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THEORETICAL ARTICLE

A two-level supply chain coordination model for perishable products under optimal markdown time and trade credit policies

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Abstract
This study investigates the effects of markdown and credit policies on perishable products in a two-level supply chain. As a significant funding source, trade credit allows the retailer to receive the products from the manufacturer and pay for them later. The retailer deposits the sales payment in the bank and earns interest while paying interest after the credit's expiration. In addition to financial concerns, if products are perishable and the demand varies over time, there will be a severe challenge for the retailer to adjust the sales and ordering policies to manage the inventory cost and revenues. Given the importance of price, the retailer examines two sales policies and chooses the more profitable one, either selling all the products at a fixed price or marking down the prices at an appropriate time to boost the demand. Assuming that demand is time-varying, this study determines the cycle length, markdown time, preservation techniques, investment, and trade credit size by considering their relationships in different decision models. Several methods are utilized in an analytical approach to solve the models and find the optimal solutions. Sensitive analyses are also conducted to evaluate the effects of the initial and marked-down price changes on the markdown time, order quantity, and sales volume.

Keywords Markdown · Trade credit · Channel coordination · Time-varying deterioration · Discount price · Preservation technology

1 Introduction

One of the common assumptions in inventory models is that the buyer should pay for the purchased products when he receives them, while he may need to pay a debt late due to limited financial resources. Unlike cash, delayed payment, also called trade credit, involves a specific time granted to the buyer to pay off his debt. Trade credit lifts the financial pressure off the buyer and allows him more time and flexibility to allocate funds. In addition, this kind of funding helps the seller to attract

Extended author information available on the last page of the article more buyers and have more stable interactions with customers. Trade credit is a significant short-term funding source for most businesses, mainly growing companies and startups. According to the Financial Times, 90% of the world's $14 trillion business transactions in 2007 were based on this type of funding [87].

Furthermore, the quality and value of products have been considered unchanged in most of the studies conducted on inventory management, while some products are perishable and have a limited lifetime. These products cannot be used after a certain time for different reasons, such as perishability and obsolescence. Products such as medicine, blood, chemicals, food, and fruits change their physical and internal conditions. Electronic and technology-oriented products and fashion goods become obsolete with the advent of newer products, and the demand for them suddenly drops [2, 80]. Decreased consumption over time is an inherent feature of these products, and customers are sensitive to the remaining time for consumption. Limited consumption time, uncertain demand, and nominal value at the end of the sales cycle are the significant characteristics of perishable products compared to unperishable ones. Perishability and time-dependent demand make inventory management highly difficult. In this case, the products lose their consumption value, affecting the system costs if orders and sales policies are incorrectly determined. Since price affects customers' choices, reducing the price of the products left on the shelf at a specific time can attract more customers and improve sales [41]. Markdown is an excellent policy to level up the demand to increase sales and reduce the losses caused by the devaluation of the products due to perishability or obsolescence [80]. Investing in preservation technology is another policy to slow down the rate of perishability and prolong the product's lifetime.

Moreover, a supply chain consists of interdependent members constantly seeking to optimize their goals and objectives, sometimes leading to conflict of interests and decentralized decision-making. Supply chain coordination is a concept that encourages the members of a chain to align their individual decisions [39]. Aligning the decisions not only increases the overall profitability of the supply chain but also ensures a sustainable relationship among the supply chain members and their final success in the long term. In other words, coordination is a mechanism that encourages the chain members to make their decisions optimal for better profitability for the whole chain [34]. Channel coordination is possible when the goals of all the members are taken into account simultaneously. Encouraging the members to make consistent decisions and ensuring that they all benefit from coordination is one of the significant challenges for supply chain managers [35].

In summary, trade credit, as a standard funding method in supply chains, helps retailers with limited budgets obtain the necessary financial credit. Manufacturers with higher financial ability give retailers trade credits, allowing them to pay for the products with delay. Although offering trade credits imposes the cost of opportunity investment on manufacturers, it lets them have stable interactions with retailers. Furthermore, optimal ordering and selling decisions are essential for the sound performance of firms [29]. Determining the optimal ordering and selling policies of products is very challenging when the products are time-varying and perishable, or their demand varies over time. The price and freshness are crucial factors to consider while discussing the customer's desire to purchase.

Markdown and investing in preservation techniques are valuable strategies that help firms properly manage the ordering and selling policies of deteriorating inventory. The markdown policy aims to increase the sales by encouraging the customers to buy. Reducing the price at the right time may mainly prevent a demand reduction. Also, the preservation technology extends the expiration time of the product.

In reality, the concurrent application of trade credit and markdown policies is prevalent in many supply chains. For example, in grocery supply chains, the retailers demand a trade credit from the upper-stream manufacturers and offer a price markdown to the downstream customers to raise the sale volume. Also, they use freezing, packaging, and drying as preservation techniques to keep the products fresh [24].

The motivation behind this research is that many academic studies have dealt with trade credit and markdown policies independently in inventory or supply chain models. However, there is no study to consider these issues simultaneously. What fills the gap is the study of the combined effects of trade credit, markdown, deterioration, and preservation technology through a supply chain.

The present study seeks to bridge the research gap by presenting a practical supply chain model considering the mentioned issues. The study concerns the following items:

- Markdown policy applied by the retailer
- Investment in preservation techniques
- Trade credit policy offered by the manufacturer
- Time-varying deterioration
- different decision-making structures

This study aims to determine the time of markdown, the period of trade credit, the cost of preservation, and the duration of sales cycles under two decentralized and centralized decision-making models. It finally examines the channel coordination issue with appropriate contracts. An analytical solution approach serves to optimize the manufacturer and retailer's decisions. The optimal solutions are obtained by a differential calculus method, and the concavity of the objective function of the supply chain is shown graphically.

The rest of this article is as follows. Section 2 provides a review of some representative studies in the field. Section 3 introduces the notations and assumptions of the proposed models and presents mathematical formulations for the retailer and the manufacturer based on the relationships among markdown time, trade credit, and sales cycle. In Sect. 4, efficient algorithms are offered to find optimal solutions in centralized and decentralized models. In Sect. 5, the proposed models are validated by different examples, and critical insights are provided through sensitivity analyses. Section 6 presents and discusses the results obtained from sensitivity analyses. Finally, Sect. 7 concludes the study and makes suggestions for future research.

2 Literature review

Markdown policy on perishable products and trade credit policy are the two main issues in the literature related to this study. The first issue concerns cost and revenue management. Markdown is a standard, effective policy to increase sales so that the products can be offered to customers before they lose their value. The primary justification for markdown is that products have no value after the expiration date or their salvage value is so low that the seller prefers to sell them at a lower price instead of throwing them away. Pashigian [63] was one of the first researchers who studied markdown policies for fashion products in a two-period model. Taudes and Rudloff [69] proposed a two-period model of determining optimal ordering policies with price markdown. To this end, the demand depending on current and reference prices was taken into consideration. Chew et al. [12] calculated the optimal order quantity of new products in a multi-period model for perishables. To do so, they adopted the markdown strategy for old products of different ages.

Assuming that customers would have different behaviors in product selection, some researchers investigated the optimal markdown policies to increase sales. In this case, one may refer to [20, 23, 43, 74, 88]. Chintapalli [13] and Azadi et al. [3] also examined the markdown policy in a model where old products were sold simultaneously as new products. The demand was assumed to be a function of the price of the latest and old products. While the inventory level affected the demand quantity, Nagare and Dutta [62] obtained the optimal order quantity of perishable products. Assuming that demand depended on the price and product quality, some authors, such as [9, 73, 75], examined the effect of markdown policies on retailer decisions. Piramuthu and Zhou [65] provided a model which determined optimal inventory policies. Demand was assumed as a function of the quality and amount of shelf space allocated to each product. Regarding price and time-dependent demand, Kaya and Polat [41] designed a perishable inventory model to determine the optimal time of price change, the optimal number of markdowns, and the product quantity.

In the studies reviewed above, price reduction policies were only adopted for the retailer. Numerous studies have evaluated the effects of markdown policies all across the supply chain. For example, Chen et al. [5] designed a two-level supply chain. They obtained the optimal wholesale price for the manufacturer and optimal ordering and markdown policies for the retailer by considering different customer behaviors and the reference price. Some researchers, such as [2, 79, 81], also referred to various contracts and studied the optimal decisions of supply chain members under the markdown strategy. [14, 15] considered the price discount policy for the supplier and the retailer in a supply chain. Jadidi et al. [36] examined the behavior of the retailer and the manufacturer in a two-level supply chain by considering the markdown policy under wholesale and rebate contracts. Based on a study conducted by [36], Xu et al. [78] evaluated the effect of the markdown strategy on the profit of the supply chain members under the wholesale price and consignment contracts, while the demand was dependent on time and price.

One of the basic assumptions in the reviewed studies is that the retailer should pay for the products as soon as they are received. Specific payment policies, such as trade credit, have not been considered. The trade credit policy is, indeed, the second issue examined in the review of the literature here. Goyal [28] was the first to study an economic order model (EOQ) focusing on trade credit. Then, different models were developed with trade credit policies included. In similar studies, [26, 37, 40, 50] examined optimal inventory policies under payment delay, assuming shortages were allowed. Some studies, such as [8, 18, 49, 66, 70, 72, 83], also analyzed optimal lot-sizing decisions for perishable products by considering a permissible delay in the payment policy. Tripathy et al. [71] developed trade credit financing for an inventory system in which the items are non-instantaneous and perishable. Some authors, such as [54, 55], evaluated the effect of permissible delay in a production-inventory system under different assumptions like deterioration, shortage, warranty period, and two-level credit policy.

Moradi et al. [60] considered the partial credit policy in an imperfect inventory model while assuming the inspection period depends on learning effects. Given that the products were non-instantaneous perishable, Jani et al. [38]evaluated dynamic pricing policies under trade credit. The retailer's earned interest was calculated only according to the initial price, i.e., without considering the price changes. Bi et al. [4] obtained optimal ordering policies based on the assumption that demand was uncertain and trade credit was dependent. Unlike the studies mentioned above, which evaluated the effect of trade credit on the decisions of only one member of the supply chain, different models discussed various credit policies across integrated supply chains [17, 25, 27, 48, 86]. Esmaeili and Nasrabadi [22] formulated a Stackelberg model under trade credit to determine optimal inventory policies. Furthermore, Wu et al. [76] and Zhang et al. [87] obtained the optimal decisions of a two-level supply chain under centralized and decentralized decision-making by assuming that demand was dependent on trade credit. Abdul-Jalbar et al. [1] evaluated the effect of trade credit on the profit of a two-level supply chain, including one seller and two buyers, under centralized and decentralized models. Chen and Kang [6] presented an inventory model considering a one-level trade credit under decentralized and centralized decision-making. A price negotiation procedure was also proposed to coordinate the decisions. Chen and Kang [7] developed their previous model by assuming that the retailer would credit customers whose demand depended on price. Similarly, a two-level supply chain was presented with the assumption that demand was an exponential function of trade credit [35, 39]. The manufacturer proposed a delay in the payment contract to make coordinated decisions. In some other studies [10, 44, 46], trade credit was compared with bank credit in a two-level supply chain using the newsvendor model. Chen [10] noted that trade credit was more profitable for retailers and manufacturers than bank credit. Firms that consider sustainability in their activities experience better performance and market values [30, 31]. Recently, a sustainable supply chain was developed under a trade credit policy by considering different carbon emission controls [19, 45, 68]. Yang et al. [85] formulated a green supply chain model with budget constraints, including one manufacturer and two retailers. They investigated the effects of trade credit and bank credit strategies on the supply chain profitability. Others considered investment in the preservation technology for perishable products under trade credit [33, 56, 57, 82]. Shah et al. [67] developed a trade credit inventory model with an uncertain demand by investing in preservation techniques to reduce the deterioration rate. Handa et al. [32] proposed a sustainable supply chain model by incorporating the impacts of trade credit financing, inflation rate, and investment in preservation technology. Mahato et al. [52] proposed a non-instantaneous deteriorating inventory model while considering a carbon tax policy, two-level credit financing, and investment in preservation technology.

The review of the related studies shows that the literature on trade credit is rich and has been studied under different assumptions. Also, many studies have been conducted on markdown policy and fast expansion, especially in inventory systems. However, no attempt has been made to address the combined impacts of trade credit, markdown, and preservation policies on a two-level supply chain for perishable items. Table 1 provides a summary of the reviewed papers and the main contributions of the present study.

Table 1 Comparison of this study with conducted research in the literature

References	Year	Deterioration rate	Delay-in-payment	Markdown	Preservation technology	Model type
Xu et al. [78]	2023	Time-varying	–	✓	–	Decentralized
Shah et al. [67]	2023	Constant	✓	✓	✓	EOQ
Moradi et al. [60]	2023	–	✓	–	–	EOQ
Mahato et al. [53]	2023	–	✓	–	–	EPQ
Mahato et al. [52]	2023	Time-varying	✓	–	✓	EOQ
Mahato and Mahata [51]	2023	–	✓	–	–	EOQ
Mahato and Mahata [50]	2023	–	✓	–	–	EOQ
Hatibaruah and Saha [33]	2023	Time-varying	✓	–	✓	EPQ
Zhao et al. [88]	2022	–	–	✓	–	EOQ
Wang et al. [73]	2021	Time-varying	–	✓	–	EOQ
Chen et al. [11]	2019	Time-varying	–	✓	–	Decentralized
Chen and Chen [9]	2021	Time-varying	–	✓	–	EOQ
Dai and Wang [16]	2022	Constant	✓	–	–	Integrated
Khan et al. [42]	2020	–	✓	–	–	EOQ
Mashud et al. [58]	2020	Constant	✓	–	–	EOQ
Murmu et al. [61]	2023	Constant	✓	–	✓	EOQ
Momena et al. [59]	2023	Constant	✓	–	–	EOQ
Yang [84]	2022	Constant	✓	–	–	EOQ
This paper	–	Time-varying	✓	✓	✓	Coordination

Therefore, analyzing these policies in a coordinated two-level supply chain can add value and richness to the existing literature. To our knowledge, the present study is the first attempt to consider this issue. The contributions of the study are as follows:

- All the possible relationships among the sales cycle, time of markdown, and trade credit are dealt with.
- Demand is linearly dependent on price and time variation.
- The product perishability rate is assumed to be time-dependent.
- Decisions of supply chain members under decentralized and centralized decision models are evaluated, and channel coordination is established through wholesale price contracts.
- Preservation technology is considered to extend the product life cycle.

