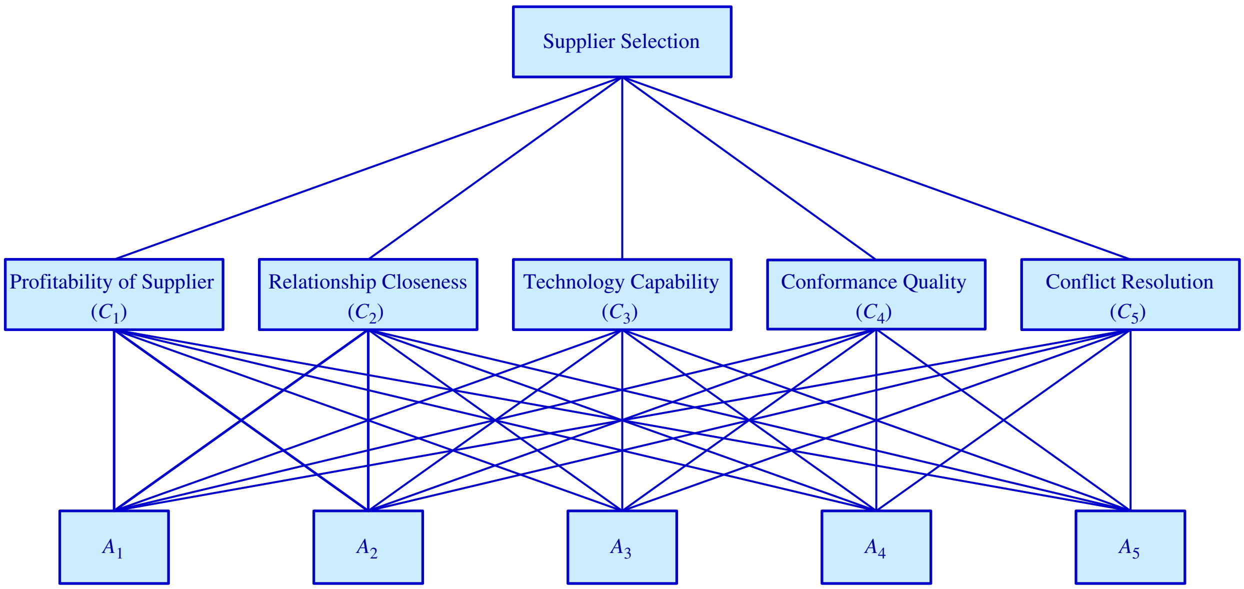

Fig. 3. The supplier selection hierarchical structure.

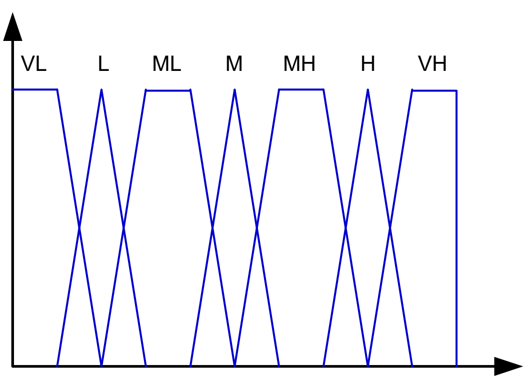

Fig. 4. The seven levels of the linguistic scale used for the importance weights.

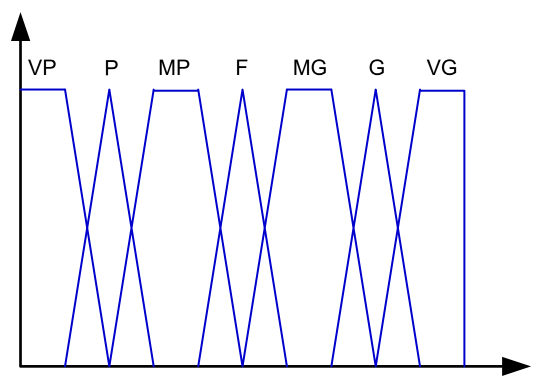

Fig. 5. The seven levels of the linguistic scale used for the ratings.

