Information Sciences 317 (2015) 295–314

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

An ordinal ranking criterion for the subjective evaluation of alternatives and exchange reliability

Madjid Tavana [a,b,*], Debora Di Caprio [c,d], Francisco J. Santos-Arteaga [e]

[a] *Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA*
[b] *Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany*
[c] *Department of Mathematics and Statistics, York University, Toronto M3J 1P3, Canada*
[d] *Polo Tecnologico IISS G. Galilei, Via Cadorna 14, 39100 Bolzano, Italy*
[e] *Departamento de Economía Aplicada II, Facultad de Económicas, Universidad Complutense de Madrid, Campus de Somosaguas, 28223 Pozuelo, Spain*

A R T I C L E I N F O

Article history:
Received 25 December 2014
Received in revised form 13 March 2015
Accepted 2 May 2015
Available online 8 May 2015

Keywords:
Decision analysis
Multi-attribute alternative
Subjective perception
Subjective description
Ordinal ranking
Expected utility

A B S T R A C T

We consider the problem of a decision maker (DM) who must choose among a set of alternatives offered by different information senders (ISs). Each alternative is characterized by finitely many characteristics. We assume that the DM and the ISs have their own perception of the available alternatives. These perceptions are reflected by the evaluations provided for the characteristics of the alternatives and the order of importance assigned to the characteristics. Due to these subjective components, the DM may not envision the exact alternative that an IS describes, even when a complete description of the alternative is provided. These subjective biases are common in the literature analyzing the effect of framing on the behavior of the DMs. This paper provides a normative setting illustrating how the DMs should consider these differences in perception when interacting with other DMs. We design an evaluation criterion that allows the DM to generate a reliability ranking on the set of ISs and, hence, to quantify the likelihood of choosing any alternative. This ranking is based on the existing differences between the preference order of the DM and those of the ISs. Our results constitute a novel approach to choice and search under uncertainty that enhances the findings of the expected utility literature. We provide several examples to demonstrate the applicability of the method proposed and exhibit the efficacy of the ranking criterion designed.

1. Introduction

1.1. Motivation

It is widely known in the social sciences that decision makers (DMs) are highly sensitive to the way and the order in which information is presented to them. This fact is referred to as the framing effect and imposes a subjective bias in the

* Corresponding author at: Business Systems and Analytics Department, Lindback Distinguished Chair of Information Systems and Decision Sciences, La Salle University, Philadelphia, PA 19141, USA. Tel.: +1 215 951 1129; fax: +1 267 295 2854.

E-mail addresses: tavana@lasalle.edu (M. Tavana), dicaper@mathstat.yorku.ca, debora.dicaprio@istruzione.it (D. Di Caprio), fransant@ucm.es (F.J. Santos-Arteaga).

URL: http://tavana.us/ (M. Tavana).

http://dx.doi.org/10.1016/j.ins.2015.05.011
0020-0255/

choices of DMs. The literature on the framing effect encompasses several disciplines, ranging from psychology to economics. Kahneman and Tversky [28] provide an extensive description of the literature in both fields. In particular, as emphasized by Gächter et al. [20], this effect prevails even among those academics who study it, mainly experimental economists.

Thus, the order in which the characteristics of a choice object are described to the DMs is fundamental in determining their behavior. Similarly, the way in which an information sender (IS) describes a given choice object reflects the subjective importance assigned to each one of the characteristics of the object. To our knowledge, a formalization of the normative framework determining the behavior of the DMs when interacting with others while acknowledging this fact has not been provided in the literature.

The aim of this paper is to define an ordinal criterion that allows the DMs to generate a ranking based on the differences existing between their preference order and those of the ISs. Indeed, the main characteristics of a product according to a DM are generally given a more detailed and careful description than those considered as secondary. For example, when writing a paper, we academics highlight those results and implications that *we* consider more relevant and develop and explain them to a greater extent than those *we* consider as secondary. When selecting a research paper or book to read, we concentrate on the main features described by the authors and compare them to those *we* prioritize in our own research. In the same way, preference similarity and common reactions to news, illustrating the common/coordinated interests existing between investors, have been shown to determine their investment behavior [1,26].

In the current paper, the order in which information is displayed by the ISs will directly affect the choices made by the DMs. Operational researchers and computer scientists have already started considering the elicitation of product characteristics and valuations from the text that describes it. For example, [21] illustrate how characteristic and value pairs can be reliably extracted from textual product descriptions.

1.2. State of the art

Several academic disciplines deal with the search, choice and exchange problems of DMs. These problems are analyzed from substantially different perspectives depending on the discipline. We will concentrate on the main findings provided by the consumer choice, psychology, marketing and economic literatures from an applied perspective. A common starting point to all these disciplines is the fact that the DMs must decide whether or not to purchase a product given their evaluation of its characteristics. These evaluations are assumed to be based on the DMs' subjective perception of the product and determine the likelihood of the DMs purchasing the product.

In *consumer choice*, the acquisition of a product and the basis of models such as the technology adoption one rely on the relevant characteristics of the product and their importance considered *from the DM's point of view*. See, among others, Brown et al. [8], Blackwell et al. [7], Davis [11], Davis et al. [12], Dillon and Morris [17], Hess et al. [24], Shackel [41], and Venkatesh et al. [48].

Psychologists studying the behavior of consumers have found that characteristics are ranked based on their order of importance and their influence on the attitude of the DMs towards the product. See, for instance, Jaccard et al. [27] and Kimmel [29]. In particular, Fishbein [18] shows that the attitude towards a product is generally defined as the weighted sum of *the DM's beliefs* regarding its salient characteristics.

Several *marketing* papers [5–7,22,33,39] have illustrated empirically how the attitude of the DMs towards a product can be described as a function of the relative importance of each characteristic and the DMs' belief regarding the performance of the product on each characteristic.

The *economic* literature has given considerable importance to the improvement of characteristics when dealing with the introduction of new products in the market [23]. Furthermore, it introduced the idea of defining products as vectors of characteristics [31]. Nevertheless, it has not pursued this specific line of research when analyzing market exchange environments.

Finally, *information scientists* have also dealt with the behavior of DMs when facing an uncertain environment with choices determined by multiple characteristics. Their contributions are developed at a more formal level and generally follow a fuzzy environment approach to account for the frictions caused by the uncertainty. See, among others, Aliev et al. [3,4], Di Caprio et al. [15] and Tavana et al. [44].

1.3. Contribution

We consider the problem of a DM who must rank several alternatives described by other DMs, who are referred to as information senders (ISs). Each alternative consists of a fixed number of characteristics. We assume that each IS is endowed with a preference order on the characteristics of the alternative he is offering to the DM and that this fact affects their description of the alternative. The preference order of each IS will generally differ from that of the DM and those of the other ISs.

Thus, a DM whose preference order does not exactly coincide with that of the IS, a fact that is directly reflected in the order in which the characteristics are described, should expect a larger friction in the evaluation of the alternative than if the orders were to exactly coincide. In this latter case, the IS will provide a more detailed and careful description of the characteristics that the DM considers more important. Detailed descriptions allow the DM to form a more reliable expectation of the product to be obtained from an exchange.

Following the consumer behavior and marketing approaches to choice under uncertainty [34,38], we will assume that the DM chooses to interact with the IS that he considers most reliable while providing the highest expected utility.

Tavana et al. [44] propose a theoretical model where each DM is endowed with a product. The DMs are aware of the fact that they do not perceive the characteristics of the products in the same way and define their beliefs based on these differences in perception. The DMs' beliefs are used to construct their subjective expected utility derived from each alternative. The authors illustrate how the DMs may agree to exchange products and end up with a less preferred product that the one they initially owned. This is the case even if strategic reporting is excluded from the model. However, Tavana et al. [44] make no attempt to quantify the distance among the perceptions of the DMs. Also, the authors do not consider the problem of ranking several alternatives.

We introduce a novel and easy to implement ranking criterion on which the DMs can base their choice from a set of alternatives described by different ISs. We will base the reliability between the DMs and the ISs on the ordinal properties of the preferences defining the utility functions of the DMs [14]. This will be done in a fully cooperative environment where all valuation frictions are reflected in the order of the characteristics chosen to describe the alternatives being considered. That is the information transmitted by the ISs is assumed to be truthful and reliable in the following sense.

(a) We do not consider strategic reporting between the ISs and the DMs. The capacity of the ISs to misrepresent the information content transmitted to the DMs has been widely studied in the economic literature [2,13,32].
(b) We do not consider linguistic imprecisions in the description of the alternatives by the ISs as is the case in several fuzzy variants of the decision making literature; see, among others, Aliev et al. [3,4].

We introduce two novel indexes to quantify the differences between the order assigned by the DM on the characteristics of an alternative and that assigned by the ISs. Namely,

- A coordination index, in order to measure the shortest distance between the position of the most valued characteristic in the DM's order and that of the same characteristic in the order of each IS.
- A synchronization index, in order to measure the pairwise distances between the order position assigned by the DM to each characteristic and the order positions assigned to the same characteristics by each IS.

Both these indexes will be combined to define a composite index that allows the DMs to weight and rank the expected utility values of the alternatives proposed by the ISs. We will provide several examples to demonstrate the applicability of the method proposed and exhibit the efficacy of the ranking criterion designed.

The main research areas where the results obtained can be directly applied are the following.

(a) The analysis of (bilateral or multilateral) economic exchange settings where an ordered description of a set of product characteristics is provided by the ISs to the DMs [44]. This type of information structure applies also to online search environments, where consumers are presented with a set of product characteristics displayed within several links when browsing through different websites [10].
(b) A second area of application follows from online search environments and concentrates on the agent recommendations provided to the DMs based on their purchasing history [35,40,43]. These recommendations involve the description of a series of product characteristics by unknown third parties to the DM. Absent credibility and trust considerations [19,30], the DM must choose which one among the recommendations available to follow.
(c) Finally, a similar approach must be considered when deciding what project to undertake or how to proceed with one already being developed based on the different descriptions provided by third parties [37,47]. This scenario can take place within a given production chain [42,36] with DMs subject to time pressure constraints [25,9].

The paper proceeds as follows. In Section 2, we define the basic concepts and notations. In Section 3, we model the point of view of the ISs and the DM. We introduce relative coordination and synchronization indexes in Sections 4 and 5, respectively, and compare them numerically in Section 6. Both these indexes are combined together in Section 7 to define the ordinal reliability ranking criterion proposed in the paper. Section 8 summarizes the main findings and suggests potential extensions.

2. Basic concepts and notations

We consider the problem of a DM who must rank n alternatives: all alternatives belong to the same category of choice objects; each alternative consists of a certain number of characteristics; each alternative is offered by an IS; each IS provides the DM with a description of the characteristics of the alternative that he is offering.

If, for instance, the DM is a consumer interested in purchasing a bottle of wine from a set, then the set of alternatives is a set of n bottles of wine each of them produced by a certain seller. If the DM is the head manager of a firm/corporation who must select a project to finance, then the set of alternatives is a set of n project portfolios/folders each of them proposed by a certain applicant.

Each alternative is characterized by finitely many characteristics that the corresponding IS must describe to the DM. In the case of a consumer buying a wine bottle, for example, the alternatives consist of several characteristics, such as $\%vol, price, color, flavor$ and *smell*. Similarly, in the case of a head manager selecting a project, the alternatives consist of characteristics such as *cost*, *time*, *manpower*, *quality* and *reliability*.