These five features make the present study unique in the supply chain literature.

3 Problem formulation

3.1 Notations and assumptions

The notations are summarized in Table 2, and the mathematical relations presented in Sect. 4 are based on the following assumptions:

(1) $d\left(p_i, t\right) = a - \beta p_i - kt$ as presented in [41].
(2) $\theta(t) = \frac{1}{(1+n-t)}$ where n is the maximum lifetime of the product.
(3) Delivery of the products from the manufacturer to the retailer is immediate, and the delivery time is zero.
(4) No shortage is allowed.
(5) Since the retailer has financial constraints, the manufacturer offers him a trade credit.
(6) Wholesale price (w) is a function of the trade credit period (M): $w = g + lM$. g > 0 represents the cost of each product unit in the absence of trade credit,and $l > 0$ represents the sensitivity factor [17, 25].
(7) The cost of investment opportunity is imposed on the manufacturer due to offering trade credit.
(8) Investment in preservation technology reduces the rate of product loss: $Z(\delta) = 1 - e^{-\lambda\delta}$.

3.2 Model description

The proposed supply chain involves a manufacturer as a leader and a retailer with financial constraints as a follower. The manufacturer delivers product unit Q_0 to the

Table 2 Model symbols

Ic	Earned interest rate per year	Ar	Retailer's ordering cost per order
Ie	Paid interest rate per year	As	Manufacturer's ordering cost per order
Ii	Investment opportunity cost rate per year	h	Holding cost per unit
a	Initial demand	c	Manufacturer's production cost per unit
k	The coefficient for the sensitivity of demand to time	β	The coefficient for the sensitivity of demand to price
P_2	Marked down price per unit	P_1	The initial price per unit
T	Sales cycle length (decision variable) in month	td	Markdown time (decision variable) in month
w	Wholesale price (decision variable) per unit	M	Trade credit period (decision variable) in month
d_1	Demand rate before markdown	d_2	Demand rate after markdown
Q_1	Amount of the products sold before markdown	Q_2	Amount of the products sold after markdown
TPs_i	Manufacturer's total profit under model i	Q_0	Retailer's order quantity
TPr_{ij}	Retailer's total profit in model i under case j	$TPrs_{ij}$	Supply chain total profit in model i under case j
λ	Efficiency coefficient for preservation technology	δ	Preservation technology cost

retailer at wholesale price w per unit, giving him the trade credit to pay for the products after the expiration of period M. The trade credit allows the retailer to deposit the total revenue from the sale of the products in the bank until the expiration period M and receive an interest at the rate of Ie. With the expiration of moment M, the purchase price of the products must be paid to the manufacturer, but the retailer can only pay the purchase price of the products sold until M. After moment M, for the amount of the products left in stock, the purchase price must be paid with interest at the rate of Ic as soon as the products are sold.

Time-sensitive demand and time-varying perishability make it challenging to manage sales. Therefore, retailers adopt two strategies to improve their cost and revenue management. The strategies include a) investing in the preservation technology to reduce the deterioration rate and b) selling all the products at the same price or discounting the product price at an appropriate time to reinforce the demand rate and the sales volume. With the markdown policy, the retailer starts selling the products at the price of p_1 and then, at time $td(< T)$, reduces the price of the products, and sells them at the price $p_2\left(< p_1\right)$. In other words, the retailer sells a part of the products at the price of p_1 and the demand rate of $d_1(p_1, t)$ and another part of the product at the price of p_2 and the demand rate of $d_2(p_2, t)$. When $td = T$, all the product is sold at the price of p_1. When $td = 0$, all the product is sold at the price p_2. Depending on the retailer's chosen policy and the possible relationships among T, M, and td, there are several cases, each of which involves a specific earned and paid interest:

$case\ 1: M \le td < T$

$case\ 2: td \le M \le T$

$case\ 3: td < T \le M$

The following introduces the profit relationship between the manufacturer and the retailer in different cases. Then, the optimal values of the variables are obtained in decentralized and centralized decision-making models.

3.2.1 Manufacturer problem

The manufacturer's total profit is:

$$TPs_i = \frac{(w-c)Q_0}{T} - \frac{As}{T} - \frac{IiMQ_0w}{T} \quad i = 1,2,3 \tag{1}$$

The first and the second terms represent the revenue from the sale of products and the cost of ordering, and the last term represents the cost of the manufacturer's investment opportunity due to granting trade credit.

3.2.2 Retailer problem

If the retailer chooses the markdown policy, the demand in each sales cycle is as follows:

$$d = \begin{cases} d_1(t) = d_1\left(p_1, t\right) & 0 \le t < td \\ d_2(t) = d_2\left(p_2, t\right) & td \le t \le T \end{cases}$$

During the [0 T] period, the inventory level decreases due to the demand and product perishability. Thus, the inventory level for the retailer at any time is as follows. With the boundary condition $I_2(T) = 0$ and $I_1(td) = I_2(td)$, there are:

$$\frac{dI_1(t)}{dt} = -d_1\left(p_1, t\right) - (1 - Z(\delta))\theta(t)I_1(t) \quad 0 \le t \le td$$

$$\frac{dI_2(t)}{dt} = -d_2\left(p_2, t\right) - (1 - Z(\delta))\theta(t)I_2(t) \quad td \le t \le T$$

$$I_1(t) = e^{-v(t)} \int_0^t -d_1\left(p_1, s\right)e^{v(s)}ds + Q_0e^{-v(t)},$$

$$I_2(t) = e^{-v(t)} \int_t^T d_2\left(p_2, s\right)e^{v(s)}ds$$

$$v(t) = (1 - Z(\delta)) \int_0^t \theta(s)ds$$

$$\theta(t) = 1/(1 + n - t)$$

In addition, $I_1(0) = Q_0$ represents the inventory at the moment $t = 0$ or the retailer's order quantity received from the manufacturer.

The total holding cost is:

$$HC = h\left(\int_0^{td} I_1(t)dt + \int_{td}^{T} I_2(t)dt\right)$$

The revenue of the products sold before the markdown (Q_1) and after the markdown (Q_2) is as follows:

$$p_1Q_1 = p_1\int_0^{td} d_1(p_1, t)dt \quad p_2Q_2 = p_2\int_{td}^{T} d_2(p_2, t)dt$$

The interest earned and paid differs depending on each case:

Case 1:$M \leq td < T$

Case 1 indicates that the retailer reduces the product's sales price at moment M or later. The retailer deposits the sales revenue in the bank to earn an interest at the rate of Ie. The earned interest is as follows [21].

$$IE_1 = Iep_1\left(\int_0^{M}\int_0^{t} d_1(p_1, u)dudt\right)$$

Since the trade credit period expires before the end of the sales cycle, the retailer only pays the purchasing price of the products sold until moment M to the manufacturer and pays the interest on the products left in stock at the rate of Ic. The paid interest is equal to:

$$IP_1 = Icw\left(\int_M^{td} I_1(t)dt + \int_{td}^{T} I_2(t)dt\right)$$

Finally, the retailer's total profit in the first case is as follows:

$$\begin{aligned}&TPr_{i1} = \frac{1}{T}\left(p_1Q_1 + p_2Q_2 - wQ_0 - Ar - HC + IE_1 - IP_1 - \delta\right)\ i = 1, 2, 3\\&S.t.\\&M \leq td < T, \delta \geq 0\end{aligned} \tag{2}$$

Case 2:$td \leq M \leq T, td \neq T$

Case 2 implies that the retailer reduces the price of the products before the expiration of period M. Thus, the interest earned is equal to:

$$\begin{aligned}IE_2 =&Ie\Bigg[p_1\left(\int_0^{td}\int_0^{t} d_1(p_1, u)dudt + (M - td)\int_0^{td} d_1(p_1, u)du\right)\\&+p_2\left(\int_{td}^{M}\int_{td}^{t} d_2(p_2, u)dudt\right)\Bigg]\end{aligned}$$

The paid interest is equal to:

$$IP_2 = Icw\int_M^T I_2(t)dt$$

Thus, the retailer's total profit in the second case is as follows:

$$\begin{aligned} & TPr_{i2} = \frac{1}{T}\left(p_1Q_1 + p_2Q_2 - wQ_0 - Ar - HC + IE_2 - IP_2 - \delta\right) \quad i = 1,2,3 \\ & S.t. \\ & 0 \le td \le M \le T, \delta \ge 0 \end{aligned} \tag{3}$$

Case 3:$td < T \le M$

Case 3 expresses that all the products are sold before the expiration of the trade credit period. So, there is no interest paid. The interest earned is equal to:

$$\begin{aligned} IE_3 = Ie\Big[& p_1\Big(\int_0^{td}\int_0^t d_1\left(p_1, u\right)dudt + (M - td)\int_0^{td} d_1\left(p_1, u\right)du\Big) \\ & + p_2\Big(\int_{td}^T\int_{td}^t d_2\left(p_2, u\right)dudt + (M - T)\int_{td}^T d_2\left(p_2, u\right)du\Big)\Big] \end{aligned}$$

$$IP_3 = 0$$

Finally, the retailer's total profit in the third case is equal to:

$$\begin{aligned} & TPr_{i3} = \frac{1}{T}\left(p_1Q_1 + p_2Q_2 - wQ_0 - Ar - HC + IE_3 - \delta\right) \quad i = 1,2,3 \\ & S.t. \\ & 0 \le td < T \le M, \delta \ge 0 \end{aligned} \tag{4}$$

4 Proposed method

This section examines the behavior of the manufacturer and the retailer in centralized and decentralized decision-making models.

4.1 Decentralized decision-making (Model 1)

As the leader, the manufacturer first announces the values of M and w. Then the retailer, as the follower, evaluates the three cases and finally selects the case with the most profit. If $T \neq td$, the retailer chooses the markdown policy and then announces the optimal values of td and T. If $T = td > 0$ or $td = 0$, it means the retailer should choose a fixed price policy and only announce the optimal value of T. The following is a study of the optimal decisions made by the manufacturer and the retailer in a decentralized decision-making model.

4.1.1 Retailer's optimal solution

With specific values of M and w, the retailer calculates the maximum profit in each case separately. To this end, all the feasible interior and boundary solutions are evaluated, and the solution with the maximum value of the objective function is introduced as the optimal solution for that case [4]. The following proves that each case has only one interior solution, and all the boundary solutions are demonstrated.

The optimal solution of the retailer in the first case ($M \le td < T$).

Theorem 1 (1–1) *For the given values of* $\delta \ge 0$, $M \ge 0$, *and* $td \ge 0$, TPr_{11} *is strictly pseudo-concave to T.*

(1–2) For the given values of $\delta \ge 0$, $M \ge 0$, and $T \ge 0$, TPr_{11} is concave to td.

(1–3) For the given values of $M \ge 0$, $td \ge 0$, and $T \ge 0$, TPr_{11} is concave to δ.

Proof: See Appendix A.

Based on Theorem 1, TPr_{11} has an extreme point. By the simultaneous solving of Eqs. (5–7) the interior solution $(td_{11}, T_{11}, \delta_{11})$ for the function TPr_{11} is feasible if the constraint $M \le td < T, \delta \ge 0$ is satisfied.

$$\begin{aligned}\frac{\partial TPr_{11}}{\partial T} =& \frac{1}{T^2}\Bigg[-Icw\left(\int_M^{td} A_1{}^{-1}(t)\left(-\int_{td}^{T} d_2(s)A_1(s)ds + \int_{td}^{t} d_1(s)A_1(s)ds\right)dt\right) \\ & -h\left(\int_0^{td} A_1{}^{-1}(t)\left(-\int_{td}^{T} d_2(s)A_1(s)ds + \int_{td}^{t} d_1(s)A_1(s)ds\right)dt\right) \\ & +(Icw+h)\left(\int_{td}^{T}\int_t^T A_7(t)A_6(s)dsdt\right) - Icd_2(T)Tw\int_M^T \frac{A_1(T)}{A_1(t)}dt \\ & -d_2(T)Th\int_0^T \frac{A_1(T)}{A_1(t)}dt - A_6(T)\,Tw - Iep_1\int_0^M\int_0^t d_1(s)dsdt \\ & +d_2(T)Tp_2 - p_1\int_0^{td} d_1(t)dt - p_2\int_{td}^{T} d_2(t)dt \\ & +w\left(\int_0^t d_1(t)A_1(t)dt + \int_{td}^{T} d_2(s)A_1(s)ds\right) + \delta + Ar\Bigg] = 0\end{aligned} \tag{5}$$

$$\frac{\partial TPr_{11}}{\partial td} = \frac{1}{T}\left[\begin{array}{c} -\mathrm{wIc}\big(d_1(td)-d_2(td)\big)\int_M^{td}\frac{A_1(\mathrm{td})}{A_1(\mathrm{s})}ds - h(d_1(td)-d_2(td))\int_0^{td}\frac{A_1(\mathrm{td})}{A_1(\mathrm{s})}ds \\ -w\big(d_1(td)-d_2(td)\big)A_1(td) + d_1(td)p_1 - d_2(td)p_2 \end{array}\right] = 0 \tag{6}$$

$$\frac{\partial TPr_{11}}{\partial \delta}=\frac{1}{T}\Bigg[Icw\lambda\bigg(\int_M^{td}\bigg(-A_7(t)\bigg(\int_{td}^{t}A_3(s)ds\bigg)$$
$$+A_7(t)\int_{td}^{T}A_4(s)ds+A_2(t)e^{-2\theta(t)}\bigg(-\bigg(\int_{td}^{T}A_6(s)ds\ \ +\int_{td}^{t}A_5(s)ds\bigg)\bigg)dt\bigg)$$
$$+h\lambda\bigg(\int_0^{td}\big(-A_7(t)(\ \int_{td}^{t}A_3(s)ds\big)\ +A_7(t)\bigg(\int_{td}^{T}A_4(s)\,ds)+A_2(t)e^{-2\theta(t)}\bigg(-\int_{td}^{T}A_6(s)ds\ \ +\int_{td}^{t}A_5(t)ds\bigg)\bigg)dt\bigg)$$
$$+\lambda(Icw+h)\bigg(\int_{td}^{T}\bigg(-A_2(t)e^{-2\theta(t)}\int_t^T A_6(s)ds+A_7(t)\quad\int_t^T A_4(s)ds\bigg)dt\bigg)$$
$$+w\lambda\int_0^{td}A_3(t)dt+w\lambda\int_{td}^{T}A_4(t)dt-1\Bigg]=0 \tag{7}$$

$A_1, A_2, ...A_{13}$ are demonstrated in Appendix C.