Table 1
The linguistic variables for the importance weights of the five criteria.

Linguistic variable	Fuzzy number
Very low (VL)	(0, 0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.2, 0.3)
Medium low (ML)	(0.2, 0.3, 0.4, 0.5)
Medium (M)	(0.4, 0.5, 0.5, 0.6)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1, 1)

calculated using the Hamming distance method. Note that in Table 8, the first number and the second in each cell represent $d(\max(\tilde{v}_{gj},\tilde{v}_{fj}),\tilde{v}_{gj})$ and $d(\max(\tilde{v}_{gj},\tilde{v}_{fj}),\tilde{v}_{fj})$, respectively.

Steps* 10 and 11*: Next, we use Eq. (13) to obtain the concordance matrix. This matrix exhibits the preference value between two given actions regarding each criterion. For example, consider two actions A_1 and A_4. We can assert that action A_1 outranks action A_4 (that is, A_1 is at least as good as A_4) denoted by A_1SA_4. According to Table 9, three criteria,C_1, C_4 and C_5; belong to the concordant coalition with the outranking relation A_1SA_4 and its value of (2.1, 2.4, 2.4, 2.7). Similarly, we obtain the discordance matrix using Eq. (14). The concordance and discordance matrices are presented in Tables 9 and 10, respectively.

Table 2
The linguistic variables for the performance ratings.

Linguistic variable	Fuzzy number
Very poor (VP)	(0, 0, 1, 2)
Poor (P)	(1, 2, 2, 3)
Medium poor (MP)	(2, 3, 4, 5)
Fair (F)	(4, 5, 5, 6)
Medium good (MG)	(5, 6, 7, 8)
Good (G)	(7, 8, 8, 9)
Very good (VG)	(8, 9, 10, 10)

Table 3
The importance weights of the five criteria by three DMs.

Criteria	DMs		
	e_1	e_2	e_3
C_1	H	H	H
C_2	VH	VH	VH
C_3	VH	VH	H
C_4	H	H	H
C_5	H	H	H

***Step* 12:** Tables 11 and 12 present the Boolean matrices B and H based on the minimum concordance level and minimum discordance level, respectively. Note that the minimum concordance and discordance levels are (2.12, 2.45, 2.54, 2.76) and 0.56, respectively.

***Step* 13:** Next, we construct the global matrix Z presented in Table 13 by multiplying matrices B and G in order to disregard the effects of the Boolean matrices B and H separately. This signifies that matrix Z is the aggregation matrix that includes all the necessary data for constructing the decision graph from matrices B and H.

***Step* 14:** Finally, we construct the decision graph presented in Fig. 6. This decision graph, derived from a great deal of imprecise data, shows which action is preferable, incomparable or indifferent.

In Fig. 6 we see that there are a total of eight relationships between A_1, A_2, A_3, A_4 and A_5. A_1 is preferred to A_5; A_2 is preferred to A_1, A_4 and A_5; A_3 is preferred to A_1, A_4 and A_5; and A_4 is preferred to A_5. There actually are only six basic relationships provided by this graph since two of the relationships can be derived by using the concept of transitivity. The fact that the relationship A_2 is preferred to A_5 can be inferred from the following two relationships: A_2 is preferred to A_1 and A_1 is preferred to A_5. Moreover, the fact that the relationship A_3 is preferred to A_5 can be inferred from the following two relationships: A_3 is preferred to A_4 and A_4 is preferred to A_5.

5. Discussion

The purpose of this paper is to propose an alternative outranking method by extending the Electre I method for group decision-making under a fuzzy environment. We considered the numerical example used by Chen et al. [98] to demonstrate the details of the proposed fuzzy Electre I method in the previous section. This step by step demonstration of the proposed method was not intended to compare our results with those of Chen et al. [98]. Such a comparison may be pointless as different multi-criteria decision making methods may yield inconsistent results when applied to the same problem. Zanakis et al. [100] explain that the inconsistency in results occurs because (1) the techniques use weights differently; (2) algorithms differ in their approach to

Table 4
The rating of the five actions with respect to the five criteria by three DMs.