Henceforth, we will use the following notations:

- D will denote the DM.
- n will denote a fixed, but arbitrary, integer value indicating the number of alternatives and, hence, of ISs.
- For every $i = 1, \dots, n, S_i$ will denote the i-th IS.
- For every $i = 1, \dots, n, A_i$ will denote the alternative that S_i offers and describes to D.
- Γ will denote the set of all existing alternatives; the alternatives A_i offered by the ISs form only a subset of Γ.
- Δ will denote the set all the characteristics of the alternatives in Γ.
- $|\Delta|$ will denote the cardinality of the set Δ.
- For every $\delta \in \Delta, X_\delta$ will denote the set of all possible values that can be assigned to the δ-th characteristic of the alternatives in Γ.
- For every $\delta \in \Delta$ and every $i = 1, \dots, n, \overset{i}{x}_\delta [A_i]$ will denote the value in X_δ that S_i assigns to the δ-th characteristic of the alternative A_i.
- For every $\delta \in \Delta$ and every $A \in \Gamma, \overset{D}{x}_\delta [A]$ will denote the value in X_δ that D would assign to the δ-the characteristic of the alternative A.
- R will denote the set of real numbers.
- The symbol $>$ stands for the standard linear order on the set R.

Remark 1. The set X_δ contains all possible evaluations that each S_i can provide for the δ-th characteristic of the alternative A_i. In real life problems, the set X_δ consists of either quantitative or qualitative evaluations. Suppose, for example, that A_i is a bottle of wine. Then, a quantitative characteristic could be the price, while an example of qualitative characteristic could be the *flavor*. The price could range from \$10 to \$100, but the values that may be assigned to *flavor* cannot be quantified exactly. The evaluations for *flavor* can be assumed to belong to a set of the form $\{\textit{not}\ a(\delta),\ \textit{almost not}\ a(\delta),\ \textit{lowly}\ a(\delta),\ \textit{not very}\ a(\delta),\ \textit{regularly}\ a(\delta),\ \textit{very}\ a(\delta),\ \textit{highly}\ a(\delta),\ \textit{extremely}\ a(\delta)\}$, where $a(\delta)$ stands for an adjective describing the *flavor*. Thus, if δ is a quantitative characteristic, X_δ can be easily identified with an interval of real numbers. On the other hand, if δ is a qualitative characteristic, X_δ can be considered a linguistic variable. Linguistic values similar to the ones used above are common in the literature on fuzzy decision making. See, among the most recent papers, [45,46,44]. In this paper, X_δ is just a set of values: the fact that this values can be numerical or linguistic does not play any role in the development of our results. □

3. Subjective perceptions and subjective descriptions

The basic assumption we consider in this paper is the following:

Assumption 0. Each one among $D, S_1, \dots, S_n$ has a subjective perception of each of the alternatives and, hence, of the values to assign to the characteristics of the alternatives. □

As a consequence, S_i will describe the alternative A_i using a set of evaluations (one per characteristic) in general different from the one that D would use. The order in which S_i decides to describe the characteristics will also be different from the one that D would use.

Given Assumption 0, it is natural to analyze both how the ISs proceed in order to construct the descriptions of the alternatives that are subsequently checked by the DM and how the DM interprets and uses such descriptions. We propose the following theoretical framework.

3.1. Information senders' viewpoint

Let us consider first S_i's perception of alternative A_i. The subjective perception that S_i has of A_i must be reflected both by the evaluations that S_i provides for the characteristics of A_i and by the order in which he provides them.

Thus, we can assume the ISs to behave according to the following two working assumptions.

Assumption IS.1. For every $i = 1, \dots, n, S_i$ defines a linear order $\triangleright_i$ on Δ, that is, a binary relation on Δ satisfying irreflexivity ($\forall \delta \in \Delta, \delta \triangleright_i \delta$ does not hold), comparability ($\forall \delta, \delta' \in \Delta$, either $\delta \triangleright_i \delta'$ or $\delta' \triangleright_i \delta$) and transitivity ($\forall \delta, \delta', \delta'' \in \Delta, \delta \triangleright_i \delta'$ and $\delta' \triangleright_i \delta''$ imply $\delta \triangleright_i \delta''$). On the basis of the linear order $\triangleright_i, S_i$ linearly orders the characteristics of A_i from the most to the less preferred one as follows:

$$\delta_1^i \triangleright_i \delta_2^i \triangleright_i \cdots \triangleright_i \delta_{|\Delta|}^i. \qquad \square \tag{1}$$

Assumption IS.2. For every $i = 1, \ldots, n, S_i$ identifies A_i with the list of $|\Delta|$ evaluations that he assigns to the characteristics of A_i. S_i lists the evaluations from the one corresponding to the most preferred characteristic to the one corresponding to the less preferred one. In symbols:

$$A_i \overset{S_i}{\equiv\equiv} \left(\overset{i}{x}_{\delta^i_1}[A_i], \overset{i}{x}_{\delta^i_2}[A_i], \ldots, \overset{i}{x}_{\delta^i_{|\Delta|}}[A_i] \right) \tag{2}$$

where the symbol $\overset{S_i}{\equiv\equiv}$ means "identification according to S_i's viewpoint". □

Definition 1. For every $i = 1, \ldots, n$, the ordered $|\Delta|$-tuple of Eq. (2) will be referred to as *the full and truthful description* of the alternative A_i according to S_i's viewpoint. □

Assumption IS.3. For every $i = 1, \ldots, n, S_i$ makes available to D the full and truthful description of the alternative A_i from his viewpoint. □

If D checks the full description of the alternative A_i, then he knows not only the values that S_i assigns to the characteristics of A_i, but also which order of importance S_i gives to the characteristics. If D decides (or is forced) to check only an initial segment of this full description, then he will know the values only of the characteristic which matter more to S_i.

3.2. Decision maker's viewpoint

In this section, we complete the behavioral assumptions of our model by considering the viewpoint of the DM D. We assume the following.

Assumption D.1. The DM D defines his own linear order $\triangleright$ on Δ, according to which he orders the characteristics of all the alternatives in Γ (and, hence, those of all the A_i) from the most to the less preferred one as follows:

$$\delta^D_1 \triangleright \delta^D_2 \triangleright \cdots \triangleright \delta^D_{|\Delta|}. \qquad \square \tag{3}$$

Assumption D.2. The DM D identifies each A in Γ with the list of $|\Delta|$ evaluations that he assigns to the characteristics of A. D lists the evaluations from the one corresponding to the most preferred characteristic to the one corresponding to the less preferred one. In symbols:

$$A \overset{D}{\equiv\equiv} \left(\overset{D}{x}_{\delta^D_1}[A], \overset{D}{x}_{\delta^D_2}[A], \ldots, \overset{D}{x}_{\delta^D_{|\Delta|}}[A] \right) \tag{4}$$

where the symbol $\overset{D}{\equiv\equiv}$ means "identification according to D's viewpoint". □

Assumption D.3. The DM D defines a strict preference relation $\succ$ on Γ, that is, a binary relation on Γ satisfying irreflexivity ($\forall A \in \Gamma, A \succ A$ does not hold), completeness ($\forall A, A' \in \Gamma, A \succ A'$ or $A' \succ A$ (but not both)) and transitivity ($\forall A, A', A'' \in \Gamma, A \succ A'$ and $A' \succ A''$ imply $A \succ A''$). The relation $\succ$ is represented by a utility function $u : \Gamma \to R$, that is, a strictly increasing real-valued function on the preference order $(\Gamma, \succ)$, that expresses the importance that D assigns to each alternative in Γ. Thus,

$$\forall A, A' \in \Gamma, \quad A \succ A' \quad \Longleftrightarrow \quad u(A) > u(A'). \qquad \square \tag{5}$$

Assumption D.4. The DM D fixes a number $J, 1 \leqslant J \leqslant |\Delta|$, and checks the first J values composing the full descriptions provided by all the ISs. In other words, the DM fixes a number of characteristics to check in order to compare the available alternatives. In symbols:

$$\exists\, J \in \{1, \ldots, |\Delta|\} \text{ s.t. } \forall i = 1, \ldots, n, D \text{ checks } \left(\overset{i}{x}_{\delta^i_1}[A_i], \overset{i}{x}_{\delta^i_2}[A_i], \ldots, \overset{i}{x}_{\delta^i_J}[A_i] \right). \qquad \square \tag{6}$$

Definition 2. For every $i = 1, \ldots, n$, we will refer to the J-tuple $\left(\overset{i}{x}_{\delta^i_1}[A_i], \overset{i}{x}_{\delta^i_2}[A_i], \ldots, \overset{i}{x}_{\delta^i_J}[A_i] \right)$ as the *J-description* of the alternative A_i. In particular, a *J-description* corresponds to the full description when $J = |\Delta|$. □

Henceforth, in both description scenarios above, we will use $\left(\overset{i}{x}_{\delta^i_1}, \overset{i}{x}_{\delta^i_2}, \ldots, \overset{i}{x}_{\delta^i_J} \right)$ in place of $\left(\overset{i}{x}_{\delta^i_1}[A_i], \overset{i}{x}_{\delta^i_2}[A_i], \ldots, \overset{i}{x}_{\delta^i_J}[A_i] \right)$, since it is clear that S_i is describing the alternative A_i (and that he is doing so from his point of view; see Assumptions IS.2 and IS.3).

We will analyze the ranking problem faced by the DM D both when he can (or decides to) check the values of all the characteristics of the available alternatives and when he must (or decides to) check the values of only an initial segment of all the characteristics of the alternatives. In the first case, we will say that D checks the ***full description*** of all the alternatives (i.e., $J = |\Delta|$); in the latter case, we will say that D checks a ***partial description*** (i.e., $J < |\Delta|$).

3.3. Subjective perceptions and expected utilities

The fact that D checks either the full description or a partial description of length $J < |\Delta|$ of the alternative A_i (described by S_i) does not imply that D actually envisions the alternative A_i.

Suppose, for instance, that D checks the first value of the full description of A_i provided by S_i. This value corresponds to a certain δ-characteristic of A_i. After checking this value, given his subjective perception of the alternatives, D is induced to consider all the alternatives in Γ to whose δ-characteristic he would assign the same value (from his viewpoint). Similarly, if D checks the values assigned by S_i to more than one characteristic, i.e. the values of a J-description $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \ldots, \overset{i}{x}_{\delta_J^i}\right)$, then D is naturally induced to think of all the alternatives A in Γ such that, for every $j = 1, \ldots, J, \overset{D}{x}_{\delta_j^i}[A] = \overset{i}{x}_{\delta_j^i}$.

Thus, despite the fact that the order in which S_i provides the evaluations of the characteristics of the alternative A_i does not in general coincide with the one that D would follow to describe the same alternative, D can collect together all the alternatives that he believes to correspond to the list of evaluations provided by S_i and, hence, to be the actual alternative offered by S_i. Following Tavana et al. [44], we can formalize the set of alternatives collected by D as follows.

Definition 3. For every $i = 1, \ldots, n$, every $J = 1, \ldots, |\Delta|$ and every $j = 1, \ldots, J$, let s_{ij} be the position that the j-th characteristic in the linear order of S_i occupies in the linear order of D. In other words, s_{ij} is the index in $\{1, \ldots, |\Delta|\}$ such that $\delta_j^i = \delta_{s_{ij}}^D$.