Then, the only boundary to be considered is as follows:

$$a1)M = td, td < T$$

As this boundary is placed in Eqs. (5) and (7), the boundary solution $({td_{11}}^{b} = M, {T_{11}}^{b}, {\delta_{11}}^{b})$ is obtained. This solution is accepted if the constraint $td < T, \delta \geq 0$ is satisfied. Finally, the profits gained from the feasible solutions are compared, and the retailer's maximum profit is determined in the first case.

The optimal solution of the retailer in the second case ($0 \leq \mathrm{td} \leq \mathrm{M} \leq \mathrm{T}, \mathrm{td} \neq \mathrm{T}$).

Theorem 2 (2–1)*For the given values of* $\delta \geq 0$, $M \geq 0$, *and* $td \geq 0$, TPr_{12} *is strictly pseudo-concave to* T.

(2–2)For the given values of $\delta \geq 0$, $M \geq 0$, and $T \geq 0$, TPr_{12} is concave in td.

(2–3)For the given values of $M \geq 0$, $td \geq 0$, and $T \geq 0$, TPr_{12} is concave to δ.

The proof is similar to that of theorem 1.

Based on Theorem 2, TPr_{12} has an extreme point. By the simultaneous solving of Eqs. (9–11) The interior solution $(td_{12}, T_{12},\delta_{12})$ for the function TPr_{12} is feasible if it satisfies the constraint $0 \leq td \leq M \leq T, td \neq T, \delta \geq 0$.

$$\begin{aligned}\frac{\partial TPr_{12}}{\partial T} = \frac{1}{T^2}\Bigg[&-h(\int_0^{td} A_7(t)\left(-\int_{td}^{T} A_6(s)ds + \int_{td}^{t} A_5(s)ds\right)dt) \\ &+h\left(\int_{td}^{T}\int_{t}^{T} A_7(t)A_6(s)dsdt\right) + Icw\left(\int_{M}^{T}\int_{t}^{T} A_7(t)A_6(s)dsdt\right) \\ &-Icd_2(T)Tw\int_M^T \frac{A_7(t)}{A_7(T)}dt - d_2(T)Th\int_0^T \frac{A_7(t)}{A_7(T)}dt \\ &+w\left(\int_{td}^{T} A_6(s)ds + \int_0^{td} A_5(t)dt\right) - Iep_2\int_{td}^{M}\int_{td}^{t} d_2(s)dsdt \\ &-Iep_1\int_0^{td}\int_0^{t} d_1(s)dsdt - A_7^{-1}(T)d_2(T)Tw \\ &-(1+(M-td)Ie)p_1\int_0^{td} d_1(t)dt + d_2(T)Tp_2 \\ &-p_2\int_{td}^{T} d_2(t)dt + \delta + Ar\Bigg] = 0\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial TPr_{12}}{\partial td} = \frac{1}{T}\Bigg[&-h(d_1\dot{(}td) - d_2(td))\left(\int_0^{td}\frac{A_7(s)}{A_7(td)}ds\right) \\ &-w\big(-d_2(td) + d_1(td))A^{-1}{}_7(td) + \big(p_1d_1(td) \\ &-p_2d_2(td)\big)(1+(M-td)Ie)\Bigg] = 0\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial TPr_{12}}{\partial \delta} = \frac{1}{T}\Bigg[&-Icw\lambda\left(\int_M^T\left(A_8(t)\theta(t)\int_t^T A_6(s)ds - A_7(t)\int_t^T A_4(s)ds\right)dt\right) \\ &-h\lambda(\int_0^{td}(A_7(t)\int_{td}^{t} A_3(t)ds - A_7(t)\int_{td}^{T} A_4(s)ds - A_8(t)(-\int_{td}^{T} A_6(s)ds \\ &+\int_{td}^{t} A_5(s)ds))dt) - h\lambda(\int_{td}^{T}\left(A_8(t)\theta(t)\int_t^T A_6(t)ds\right. \\ &\left.-A_7(t)\int_t^T A_4(s)ds\right)dt) + w\lambda\left(\int_{td}^{T} A_4(t)dt + \int_0^{td} A_3(s)dt\right) - 1\Bigg] = 0\end{aligned} \tag{10}$$

Then, the boundary solutions $(td_{12}{}^{b}, T_{12}{}^{b}, \delta_{12}{}^{b})$ are examined based on the following constraints:

$$b1)td = 0, M < T$$

$$b2)0 < t, M = T$$

$$b3)td = M, M < T$$

$$b4)td = 0, M = T$$

For b1, by the placement of $td = 0$ in Eqs. (8) and (10), the solution $({td_{12}}^{b} = 0, {T_{12}}^{b}, {\delta_{12}}^{b})$ is feasible if it makes sense under $M < T, \delta \geq 0$. For the b2 case, the solution $({td_{12}}^{b}, {T_{12}}^{b} = M, {\delta_{12}}^{b})$ is obtained by the placement of $M = T$ in Eqs. (9) and (10), and this solution is feasible if $0 < td$. For the case b3, the solution $({td_{12}}^{b} = M, {T_{12}}^{b}, {\delta_{12}}^{b})$ obtained by the placement of $td = M$ in Eqs. (8) and (10) are accepted if $M < T, \delta \geq 0$. The case b4 is always a feasible solution. Finally, the optimal solution of the retailer in the second case is obtained by comparing the profits of feasible solutions.

The optimal solution of the retailer in the third case (0 ≤ td < T ≤ M).

Theorem 3 (3–1) *For the given values of $\delta \geq 0$, $M \geq 0$, and $td \geq 0$, TPr_{13} is strictly pseudo-concave to T.*

(3–2) For the given values of $\delta \geq 0, M \geq 0$, and $T \geq 0, TPr_{13}$ is concave to td.

(3–3) For the given values of $M \geq 0, td \geq 0$, and $T \geq 0, TPr_{13}$ is concave to δ.

The proof is similar to that of theorem 1.

Theorem 3 demonstrates that, TPr_{13} has an extreme point. By the simultaneous solving of Eqs. (11–13), the interior solution $\left(td_{13}, T_{13}, \delta_{13}\right)$ for the function TPr_{13} is accepted if it is true in the constraint $0 \leq td < T \leq M, \delta \geq 0$.

$$\frac{\partial TPr_{13}}{\partial T} = \frac{1}{T^2}\left[\begin{array}{l} h\left(\int_0^{td} A_7(t)\left(\int_{td}^{T} A_6(s)\mathrm{d}s + \int_t^{td} A_5(s)\mathrm{d}s\right)\mathrm{d}t\right) + h\int_{td}^{T}\int_t^{T} A_7(t)A_5(s)\mathrm{d}s dt \\ -h\left(\int_0^{td} A_7(t)\left(\int_{td}^{T} A_6(s)\mathrm{d}s + \int_t^{td} A_5(s)\mathrm{d}s\right)\mathrm{d}t\right) + h\int_{td}^{T}\int_t^{T} A_7(t)A_5(s)\mathrm{d}s dt \\ -Iep_1\int_0^{td}\int_0^{t} d_1(s)\mathrm{d}s\mathrm{d}t - Td_2(T)wA_7^{-1}(T) - p_2(1 + (M-T)Ie)\int_{td}^{T} d_2(s)\mathrm{d}s \\ -p_1(1 + (M-td)Ie)\int_0^{td} d_1(t)\mathrm{d}t + p_2T(1 + (M-T)Ie)d_2(T) + Ar + \delta \end{array}\right] = 0 \tag{11}$$

$$\begin{aligned} \frac{\partial TPr_{13}}{\partial td} = \frac{1}{T}\Big[&-\big(d_1(td) - d_2(td)\big)\left(\int_0^{td}\frac{A_7(s)}{A_7(td)}ds\right)h \\ &-w\big(d_1(td) - d_2(td)\big){A_7}^{-1}(td) + \big(p_1d_1(td) \\ &-p_2d_2(td)\big)(1 + (M-td)Ie)\Big] = 0 \end{aligned} \tag{12}$$

$$\frac{\partial TPr_{13}}{\partial \delta} = \frac{1}{T}\left[\begin{array}{c} -h\lambda\left(\begin{array}{c}\int_0^{td}(-A_7(t)\int_t^{td}A_3(s)ds - A_7{}^{-1}(t)\int_{td}^{T}A_4(s)ds+ \\ A_4(t)\theta(t)\left(\int_{td}^{T}A_6(t)ds + \int_t^{td}A_5(s)ds\right)dt\end{array}\right) \\ -h\lambda\left(\int_{td}^{T}\left(A_8(t)\theta(t)\int_t^{T}A_6(s)ds - A_7(t)\int_t^{T}A_4(t)ds\right)dt\right) + \lambda w\int_{td}^{T}A_4(s)ds \\ \\ -1 + \lambda w\int_0^{td}A_3(s)ds \end{array}\right] = 0 \tag{13}$$

Then, the boundary solutions are examined according to the following constraints:

$$c1)td = 0, T < M$$

$$c2)0 < td, M = T$$

$$c3)td = 0, T = M$$

To examine c1, by the placement of $td = 0$ in Eqs. (11) and (13), $({td_{13}}^b = 0, {T_{13}}^b, {\delta_{13}}^b)$ is feasible if it is true in $T < M, \delta \geq 0$. The boundary solutions c2 and c3 are the same as those of b2 and b4. The optimal value of the retailer in the third case is obtained by comparing the profits of feasible solutions.

Finally, a combination of the above results yields the retailer's maximum total profit as follows:

$$TPr_1\left({td_1}^*, {T_1}^*, {\delta_1}^*\right) = \max\left(TPr_{11}, TPr_{12}, TPr_{13}\right) \tag{14}$$

4.1.2 Manufacturer's optimal solution

Using the following algorithm, the manufacturer problem is solved, and the optimal values $(M^*, {td_1}^*, {T_1}^*, {\delta_1}^*)$ are ultimately determined for both members of the supply chain:

Step 1 Input the values of all the parameters.

Step 2 Assign $M = 0$.

Step 3 Find $TPr_1\left({td_1}^*, {T_1}^*, {\delta_1}^*\right)$.

Step 4 Substitute $\left({td_1}^*, {T_1}^*, {\delta_1}^*\right)$ in the manufacturer problem and save the manufacturer's profit using Eq. (1) for the current solution $\left(M, {td_1}^*, {T_1}^*, {\delta_1}^*\right)$.

Step 5: If $M < M_{max}$, then $M = M + \delta$ (δ *is* a *small prameter*). Go to Step 3.

Step 6: A combination of $\left(M^*, {td_1}^*, {T_1}^*, {\delta_1}^*\right)$ with the greatest manufacturer's profit is optimal.

4.2 Centralized (integrated) decision-making (Model 2)

In Model 2, all the decisions of the supply chain members are made in an integrated manner. The manufacturer and the retailer are considered units, and all the decisions are optimized from the whole supply chain perspective.

$$TPrs_{2i} = TPs_2 + TPr_{2i} \quad i = 1..3$$

The optimal solution of the first case $(0 \le M \le td < T)$

$$\begin{aligned} &Max\,TPrs_{21}(M, td, T, \delta) = TPs_2 + TPr_{21} \\ &s.t. \\ &0 \le M \le td < T, \delta \ge 0 \end{aligned} \tag{15}$$

Theorem 4 (4–1) *For the given values of* $\delta \ge 0, td \ge 0,$ *and* $M \ge 0, TPrs_{21}$ *is strictly pseudo-concave to T.*

(4–2) For the given values of $\delta \ge 0, M \ge 0,$ and $T \ge 0, TPrs_{21}$ is concave to td.
(4–3) For the given values of $\delta \ge 0, td \ge 0,$ and $T \ge 0, TPrs_{21}$ is concave to M.
(4–4) For the given values of $M \ge 0, td \ge 0$ and $T \ge 0, TPrs_{21}$ is concave to δ.
Proof: See Appendix B.