Criteria	Suppliers	DM		
		D_1	D_2	D_3
C_1	A_1	MG	MG	MG
	A_2	G	G	G
	A_3	VG	VG	G
	A_4	G	G	G
	A_5	MG	MG	MG
C_2	A_1	MG	MG	VG
	A_2	VG	VG	VG
	A_3	VG	G	G
	A_4	G	G	MG
	A_5	MG	G	G
C_3	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	VG	VG	G
	A_4	MG	MG	G
	A_5	MG	MG	MG
C_4	A_1	G	G	G
	A_2	G	VG	VG
	A_3	VG	VG	VG
	A_4	G	G	G
	A_5	MG	MG	G
C_5	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	G	VG	G
	A_4	G	G	VG
	A_5	MG	MG	MG

Table 5
The fuzzy decision matrix and the fuzzy weight of each criterion.

	C_1	C_2	C_3	C_4	C_5
A_1	(5, 6, 7, 8)	(5, 7, 8, 10)	(7, 8, 8,9)	(7, 8, 8, 9)	(7, 8, 8, 9)
A_2	(7, 8, 8, 9)	(8, 9, 10, 10)	(8, 9, 10, 10)	(7, 8.7, 9.3, 10)	(8, 9, 10, 10)
A_3	(7, 8.7, 9.3, 10)	(7, 8.3, 8.7, 10)	(7, 8.7, 9.3, 10)	(8, 9, 10, 10)	(7, 8.3, 8.7, 10)
A_4	(7, 8, 8, 9)	(5, 7.3, 7.7, 9)	(5, 6.7, 7.3, 9)	(7, 8, 8, 9)	(7, 8.3, 8.7, 10)
A_5	(5, 6, 7, 8)	(5, 7.3, 7.7, 9)	(5, 6, 7, 8)	(5, 6.7, 7.3, 9)	(5, 6, 7, 8)
Weight	(0.7, 0.8, 0.8,0.9)	(0.8, 0.9, 1,1)	(0.7, 0.87, 0.93,1)	(0.7, 0.8, 0.8,0.9)	(0.7, 0.8, 0.8,0.9)

Table 6
The normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.7, 0.8, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
A_2	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)	(0.8, 0.9, 1, 1)	(0.7, 0.87, 0.93, 1)	(0.8, 0.9, 1, 1)
A_3	(0.7, 0.87, 0.93, 1)	(0.7, 0.83, 0.87, 1)	(0.7, 0.87, 0.93, 1)	(0.8, 0.9, 1, 1)	(0.7, 0.83, 0.87, 1)
A_4	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.73, 0.77, 0.9)	(0.5, 0.67, 0.73, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.87, 1)
A_5	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.73, 0.77, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.67, 0.73, 0.9)	(0.5, 0.6, 0.7, 0.8)

Table 7
The weighted normalized fuzzy decision matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.35,0.48,0.56,0.72)	(0.4,0.63,0.8,1)	(0.49,0.7,0.74,0.9)	(0.49,0.64,0.64,0.81)	(0.49,0.64,0.64,0.81)
A_2	(0.49,0.64,0.64,0.81)	(0.64,0.81,1,1)	(0.56,0.78,0.93,1)	(0.49,0.7,0.74,0.9)	(0.56,0.72,0.8,0.9)
A_3	(0.49,0.7,0.74,0.9)	(0.56,0.75,0.87,1)	(0.49,0.76,0.86,1)	(0.56,0.72,0.8,0.9)	(0.49,0.66,0.7,0.9)
A_4	(0.49,0.64,0.64,0.81)	(0.4,0.66,0.77,0.9)	(0.35,0.58,0.68,0.9)	(0.49,0.64,0.64,0.81)	(0.49,0.66,0.7,0.9)
A_5	(0.35,0.48,0.56,0.72)	(0.4,0.66,0.77,0.9)	(0.35,0.52,0.65,0.8)	(0.35,0.54,0.58,0.81)	(0.35,0.48,0.56,0.72)