For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, let:

$$\Psi_i(J) = \left\{A \in \Gamma : \forall j = 1, \ldots, J, \overset{i}{x}_{\delta_j^i} = \overset{D}{x}_{\delta_{s_{ij}}^D}[A]\right\}. \qquad \square \tag{7}$$

Interpretation of $\Psi_i(J)$**.** For every $i = 1, \ldots, n$, $\Psi_i(J)$ is the set of all alternatives whose s_{ij}-characteristic, with $j = 1, \ldots, J$, satisfies the value $x^i_{\delta_j^i}$ according to D's viewpoint. In other words, $\Psi_i(J)$ contains all alternatives that D believes could be the alternative A_i after checking the J-description $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \ldots, \overset{i}{x}_{\delta_J^i}\right)$. $\square$

Note that by the Assumption (D.3), D also has a strict preference relation on each set of the form $\Psi_i(J)$, namely, the restriction of $\succ$ to $\Psi_i(J)$.

Finally, by checking just the evaluations included in a J-description of the alternative A_i, i.e. $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \ldots, \overset{i}{x}_{\delta_J^i}\right)$, D has no objective way to know which one of the alternatives he may think of is the actual one offered by S_i. Indeed, it could also be the case that none of the alternatives that D thinks of coincide with the one actually described by S_i. Nevertheless, we can assume D to have subjective beliefs about which alternative is more probable. This can be done by endowing D with a probability function on $\Psi_i(J)$, whose values numerically express how much D believes each of the alternatives in $\Psi_i(J)$ to actually be the alternative offered by S_i. Thus, we introduce the following assumption.

Assumption D.5. For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, D defines a subjective probability function $\mu_i(\cdot|J)$ on $\Psi_i(J)$. For every $A \in \Gamma$, the value $\mu_i(A|J)$ is the probability assigned by D to the event "A actually is the alternative described by S_i" after checking the J-description $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \ldots, \overset{i}{x}_{\delta_J^i}\right)$. $\square$

Once D has defined his beliefs on the basis of the observed J-description, he is able to evaluate the utility he is to expect from accepting the alternative A_i offered by S_i. We define this expected utility as follows.

Definition 4. For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, the *expected utility* induced by the J-description $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \ldots, \overset{i}{x}_{\delta_J^i}\right)$ is the following sum:

$$E(u, \mu_i|J) = \sum_{A \in \Psi_i(J)} u(A) \cdot \mu_i(A|J). \qquad \square \tag{8}$$

To simplify notations, we will use $\Psi_i, \mu_i(A)$ and $E(u, \mu_i)$ in place of $\Psi_i(J), \mu_i(A|J)$ and $E(u, \mu_i|J)$, respectively, whenever $J = |\Delta|$.

4. Introducing the relative coordination index

In this section, we introduce a *relative coordination index* in order to allow the DM to measure how much his preference order on the set of characteristics differs from those of the ISs and, at the same time, to make an *inner comparison* among the different ISs.

By Assumptions (IS.1) and (D.1), there are $n+1$ linear orders on the set Δ of all the characteristics composing an alternative in Γ, one for each IS and one for the DM.

If one or more of the linear orders $\rhd_1, \ldots, \rhd_n$ on Δ coincide with the DM's linear order $\rhd$ on Δ, then the DM and the ISs who share the same linear order on Δ can be considered completely coordinated.

If none of the linear orders $\rhd_1, \ldots, \rhd_n$ on Δ coincides with the DM's linear order $\rhd$ on Δ, then the DM and any of the ISs can be thought of as uncoordinated. Nevertheless, it is reasonable to allow for the existence of different levels of incoordination. To keep it operationally useful, in the following, we will relate the level of incoordination between D and S_i to the problem of existence of fixed points between the linear orders $(\Delta, \rhd)$ and $(\Delta, \rhd_i)$.

More precisely, we will introduce a method for D to measure his incoordination level with each S_i. This method will exploit the differences between the linear orders $(\Delta, \rhd)$ and $(\Delta, \rhd_i)$ and the existence of a fixed point between these linear orders.

4.1. Fixed points

We will use the following additional notations. For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$,

- $\Delta[D,J]$ denotes the set composed by the first J elements of the linear order $(\Delta, \rhd)$, that is: $\Delta[D,J] = \{\delta_j^D : j = 1, \ldots, J\}$;
- $\Delta[i,J]$ denotes the set composed by the first J elements of the linear order $(\Delta, \rhd_i)$, that is: $\Delta[i,J] = \{\delta_j^i : j = 1, \ldots, J\}$.
- $F_{i|J} : \Delta[D,J] \to \Delta[i,J]$ is the function defined by

$F_{i|J}(\delta_j^D) = \delta_j^i$, for every $j = 1, \ldots, J$.

In particular, when $J = |\Delta|$, we will use $\Delta[D], \Delta[i]$ and F_i in place of $\Delta[D, |\Delta|], \Delta[i, |\Delta|]$ and $F_{i||\Delta|}$, respectively.

Remark 2. Clearly, for every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, the function $F_{i|(J-1)}$ is the restriction of the function $F_{i|J}$ to the subset of Δ consisting of its first J characteristics with respect to $\rhd$. It is also trivial to check that for every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, the function $F_{i|(J-1)}$ is order preserving. □

The concept of fixed point for a function of a set in itself is well-known. In our setting, a characteristic $\delta \in \Delta$ is a fixed point for $F_{i|J}$ if $F_{i|J}(\delta) = \delta$. In particular, in the J-description scenario, we can interpret a fixed point for $F_{i|J}$ as a characteristic to which both D and S_i assign the same position in their own linear order.

Definition 5. For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, a fixed point of the function $F_{i|J}$ is a characteristic δ_j^D, where $j \in \{1, \ldots, J\}$, such that:

$$\delta_j^D = F_{i|J}(\delta_j^D) = \delta_j^i. \qquad \square \tag{9}$$

Motive to consider fixed points: Our key idea to measure the level of incoordination between D and S_i is the following. Given $J = 1, \ldots, |\Delta|$, if $F_{i|J}$ admits fixed points, we can naturally interpret the lowest value of j such that δ_j^D is a fixed point for the function $F_{i|J}$ as a measure of the *coordination gap* existing between the DM D and the IS S_i. If $F_{i|J}$ does not have fixed points in the classical sense, then we must somehow induce the existence of fixed-like points.

Drawbacks: The concept of fixed point loses completely its meaning if the domain and range of the function $F_{i|J}$ do not intersect, as it could be the case in a partial description scenario. Thus, while, in the full description scenario, the problem of finding or creating a fixed point for the function $F_{i|J}$ can be generally considered to be well-posed, in a partial description scenario, it turns out to be totally artificial.

Thus, we proceed to the analysis of the full description and the partial scenarios, separately.

4.1.1. Rightward shifts and induced fixed points: full description scenario

Suppose that, for every $i = 1, \ldots, n, D$ checks the full description $\left(x^i_{\delta^i_1}, x^i_{\delta^i_2}, \ldots, x^i_{\delta^i_{|\Delta|}}\right)$ provided by S_i.

As discussed above, we need each function F_i to admit a fixed-like point, even when F_i does not have a fixed point in the standard sense. The idea is to use the position of the least fixed-like point as a measure of the *coordination gap* existing between D and S_i.

In order to introduce a fixed-like point for every function F_i, we define a family of translation functions that allows D to suitably shift rightwards his linear order until a fixed point is reached. These translation functions are formalized in the following definition.

Definition 6. For every $k = 0, 1, \dots, |\Delta| - 1$, let $T_k : (\{\delta_1^D, \dots, \delta_{|\Delta|-k}^D\}, \rhd) \to (\{\delta_{1+k}^D, \dots, \delta_{|\Delta|}^D\}, \rhd)$ be the function defined by:

$$\forall j = 1, \dots, |\Delta| - k, \quad T_k(\delta_j^D) = \delta_{j+k}^D. \tag{10}$$

We will refer to T_k as *the rightward shift of* $(\Delta, \rhd)$ *of* k *positions* (in short: k-**RWS**). The family of functions $\{T_k : k = 1, \dots, |\Delta| - 1\}$ is the family of all possible rightward shifts of $(\Delta, \rhd)$. In particular, the 0-RWS, T_0, coincides with the identity map of $(\Delta, \rhd)$; this RWS will be referred to as *the trivial RWS.* □

Interpretation of RWS: The function T_k formalizes a shift of k positions towards the right of the linear order $(\Delta, \rhd)$ with respect to the linear order $(\Delta, \rhd_i)$. By means of this shift, the linear order $(\Delta, \rhd)$ is displaced in a way that the j-th characteristic of the DM "advances" k positions and, hence, corresponds to the $(j + k)$-th characteristic of S_i under the function F_i. The following simple example provides further intuition for the use of RWSs. □

Example 1. Suppose that the set of alternatives Γ consists of wine bottles and that each bottle can be described by a maximum of four characteristics:

$$\Delta = \{alcohol\ content, color, smell, aftertaste\}. \tag{11}$$

Suppose that the DM linearly orders the set of characteristics, Δ, as follows:

$$aftertaste \rhd color \rhd alcohol\ content \rhd smell. \tag{16}$$

Thus, $aftertaste = \delta_1^D, color = \delta_2^D, alcohol\ content = \delta_3^D, smell = \delta_4^D, |\Delta| = 4$ and $k = 0, 1, 2, 3$. It is possible to define up to three non-trivial RWS of $(\Delta, \rhd)$, namely:

- $T_1 : (\{\delta_1^D, \delta_2^D, \delta_3^D\}, \rhd) \to (\{\delta_2^D, \delta_3^D, \delta_4^D\}, \rhd)$ defined by:

  $T_1(\delta_1^D) = \delta_2^D, \quad T_1(\delta_2^D) = \delta_3^D, \quad T_1(\delta_3^D) = \delta_4^D;$
- $T_2 : (\{\delta_1^D, \delta_2^D\}, \rhd) \to (\{\delta_3^D, \delta_4^D\}, \rhd)$ defined by:

  $T_2(\delta_1^D) = \delta_3^D, \quad T_2(\delta_2^D) = \delta_4^D;$
- $T_3 : (\{\delta_1^D\}, \rhd) \to (\{\delta_4^D\}, \rhd)$ defined by:

  $T_3(\delta_1^D) = \delta_4^D.$

Fig. 1 represents this family of functions.□

Clearly, for every $i = 1, \dots, n$ and every $k = 0, 1, \dots, |\Delta| - 1$, it is possible to compose the function F_i with the k-RWS, T_k. Fig. 2 shows the composition $F_i \circ T_1$ when, as in Example 1, the alternatives can be described by a maximum of four characteristics.

It is clear that:

Proposition 1. *F_i has a fixed point if and only if $F_i \circ T_0$ has a fixed point.* □

Theorem 1. *For every $i = 1, \dots, n$ and every $j = 1, 2, \dots, |\Delta|$, the following are equivalent:*

(a) $\exists h \geqslant j$ such that $\delta_j^D = \delta_h^i$;

(b) $\exists k_{ij} \in \{0, 1, \dots, |\Delta| - 1\}$ such that the characteristic δ_j^D is a fixed point of the function $F_i \circ T_{k_{ij}}$, that is, $\delta_j^D = F_i(T_{k_{ij}}(\delta_j^D)) = \delta_{j+k_{ij}}^i$.

Proof. Fix $i \in \{1, \dots, n\}$ and $j \in \{1, 2, \dots, |\Delta|\}$. We proceed by showing $(a) \Rightarrow (b)$ and *negation of* $(a) \Rightarrow$ *negation of* (b).

"$(a) \Rightarrow (b)$": Suppose that there exists $h \geqslant j$ such that $\delta_j^D = \delta_h^i$. If $\delta_j^D = \delta_j^i$, then δ_j^D is a fixed point for $F_i = F_i \circ T_0$ and we are done (the claim is true for $k_{ij} = 0$). If $\delta_j^D = \delta_h^i$ for some $h > j$, then we have $\delta_j^D = F_i(T_{h-1}(\delta_j^D)) = \delta_{j+h-1}^i$, that is, δ_j^D is a fixed point for $F_i \circ T_{h-1}$. Hence, the claim is true for $k_{ij} = h - 1$.