Based on Theorem 4, $TPrs_{21}$ has an extreme point. By the simultaneous solving of Eqs. (16–19), the interior point $(M_{21}, td_{21}, T_{21}, \delta_{21})$ is feasible if it is true in the constraint $0 \le M \le td < T, \delta \ge 0$.

$$\begin{aligned} \frac{\partial TPrs_{21}}{\partial T} = \frac{1}{T^2}\Bigg[& Icw\left(\int_M^{td} A_7(t)\left(\int_{td}^{T} A_6(t)ds + \int_t^{td} A_5(t)\right)dt\right) \\ & + h\int_0^{td} A_7(t)\left(\int_{td}^{T} A_6(t)ds + \int_t^{td} A_5(t)ds\right)dt\Bigg) \\ & + (Icw + h)\int_{td}^{T} A_7(t)\int_t^{T} A_6(t)dsdt\Bigg) - Icd_2(T)Tw\int_M^{T} \frac{A_7(t)}{A_7(T)}dt \\ & - d_2(T)Th\int_0^{T} \frac{A_7(t)}{A_7(T)}dt + (IiwM + c)\int_{td}^{T} A_6(t)ds \\ & + (\mathrm{Ii}wM + c)\int_0^{td} A_5(t)dt - \mathrm{Ie}p_1\int_0^{M}\int_0^{t} d_1(s)dsdt \\ & - Td_2(T)(\mathrm{Ii}wM + c)A_7{}^{-1}(T) + d_2(T)Tp_2 \\ & - p_1\int_0^{td} d_1(t)dt - p_2\int_{td}^{T} d_2(t)dt + Ar + As + \delta\Bigg] = 0 \end{aligned} \tag{16}$$

$$\frac{\partial TPrs_{21}}{\partial td} = \frac{1}{T}\Big[-Ic\big(d_1(td) - d_2(td)\big)\int_M^{td}\frac{A_7(s)}{A_7(td)}dsw - \big(-h\big(-d_2(td) + d_1(td)\big)\int_0^{td}\frac{A_7(s)}{A_7(td)}ds - \big(-d_2(td) + d_1(td)\big)(IiMw + c)A_7{}^{-1}(td) + d_1(td)p_1 - d_2(td)p_2\Big] = 0 \tag{17}$$

$$\frac{\partial TPrs_{21}}{\partial M} = \frac{1}{T}\left[w\big(A_7(M)Ic - Ii\big)\int_{td}^{T}A_6(s)ds - IcwA_7(M)\int_{td}^{M}A_5(s)ds + Iep_1\int_0^M d_1(s)ds - Iiw\int_0^{td}A_5(s)ds\right] = 0 \tag{18}$$

$$\begin{aligned}\frac{\partial TPrs_{21}}{\partial \delta} = \frac{1}{T}\Bigg[&Icw\lambda\Bigg(\int_M^{td}\Bigg(-A_7(t)\int_t^{td}A_5(s)ds \\ &-A_7(t)\int_{td}^{T}A_4(s)ds + A_8(t)\theta(t)\left(\int_{td}^{T}A_6(s)ds + \int_t^{td}A_5(t)ds\right)\Bigg)dt\Bigg) \\ &-h\lambda\Bigg(\int_0^{td}\Bigg(-A_7(t)\int_t^{td}A_3(s)ds - A_7(t) \\ &\int_{td}^{T}A_4(t)ds + A_8(t)\theta(t)\left(\int_{td}^{T}A_6(t)ds + \int_t^{td}A_5(t)ds\right)dt\Bigg) \\ &-\lambda(Icw + h)\left(\int_{td}^{T}\left(-A_8(t)\theta(t)\int_t^{T}A_6(t)ds - A_7(t)\int_t^{T}A_4(t)ds\right)dt\right) \\ &+\lambda(IiMw + c)\int_{td}^{T}A_4(t)dt + (IiMw + c)\lambda\int_0^{td}A_3(t)dt - 1\Bigg] = 0\end{aligned} \tag{19}$$

Then, boundary solutions are considered with respect to the following constraints:

$$d1)M = 0, td < T$$

$$d2)td = M, M < T$$

For the case d1, when $M = 0$ is replaced in Eqs. (17–20), the solution $({M_{21}}^b = 0, {td_{21}}^b, {T_{21}}^b, {\delta_{21}}^b)$ is feasible if it is true in the constraint $td < T, \delta \geq 0$. For the case d2, when $td = M$ is replaced in Eqs. (17), (18) and (20) the solution $({M_{21}}^b, {td_{21}}^b = {M_{21}}^b, {T_{21}}^b, {\delta_{21}}^b)$ is feasible if it makes sense when $M < T, \delta \geq 0$. The optimal solution results from comparing the profits of the three possible solutions.

The optimal solution of the second case $(0 \leq td \leq M \leq T, td \neq T)$

Table 3 The parameter values of example 1

a	100	Ar	4000	h	0.1	Ic	0.15
β	0.85	As	800	c	1	Ie	0.01
k	4	p_1	60	g	15	Ii	0.15
n	12	p_2	48	l	0.3	λ	0.05

Table 4 The optimal solutions for models 1, 2, and 3

Model	Optimal case	M	T	td	δ	TPs	TPr	$TPrs$
Decentralized	*Case*3	10.92	6.84	6.00	84.51	425.93	949.89	1375.82
Centralized	*Case*3	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
Coordination		9.68	6.88	2.77	54.82	438.91	983.76	1422.67
		$x \in [0.88\ 0.94] x_0 = 0.9$						

$$\begin{aligned} &Max\ TPrs_{22}(M, td, T, \delta) = TPs_2 + TPr_{22} \\ &s.t. \\ &0 \le td \le M \le T, td \ne T, \delta \ge 0 \end{aligned} \tag{20}$$

Theorem 5 (5–1) *For the given values of* $\delta \ge 0, td \ge 0,$ *and* $M \ge 0, TPrs_{22}$ *is strictly pseudo-concave to* T.

(5–2) For the given values of $\delta \ge 0, M \ge 0$, and $T \ge 0$, $TPrs_{22}$ is concave to td.
(5–3) For the given values of $\delta \ge 0, td \ge 0$, and $T \ge 0$, $TPrs_{22}$ is concave to M.
(5–4) For the given values of $M \ge 0, td \ge 0$, and $T \ge 0$, $TPrs_{22}$ is concave to δ.
The proof is similar to that of theorem 4.

Based on Theorem 5, $TPrs_{22}$ has an extreme point. Once Eqs. (22–25) are simultaneously solved, the interior point $(M_{22}, td_{22}, T_{22}, \delta_{22})$ is feasible if the constraint $0 \le td \le M \le T, td \ne T, \delta \ge 0$ is met. Then, all the boundary solutions

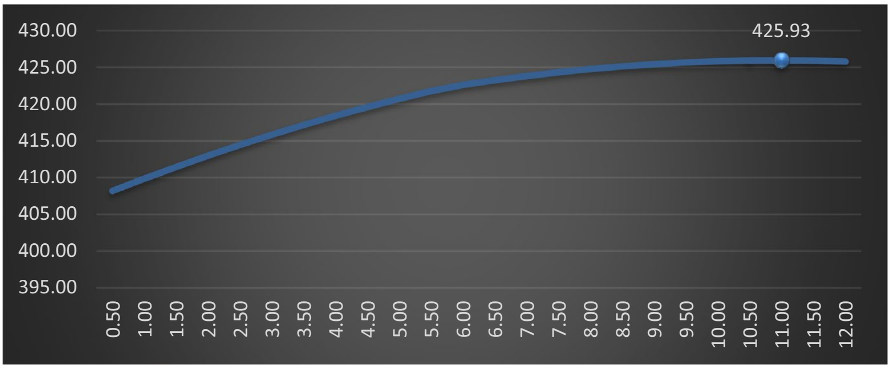

Fig. 1 Changes in the manufacturer's profit versus the changes of M in model 1

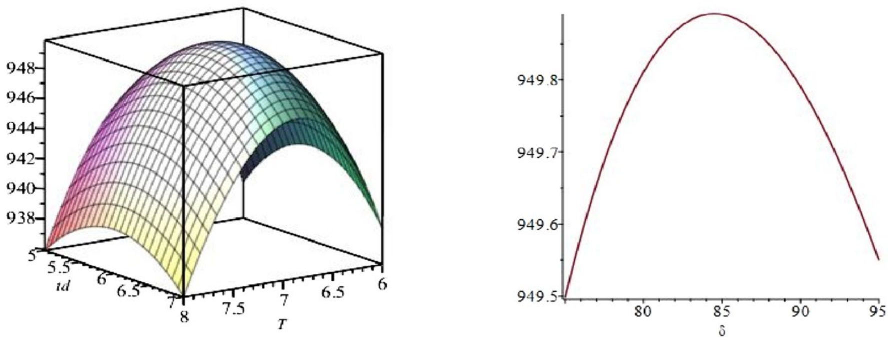

Fig. 2 Concavity of the retailer function at $T=6.84$, $td=6.00$ and $\delta = 84.51$ in model 1 when $M = 10.92$

from b1 to b4 are checked. An optimal solution results from comparing the profits of the feasible solutions.

$$\begin{aligned}\frac{\partial TPrs_{22}}{\partial T} = \frac{1}{T^2}\Bigg[\Bigg(&-h\bigg(\int_0^{td} A_7(t)\bigg(-\bigg(\int_{td}^{T} A_6(s)ds\bigg) + \int_{td}^{t} A_5(s)ds\bigg)dt\bigg) \\ &+h\bigg(\int_{td}^{T} A_7(t)\int_t^T A_6(s)ds\ dt) + Icw\bigg(\int_M^T A_7(t)\int_t^T A_6(s)dsdt\bigg) \\ &-Icd_2(T)Tw\int_M^T \frac{A_7(t)}{A_7(T)}dt - d_2(T)Th\int_0^T \frac{A_7(t)}{A_7(T)}dt \\ &+(IiMw + c)\bigg(\int_{td}^{T} A_6(s)\ ds + \int_0^{td} A_5(s)ds\bigg) \\ &-Iep_2\int_{td}^{M}\int_{td}^{t} d_2(s)dsdt - Iep_1\int_0^{td}\int_0^{t} d_1(s)dsdt \\ &-Td_2(T)(IiMw + c)A_7(T) - p_1(1 + (M - td)Ie)\int_0^{td} d_1(s)ds + d_2(T)Tp_2 \\ &-p_2\int_{td}^{T} d_2(s)ds + Ar + As + \delta\Bigg] = 0\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial TPrs_{22}}{\partial td} = \frac{1}{T}\Bigg[&h\big(d_1(td) - d_2(td)\big)\bigg(\int_0^{td}\frac{A_7(s)}{A_7(td)}ds\bigg) \\ &-\big(d_1(td) - d_2(td)\big)(IiMw + c)A_7{}^{-1}(td) \\ &+(1 + (M - td)Ie)\big(p_1 d_1(td) - d_2(td)p_2\big)\Bigg] = 0\end{aligned} \tag{22}$$

$$\frac{\partial TPrs_{22}}{\partial M} = \frac{1}{T}\left[\begin{array}{c} IcwA_7(M)\Big(\int_M^T A_6(s)ds\Big) - Iiw\Big(\int_{td}^T A_6(s)ds\Big) - Iiw\int_0^{td} A_5(t)ds + \\ Iep_2\int_{td}^M d_2(s)ds + p_1 Ie\int_0^{td} d_1(s)ds \end{array}\right] = 0 \tag{23}$$

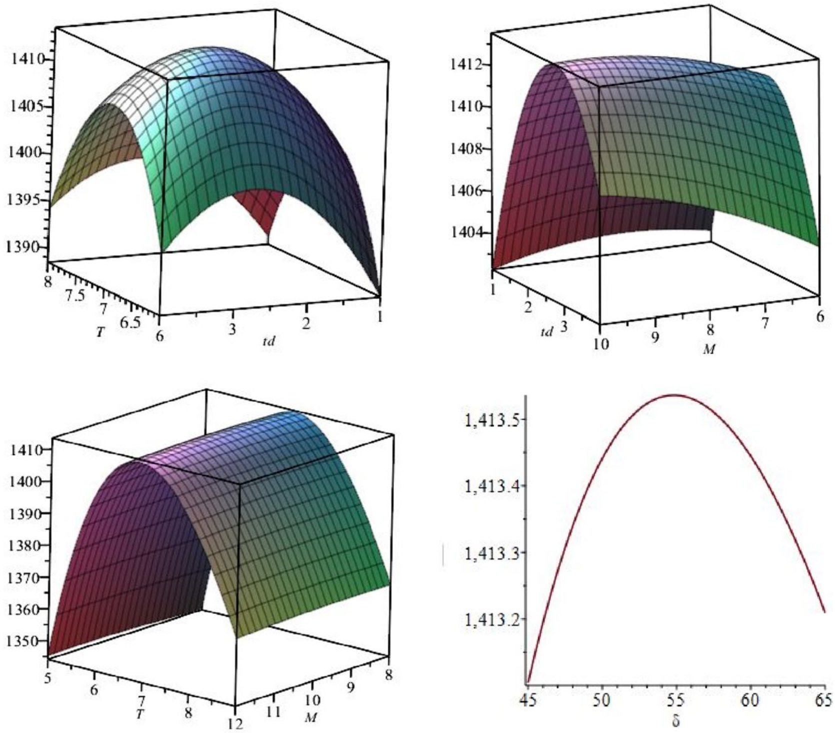

Fig. 3 Concavity of supply chain function at $M = 9.68$, $T = 6.88$, $td = 2.77$ and $\delta = 54.82$ in Model 2

Table 5 The optimal solutions, according to models 1, 2, and 3

Model	Optimal case	M	T	td	δ	TPs	TPr	$TPrs$
Decentralized	*Case*3	10.54	7.05	6.03	85.70	426.10	910.07	1336.17
Centralized	*Case*1	0.00	6.94	2.42	41.99	492.43	915.37	1407.79
Coordination		0.00	6.94	2.42	41.99	492.43	915.37	1409.12
		$x \in [0.898\ 1] x_0 = 0.95$						

$$\begin{aligned}\frac{\partial TPrs_{22}}{\partial \delta} = \frac{1}{T}\Bigg[& h\lambda\Bigg(\int_0^{td}\Big(-A_7(t)\int_{td}^{t}A_3(s)\mathrm{d}s \\ & + A_7(t)\textstyle\int_{td}^{T}A_3(s)ds + A_8(t)\Big(\int_{td}^{t}A_5(s)\mathrm{d}s - \int_{td}^{T}A_6(s)\mathrm{d}s\Big)\Big)\mathrm{d}t\Bigg) \\ & -h\lambda\Bigg(\int_{td}^{T}\Bigg(A_8(t)\theta(t)\int_t^{T}A_6(s)\mathrm{d}s - A_7(t)\int_t^{T}A_4(s)ds\Bigg)dt\Bigg) \\ & -Icw\lambda\Bigg(\int_M^{T}\Bigg(A_8(t)\theta(t)\int_t^{T}A_6(s)\mathrm{d}s - A_7(t)\int_t^{T}A_4(s)ds\Bigg)dt\Bigg) \\ & +\lambda(IiMw + c)\int_{td}^{T}A_4(t)\mathrm{d}t - 1 + \lambda(IiMw + c)\int_0^{td}A_3(t)dt\Bigg] = 0\end{aligned} \tag{24}$$

Table 6 The optimal solutions, according to models 1, 2 and 3

Model	Optimal case	M	T	td	δ	TPs	TPr	$TPrs$
Decentralized	$Case3$	10.68	6.98	6.02	85.31	426.07	922.75	1348.83
Centralized	$Case2$	4.55	6.90	2.52	46.69	504.08	897.84	1401.92
Coordination		4.55	6.90	2.52	46.69	46.69	968.61	1406.39
		$x \in [0.882\ 0.964] x_0 = 0.9$						

The optimal solution of the third case $(0 \le td < T \le M)$

$$Max\ TPrs_{23}(M, td, T, \delta) = TPs_2 + TPr_{23} \tag{25}$$

Theorem 6 (6–1) *For the given values of* $\delta \ge 0$, $td \ge 0$, *and* $M \ge 0$, $TPrs_{23}$ *is strictly pseudo-concave to* T.