Table 8
The distances between two actions g and f with respect to each criterion.

	x_{11}	x_{21}	x_{31}	x_{41}	x_{51}		x_{12}	x_{22}	x_{32}	x_{42}	x_{52}
x_{11}	–	(0.065,0)	(0,0)	(0.065,0)	(0,0)	x_{12}	–	(0.205,0)	(0.105,0)	(0.035,0.065)	(0.035,0.065)
x_{21}	–	–	(0.065,0)	(0,0)	(0,0.065)	x_{22}	–	–	(0,0.1)	(0,0.105)	(0,0.105)
x_{31}	–	–	–	(0,0.065)	(0,0)	x_{32}	–	–	–	(0,0.005)	(0,0.005)
x_{41}	–	–	–	–	(0,0.065)	x_{42}	–	–	–	–	(0,0)
x_{51}	–	–	–	–	–	x_{52}	–	–	–	–	–
	x_{13}	x_{23}	x_{33}	x_{43}	x_{53}		x_{14}	x_{24}	x_{34}	x_{44}	x_{54}
x_{13}	–	(0.07,0)	(0.08,0)	(0,0.1)	(0,0.065)	x_{14}	–	(0.065,0)	(0.05,0)	(0,0)	(0,0.09)
x_{23}	–	–	(0,0.1)	(0,0.03)	(0,0.005)	x_{24}	–	–	(0.015,0)	(0,0.065)	(0,0.025)
x_{33}	–	–	–	(0,0.02)	(0,0.015)	x_{34}	–	–	–	(0,0.05)	(0,0.04)
x_{43}	–	–	–	–	(0,0.039)	x_{44}	–	–	–	–	(0,0.09)
x_{53}	–	–	–	–	–	x_{54}	–	–	–	–	–
	x_{15}	x_{25}	x_{35}	x_{45}	x_{55}						
x_{15}	–	(0.05,0)	(0.065,0)	(0.065,0)	(0,0.065)						
x_{25}	–	–	(0,0.015)	(0,0.015)	(0,0.015)						
x_{35}	–	–	–	(0,0)	(0,0)						
x_{45}	–	–	–	–	(0,0)						
x_{55}	–	–	–	–	–						

Table 9
The concordance matrix.

	A_1	A_2	A_3	A_4	A_5
A_1	–	(0,0,0,0)	(0.7,0.8,0.8,0.9)	(2.2,2.57,2.73,2.9)	(3.6,4.17,4.33,4.7)
A_2	(3.6,4.17,4.33,4.7)	–	(2.2,2.57,2.73,2.9)	(3.6,4.17,4.33,4.7)	(3.6,4.17,4.33,4.7)
A_3	(3.6,4.17,4.33,4.7)	(1.4,1.6,1.6,1.8)	–	(3.6,4.17,4.33,4.7)	(3.6,4.17,4.33,4.7)
A_4	(2.1,2.4,2.4,2.7)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)	–	(3.6,4.17,4.33,4.7)
A_5	(0.7,0.8,0.8,0.9)	(0,0,0,0)	(1.4,1.6,1.6,1.8)	(1.5,1.7,1.8,1.9)	–

Table 10
The discordance matrix.

	A_1	A_2	A_3	A_4	A_5
A_1	–	1	1	0.65	0
A_2	0	–	0.65	0	0
A_3	0	1	–	0	0
A_4	1	1	1	–	0
A_5	1	1	1	1	–

Table 11
Boolean matrix B based on the minimum concordance level.

	A_1	A_2	A_3	A_4	A_5
A_1	–	0	0	1	1
A_2	1	–	1	1	1
A_3	1	0	–	1	1
A_4	0	0	0	–	1
A_5	0	0	0	0	–

Table 12
Boolean matrix H based on the minimum discordance level.