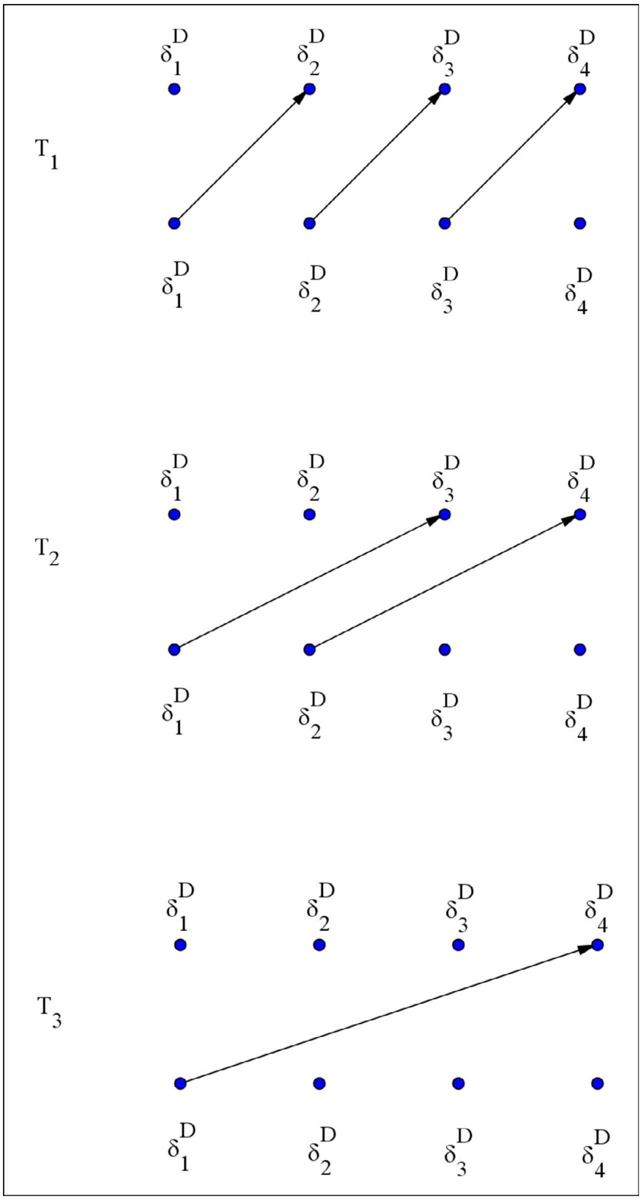

Fig. 1. Non-trivial RWS of $(\Delta, \triangleright)$ when Δ consists of 4 characteristics.

"*negation of* $(a) \Rightarrow$ *negation of* (b)": Suppose now that there exists $h < j$ such that $\delta_j^D = \delta_h^i$. Then, the characteristic δ_j^D occupies the j-th position w.r.t. the linear order $(\Delta, \triangleright)$ of D and a position on the left of the j-th one in the linear order $(\Delta, \triangleright_i)$ of S_i. Hence, no RWS allows D to obtain δ_j^D as a fixed point w.r.t. S_i. □

Corollary 1. *For every* $i = 1, \ldots, n$, *there exists* $k_{i1} \in \{0, 1, \ldots, |\Delta| - 1\}$ *such that the first characteristic* δ_1^D *is a fixed point of the function* $F_i \circ T_{k_{i1}}$.

Proof. Fix $i \in \{1, \ldots, n\}$. It is immediate to observe that there exists $h \geqslant 1$ such that $\delta_1^D = \delta_h^i$. Apply Theorem 1. □

Corollary 2. *For every* $i = 1, \ldots, n$, *there exists* $k_i^* \in \{0, 1, \ldots, |\Delta| - 1\}$ *such that the function* $F_i \circ T_{k_i^*}$ *has a fixed point, that is:*

$$\exists k_i^* \in \{0, 1, \ldots, |\Delta| - 1\} \quad \exists j \in \{1, \ldots, |\Delta|\} : \quad \delta_j^D = F_i(T_{k_i^*}(\delta_j^D)) = \delta_{j+k_i^*}^i. \qquad \square$$

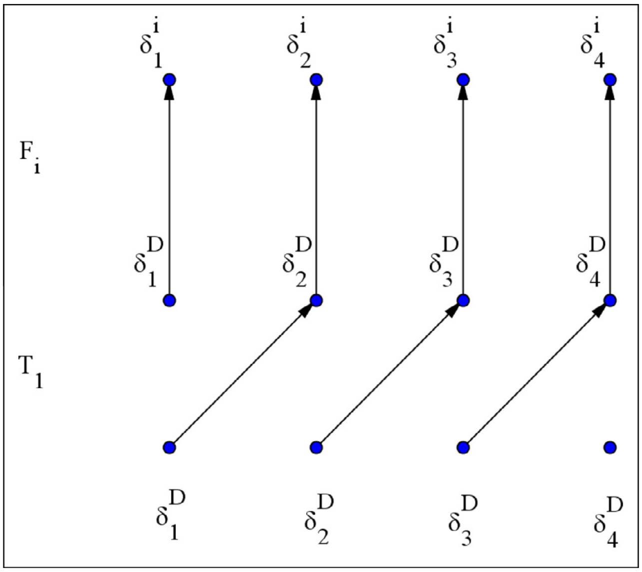

Fig. 2. Composite function $F_i \circ T_1$ when Δ consists of 4 characteristics.

Example 2. As in Example 1, let the set of alternatives Γ consist of wine bottles, each of which can be described by a maximum of four characteristics, $\Delta = \{alcohol\ content, color, smell, aftertaste\}$, and let D order the characteristics as follows, "*aftertaste* $\triangleright$ *color* $\triangleright$ *alcohol content* $\triangleright$ *smell*". Thus, $aftertaste = \delta_1^D, color = \delta_2^D, alcohol\ content = \delta_3^D, smell = \delta_4^D$.

Suppose that D consults three ISs, each of them endowed with a linear order on Δ different from the one of D. More precisely, assume that:

- S_1 orders Δ as follows, "*alcohol content* $\triangleright_1$ *color* $\triangleright_1$ *smell* $\triangleright_1$ *aftertaste*", that is:

$$alcohol\ content = \delta_1^1, \quad color = \delta_2^1, \quad smell = \delta_3^1, \quad aftertaste = \delta_4^1.$$

- S_2 orders Δ as follows, "*smell* $\triangleright_2$ *alcohol content* $\triangleright_2$ *color* $\triangleright_2$ *aftertaste*", that is:

$$smell = \delta_1^2, \quad alcohol\ content = \delta_2^2, \quad color = \delta_3^2, \quad aftertaste = \delta_4^2.$$

- S_3 orders Δ as follows, "*color* $\triangleright_3$ *alcohol content* $\triangleright_3$ *smell* $\triangleright_3$ *aftertaste*", that is:

$$color = \delta_1^3, \quad alcohol\ content = \delta_2^3, \quad smell = \delta_3^3, \quad aftertaste = \delta_4^3.$$

The function F_1 already has a fixed point, i.e. the characteristic *color*. Thus, the composite function $F_1 \circ T_0$, where T_0 is the trivial RWS, has the same fixed point. See Fig. 3(a).

To obtain a fixed point w.r.t. S_2, D may apply the RWS of 1 position, T_1, followed by F_2. In this case, the characteristic *color* is the fixed point of $F_2 \circ T_1$. Alternatively, D may also apply the RWS of 3 positions, T_3, followed by F_2. In this case, the characteristic *aftertaste* is the fixed point of $F_2 \circ T_3$. See Fig. 3(b).

Finally, to get to a fixed point w.r.t. S_3, D must apply the RWS of 3 positions, T_3, followed by F_3. The characteristic *aftertaste* is the fixed point of $F_3 \circ T_3$. See Fig. 3(c). □

Definition 7. For every $i = 1, \dots, n$, a fixed point of the function $F_i \circ T_{k_i^*}$ will be called a (k_i^*)-*RWS-induced fixed point* of D w.r.t. S_i. □

4.1.2. Rightward shifts and induced fixed points: partial description scenario

Suppose now that, for every $i = 1, \dots, n$, the DM checks a partial description of fixed length $J < |\Delta|$ provided by S_i, that is, $\left(x^i_{\delta^i_1}, x^i_{\delta^i_2}, \dots, x^i_{\delta^i_J}\right)$. In this case, Theorem 1 still guarantees that a fixed point can be reached, but the DM may not have enough information to actually reach it.

Consider, for example, D and the ISs of Example 2. If D checks only the first characteristic of each available alternative, that is, a partial description of length $J = 1 < 4 = |\Delta|$, then D is unable to establish the existence of a fixed point, or to find the RWS necessary to induce one. Similar considerations hold, particularly for S_2 and S_3, if we assume that D checks a partial description of length $J = 2 < 4 = |\Delta|$.

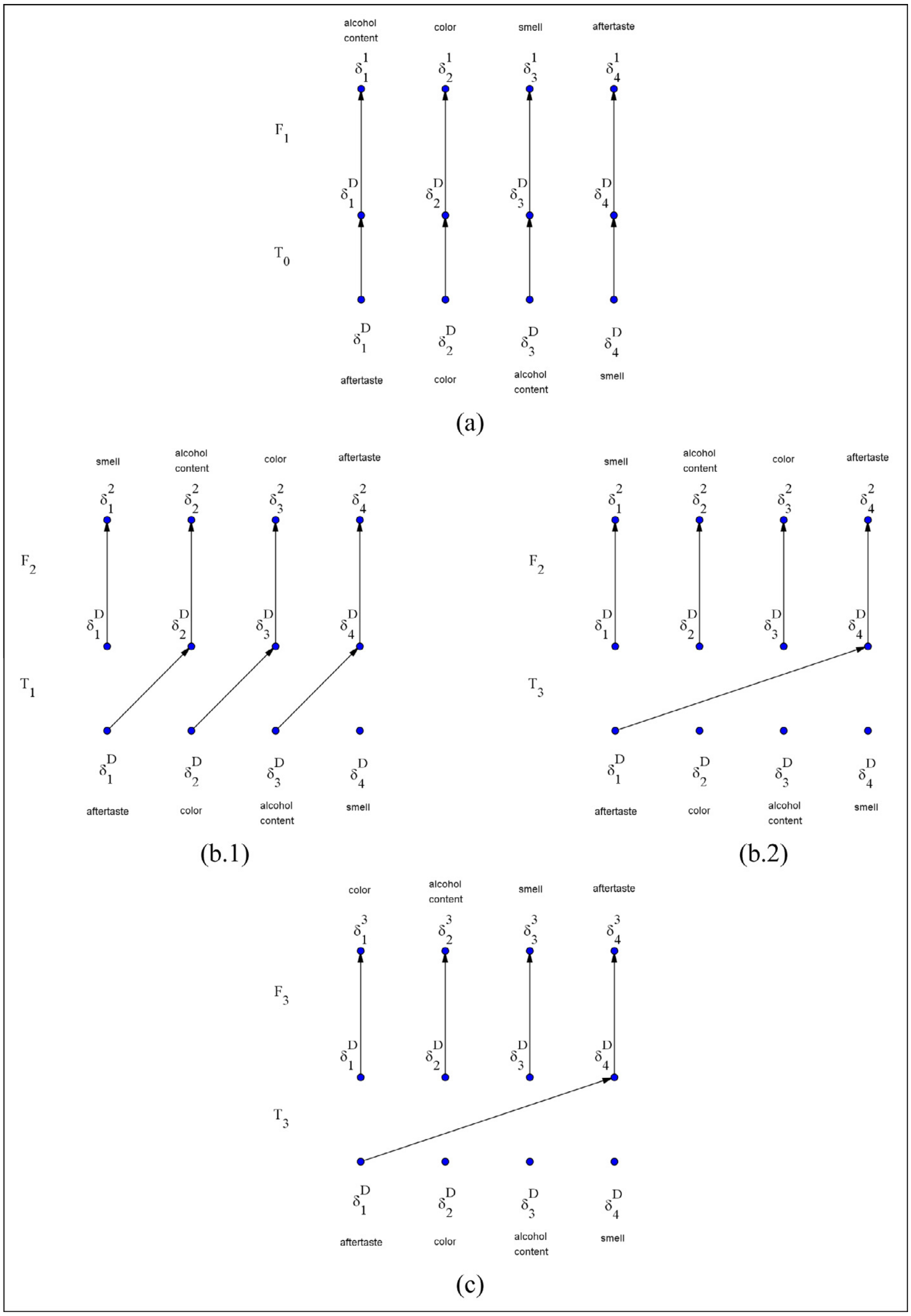

Fig. 3. Composite functions $F_i \circ T_k$ for the DM to compare three different sources when $|\Delta| = 4$.