(6–2) For the given values of $\delta \ge 0$, $M \ge 0$, and $T \ge 0$, $TPrs_{23}$ is concave to td.
(6–3) For the given values of $\delta \ge 0$, $td \ge 0$, and $T \ge 0$, $TPrs_{23}$ is concave to M.
(6–4) For the given values of $M \ge 0$, $td \ge 0$, and $T \ge 0$, $TPrs_{23}$ is concave to δ.
The proof is similar to that of theorem 4.

Based on Theorem 4, $TPrs_{23}$ has an extreme point. By the simultaneous solving of Eqs. (27–30), the interior point $(M_{23}, td_{23}, T_{23}, \delta_{23})$ is accepted if the constraint $0 \le td < T \le M$ is satisfied.

Table 7 The sensitivity analysis concerning a, β, k, p_1 and p_2

Model	Decentralized			Centralized						
Parameters	TPs	TPr	$TPrs$	M	T	td	δ	TPs	TPr	$TPrs$
$a = 95$	381.21	747.07	1128.27	7.90	7.02	1.39	50.66	458.80	706.52	1165.32
100	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
105	485.55	1165.99	1651.54	11.48	6.68	4.15	57.65	550.02	1125.03	1675.05
$\beta = 0.8$	455.77	1078.80	1534.57	11.55	6.68	4.21	56.59	515.89	1041.49	1557.39
0.85	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
0.9	410.95	831.98	1242.93	7.84	7.00	1.32	51.91	496.83	787.22	1284.05
$k = 3$	453.99	1099.85	1553.84	10.36	7.79	3.77	62.80	536.22	1052.12	1588.34
4	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
5	404.05	821.39	1225.44	9.23	6.23	2.19	48.41	475.30	783.51	1258.81

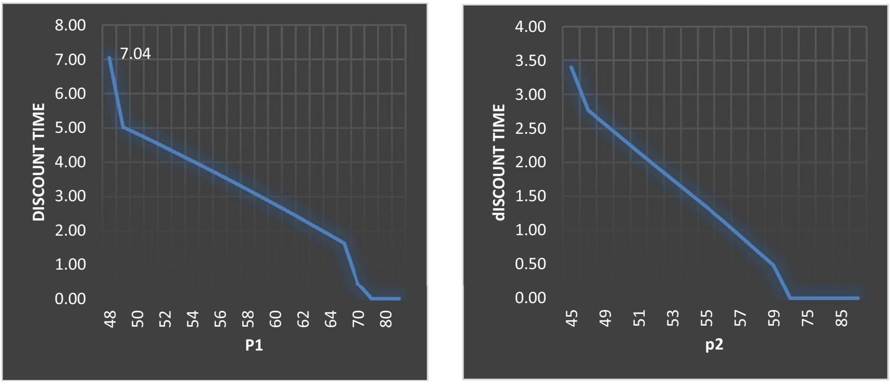

Fig. 4 Changes in *td* versus changes in p_1 and p_2

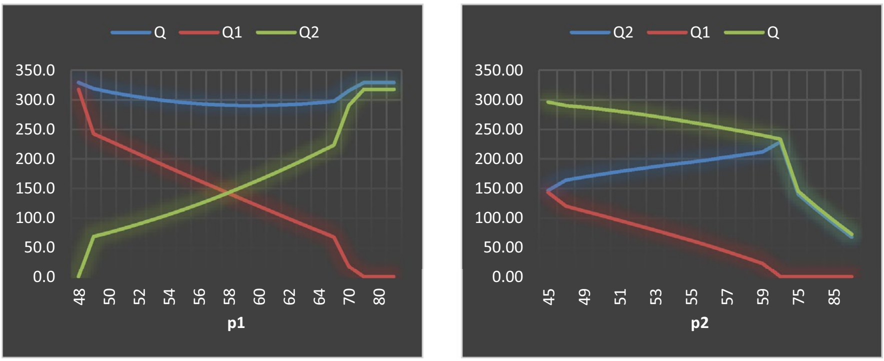

Fig. 5 Impact of the p_1 and p_1 changes on Q_0, Q_1 and Q_2

$$\frac{\partial TPrs_{23}}{\partial T} = \frac{1}{T^2}\Bigg[h\left(\int_0^{td} A_7(t)\left(\int_{td}^{T} A_6(s)\mathrm{d}s - \left(\int_{td}^{t} A_5(s)\mathrm{d}s\right)\right)\mathrm{d}t\right) + h\int_{td}^{T}\int_{t}^{T} A_7(t)A_6(t)\,\mathrm{d}s)\mathrm{d}t) - d_2(T)Th\left(\int_0^{T} \frac{A_7(t)}{A_7(T)}\mathrm{d}t\right) + \left(IiM^2l + IiMg + c\right)\int_{td}^{T} A_6(t)\mathrm{d}s + \left(IiM^2l + IiMg + c\right)\int_0^{td} A_5(s)\mathrm{d}s - Iep_2\int_{td}^{T}\int_{td}^{t} d_2(s)\mathrm{d}s\mathrm{d}t - Iep_1\int_0^{td}\int_0^{t} d_1(s)\mathrm{d}sdt - Td_2(p,T)\left(IiM^2l + IiMg + c\right)A_7^{-1}(T) - p_2(IeM - IeT + 1)\int_{td}^{T} d_2(s)\mathrm{d}s - (IeM - Ietd + 1)\, p_1\int_0^{td} d_1(s)\mathrm{d}s + p_2T(IeM - IeT + 1)d_2(T) + Ar + As + \delta\Bigg] = 0 \tag{26}$$

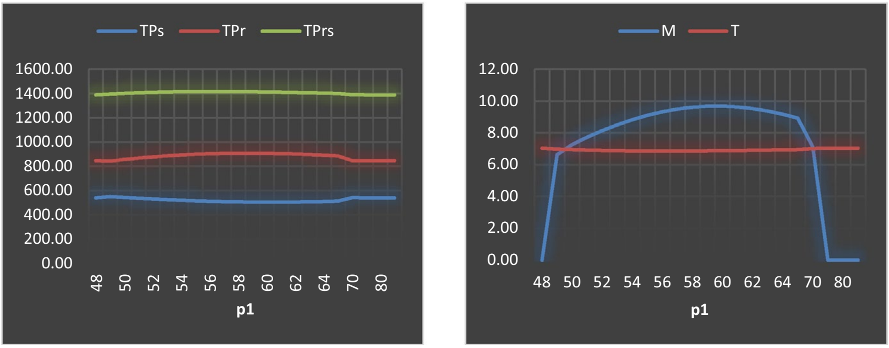

Fig. 6 Impact of p_1 changes on M, T, and all the member profits

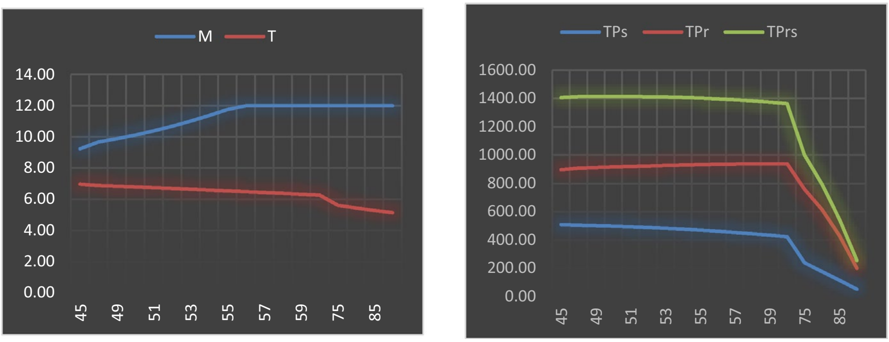

Fig. 7 Impact of p_2 changes on M, T and all the member profits

$$\begin{aligned}\frac{\partial TPrs_{23}}{\partial td} = \frac{1}{T}\Big[&-\big(d_1(td)-d_2(td)\big)\int_0^{td}\frac{A_7(s)}{A_7(td)}dsh \\ &-\big(d_1(td)-d_2(td)\big)\big(IiM^2l+IiMg+c)A_7{}^{-1}(td) \\ &+\big(p_1d_1(td)-p_2d_2(td)\big)(\text{ Ie } M-\text{ Ie } td+1)\Big]=0\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial TPrs_{23}}{\partial M} = \frac{1}{T}\Big[&-2\Big(Ml+\frac{g}{2}\Big)Ii\int_{td}^{T}A_6(s)ds-2\Big(Ml+\frac{g}{2}\Big) \\ &Ii\int_0^{td}A_5(t)ds+Ie(p_1\int_0^{td}d_1(s)ds+p_2\int_{td}^{T}d_2(s)ds)\Big]=0\end{aligned} \tag{28}$$

$$\frac{\partial TPrs_{23}}{\partial \delta} = \frac{1}{T}\Bigg(-h\lambda\Bigg(\int_0^{td} A_7(t)\int_{td}^{t} A_3(s)\mathrm{d}s - A_7(t)\int_{td}^{T} A_4(s)ds$$
$$-A_8(t)\Bigg(-\int_{td}^{T} A_6(s)\mathrm{d}s + \int_{td}^{t} A_5(s)\mathrm{d}s\Bigg)\Bigg)\mathrm{d}t\Bigg)$$
$$-h\lambda\Bigg(\int_{td}^{T}\Bigg(\theta(t)A_8(t)\int_t^T A_5(t)\mathrm{d}s - A_7(t)\int_t^T A_4(s)\mathrm{d}s\Bigg)dt\Bigg)$$
$$+\lambda(MwIi + c)\Bigg(\int_{td}^{T} A_4(s)\mathrm{d}s + \int_0^{td} A_3(s)ds\Bigg) - 1\Bigg) = 0 \tag{29}$$

The only task to do at this stage is to evaluate the boundary solution c1. In the third case, the optimal retailer value is obtained by comparing the profits of the feasible solutions. Finally, the maximum total profit of the supply chain in the centralized model is equal to:

$$TPrs_2 = \mathrm{Max}\big(TPrs_{21}, TPrs_{22}, TPrs_{23}\big) \tag{30}$$

4.3 Coordination (Model 3)

The manufacturer's profit increases, while the retailer loses with the change of the decision structure from decentralized to centralized. This makes the retailer reluctant to make decisions consistent with the manufacturer. Since the total profit of the supply chain in the centralized model is more than that in the decentralized one, channel coordination can be established throughout the supply chain by sharing the increased profit among members. If a discount is offered on the wholesale price, the manufacturer encourages the retailer to cooperate for channel coordination. The retailer and the manufacturer are willing to coordinate if the minimum profit received in the centralized model equals the profit obtained in the decentralized model. In other words, the following relationships should be established for the

Table 8 The sensitivity analysis concerning *Ic*, *Ie* and *Ii*

Model	Decentralized			Centralized						
Parameters	*TPs*	*TPr*	*TPrs*	*M*	*T*	*td*	*δ*	*TPs*	*TPr*	*TPrs*
$Ic = 0.1$	425.93	949.89	1375.82	0.00	6.97	2.38	40.37	495.10	921.53	1416.63
0.15	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
0.2	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
$Ie = 0.01$	426.06	887.02	1313.08	0.00	6.94	2.42	41.99	492.43	915.37	1407.79
0.06	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
0.12	424.76	1041.18	1465.94	12.00	6.58	2.85	56.31	505.78	1006.20	1511.97
$Ii = 0.12$	446.23	949.59	1395.82	12.00	6.85	2.77	54.98	531.74	904.50	1436.23
0.15	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
0.17	416.44	951.32	1367.76	0.00	6.94	2.42	41.99	492.43	915.37	1407.79

Table 9 Sensitivity analysis according to the other parameters

Model	Decentralized			Centralized						
Parameters	*TPs*	*TPr*	*TPrs*	*M*	*T*	*td*	δ	*TPs*	*TPr*	*TPrs*
$h = 0.05$	427.64	955.57	1383.21	9.61	6.91	2.74	54.13	506.27	914.13	1420.40
0.1	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
0.2	424.15	944.27	1368.42	9.75	6.86	2.80	55.47	504.31	902.39	1406.70
$C = 0.5$	444.30	949.92	1394.22	9.41	6.86	2.65	51.19	531.32	903.40	1434.71
1	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
2	389.20	949.83	1339.03	10.10	6.94	3.00	60.48	456.06	915.63	1371.69
$Ar = 3500$	419.94	1026.44	1446.38	9.82	6.49	2.78	52.74	506.87	981.45	1488.32
4000	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
4500	428.21	879.35	1307.56	9.56	7.26	2.76	56.68	502.72	840.11	1342.83
$As = 750$	433.25	949.87	1383.12	9.69	6.85	2.7685	54.62	512.79	908.03	1420.82
800	425.93	949.89	1375.82	9.68	6.88	2.7677	54.82	505.28	908.26	1413.54
850	418.62	949.90	1368.52	9.67	6.92	2.7670	55.02	497.84	908.46	1406.29
$g = 12$	343.71	1061.21	1404.92	12.00	6.85	2.80	55.60	399.84	1031.84	1431.68
15	425.93	949.89	1375.82	9.68	6.88	2.77	54.82	505.28	908.26	1413.54
17	480.60	877.89	1358.49	5.86	6.91	2.61	49.94	582.36	822.88	1405.25

two levels to achieve channel coordination under a contract for a wholesale price discount:

$$\begin{aligned} TPr_2\left(M_2, td_2, T_2, \delta_2, x.w_2\right) &\geq TPr_1\left(M_1, td_1, T_1, \delta_1, w_1\right) \\ TPs_2\left(M_2, td_2, T_2, x.w_2\right) &\geq TPs_1\left(M_1, td_1, T_1, w_1\right) \end{aligned} \tag{31}$$

Parameter x is a coefficient belonging to the range [0 1]. It is multiplied by wholesale price w, resulting in a discounted wholesale price ($x.w$). As the two inequalities above are solved based on x, the minimum and maximum amounts of the discount in the wholesale price can be obtained. The lower the x, the lower the discount, which is better for the manufacturer, but the retailer is reluctant to do it, and vice versa.