	A_1	A_2	A_3	A_4	A_5
A_1	–	0	0	0	1
A_2	1	–	0	1	1
A_3	1	0	–	1	1
A_4	0	0	0	–	1
A_5	0	0	0	0	–

Table 13
The global matrix Z.

	A_1	A_2	A_3	A_4	A_5
A_1	–	0	0	0	1
A_2	1	–	0	1	1
A_3	1	0	–	1	1
A_4	0	0	0	–	1
A_5	0	0	0	0	–

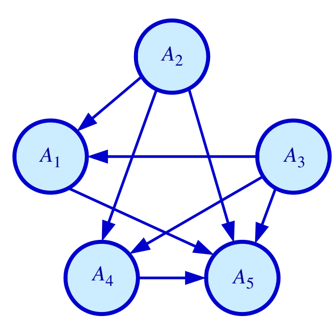

Fig. 6. The decision graph for the numerical example.

selecting the 'best' solution; (3) many algorithms attempt to scale the objective; and (4) some algorithms introduce additional parameters that affect which solution will be chosen.

The Electre I method is often comprised of two steps: (1) the construction of one or several outranking relations and (2) the derivation of a recommendation based on the outranking relations [101]. The TOPSIS method consists of three steps: (1) the construction of the normalized decision matrices; (2) the determination of the closeness coefficient to the ideal solution; and (3) the derivation of a preference rank order based on the separation measures [5]. Generally speaking, the concordance and discordance indices in Electre are analogous to the Euclidean distances of each action to the ideal solution and the nadir ideal solution in TOPSIS. Accordingly, the preferred actions by Electre and TOPSIS are normally in agreement for a given problem, particularly for problems with few criteria ([100], p. 511).

Cheng et al. [102] used Electre and TOPSIS to evaluate the landfill site actions considered in solid waste management and compared the results from the two approaches. They argue that since each method reflects different characteristics and assumptions, using a single method may not give satisfactory results ([102], p. 553). Amiri et al. [103] also applied Electre and TOPSIS to a portfolio selection problem and compared results from the two methods. They also advocated using multiple MCDA methods to solve the same problem. In spite of that, Gershon and Duckstein [104] warn that different MCDA techniques may yield different results when applied to the same problem, apparently under the same assumptions and by a single DM. Other researchers have argued the opposite; namely that, given different types of problems, the solutions obtained by different MCDA methods are essentially the same [105–109].

There is no one optimal method for a given MCDA problem and the numerical comparison is not usually enough to determine which method is the most appropriate. However, it is worthwhile to examine different models from different perspectives. Deng and Wibowo [110] have provided guidelines for choosing between the Electre and TOPSIS methods on the basis of the criteria weights, the performance ratings, the criteria information processing, features, solution aims and the common scale. In Table 14, we expand the guidelines provided by Deng and Wibowo [110] and present a side-by-side comparison of the Electre and TOPSIS methods.

As shown in Table 14:

- The fuzzy Electre I model proposed in this study and the fuzzy TOPSIS model used by Chen et al. [98] both used fuzzy criteria weights and fuzzy performance ratings.
- The criteria information processing is partially compensatory in Electre I and fully compensatory in TOPSIS [5,111]. Unlike the compensatory method of TOPSIS, an advantage of the Electre method is that a significantly weak criterion value of an action cannot directly be compensated for by other good criteria values. On the other hand, the solution mechanism in Electre method is not as extreme as purely non-compensatory methods [112].
- The solution aim is "choice problematic" for Electre I and "ranking problematic" for TOPSIS. In choice problematic, the solution aim is oriented towards the selection of a small set of "good" actions in such a way that a single action may finally be chosen. In contrast, the solution aim in ranking problematic is oriented towards the selection of a complete or partial order of the actions [4].
- The concordance and discordance indices in Electre are analogous to the Euclidean distances of each action to the ideal solution and the nadir ideal solution in TOPSIS ([100], p. 511).
- Electre is an outranking method and TOPSIS is based on an aggregating function representing 'closeness to the ideal', which originated in the compromise programming method.
- Both Electre I and TOPSIS use normalized scales [5,111].
- As the number of actions increases, Electre and TOPSIS tend to produce similar final weights, but dissimilar rankings, and more rank reversals (fewer reversals for Electre). As the number of criteria increases, Electre exhibits more rank reversals compared with TOPSIS [100].