The case where D checks a partial description of length $J = 3 < 4 = |\Delta|$ is slightly different: even though D does not check the value of the last characteristic, he can easily deduce whether this last characteristic is a fixed point or a RWS-induced fixed point for the function F_i. In particular, after checking the first three characteristics displayed by S_3, D knows that, according to S_3's linear order, the fourth characteristic is *aftertaste* and, hence, he can deduce that the characteristic *aftertaste* is the 3-RWS-induced fixed point for F_3.

4.2. Coordination gaps and relative coordination indexes

On the basis of all the considerations above, we can now formulate the definition of coordination gap and relative coordination index in a generic J-description scenario, where $J \leqslant |\Delta|$.

Firstly, to guarantee that the relative coordination index is well-defined (that is, not ambiguously defined), we need to formulate a definition that relies on the "smallest" possible RWS necessary for D to reach a fixed point. The following definition and the proposition after it are introduced to this end.

Definition 8. For every $i = 1, \dots, n$ and $J = 1, \dots, |\Delta|$, let

- $U_{i|J}$ denote the set of indexes j corresponding to the position of a fixed point of $F_{i|J}$ in the restricted linear order $(\{\delta_1^D, \dots, \delta_J^D\}, \triangleright)$; that is:

$$U_{i|J} \stackrel{def}{=} \{j : 1 \leqslant j \leqslant J \wedge \delta_j^D \text{ is a fixed point for } F_{i|J}\}; \tag{12}$$

- $W_{i|J}$ denote the set of indexes k_i^* corresponding to the position of a (k_i^*)-RWS-induced fixed point for $F_{i|J}$ in the restricted linear order $(\{\delta_1^D, \dots, \delta_J^D\}, \triangleright)$; that is:

$$W_{i|J} \stackrel{def}{=} \{k_i^* : 1 \leqslant k_i^* \leqslant J - 1 \wedge F_{i|J} \circ T_{k_i^*} \text{ has a fixed point}\}. \qquad \square \tag{13}$$

Note that, for every $J = 1, \dots, |\Delta|, F_{i|J}$ having a fixed point is equivalent to F_i having a fixed point and $\min U_{i|J} = \min U_{i||\Delta|} \leqslant J$. On the other hand, if $F_{i|J}$ does not admit a fixed point, then:

(a) in the full description scenario, we can apply Theorem 1 and obtain a (k_i^*)-RWS-induced fixed point of D w.r.t. S_i (hence, $W_{i|J}$ is surely non-empty);
(b) in the a partial description scenario, there is no guarantee of finding a RWS-induced fixed point (i.e., $W_{i|J}$ could be empty).

More precisely, except for the cases when $J = |\Delta| - 1$ and $J = |\Delta|, U_{i|J} = \emptyset$ does not imply $W_{i|J} \neq \emptyset$.
The following proposition is now easy to check.

Proposition 2. *Let $i = 1, \dots, n$.*

(a) *For every $J = 1, \dots, |\Delta|, F_{i|J}$ admits a fixed point if and only if $U_{i|J} \neq \emptyset$;*
(b) *For every $J = 1, \dots, |\Delta|$, $\exists k_i^* \in 1, \dots, J - 1$ s.t. $F_{i|J} \circ T_{k_i^*}$ admits a fixed point if and only if $W_{i|J} \neq \emptyset$;*
(c) *If $U_{i|J} = \emptyset$ and either $J = |\Delta|$ or $J = |\Delta| - 1$, then $W_{i|J} \neq \emptyset$.*
(d) *If $U_{i|J} = \emptyset$ and $J < |\Delta| - 1$, then both $W_{i|J} \neq \emptyset$ and $W_{i|J} = \emptyset$ are possible.* $\square$

We propose the following method to evaluate the coordination level between the DM and any of the ISs.

Definition 9. For every $i = 1, \dots, n$ and $J = 1, \dots, |\Delta|$, the *coordination gap between D and S_i induced by the J-description* $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \dots, \overset{i}{x}_{\delta_J^i}\right)$, denoted by $\gamma(D, S_i|J)$, is calculated as follows:

$$\gamma(D, S_i|J) = \begin{cases} (\min U_{i|J}) - 1, & \text{if } U_{i|J} \neq \emptyset \\ (\min W_{i|J}) + |\Delta| - 1, & \text{if } U_{i|J} = \emptyset \wedge (J = |\Delta| - 1 \vee (J < |\Delta| - 1 \wedge \mathrm{W}_{i|J} \neq \emptyset)) \\ 2(J-1) + 1, & \text{if } U_{i|J} = \emptyset \wedge J < |\Delta| - 1 \wedge \ \mathrm{W}_{i|J} = \emptyset \end{cases} \tag{14}$$

where $\min U_{i|J}$ and $\min W_{i|J}$ are calculated w.r.t. the standard linear order of R. When $J = |\Delta|$ (i.e. full description scenario), we will use $\gamma(D, S_i)$ in place of $\gamma(D, S_i||\Delta|)$. $\square$

Remark 3. The intuition behind this definition: By Definition 9, $\min U_{i|J}$ is the position of the first fixed point of $F_{i|J}$ to be met in the linear order $\delta_1^D \triangleright \delta_2^D \triangleright \dots \triangleright \delta_{|\Delta|}^D$, while $\min W_{i|J}$ is the minimal number of positions that D must shift rightwards in order to reach a fixed point w.r.t. S_i. When $U_{i|J} \neq \emptyset$, we subtract -1 to ensure that there is no coordination gap when the fixed point is the first characteristic. When $U_{i|J} = \emptyset$, we add $|\Delta| - 1$ to account for the fact that there is no fixed point among the first J characteristics described by S_i and, as a result, D must start shifting rightwards. The third case implies that if no fixed point can be found after shifting rightwards through the set of J characteristics checked by the DM, the value $2(J-1)+1$ is assigned by default to measure the coordination gap. This value is the minimum integer indicating that there is no fixed point or induced fixed point among the first J characteristics observed by the DM when $J < |\Delta| - 1$. $\square$

Finally, we define the DM's relative coordination index, that is, an index that allows D to rank the alternatives offered by the different ISs in any description scenario. The higher is the gap in coordination between D and the source, the lower D will rank the source.

Definition 10. For every $J = 1, \ldots, |\Delta|$, let $I^{\gamma|J} : \{S_i : i = 1, \ldots, n\} \to [0, 1]$ be the function defined as follows:

$$I^{\gamma|J}(S_i) = \begin{cases} 1 - \frac{\gamma(D,S_i|J)}{\max\{1,\max\{\gamma(D,S_i|J):i=1,\ldots,n\}\}}, \\ \text{if } U_{i|J} \neq \emptyset \vee (U_{i|J} = \emptyset \wedge (J = |\Delta| \vee J = |\Delta| - 1 \vee (J < |\Delta| - 1 \wedge W_{i|J} \neq \emptyset))) \\ 1 - \frac{\gamma(D,S_i|J)}{\max\{1,\max\{\gamma(D,S_i|J):i=1,\ldots,n\}\}} \cdot \frac{J}{|\Delta|}, \\ \text{if } U_{i|J} = \emptyset \wedge J < |\Delta| - 1 \wedge W_{i|J} = \emptyset \end{cases} \quad (15)$$

$I^{\gamma|J}(S_i)$ will be referred to as *D 's relative coordination index for S_i induced by the J-description*. When $J = |\Delta|$ (i.e. full description scenario), we will use $I^{\gamma}(S_i)$ in place of $I^{\gamma||\Delta|}(S_i)$. □

Note that the maximum value of the index $I^{\gamma|J}(S_i)$ is 1 (for $\gamma(D, S_i|J) = 0$), while its minimum value is 0 (for $\gamma(D, S_i|J) = \max\{\gamma(D, S_i|J) : i = 1, \ldots, n\}$) provided that $J = |\Delta|$.

Remark 4. The intuition behind this definition: In Eq. (15), the denominators on the right-hand-side have been defined so as to avoid divisions by zero. The factor $\frac{J}{|\Delta|}$ weights the second case based on the number of observations acquired by the DM whenever an induced fixed point is not reached. That is, the shorter is the *J*-description checked per alternative, the lower weight is given to the fact of not being able to reach a fixed point within the characteristics observed. □

Clearly, $\gamma(D, S_i|J)$ and $I^{\gamma|J}(S_i)$ coincide with $\gamma(D, S_i)$ and $I^{\gamma}(S_i)$, respectively, when

- $J = |\Delta|$, i.e. the DM checks the full description provided by each source;
- $J < |\Delta|$ and the DM finds either a fixed point or an RWS-induced fixed point within the first *J* characteristics.

Thus, we can state the following criterion that the DM can apply in all description scenarios.

> Coordination Ranking Criterion
> Let $J \in \{1, \ldots, |\Delta|\}$ and the DM *D* check the *J*-description provided by all the ISs, $S_1, \ldots, S_n$. *D* calculates and ranks the values $I^{\gamma|J}(S_1)$, ..., $I^{\gamma|J}(S_n)$ from the highest to the lowest one. The closer the value $I^{\gamma|J}(S_i)$ is to 1, the higher the synchronicity level between *D* and S_i.

Example 3. Let Γ, Δ and the linear orders of D, S_1, S_2, S_3 be defined as in Example 2.

Suppose first that *D* checks the full description of the alternatives offered by the ISs. The coordination gaps between *D* and each IS are the following:

$$\begin{aligned} &\gamma(D, S_1) = 1 \\ &\gamma(D, S_2) = 1 + |\Delta| - 1 = 4 \\ &\gamma(D, S_3) = 3 + |\Delta| - 1 = 6. \end{aligned} \quad (16)$$

Hence, *D*'s relative coordination indexes for S_1, S_2, S_3 are:

$$I^{\gamma}(S_1) = 1 - \frac{1}{6} = \frac{5}{6}, \qquad I^{\gamma}(S_2) = 1 - \frac{4}{6} = \frac{1}{3}, \qquad I^{\gamma}(S_3) = 1 - \frac{6}{6} = 0. \quad (17)$$

Thus, in the full description scenario, *D* ranks S_1 as the IS with who he shares the highest level of coordination.