5 Numerical results

The proposed framework can be used in the supply chains of foods and farming products. For instance, through its agent in Iran, Nestlé Company (one of the greatest food producers in the world) grants trade credit to local retailers such as supermarkets to cover their money shortage. Supermarkets may use food preservation tools to keep products fresh, reduce prices to manage their inventories better and enhance sales. In this section, the proposed models are examined through three hypothetical numerical examples, and the optimal values of the sales cycle length, discount time, preservation cost, and trade credit are obtained in each case separately.

Example 1 The parameter values are presented in Table 3, and the optimal solutions for models 1, 2, and 3 are given in Table 4.

In the decentralized model, for each value of M declared by the manufacturer, the retailer obtains the optimal values (T, td, δ) and announces them to the manufacturer. The manufacturer also calculates his profit according to (M, T, td, δ), and the optimal M is the one that has the highest profit for him. Figure 1 displays the manufacturer's profit under different values of M in the decentralized model. As can be seen, the manufacturer's maximum profit is 425.93 at $M = 10.92$. If the manufacturer announces $M = 10.92$ to the retailer, comparing three profit cases, the retailer selects case 3 with the values $td = 6.00$ and $T = 6.84$ as well as profit 949.89. The concavity of the retailer objective function at the optimal point is shown in Fig. 2.

In the centralized model, the third case has the maximum profit of 1413.54 at $M = 9.68, T = 6.88, \mathrm{td} = 2.77, \mathrm{and} \delta = 54.82$; the profit of the manufacturer, the retailer, and total supply chain is 505.28, 908.26 and 1413.54, respectively. The concavity of the supply chain function at the optimal solution is depicted in Fig. 3. A comparison of the profits of the individuals and the whole supply chain in the centralized and decentralized models indicates that the profits of the supply chain and the manufacturer have increased in the centralized model. At the same time, the retailer faces a reduced profit. The manufacturer offers a wholesale price discount contract to encourage the retailer to decide in coordination with the manufacturer. If the coefficient x is in the range of [0.88 0.94], the two levels reach the channel coordination. With $x_0 = 0.9$, the profit of the manufacturer, the retailer, and the total supply chain in the coordination model is 438.91, 983.76, and 1422.67, respectively.

Example 2 Table 5 indicates the optimal solutions using the same parameter values as in Example 1, except for Ic = 0.04.

Example 3 Table 6 shows the optimal solutions using the same parameter values as in Example 1, except for $Ic = 0.04$ and $Ie = 0.1$.

5.1 Sensitivity analysis

The data in example 1 are used to analyze the effects of the parameter changes on the supply chain members' decisions and profits. By comparing the profits of models 1 and 2, the supply chain members make decisions based on the centralized model. Therefore, the only values presented here are the optimal values of the variables under the centralized model. This sub-section discusses the results obtained under the centralized model. Table 7 presents the results of the sensitivity analysis of the parameters affecting the demand (a, β, k, p_1, p_2).

Based on Table 7, the following results can be beneficial for managers to make better decisions on selling their products:

- The changes of td, M, and δ, except for T, are consistent with the changes of the initial demand a. A larger initial market size means more potential demand and selling. Therefore, the price reduction to boost the demand rate happens with a delay, and, as a result, discount time (td) increases. As the initial demand enlarges, sales grow, and the profits of the retailer, the manufacturer, and the total supply chain increase. Hence, it is advisable to follow commercial strategies to make a high initial demand.
- An increase of p_1 or p_2 makes td decrease. This is because d_1 and d_2 decline in response to an increase in p_1 and p_2. The decline in d_1 and d_2 results in a reduction in sales. To partially compensate for the negative impact of the price increase on demand, the price should be reduced earlier to level up the demand rate and the selling volume. So, markdown time (td) decreases and moves closer to the beginning of the sales cycle. The effects of β and k on td are the same as those of p_1 and p_2 on td. With an increase in one of the parameters related to the demand, it is wise to discount the price sooner to cope with any demand reduction.
- A longer interval is considered for a better analysis of the effect of price changes on the markdown time and the number of orders and sales. According to Figs. 4 and 5, when all the products are sold at a discounted price ($p_1 = p_2 = 48$), the markdown time and the cycle length are the same ($T = td = 7.04$). As a result, the quantity of the products sold before the markdown ($Q_1 = 317.8$) and after it ($Q_2 = 0$) reaches its maximum and minimum values. When p_1 starts to increase and $p_2 (= 48)$ is fixed, td decreases. When p_1 takes a value more than or equal to 75, td is zero. It means that the amount of the sale at the price of p_1 reaches zero, and all the products are sold at the price of $p_2 (= 48)$. In other words, when $p_1 \geq 75$, the quantities of the products sold before and after markdown are not sensitive to p_1 increase. They are fixed and equal to 329.2, 0, and 317.8, respectively.
- If p_2 increases (i.e., if it moves closer to p_1) and $p_1 (= 60)$ is fixed, d_2 declines. This negatively affects the total sales volume. Since d_2 is still higher than d_1, selling products with a demand rate of d_2 for a longer time can recoup the decrease of d_2. Therefore, td moves closer to the beginning of the sale cycle. Based on Fig. 5, when $p_2 < 60$, Q_2 increases but Q_1 decreases. At $p_2 = 60$, Q_1 reaches zero, and Q_2 takes its maximum value. When $p_2 > 60$, Q_2 decreases. Figures 6 and 7 show the effects of the changes of p_1 and p_2 on M, T, and all the member profits.
- In response to the increase of β or k, the values of M and δ increase. Although an increase in β has a direct impact on the sale cycle, k has an opposite effect on T. As observed in Table 6, an increase in β or k is not in the interest of any member of the supply chain.

Table 8 indicates the effects of the change in Ic, Ie and Ii on the variables and the profits of the supply chain members.

As shown in Table 8, the changes of td, M, and δ are positively correlated with the changes of Ic and Ie, but the changes in T are the opposite of those in Ic and

Ie. When *Ic* low, case 1 is the optimal case for the whole supply chain. Since an increase in *Ic* means an increase in the paid interest, the optimal case changes from case 1 to case 3 to reduce the paid interest. When $Ic \geq 0.15$, more increase in *Ic* does not affect the variables and the members' profit; this is because the paid interest in case 3 is zero.

Considering that an increase in *Ie* results in more earned interest, the optimal case changes from 1 to 3. As it is found, with an increase in *Ie*, the profits of all the members and, ultimately, the profit of the supply chain are improved. Setting the sales cycle length shorter than the trade credit period, when the earned or paid interest is high, is advisable.

In addition, a rise in *Ii* increases *T* but decreases *M*, *td*, and δ. An increase in *Ii* causes more investment opportunity costs for the manufacturer. So, *M* reduces to be closer to zero, and the optimal case changes from 3 to 1. As *Ii* increases, the retailer's profit increases too, but the profits of the manufacturer and the supply chain decrease. When $Ii \geq 0.17$, the amount of the trade credit is zero ($M = 0$), the manufacturer is reluctant to grant a trade credit to the retailer. So, the retailer has to pay the product purchase costs in cash. Furthermore, all the variables and profits are insensitive to the changes of *Ii*. With an increase in the investment opportunity cost, paying simultaneously as receiving the products is advisable.

Table 9 shows the effects of the other parameter changes on the behavior of the supply chain members.

As shown in Table 9, the changes of *td* and *M* are in line with the changes of *h* and *c* but are inverse to those of Ar, As, and g. T picks larger values as Ar, As, c, and g increase or h decreases. However, the changes of δ are in line with those of h, c, Ar, As, and As, except for the parameter g. Except for the parameters c and As, any increase of h, Ar, and g has a negative effect on the retailer's profit. The parameters h, c, Ar, and As have opposite impacts on the manufacturer's profit, but the parameter g has a direct impact on it. Finally, all the parameters h, c, Ar, As, and g have negative effects on the supply chain profit. Based on Tables 7, 8, and 9, the profits of the supply chain and the manufacturer in model 2 are more than those in model 1. It is wise to use an appropriate contract, as for wholesale price discounts, to make the decisions of the supply chain members consistent and direct them to the interest of all the members.

6 Discussion

This section discusses the results of the numerical examples and the sensitivity analyses. In example 1, the results show that the extreme points are optimal solutions, and case 3 is optimal for decentralized and centralized models. According to case 3, the markdown time and the sale cycle length should be shorter than the trade credit period.

Example 2 uses the same parameters as in example 1, except that the rate of the paid interest changes from 0.15 to 0.04. The optimal case of the centralized model shifts from 3 to 1, but that of the decentralized model is unchanged. For both models, the extreme point is the optimal solution. The optimal solution of model 1 denotes $M > 0$. This means the manufacturer provides trade credit for the retailer in a decentralized structure. Following case 1, the retailer should set the markdown time and the sales cycle shorter than the trade credit.

In the optimal solution of the centralized model, $M = 0$ denotes that the trade credit is not offered by the manufacturer, so the retailer has to pay the purchase cost when he receives the products. Example 3 is based on example 2, except that the rate of the earned interest changes from 0.01 to 0.04. In this example, cases 2 and 3 are optimal for the decentralized and centralized models, respectively. The extreme points are optimal for both models.

In all three examples, the profits of the supply chains and manufacturers under the centralized model are better than those in the decentralized model. Still, the retailer's profit in the decentralized model is more than that in the centralized model. Then, the manufacturer proposes an appropriate contract to persuade the retailer to make decisions according to a coordination model. This model improves the retailer's profit by compensating his losses by proposing a wholesale price discount by the manufacturer. This discount can change the retailer's decisions. In general, when all the supply chain members accept to make their decisions according to the coordinated decision structure, the members' profit is enhanced, compared to the decentralized decision structure [39].

Based on the numerical results and the sensitivity analyses, the following key managerial insights can help supply chain managers make proper decisions:

- The proposed framework applies to many fields like agriculture and food and chemical industries.
- This study noticeably demonstrates that a combination of trade credit and markdown policies positively impact supply chain profits. Thus, supply chain managers should provide a permissible delay period as a payment mechanism [53, 67] and consider the markdown policy as a proportional marketing strategy [47].
- Markdown pricing should be done appropriately during the sale cycle to guarantee a maximum profit [73]. The presented model helps managers know what time they should discount the price.
- The main purpose of adopting a markdown policy is to persuade new customers to purchase and raise the market demand. Thus, managers are advised to discount the initial price with a delay when the initial market size is high.
- A suggestion is made for the managers to offer a discount on the price sooner when either the initial price or markdown price increases.
- Perishability is an unavoidable factor in deteriorating inventory models. Due to deterioration, products lose their quality and consumption value. Thus, the retailer should consider the deterioration process and try to control it. It is also suggested that the retailer invest in preservation strategies to prevent the deterioration rate [33, 52, 56]. The numerical results prove that investing in preservation technologies can increase the total profit of supply chains.
- As the results of the model implementation show, a coordinated decision structure generates more profits for the whole supply chain than a decentral-

ized decision structure [39]. Therefore, the managers are advised to align and coordinate the decisions of all the members through an appropriate contract [64].

7 Conclusion

This study has proposed a two-level coordinated supply chain for perishable products. It includes a manufacturer and a retailer and operates under trade credit, markdown, and preservation technology investment policies. Although the trade credit removes the retailer's financial constraints, it costs the manufacturer the opportunity to invest. The retailer invests in preservation technologies and adopts markdown policies to reduce deterioration and uplift the demand rate. The proposed supply chain has been modeled in decentralized, centralized, and coordinated decision-making formats. As the leader, the manufacturer first announces the trade credit period. Then, the retailer, which is faced with a price-sensitive and time-varying demand, decides on the length of the sales cycle and the appropriate time to reduce the price and the preservation cost. Three possible decision-making cases have been examined in decentralized and centralized models, and certain solution procedures have been developed to obtain optimal two-level decisions. Then, the proposed models are solved with different numerical examples. The sensitivity analysis on each parameter provides managerial insights. The numerical results indicate that implementing markdown and trade credit policies is more profitable for the two members. During the model implementation in this study, the retailer faced profit reduction, while the supply chain and manufacturer profits increased under centralized decision-making. A wholesale price discount was also proposed to coordinate the decisions at both levels of the supply chain.