According to Kim et al. [113] and Shih et al. [114], at least three TOPSIS advantages can be identified: (1) a sound logic that simulates the rationale of human choice; (2) a scalar value that accounts for both the best and worst actions simultaneously; and (3) a simple computation process that can be easily programmed into a spreadsheet. In spite of that, Roghanian et al. [115] have reported that the aggregating function of the TOPSIS method does not produce results such that the highest ranked action is simultaneously the closest to the ideal solution and the furthest from the nadir solution since these criteria can be conflicting. This issue is dealt with arbitrarily by the original TOPSIS method through the use of the notion of closeness coefficient which is a measure of the relative distance between a certain action and the ideal and the nadir solutions.

Chen et al. [98] proposed a new fuzzy TOPSIS method. They demonstrated the details of their method in a MCDA problem where a high-technology manufacturing company desires to select a suitable material supplier among five candidates. Their results are presented in Table 15.

The *closeness coefficient* (CC) for the five suppliers are 0.5 for A_1, 0.64 for A_2, 0.62 for A_3, 0.51 for A_4 and 0.4 for A_5 which results in the ranking of $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$, where the symbol '$\succ$' means 'is superior'. Next, Chen et al. [98] divide the [0,1] interval into five sub-intervals (classes) to state the evaluation status of the suppliers as linguistic variables presented in Table 16. According to this classification scheme, if multiple suppliers fall in the same class, their CC values are used to rank them within that class. As shown in Table 15, suppliers A_2 and A_3 are classified in class V and suppliers A_1, A_4 and A_5 are classified in class III based on their CC values and the classification scheme proposed in Table 16. With respect to the CC values, the five suppliers are ranked as follows: $\{A_2 \succ A_3\} \succ \{A_4 \succ A_1 \succ A_5\}$. Furthermore, Chen et al. [98] translate

Table 14
Problem requirements and characteristics of the Electre and TOPSIS methods.

Problem requirements and characteristics	MCDA method	
	Electre	TOPSIS
Criteria weights	Crisp, interval or fuzzy	Crisp, interval or fuzzy
Performance ratings	Crisp, interval or fuzzy	Crisp, interval or fuzzy
Criteria information processing	Partially compensatory	Fully compensatory
Solution aims	Choice problematic	Ranking problematic
Features	Outranking	Ideal solution
Orientation	Outranking	Compromise programming
Common scale	Normalized scale	Normalized scale
Rank reversal (Actions)	More actions—small rank reversal	More actions—large rank reversal
Rank reversal (Criteria)	More criteria—large rank reversal	More criteria—small rank Reversal

Table 15
The results of Chen et al.'s [98] TOPSIS method.

Suppliers	CC_i	Ranking	Class
A_1	0.5	4	III
A_2	0.64	1	IV
A_3	0.62	2	IV
A_4	0.51	3	III
A_5	0.4	5	III

Table 16
The decision rules of Chen et al.'s [98] TOPSIS method.

Class	Closeness coefficient	Assessment status
I	[0, 0.2]	Do not recommend
II	[0.2, 0.4]	Recommend with high risk
III	[0.4, 0.6]	Recommend with low risk
IV	[0.6, 0.8]	Approved
V	[0.8, 1]	Approved and preferred

these rankings into linguistic terms such as "*Do not recommend*" or "*Recommend with high risk*". While linguistic terms are easy to understand, they do not express the outranking relations.