Suppose now that *D* checks a partial description of length $J = 2 < 4 = |\Delta|$ for each of the three available alternatives A_1, A_2, A_3. The coordination gaps between *D* and each IS induced by the corresponding 2-descriptions are the following:

$$\gamma(D, S_1|2) = 1$$

(since $F_{1|2}$ has a fixed point)

$$\gamma(D, S_2|2) = 2(J - 1) + 1 = 3$$

(since $F_{2|2}$has no fixed point, $\quad J = 2 < 3 = |\Delta| - 1$ and $W_{2|2} = \emptyset$) (18)

$$\gamma(D, S_3|2) = 2(J - 1) + 1 = 3$$

(since $F_{3|2}$ has no fixed point, $J = 2 < 3 = |\Delta| - 1$ and $W_{3|2} = \emptyset$).

Hence, *D*'s relative coordination indexes for S_1, S_2, S_3, induced by the corresponding 2-descriptions, are:

$$I^{\gamma|2}(S_1) = 1 - \frac{1}{3} = \frac{2}{3}, \qquad I^{\gamma|2}(S_2) = 1 - \frac{3}{3} \cdot \frac{2}{4} = \frac{1}{2}, \qquad I^{\gamma|2}(S_3) = 1 - \frac{3}{3} \cdot \frac{2}{4} = \frac{1}{2}. \quad (19)$$

It follows that, in the 2-description scenario, D ranks again S_1 as the IS with who he shares the highest level of coordination, while S_2 and S_2 are both ranked second. □

5. The relative synchronization index

To simplify notations, in this section, we identify the j-th characteristic δ_j^D with its position in the preference order $(\Delta, \triangleright)$ of the DM D, that is, with the position j. At the same time, we also identify the j-th characteristic δ_j^i of the alternative A_i of S_i with the position it occupies in the preference order of D, that is, with the position s_{ij} such that $\delta_j^i = \delta_{s_{ij}}^D$. This allows us to schematize the DM's comparisons between his way of ordering the characteristics and that of each of the available ISs using a $(n+1) \times (|\Delta|+1)$ matrix. The last row of this matrix will consist of D and the list of positions $1, \ldots, |\Delta|$. The i-th row will display S_i, where $i = 1, \ldots, n$, and the characteristics as ordered by S_i but seen from the point of view of D. That is, the i-th row of the matrix will be as follows: $S_i, s_{i1}, \ldots, s_{i|\Delta|}$.

Example 4. Consider the situation described in Examples 2 and 3. We can schematize and analyze D's viewpoint by using the following matrix:

$$\left[\begin{array}{c|cccc} S_1 & 3 & \langle 2 \rangle & 4 & 1 \\ S_2 & 4 & 3 & 2 & 1 \\ S_3 & 2 & 3 & 4 & 1 \\ D & 1 & 2 & 3 & 4 \end{array}\right] \tag{20}$$

The element between the angle brackets in the row corresponding to S_1 signals the fixed point of D with respect to S_1, that is, the fixed point of F_1. □

We design now a synchronization index that will account for the differences between the rankings assigned to the characteristics by D and the ISs. This synchronization index is a purely ordinal measure of these differences and is based on the "vertical distances" existing between the order assigned by the DM to the characteristics of the alternatives and the order assigned by ISs.

The definition we propose applies both to the full description scenario and to the partial description one.

Definition 11. For every $i = 1, \ldots, n$ and $J = 1, \ldots, |\Delta|$, the *synchronicity gap between D and S_i induced by the J-description* $\left(x^i_{\delta^i_1}, x^i_{\delta^i_2}, \ldots, x^i_{\delta^i_J}\right)$, denoted by $\sigma(D, S_i|J)$, is calculated as follows:

$$\sigma(D, S_i|J) = \begin{cases} \sum_{j=1}^{|\Delta|} |s_{ij} - j|, & \text{if } J = |\Delta| \vee J = |\Delta| - 1 \\ \sum_{j=1}^{J} |s_{ij} - j|, & \text{if } J < |\Delta| - 1 \end{cases} \tag{21}$$

where, for every $j = 1, \ldots, J, s_{ij}$ is such that $\delta^i_j = \delta^D_{s_{ij}}$. In the case when $J = |\Delta|$ (i.e. full description scenario), we will use $\sigma(D, S_i)$ in place of $\sigma(D, S_i||\Delta|)$. □

Remark 5. The intuition behind this definition: As stated in Section 4.1.2, in case the DM checks a J-description $\left(x^i_{\delta^i_1}, x^i_{\delta^i_2}, \ldots, x^i_{\delta^i_J}\right)$ where $J = |\Delta| - 1$, he can easily deduce the missing characteristic. This is the reason for the two cases in Eq. (21). □

A relative synchronicity index can be defined on the basis of the highest gap and used as a base relative to which the other distances will be calculated.

Definition 12. For every $J = 1, \ldots, |\Delta|$, let $I^{\sigma|J} : \{S_i : i = 1, \ldots, n\} \to [0, 1]$ be the function defined as follows:

$$I^{\sigma|J}(S_i) = \begin{cases} 1 - \frac{\sigma(D,S_i|J)}{\max\{1, \max\{\sigma(D,S_i|J) : i=1,\ldots,n\}\}}, & \text{if } J = |\Delta| \vee J = |\Delta| - 1 \\ \left(1 - \frac{\sigma(D,S_i|J)}{\max\{1, \max\{\sigma(D,S_i|J) : i=1,\ldots,n\}\}}\right) \cdot \frac{J}{|\Delta|}, & \text{if } J < |\Delta| - 1 \end{cases} \tag{22}$$

We will refer to the value $I^{\sigma|J}(S_i)$ as *D's relative synchronicity index for S_i induced by the J-description* $\left(x^i_{\delta^i_1}, x^i_{\delta^i_2}, \ldots, x^i_{\delta^i_J}\right)$. In the case when $J = |\Delta|$ (i.e. full description scenario), we will use $I^{\sigma}(S_i)$ in place of $I^{\sigma||\Delta|}(S_i)$. □

Remark 6. The intuition behind this definition: The cases described in Eq. (22) follow the same intuition as the cases introduced in Eq. (15). The factor $\frac{J}{|\Delta|}$ in the second case of Eq. (22) weights the value of the relative synchronicity index based on the number of observations acquired by the DM w.r.t. the total number of characteristics of an alternative. The lower the number of observations acquired, the lower the number of comparisons performed by the DM and, therefore, the strength of the index. □

> Synchronicity Ranking Criterion
>
> Let $J \in \{1, \ldots, |\Delta|\}$ and the DM D check the J-description provided by all the ISs, $S_1, \ldots, S_n$.
> D calculates and ranks the values $I^{\sigma|J}(S_1), \ldots, I^{\sigma|J}(S_n)$ from the highest to the lowest one. The closer the value $I^{\sigma|J}(S_i)$ is to 1, the higher the synchronicity level between D and S_i.

Example 5. Consider the situation described in Examples 2 and 3 and the corresponding matrix (already introduced in Example 4):

$$\left[\begin{array}{c|cccc} S_1 & 3 & \langle 2 \rangle & 4 & 1 \\ S_2 & 4 & 3 & 2 & 1 \\ S_3 & 2 & 3 & 4 & 1 \\ D & 1 & 2 & 3 & 4 \end{array}\right] \tag{23}$$

Suppose that D checks the full description provided by each S_i. The synchronicity gaps between D and the ISs are the following:

$$\begin{aligned} \sigma(D, S_1) &= |3-1| + |2-2| + |4-3| + |1-4| = 6 \\ \sigma(D, S_2) &= |4-1| + |3-2| + |2-3| + |1-4| = 8 \\ \sigma(D, S_3) &= |2-1| + |3-2| + |4-3| + |1-4| = 6. \end{aligned} \tag{24}$$

Hence, D's relative synchronicity indexes for S_1, S_2, S_3 are:

$$I^{\sigma}(S_1) = 1 - \frac{6}{8} = \frac{1}{4}, \quad I^{\sigma}(S_2) = 1 - \frac{8}{8} = 0, \quad I^{\sigma}(S_3) = 1 - \frac{6}{8} = \frac{1}{4}. \tag{25}$$

Suppose now that D checks the 2-description provided by each S_i. The synchronicity gaps between D and the ISs decrease:

$$\begin{aligned} \sigma(D, S_1|2) &= |3-1| + |2-2| = 2 \\ \sigma(D, S_2|2) &= |4-1| + |3-2| = 4 \\ \sigma(D, S_3) &= |2-1| + |3-2| = 2 \end{aligned} \tag{26}$$

while D's relative synchronicity indexes for S_1, S_2, S_3 clearly increase:

$$I^{\sigma|2}(S_1) = \left(1 - \frac{2}{4}\right) \cdot \frac{2}{4} = \frac{1}{4}, \quad I^{\sigma|2}(S_2) = \left(1 - \frac{4}{4}\right) \cdot \frac{2}{4} = 0, \quad I^{\sigma|2}(S_3) = \left(1 - \frac{2}{4}\right) \cdot \frac{2}{4} = \frac{1}{4}. \tag{27}$$

Thus, in both the full description and the 2-description scenarios, D ranks S_1 and S_3 as the most synchronized ISs. □

6. Coordination vs synchronicity at work: a numerical example

Consider the order situation represented by the following matrix and suppose that D checks the full description provided by each S_i.

$$\left[\begin{array}{c|cccccccccc} S_1 & 5 & 10 & \langle 3 \rangle & 9 & 6 & 7 & 2 & 4 & 5 & 8 \\ S_2 & 3 & 8 & 9 & 1 & 2 & \vec{4} & \vec{5} & 10 & 6 & 7 \\ S_3 & 3 & 7 & 6 & 2 & \langle 5 \rangle & 9 & 1 & 4 & 10 & 8 \\ S_4 & 10 & 9 & 8 & 7 & 6 & \vec{5} & 4 & 3 & 2 & 1 \\ D & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array}\right] \tag{28}$$

The elements between the angle brackets in the rows corresponding to S_1 and S_3 signal the fixed points of D with respect to S_1 and S_3, respectively. That is, the characteristic δ_3^D is a fixed point for F_1, while the characteristic δ_5^D is a fixed point for F_3.

The elements over-lined by an arrow indicate the closest position of the fixed point that D can obtain after a RWS w.r.t. S_2 and S_4, respectively. More precisely, we have $\min W_{2||\Delta|} = 2$ and $\min W_{4||\Delta|} = 1$. Thus, D needs to apply the 2-RWS to reach a fixed point w.r.t. S_2 (hence, a fixed point of $F_2 \circ T_2$) and the 1-RWS to reach a fixed point w.r.t. S_4 (hence, a fixed point of $F_4 \circ T_1$).

Since the DM has just one fixed point w.r.t. S_1 located in position 3 and just one fixed point w.r.t. S_3 in position 5, we have:

$$\gamma(D, S_1) = 2 \quad \text{and} \quad \gamma(D, S_3) = 4. \tag{29}$$

However, the synchronicity gap between D and S_3 is smaller than the one between D and S_1:

$$\sigma(D, S_1) = 34 \quad \text{and} \quad \sigma(D, S_3) = 28. \tag{30}$$

Similarly, the DM reaches the closest possible fixed point w.r.t. S_4 after a rightward shift of 1 position, while he needs to shift rightwards 2 positions in order to obtain the closest possible fixed point w.r.t. S_2. Hence:

$$\gamma(D, S_2) = 2 + |\Delta| - 1 = 11 \quad \text{and} \quad \gamma(D, S_4) = 1 + |\Delta| - 1 = 10. \tag{31}$$

At the same time, the synchronicity gap between D and S_2 is smaller than the one between D and S_4:

$$\sigma(D, S_2) = 32 \quad \text{and} \quad \sigma(D, S_4) = 50. \tag{32}$$

Finally, note that there is no direct link between the position of the fixed point (original or induced) w.r.t. a source and the level of synchronicity with the source. In particular, the fixed point converging to position 1 does not imply the synchronicity gap converging to 0 (i.e., the relative synchronicity index converging to 1). Consider, for example, S_1 and S_2. D assigns to S_2 a relative synchronicity index of $I^\sigma(S_2) = 1 - \frac{32}{50} = \frac{9}{25}$, which is higher than the one he assigns to $S_1, I^\sigma(S_1) = 1 - \frac{34}{50} = \frac{8}{25}$, despite the latter source allowing for a fixed point located almost immediately after position 1.