Although this study focuses on a coordinated two-level supply chain, it has a few limitations. First of all, price is considered as a parameter. Secondly, the models deal with only a one-level trade credit; the manufacturer gives credit to the retailer, and customers should pay for the product as soon as they receive it. Thirdly, the manufacturer provides the retailer full credit, while the manufacturer may receive a portion of the cost of purchasing the products at delivery. Concerning these limitations, this study may be expanded in several ways. First, assuming that the retailer offers trade credits to customers, it is possible to study the optimal behavior of the supply chain members under two-level trade credits. Also, considering price as a variable, the optimal price should be obtained before and after a product discount. Furthermore, evaluating markdown policies under different payment strategies, such as advance payment or partial credit, is beneficial.

Appendix A

Proof of (1–1)

We define $TPr_{11} = \frac{f_1(T)}{f_2(T)}$:

$f_1(T) = p_1Q_1 + p_2Q_2 - wQ_0 - Ar - HC + IE_1 - IP_1 - \delta$ and $f_2(T) = T \geq 0$.

Taking the first-order and second-order partial derivatives of f_1 *(T) w.r.t T*, we have

$$f_1{}'(T) = -d_2(p,T)\left(h\int_0^T \frac{A_1(T)}{A_1(t)}dt + Icw\int_M^T \frac{A_1(T)}{A_1(t)}dt + A_1(T)w - p_2\right)$$

$$\begin{aligned} f_1{}''(T) = \Bigg[&-Icw\int_M^T \left(d_2{}'(T)\frac{A_7(t)}{A_7(T)} + d_2(T)\theta'(T)A_9(T,t)\right)dt \\ &-h\int_0^T \left(d_2{}'(T)\frac{A_7(t)}{A_7(T)} + d_2(T)\theta'(T)A_9(T,t)\right)dt \\ &-d_2(T)\theta'(T)(w)A_2(T) - d_2{}'(T)\left((w)A_7{}^{-1}(T) - p_2\right) - d_2(T)(Icw+h)\Bigg] \end{aligned}$$

$TPr_{11} = \frac{f_1(T)}{f_2(T)}$ is a strictly pseudo-concave function in T when $f_1{}''(T) < 0$ [77].

For $f_1{}''(T) < 0$, the $wA_7{}^{-1}(T) - p_2 \leq 0$ should be true.

Proof of (1–2)

Taking the second-order partial derivatives of TPr_{11} *w.r.t td*,

$$\frac{\partial^2 TPr_{11}}{\partial td^2} = \frac{\left(p_1 - p_2\right)(\theta'(td)\beta(wIc\int_M^{td} A_9(td,s)ds + h\int_0^{td} A_9(td,s)ds + A_2(td)w) + (Icw+h)\beta - k)}{T}$$

When $\theta'(td)\beta(wIc\int_M^{td} A_9(td,s)ds + h\int_0^{td} A_9(td,s)ds + A_2(td)w) + (Icw+h)\beta \leq k$ is satisfied, we have $\frac{\partial TPr_{11}}{\partial td^2} \leq 0$.

Proof of (1–3)

Taking the second-order partial derivatives of TPr_{11} *w.r.t* δ, we have

$$\frac{\partial^2 TPr_{11}}{\partial \delta^2} = \frac{\lambda}{T}(B)$$

$$B = \begin{array}{c} Icw\left(\int_M^{td}\left(\begin{array}{c} -A_7(t)\int_{td}^T A_{12}(s)ds + A_7(t)\int_{td}^t A_{11}(s)ds \\ -2A_8(t)\int_{td}^t A_3(s)ds + 2A_8(t)\int_{td}^T A_4(s)ds + \theta(t)\left(\begin{array}{c} -\int_{td}^T A_6(s)ds + \\ \int_{td}^t A_5(s)ds \end{array}\right)A_{12}(t) \end{array}\right)dt\right. \\ +h\int_0^{td}\left(\begin{array}{c} -A_7(t)\int_{td}^T A_{12}(t)ds + A_7(t)\int_{td}^t A_{11}(t)ds - 2A_8(t)\int_{td}^t A_3(s)ds + \\ 2A_8(t)\int_{td}^T A_4(s)ds\left(-\int_{td}^T A_6(s)\mathrm{d}s\right) + \int_{td}^t A_5(s)\mathrm{d}s\Big)A_{13}(t)\theta(t) \end{array}\right)dt \\ +(Icw+h)(\int_{td}^T\left(-A_7(t)\int_t^T A_{12}(s)ds - \theta(t)(-2A_8(t)\left(\int_t^T A_7(t)\mathrm{d}s\right) + A_{13}(t)\int_t^T A_6(s)ds\right) - \\ w\left(\int_0^{td} A_{11}(t)dt + \int_{td}^T A_{12}(t)dt\right) \end{array}$$

If $B < 0$, then TPr_{11} is a concave function in δ.

Appendix B

Proof of (4–1)

We define $TPrs_{21} = \frac{V_1(T)}{V_2(T)}$

$V_1(T) = T.\left(TPs + TPr_{21}\right), V_2(T) = T.$

Taking the first-order and second-order partial derivatives of $V_1(T)$ *w.r.t* T, we have.

$$V_1{}'(T) = -d_2(p,T)\left(Icw\left(\int_M^T \frac{A_7(t)}{A_7(T)}dt\right) + h\left(\int_0^T \frac{A_7(t)}{A_7(T)}dt\right) + (IiMw+c)A_7{}^{-1}(T) - p_2\right)$$

$$\begin{aligned} V_1{}''(T) = &-Icw\left(\int_M^T\left(d_2{}'(T)\frac{A_7(t)}{A_7(T)} + d_2(T)\theta'(T)A_9(T,t)\right)dt\right) \\ &-h\left(\int_0^T\left(d_2{}'(T)\frac{A_7(t)}{A_7(T)} + d_2(T)\theta'(T)A_9(T,t)\right)dt\right) \\ &-d_2(T)\theta'(T)(IiMw+c)A_2(T) - d_2{}'(T)\left((IiMw+c)A_7{}^{-1}(T) - p_2\right) \\ &-d_2(T)(Icw+h) \end{aligned}$$

$TPrs_{21} = \frac{V_1(T)}{V_2(T)}$ Is a strictly pseudo-concave function in T, if $V_1{}''(T) < 0$. So $(IiMw+c)A_7{}^{-1}(T) - p_2 \geq 0$ should be satisfied.

Proof of (4–2)

$$\frac{\partial^2 TPrs_{21}}{\partial td^2} = \frac{\partial^2 TPr_{11}}{\partial td^2} - \frac{\theta'(td)\beta\left(p_1 - p_2\right)(-w - IiMw - c)A_2(t)\theta^{-1}(t)}{T}$$

When $\frac{\partial^2 TPrs_{21}}{\partial td^2} \leq \frac{\theta'(td)\beta\left(p_1 - p_2\right)(-w - IiMw - c)A_2(t)\theta^{-1}(t)}{T}$ is true; we have $\frac{\partial TPrs_{21}}{\partial td^2} \leq 0$.

Proof of (4–3)

$$\frac{\partial^2 TPrs_{21}}{\partial M^2} = \frac{1}{T}\left[-Icw\theta\prime(M)\left(\int_{td}^{T} A_6(s)ds - \int_{td}^{M} A_5(s)ds\right)A_8(M) + d_1(M)(Iep_1 - Icw)\right]$$

For $\frac{\partial^2 TPrs_{21}}{\partial M^2} \leq 0$, the following inequality should be satisfied $-Icw\theta\prime(M)\left(\int_{td}^{T} A_6(s)ds - \int_{td}^{M} A_5(s)ds\right)A_8(M) + d_1(M)\left(Iep_1 - Icw\right) \leq 0$.

Proof of (4–4)

$$\frac{\partial^2 TPrs_{21}}{\partial \delta^2} = \frac{\partial^2 TPr_{11}}{\partial \delta^2} + -\frac{\lambda^2}{T}\left(\int_{0}^{td} A_{11}(t)dt + \int_{td}^{T} A_{12}(t)dt\right)(IiMw + c - w)$$

If $\frac{\partial^2 TPrs_{21}}{\partial \delta^2} = -\frac{\lambda^2}{T}\left(\int_0^{td} A_{11}(t)dt + \int_{td}^{T} A_{12}(t)dt\right)(IiMw + c - w) \leq 0$ is satisfied; we have $\frac{\partial^2 TPrs_{21}}{\partial \delta^2} \leq 0$.

Appendix C

$$A_1(t) = e^{-\theta(t)(-1+z(\delta))},\ \ A_2(t) = \theta(t)e^{-\lambda\delta+\theta(t)e^{-\lambda\delta}}$$

$$d_1\left(p_1, t\right) = d_1(t),\ \ A_3(t) = A_2(t)d_1(t),\ \ A_4(t) = A_2(t)d_2(t)$$

$$d_2\left(p_2, t\right) = d_2(t), A_5(t) = d_1(s)e^{\theta(s)e^{-\lambda\delta}}, A_6(t) = d_2(s)e^{\theta(s)e^{-\lambda\delta}}$$

$$A_7(t) = e^{-\theta(t)e^{-\lambda\delta}},\ \ A_8(t) = e^{-\lambda\delta-\theta(t)e^{-\lambda\delta}}, A_9(t, s) = e^{(\theta(t)-\theta(s))e^{-\lambda\delta}-\lambda\delta}$$

$$A_{11}(t) = d_1(p, s)\theta(t)\left(e^{-2\lambda\delta+\theta(t)\mathrm{e}^{-\lambda\delta}}\theta(t) + e^{-\lambda\delta+\theta(t)\mathrm{e}^{-\lambda\delta}}\right)$$

$$A_{12}(t) = d_2(p, s)\theta(t)\left(e^{-2\lambda\delta+\theta(t)\mathrm{e}^{-\lambda\delta}}\theta(t) + e^{-\lambda\delta+\theta(t)\mathrm{e}^{-\lambda\delta}}\right)$$

$$A_{13}(t) = \mathrm{e}^{-2\lambda\delta-\theta(t)\mathrm{e}^{-\lambda\delta}}\theta(t) - \mathrm{e}^{-\lambda\delta-\theta(t)\mathrm{e}^{-\lambda\delta}}$$

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Declarations

Conflict of interest The authors declare that they have no competing interests.

References

1. Abdul-Jalbar, B., Colebrook, M., Dorta-Guerra, R., Gutierrez, J.M.: Centralized and decentralized inventory policies for a single-vendor two-buyer system with permissible delay in payments. Comput. Oper. Res. **74**, 187–195 (2016)
2. Avittathur, B., Biswas, I.: A note on limited clearance sale inventory model. Int. J. Prod. Econ. **193**, 647–653 (2017)
3. Azadi, Z., Eksioglu, S.D., Eksioglu, B., Palak, G.: Stochastic optimization models for joint pricing and inventory replenishment of perishable products. Comput. Ind. Eng. **127**, 625–642 (2019)
4. Bi, G., Wang, P., Wang, D., Yin, Y.: Optimal credit period and ordering policy with credit-dependent demand under two-level trade credit. Int. J. Prod. Econ. **242**, 108311 (2021)
5. Chen, K., Zha, Y., Alwan, L.C., Zhang, L.: Dynamic pricing in the presence of reference price effect and consumer strategic behaviour. Int. J. Prod. Res. **58**, 546–561 (2020)
6. Chen, L.-H., Kang, F.-S.: Integrated inventory models considering permissible delay in payment and variant pricing strategy. Appl. Math. Modell **34**, 36–46 (2010)
7. Chen, L.-H., Kang, F.-S.: Integrated inventory models considering the two-level trade credit policy and a price-negotiation scheme. Eur. J. Oper. Res. **205**, 47–58 (2010)
8. Chen, S.-C., Teng, J.-T.: Inventory and credit decisions for time-varying deteriorating items with up-stream and down-stream trade credit financing by discounted cash flow analysis. Eur. J. Oper. Res. **243**, 566–575 (2015)
9. Chen, S.-P., Chen, C.-Y.: Dynamic markdown decisions based on a quality loss function in on-site direct-sale supply chains for perishable food. J. Oper. Res. Soc. **72**, 822–836 (2021)
10. Chen, X.: A model of trade credit in a capital-constrained distribution channel. Int. J. Prod. Econ. **159**, 347–357 (2015)
11. Chen, X., Wu, S., Wang, X., Li, D.: Optimal pricing strategy for the perishable food supply chain. Int. J. Prod. Res. **57**, 2755–2768 (2019)
12. Chew, E.P., Lee, C., Liu, R., Hong, K.-S., Zhang, A.: Optimal dynamic pricing and ordering decisions for perishable products. Int. J. Prod. Econ. **157**, 39–48 (2014)
13. Chintapalli, P.: Simultaneous pricing and inventory management of deteriorating perishable products. Ann. Oper. Res. **229**, 287–301 (2015)
14. Chung, W., Talluri, S., Narasimhan, R.: Price markdown scheme in a multi-echelon supply chain in a high-tech industry. Eur. J. Oper. Res. **215**, 581–589 (2011)
15. Chung, W., Talluri, S., Narasimhan, R.: Optimal pricing and inventory strategies with multiple price markdowns over time. Eur. J. Oper. Res. **243**, 130–141 (2015)
16. Dai, Z., Wang, Y.: A production and inventory model for deteriorating items with two-level partial trade credit and stochastic demand in a supply chain. Kybernetes **52**(11), 4846–4875 (2022)
17. Das, B.C., Das, B., Mondal, S.K.: Integrated supply chain model for a deteriorating item with procurement cost dependent credit period. Comput. Ind. Eng. **64**, 788–796 (2013)
18. Das, S., Choudhury, M., Mahata, G.C.: Optimal inventory policies for time varying deteriorating items with dynamic demand under upstream and downstream trade credit by discounted cash-flow analysis. Opsearch **61**(1), 1–32 (2023)