The ranking methods are frequently used in multi-criteria decision making to rank order the potential actions based on the DM's preferences where ties among various actions are also allowed. Although the ranking methods are simple to use, they do not capture the preference intensity of the DMs and the numerical values obtained from the ranking of the actions are non-intuitive to the DM, especially for visual representations. As shown in Table 15, A_2 is ranked ahead of A_3 and only 0.02 points separate their CC values. This could be problematic, especially, in a fuzzy environment where the values are not necessarily precise. Furthermore, the difference between the CCvalues for A_1 and A_4 is only 0.01 points in the fuzzy TOPSIS method proposed by Chen et al. [98]. It does not seem logical to rank A_1 and A_4 differently because of the potential variations in the imprecise data. Contrary to the TOPSIS rankings, suppliers A_1 and A_4 are categorized in class III in the proposed fuzzy Electre I method. Although the fuzzy Electre I does not rank order the actions similar to the fuzzy TOPSIS method proposed by Chen et al. [98], it does provide insightful information about the relationship among the actions especially in cases where no action outranks the others and no decision can be made.

We used the fuzzy Electre I method in the previous section and derived the decision graph presented in Fig. 6. As shown in this figure, suppliers A_2 and A_3 are categorized in the first rank, because three arcs derive from the nodes A_2 and A_3. It means that A_2 and A_3 are preferred to A_1, A_4 and A_5. A_2 and A_3 are also not comparable because there is no arc between them. Furthermore, suppliers A_1 and A_4 are categorized in the second rank. In fact, A_1 and A_4 are preferred to A_5, but they are also incomparable. The last prioritization belongs to supplier A_5, because all actions are dominated on A_5. The results of our fuzzy Electre I method are presented in Table 17.

A comparison between Tables 15 and 17 shows that there is a similarity between our prioritization and the rankings provided by Chen et al. [98]. This similarity was further validated from the pictorial representation of the results provided in Fig. 7. In addition, we applied the Spearman's rank correlation coefficient to measure the correlation between the two results. The Spearman's rank correlation coefficient of 0.80 also supported the similarity of the results.

In the approach proposed by Chen et al. [98], the Euclidean distances of each action to the ideal solution and nadir solution are both crisp values. This leads to a crisp point estimate for the

Table 17
The results of the proposed Electre I method.

Action	Incomparable action	Submissive action	Ranking
A_1	A_4	A_5	3
A_2	A_3	A_1,A_4,A_5	1
A_3	A_2	A_1,A_4,A_5	1
A_4	A_1	A_5	3
A_5	–	–	5

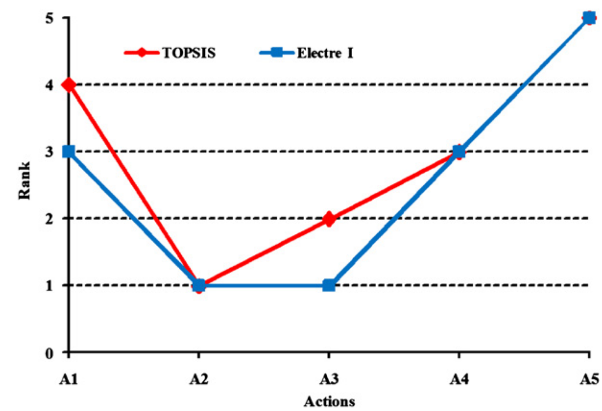

Fig. 7. The pictorial representation of the Electre I and TOPSIS rankings.

closeness coefficient of each action. However, our approach overcomes this shortcoming by using a logical outranking relation. In other words, our approach provides more meaningful and useful information by revealing which supplier is preferable, incomparable or indifferent, whereas their approach only provides a simple ranking of the suppliers.

In summary, the Electre and TOPSIS methods produce similar results [116]. However, the outranking result obtained from the Electre I method reveals more useful information such as the preferability, incomparability or indifference among the actions. This advantage is especially more valuable in problems with a large number of actions [117]. In addition, Electre is the preferred method for problems with a large set of actions and few criteria [118].