Similar considerations apply in any partial description scenario.

7. Main result: ordinal reliability rankings

Fix $J \in \{1, \dots, |\Delta|\}$ and consider the general case when, for every $i = 1, \dots, n$, the DM checks the J-description provided by S_i, that is, $\left(\overset{i}{x}_{\delta_1^i}, \overset{i}{x}_{\delta_2^i}, \dots, \overset{i}{x}_{\delta_J^i} \right)$.

After the DM D has observed the J-description provided by each source, he can calculate the expected utility (see Definition 4) of each of the available alternatives on the basis of his subjective beliefs (see Assumption (D.5)) and then rank the expected utility values obtained.

We propose the following criterion in order to obtain a ranking as satisfying as possible: D must scale the expected utility values induced by the J-descriptions using the corresponding relative coordination and synchronicity indexes. The ranking criterion proposed is schematized below.

Ordinal Reliability Ranking (ORR) Criterion
Let $J \in \{1, \dots, |\Delta|\}$ and the DM D check the J-description provided by all the ISs, $S_1, \dots, S_n$.

Step I
D fixes two weights $w_\gamma, w_\sigma \in [0, 1]$ such that $w_\gamma + w_\sigma = 1$.

Step II
For every $i = 1, \dots, n, D$ calculates the following weighted expected utility:

$$\Omega(S_i) \overset{def}{=} \left(w_\gamma I^{\gamma|J}(S_i) + w_\sigma I^{\sigma|J}(S_i) \right) \cdot E(u, \mu_i|J) = \left(w_\gamma I^{\gamma|J}(S_i) + w_\sigma I^{\sigma|J}(S_i) \right) \cdot \sum_{A \in \Psi_i(J)} u(A) \cdot \mu_i(A|J)$$

Step III
D ranks the values $\Omega(S_1), \dots, \Omega(S_n)$ from the highest to the lowest one.

If, in particular, $J = |\Delta|$, then the weighted expected utilities $\Omega(S_i), i = 1, \dots, n$, can be written as follows:

$$\Omega(S_i) \overset{def}{=} \left(w_\gamma I^\gamma(S_i) + w_\sigma I^\sigma(S_i) \right) \cdot E(u, \mu_i) = \left(w_\gamma I^\gamma(S_i) + w_\sigma I^\sigma(S_i) \right) \cdot \sum_{A \in \Psi_i} u(A) \cdot \mu_i(A)$$

Interpretation of weights. The weights $w_\gamma, w_\sigma \in [0, 1]$ have been introduced to account for any potential subjective bias on the importance of the indexes considered by the DM. □

Fix $i = 1, \dots, n$ and let $J = |\Delta|$. It may happen that D's linear order $\triangleright$ coincides with S_i's linear order $\triangleright_i$. In this case, both the relative coordination index $I^\gamma(S_i)$ and the relative synchronicity index $I^\sigma(S_i)$ that D assigns to S_i would equal 1 (see Definition

12). As a consequence, independently from the weights $w_\gamma, w_\sigma \in [0,1]$ assigned by the DM, the total weight assigned to the expected utility $E(u, \mu_i)$ will be 1, that is:

$$\left(w_\gamma I^\gamma(S_i) + w_\sigma I^\sigma(S_i)\right) = 1. \tag{33}$$

It follows that:

$$\Omega(S_i) \overset{def}{=} \left(w_\gamma I^\gamma(S_i) + w_\sigma I^\sigma(S_i)\right) \cdot E(u,\mu_i) = \left(w_\gamma + w_\sigma\right) \cdot E(u,\mu_i) = E(u,\mu_i). \tag{34}$$

Hence, D will have to rank the alternative offered by S_i using the "full" expected utility delivered by this alternative. Note that Eqs. (33) and (34) still hold for $I^{\gamma|J}(S_i)$ and $I^{\sigma|J}(S_i)$ when $J = |\Delta| - 1$ and D's linear order $\rhd$ coincides with S_i's linear order $\rhd_i$. That is:

$$\begin{aligned} &\left(w_\gamma I^{\gamma||\Delta|-1}(S_i) + w_\sigma I^{\sigma||\Delta|-1}(S_i)\right) = 1 \\ &\Omega(S_i) \overset{def}{=} \left(w_\gamma I^{\gamma||\Delta|-1}(S_i) + w_\sigma I^{\sigma||\Delta|-1}(S_i)\right) \cdot E(u,\mu_i) = \left(w_\gamma + w_\sigma\right) \cdot E(u,\mu_i) = E(u,\mu_i). \end{aligned} \tag{35}$$

Finally, note that when $J < |\Delta| - 1$, in order for D to rank the alternative using its "full" expected utility it would not suffice that $\rhd$ and $\rhd_i$ coincide on the first J characteristics. Indeed, by Definition 12, $I^{\sigma|J}(S_i) \neq 1$ whenever $J < |\Delta| - 1$.

Thus, the following corollary of the weighted expected utility criterion holds.

Ranking Criterion for Coordinated and Synchronized Sources

If:

- $J = |\Delta|$ or $J = |\Delta| - 1$;
- D check the J-description provided by all the ISs, $S_1, \ldots, S_n$;
- for every $i = 1, \ldots, n$, the first J elements in Δ coincide for both D (w.r.t. the linear order $\rhd$) and S_i (w.r.t. the linear order $\rhd_i$).

Then:

Step 1.I

For every $i = 1, \ldots, n, D$ calculates the weighted expected utility: $\Omega(S_i) = E(u, \mu_i)$.

Step 1.II

D ranks the values $\Omega(S_1), \ldots, \Omega(S_n)$ from the highest to the lowest one.

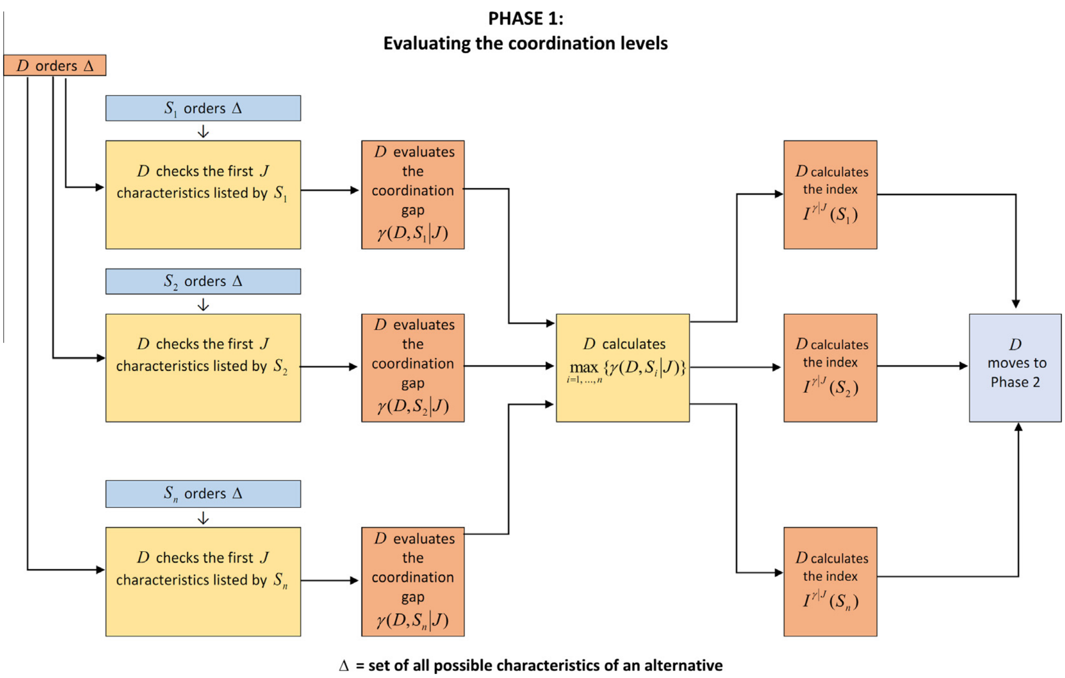

Fig. 4. DM evaluates his coordination levels with respect to the information senders.

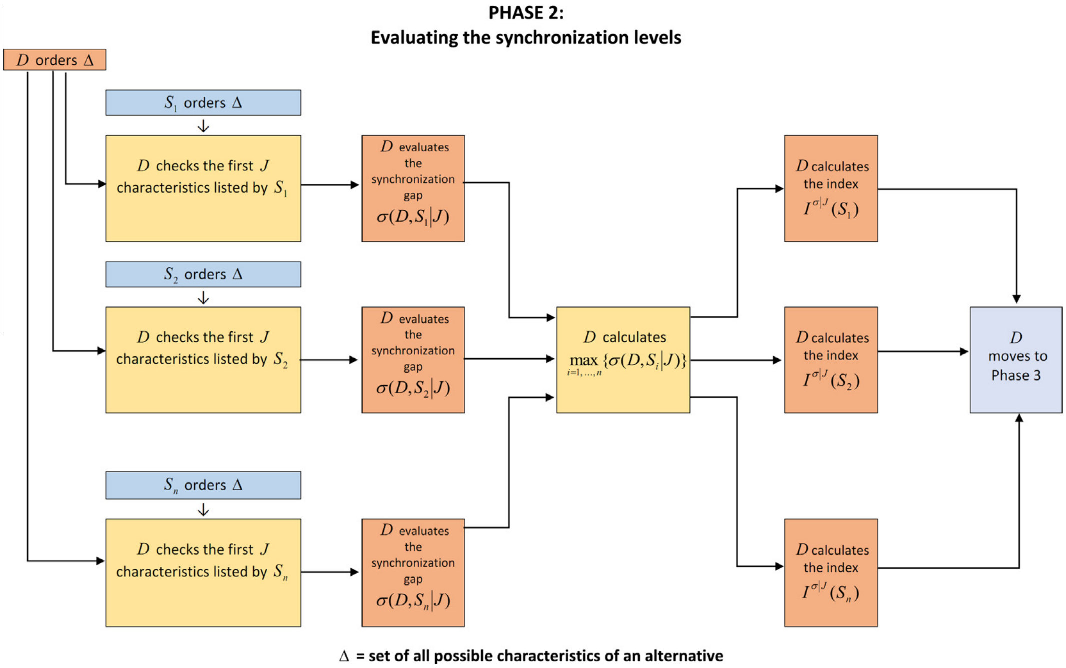

Fig. 5. DM evaluates his synchronization levels with respect to the information senders.