19. Dey, B.K., Datta, A., Sarkar, B.: Effectiveness of carbon policies and multi-period delay-in-payments in a global supply chain under remanufacturing consideration. J. Cleaner Prod. **402**, 136539 (2023)
20. Dong, J., Wu, D.D.: Two-period pricing and quick response with strategic customers. Int. J. Prod. Econ. **215**, 165–173 (2019)
21. Duary, A., Das, S., Arif, M.G., Abualnaja, K.M., Khan, M.A.-A., Zakarya, M., Shaikh, A.A.: Advance and delay in payments with the price-discount inventory model for deteriorating items under capacity constraint and partially backlogged shortages. Alex. Eng. J. **61**, 1735–1745 (2022)
22. Esmaeili, M., Nasrabadi, M.: An inventory model for single-vendor multi-retailer supply chain under inflationary conditions and trade credit. J. Ind. Prod. Eng. **38**, 75–88 (2021)
23. Farshbaf-Geranmayeh, A., Zaccour, G.: Pricing and advertising in a supply chain in the presence of strategic consumers. Omega **101**, 102239 (2021)
24. Gautam, P., Maheshwari, S., Kausar, A., Jaggi, C.K.: Sustainable retail model with preservation technology investment to moderate deterioration with environmental deliberations. J. Cleaner Prod. **390**, 136128 (2023)
25. Giri, B., Sharma, S.: An integrated inventory model for a deteriorating item with allowable shortages and credit linked wholesale price. Optim. Lett. **9**, 1149–1175 (2015)
26. Giri, B., Sharma, S.: Optimal ordering policy for an inventory system with linearly increasing demand and allowable shortages under two levels trade credit financing. Oper. Res. **16**, 25–50 (2016)
27. Giri, B., Sharma, S.: Optimising an integrated production–inventory system under cash discount and retailer partial trade credit policy. Int. J. Syst. Sci. Oper. **6**, 99–118 (2019)
28. Goyal, S.K.: Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. **36**(4), 335–338 (1985)
29. Habib, A.M.: Does the efficiency of working capital management and environmental, social, and governance performance affect a firm's value? Evidence from the United States, Financial Markets. Institutions and Risks (FMIR) **6**, 18–25 (2022)
30. Habib, A.M.: Do business strategies and environmental, social, and governance (ESG) performance mitigate the likelihood of financial distress? A multiple mediation model. Heliyon **9**, e17847 (2023)
31. Habib, A.M.: Does real earnings management affect a firm's environmental, social, and governance (ESG), financial performance, and total value? A moderated mediation analysis, Environment. Dev. Sustain. (2023). https://doi.org/10.1007/s10668-023-03809-6
32. Handa, N., Singh, S., Punetha, N.: Impact of carbon emission in two-echelon supply chain inventory decision with controllable deterioration and two-level trade-credit period. Opsearch **59**, 1555–1586 (2022)
33. Hatibaruah, A., Saha, S.: An inventory model for two-parameter Weibull distributed ameliorating and deteriorating items with stock and advertisement frequency dependent demand under trade credit and preservation technology. Opsearch **60**, 1–52 (2023)
34. Heydari, J.: Supply chain coordination using time-based temporary price discounts. Comput. Ind. Eng. **75**, 96–101 (2014)
35. Heydari, J., Rastegar, M., Glock, C.H.: A two-level delay in payments contract for supply chain coordination: the case of credit-dependent demand. Int. J. Prod. Econ. **191**, 26–36 (2017)
36. Jadidi, O., Taghipour, S., Zolfaghari, S.: A two-price policy for a newsvendor product supply chain with time and price sensitive demand. Eur. J. Oper. Res. **253**, 132–143 (2016)
37. Jaggi, C.K., Yadavalli, V.S., Verma, M., Sharma, A.: An EOQ model with allowable shortage under trade credit in different scenario. Appl. Math. Comput. **252**, 541–551 (2015)
38. Jani, M.Y., Betheja, M.R., Bhadoriya, A., Chaudhari, U., Abbas, M., Alqahtani, M.S.: Optimal pricing policies with an allowable discount for perishable items under time-dependent sales price and trade credit. Mathematics **10**, 1948 (2022)
39. Johari, M., Hosseini-Motlagh, S.-M., Nematollahi, M., Goh, M., Ignatius, J.: Bi-level credit period coordination for periodic review inventory system with price-credit dependent demand under time value of money. Transp. Res. E: Logist. Transp. **114**, 270–291 (2018)
40. Kaushik, J.: An inventory model with permissible delay in payment and different interest rate charges. Decision Analytics Journal **6**, 100180 (2023)
41. Kaya, O., Polat, A.L.: Coordinated pricing and inventory decisions for perishable products. OR Spectr. **39**, 589–606 (2017)

42. Khan, M.A.-A., Shaikh, A.A., Cárdenas-Barrón, L.E.: An inventory model under linked-to-order hybrid partial advance payment, partial credit policy, all-units discount and partial backlogging with capacity constraint. Omega **103**, 102418 (2021)
43. Khouja, M., Liu, X.: A price adjustment policy for maximizing revenue and countering strategic consumer behavior. Int. J. Prod. Econ. **236**, 108116 (2021)
44. Kouvelis, P., Zhao, W.: Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts. Oper. Res. **60**, 566–580 (2012)
45. Kumari, M., De, P.K., Narang, P., Shah, N.H.: Integrated optimization of inventory, replenishment, and vehicle routing for a sustainable supply chain utilizing a novel hybrid algorithm with carbon emission regulation. Expert Syst. Appl. **220**, 119667 (2023)
46. Li, G., Wu, H., Xiao, S.: Financing strategies for a capital-constrained manufacturer in a dual-channel supply chain. Int. Tran. Oper. Res. **27**, 2317–2339 (2020)
47. Liang, C.-C., Lin, Y.-W.: Online promotion effects under time limitation-A study of survey and physiological signals. Decis. Support. Syst. **170**, 113963 (2023)
48. Lin, F., Jia, T., Wu, F., Yang, Z.: Impacts of two-stage deterioration on an integrated inventory model under trade credit and variable capacity utilization. Eur. J. Oper. Res. **272**, 219–234 (2019)
49. Mahato, C., Mahata, G.C.: Optimal inventory policies for deteriorating items with expiration date and dynamic demand under two-level trade credit. Opsearch **58**, 994–1017 (2021)
50. Mahato, C., Mahata, G.C.: Optimal ordering policy under order-size dependent trade credit and complete backlogging derived algebraically. Opsearch **60**, 420–444 (2023)
51. Mahato, C., Mahata, G.C.: Sustainable partial backordering inventory model under linked-to-order credit policy and all-units discount with capacity constraint and carbon emissions. Flex. Serv. Manuf. J. **35**, 896–944 (2023)
52. Mahato, C., Mahato, F., Mahata, G.C.: Bi-level trade credit policy under pricing and preservation technology in inventory models for non-instantaneous deteriorating items under carbon tax policy. Sādhanā **48**, 103 (2023)
53. Mahato, F., Mahato, C., Mahata, G.C.: Sustainable optimal production policies for an imperfect production system with trade credit under different carbon emission regulations. Environ. Dev. Sustain. **25**, 10073–10099 (2023)
54. Maji, A., Bhunia, A.K., Mondal, S.K.: A production-reliability-inventory model for a series-parallel system with mixed strategy considering shortage, warranty period, credit period in crisp and stochastic sense. Opsearch **59**, 862–907 (2022)
55. Majumder, P., Bera, U., Maiti, M.: An EPQ model of substitutable products under trade credit policy with stock dependent and random substitution. Opsearch **57**, 1205–1243 (2020)
56. Mashud, A.H.M., Sarkar, B.: Retailer's joint pricing model through an effective preservation strategy under a trade-credit policy. RAIRO Oper. Res **55**, 1799–1823 (2021)
57. Mashud, A.H.M., Wee, H.-M., Huang, C.-V.: Preservation technology investment, trade credit and partial backordering model for a non-instantaneous deteriorating inventory. RAIRO Oper. Res **55**, S51–S77 (2021)
58. Mashud, A.H.M., Wee, H.-M., Sarkar, B., Li, Y.-H.C.: A sustainable inventory system with the advanced payment policy and trade-credit strategy for a two-warehouse inventory system. Kybernetes **50**, 1321–1348 (2020)
59. Momena, A.F., Haque, R., Rahaman, M., Mondal, S.P.: A two-storage inventory model with trade credit policy and time-varying holding cost under quantity discounts. Logistics **7**, 77 (2023)
60. Moradi, S., Gholamian, M.R., Sepehri, A.: An inventory model for imperfect quality items considering learning effects and partial trade credit policy. Opsearch **60**, 276–325 (2023)
61. Murmu, V., Kumar, D., Jha, A.K.: Optimal joint pricing and lot sizing policies for perishables under periodic inspections, and trade credit financing schemes. Int. J. Appl. Comput. Math. **9**, 92 (2023)
62. Nagare, M., Dutta, P.: Single-period ordering and pricing policies with markdown, multivariate demand and customer price sensitivity. Comput. Ind. Eng. **125**, 451–466 (2018)
63. Pashigian, B.P.: Demand uncertainty and sales: a study of fashion and markdown pricing. Am. Econ. Rev. **78**, 936–953 (1988)
64. Phan, D.A., Vo, T.L.H., Lai, A.N.: Supply chain coordination under trade credit and retailer effort. Int. J. Prod. Res. **57**, 2642–2655 (2019)
65. Piramuthu, S., Zhou, W.: RFID and perishable inventory management with shelf-space and freshness dependent demand. Int. J. Prod. Econ. **144**, 635–640 (2013)

66. Pramanik, P., Maiti, M.K.: An inventory model for deteriorating items with inflation induced variable demand under two level partial trade credit: A hybrid ABC-GA approach. Eng. Appl. Artif. Intell. **85**, 194–207 (2019)
67. Shah, N.H., Keswani, M., Khedlekar, U.K., Prajapati, N.M.: Non-instantaneous controlled deteriorating inventory model for stock-price-advertisement dependent probabilistic demand under trade credit financing. Opsearch **61**(1), 1–39 (2023)
68. Taleizadeh, A.A., Aliabadi, L., Thaichon, P.: A sustainable inventory system with price-sensitive demand and carbon emissions under partial trade credit and partial backordering. Oper. Res. **22**(4), 1–46 (2022)
69. Taudes, A., Rudloff, C.: Integrating inventory control and a price change in the presence of reference price effects: a two-period model. Math. Methods Oper. Res. **75**, 29–65 (2012)
70. Tripathi, R., Kumar, M.: A new model for deteriorating items with inflation under permissible delay in payments. Int. J. Ind. Eng. Comput. **5**, 365–374 (2014)
71. Tripathy, M., Sharma, G., Sharma, A.K.: An EOQ inventory model for non-instantaneous deteriorating item with constant demand under progressive financial trade credit facility. Opsearch **59**, 1215–1243 (2022)
72. Tsao, Y.-C., Zhang, Q., Fang, H.-P., Lee, P.-L.: Two-tiered pricing and ordering for non-instantaneous deteriorating items under trade credit. Oper. Res. **19**, 833–852 (2019)
73. Wang, J.-T., Zhang, S., Wu, C.-H., Yu, J.-J., Chen, C.-B., Tsai, S.-B.: Time-sensitive markdown strategies for perishable products based on dynamic quality evaluation. Kybernetes **50**, 165–180 (2021)
74. Wang, X., Fan, Z.-P., Liu, Z.: Optimal markdown policy of perishable food under the consumer price fairness perception. Int. J. Prod. Res. **54**, 5811–5828 (2016)
75. Wang, X., Li, D.: A dynamic product quality evaluation based pricing model for perishable food supply chains. Omega **40**, 906–917 (2012)
76. Wu, C., Zhao, Q., Xi, M.: A retailer-supplier supply chain model with trade credit default risk in a supplier-Stackelberg game. Comput. Ind. Eng. **112**, 568–575 (2017)
77. Wu, J., Ouyang, L.-Y., Cárdenas-Barrón, L.E., Goyal, S.K.: Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. Eur. J. Oper. Res. **237**, 898–908 (2014)
78. Xu, C., Fan, T., Zheng, Q., Song, Y.: Contract selection for fresh produce suppliers cooperating with a platform under a markdown-pricing policy. Int. J. Prod. Res. **61**, 3756–3780 (2023)
79. Xue, M., Tang, W., Zhang, J.: Supply chain pricing and coordination with markdown strategy in the presence of conspicuous consumers. Int. Tran. Oper. Res. **23**, 1051–1065 (2016)
80. Yaghin, R.G., Torabi, S., Ghomi, S.F.: Integrated markdown pricing and aggregate production planning in a two echelon supply chain: A hybrid fuzzy multiple objective approach. Appl. Math. Modell **36**, 6011–6030 (2012)
81. Yan, B., Chen, X., Cai, C., Guan, S.: Supply chain coordination of fresh agricultural products based on consumer behavior. Comput. Oper. Res. **123**, 105038 (2020)
82. Yang, C.-T., Dye, C.-Y., Ding, J.-F.: Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. Comput. Ind. Eng. **87**, 356–369 (2015)
83. Yang, H.-L.: Retailer's ordering policy for demand depending on the expiration date with limited storage capacity under supplier credits linked to order quantity and discounted cash flow. Int. J. Syst. Sci. Oper. **8**, 136–153 (2021)
84. Yang, H.-L.: A note on'a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation'. Int. J. Oper. Res. **45**, 141–160 (2022)
85. Yang, H., Miao, L., Zhao, C.: The credit strategy of a green supply chain based on capital constraints. J. Cleaner Prod. **224**, 930–939 (2019)
86. Yao, M.-J., Lin, J.-Y., Lee, C., Wang, P.-H.: Optimal replenishment and inventory financing strategy in a three-echelon supply chain under the variable demand and default risk. J. Oper. Res. Soc. **72**, 2196–2210 (2021)
87. Zhang, C., You, M., Han, G.: Integrated supply chain decisions with credit-linked demand: a Stackelberg approach. Sci. Iran. **28**, 927–949 (2021)
88. Zhao, N., Wang, Q., Wu, J.: The reference effect on a retailer's selling strategy with limited initial inventory level and strategic consumer behavior. Comput. Ind. Eng. **169**, 108260 (2022)

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