6. Conclusions and future research directions

Many real-world decision problems take place in a complex environment and involve conflicting systems of criteria, uncertainty and imprecise information. Numerous methods have been developed to solve multi-criteria problems when available information is precise. However, uncertainty and fuzziness inherent in the structure of information make rigorous mathematical models unsuitable for solving multi-criteria problems with imprecise information [3,67,70,71].

MCDA forms an important part of the decision process for complex problems and the theory of fuzzy set is well-suited to handle the ambiguity and impreciseness inherent in multi-criteria decision problems. Electre is a well-established MCDA method that has a history of successful real-world applications [99]. In this paper, we proposed a methodological and computational enhancement of the Electre I decision aid method for processing fuzzy preferential information as is common in the context of multi-criteria preference modeling. We utilized a decision graph to identify preferable, incomparable or indifferent actions. The proposed algorithm for group decision-making under fuzzy environments is flexible and could be applied easily to other managerial decision-making problems.

The contribution of this paper is fivefold: (1) we addressed the gap in the Electre literature for problems involving conflicting systems of criteria, uncertainty and imprecise information; (2) we extended the Electre I method to take into account the uncertain, imprecise and linguistic assessments; (3) we defined outranking relations by pairwise comparisons and used decision graphs to determine which action is preferable, incomparable or indifferent in the fuzzy environment; (4) we showed that contrary to the TOPSIS rankings, the Electre approach reveals more useful information including the incomparability among the actions; and (5) we provided a numerical example to elucidate the details of the proposed method.

When compared with the other fuzzy outranking methods, some of the features of the alternative fuzzy outranking method proposed in this study are:

1. The proposed method is composed of two phases: the *construction* of a global outranking relation and the *exploitation* of this relation [119]. The construction phase of our method is composed of two steps: the construction of an outranking relation with respect to each criterion and the aggregation of the single outranking relation into a global outranking relation. The exploitation phase of our method processes global preference relations resulting from the previous phase in order to derive the final ranking [120].
2. In many outranking problems, in order to find the difference between two actions, the knowledge of the distance between two fuzzy sets is necessary. There are generally two approaches in the literature: Hamming distance and Euclidean distance [121,122]. We used the Hamming distance because of its simplicity and intuitively appealing properties [123].
3. The fuzzy outranking procedure used here is simple yet structured and logical. Rao ([124], p. 5) stresses the need for simple, systematic and logical outranking methods.
4. The proposed outranking method is generalized and can be applied to great variety of practical problems.

Nevertheless, we cannot claim that our alternative fuzzy outranking method produces a better solution because different MCDM methods involve various types of underlying assumptions, information requirements from a DM, and evaluation principles ([5], p. 213). For a given problem, there are both compatibilities and incompatibilities using each model. In such situations, it is reasonable to examine different models, which normally yield different solutions, before making a final decision [57,100,109,112,125,126]. As to which MCDM method(s) we should use, there are no specific rules. Different MCDM methods are introduced for different decision situations ([5], p. 210). There are many MCDM methods and models, but none can be considered the "best" and/or appropriate for all situations [127]. Some of the motivations for the development of the fuzzy outranking model proposed in this study are:

1. To provide a mathematical framework for modeling vagueness and imprecision in outranking since the subjectivity, imprecision and vagueness in the estimates of the performance ratings are often entered into the outranking problem.
2. In contrast to the valued outranking methods that are well documented and have been intensively used in practice, the fuzzy outranking methods are recent and are not well documented in the literature [119].

The Electre methods with participatory approaches have not fully emerged in the MCDM literature as many outranking methods and applications assume a single DM for simplicity [92]

A stream of future research can extend our algorithms to other variations of the Electre methods such as Electre II, III, IV, Electre IS and Electre TRI. It would also be interesting to develop hybrid approaches for the integrated use of our algorithms, not only hybrids of different outranking methods but also hybrids of MAVT and numerical optimization.

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