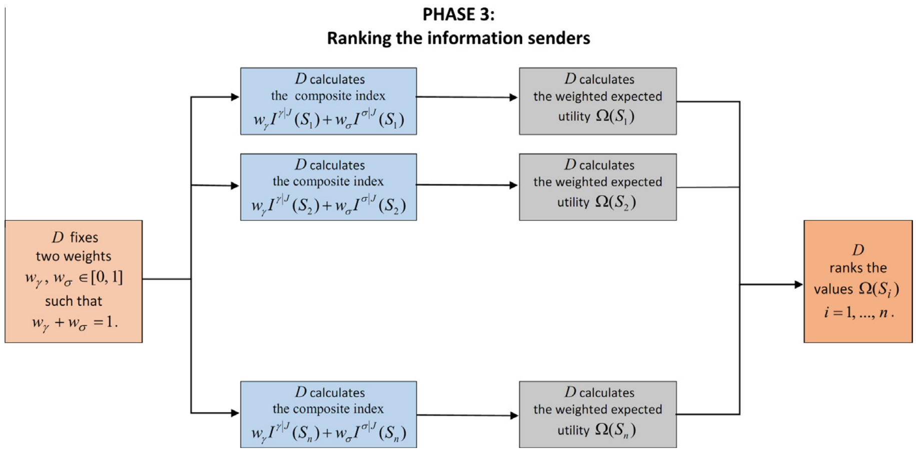

Fig. 6. DM ranks the information senders and, hence, the alternatives.

Figs. 4–6 provide a graphical description summarizing the three phases that the DM must go through in order to rank the alternatives based on the method proposed in this paper.

8. Conclusion

We have introduced a novel and easy to implement ORR criterion on which the DMs can base their choice from a set of alternatives described by different information sources. We have relied on the differences in preference orders existing between the DM and the information sources in order to determine the evaluation frictions expected to be encountered by the DM after choosing one of the alternatives presented.

The ORR criterion allows, in particular, to generalize the bilateral trade model described by Tavana et al. [44] to a multilateral setting where each DM has to rank the other DMs before deciding who among them to exchange products with. More precisely, assume that there are n DMs, $D_1, \ldots, D_n$, each of them endowed with a product that they can exchange with any of the other DMs. The "manipulation-free" requirements R.1 to R.4 assumed by Tavana et al. [44] can be easily adapted to imply the existence of $J < |\Delta|$ initial characteristics that coincide for D_i and each one of the other DMs, $D_k, k \neq i$. The ORR criterion can be then used by each D_i *to rank all the other DMs,* $D_k, k \neq i$, before exchanging products.

Finally, it should be noted that we have left aside strategic considerations regarding the information transmitted among DMs. That is, the way and order in which information is displayed can be determined strategically to influence the choice behavior of the DMs [38,13]. Choice manipulation instruments range from standard framing effects [49,28] to information display and signaling strategies when searching online [19,30,32,10,16]. In this latter case, the ORR criterion can be modified and adapted to account for reputation and trust in sequential information acquisition environments.

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

References

[1] M. Abreu, V. Mendes, Information, overconfidence and trading: do the sources of information matter?, J Econ. Psychol. 33 (4) (2012) 868–881.
[2] G.A. Akerlof, The market for 'Lemons': quality uncertainty and the market mechanism, Quart. J. Econ. 84 (3) (1970) 488–500.
[3] R.A. Aliev, W. Pedrycz, O.H. Huseynov, Decision theory with imprecise probabilities, Int. J. Inform. Technol. Decis. Mak. 11 (2) (2012) 271–306.
[4] R.A. Aliev, W. Pedrycz, O.H. Huseynov, Behavioral decision making with combined states under imperfect information, Int. J. Inform. Technol. Decis. Mak. 12 (3) (2013) 619–645.
[5] F.M. Bass, W.W. Talarzyk, An attitude model for the study of brand preference, J. Market. Res. 9 (1) (1972) 93–96.
[6] R. Batra, O.T. Ahtola, Measuring the hedonic and utilitarian sources of consumer attitudes, Market. Lett. 2 (2) (1990) 159–170.
[7] R.D. Blackwell, P.W. Miniard, J.F. Engel, Consumer Behavior, Thomson/South-Western, Mason, Ohio, 2006.
[8] S.A. Brown, V. Venkatesh, H. Hoehle, Technology adoption decisions in the household: a seven-model comparison, J. Assoc. Inform. Sci. Technol. (2014), http://dx.doi.org/10.1002/asi.23305.
[9] A.P. Ciganek, W. Haseman, K. Ramamurthy, Time to decision: the drivers of innovation adoption decisions, Enterprise Inform. Syst. 8 (2) (2014) 279–308.
[10] D. Cyr, Return visits: a review of how web site design can engender visitor loyalty, J. Inform. Technol. 29 (2014) 1–26.
[11] F.D. Davis, Perceived usefulness, perceived ease of use, and user acceptance of information technology, MIS Quart. 13 (3) (1989) 319–340.
[12] F.D. Davis, R.P. Bagozzi, P.R. Warshaw, User acceptance of computer technology: a comparison of two theoretical models, Manage. Sci. 35 (8) (1989) 982–1003.
[13] D. Di Caprio, F.J. Santos Arteaga, Strategic diffusion of information and preference manipulation, Int. J. Strategic Decis. Sci. 2 (2) (2011) 1–19.
[14] D. Di Caprio, F.J. Santos Arteaga, Cardinal versus ordinal criteria in choice under risk with disconnected utility ranges, J. Math. Econ. 47 (4–5) (2011) 588–594.
[15] D. Di Caprio, F.J. Santos-Arteaga, M. Tavana, Information acquisition processes and their continuity: transforming uncertainty into risk, Inform. Sci. 274 (2014) 108–124.
[16] D. Di Caprio, F.J. Santos Arteaga, M. Tavana, Choice manipulation through comparability in markets with verifiable multi-attribute products, Int. J. Inform. Technol. Decis. Mak., in press. doi:http://dx.doi.org/10.1142/S0219622014500886.
[17] A. Dillon, M.G. Morris, User acceptance of information technology: theories and models, in: M. Williams (Ed.), Annual Review of Information Science and Technology, vol. 31, Information Today, Medford, NJ, 1996, pp. 3–32.
[18] M. Fishbein, An investigation of the relationships between beliefs about an object and the attitude toward that object, Human Relations 16 (3) (1963) 233–239.
[19] F. Fouss, Y. Achbany, M. Saerens, A probabilistic reputation model based on transaction ratings, Inform. Sci. 180 (11) (2010) 2095–2123.
[20] S. Gächter, H. Orzen, E. Renner, C. Stamer, Are experimental economists prone to framing effects? A natural field experiment, J. Econ. Behav. Organ. 70 (3) (2009) 443–446.
[21] R. Ghani, K. Probst, Y. Liu, M. Krema, A. Fano, Text mining for product attribute extraction, SIGKDD Explorat. 8 (1) (2006) 41–48.
[22] J.L. Ginter, An experimental investigation of attitude change and choice of a new brand, J. Market. Res. 11 (1) (1974) 30–40.
[23] G. Grossman, E. Helpman, Quality ladders in the theory of growth, Rev. Econ. Stud. 58 (1) (1991) 43–61.
[24] T.J. Hess, A.L. McNab, K.A. Basoglu, Reliability generalization of perceived ease of use, perceived usefulness, and behavioral intentions, MIS Quart. 38 (1) (2014) 1–28.
[25] M.I. Hwang, Decision making under time pressure: a model for information systems research, Inform. Manage. 27 (4) (1994) 197–203.
[26] Z. Ivkovic, S. Weisbenner, Information diffusion effects in individual investors' common stock purchases: covet thy neighbors' investment choices, Rev. Finan. Stud. 20 (4) (2007) 1327–1357.
[27] J. Jaccard, D. Brinberg, L.J. Ackerman, Assessing attribute importance: a comparison of six methods, J. Consumer Res. 12 (March) (1986) 463–468.
[28] D. Kahneman, A. Tversky, Choices, Values and Frames, Cambridge University Press, 2000.
[29] A.J. Kimmel, Psychological Foundations of Marketing, Routledge, 2012.
[30] K. Kolomvatsos, C. Anagnostopoulos, S. Hadjiefthymiades, Sellers in e-marketplaces: a fuzzy logic based decision support system, Inform. Sci. 278 (10) (2014) 267–284.
[31] K.J. Lancaster, A new approach to consumer theory, J. Polit. Econ. 74 (2) (1966) 132–157.
[32] B.-C. Lee, L. Ang, C. Dubelaar, Lemons on the web: a signalling approach to the problem of trust in internet commerce, J. Econ. Psychol. 26 (5) (2005) 607–623.
[33] D.R. Lehmann, Television show preference: application of a choice model, J. Market. Res. 8 (1) (1971) 47–55.
[34] D.R. Lehmann, J. O'Shaughnessy, Difference in attribute importance for different industrial products, J. Market. 38 (2) (1974) 36–42.
[35] Y.-M. Li, C.-L. Chou, L.-F. Lin, A social recommender mechanism for location-based group commerce, Inform. Sci. 274 (1) (2014) 125–142.
[36] K. Matsui, Gray-market trade with product information service in global supply chains, Int. J. Product. Econ. 147 (B) (2014) 351–361.
[37] D. McLain, Quantifying project characteristics related to uncertainty, Project Manage. J. 40 (4) (2009) 60–73.
[38] J.A. Quelch, Measurement of the relative importance of product attribute information: a review of the information display approach, J. Consumer Policy 3 (3–4) (1979) 232–245.
[39] N.F.M. Roozenburg, J. Eekels, Product Design: Fundamentals and Methods, John Wiley & Sons Ltd, Chichester, UK, 1995.
[40] T. Sasaki, D.V. Becker, M.A. Janssen, R. Neel, Does greater product information actually inform consumer decisions? The relationship between product information quantity and diversity of consumer decisions, J. Econ. Psychol. 32 (3) (2011) 391–398.
[41] B. Shackel, Usability – context, framework, definition, design and evaluation, in: B. Shackel, S.J. Richardson (Eds.), Human Factors for Informatics Usability, Cambridge University Press, Cambridge, UK, 1991, pp. 21–37.
[42] H. Sharifi, H.S. Ismail, J. Qiu, S.N. Tavani, Supply chain strategy and its impacts on product and market growth strategies: a case study of SMEs, Int. J. Product. Econ. 145 (1) (2013) 397–408.

[43] S. Spiekermann, M. Strobel, D. Temme, Drivers and Impediments of Consumer Online Information Search: Self-controlled versus Agent-based Search in a High Involvement Context. eCommerce Research Forum, MIT, 2001.
[44] M. Tavana, D. Di Caprio, F.J. Santos-Arteaga, A bilateral exchange model: the paradox of quantifying the linguistic values of qualitative characteristics, Inform. Sci. 296 (2015) 201–218.
[45] M. Tavana, K. Khalili-Damghani, A.R. Abtahi, A hybrid fuzzy group decision support framework for advanced-technology prioritization at NASA, Expert Syst. Appl. 40 (2) (2013) 480–491.
[46] M. Tavana, R. Khanjani Shiraz, A. Hatami-Marbini, P.J. Agrell, K. Paryab, Chance-constrained DEA models with random fuzzy inputs and outputs, Knowl.-Based Syst. 52 (2013) 32–52.
[47] M. Thoumy, S. Vachon, Environmental projects and financial performance: exploring the impact of project characteristics, Int. J. Product. Econ. 140 (1) (2012) 28–34.
[48] V. Venkatesh, M.G. Morris, G.B. Davis, F.D. Davis, User acceptance of information technology: toward a unified view, MIS Quart. 27 (3) (2003) 425–478.
[49] I. Vessey, The effect of information presentation on decision making: a cost-benefit analysis, Inform. Manage. 27 (2) (1994) 103–119.